

Plan For Today

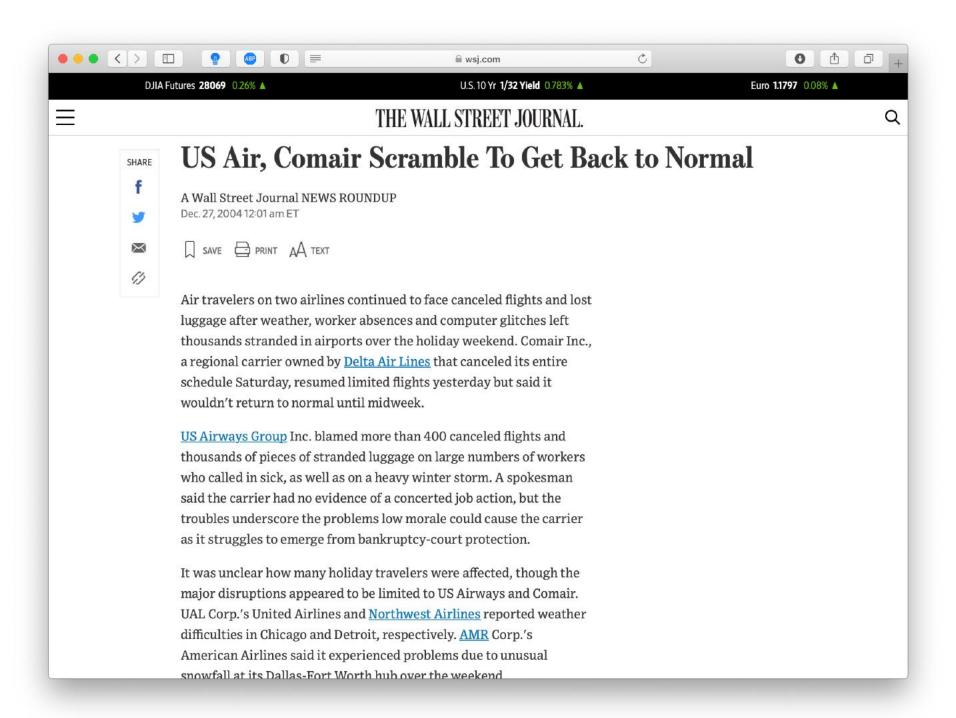
- Bits and Bytes
- Hexadecimal
- Integer Representations
- Unsigned Integers
- Signed Integers
- Overflow
- Casting and Combining Types

Disclaimer: Slides for this lecture were borrowed from

- —Nick Troccoli's Stanford CS107 class
- —Randal E. Bryant and David R. O'Hallaron's CMU 15-213 class



COMP201 Topic 1: How can a computer represent integer numbers?



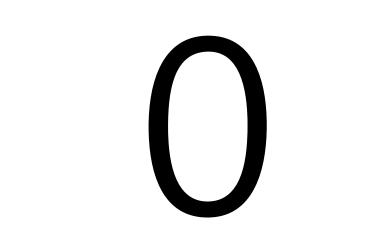
Demo: Unexpected Behavior



airline.c

Lecture Plan

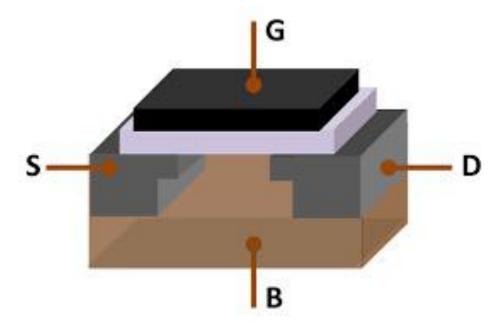
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Bits

Computers are built around the idea of two states: "on" and "off".
 Transistors represent this in hardware, and bits represent this in software!



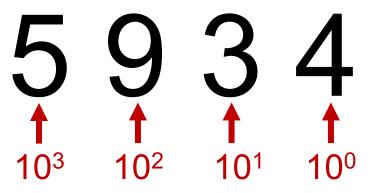
One Bit At A Time

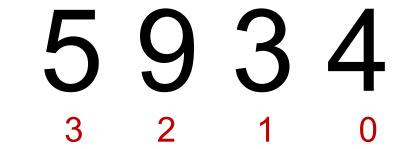
- We can combine bits, like with base-10 numbers, to represent more data. 8 bits = 1 byte.
- Computer memory is just a large array of bytes! It is *byte-addressable*; you can't address (store location of) a bit; only a byte.
- Computers still fundamentally operate on bits; we have just gotten more creative about how to represent different data as bits!
 - Images
 - Audio
 - Video
 - Text
 - And more...

5934

Digits 0-9 (0 to base-1)

$$= 5*1000 + 9*100 + 3*10 + 4*1$$





10^X:



Digits 0-1 (0 to base-1)

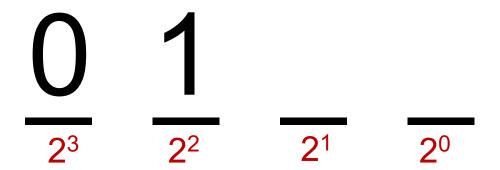




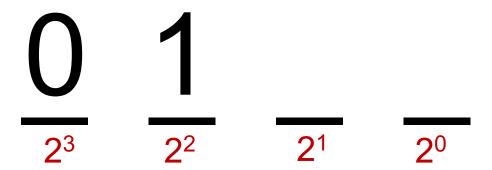
$$= 1*8 + 0*4 + 1*2 + 1*1 = 11_{10}$$

- Strategy:
 - What is the largest power of $2 \le 6$?

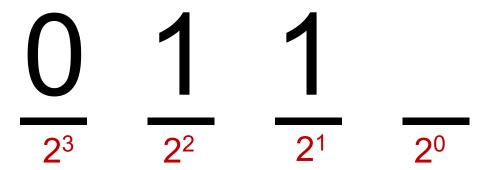
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 - What is the largest power of $2 \le 6$? $2^2=4$



- Strategy:
 - What is the largest power of $2 \le 6$? $2^2=4$
 - Now, what is the largest power of $2 \le 6 2^2$?

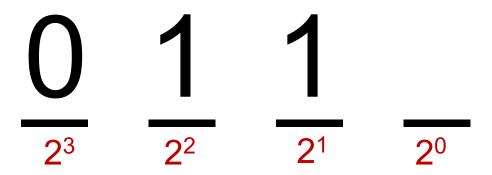


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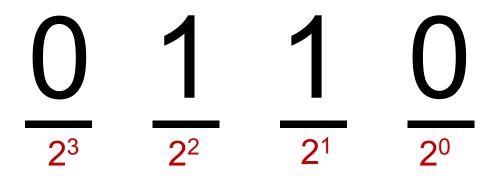


- Strategy:
 - What is the largest power of $2 \le 6$? $2^2=4$
 - Now, what is the largest power of $2 \le 6 2^2$? $2^1=2$

$$-6-2^2-2^1=0!$$



- Strategy:
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 - Now, what is the largest power of $2 \le 6 2^2$? $2^1 = 2$
 - $-6-2^2-2^1=0!$



- Strategy:
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$$-6-2^2-2^1=0!$$

Practice: Base 2 to Base 10

What is the base-2 value 1010 in base-10?

