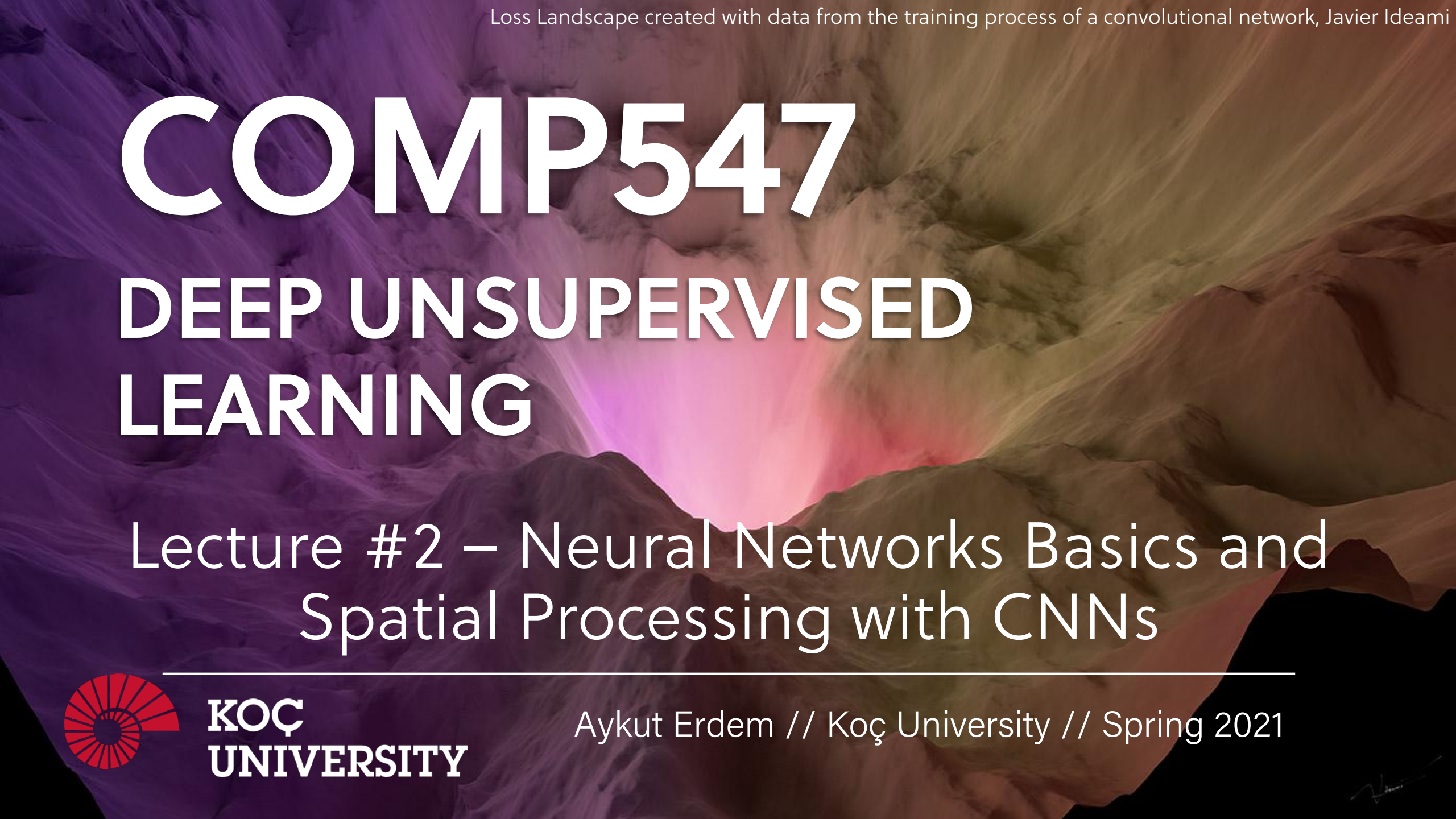


COMP547

DEEP UNSUPERVISED LEARNING

Lecture #2 – Neural Networks Basics and Spatial Processing with CNNs

KOÇ UNIVERSITY

Aykut Erdem // Koç University // Spring 2021

Previously on COMP547

- course logistics
- course topics
- what is deep unsupervised learning


Photo: Detail from Sofia Crespo's Tribute to Manolo Part 2

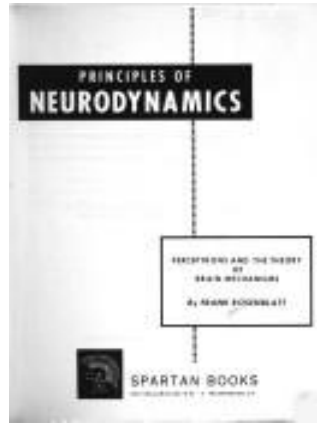
Lecture overview

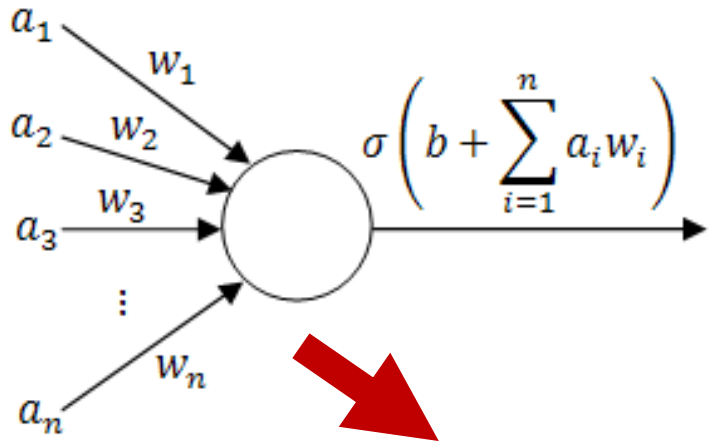
- deep learning
- computation in a neural net
- optimization
- backpropagation
- training tricks
- convolutional neural networks

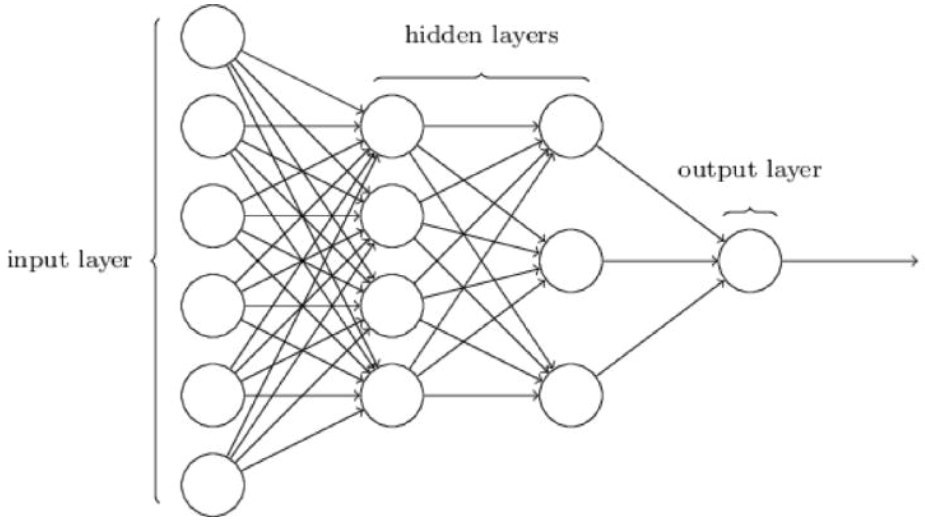
- **Disclaimer:** Much of the material and slides for this lecture were borrowed from
  —Costis Daskalakis and Aleksander Mądry's MIT 6.883 class
  —Bill Freeman, Antonio Torralba and Phillip Isola's MIT 6.869 class

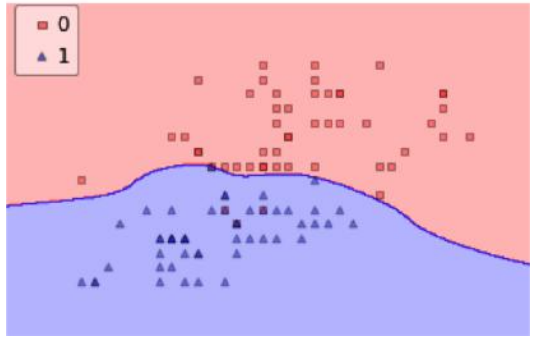
Humble beginnings

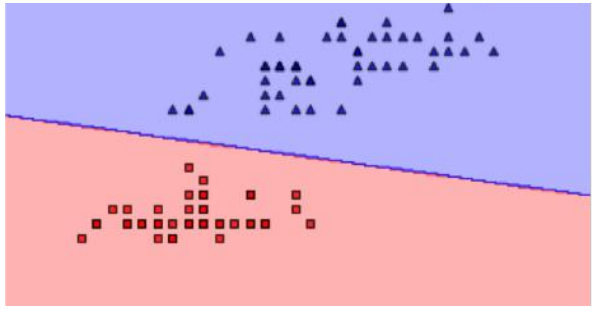
- Perceptron [Rosenblatt ‘58]



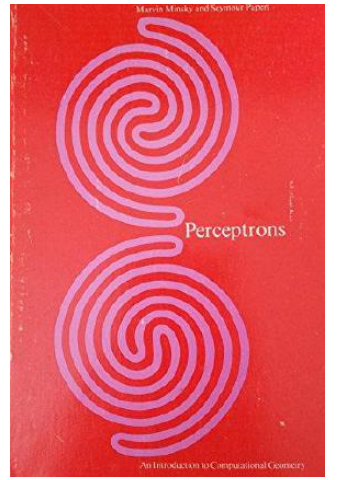




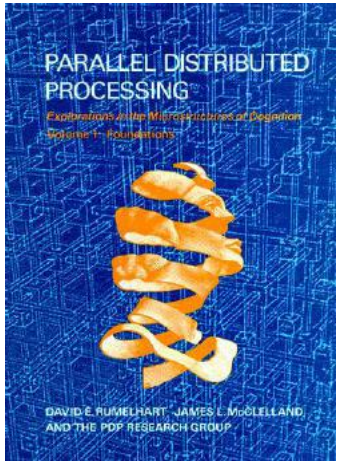




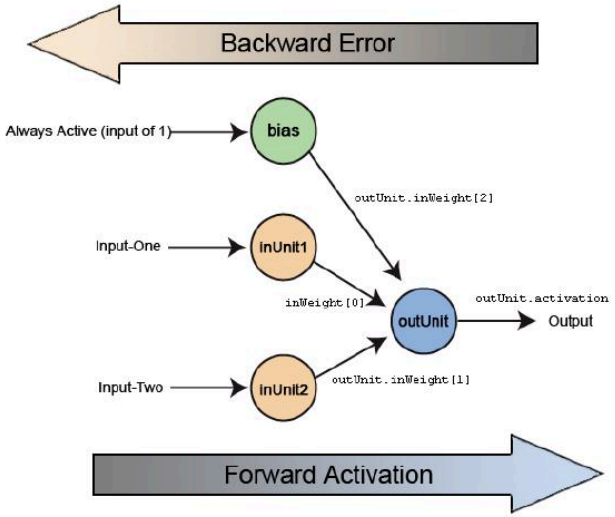
- Criticism of Perceptrons (XOR affair) [Minsky Papert ‘69]
  - Effectively causes a "deep learning winter"



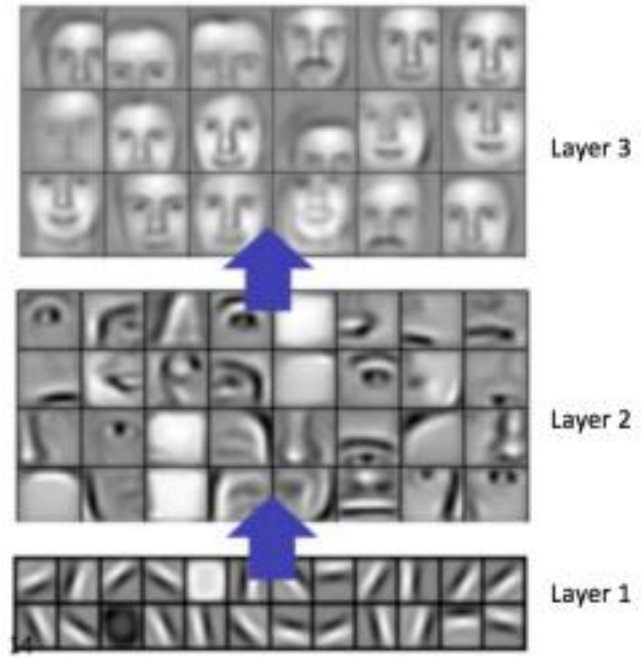
(Early) Spring



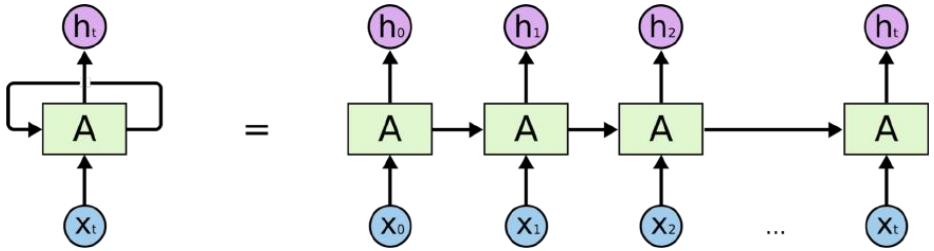
- Back-propagation [Rumelhart et al. '86, LeCun '85, Parker '85]



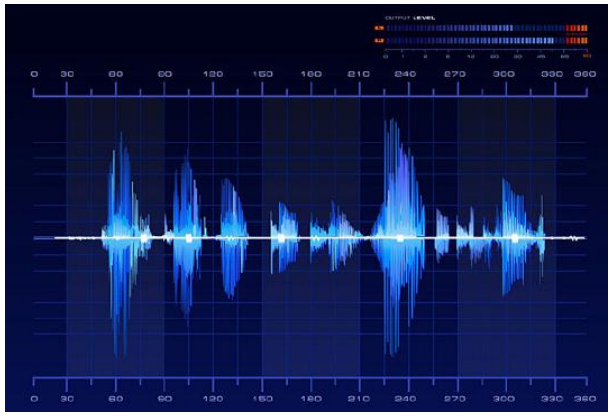
- Convolutional layers [LeCun et al. '90]



- Recurrent Neural Networks/Long Short-Term Memory (LSTM) [Hochreiter Schmidhuber '97]

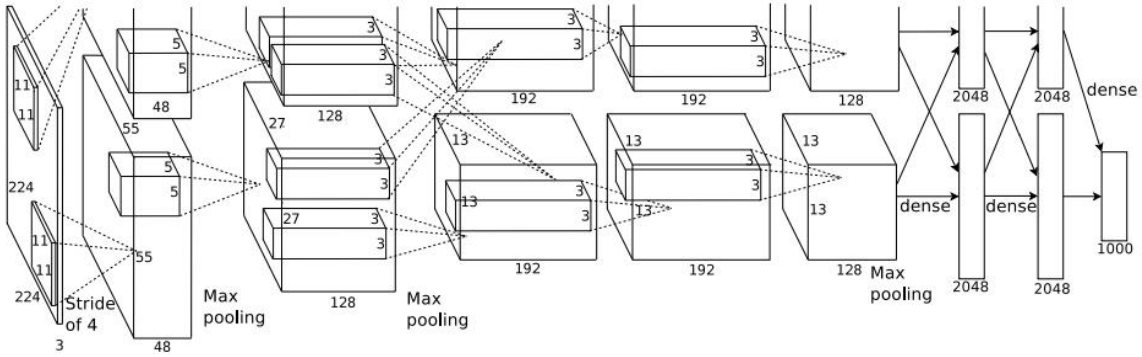


Summer

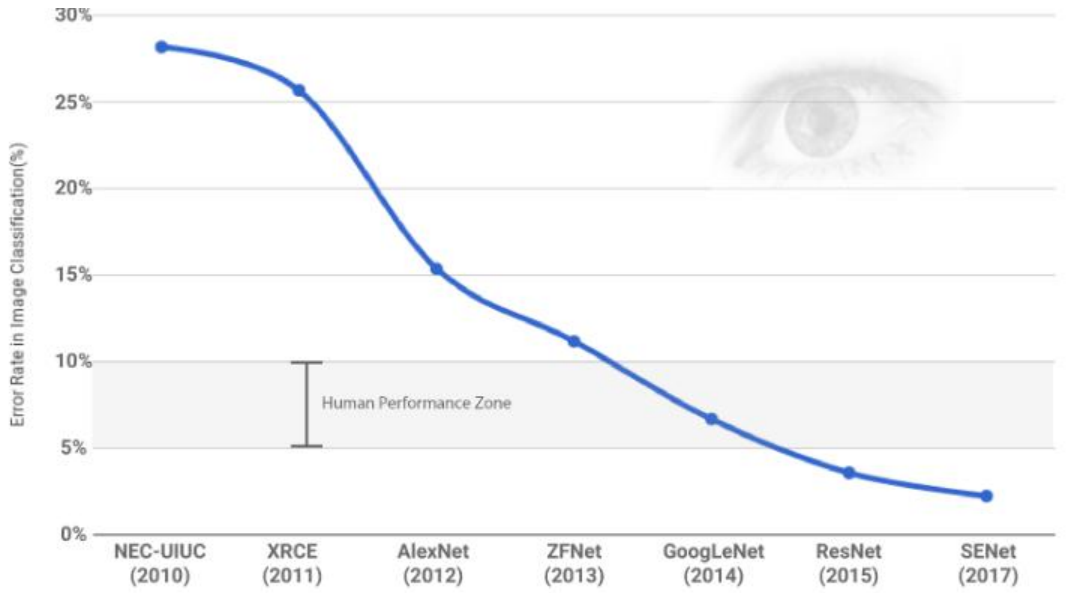


- **2006:** First big success: speech recognition

- **2012:** Breakthrough in computer vision: AlexNet [Krizhevsky et al. ‘12]



- **2015:** Deep learning-based vision models outperform humans



What enabled this success?

- Better architectures (e.g., ReLUs) and regularization techniques (e.g. Dropout)

- Sufficiently large datasets

- Enough computational power

Deep learning

- Modeling the visual world is incredibly complicated. We need high capacity models.
- In the past, we didn't have enough data to fit these models. But now we do!
- We want a class of **high capacity models** that are **easy to optimize**.

Deep neural networks!

What is deep learning?

REVIEW

Deep learning

Yann LeCun[1], Yoshua Bengio[2] & Geoffrey Hinton[3]

Deep learning allows computational models that are composed of multiple processing layers to learn representations of data with multiple levels of abstraction. These methods have dramatically improved the state-of-the-art in speech recognition, visual object recognition, object detection and many other domains such as drug discovery and genomics. Deep learning discovers intricate structure in large data sets by using the backpropagation algorithm to indicate how a machine should change its internal parameters that are used to compute the representation in each layer from the representation in the previous layer. Deep convolutional nets have brought about breakthroughs in processing images, video, speech and audio, whereas recurrent nets have shone light on sequential data such as text and speech.

Machine-learning technology powers many aspects of modern society: from web searches to content filtering on social networks to recommendations on e-commerce websites, and it is increasingly present in consumer products such as cameras and smartphones. Machine-learning systems are used to identify objects in images, transcribe speech into text, match news items, posts or products with users' interests, and select relevant results of search. Increasingly, these applications make use of a class of techniques called deep learning.

Conventional machine-learning techniques were limited in their ability to process natural data in their raw form. For decades, constructing a pattern-recognition or machine-learning system required careful engineering and considerable domain expertise to design a feature extractor that transformed the raw data (such as the pixel values of an image) into a suitable internal representation or feature vector from which the learning subsystem, often a classifier, could detect or classify patterns in the input.

Representation learning is a set of methods that allows a machine to be fed with raw data and to automatically discover the representations needed for detection or classification. Deep-learning methods are representation-learning methods with multiple levels of representation, obtained by composing simple but non-linear modules that each transform the representation at one level (starting with the raw input) into a representation at a higher, slightly more abstract level. With the composition of enough such transformations, very complex functions can be learned. For classification tasks, higher layers of representation amplify aspects of the input that are important for discrimination and suppress irrelevant variations. An image, for example, comes in the form of an array of pixel values, and the learned features in the first layer of representation typically represent the presence or absence of edges at particular orientations and locations in the image. The second layer typically detects motifs by spotting particular arrangements of edges, regardless of small variations in the edge positions. The third layer may assemble motifs into larger combinations that correspond to parts of familiar objects, and subsequent layers would detect objects as combinations of these parts. The key aspect of deep learning is that these layers of features are not designed by human engineers: they are learned from data using a general-purpose learning procedure.

Deep learning is making major advances in solving problems that have resisted the best attempts of the artificial intelligence community for many years. It has turned out to be very good at discovering intricate structures in high-dimensional data and is therefore applicable to many domains of science, business and government. In addition to beating records in image recognition[1–4] and speech recognition[5–6], it has beaten other machine-learning techniques at predicting the activity of potential drug molecules[7], analysing particle accelerator data[8,9], reconstructing brain circuits[10], and predicting the effects of mutations in non-coding DNA on gene expression and disease[11–12]. Perhaps more surprisingly, deep learning has produced extremely promising results for various tasks in natural language understanding[13], particularly topic classification, sentiment analysis, question answering[14] and language translation[15–16].

We think that deep learning will have many more successes in the near future because it requires very little engineering by hand, so it can easily take advantage of increases in the amount of available computation and data. New learning algorithms and architectures that are currently being developed for deep neural networks will only accelerate this progress.

Supervised learning

The most common form of machine learning, deep or not, is supervised learning. Imagine that we want to build a system that can classify images as containing, say, a house, a car, a person or a pet. We first collect a large data set of images of houses, cars, people and pets, each labelled with its category. During training, the machine is shown an image and produces an output in the form of a vector of scores, one for each category. We want the desired category to have the highest score of all categories, but this is unlikely to happen before training. We compute an objective function that measures the error (or distance) between the output scores and the desired pattern of scores. The machine then modifies its internal adjustable parameters to reduce this error. These adjustable parameters, often called weights, are real numbers that can be seen as 'knobs' that define the input–output function of the machine. In a typical deep-learning system, there may be hundreds of millions of these adjustable weights, and hundreds of millions of labelled examples with which to train the machine.

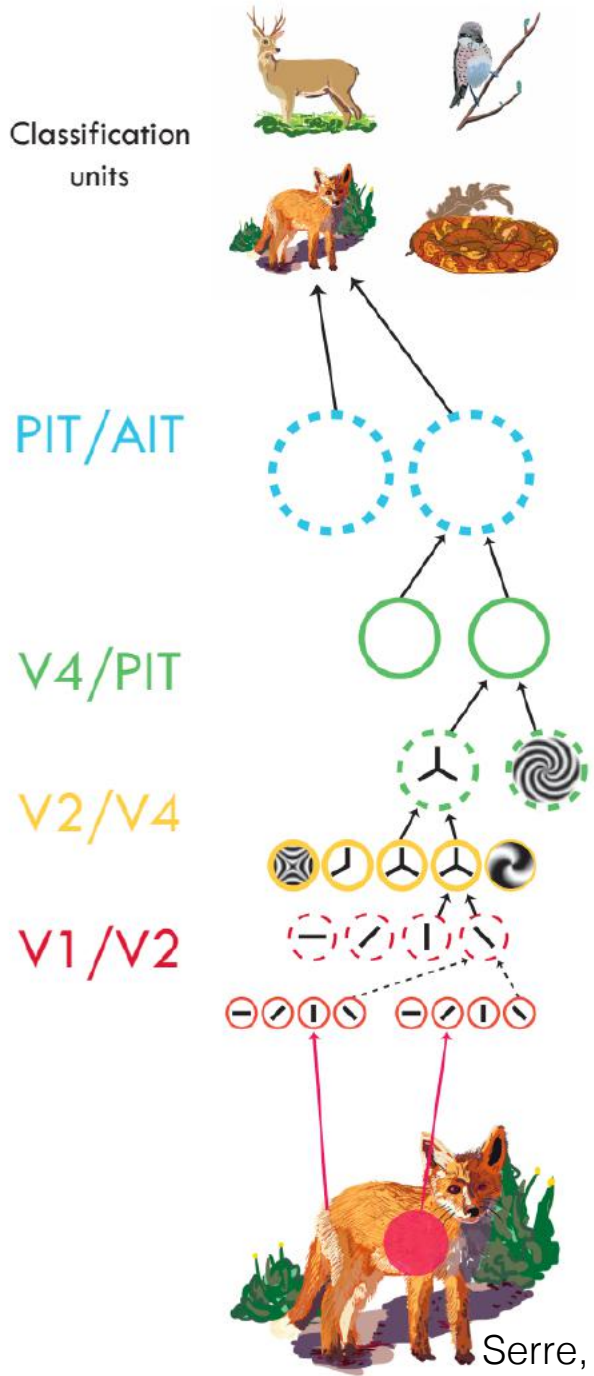
To properly adjust the weight vector, the learning algorithm computes a gradient vector that, for each weight, indicates by what amount the error would increase or decrease if the weight were increased by a tiny amount. The weight vector is then adjusted in the opposite direction to the gradient vector.

The objective function, averaged over all the training examples, can

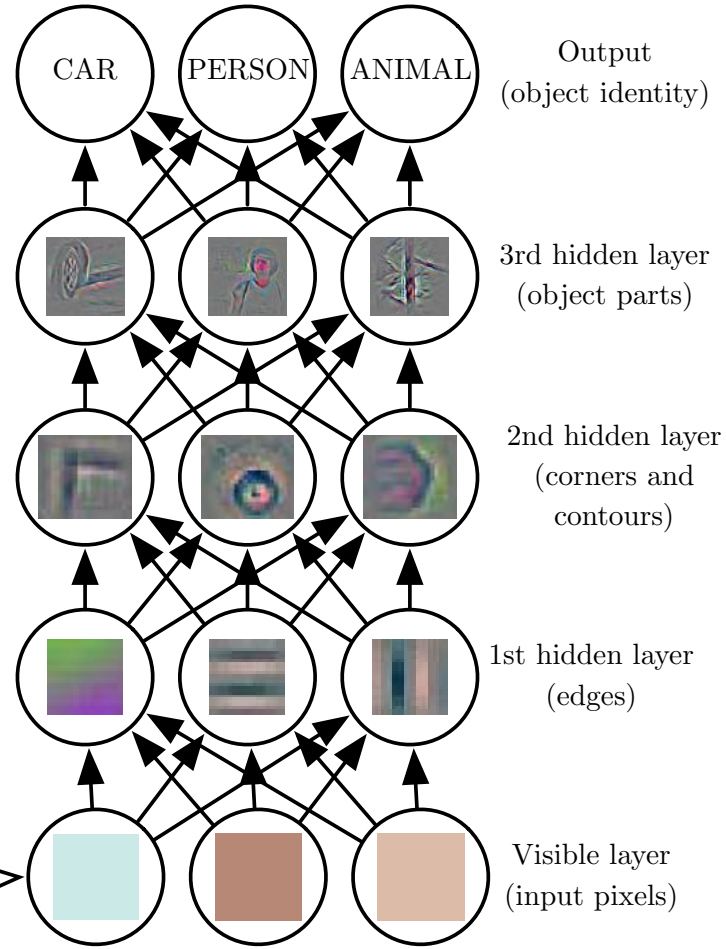
"Deep learning allows computational models that are composed of **multiple processing layers** to learn representations of data with multiple levels of abstraction."

– Yann LeCun, Yoshua Bengio and Geoff Hinton

Y. LeCun, Y. Bengio, G. Hinton, **"Deep Learning"**, Nature, Vol. 521, 28 May 2015


Classification units
PIT/AIT
V4/PIT
V2/V4
V1/V2

Serre, 2014


CAR
PERSON
ANIMAL
Output (object identity)
3rd hidden layer (object parts)
2nd hidden layer (corners and contours)
1st hidden layer (edges)
Visible layer (input pixels)

Object recognition

Edges

Texture

Colors

Segments

Parts

"clown fish"

$\phi_k(x)$

$$f_\theta(x) = \sum_{k=1}^{K} \theta_k \phi_k(x)$$

Feature extractors

Classifier

Object recognition

Edges

Texture

Colors

Learned

Segments

Parts

"clown fish"

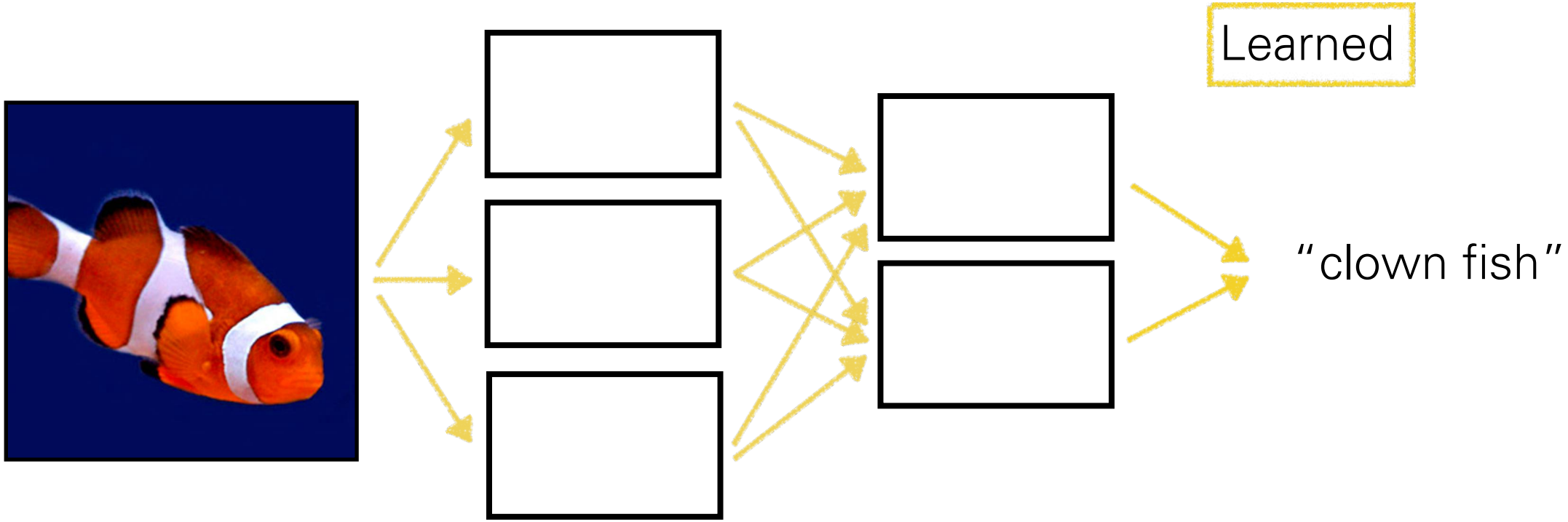
$\phi_k(x)$

$$f_\theta(x) = \sum_{k=1}^{K} \theta_k \phi_k(x)$$

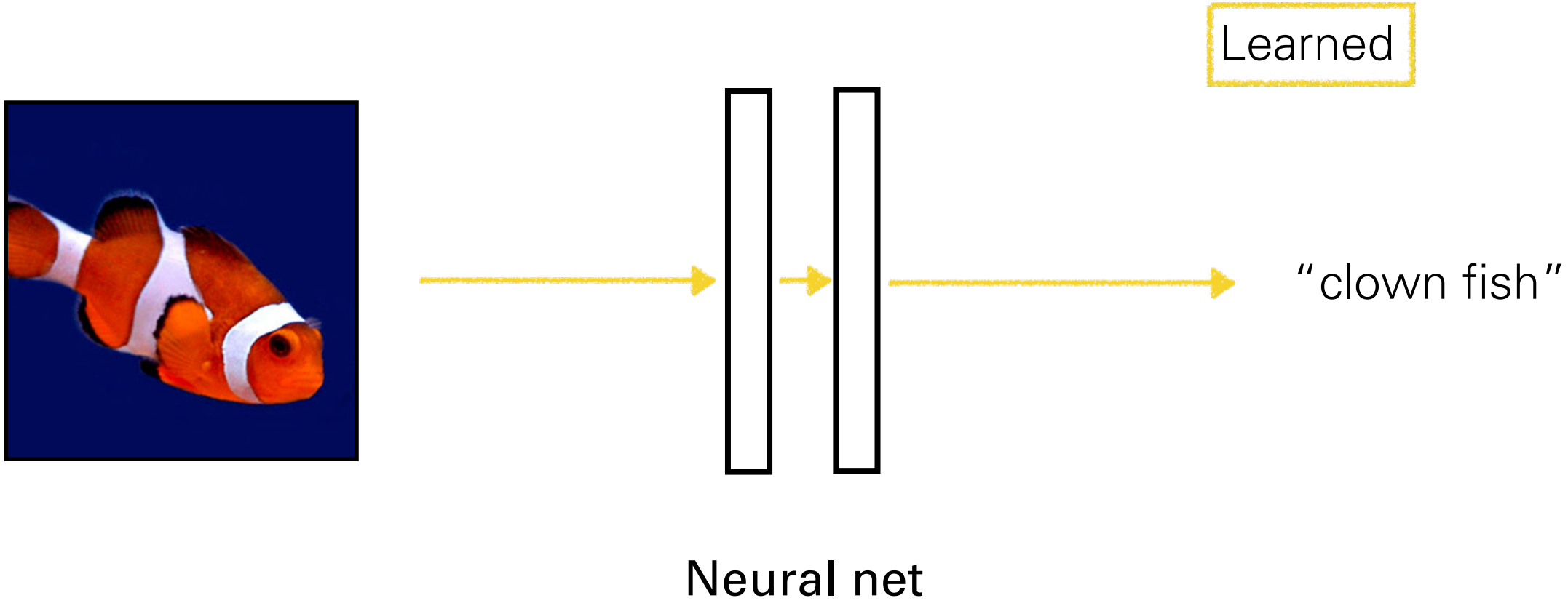
Feature extractors

Classifier

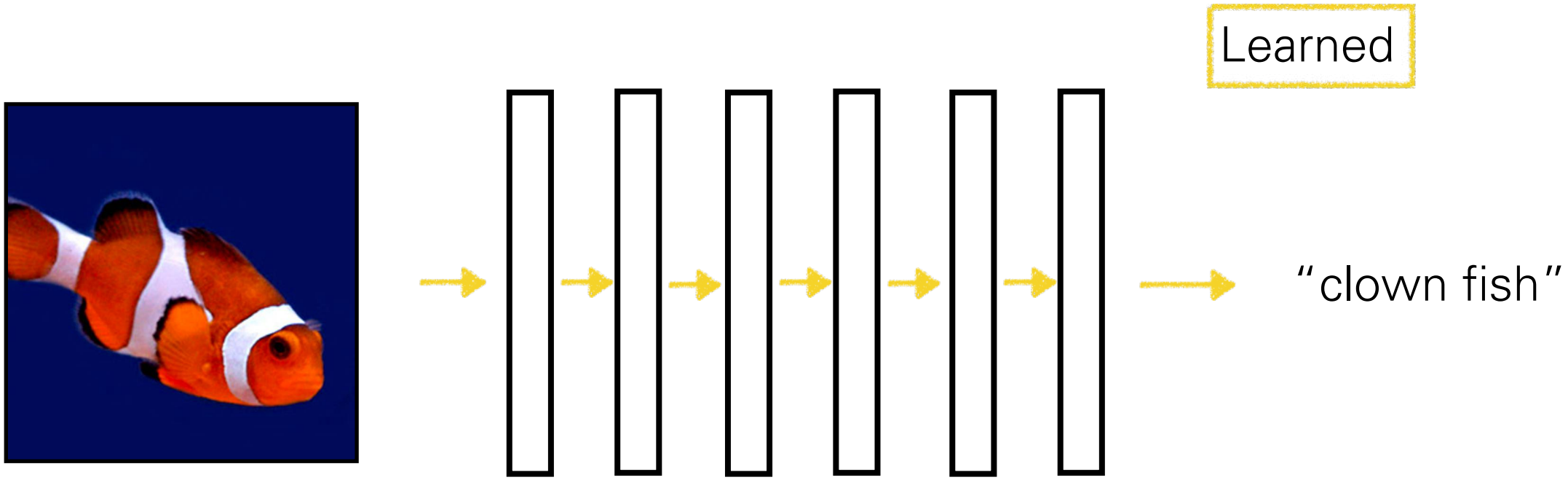
Object recognition



Object recognition



Object recognition



Deep neural net

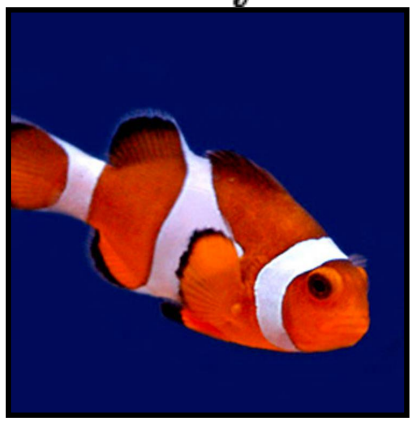
Deep learning

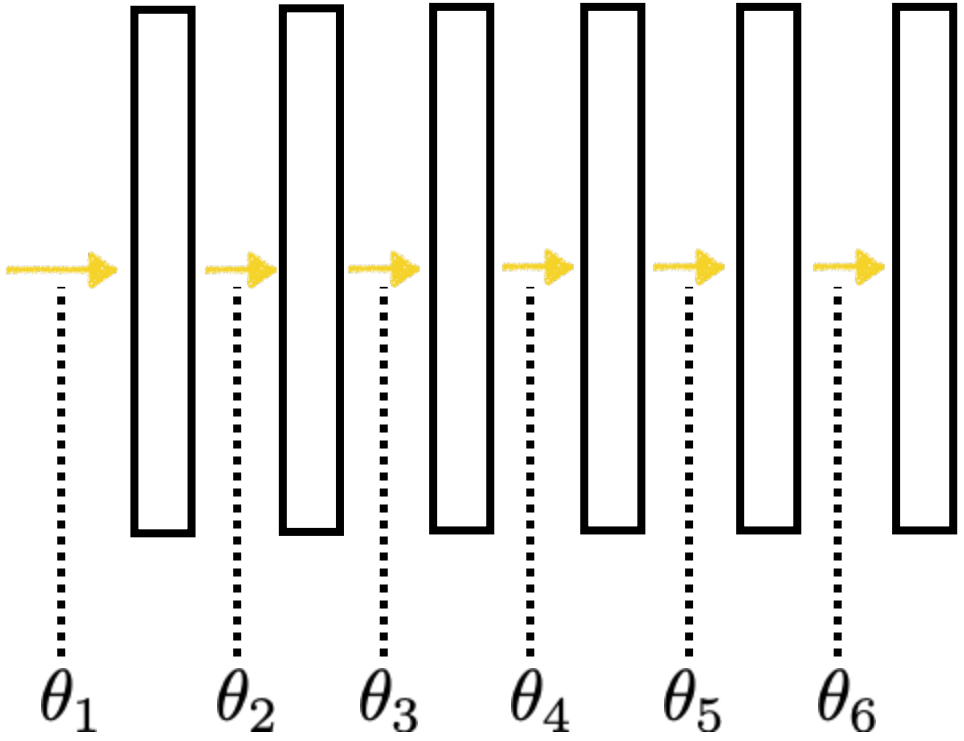
$\mathbf{y}_i$

"clown fish"

