Optimization, Separation, and Inverse Optimization

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What is an Inverse Problem?

What is an inverse problem?

Given a function, an inverse problem is that of determining *input* that would produce a given *output*.

- The input may be partially specified.
- We may want an answer as close as possible to a given *target*.

- This is precisely the mathematical notion of the inverse of a function.
- A *value function* is a function whose value is the optimal solution of an optimization problem defined by the given input.
- The inverse problem with respect to an optimization problem is to evaluate the inverse of a given *value function*.

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Why is Inverse Optimization Useful?

Inverse optimization is useful when we can observe the result of solving an optimization problem and we want to know what the input was.

Example: Consumer preferences

- Let's assume consumers are rational and are making decisions by solving an underlying optimization problem.
- By observing their choices, we try ascertain their utility function.

Example: Analyzing seismic waves

- We know that the path of seismic waves travels along paths that are optimal with respect to some physical model of the earth.
- By observing how these waves travel during an earthquake, we can infer things about the composition of the earth.

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Formal Setting

We consider the inverse of the value function

$$z_{IP}(d) = \min_{x \in \mathcal{S}} d^T x = \min_{x \in \text{conv}(\mathcal{S})} d^T x \tag{1}$$

for $d \in \mathbb{R}^n$, where

$$\mathcal{S} = \{x \in \mathbb{R}^n \mid Ax = b, x \geq 0\} \cap (\mathbb{Z}^r \times \mathbb{R}^{n-r})\}. \tag{2}$$

for $A \in \mathbb{R}^{m \times n}, b \in \mathbb{R}^m$. With respect to a given target $c \in \mathbb{R}^n$ and a given $x^0 \in \mathcal{S}$, the inverse problem is defined as

$$\min\{\|c - d\| \mid d^\top x^0 = z_{IP}(d)\} \tag{INV}$$

Assumption: $\mathcal{S}$ is bounded (for simplicity of presentation).

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Formulating as a Mathematical Program

- To formulate as a mathematical program, we need to represent the implicit constraints of (**??**) explicitly.
- This means describing the cone of feasible objective vectors.
- This cone can be described as

$$\mathcal{D} = \{d \in \mathbb{R}^n \mid d^T x^0 \leq d^T x \; \forall x \in \mathcal{S}\}. \tag{3}$$

- In the pure integer case, this is a finite number of inequalities and the above is thus an linear program (LP).
- Even in the mixed case, we need only the inequalities corresponding to extreme points of $\mathrm{conv}(\mathcal{S})$.
- This set of constraints is exponential in size, but we can generate them dynamically, as we will see.

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