- a) 20
- b) 101
- c) 10
- d) 5
- e) Other

Practice: Base 10 to Base 2

What is the base-10 value 14 in base 2?

- a) 1111
- b) 1110
- c) 1010
- d) Other

• What is the minimum and maximum base-10 value a single byte (8 bits) can store?

• What is the minimum and maximum base-10 value a single byte (8 bits) can store? minimum = 0 maximum = ?

2^x:

What is the minimum and maximum base-10 value a single byte (8 bits)
 can store? minimum = 0 maximum = ?



2^x:

What is the minimum and maximum base-10 value a single byte (8 bits)
 can store? minimum = 0 maximum = ?

• Strategy 1: $1*2^7 + 1*2^6 + 1*2^5 + 1*2^4 + 1*2^3 + 1*2^2 + 1*2^1 + 1*2^0 = 255$

What is the minimum and maximum base-10 value a single byte (8 bits)
 can store? minimum = 0 maximum = 255

- Strategy 1: $1*2^7 + 1*2^6 + 1*2^5 + 1*2^4 + 1*2^3 + 1*2^2 + 1*2^1 + 1*2^0 = 255$
- Strategy 2: $2^8 1 = 255$

Multiplying by Base

$$1450 \times 10 = 14500$$

 $1100_2 \times 2 = 11000$

Key Idea: inserting 0 at the end multiplies by the base!

Dividing by Base

$$1450 / 10 = 145$$
 $1100_2 / 2 = 110$

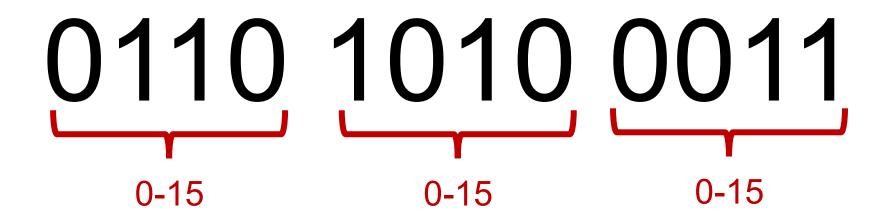
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Hexadecimal

- When working with bits, oftentimes we have large numbers with 32 or 64 bits.
- Instead, we'll represent bits in base-16 instead; this is called hexadecimal.



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- Instead, we'll represent bits in base-16 instead; this is called hexadecimal.



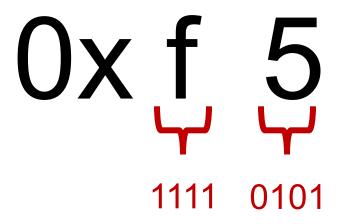
Each is a base-16 digit!

• Hexadecimal is *base-16*, so we need digits for 1-15. How do we do this?

0 1 2 3 4 5 6 7 8 9 a b c d e f

Hex digit	0	1	2	3	4	5	6	7
Decimal value	0	1	2	3	4	5	6	7
Binary value	0000	0001	0010	0011	0100	0101	0110	0111
Hex digit	8	9	Α	В	С	D	Е	F
Decimal value	8	9	10	11	12	13	14	15
Binary value	1000	1001	1010	1011	1100	1101	1110	1111

- We distinguish hexadecimal numbers by prefixing them with 0x, and binary numbers with 0b.
- E.g. **0xf5** is **0b11110101**



Practice: Hexadecimal to Binary

What is **0x173A** in binary?

Hexadecimal	1	7	3	A
Binary	0001	0111	0011	1010

Practice: Hexadecimal to Binary

What is **0b1111001010** in hexadecimal? (*Hint: start from the right*)

Binary	11	1100	1010
Hexadecimal	3	C	A

Question Break!

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Number Representations

- Unsigned Integers: positive and 0 integers. (e.g. 0, 1, 2, ... 99999...
- Signed Integers: negative, positive and 0 integers. (e.g. ...-2, -1, 0, 1,... 9999...)

• Floating Point Numbers: real numbers. (e,g. 0.1, -12.2, 1.5x10¹²)

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- Floating Point Numbers: real numbers. (e,g. 0.1, -12.2, 1.5x10¹²)
 - → More on this next week!

Number Representations

C Declaration	Size (Bytes)
int	4
double	8
float	4
char	1
char *	8
short	2
long	8

In The Days Of Yore...

C Declaration	Size (Bytes)
int	4
double	8
float	4
char	1
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short	2
long	4

Transitioning To Larger Datatypes



- Early 2000s: most computers were 32-bit. This means that pointers were 4 bytes (32 bits).
- 32-bit pointers store a memory address from 0 to 2³²-1, equaling 2³² bytes of addressable memory. This equals 4 Gigabytes, meaning that 32-bit computers could have at most 4GB of memory (RAM)!
- Because of this, computers transitioned to 64-bit. This means that datatypes were enlarged; pointers in programs were now 64 bits.
- 64-bit pointers store a memory address from 0 to 2⁶⁴-1, equaling 2⁶⁴ bytes of addressable memory. This equals 16 Exabytes, meaning that 64-bit computers could have at most 1024*1024*1024 GB of memory (RAM)!

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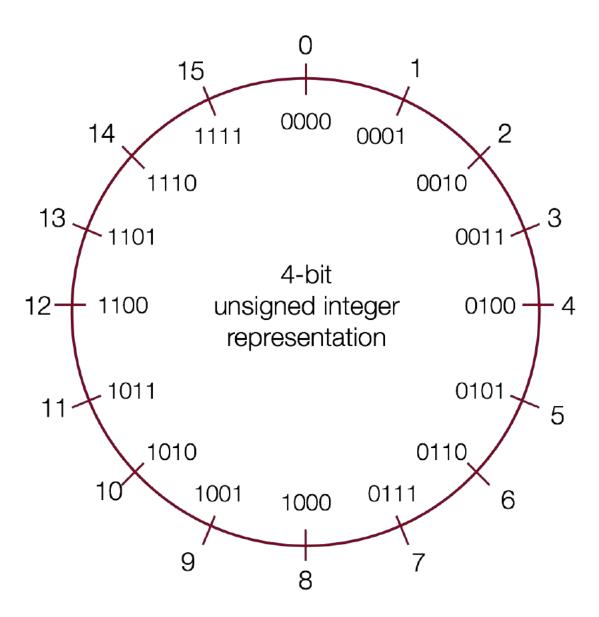
Unsigned Integers

- An unsigned integer is 0 or a positive integer (no negatives).
- We have already discussed converting between decimal and binary, which is a nice 1:1 relationship. Examples:

```
0b0001 = 1
0b0101 = 5
0b1011 = 11
0b1111 = 15
```

• The range of an unsigned number is $0 \rightarrow 2^w$ - 1, where w is the number of bits. E.g. a 32-bit integer can represent 0 to 2^{32} – 1 (4,294,967,295).