Learned

$\mathbf{x}_i$





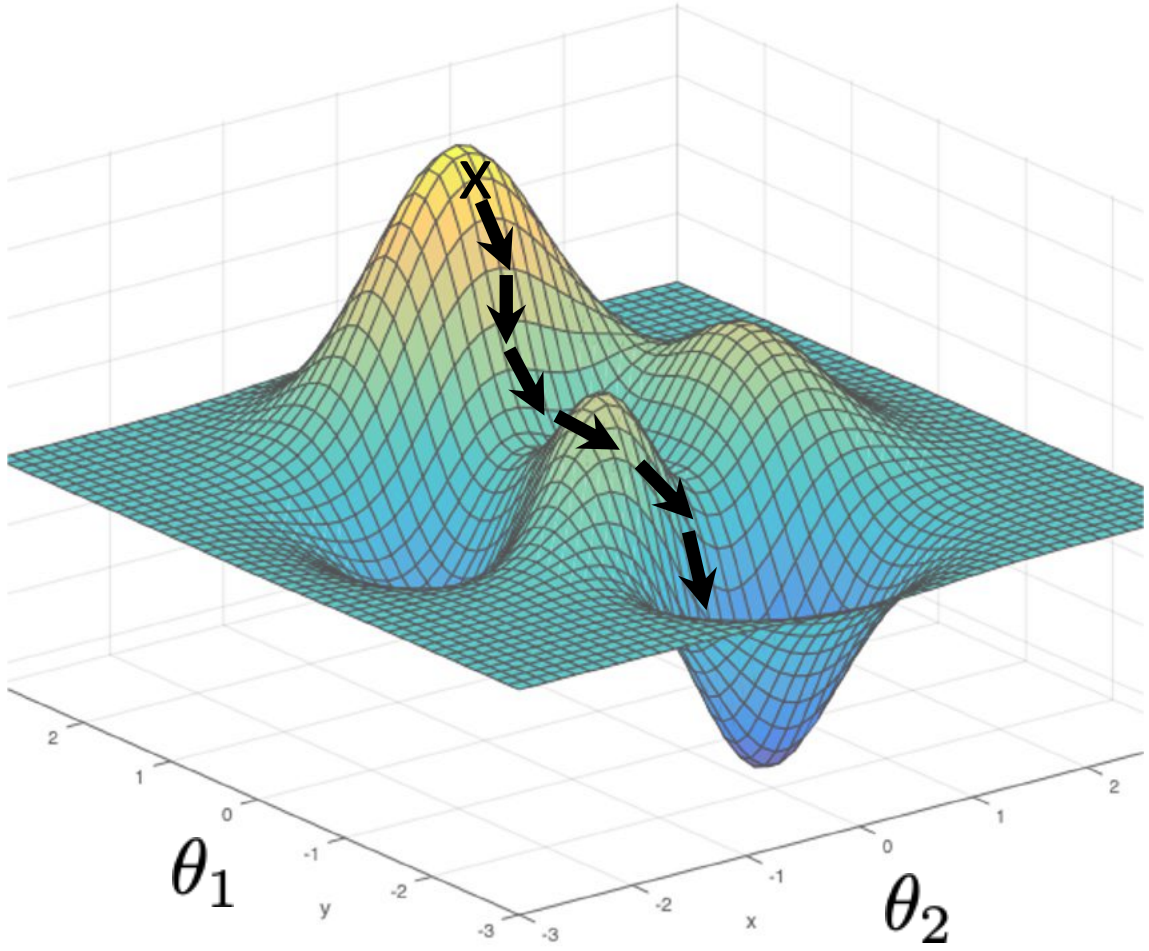
$$\theta^* = \arg\min_\theta \sum_{i=1}^{N} \mathcal{L}(f_\theta(\mathbf{x}_i), \mathbf{y}_i)$$

Gradient descent

$$\theta^* = \arg\min_{\theta} \underbrace{\sum_{i=1}^{N} \mathcal{L}(f_\theta(\mathbf{x}_i), \mathbf{y}_i)}_{J(\theta)}$$

Gradient descent

$J(\theta)$



$$\theta^* = \arg\min_{\theta} J(\theta)$$

Gradient descent

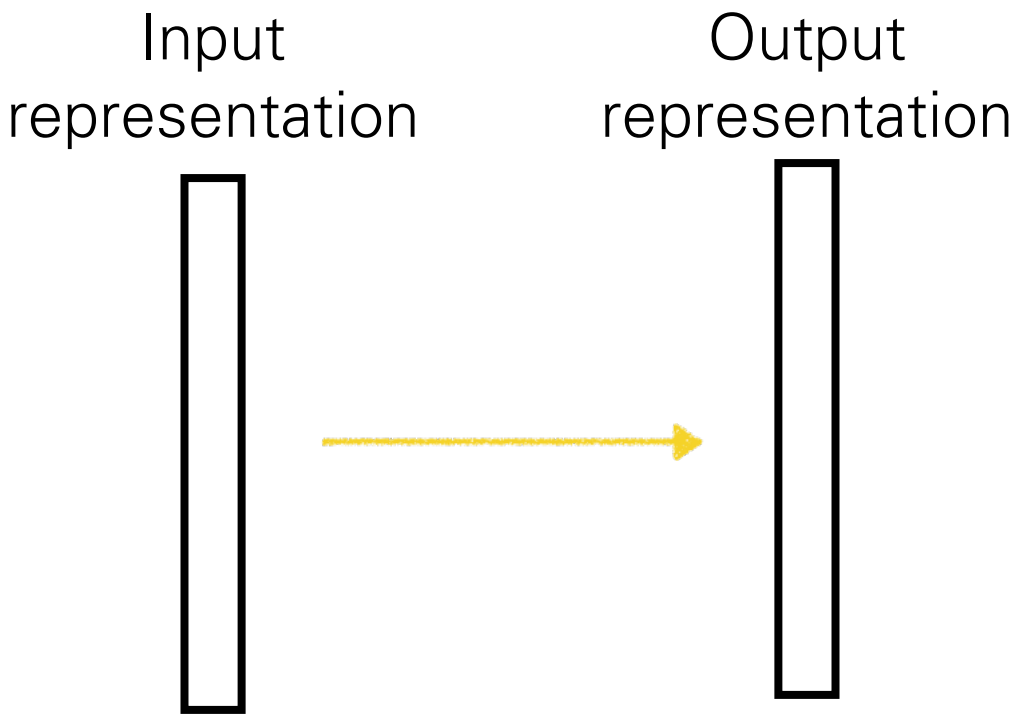
$$\theta^* = \arg\min_{\theta} \underbrace{\sum_{i=1}^{N} \mathcal{L}(f_\theta(\mathbf{x}_i), \mathbf{y}_i)}_{J(\theta)}$$

One iteration of gradient descent:

$$\theta^{t+1} = \theta^t - \eta_t \left. \frac{\partial J(\theta)}{\partial \theta} \right|_{\theta=\theta^t}$$

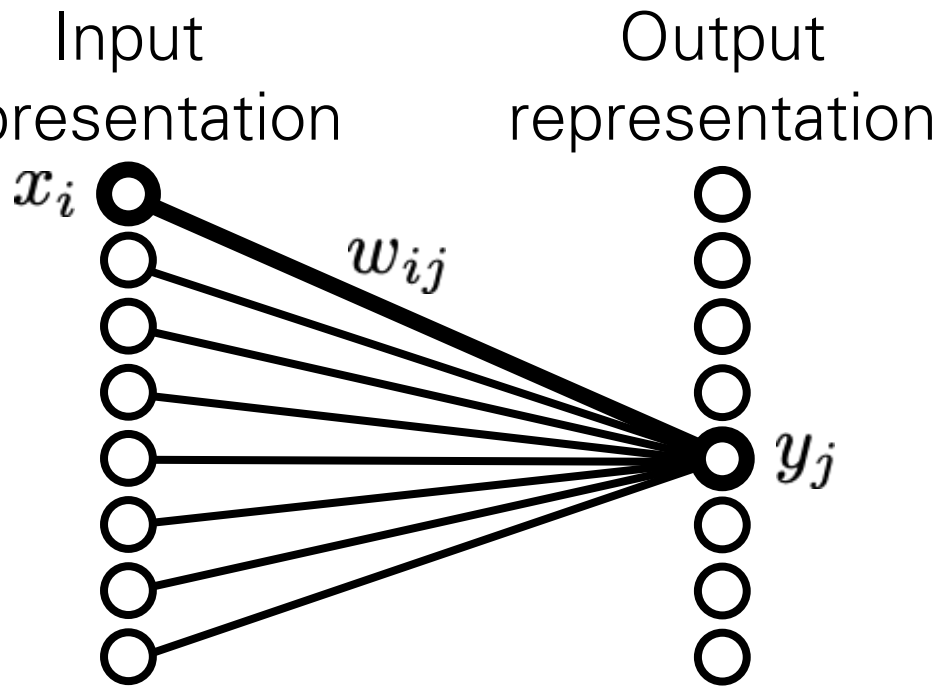
learning rate

Computation in a neural net



Computation in a neural net

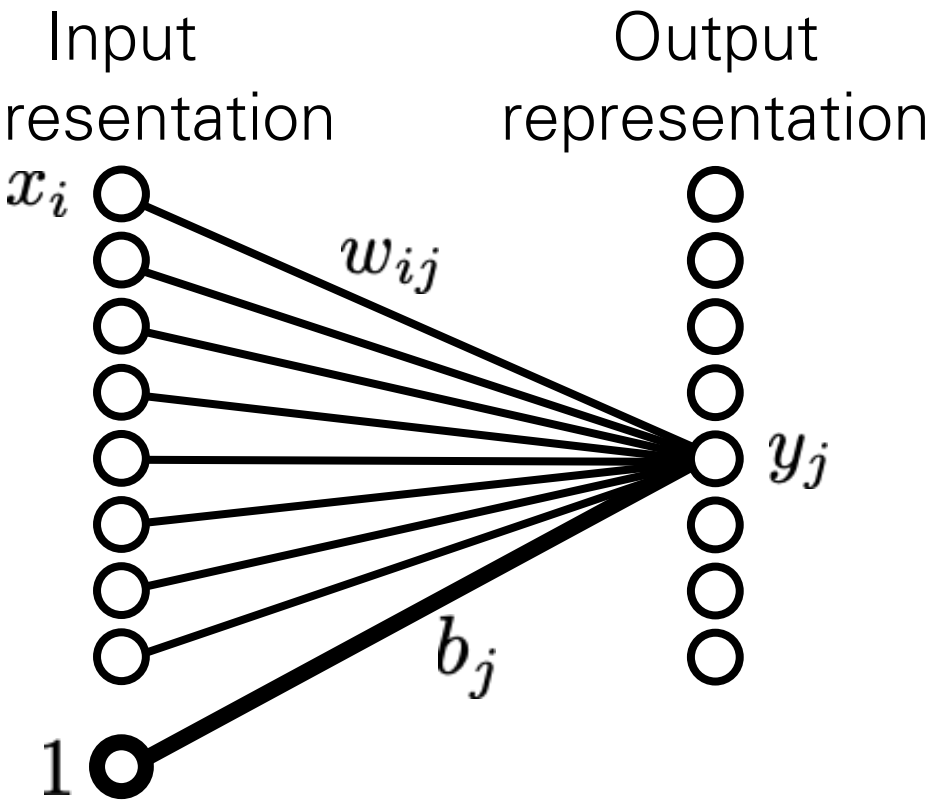
Linear layer



$$y_j = \sum_i w_{ij} x_i$$

Computation in a neural net

Linear layer



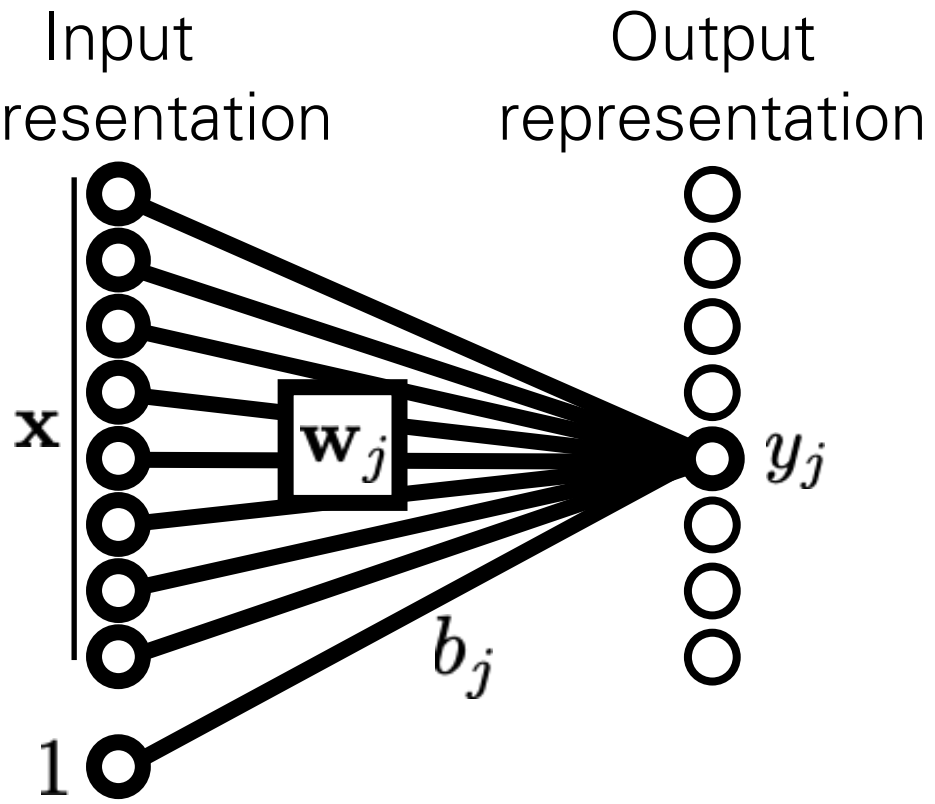
$$y_j = \sum_i w_{ij} x_i + b_j$$

weights

bias

Computation in a neural net

Linear layer



$$y_j = \mathbf{x}^T \mathbf{w}_j + b_j$$

weights

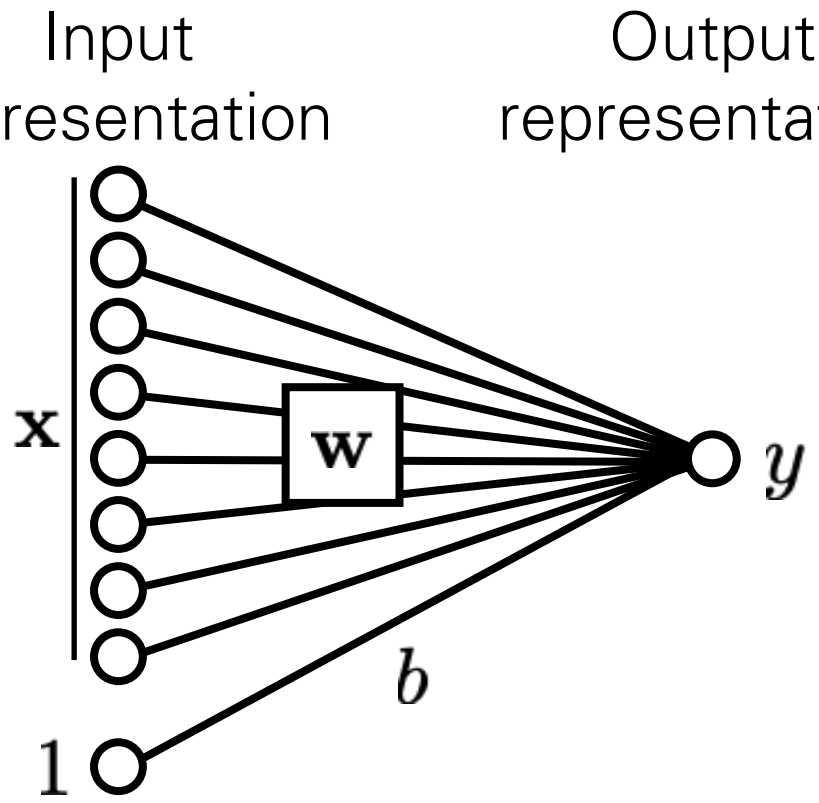
bias

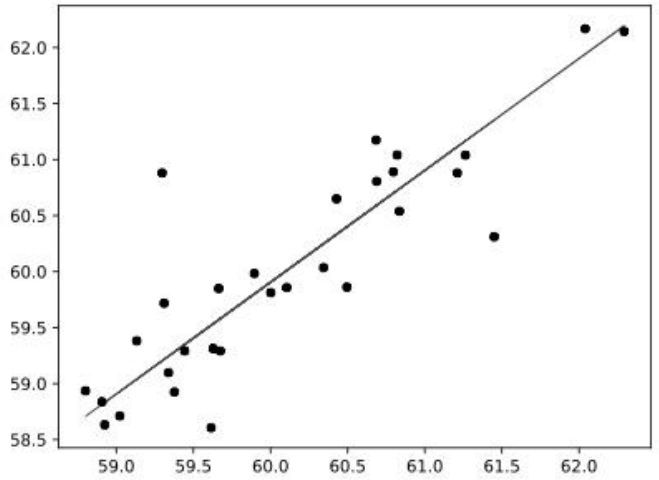
$$\theta = \{\mathbf{W}, \mathbf{b}\}$$

parameters of the model

Example: linear regression with a neural net

Linear layer

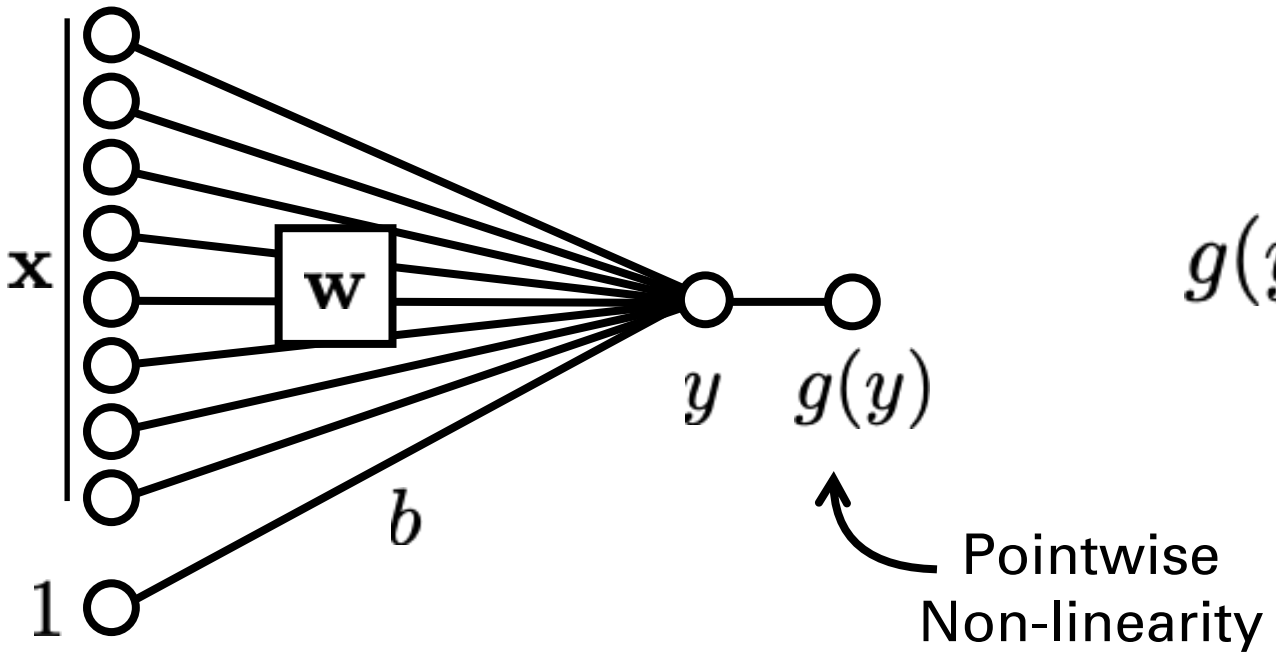




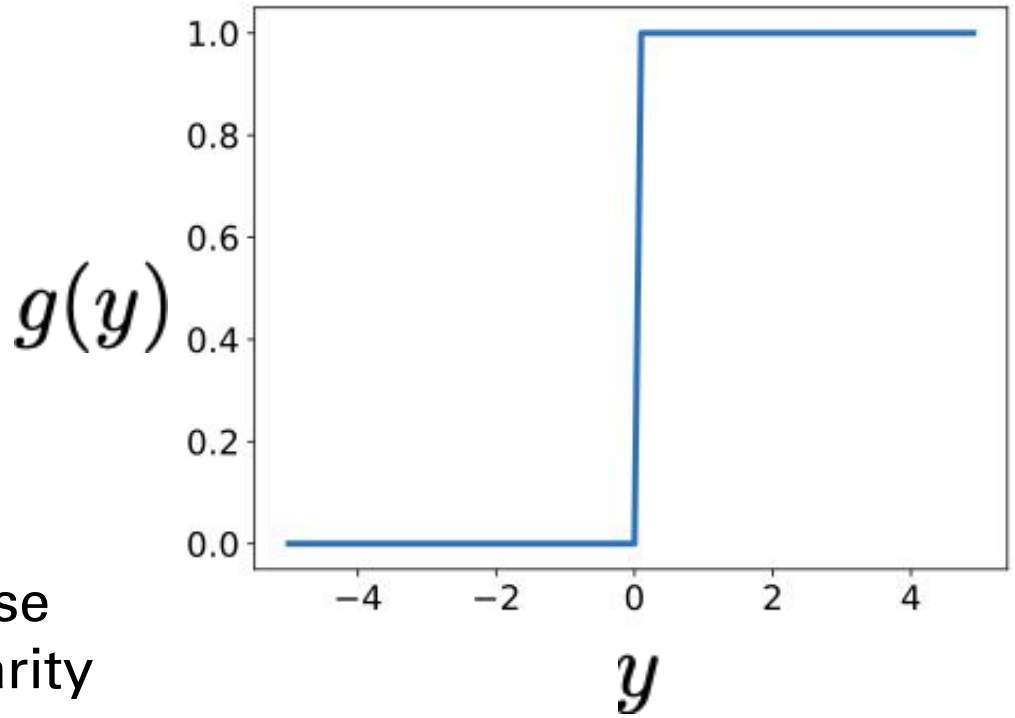
$$f_{\mathbf{w},b}(\mathbf{x}) = \mathbf{x}^T\mathbf{w} + b$$

Computation in a neural net

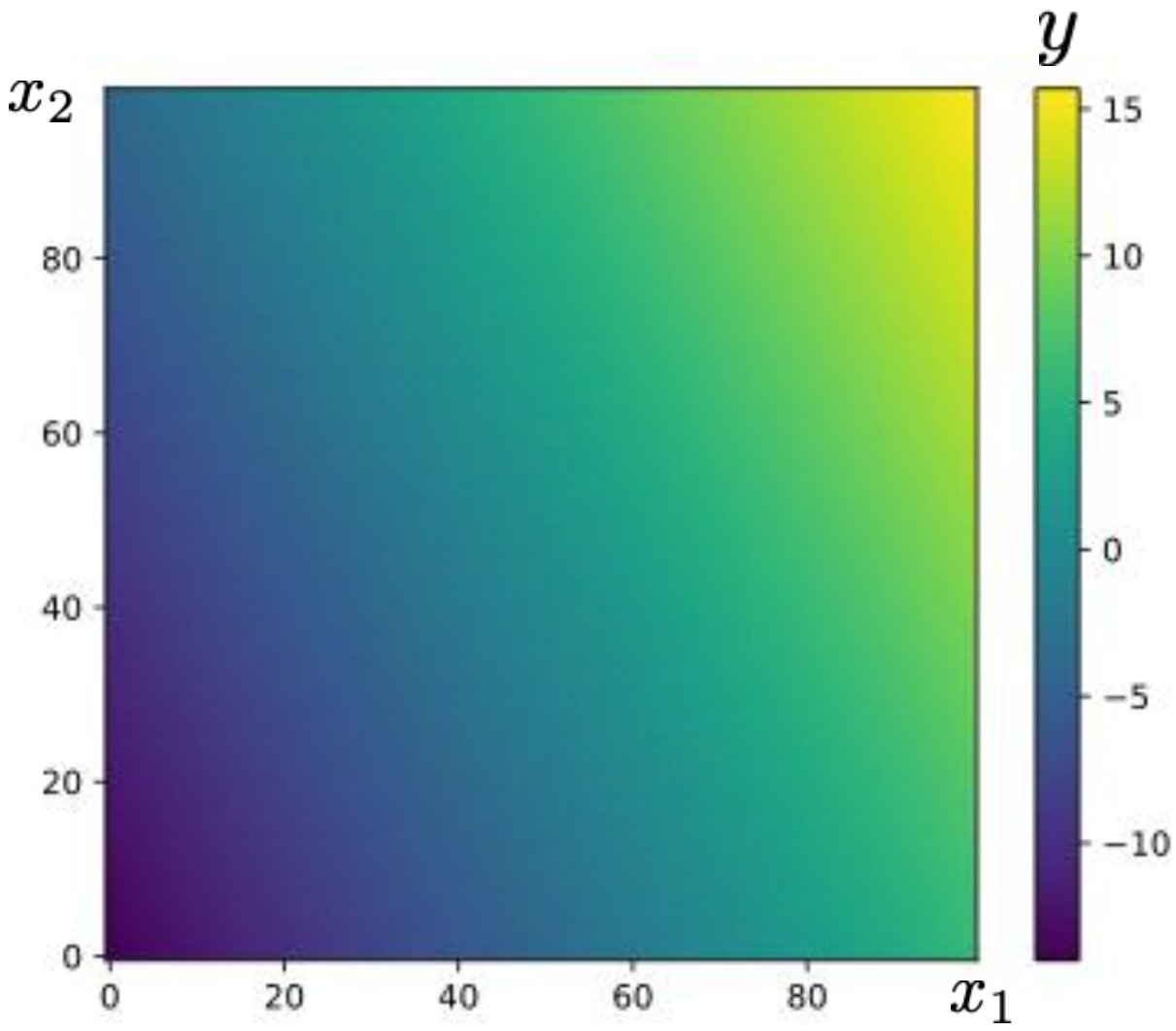
"Perceptron"



$$g(y) = \begin{cases} 1, & \text{if} \quad y > 0 \\ 0, & \text{otherwise} \end{cases}$$

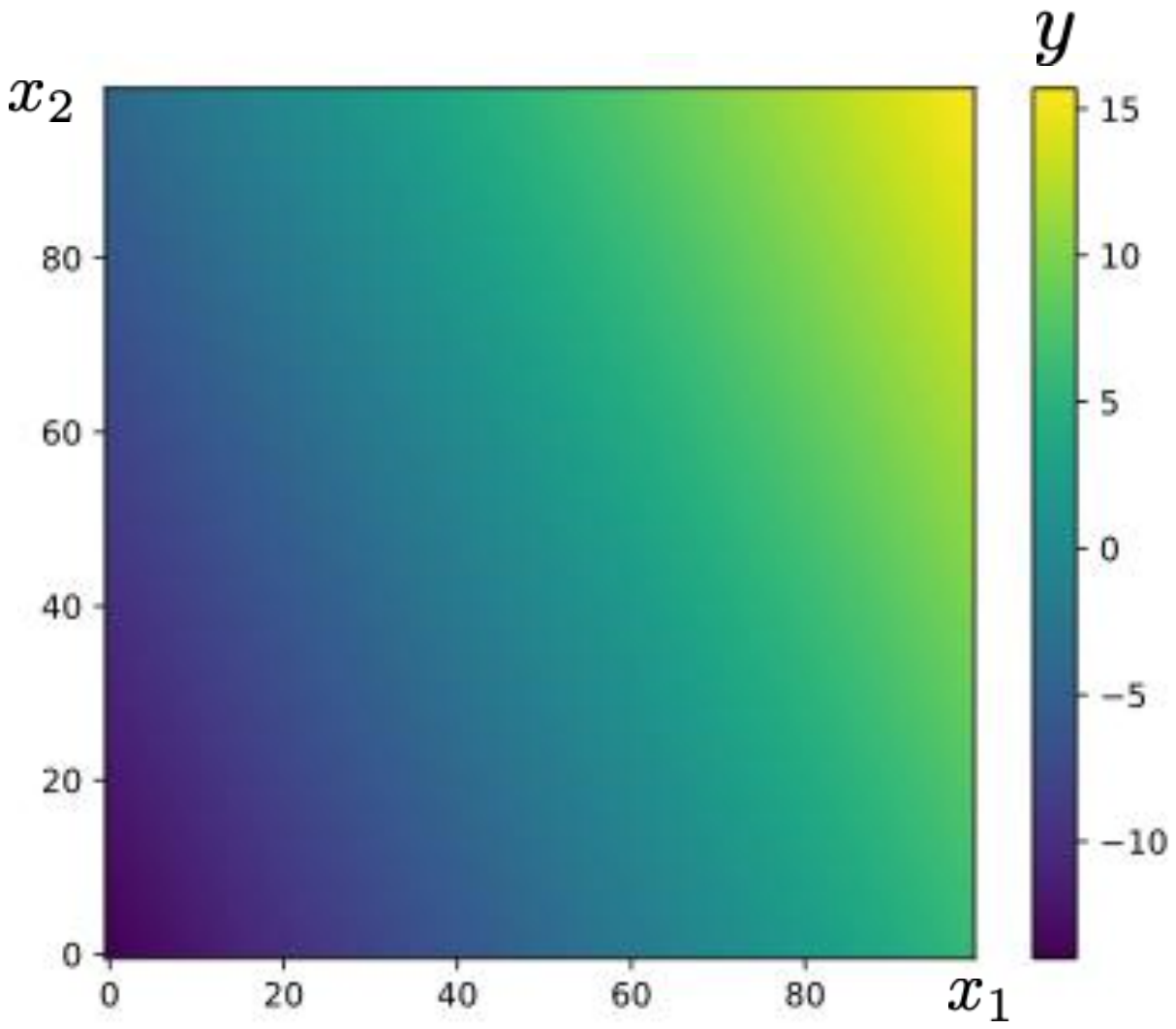


Example: linear classification with a perceptron



$$y = \mathbf{x}^T \mathbf{w} + b$$

Example: linear classification with a perceptron



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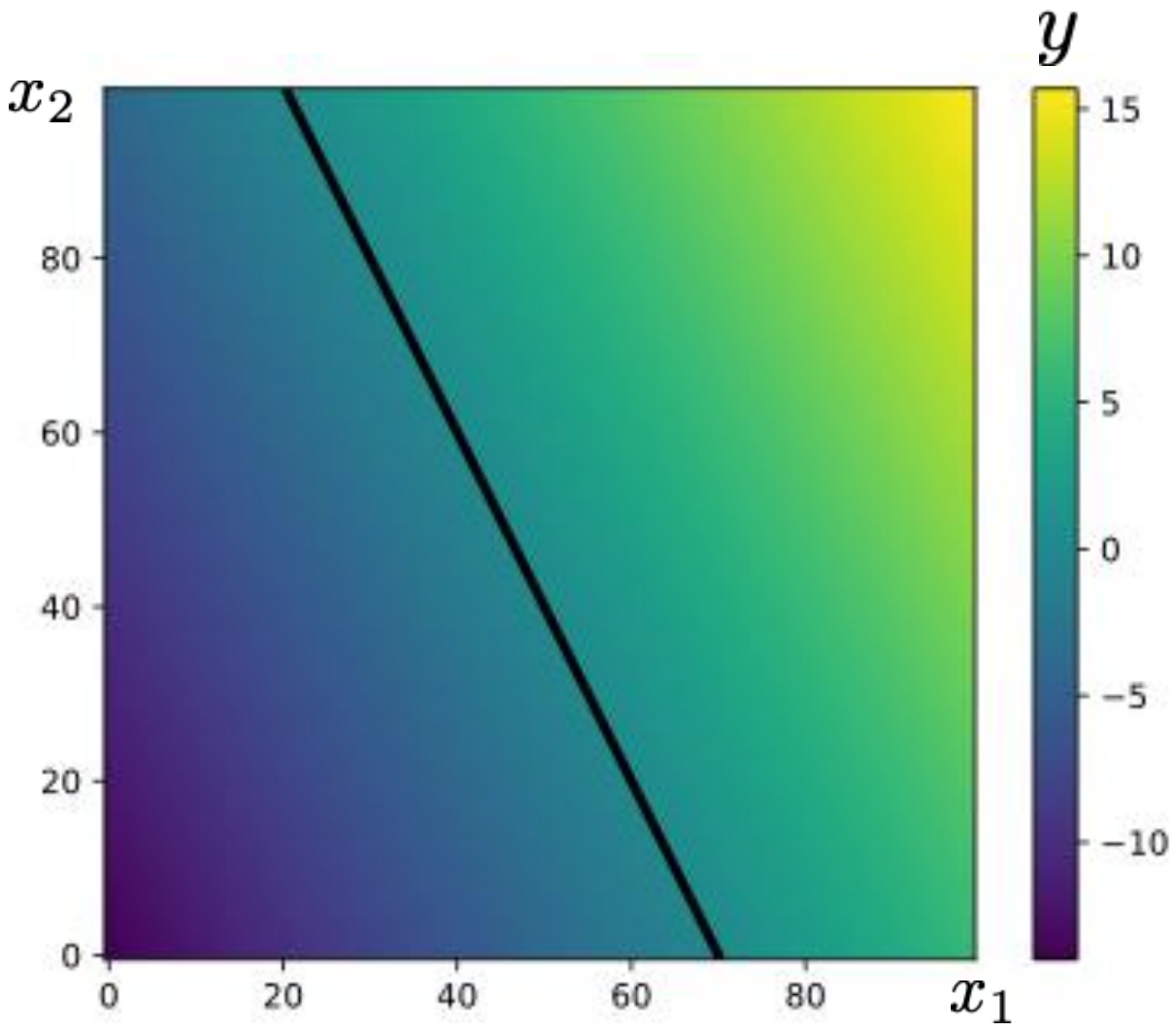
Example: linear classification with a perceptron



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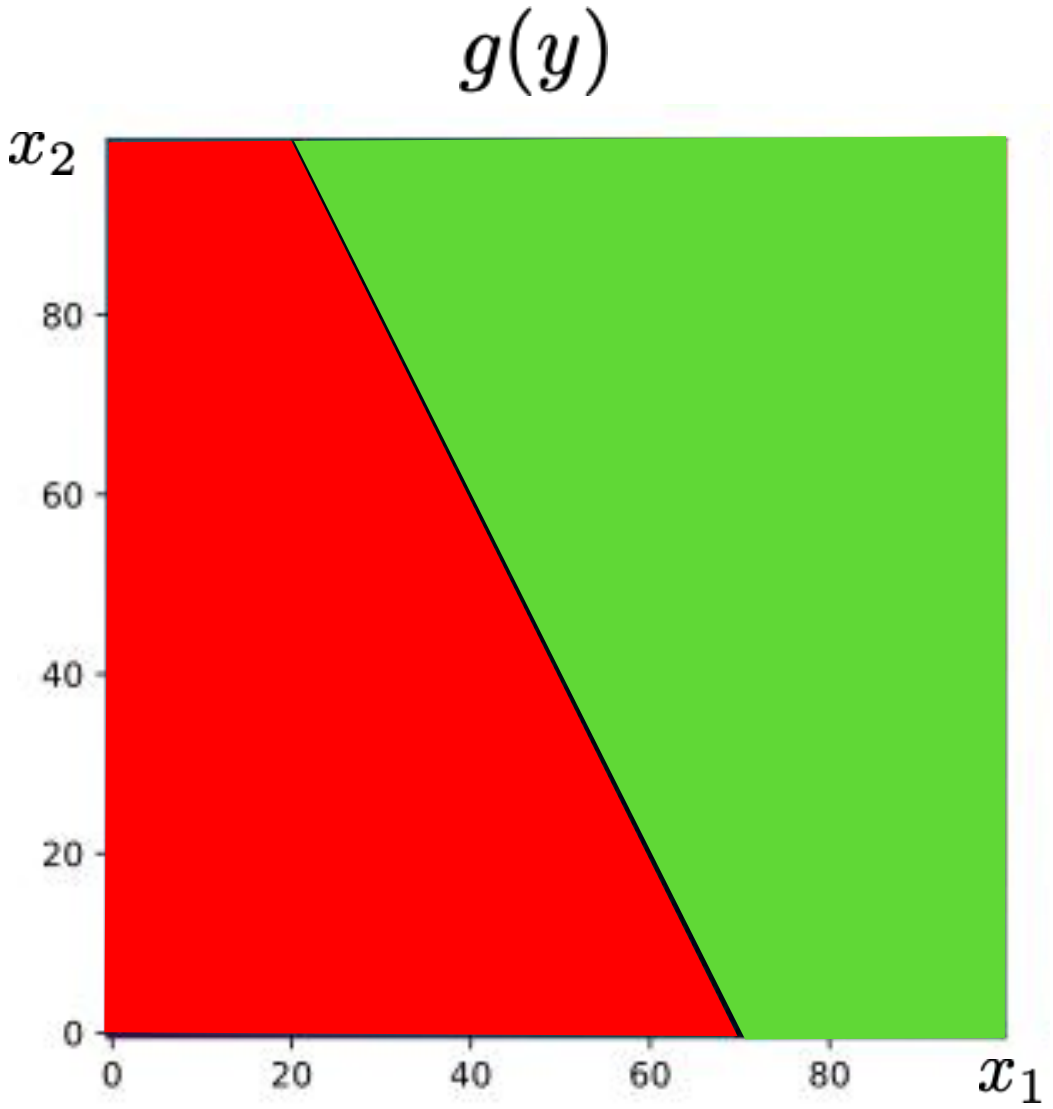
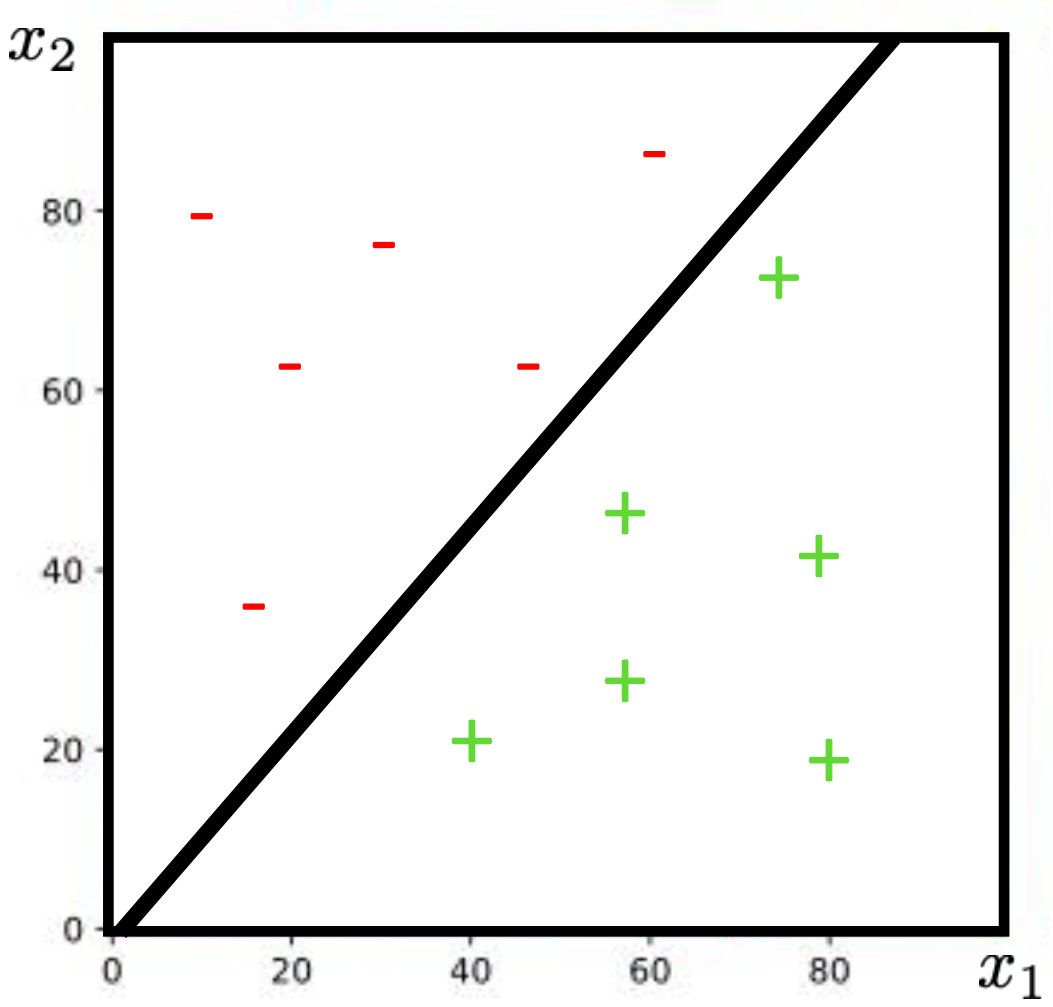
Example: linear classification with a perceptron



$$y = \mathbf{x}^T \mathbf{w} + b$$

$$g(y) = \begin{cases} 1, & \text{if} \quad y > 0 \\ 0, & \text{otherwise} \end{cases}$$

Example: linear classification with a perceptron

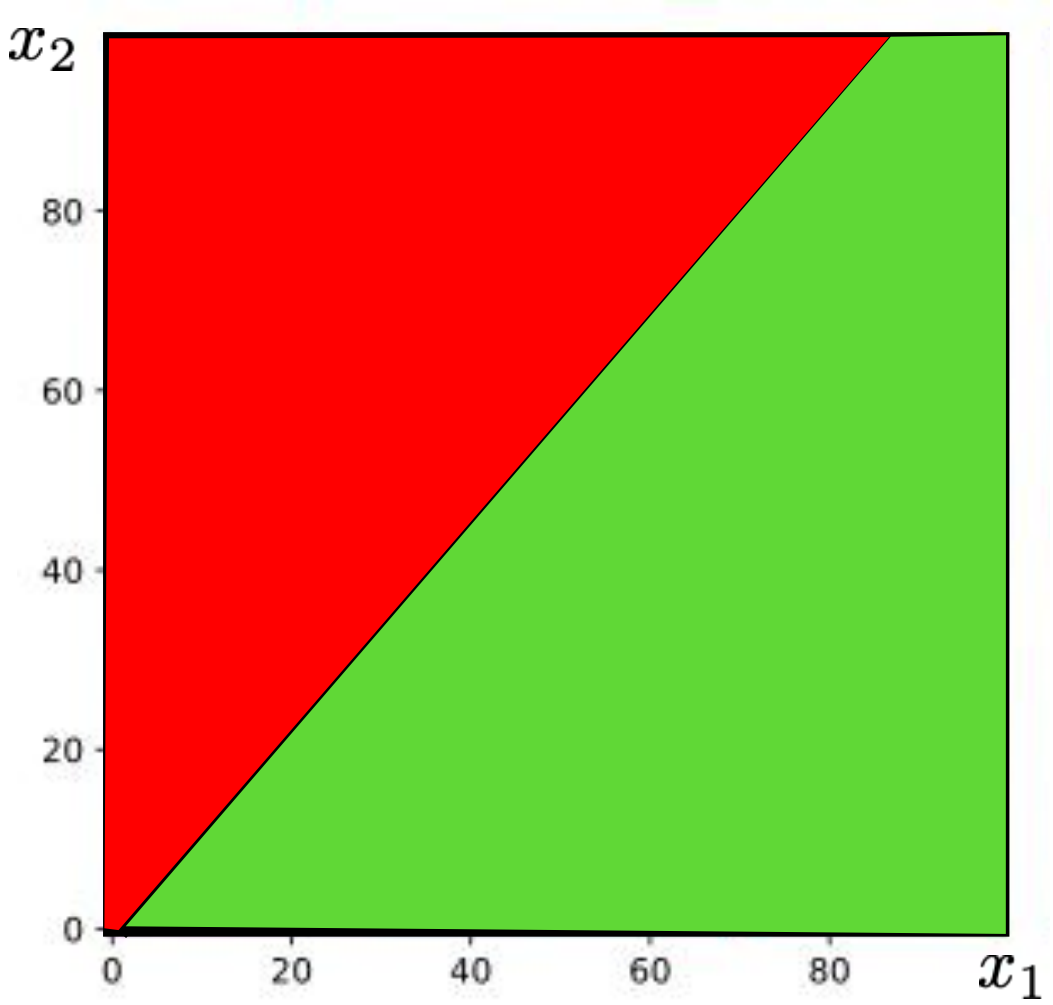


$$\hat{y} = \mathbf{x}^T \mathbf{w} + b$$

$$g(\hat{y}) = \begin{cases} 1, & \text{if} \quad \hat{y} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\mathbf{w}^*, b^* = \arg\min_{\mathbf{w}, b} \mathcal{L}(g(\hat{y}), y_i)$$

Example: linear classification with a perceptron



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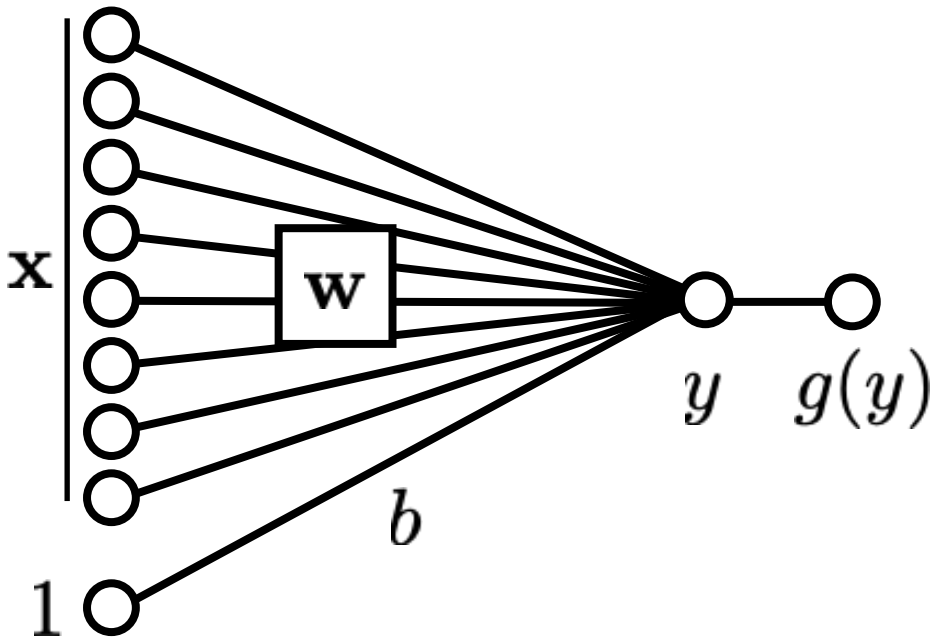
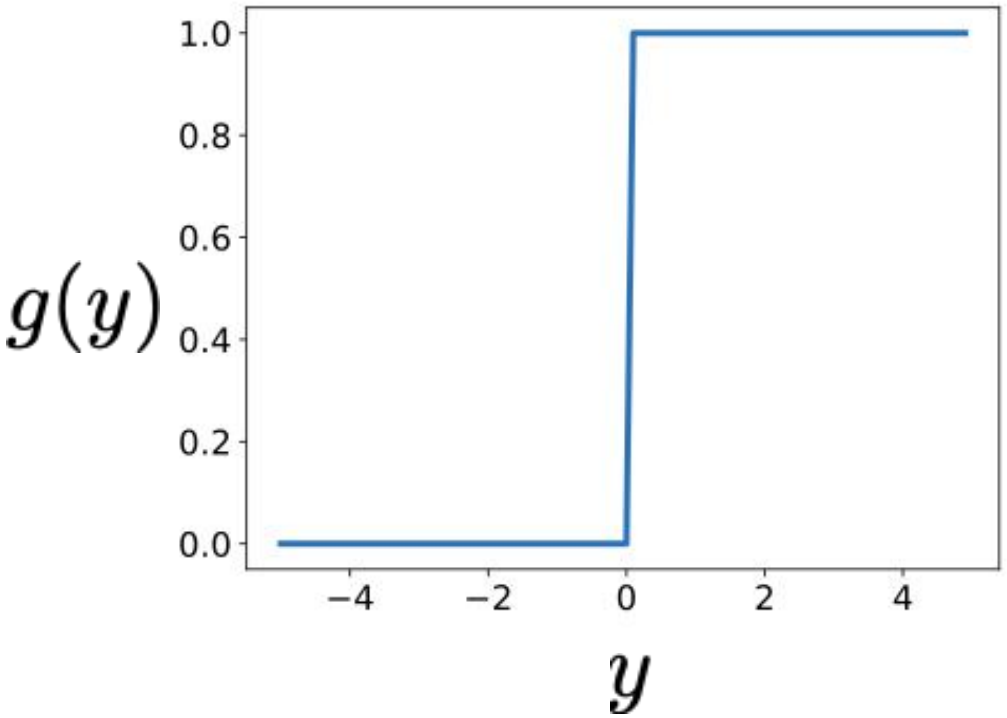
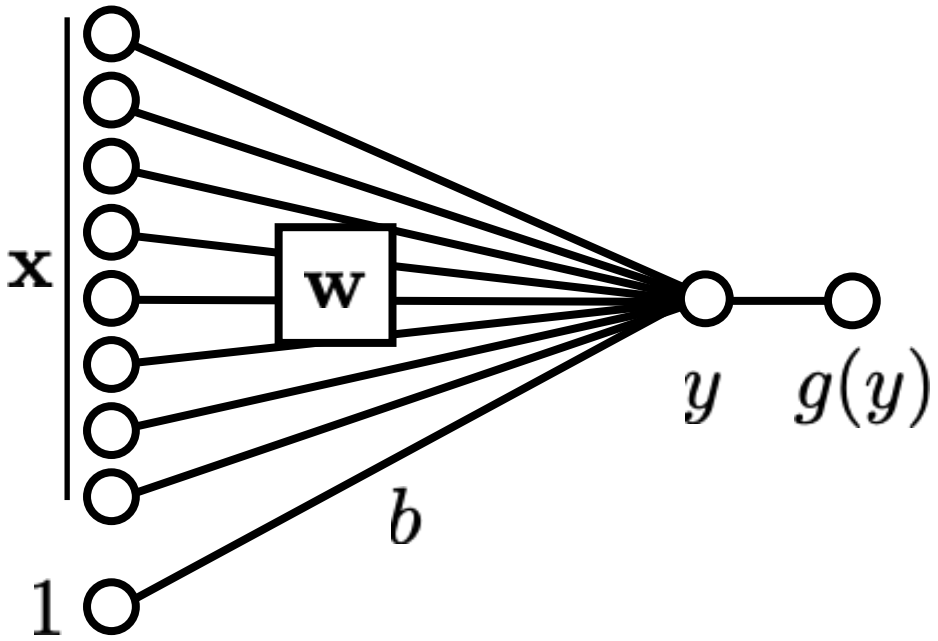
Computation in a neural net



$$g(y) = \begin{cases} 1, & \text{if} \quad y > 0 \\ 0, & \text{otherwise} \end{cases}$$

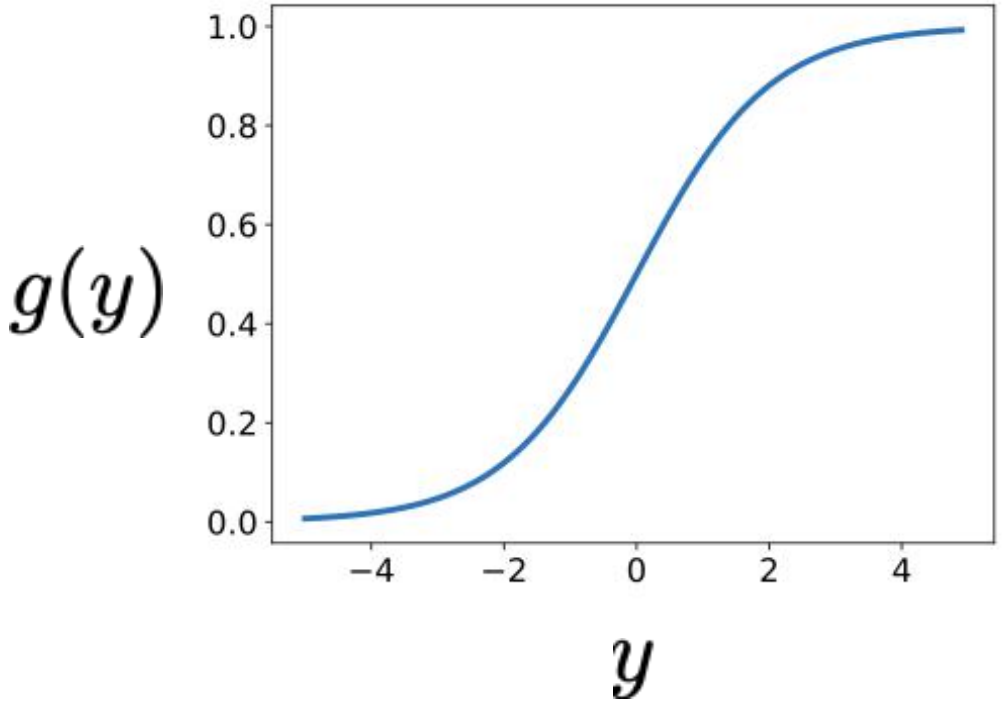


Computation in a neural net – nonlinearity



Sigmoid

$$g(y) = \frac{1}{1 + e^{-y}}$$

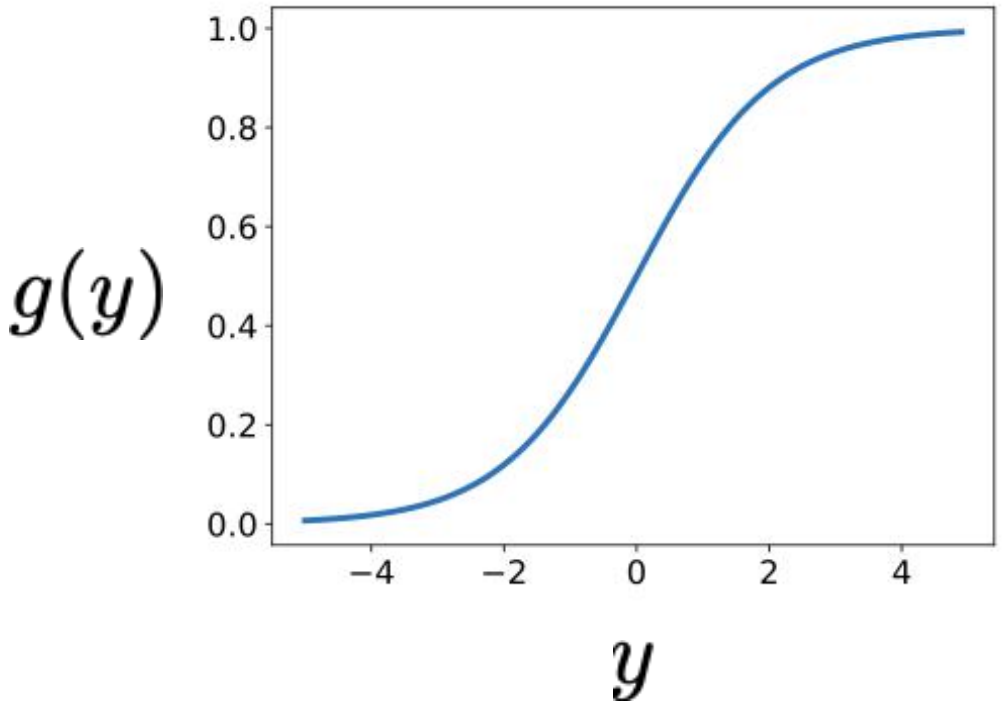


Computation in a neural net – nonlinearity

- Interpretation as firing rate of neuron
- Bounded between [0,1]
- Saturation for large +/- inputs
- Gradients go to zero
- Outputs centered at 0.5
  (poor conditioning)
- Not used in practice

Sigmoid

$$g(y) = \frac{1}{1 + e^{-y}}$$



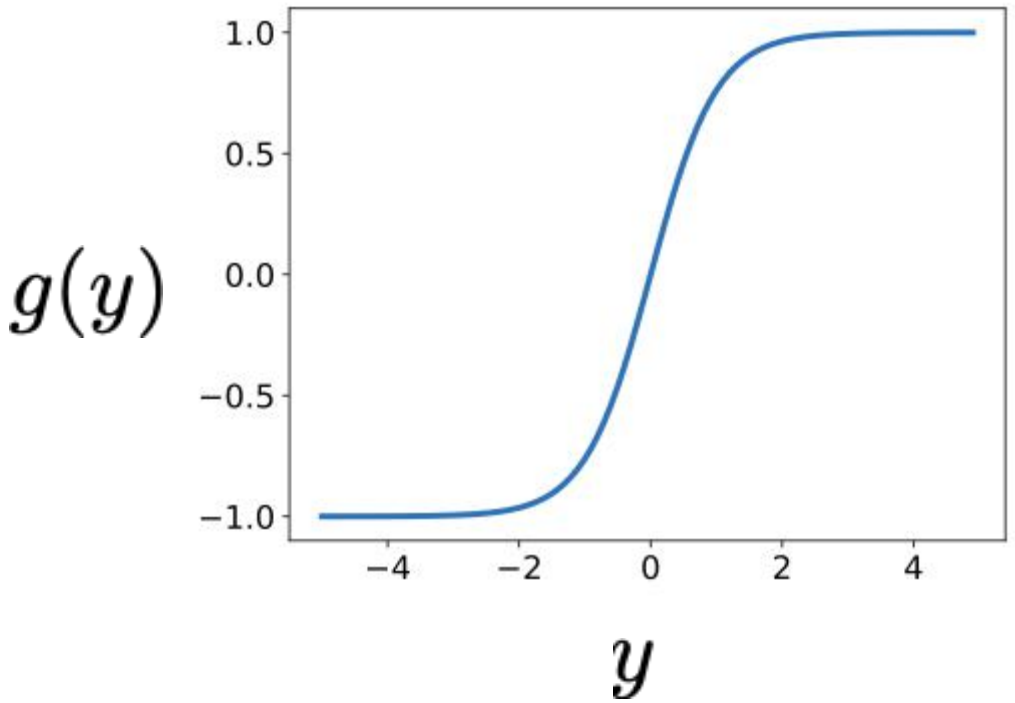
Computation in a neural net – nonlinearity

- Bounded between [-1,+1]
- Saturation for large +/- inputs
- Gradients go to zero
- Outputs centered at 0
- Preferable to sigmoid

tanh(x) = 2 sigmoid(2x) −1

Tanh

$$g(y) = \frac{e^{y} - e^{-y}}{e^{y} + e^{-y}}$$

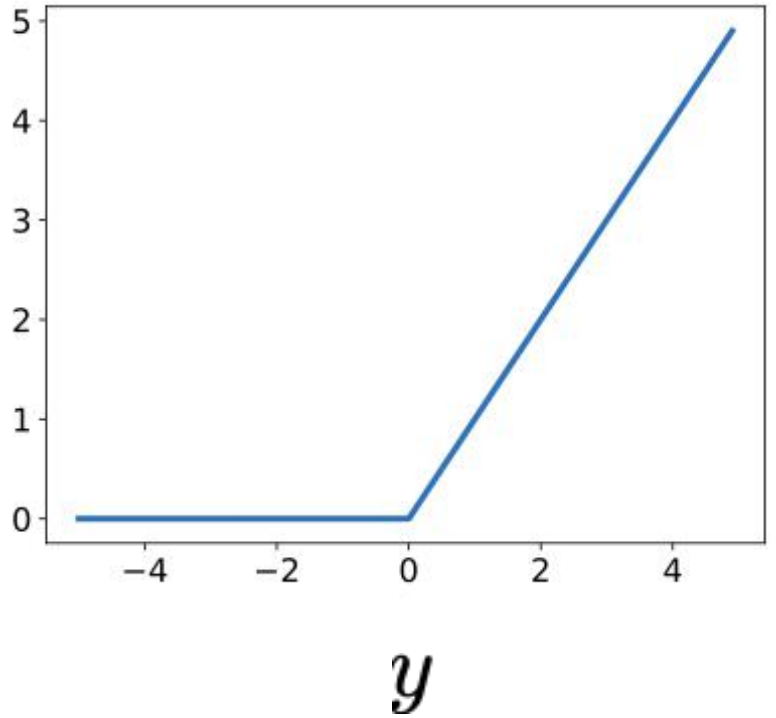


Computation in a neural net – nonlinearity

- Unbounded output (on positive side)
- Efficient to implement: $\frac{\partial g}{\partial y} = \begin{cases} 0, & \text{if} \quad y < 0 \\ 1, & \text{if} \quad y \geq 0 \end{cases}$
- Also seems to help convergence
  (see 6x speedup vs tanh in [Krizhevsky et al.])
- Drawback: if strongly in negative region, unit is dead forever (no gradient).
- Default choice: widely used in current models.

Rectified linear unit (ReLU)

$$g(y) = \max(0, y)$$

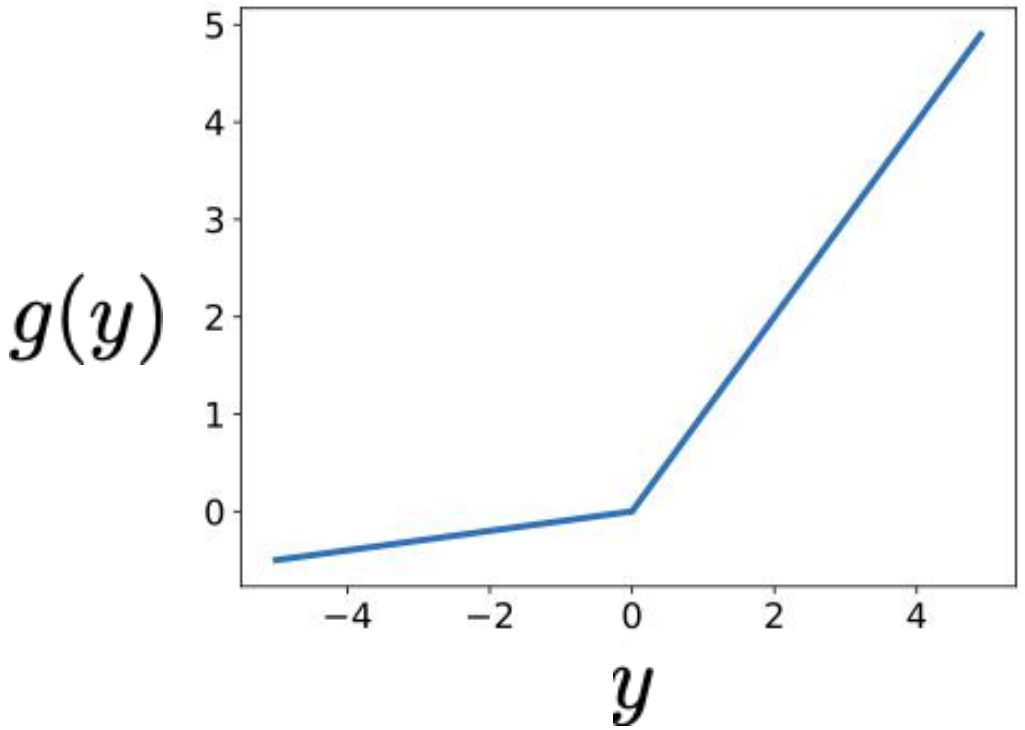


Computation in a neural net – nonlinearity

- where α is small (e.g. 0.02)
- Efficient to implement: $\frac{\partial g}{\partial y} = \begin{cases} -a, & \text{if} \quad y < 0 \\ 1, & \text{if} \quad y \geq 0 \end{cases}$
- Also known as probabilistic ReLU (PReLU)
- Has non-zero gradients everywhere (unlike ReLU)
- α can also be learned (see Kaiming He et al. 2015).