Unsigned Integers



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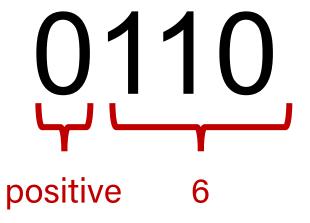
Signed Integers

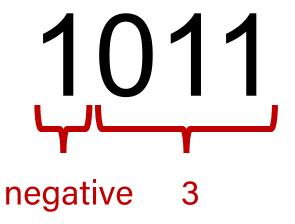
- A **signed** integer is a negative integer, 0, or a positive integer.
- *Problem:* How can we represent negative *and* positive numbers in binary?

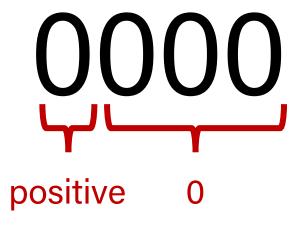
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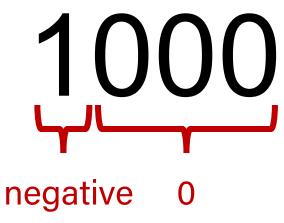
Idea: let's reserve the *most* significant bit to store the sign.











```
1\ 000 = -0 0\ 000 = 0

1\ 001 = -1 0\ 001 = 1

1\ 010 = -2 0\ 010 = 2

1\ 011 = -3 0\ 011 = 3

1\ 100 = -4 0\ 100 = 4

1\ 101 = -5 0\ 101 = 5

1\ 110 = -6 0\ 110 = 6

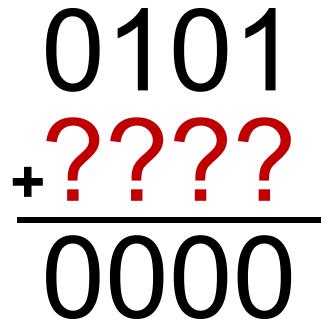
1\ 111 = -7 0\ 111 = 7
```

We've only represented 15 of our 16 available numbers!

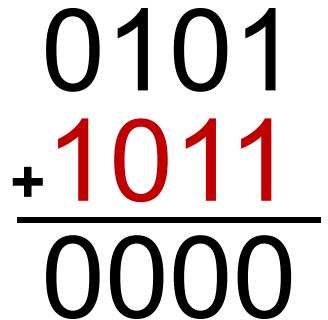
- Pro: easy to represent, and easy to convert to/from decimal.
- Con: +-0 is not intuitive
- Con: we lose a bit that could be used to store more numbers
- Con: arithmetic is tricky: we need to find the sign, then maybe subtract (borrow and carry, etc.), then maybe change the sign. This complicates the hardware support for something as fundamental as addition.

Can we do better?

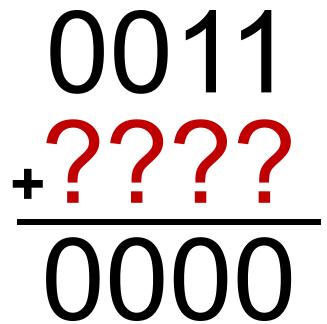
• Ideally, binary addition would *just work* **regardless** of whether the number is positive or negative.



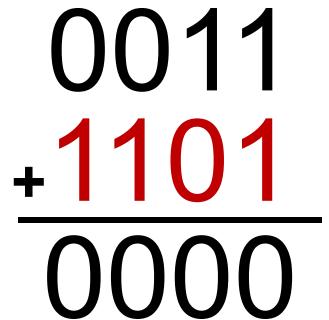
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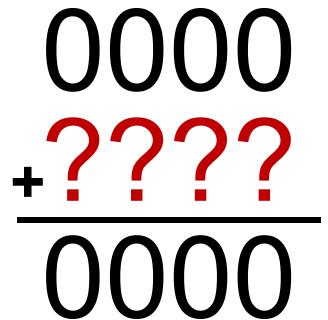
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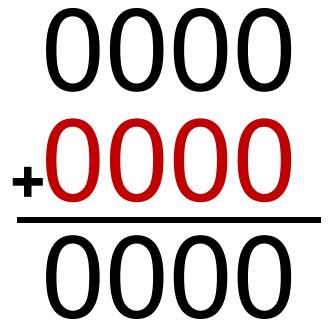
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Decimal	Positive	Negative	Decimal	Positive	Negative
0	0000	0000	8	1000	1000
1	0001	1111	9	1001 (same as -7!)	NA
2	0010	1110	10	1010 (same as -6!)	NA
3	0011	1101	11	1011 (same as -5!)	NA
4	0100	1100	12	1100 (same as -4!)	NA
5	0101	1011	13	1101 (same as -3!)	NA
6	0110	1010	14	1110 (same as -2!)	NA
7	0111	1001	15	1111 (same as -1!)	NA

There Seems Like a Pattern Here...

The negative number is the positive number inverted, plus one!

There Seems Like a Pattern Here...

A binary number plus its inverse is all 1s.

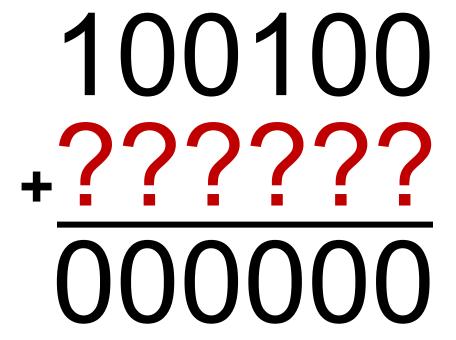
Add 1 to this to carry over all 1s and get 0!

0101
+1010
1111

1111 +001 0000

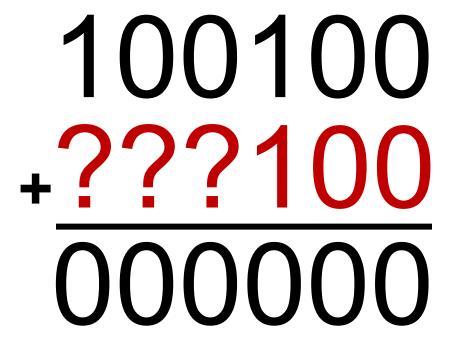
Another Trick

 To find the negative equivalent of a number, work right-to-left and write down all digits through when you reach a 1. Then, invert the rest of the digits.



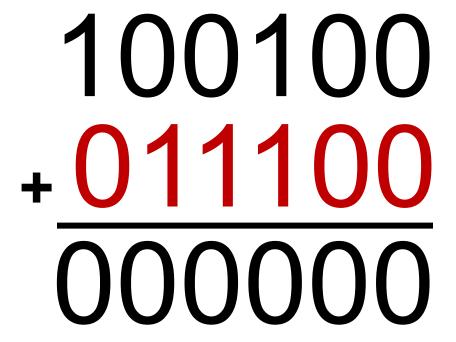
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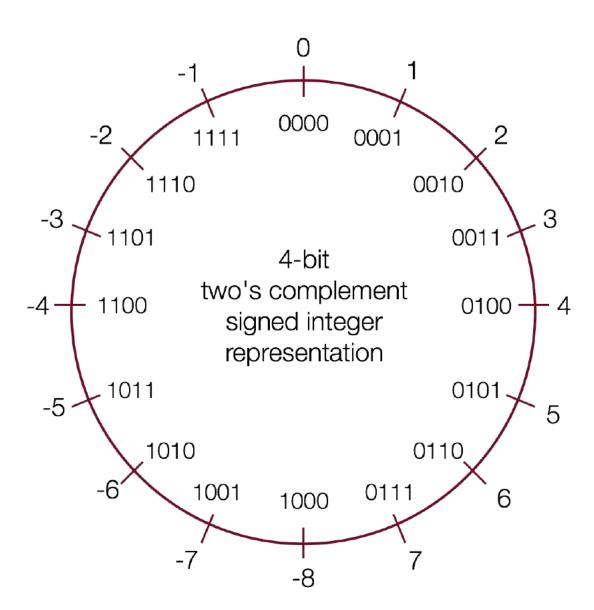