Leaky ReLU

$$g(y) = \begin{cases} \max(0, y), & \text{if} \quad y \geq 0 \\ a \min(0, y), & \text{if} \quad y < 0 \end{cases}$$

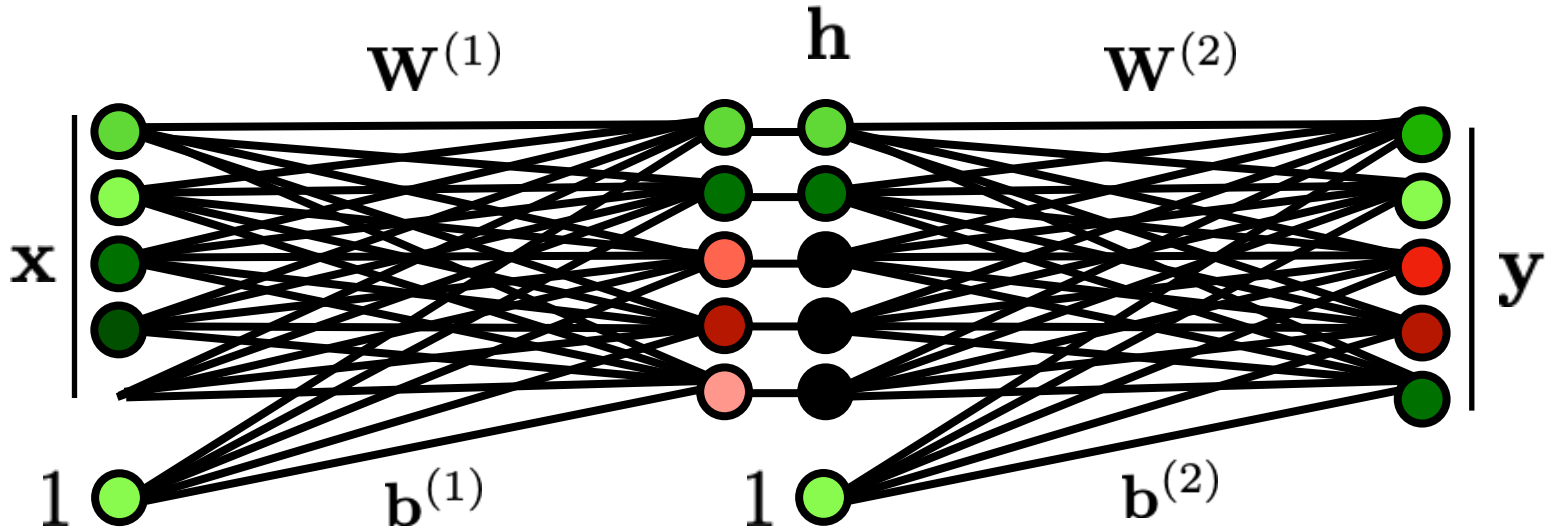


Stacking layers

Input representation

Intermediate representation

Output representation



positive

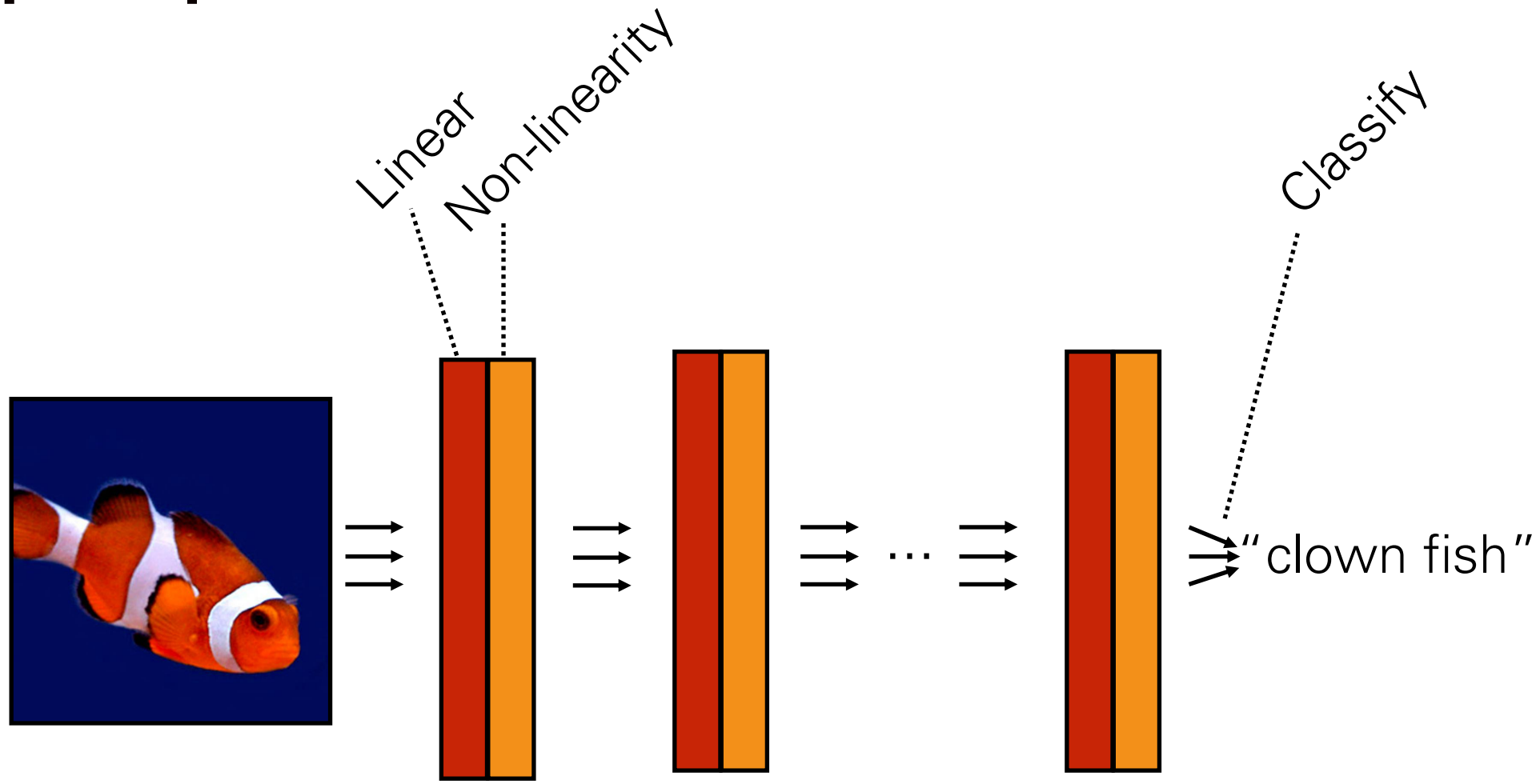
negative

$$\theta = \{\mathbf{W}^{(1)}, \ldots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \ldots, \mathbf{b}^{(L)}\}$$

Representational power

- 1 layer? Linear decision surface.
- 2+ layers? In theory, can represent any function. Assuming non-trivial non-linearity.
  - Bengio 2009, http://www.iro.umontreal.ca/~bengioy/papers/ftml.pdf
  - Bengio, Courville, Goodfellow book http://www.deeplearningbook.org/contents/mlp.html
  - Simple proof by M. Neilsen http://neuralnetworksanddeeplearning.com/chap4.html
  - D. Mackay book http://www.inference.phy.cam.ac.uk/mackay/itprnn/ps/482.491.pdf
- But issue is efficiency: very wide two layers vs narrow deep model? In practice, more layers helps.

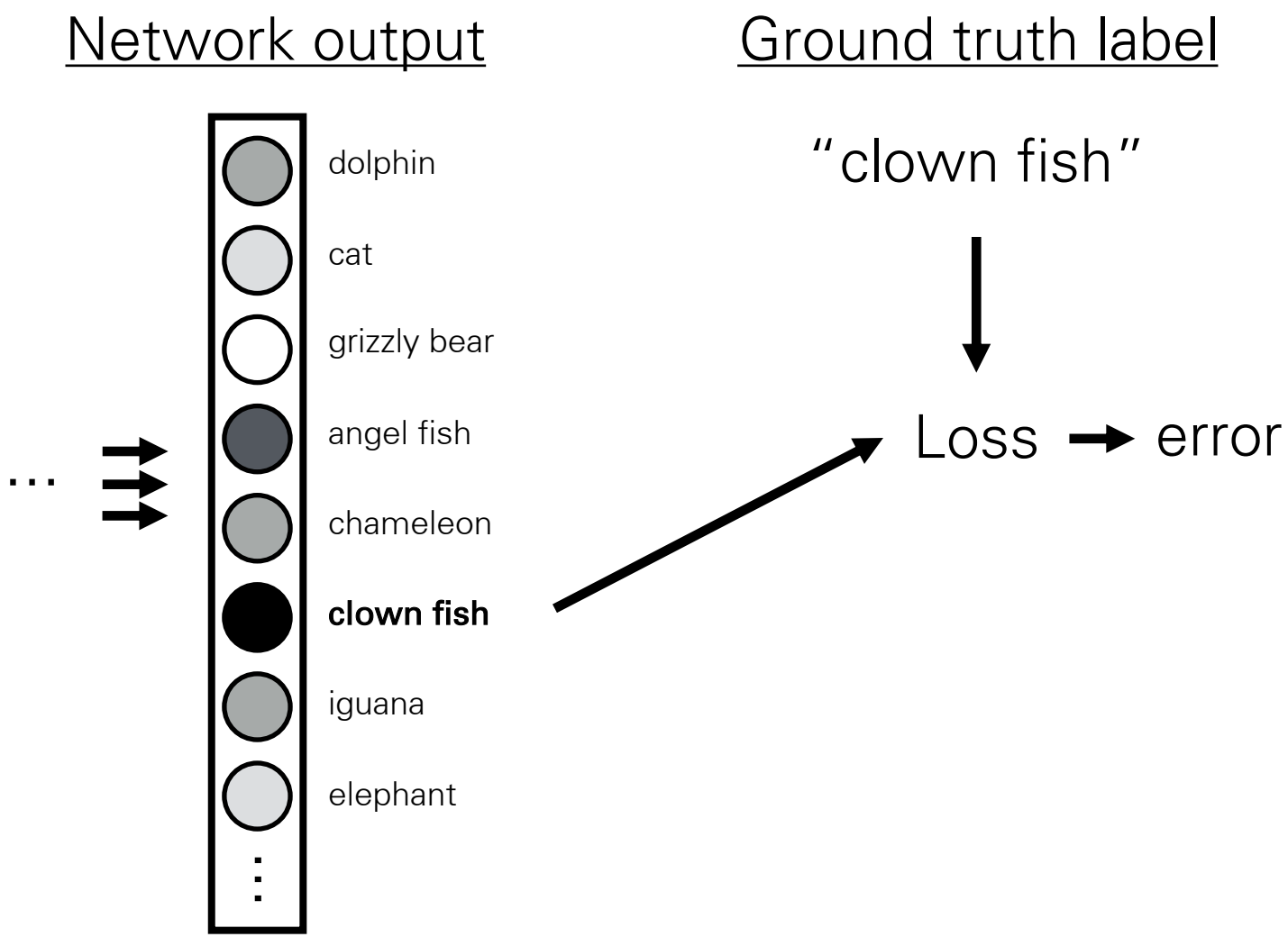
Deep supervised nets



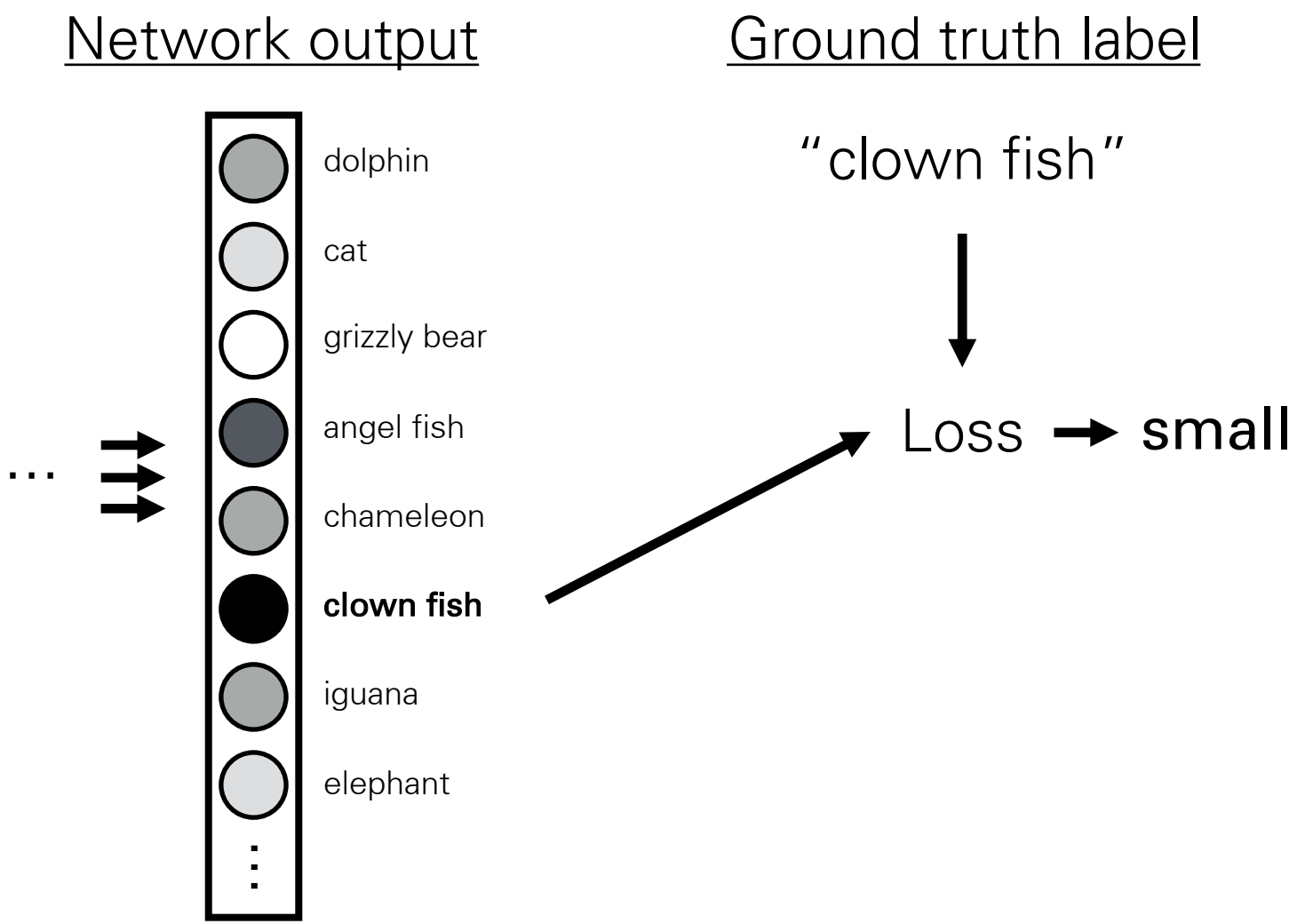
$$f(\mathbf{x}) = f_L(\ldots f_2(f_1(\mathbf{x})))$$

Classifier layer

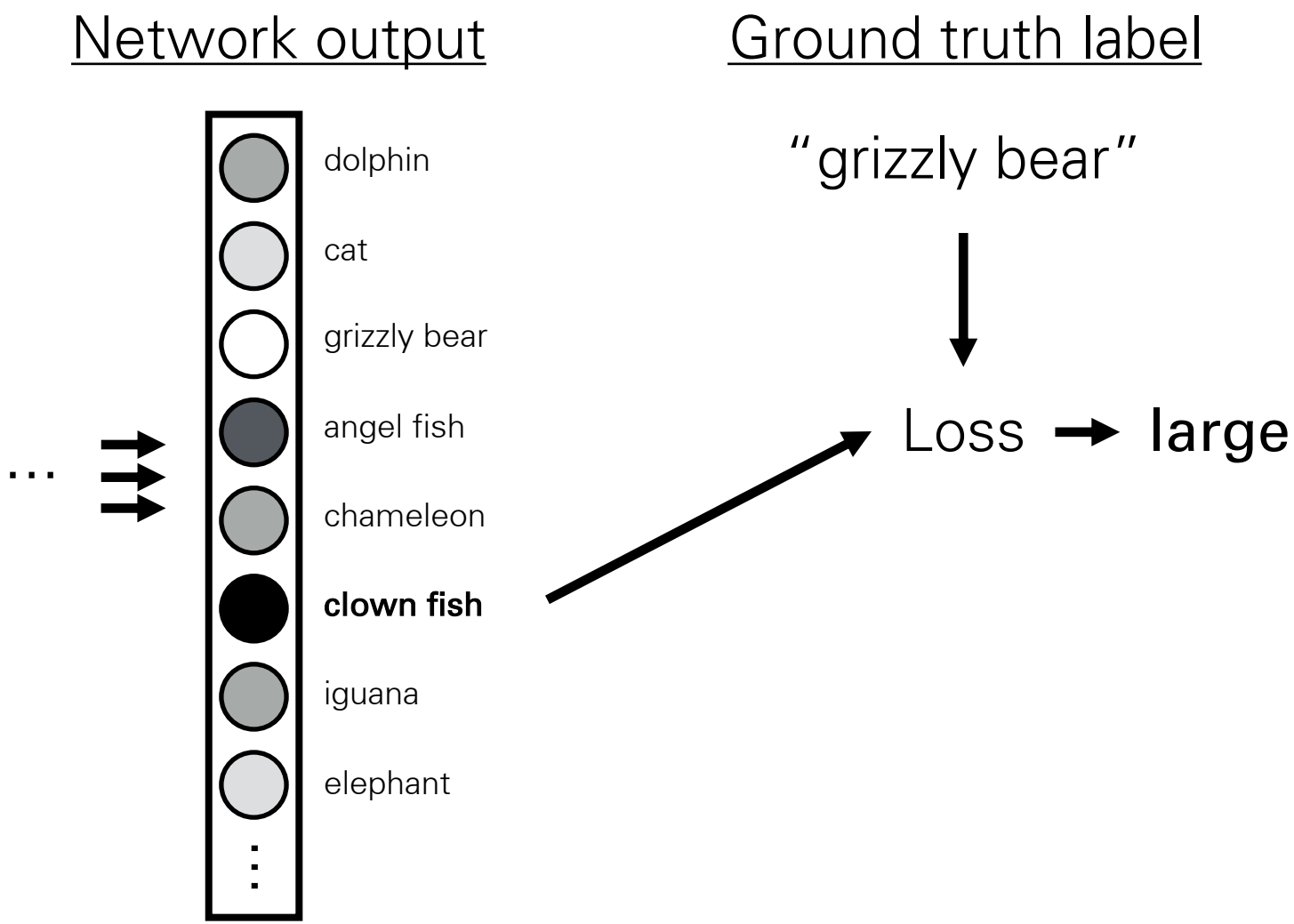
Loss function



Loss function



Loss function



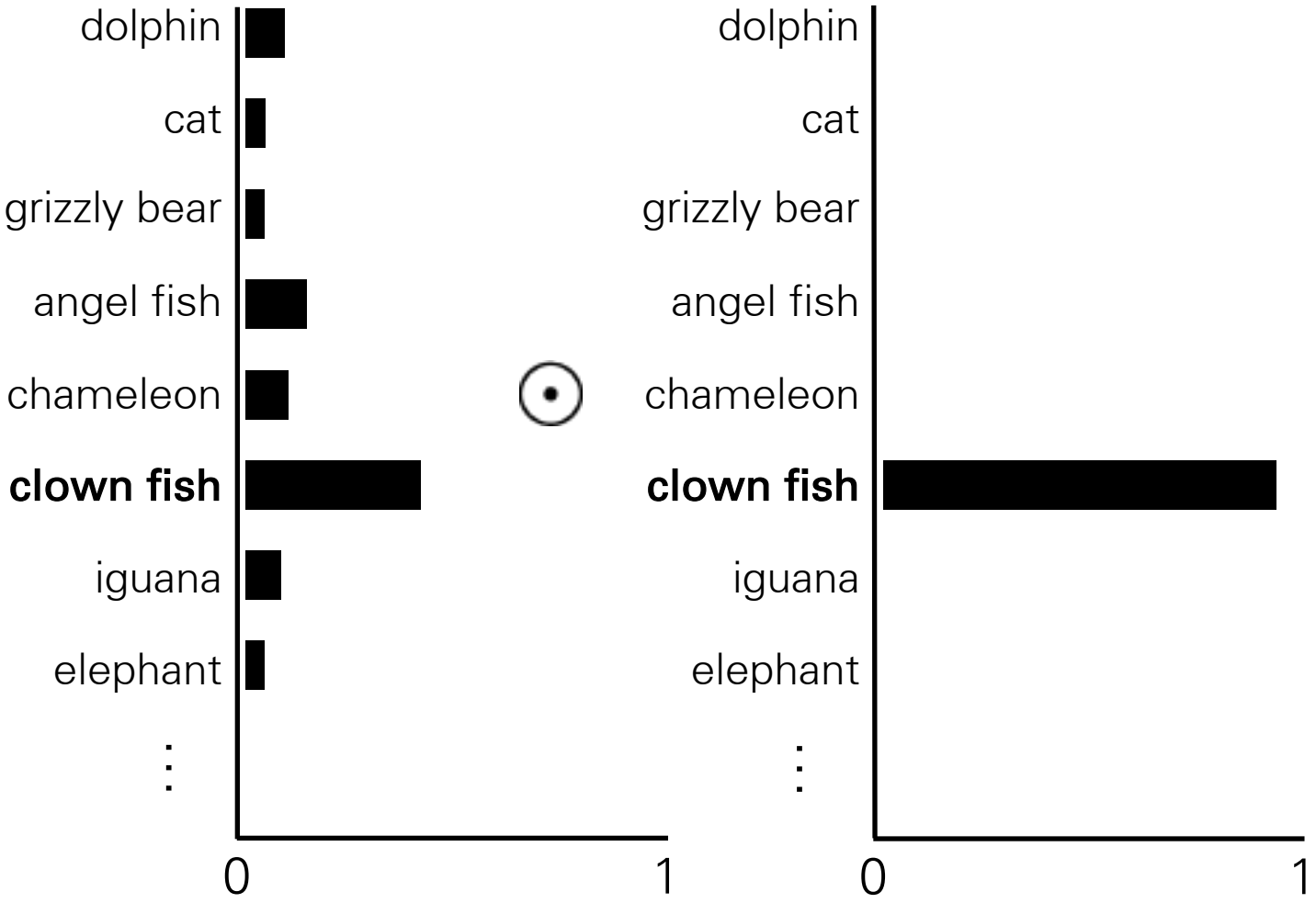
Prediction $\hat{\mathbf{y}}$

$f_\theta : X \to \mathbb{R}^K$

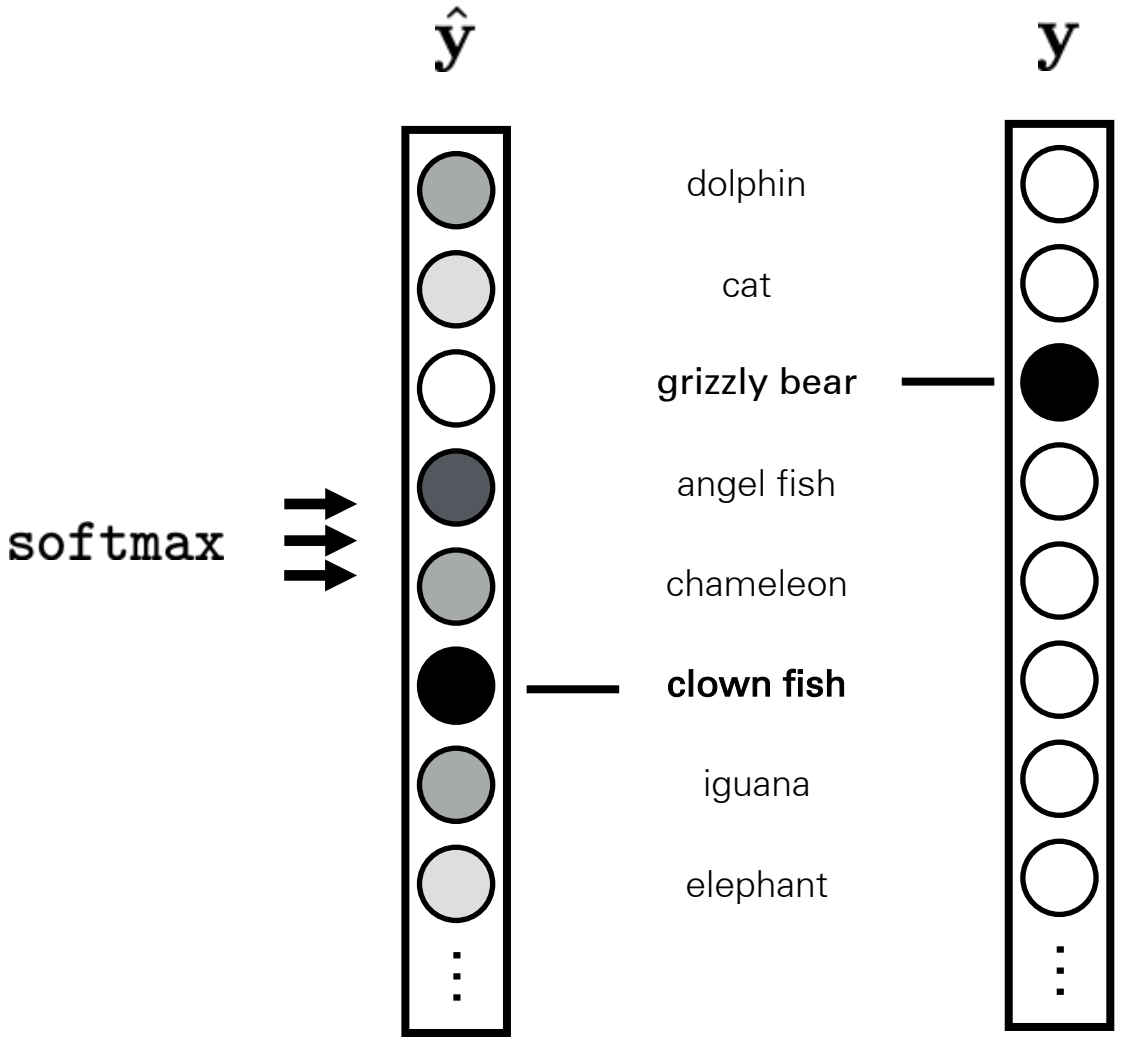
Ground truth label $\mathbf{y}$

$\mathbf{x}$

f



Network output Ground truth label



Probability of the observed data under the model

$$H(\mathbf{y}, \hat{\mathbf{y}}) = -\sum_{k=1}^{K} y_k \log \hat{y}_k$$

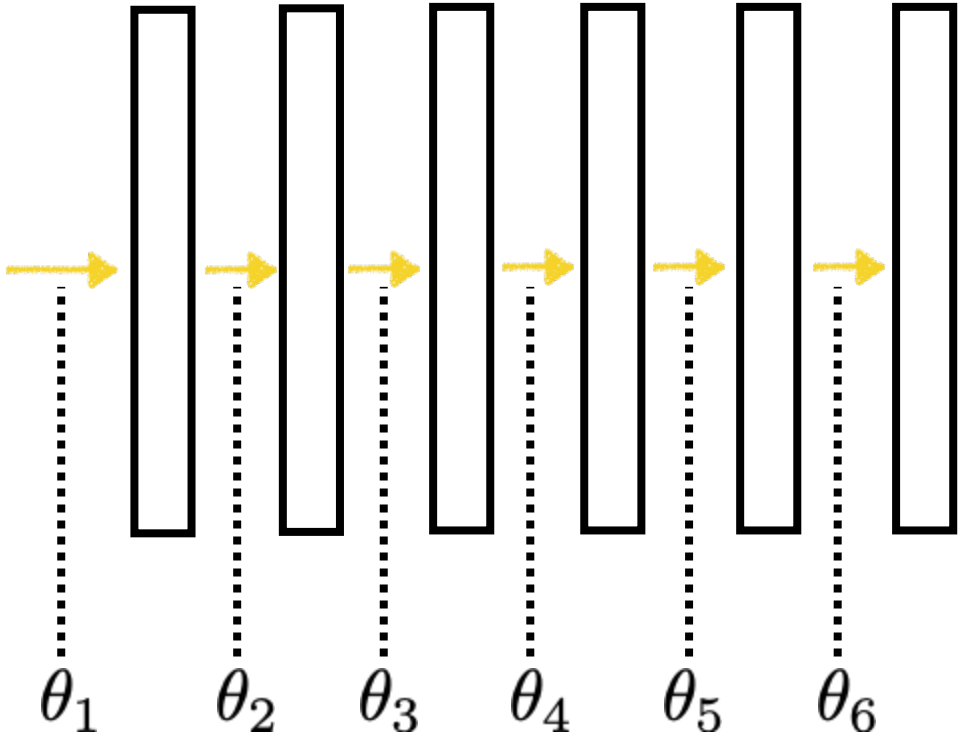
Deep learning

Learned

$\mathbf{y}_1$

"clown fish"

$\mathbf{x}_1$



$$\theta^* = \arg\min_\theta \sum_{i=1}^{N} \mathcal{L}(f_\theta(\mathbf{x}_i), \mathbf{y}_i)$$

Deep learning

$\mathbf{y}_2$

"grizzly bear"

Learned

$\mathbf{x}_2$

Loss

$\mathcal{L}(f_\theta(\mathbf{x}_2), \mathbf{y}_2)$

θ_1 θ_2 θ_3 θ_4 θ_5 θ_6

$$\theta^* = \arg\min_\theta \sum_{i=1}^{N} \mathcal{L}(f_\theta(\mathbf{x}_i), \mathbf{y}_i)$$

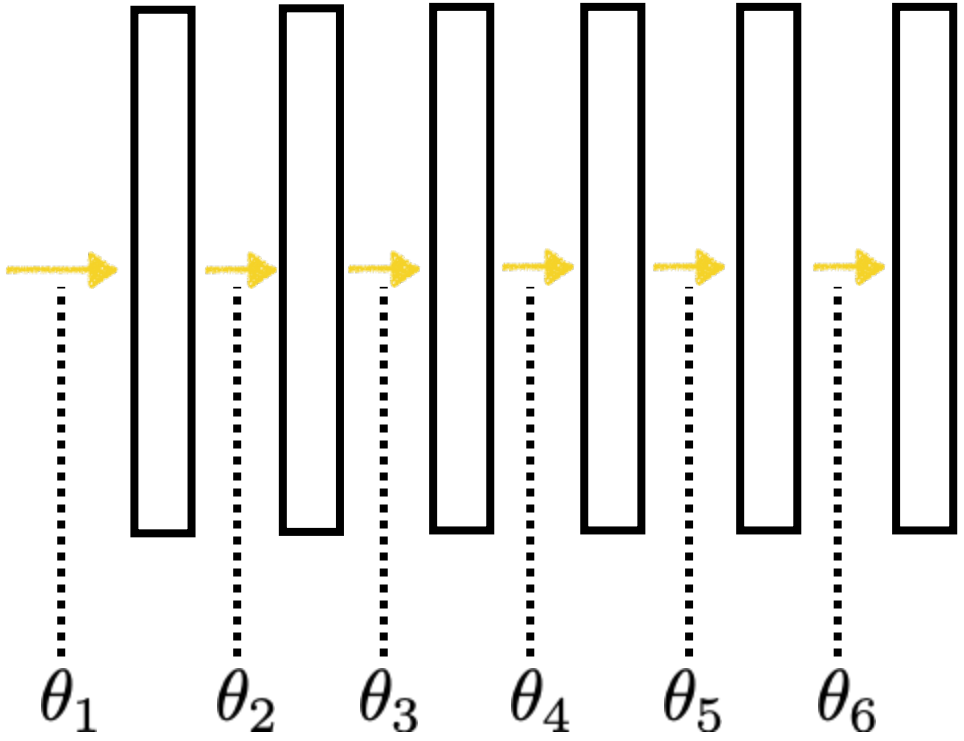
Deep learning

$\mathbf{y}_i$

"chameleon"

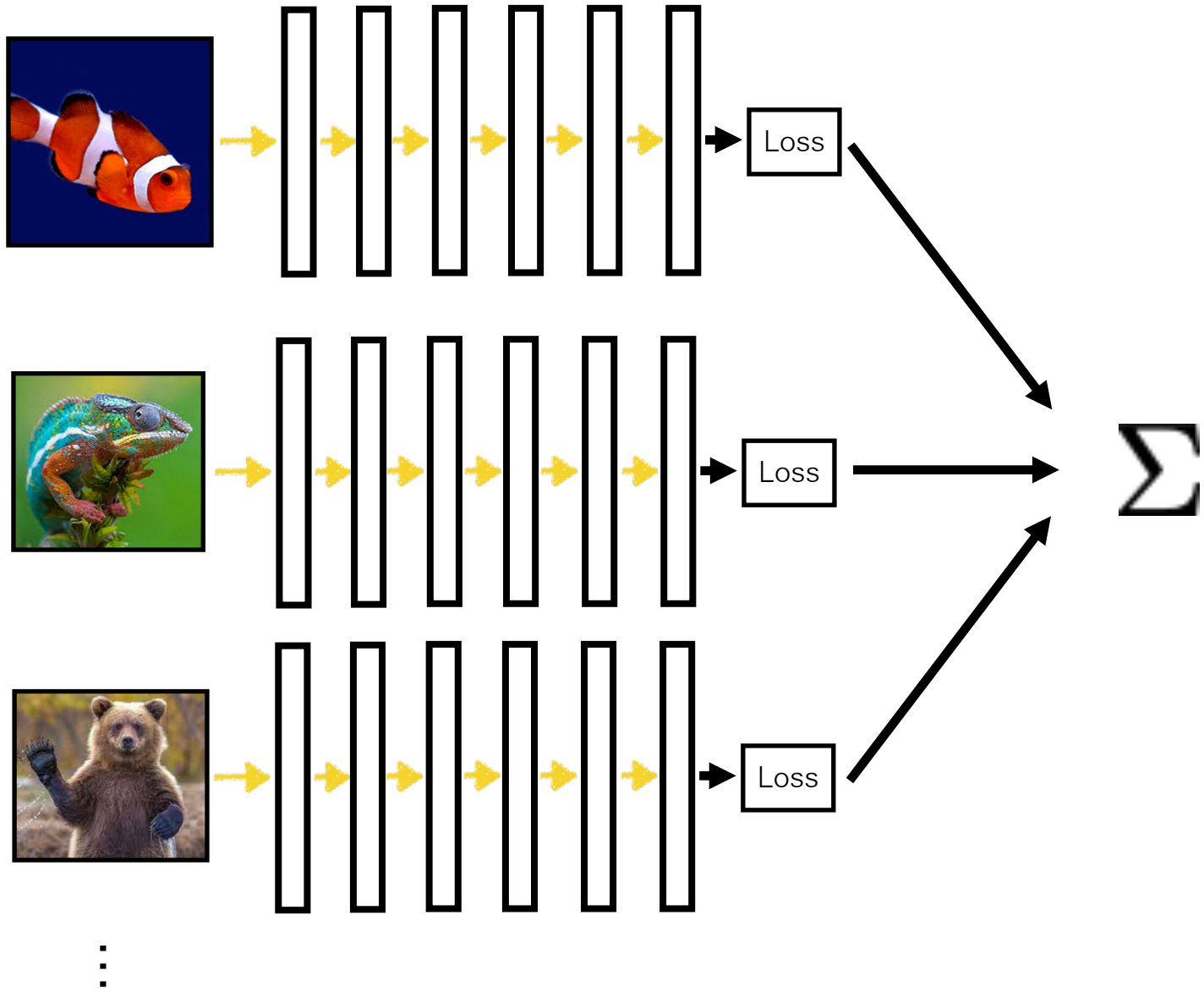
Learned

$\mathbf{x}_i$



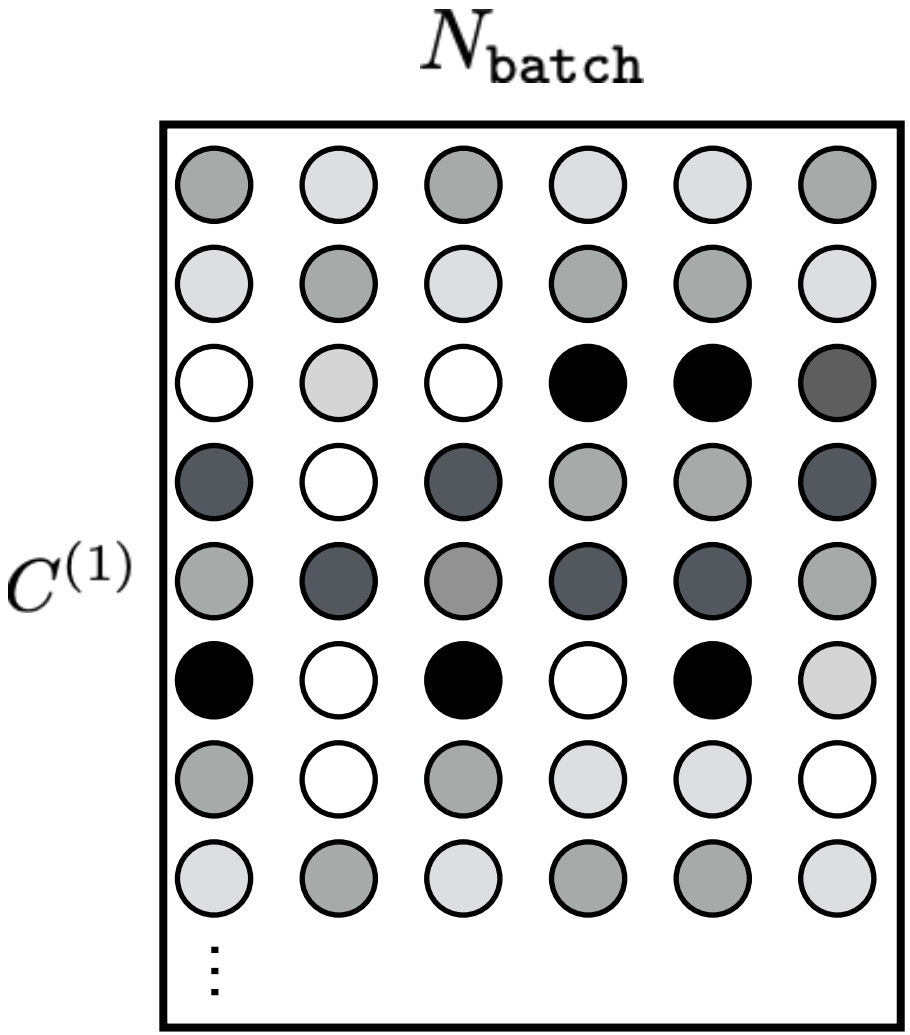
$$\theta^* = \arg\min_\theta \sum_{i=1}^{N} \mathcal{L}(f_\theta(\mathbf{x}_i), \mathbf{y}_i)$$

Batch (parallel) processing



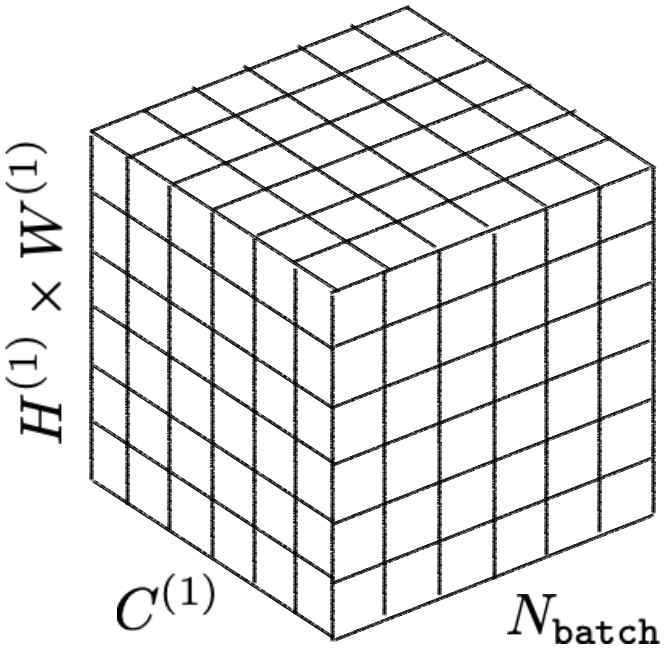
Tensors

$$\mathbf{h}^{(1)} \in \mathbb{R}^{N_{\texttt{batch}} \times C^{(1)}}$$

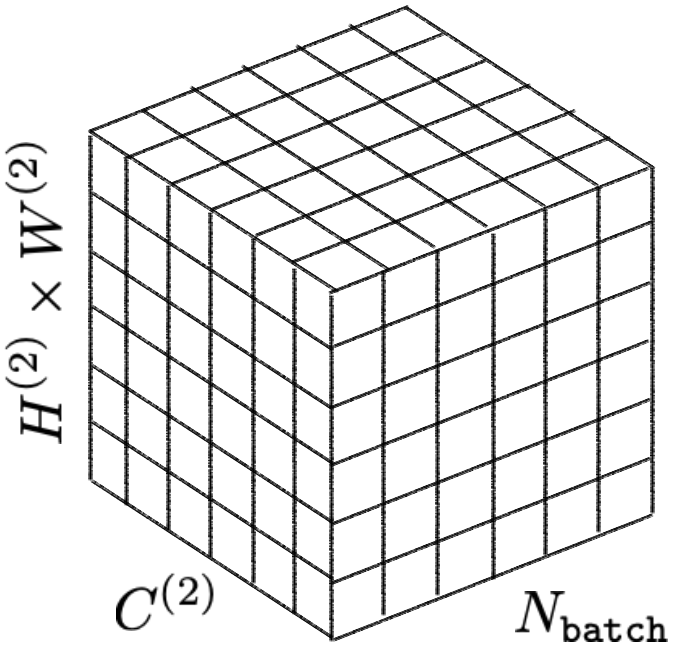


"Tensor flow"

$$\mathbf{h}^{(1)} \in \mathbb{R}^{N_{\text{batch}} \times H^{(1)} \times W^{(1)} \times C^{(1)}}$$



$$\mathbf{h}^{(2)} \in \mathbb{R}^{N_{\text{batch}} \times H^{(2)} \times W^{(2)} \times C^{(2)}}$$



Regularizing deep nets

Deep nets have millions of parameters!

On many datasets, it is easy to overfit — we may have more free parameters than data points to constrain them.

How can we regularize to prevent the network from overfitting?

1. Fewer neurons, fewer layers
2. Weight decay
3. Dropout
4. Normalization layers
5. …

Recall: regularized least squares

$$f_\theta(x) = \sum_{k=0}^{K} \theta_k x^k$$

$$R(\theta) = \lambda \|\theta\|_2^2$$ ⟵ Only use polynomial terms if you really need them! Most terms should be zero

ridge regression, a.k.a., **Tikhonov regularization**

Probabilistic interpretation: R is a Gaussian **prior** over values of the parameters.

Recall: regularized least squares

$$\theta^* = \arg\min_{\theta} \sum_{i=1}^{N} \mathcal{L}(f_\theta(\mathbf{x}_i), \mathbf{y}_i) + R(\theta)$$

$$R(\mathbf{W}) = \lambda \|\mathbf{W}\|_2^2$$ ⟵ weight decay

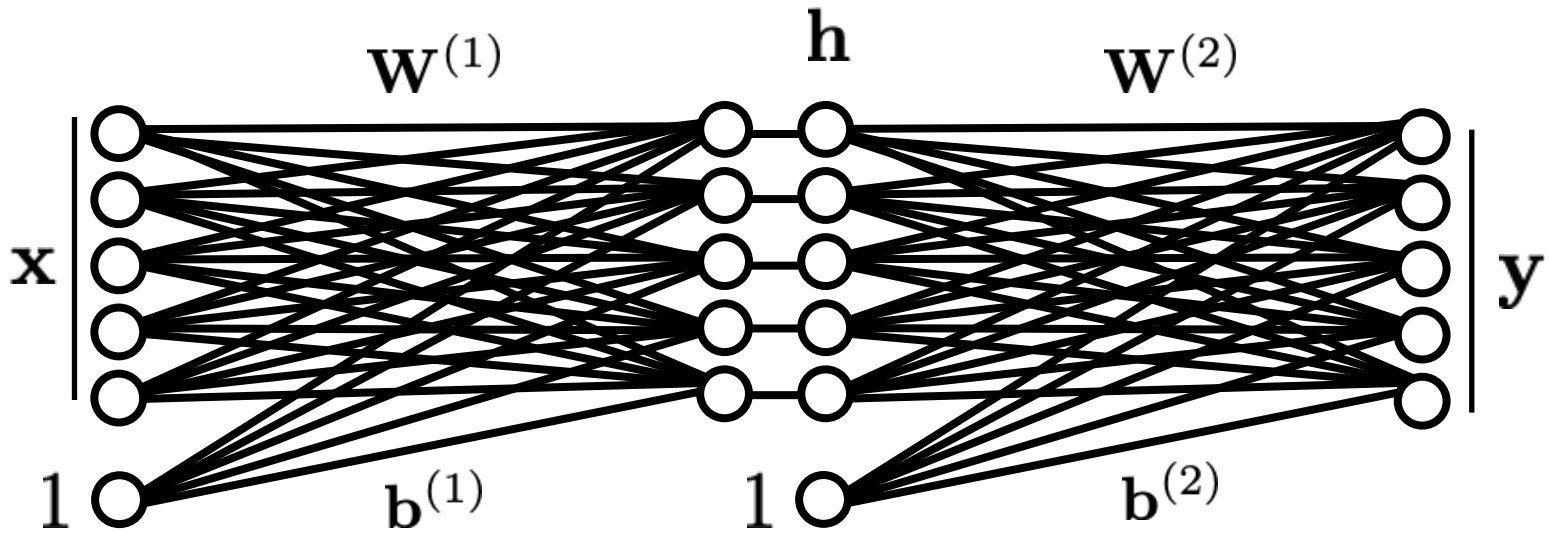
"We prefer to keep weights small."

Dropout

Input representation

Intermediate representation

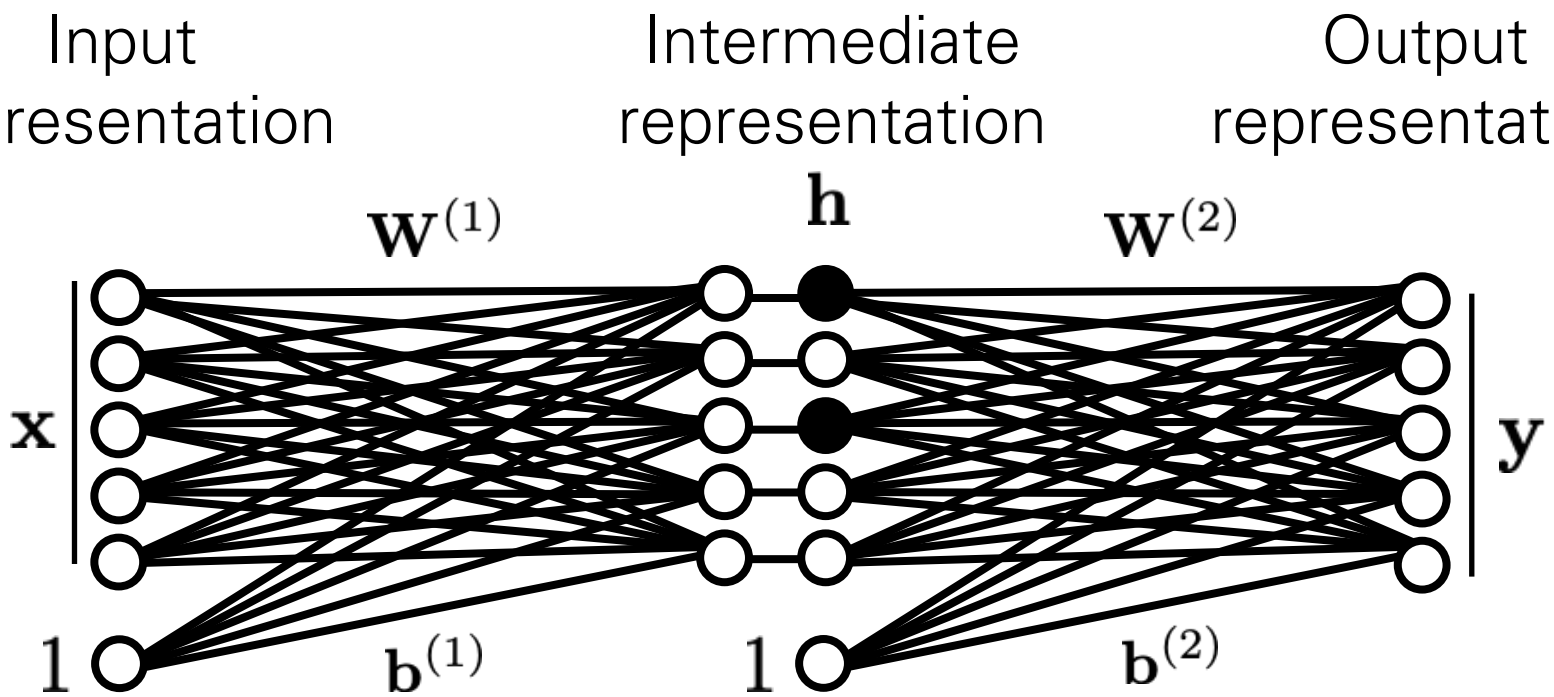
Output representation



$$\theta = \{\mathbf{W}^{(1)}, \ldots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \ldots, \mathbf{b}^{(L)}\}$$

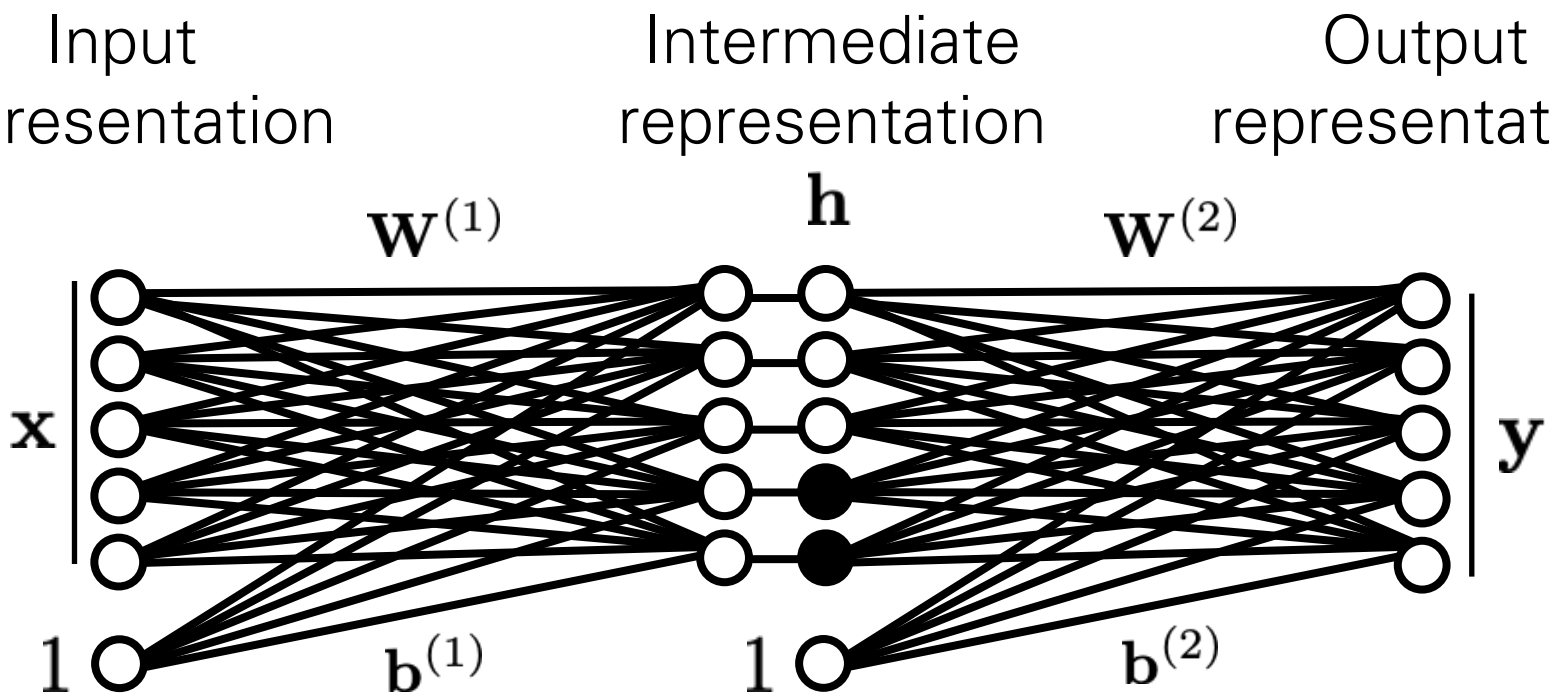
Randomly zero out hidden units.

Dropout



$$\theta = \{\mathbf{W}^{(1)}, \ldots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \ldots, \mathbf{b}^{(L)}\}$$

Dropout



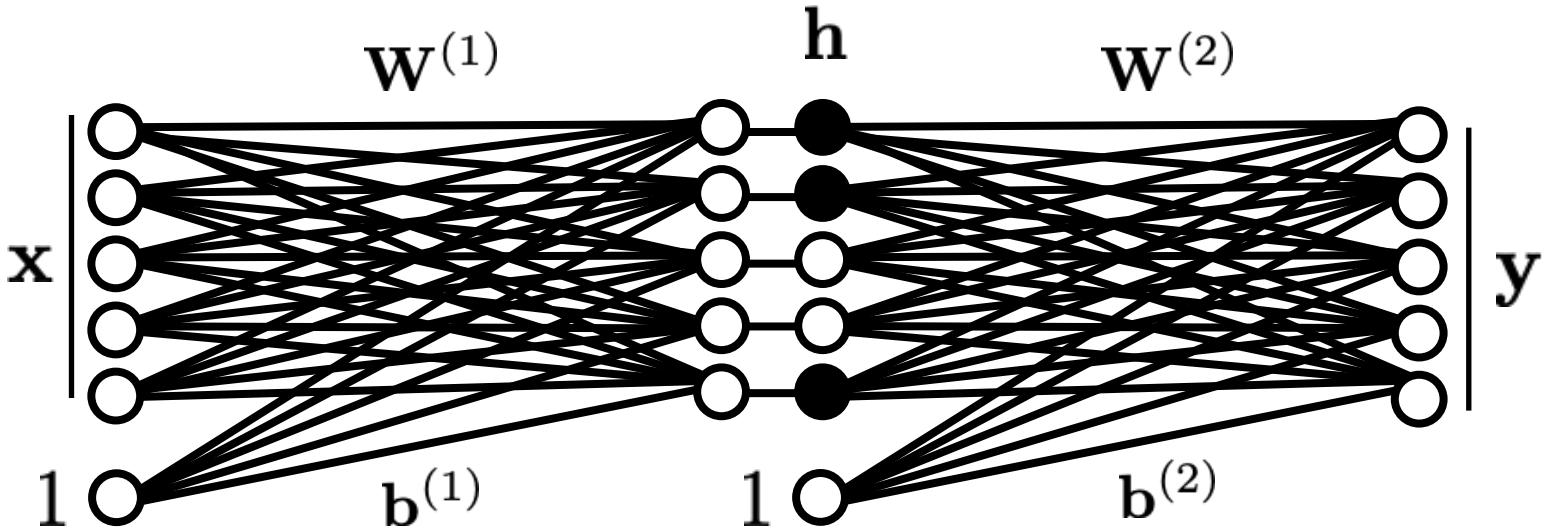
$$\theta = \{\mathbf{W}^{(1)}, \ldots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \ldots, \mathbf{b}^{(L)}\}$$

Dropout

Input representation

Intermediate representation

Output representation



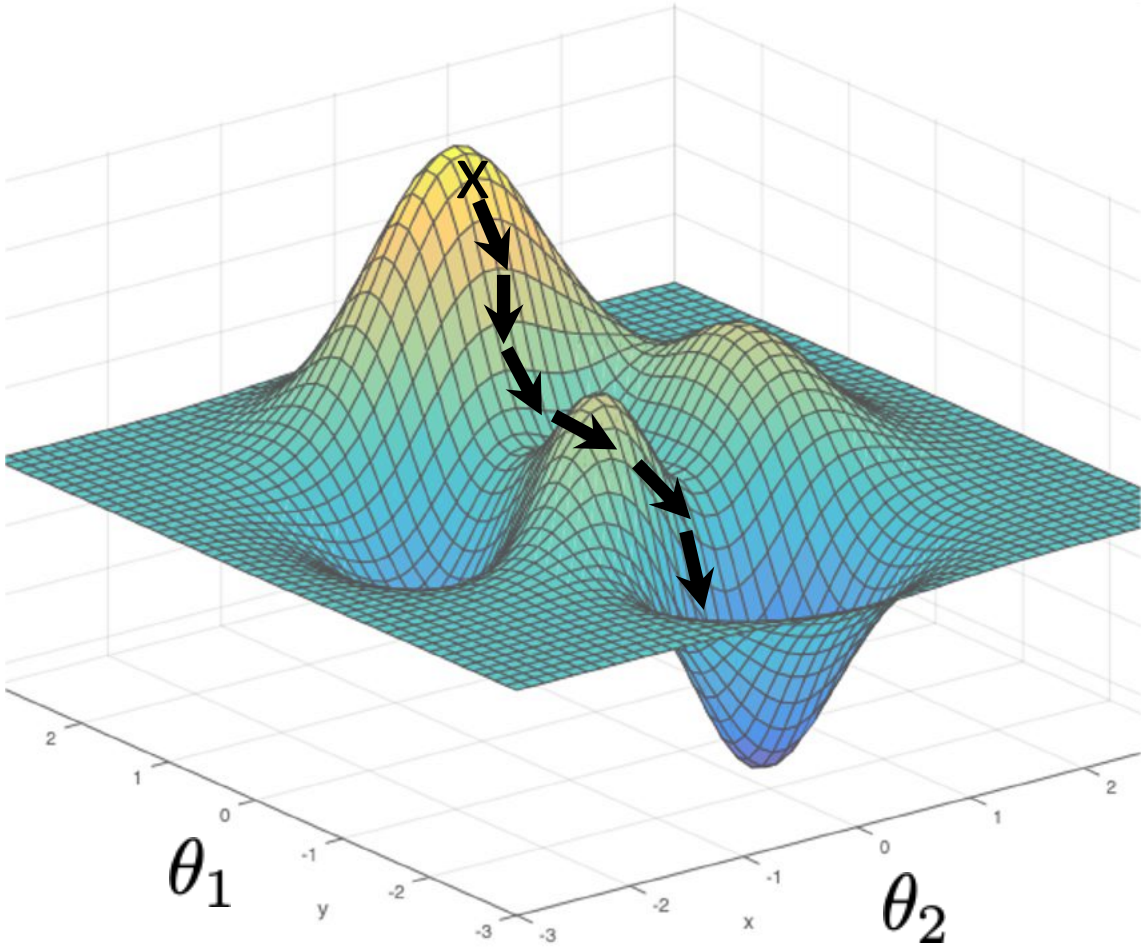
$$\theta = \{\mathbf{W}^{(1)}, \ldots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \ldots, \mathbf{b}^{(L)}\}$$

Prevents network from relying too much on spurious correlations between different hidden units.

Can be understood as averaging over an exponential **ensemble** of subnetworks. This averaging smooths the function, thereby reducing the effective capacity of the network.

Gradient descent

$J(\theta)$



$$\theta^* = \arg\min_\theta J(\theta)$$

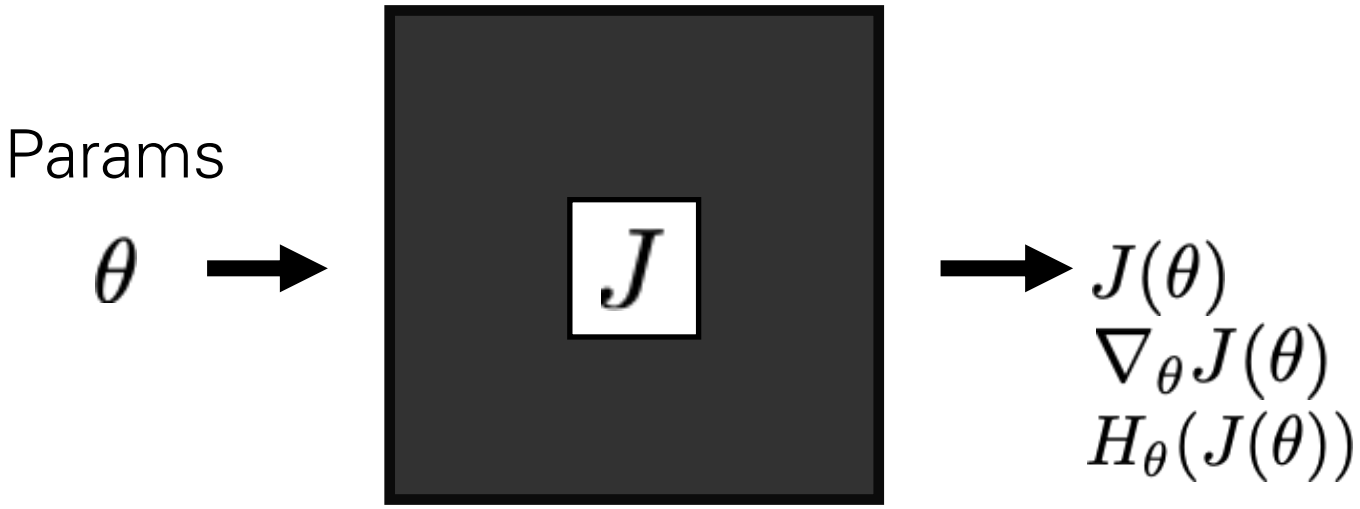
$$\theta^* = \arg\min_\theta \underbrace{\sum_{i=1}^{N} \mathcal{L}(f_\theta(\mathbf{x}_i), \mathbf{y}_i)}_{J(\theta)}$$

One iteration of gradient descent:

$$\theta^{t+1} = \theta^t - \eta_t \left.\frac{\partial J(\theta)}{\partial \theta}\right|_{\theta=\theta^t}$$

learning rate

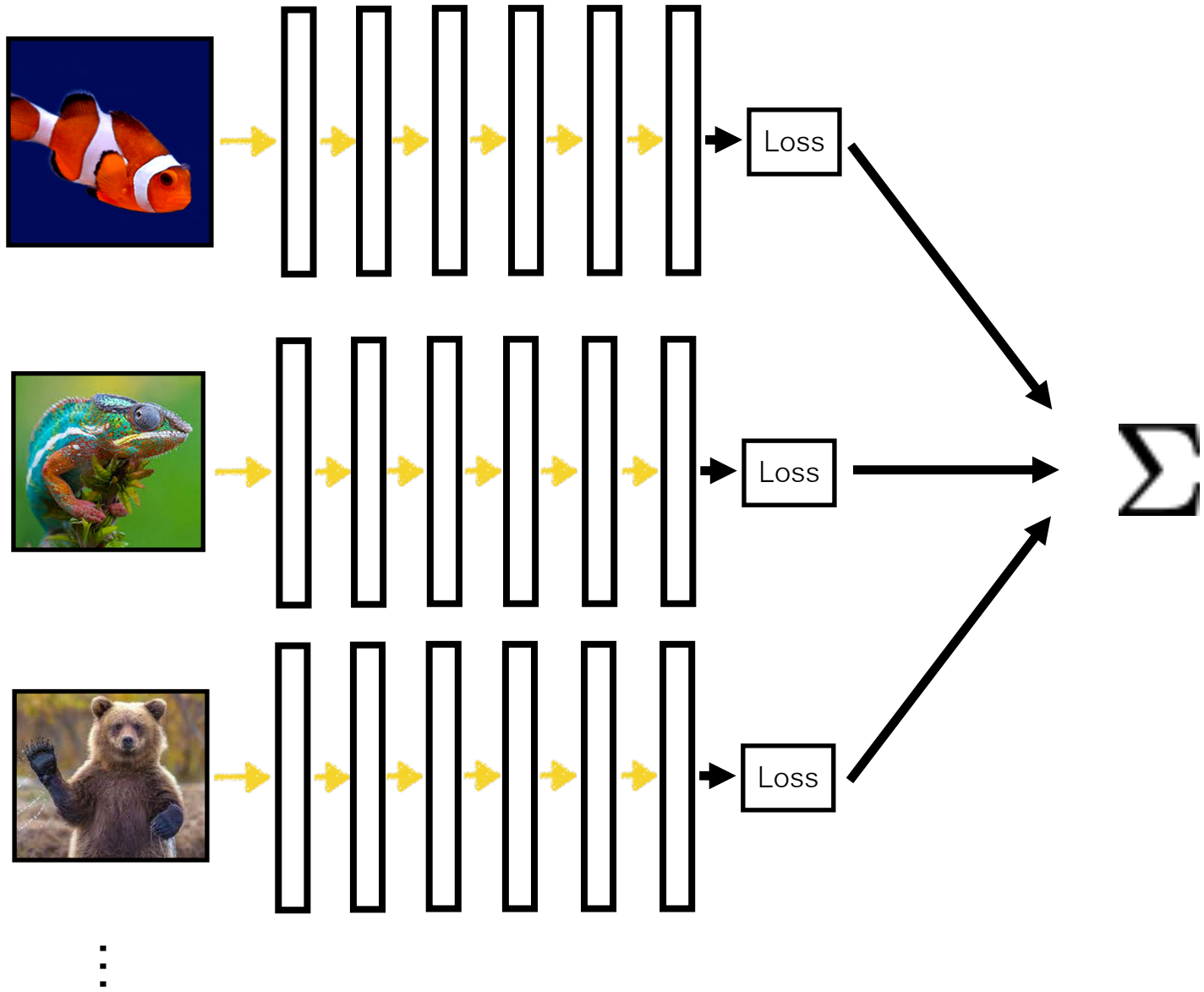
Optimization



$$\theta^* = \arg\min_\theta J(\theta)$$

- What's the knowledge we have about J?
  - We can evaluate $J(\theta)$ ← Black box optimization
  - We can evaluate $J(\theta)$ and $\nabla_\theta J(\theta)$ (Gradient) ← First order optimization
  - We can evaluate $J(\theta)$, $\nabla_\theta J(\theta)$, and $H_\theta(J(\theta))$ (Hessian) ← Second order optimization

Batch (parallel) processing



Stochastic gradient descent (SGD)

- Want to minimize overall loss function J, which is sum of individual losses over each example.
- In Stochastic gradient descent, compute gradient on sub-set (batch) of data.

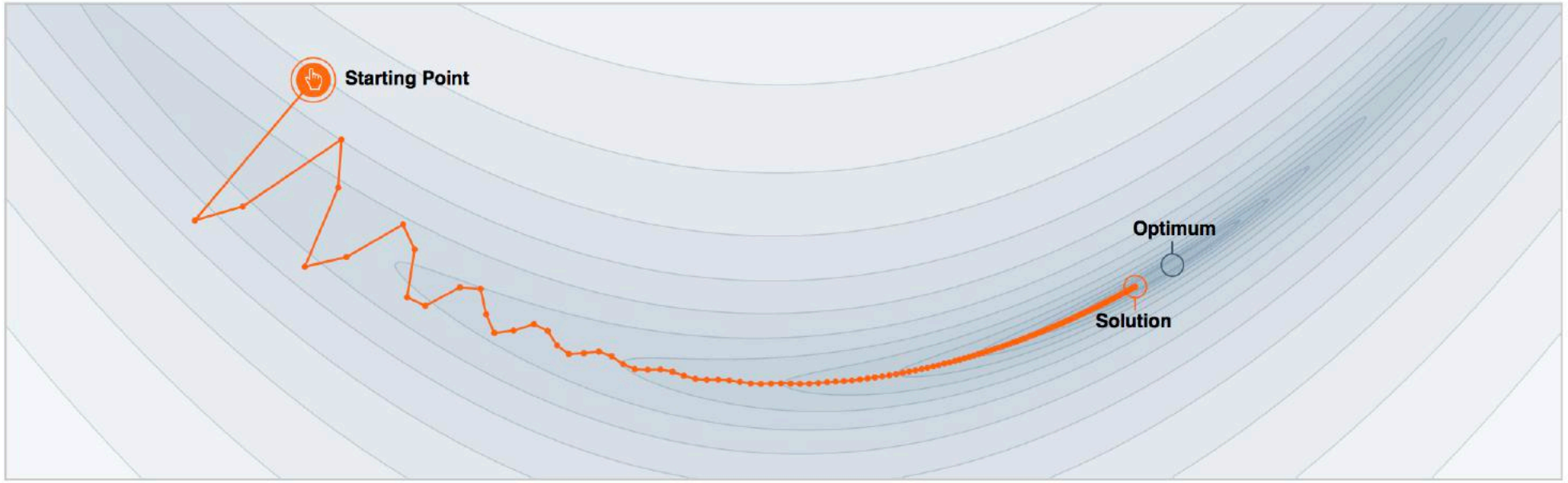
  If batchsize=1 then θ is updated after each example.

  If batchsize=N (full set) then this is standard gradient descent.
- Gradient direction is noisy, relative to average over all examples (standard gradient descent).
- **Advantages**
  - Faster: approximate total gradient with small sample
  - Implicit regularizer
- **Disadvantages**
  - High variance, unstable updates

Momentum

- **Basic idea:** like a ball rolling down a hill, we should build up speed so as to make faster progress when "on a roll"
- Can dampen oscillations in SGD updates
- Common in popular variants of SGD
  - Nesterov's method
  - RMSProp
  - Adam

Why Momentum Really Works



Step-size α = 0.02

0 0.003 0.006

Momentum β = 0.99

0.00 0.500 0.990

We often think of Momentum as a means of dampening oscillations and speeding up the iterations, leading to faster convergence. But it has other interesting behavior. It allows a larger range of step-sizes to be used, and creates its own oscillations. What is going on?