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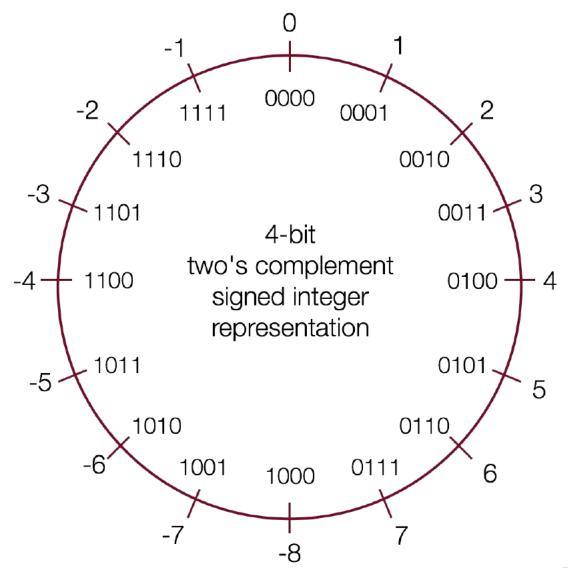
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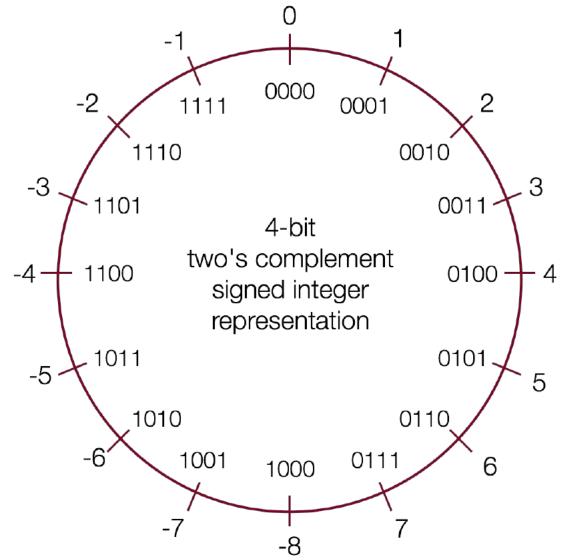
Two's Complement



- In two's complement, we represent a positive number as itself, and its negative equivalent as the two's complement of itself.
- The two's complement of a number is the binary digits inverted, plus 1.
- This works to convert from positive to negative, and back from negative to positive!



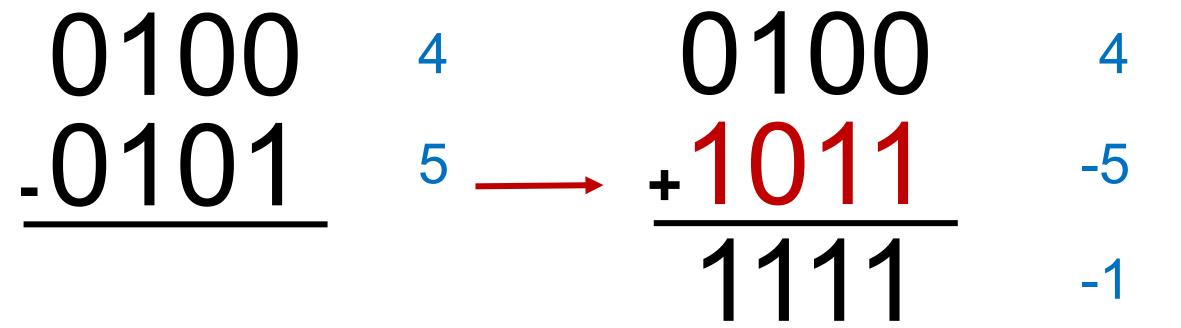
- Con: more difficult to represent, and difficult to convert to/from decimal and between positive and negative.
- Pro: only 1 representation for 0!
- Pro: all bits are used to represent as many numbers as possible
- Pro: the most significant bit still indicates the sign of a number.
- Pro: addition works for any combination of positive and negative!



 Adding two numbers is just...adding! There is no special case needed for negatives. E.g. what is 2 + -5?

0010	2
+1011	-5
1101	-3

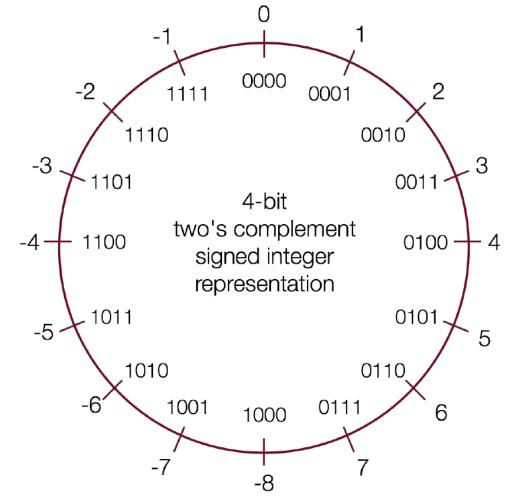
• Subtracting two numbers is just performing the two's complement on one of them and then adding. E.g. 4 - 5 = -1.



Practice: Two's Complement

What are the negative or positive equivalents of the numbers below?

- a) -4 (1100)
- b) 7 (0111)
- c) 3 (0011)
- d) -8 (1000)



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Overflow

• If you exceed the **maximum** value of your bit representation, you wrap around or overflow back to the **smallest** bit representation.

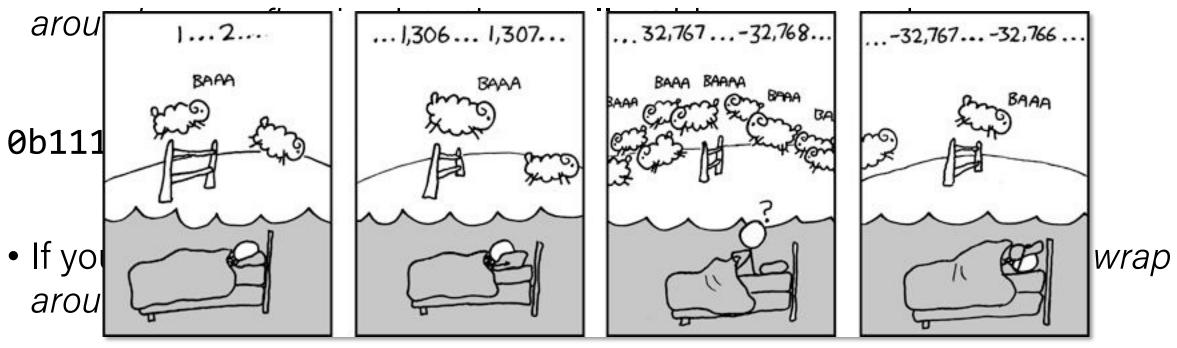
$$0b1111 + 0b1 = 0b0000$$

• If you go below the **minimum** value of your bit representation, you *wrap* around or overflow back to the **largest** bit representation.

$$0b0000 - 0b1 = 0b1111$$

Overflow

• If you exceed the maximum value of your bit representation, you wrap



https://xkcd.com/571 Can't Sleep

0b0000 - 0b1 = 0b1111

Title text: If androids someday DO dream of electric sheep, don't forget to declare sheepCount as a long int.