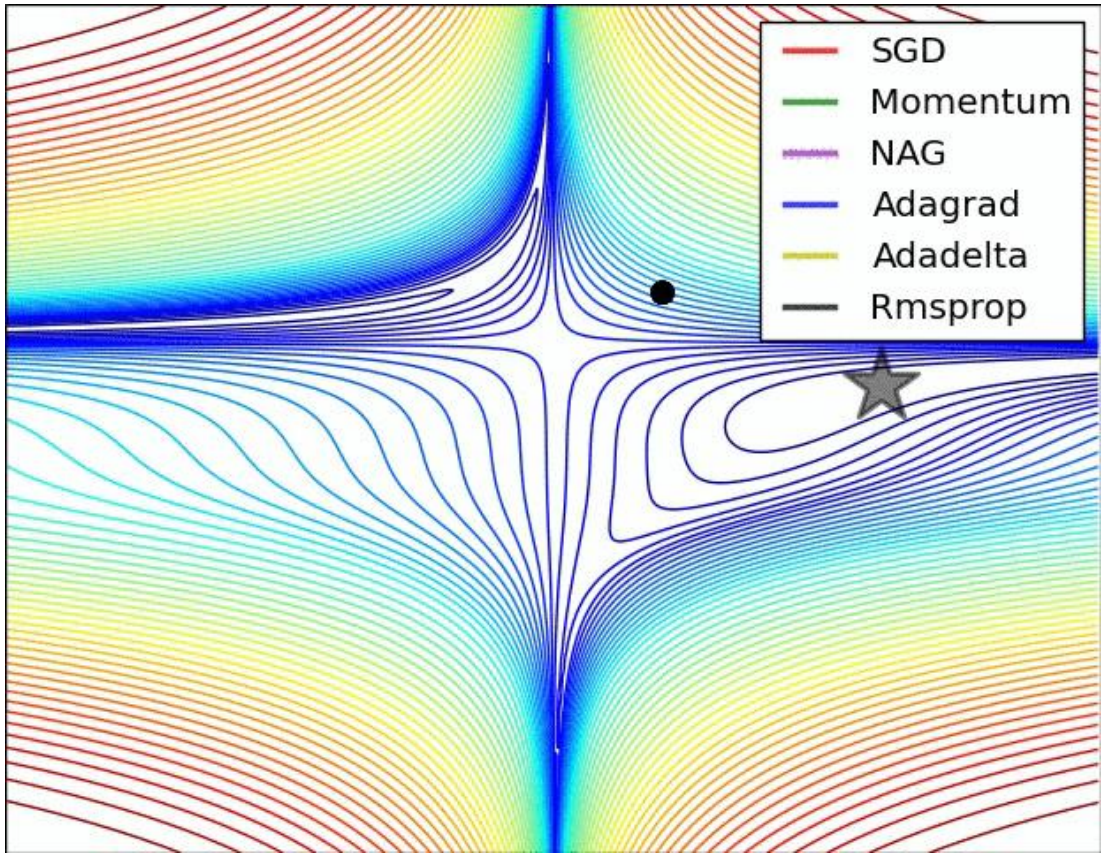
GABRIEL GOH
UC Davis

April. 4
2017

Citation:
Goh, 2017

[https://distill.pub/2017/momentum/]

Comparison of gradient descent variants



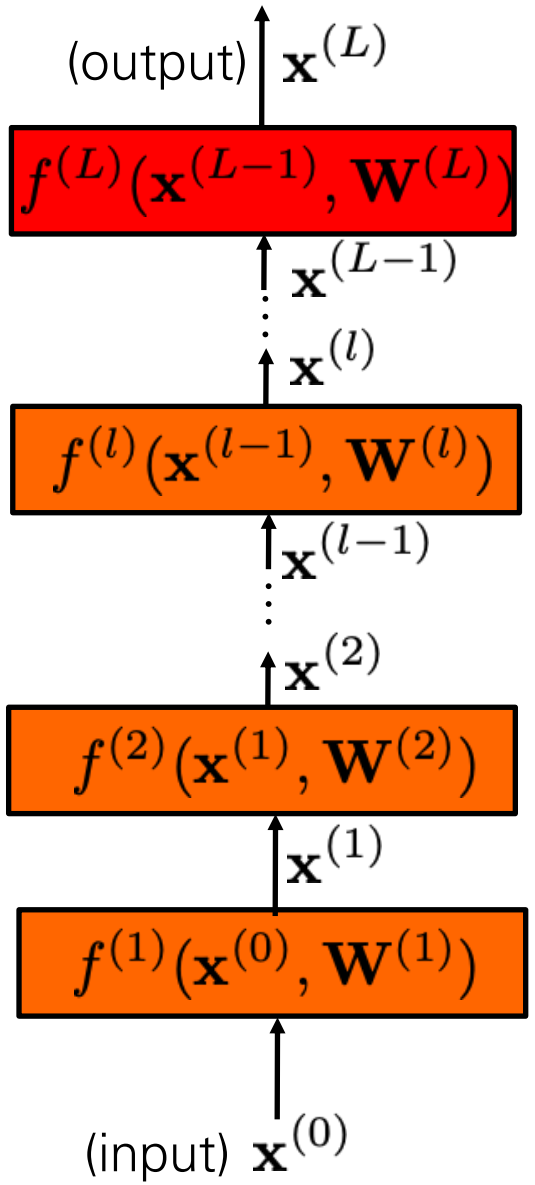
[http://ruder.io/optimizing-gradient-descent/]

Forward pass

- Consider model with L layers. Layer l has vector of weights $\mathbf{W}^{(l)}$
- **Forward pass:** takes input $\mathbf{x}^{(l-1)}$ and passes it through each layer $f^{(l)}$:

$$\mathbf{x}^{(l)} = f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})$$

- Output of layer l is $\mathbf{x}^{(l)}$.
- Network output (top layer) is $\mathbf{x}^{(L)}$.



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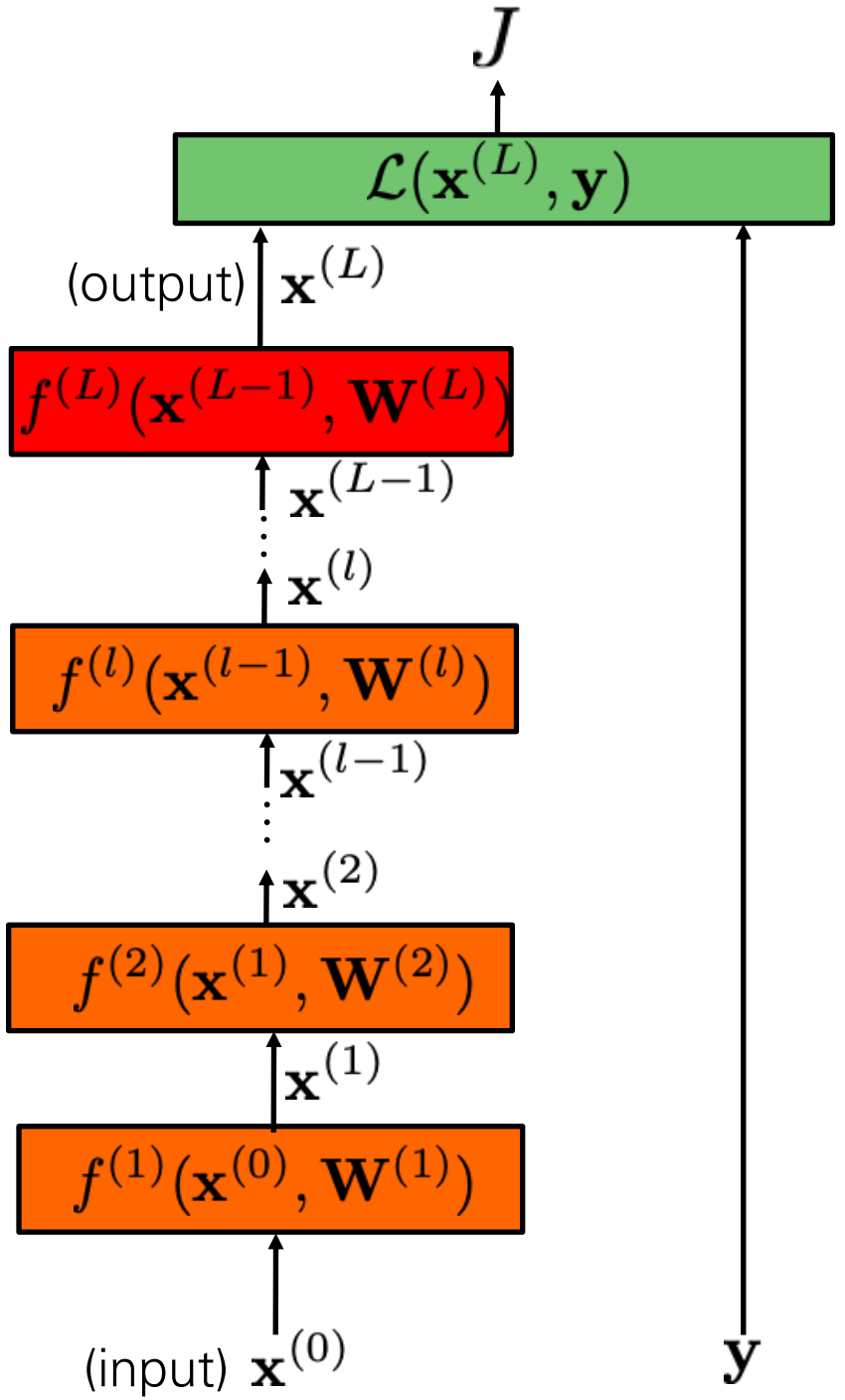
- Output of layer l is $\mathbf{x}^{(l)}$.
- Network output (top layer) is $\mathbf{x}^{(L)}$.

- **Loss function** $\mathcal{L}$ compares $\mathbf{x}^{(L)}$ to $\mathbf{y}$.
- Overall energy is the sum of the cost over all training examples:

$$J = \sum_{i=1}^{N} \mathcal{L}(\mathbf{x}_i^{(L)}, \mathbf{y}_i)$$



Gradient descent

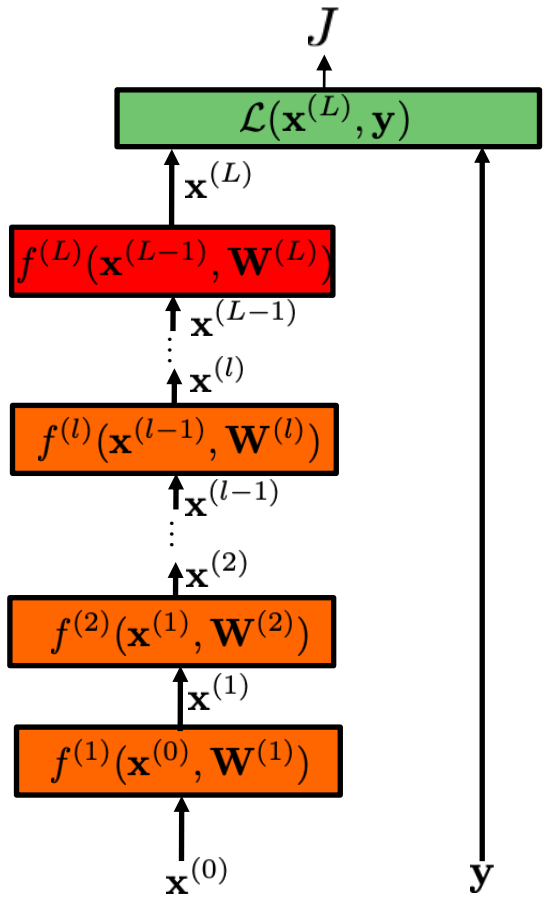
- We need to compute gradients of the cost with respect to model parameters $\mathbf{W}^{(l)}$.

- By design, each layer is differentiable with respect to its parameters and input.

Computing gradients

To compute the gradients, we could start by writing the full energy J as a function of the network parameters.

$$J(\mathbf{W}) = \sum_{i=1} \mathcal{L}(f^{(L)}(\dots f^{(2)}(f^{(1)}(\mathbf{x}_i^{(0)}, \mathbf{W}^{(1)}), \mathbf{W}^{(2)}), \dots \mathbf{W}^{(L)}), \mathbf{y}_i)$$



And then compute the partial derivatives… instead, we can use the chain rule to derive a compact algorithm: **backpropagation**

Backpropagation

- Forward pass: for each training example, compute the outputs for all layers:

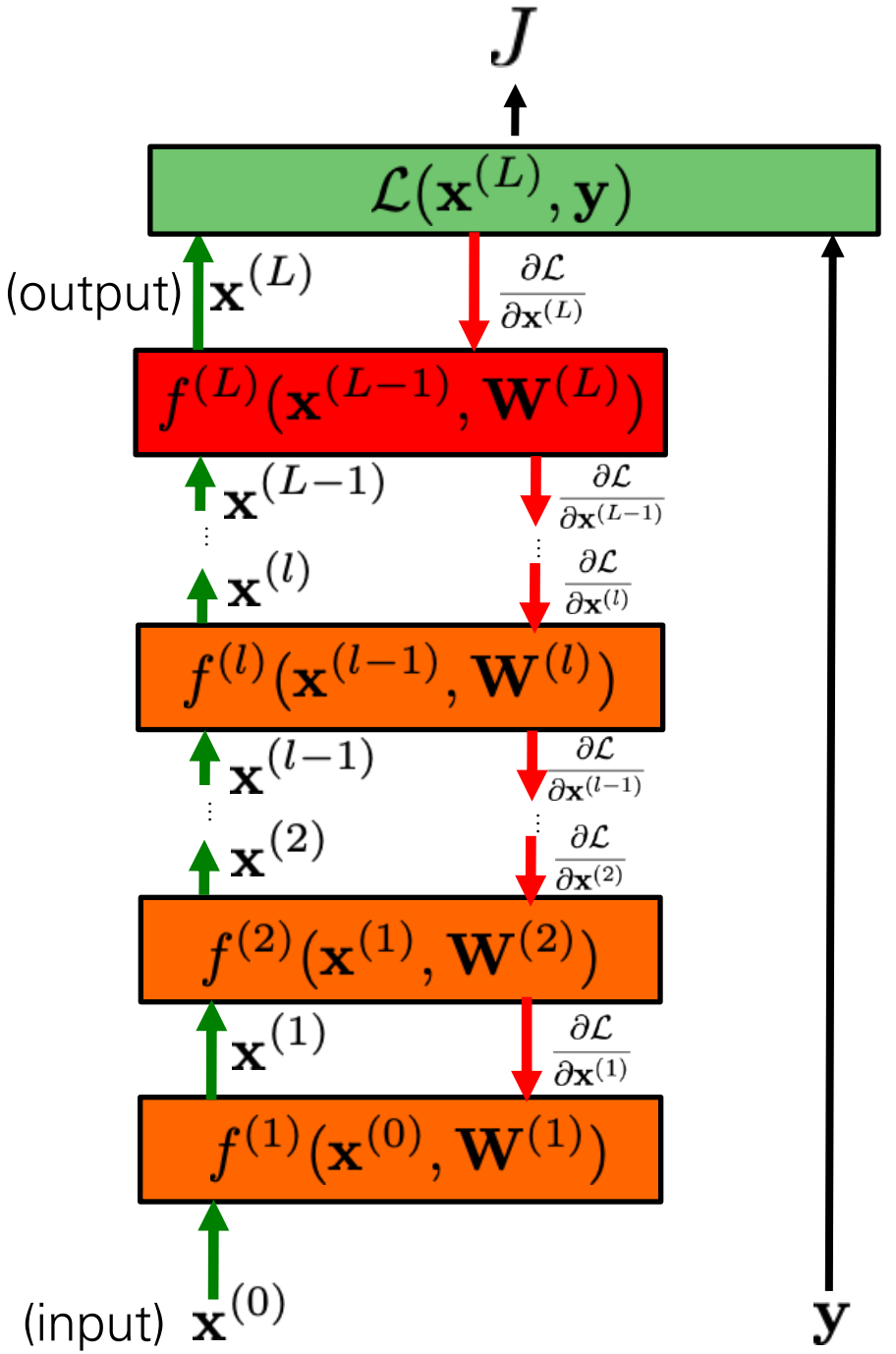
$$\mathbf{x}^{(l)} = f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})$$

- Backwards pass: compute loss derivatives iteratively from top to bottom:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l-1)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}} \cdot \frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{x}^{(l-1)}}$$

- Compute gradients w.r.t. weights, and update weights:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(l)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}} \cdot \frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{W}^{(l)}}$$



Convolutional Neural Networks

Convolutional Neural Networks

LeCun et al. 1989

Neural network with specialized connectivity

Tailored to processing natural signals with a grid topology (e.g., images).

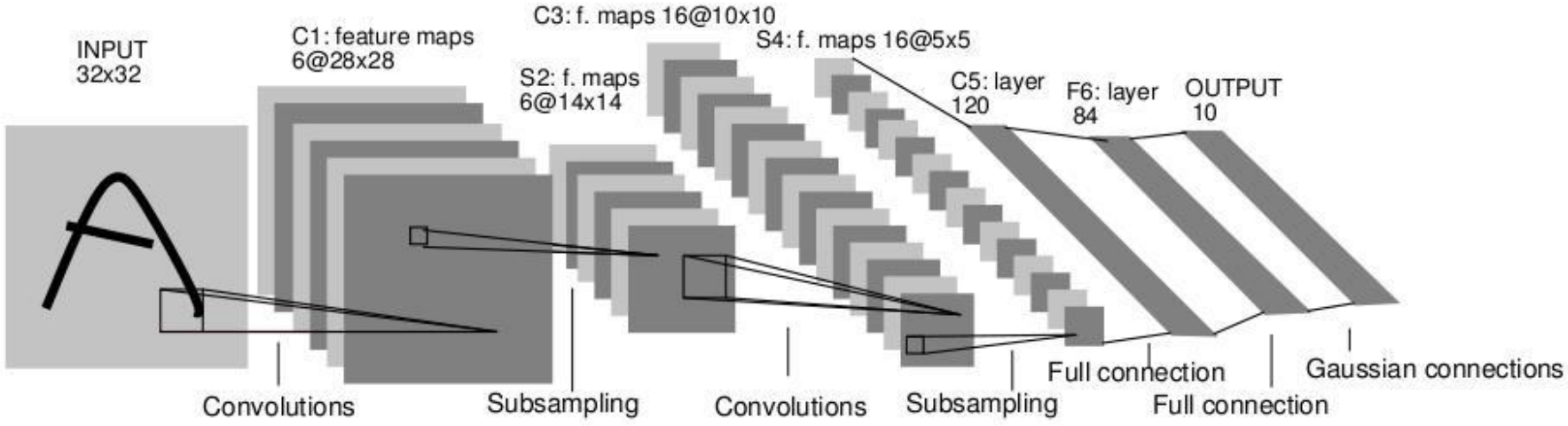


Image classification

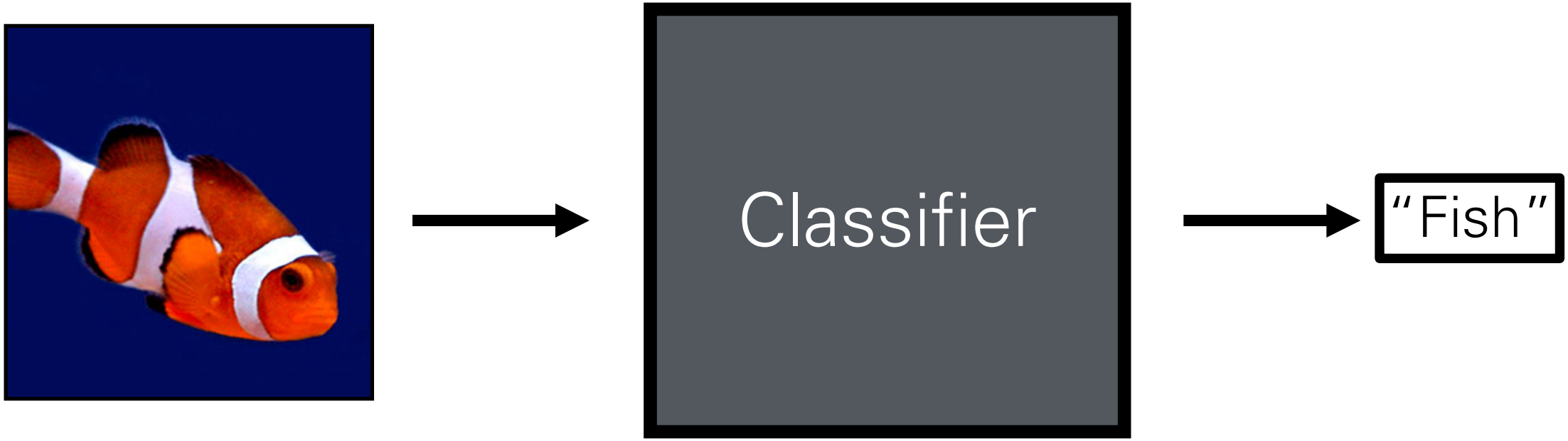
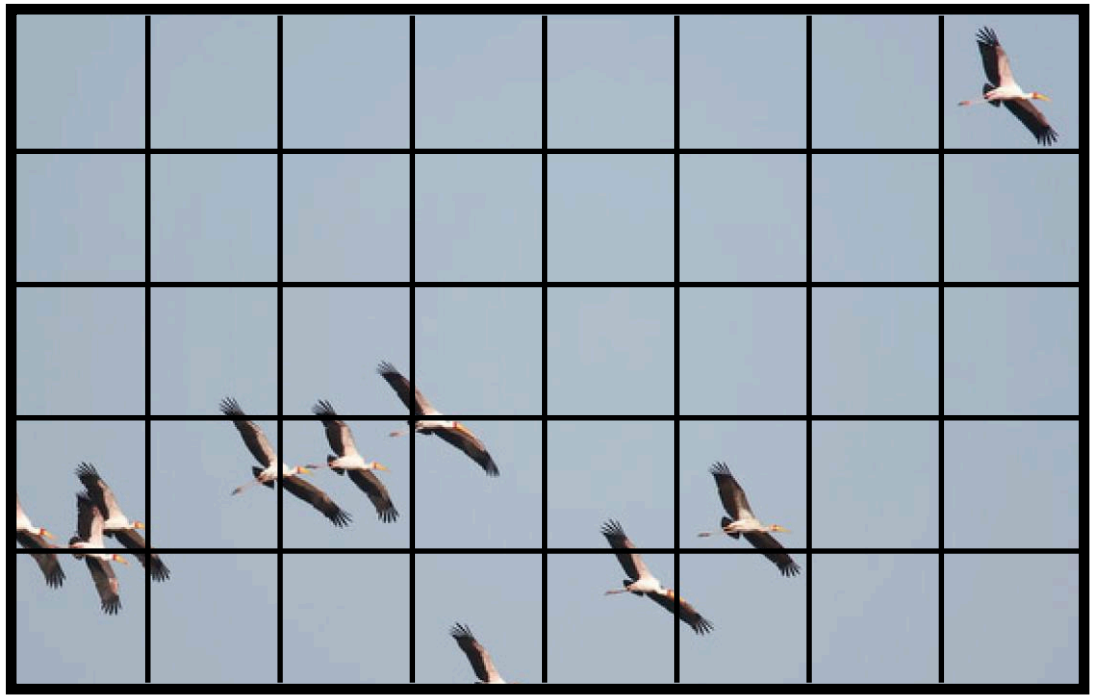
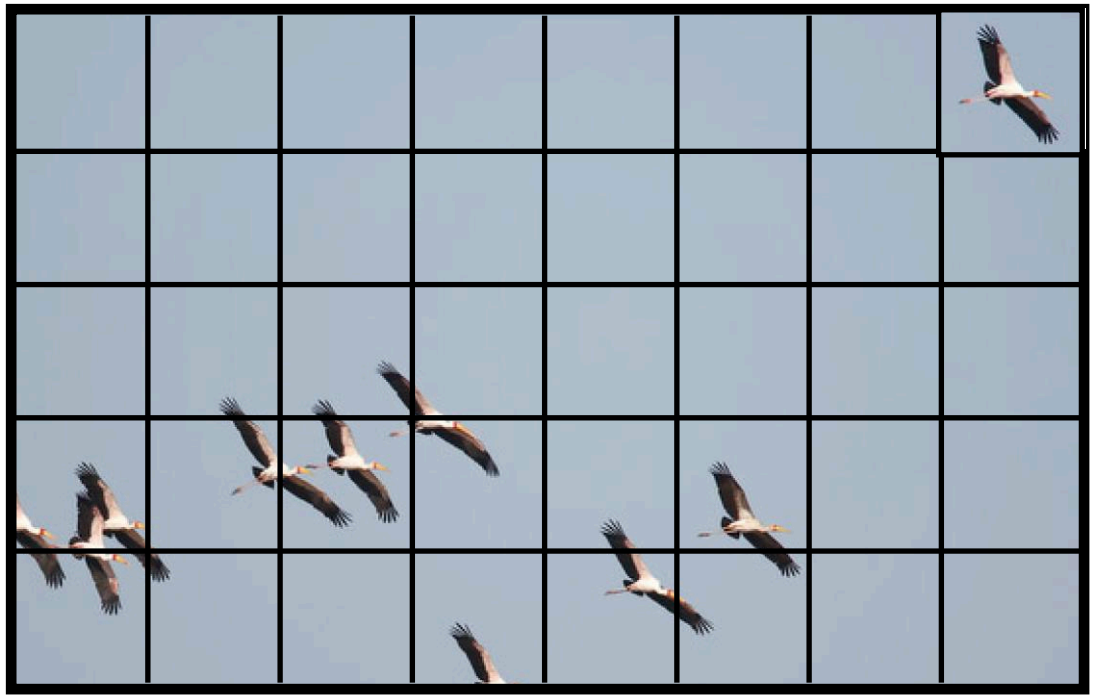


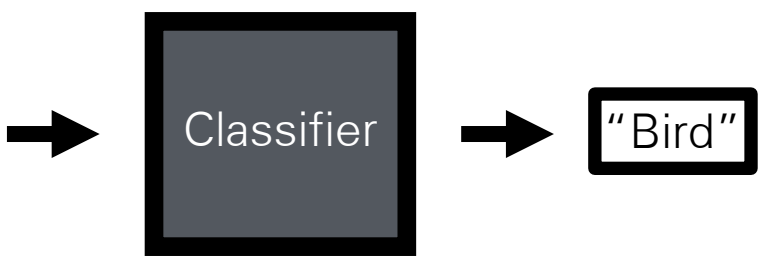
image **x**

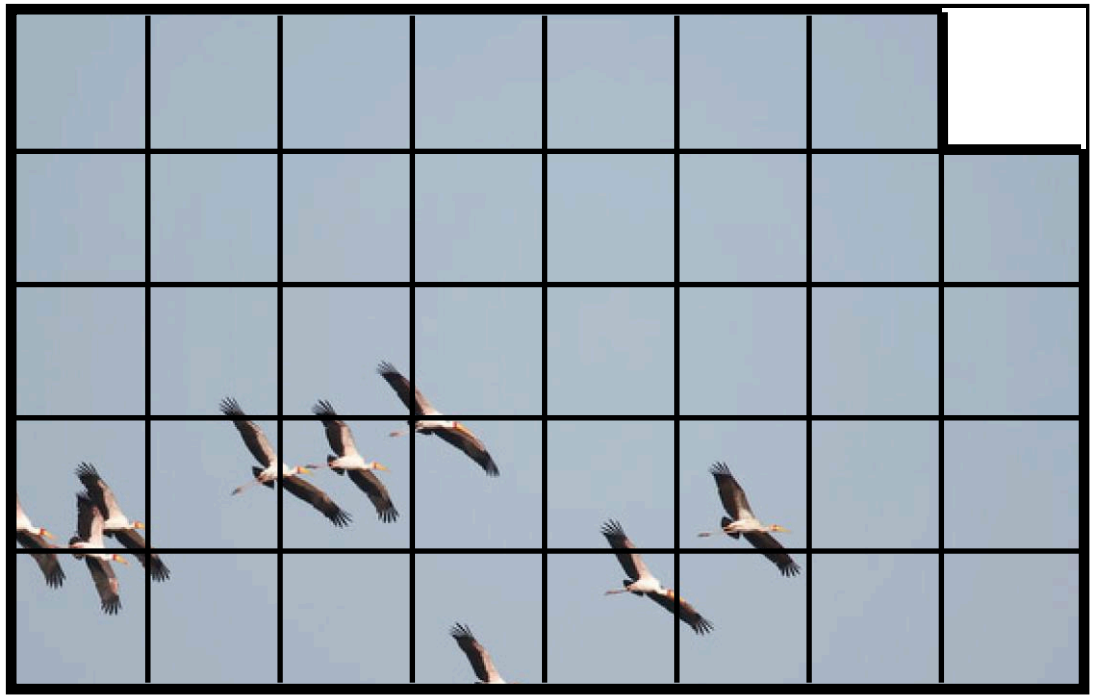
label y

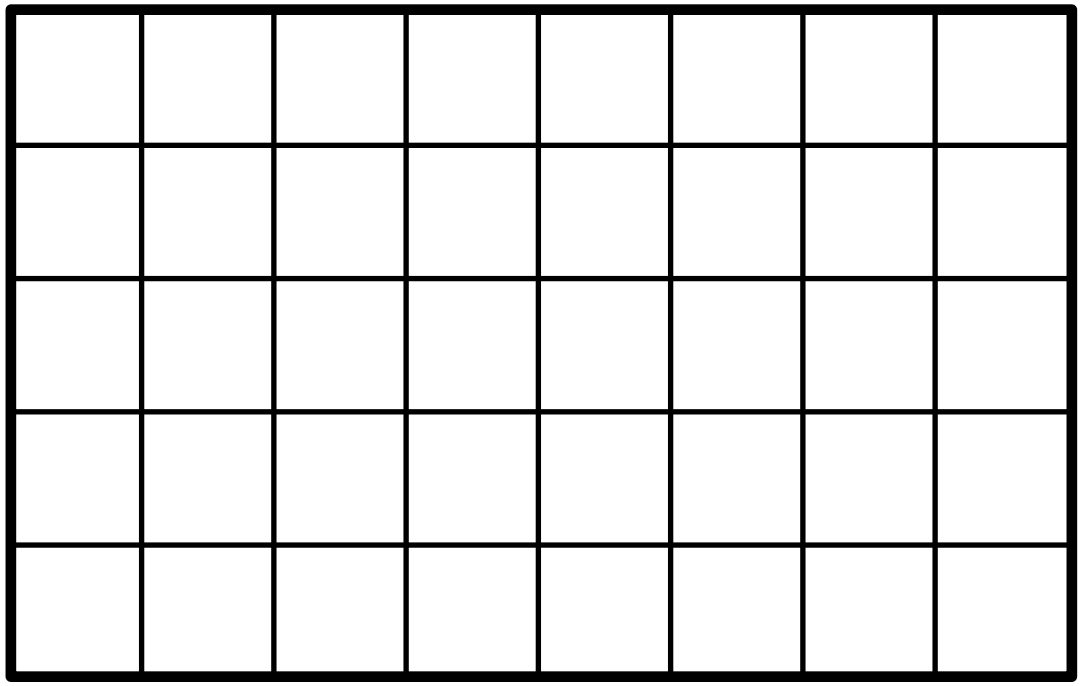
Photo credit: Fredo Durand



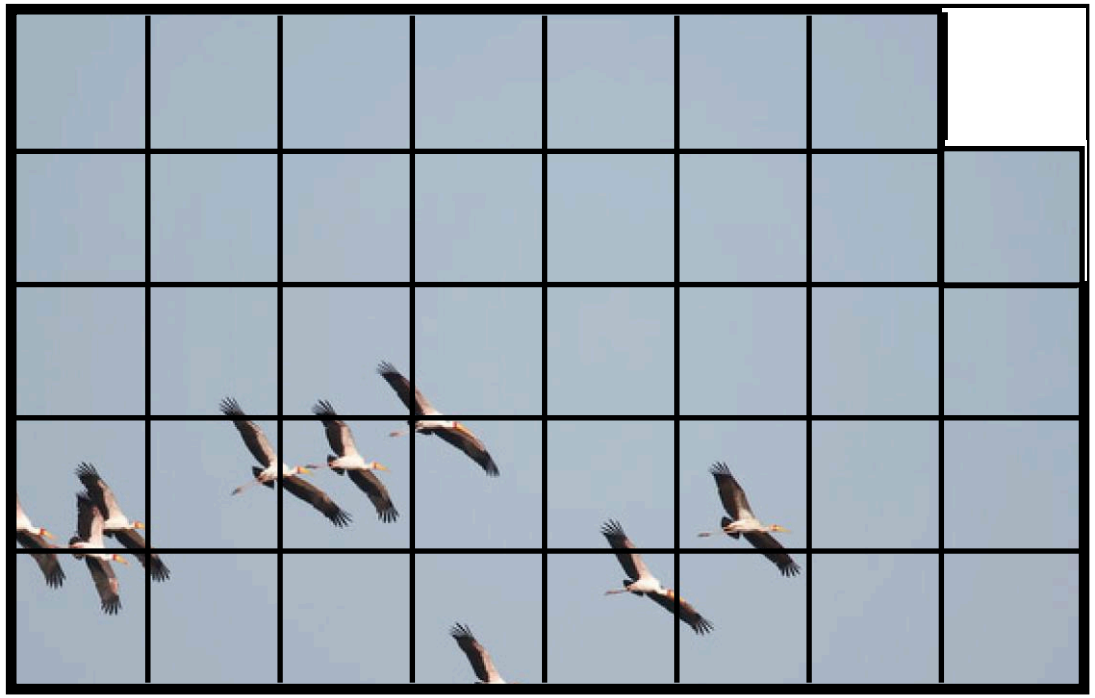


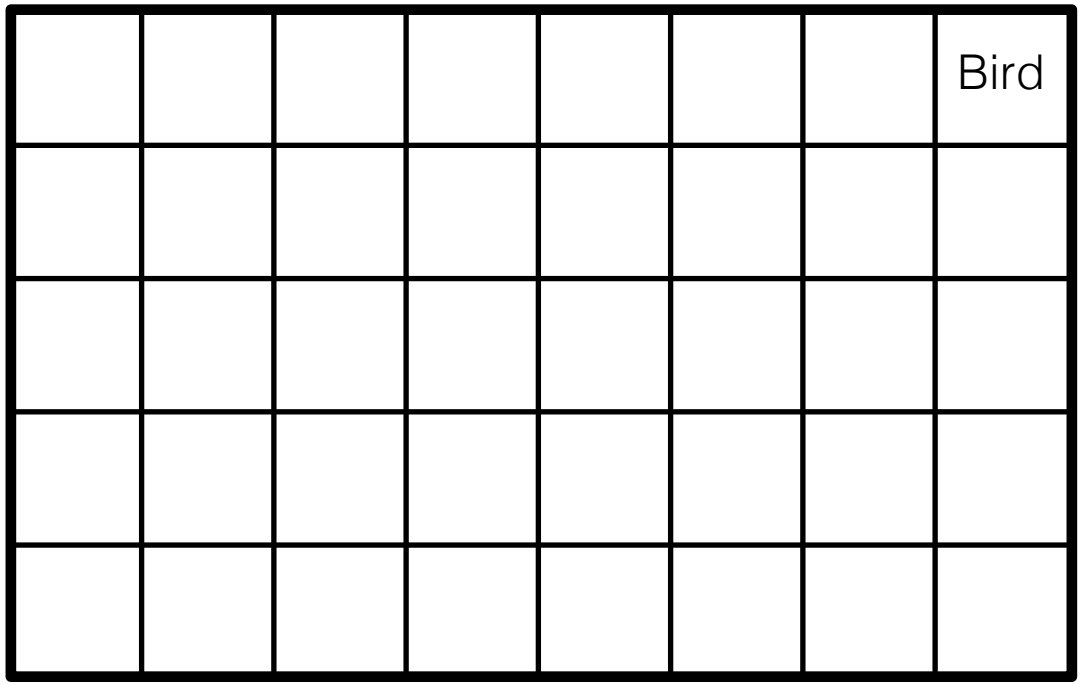

Classifier
“Bird”