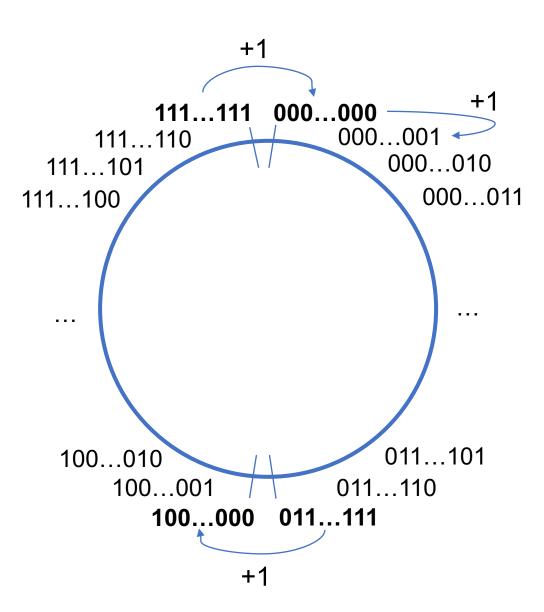
Min and Max Integer Values

Туре	Size (Bytes)	Minimum	Maximum
char	1	-128	127
unsigned char	1	0	255
short	2	-32768	32767
unsigned short	2	0	65535
int	4	-2147483648	2147483647
unsigned int	4	0	4294967295
long	8	-9223372036854775808	9223372036854775807
unsigned long	8	0	18446744073709551615

Min and Max Integer Values

INT_MIN, INT_MAX, UINT_MAX, LONG_MIN, LONG_MAX, ULONG_MAX, ...

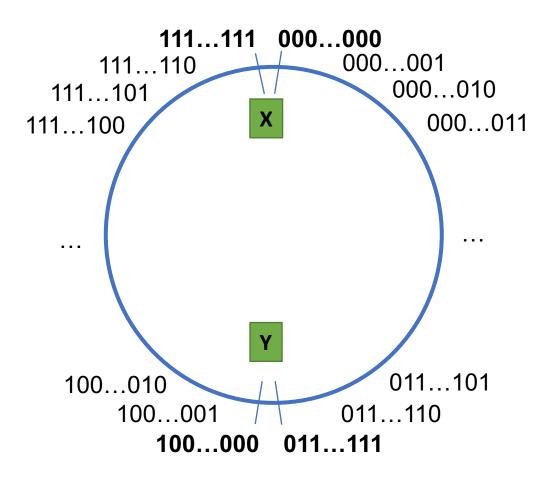
Overflow



Practice: Overflow

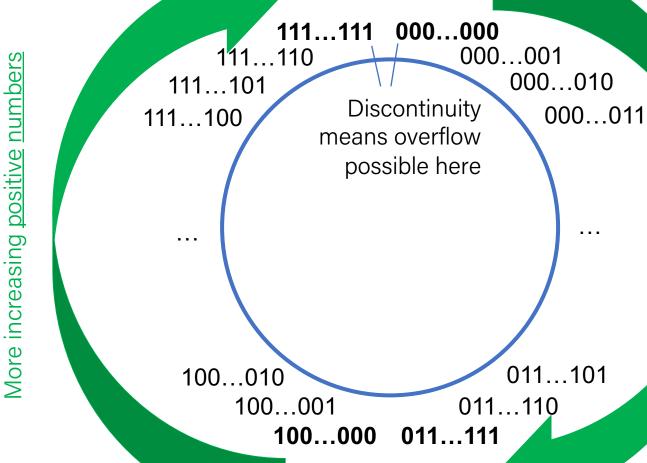
At which points can overflow occur for signed and unsigned int? (assume binary values shown are all 32 bits)

- A. Signed and unsigned can both overflow at points X and Y
- B. Signed can overflow only at X, unsigned only at Y
- C. Signed can overflow only at Y, unsigned only at X
- D. Signed can overflow at X and Y, unsigned only at X
- E. Other