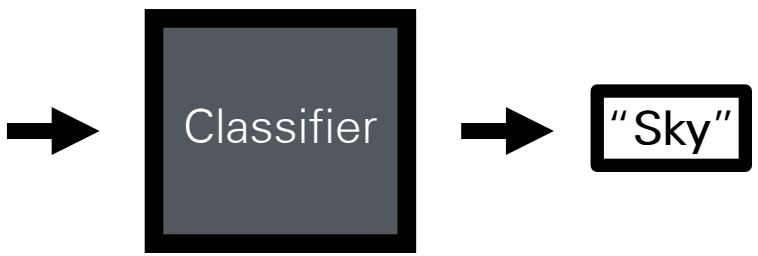


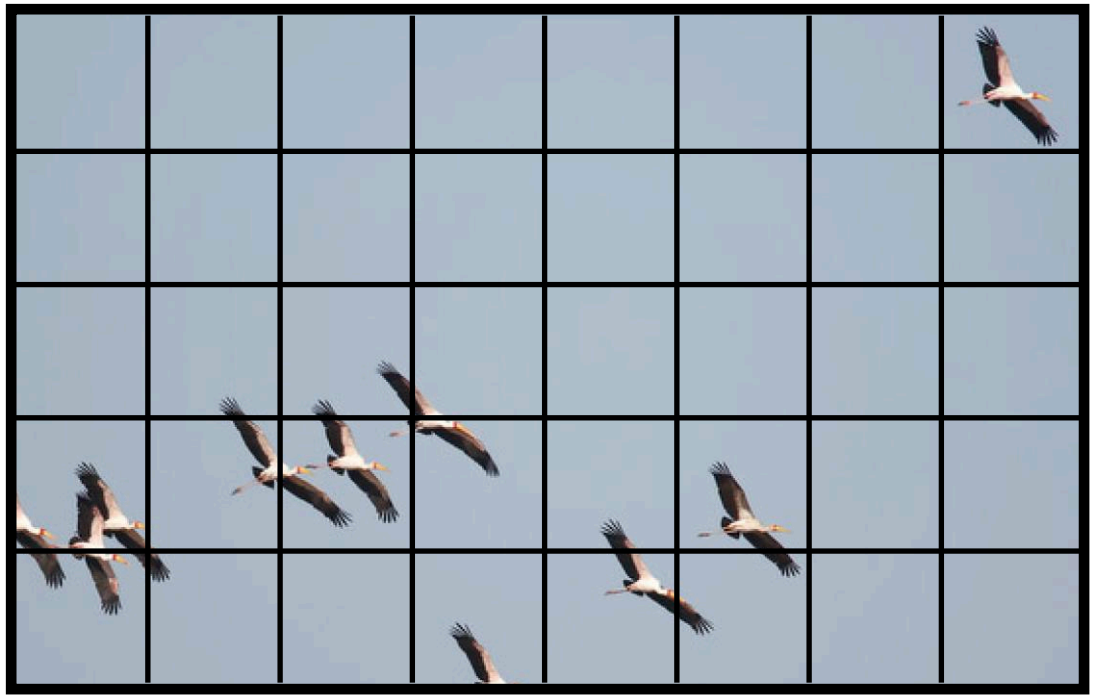



Classifier
“Bird”

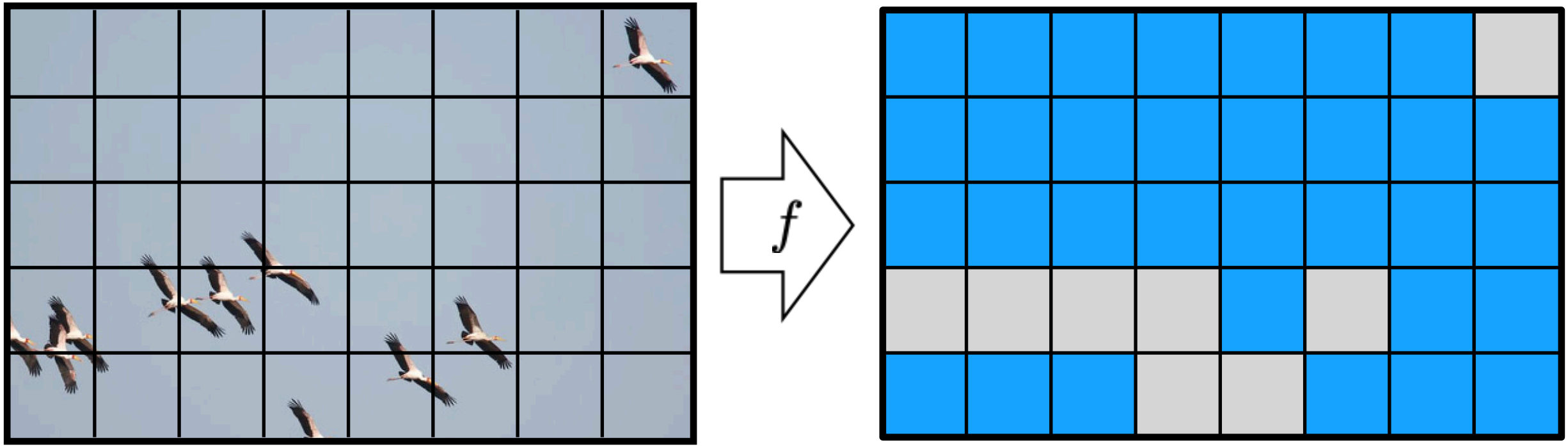


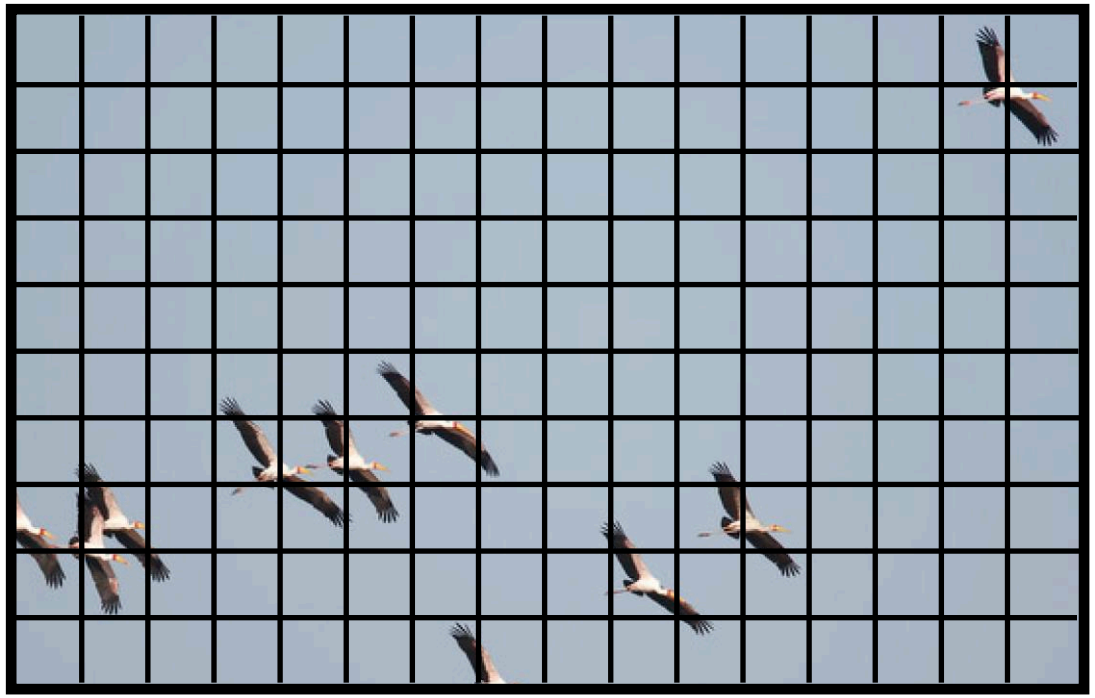

Bird


Classifier
“Sky”

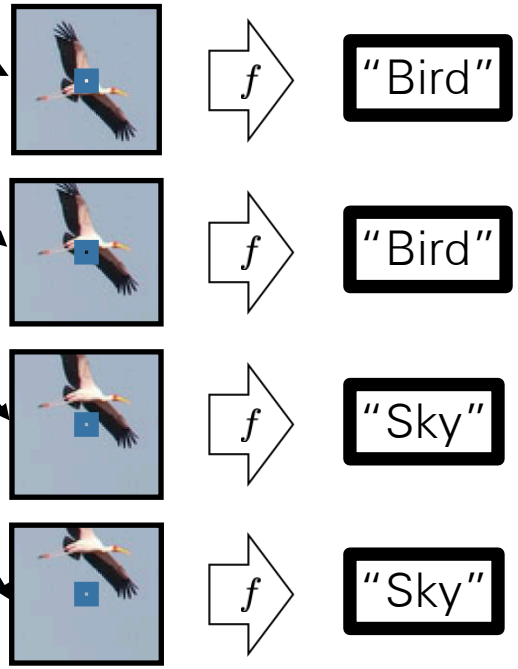


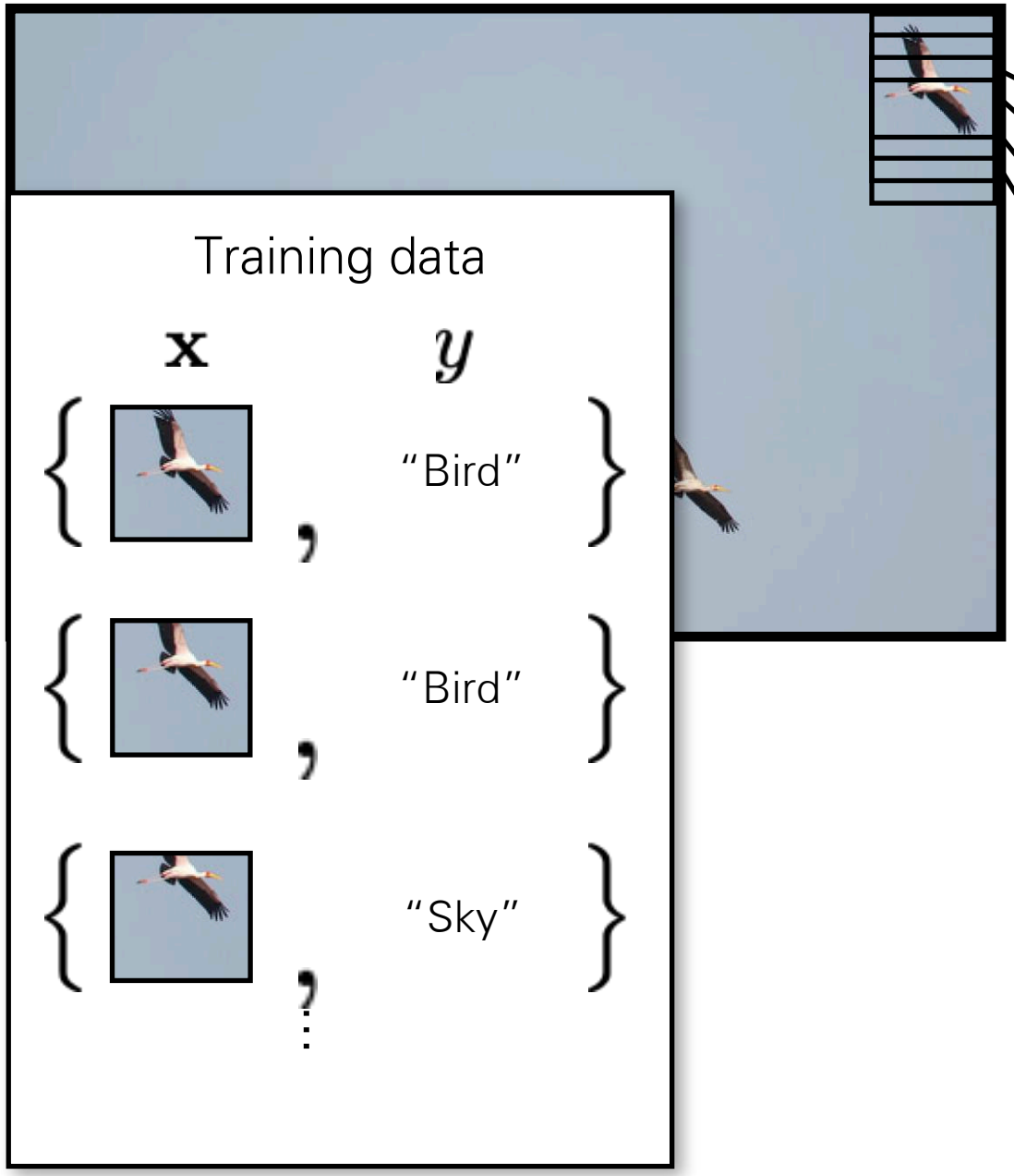
Sky	Sky	Sky	Sky	Sky	Sky	Sky	Bird
Sky	Sky	Sky	Sky	Sky	Sky	Sky	Sky
Sky	Sky	Sky	Sky	Sky	Sky	Sky	Sky
Bird	Bird	Bird	Sky	Bird	Sky	Sky	Sky
Sky	Sky	Sky	Bird	Sky	Sky	Sky	Sky


f



What's the object class of the center pixel?





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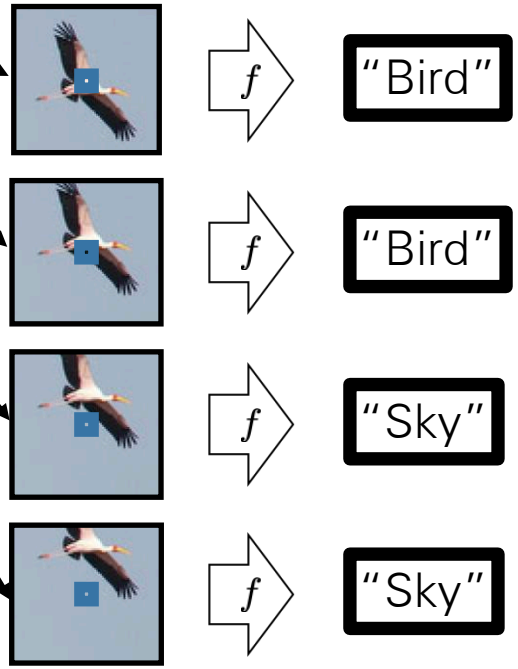
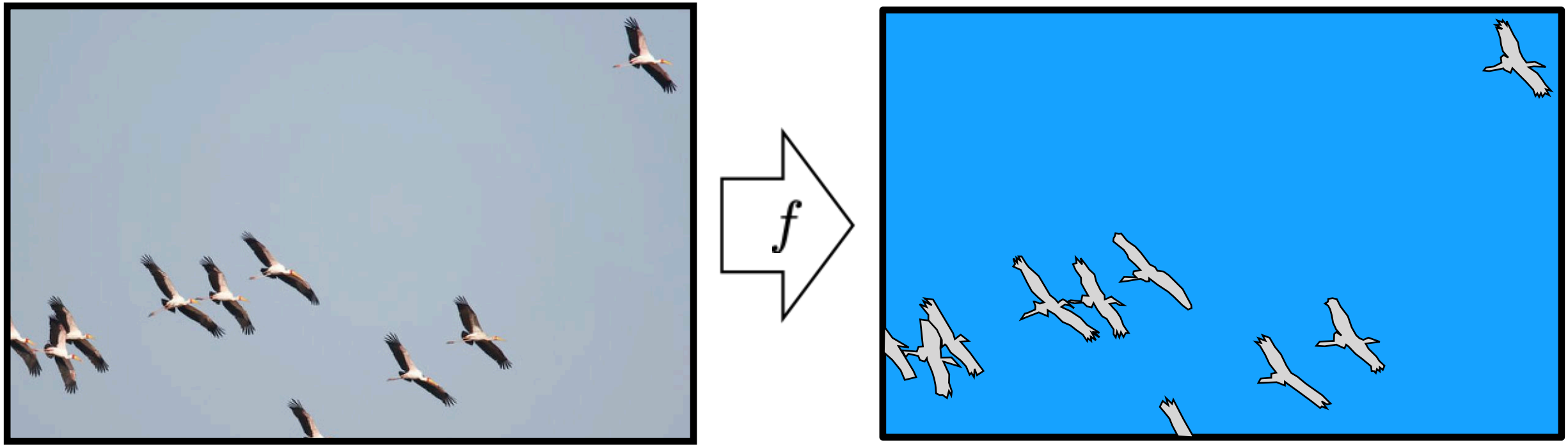
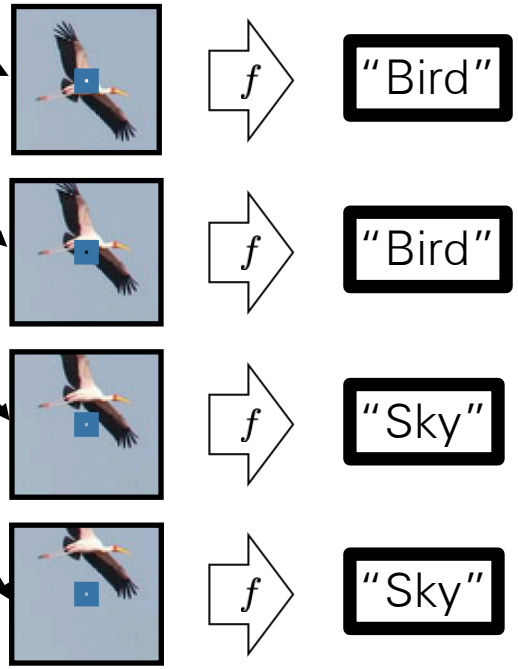




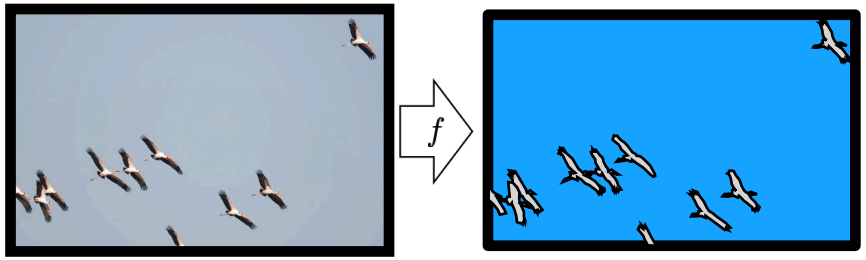
(Colors represent one-hot codes)

This problem is called **semantic segmentation**

What's the object class of the center pixel?



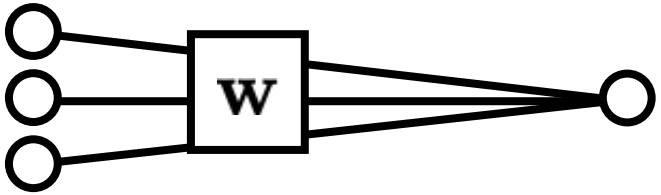
Translation invariance: process each patch in the same way.

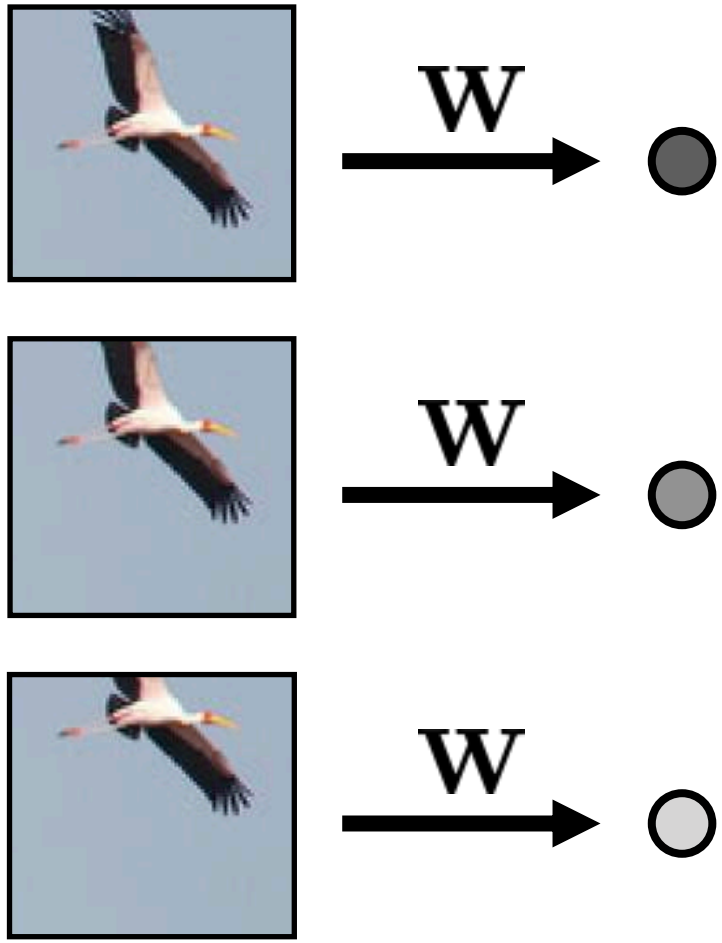


An equivariant mapping:

$$f(\texttt{translate}(x)) = \texttt{translate}(f(x))$$

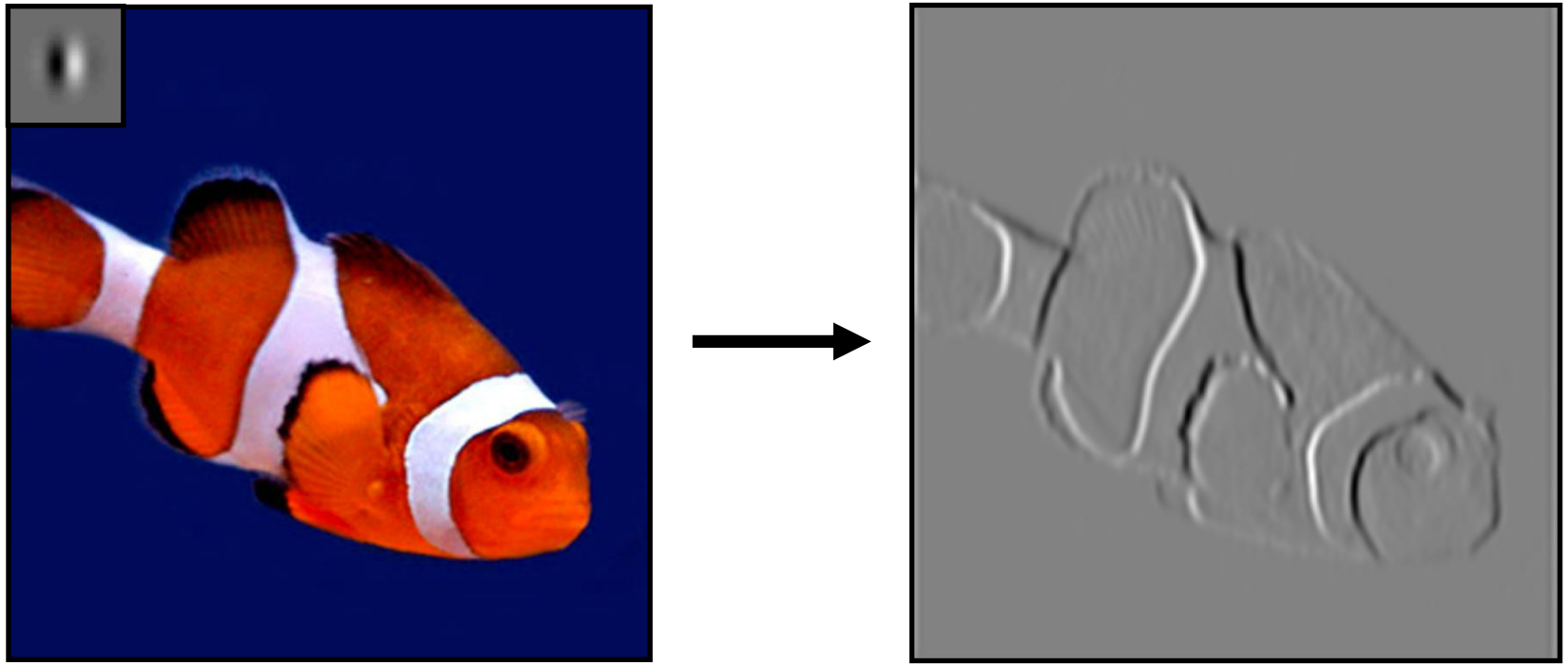
W computes a weighted sum of all pixels in the patch



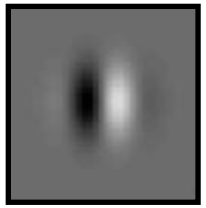


W is a convolutional kernel applied to the full image!

Convolution

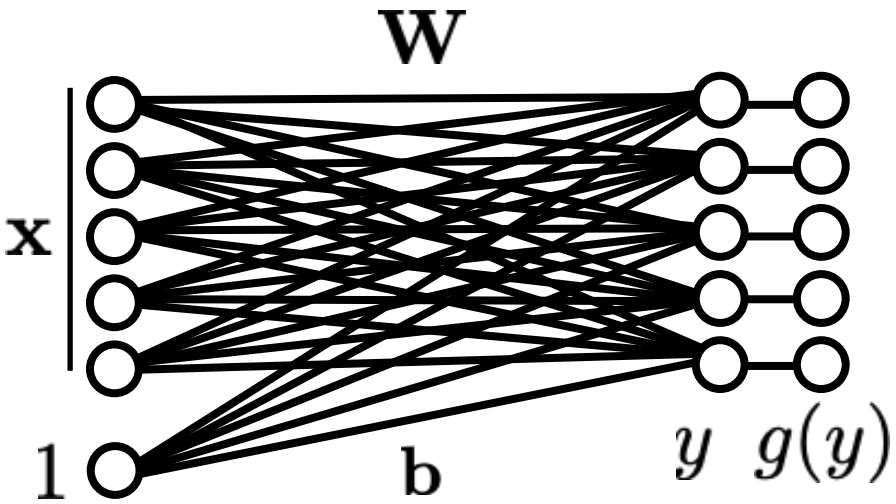


filter

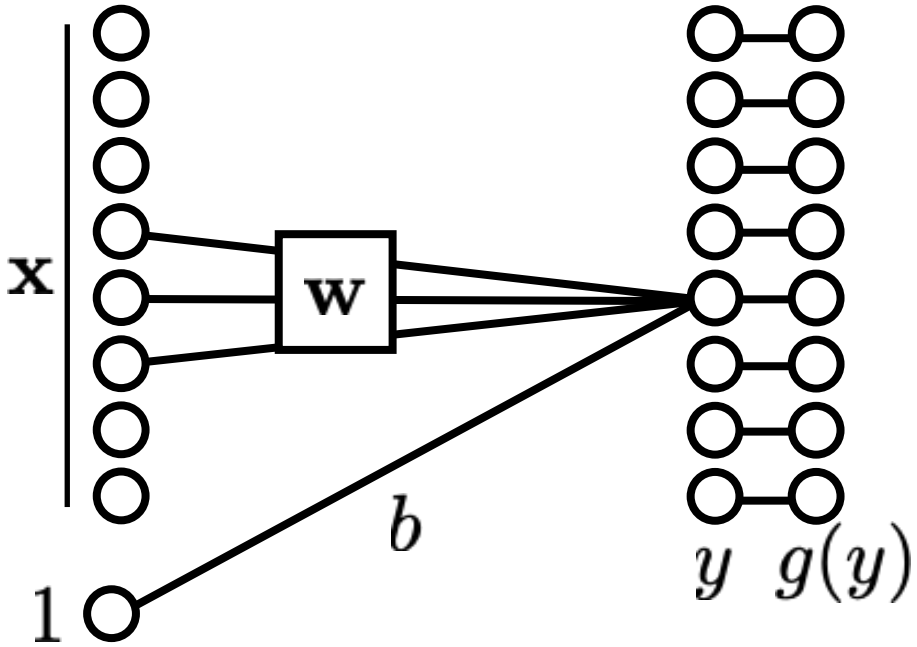


Fully-connected network

Fully-connected (fc) layer



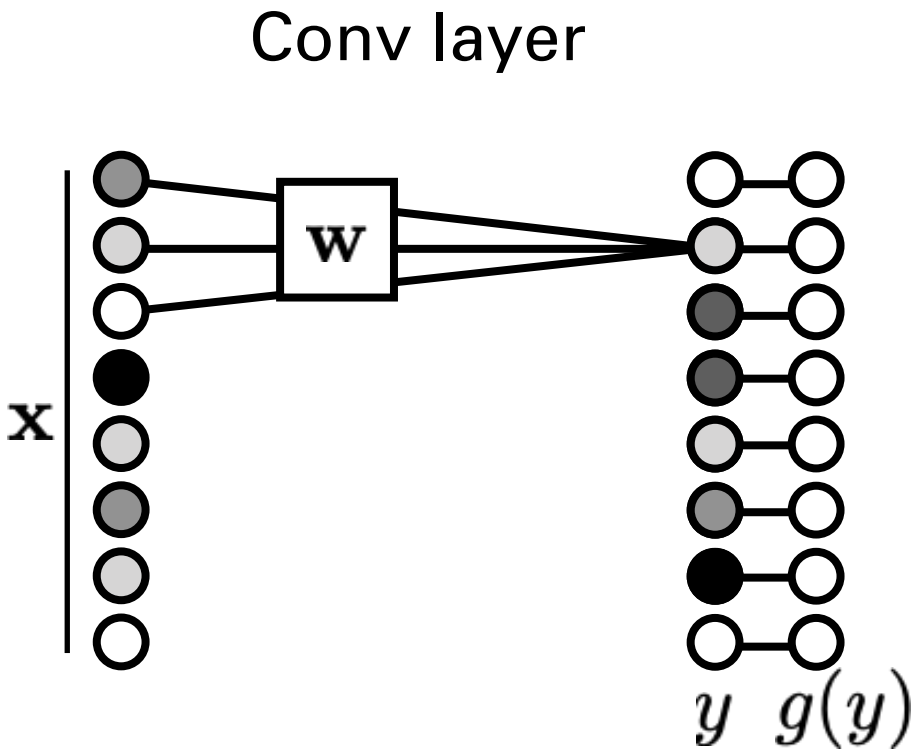
Locally connected network



Often, we assume output is a **local** function of input.

If we use the same weights (**weight sharing**) to compute each local function, we get a convolutional neural network.

Convolutional neural network



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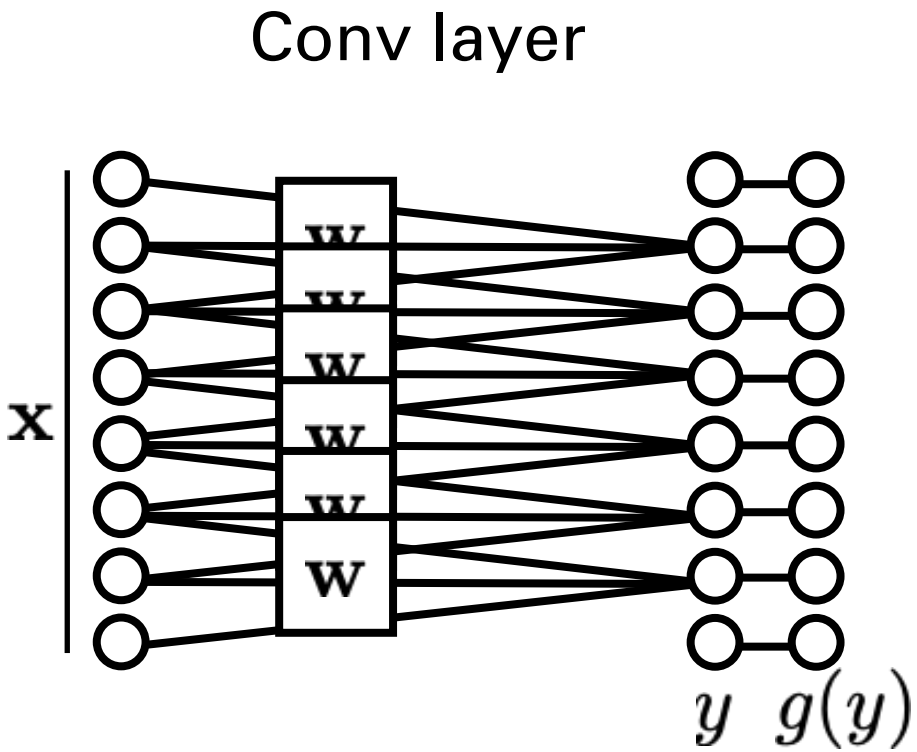
Weight sharing

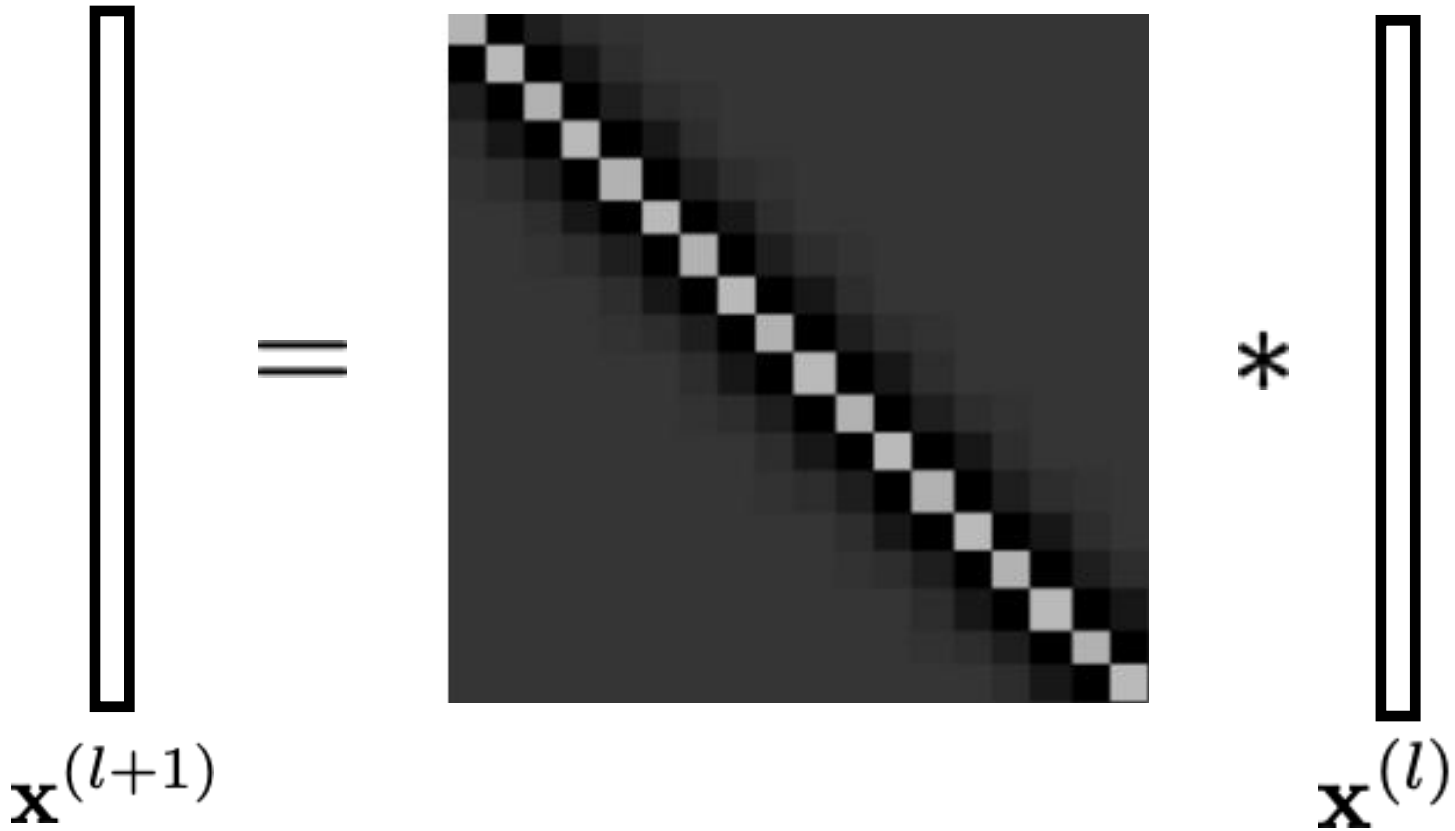


Often, we assume output is a **local** function of input.