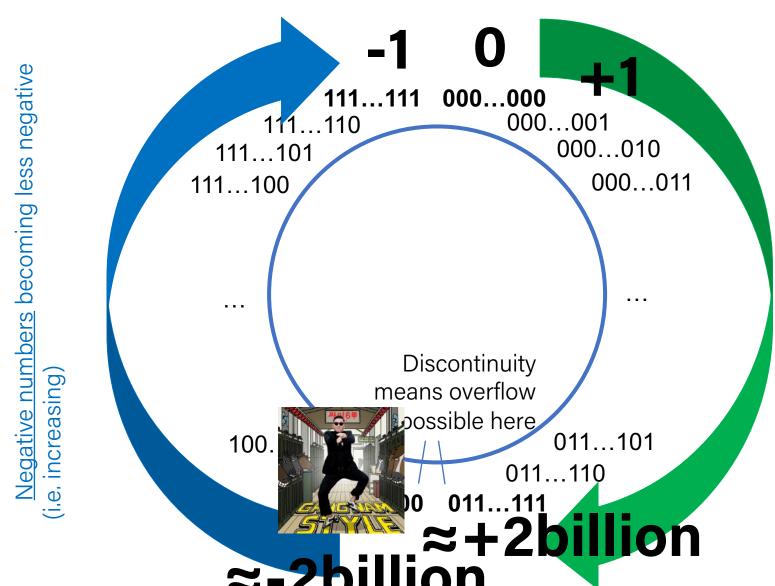
Unsigned Integers

≈+4billion 0



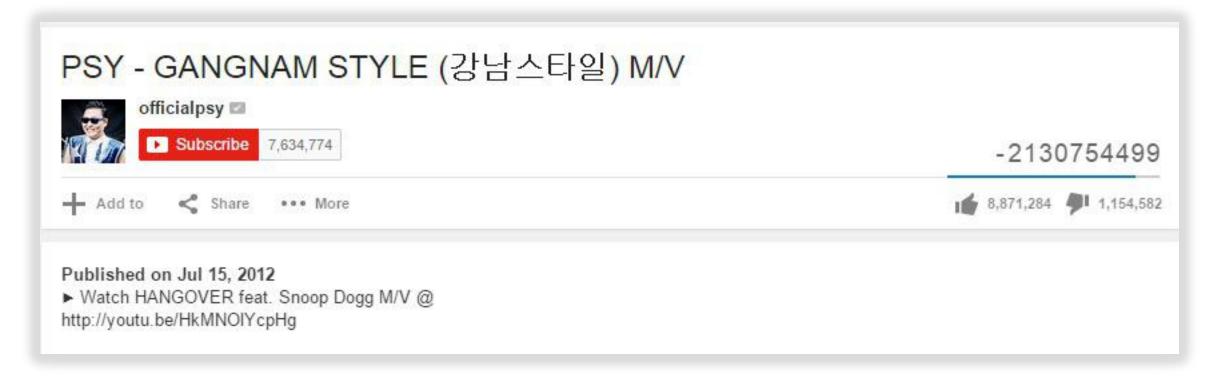
Increasing positive numbers

Signed Numbers



Increasing positive numbers

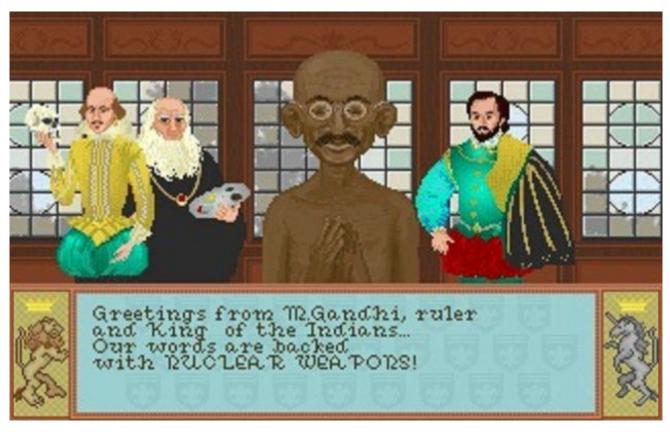
Overflow In Practice: PSY



YouTube: "We never thought a video would be watched in numbers greater than a 32-bit integer (=2,147,483,647 views), but that was before we met PSY. "Gangnam Style" has been viewed so many times we had to upgrade to a 64-bit integer (9,223,372,036,854,775,808)!"

Overflow In Practice: Gandhi

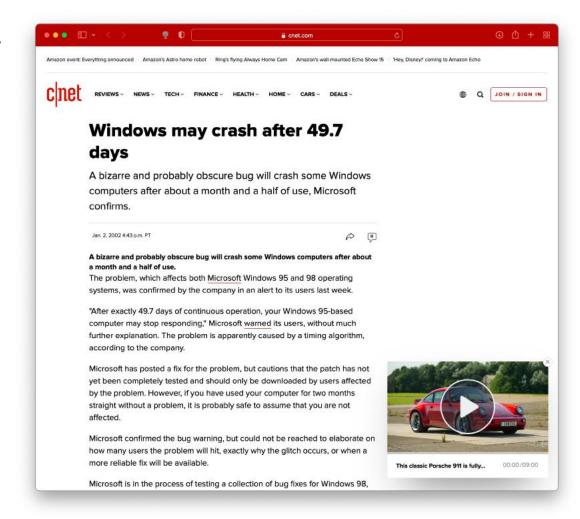
- In the game "Civilization", each civilization leader had an "aggression" rating. Gandhi was meant to be peaceful, and had a score of 1.
- If you adopted "democracy", all players' aggression reduced by 2. Gandhi's went from 1 to 255!
- Gandhi then became a big fan of nuclear weapons.

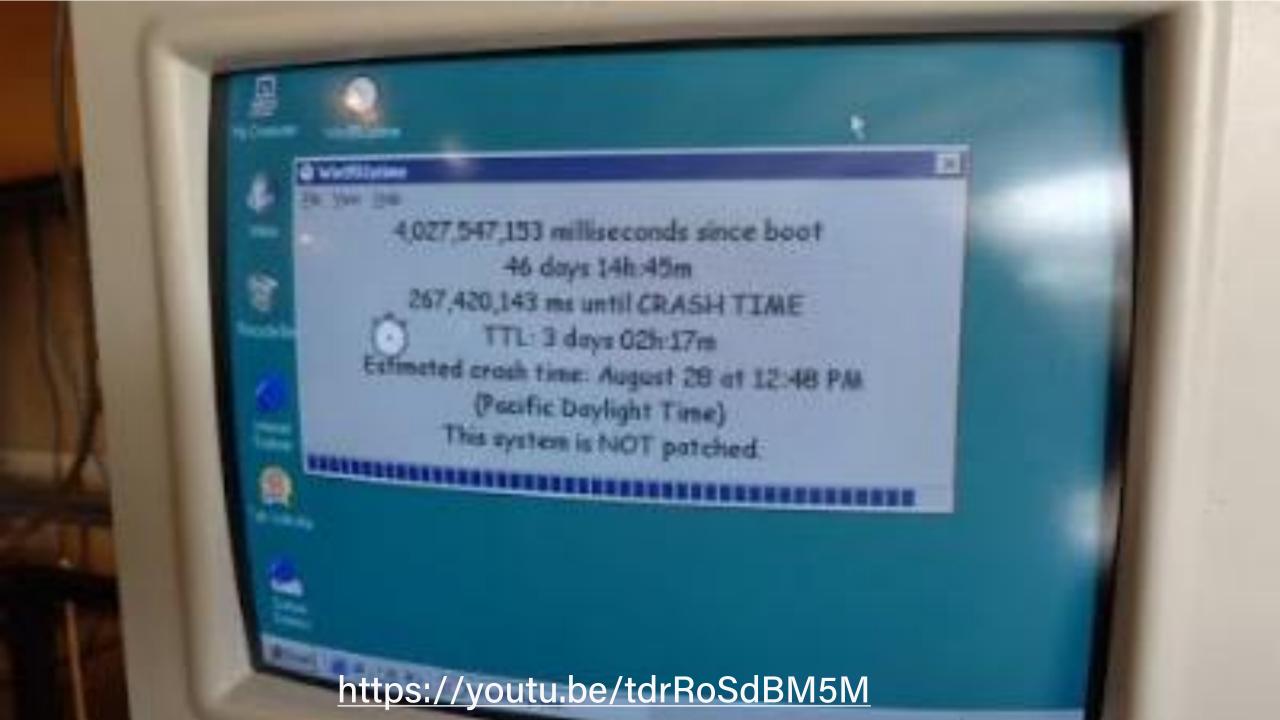


https://kotaku.com/why-gandhi-is-such-an-asshole-in-civilization-1653818245

Windows 95 can only run for 49.7 days before crashing,

- Windows 95 was unable to run longer than 49.7 days of runtime!
- There exists GetTickTime function –
 part of the Windows API which
 returns the number of milliseconds
 which has elapsed since the system
 has started up as a 32-bit uint.
- And there's 86M ms in a day, i.e. 1000
 * 60 * 60 * 24 = 86,400,000 and 32
 bits is 4,294,967,296 so 4,294,967,296
 / 86,400,000 = 49.7102696 days!





Overflow in Practice:

- Pacman Level 256
- Make sure to reboot Boeing Dreamliners every 248 days
- Comair/Delta airline had to <u>cancel thousands of flights</u> days before Christmas
- Reported vulnerability CVE-2019-3857 in libssh2 may allow a hacker to remotely execute code
- Donkey Kong Kill Screen

Demo Revisited: Unexpected Behavior



airline.c

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printf and Integers

- There are 3 placeholders for 32-bit integers that we can use:
 - %d: signed 32-bit int
 - %u: unsigned 32-bit int
 - %x: hex 32-bit int
- The placeholder—not the expression filling in the placeholder—dictates what gets printed!

Casting

What happens at the byte level when we cast between variable types?
 The bytes remain the same! This means they may be interpreted differently depending on the type.

```
int v = -12345;
    unsigned int uv = v;
    printf("v = %d, uv = %u\n", v, uv);

This prints out: "v = -12345, uv = 4294954951". Why?
```

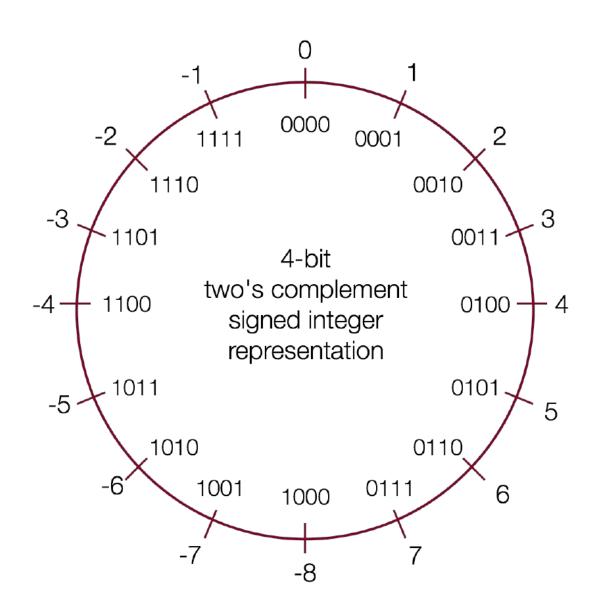
Casting

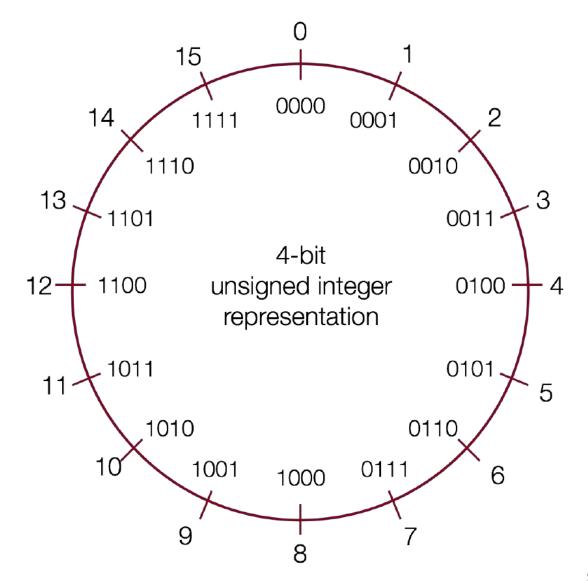
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```

If we treat this binary representation as a positive number, it's huge!

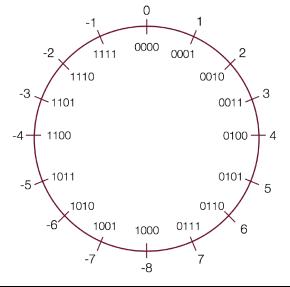
Casting





• Be careful when comparing signed and unsigned integers. C will implicitly cast the signed argument to unsigned, and then performs the operation assuming both numbers are non-negative.

Expression	Туре	Evaluation	Correct?
0 == 0U	Unsigned	1	yes
-1 < 0	Signed	1	yes
-1 < OU	Unsigned	0	No!
2147483647 > -2147483647 - 1	Signed	1	yes
2147483647U > -2147483647 - 1	Unsigned	0	No!
2147483647 > (int)2147483648U	Signed	1	No!
-1 > -2	Signed	1	yes
(unsigned)-1 > -2	Unsigned	1	yes



Туре	Size (Bytes)	Minimum	Maximum
int	4	-2147483648	2147483647
unsigned int	4	0	4294967295

Which many of the following statements are true? (assume that variables are set to values that place them in the spots shown)

```
s3 > u3
```

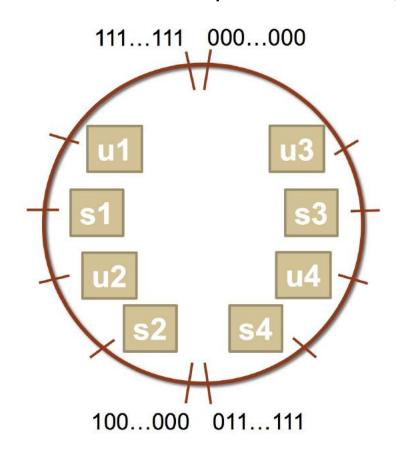
u2 > u4

s2 > s4

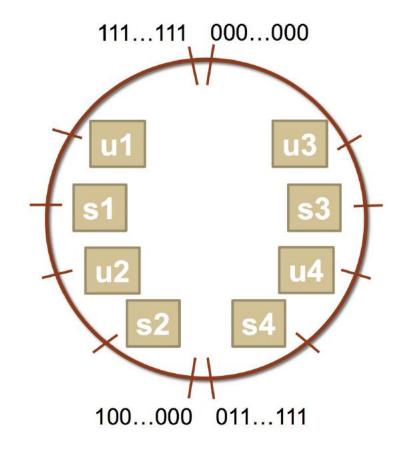
s1 > s2

u1 > u2

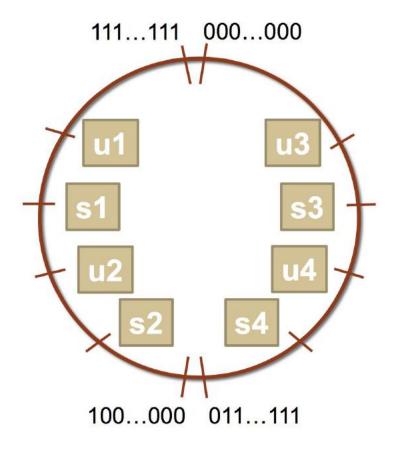
s1 > u3



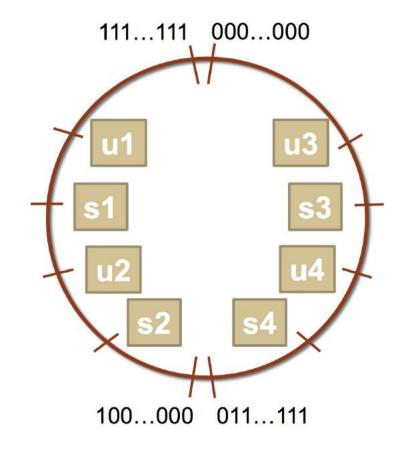
```
s3 > u3 - true
u2 > u4
s2 > s4
s1 > s2
u1 > u2
s1 > u3
```



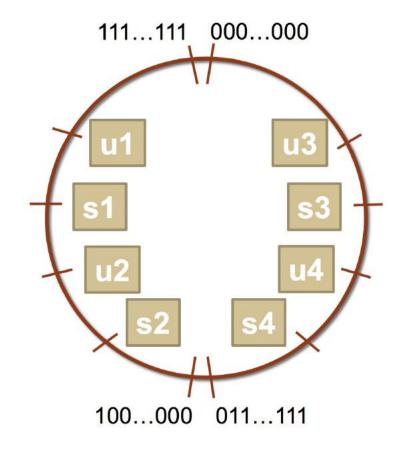
```
s3 > u3 - true
u2 > u4 - true
s2 > s4
s1 > s2
u1 > u2
s1 > u3
```



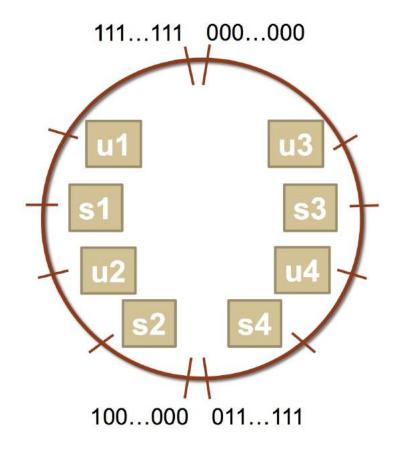
```
s3 > u3 - true
u2 > u4 - true
s2 > s4 - false
s1 > s2
u1 > u2
s1 > u3
```



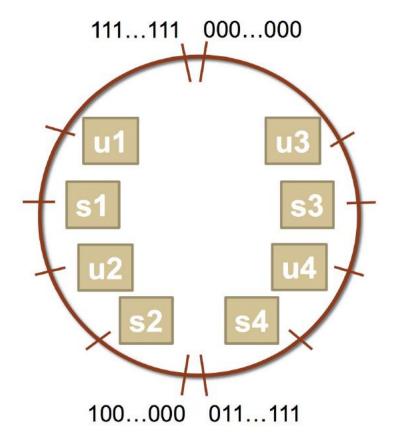
```
s3 > u3 - true
u2 > u4 - true
s2 > s4 - false
s1 > s2 - true
u1 > u2
s1 > u3
```



```
s3 > u3 - true
u2 > u4 - true
s2 > s4 - false
s1 > s2 - true
u1 > u2 - true
s1 > u3
```



```
s3 > u3 - true
u2 > u4 - true
s2 > s4 - false
s1 > s2 - true
u1 > u2 - true
s1 > u3 - true
```



Expanding Bit Representations

- Sometimes, we want to convert between two integers of different sizes (e.g. **short** to **int**, or **int** to **long**).
- We might not be able to convert from a bigger data type to a smaller data type, but we do want to always be able to convert from a smaller data type to a bigger data type.
- For **unsigned** values, we can add *leading zeros* to the representation ("zero extension")
- For **signed** values, we can *repeat the sign of the value* for new digits ("sign extension")
- Note: when doing <, >, <=, >= comparison between different size types, it will *promote to the larger type*.

Expanding Bit Representation

Expanding Bit Representation

```
short s = 4;
// short is a 16-bit format, so
                                       s = 0000 0000 0000 0100b
int i = s;
— or —
short s = -4;
// short is a 16-bit format, so
                                       s = 1111 1111 1111 1100b
int i = s;
// conversion to 32-bit int, so i = 1111 1111 1111 1111 1111 1111 1100b
```

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = 53191;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), 53191:

0000 0000 0000 0000 1100 1111 1100 0111

When we cast x to a short, it only has 16-bits, and C truncates the number:

1100 1111 1100 0111

This is -12345! And when we cast **sx** back an **int**, we sign-extend the number.

```
1111 1111 1111 1111 1100 1111 1100 0111 // still -12345
```

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
int x = -3;
short sx = x;
int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit int), -3:

```
1111 1111 1111 1111 1111 1111 1111 1101
```

When we cast x to a short, it only has 16-bits, and C truncates the number:

```
1111 1111 1111 1101
```

This is -3! If the number does fit, it will convert fine. y looks like this:

Truncating Bit Representation

If we want to **reduce** the bit size of a number, C *truncates* the representation and discards the *more significant bits*.

```
unsigned int x = 128000;
unsigned short sx = x;
unsigned int y = sx;
```

What happens here? Let's look at the bits in x (a 32-bit unsigned int), 128000:

0000 0000 0000 0001 1111 0100 0000 0000

When we cast x to a **short**, it only has 16-bits, and C *truncates* the number:

1111 0100 0000 0000

This is 62464! Unsigned numbers can lose info too. Here is what y looks like:

0000 0000 0000 1111 0100 0000 0000 // still 62464

The size of Operator

```
long sizeof(type);
```

```
// Example
long int_size_bytes = sizeof(int);  // 4
long short_size_bytes = sizeof(short); // 2
long char_size_bytes = sizeof(char); // 1
```

sizeof takes a variable type as a parameter and returns the size of that type, in bytes.

```
short x = 130; // 0b1000 0010
char cx = x;
```

```
short x = -132 // 0b1111 1111 0111 1100 char cx = x;
```

```
short x = 25; // 0b1 1001
char cx = x;
```

```
short x = 130; // 0b1000 0010
char cx = x; // -126
```

```
short x = -132 // 0b1111 1111 0111 1100
char cx = x; // 124
```

```
short x = 25; // 0b1 1001
char cx = x; // 25
```

```
short x = 390; // 0b1 1000 0110
char cx = x;
```

```
short x = -15; // 0b1111 1111 1111 0001 char cx = x;
```

```
short x = 390; // 0b1 1000 0110
char cx = x; // -122
```

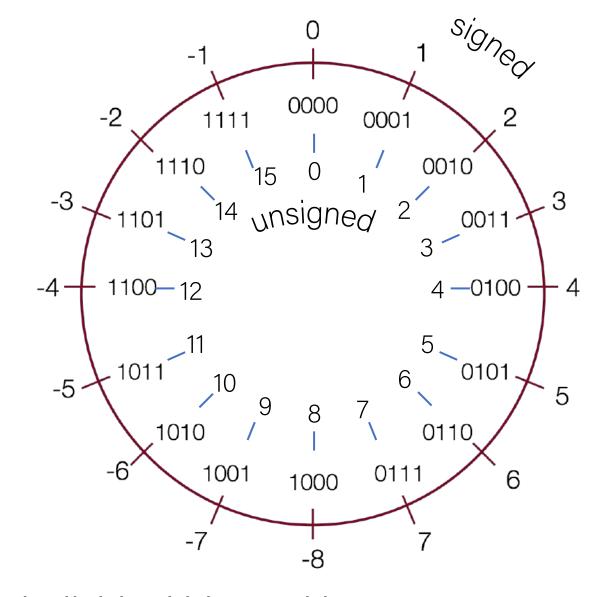
```
short x = -15; // 0b1111 1111 1111 0001 char cx = x; // -15
```

Summary: Basic Rules of Expanding, Truncating

- Expanding (e.g., short int to int)
 - Unsigned: zeros added
 - Signed: sign extension
 - Both yield expected result
- Truncating (e.g., unsigned to unsigned short)
 - Unsigned/signed: bits are truncated
 - Result reinterpreted
 - Unsigned: mod operation
 - Signed: similar to mod
 - For small (in magnitude) numbers yields expected behavior

Recap

- Getting Started With C
- Bits and Bytes
- Hexadecimal
- Integer Representations
- Unsigned Integers
- Signed Integers
- Overflow
- Casting and Combining Types



Next time: How can we manipulate individual bits and bytes? How can we represent floating point numbers?