If we use the same weights (**weight sharing**) to compute each local function, we get a convolutional neural network.

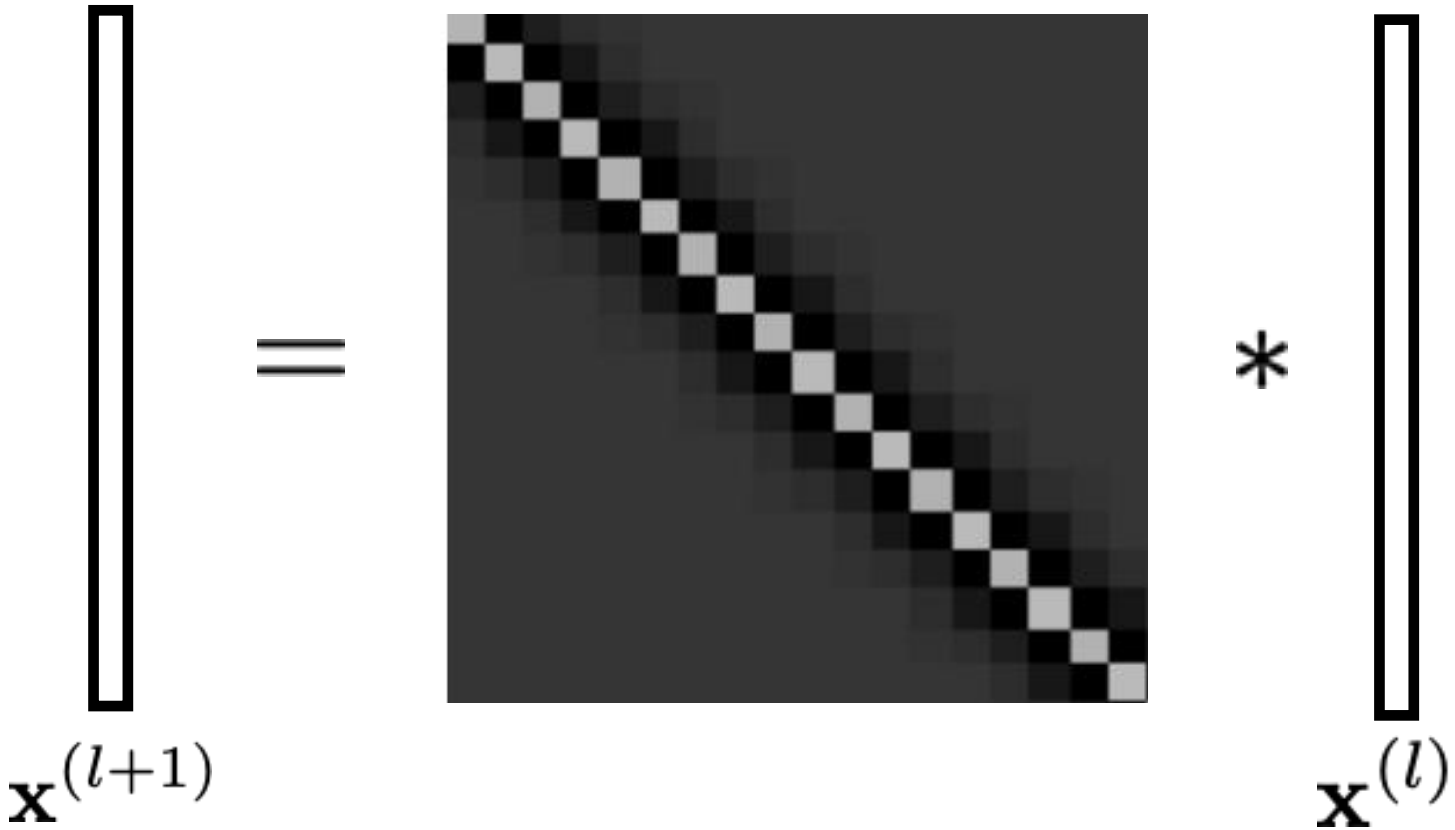
Toeplitz matrix

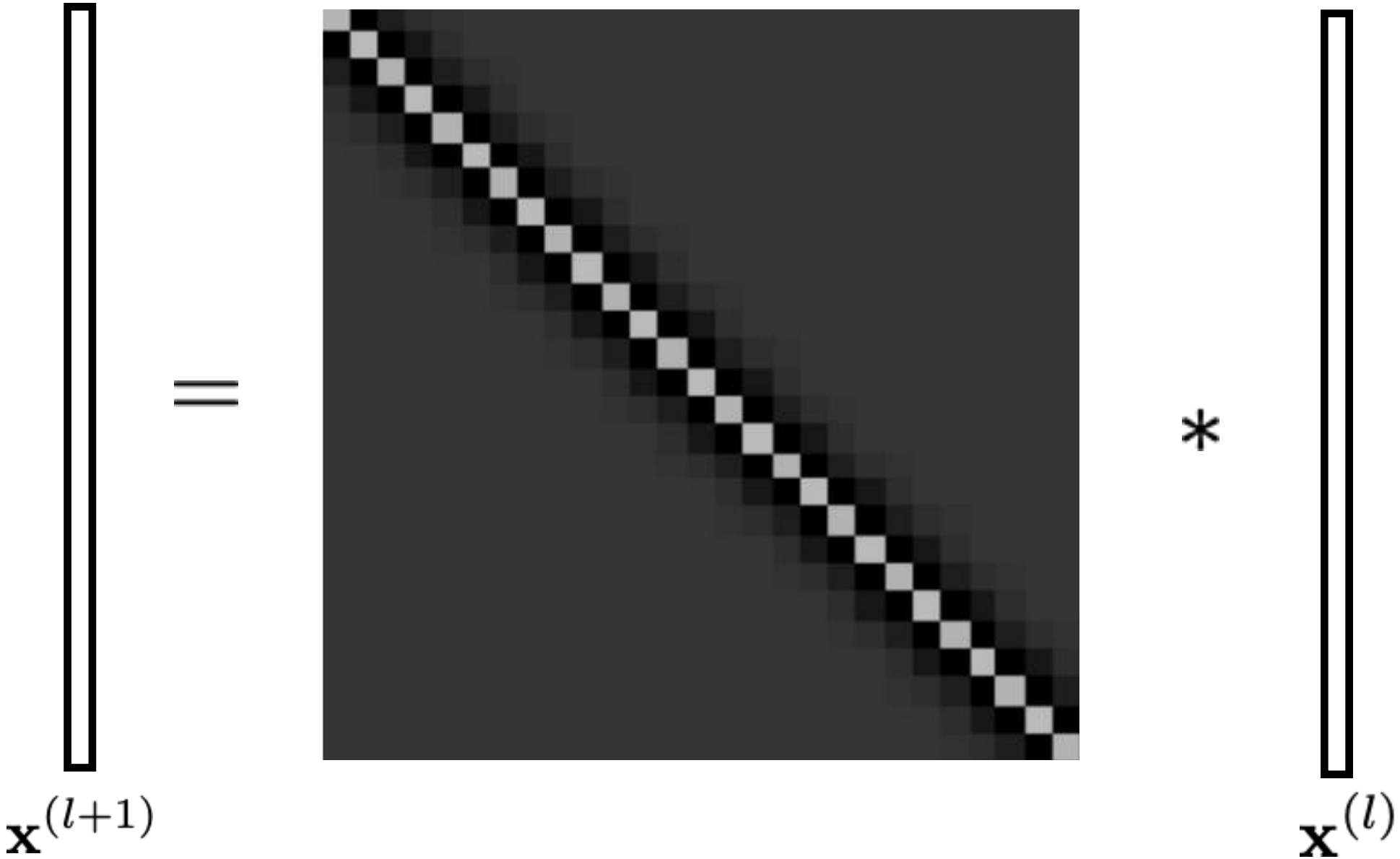
$$\begin{pmatrix} a & b & c & d & e \\ f & a & b & c & d \\ g & f & a & b & c \\ h & g & f & a & b \\ i & h & g & f & a \end{pmatrix}$$



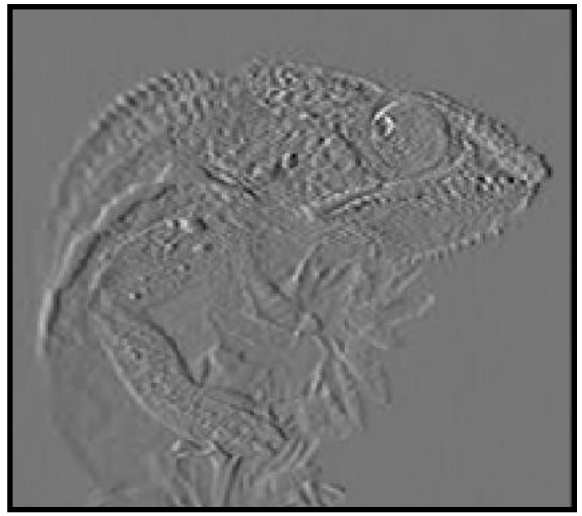
e.g., pixel image

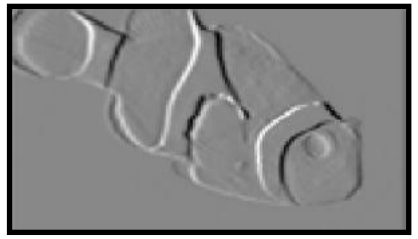
- Constrained linear layer (infinitely strong regularization)
- Fewer parameters —> easier to learn, less overfitting


$\mathbf{x}^{(l+1)}$
=
*
$\mathbf{x}^{(l)}$


$\mathbf{x}^{(l+1)}$
=
*
$\mathbf{x}^{(l)}$



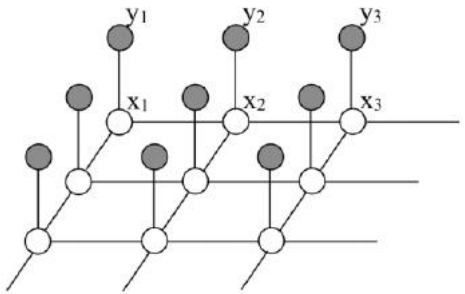


Conv layers can be applied to arbitrarily-sized inputs

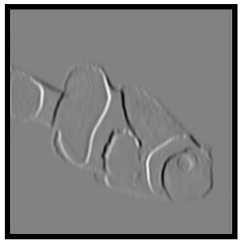
Five views on convolutional layers

1. Equivariant with translation (stationarity) $f(\texttt{translate}(x)) = \texttt{translate}(f(x))$

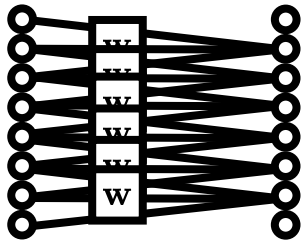


2. Patch processing (Markov assumption)

3. Image filter

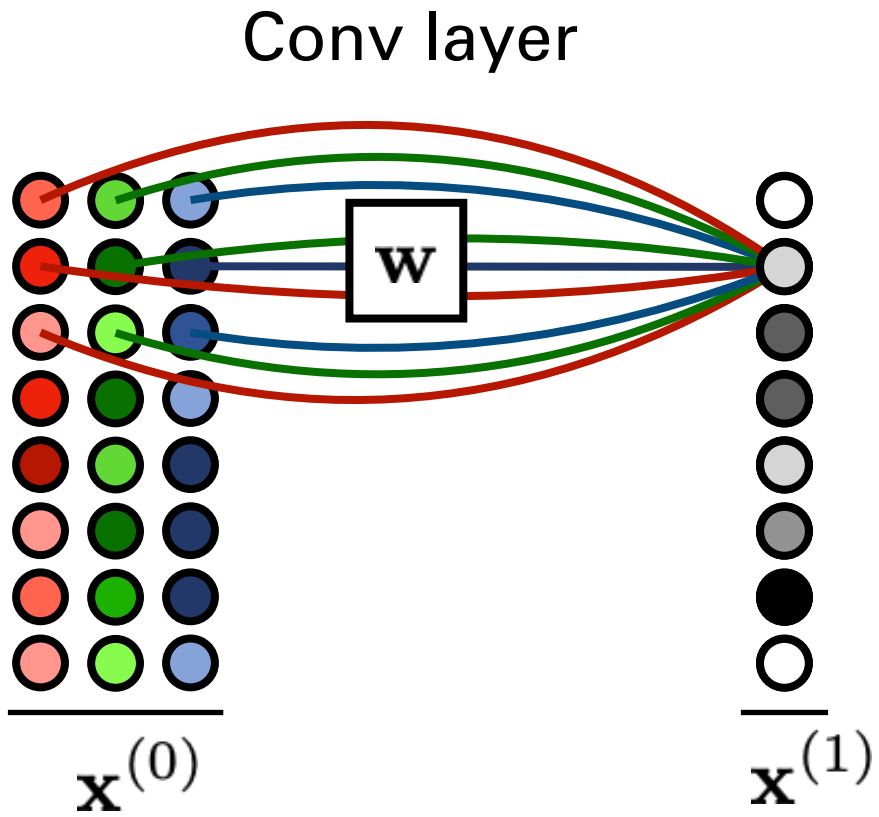


4. Parameter sharing



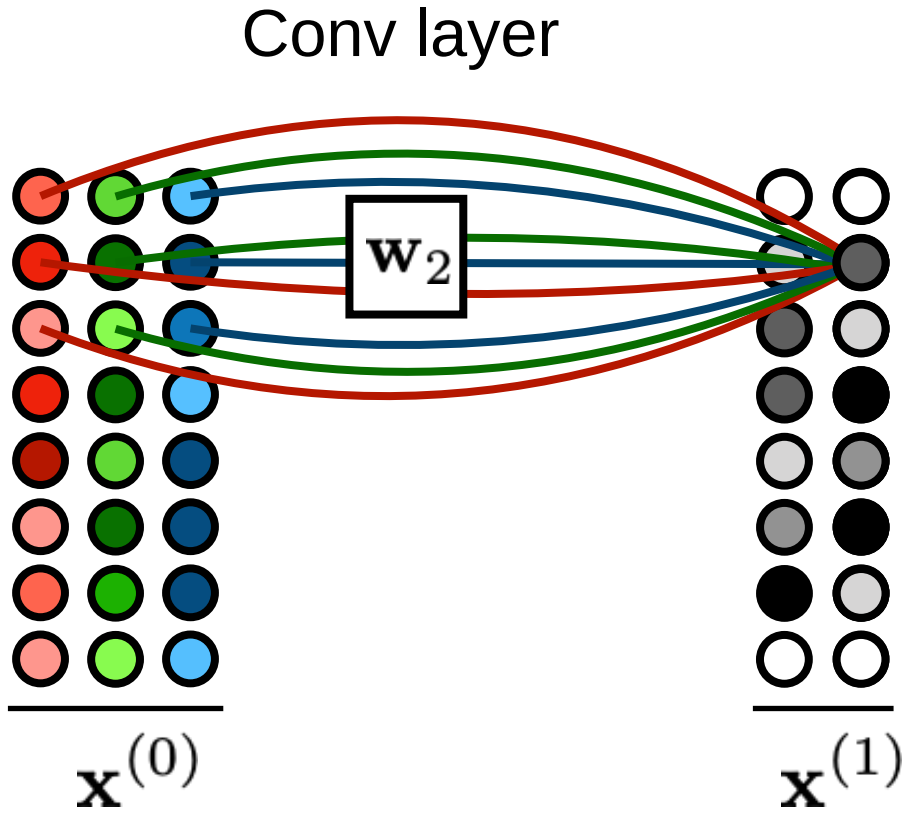
5. A way to process variable-sized tensors

Multiple channels



$$\mathbb{R}^{N \times C} \rightarrow \mathbb{R}^{N \times 1}$$

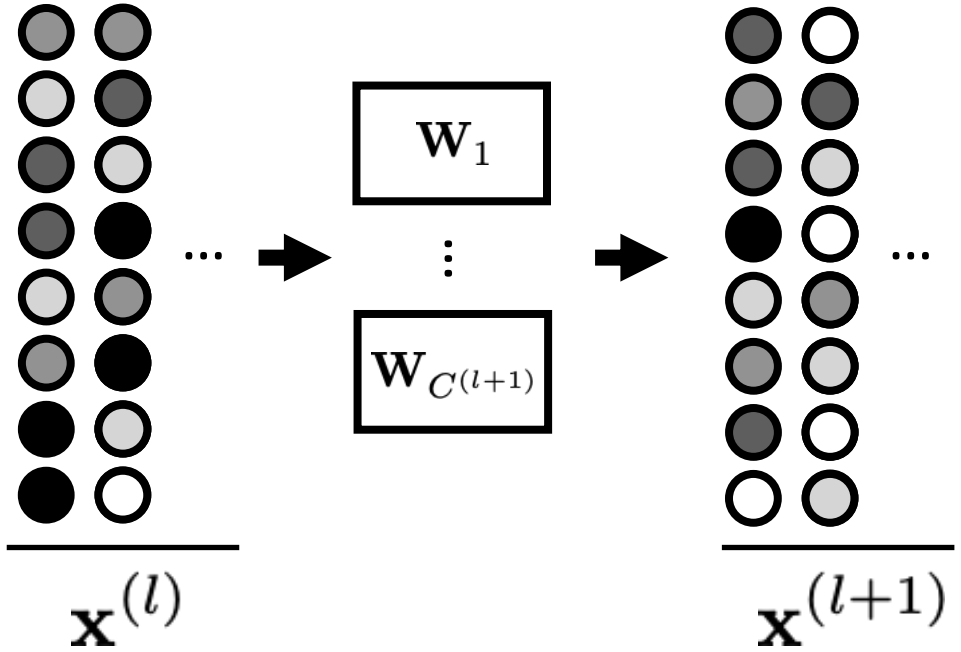
Multiple channels



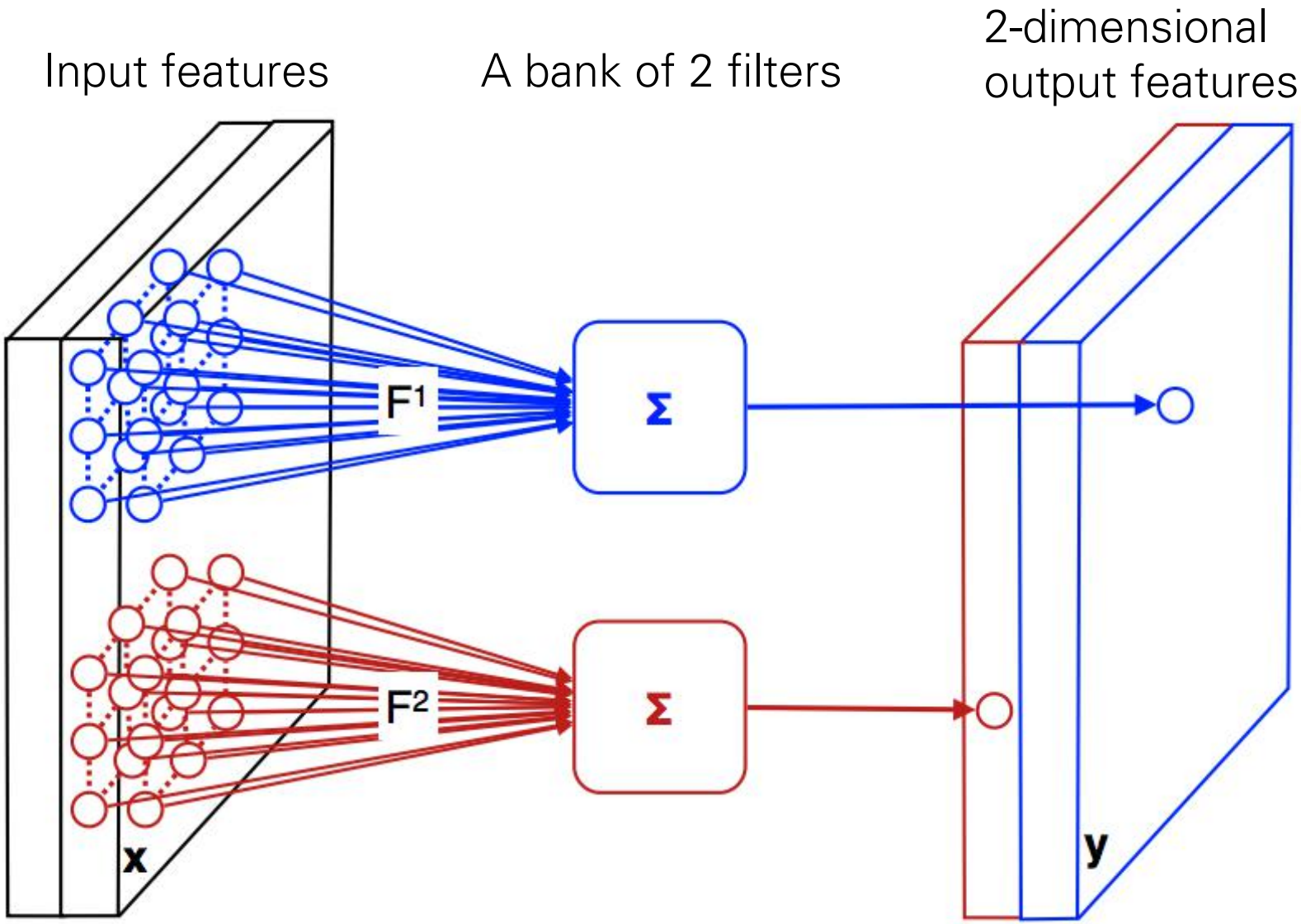
$$\mathbb{R}^{N \times C^{(0)}} \rightarrow \mathbb{R}^{N \times C^{(1)}}$$

Multiple channels

Conv layer



$$\mathbb{R}^{N \times C^{(l)}} \rightarrow \mathbb{R}^{N \times C^{(l+1)}}$$



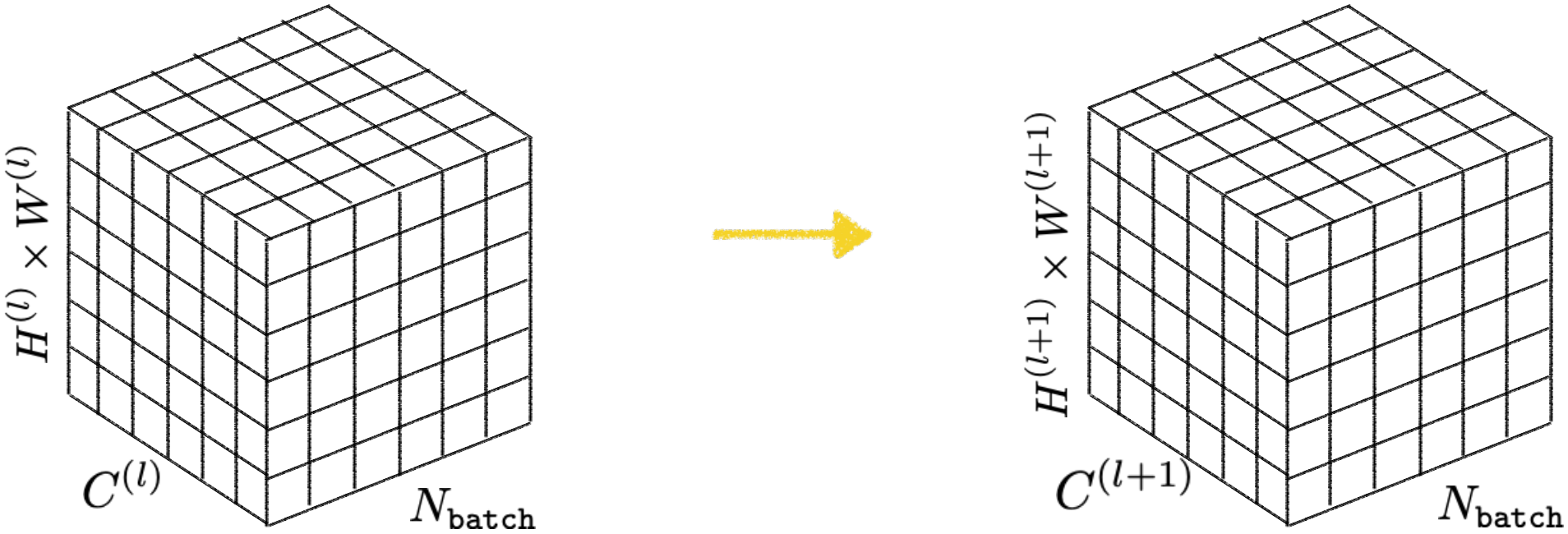
$$\mathbb{R}^{H \times W \times C^{(l)}} \rightarrow \mathbb{R}^{H \times W \times C^{(l+1)}}$$

[Figure from Andrea Vedaldi]

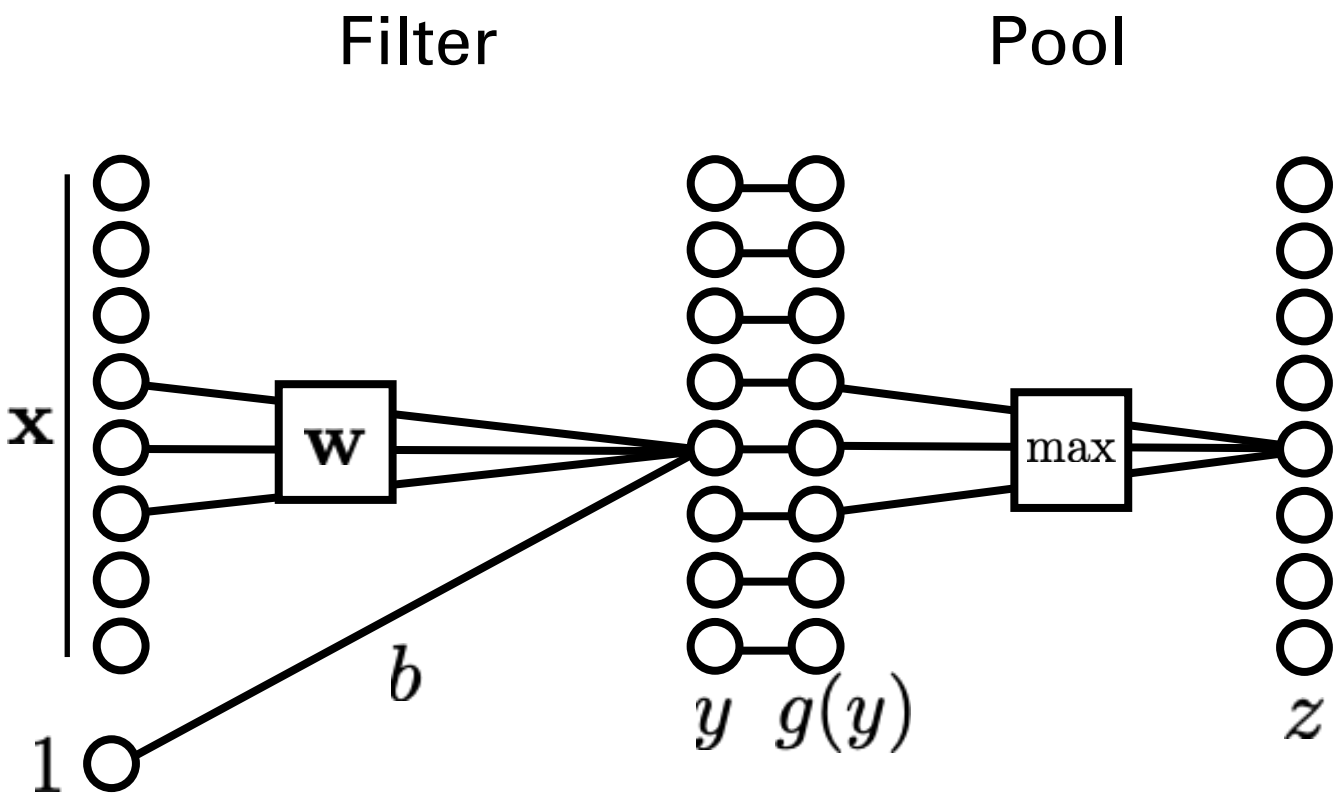
"Tensor flow"

$$\mathbf{x}^{(l)} \in \mathbb{R}^{N_{\text{batch}} \times H^{(l)} \times W^{(l)} \times C^{(l)}}$$

$$\mathbf{x}^{(l+1)} \in \mathbb{R}^{N_{\text{batch}} \times H^{(l+1)} \times W^{(l+1)} \times C^{(l+1)}}$$



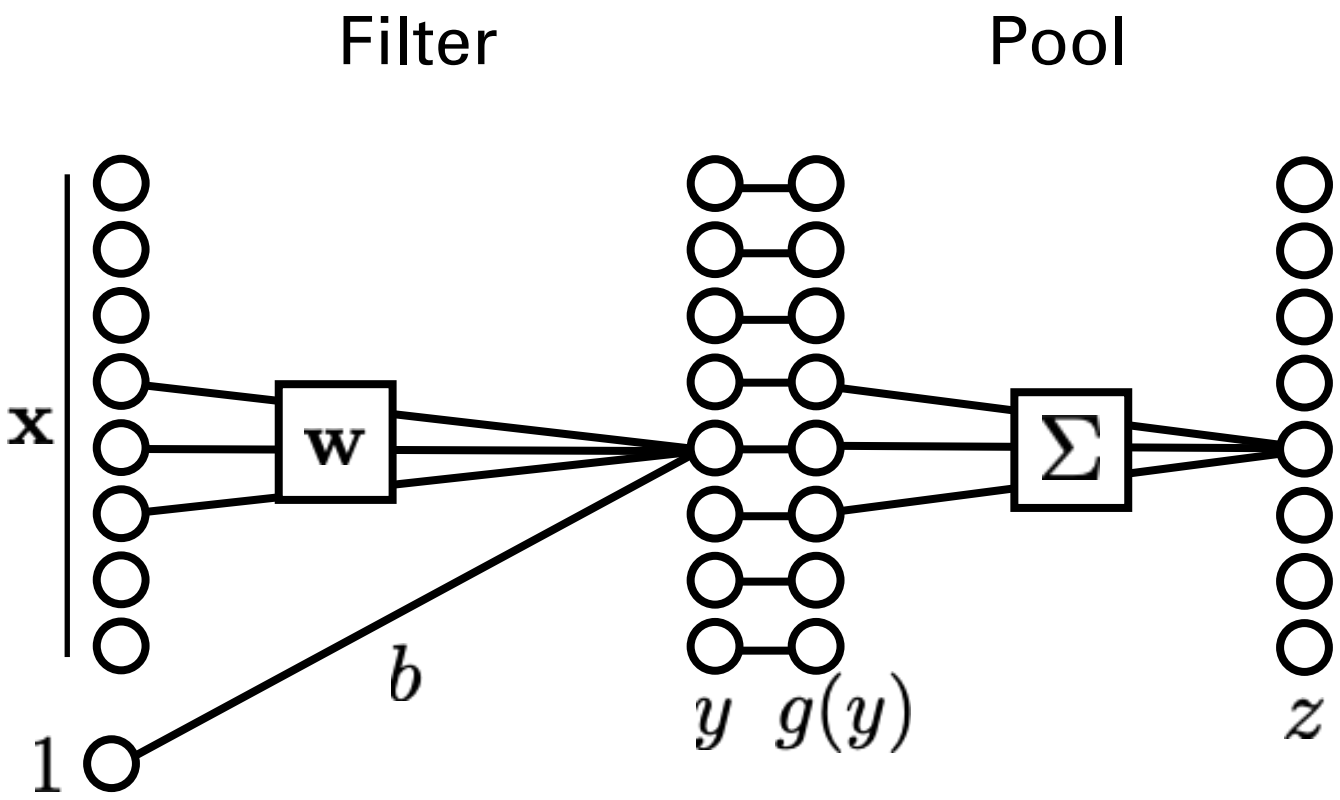
Pooling



Max pooling

$$z_k = \max_{j \in \mathcal{N}(j)} g(y_j)$$

Pooling



Max pooling

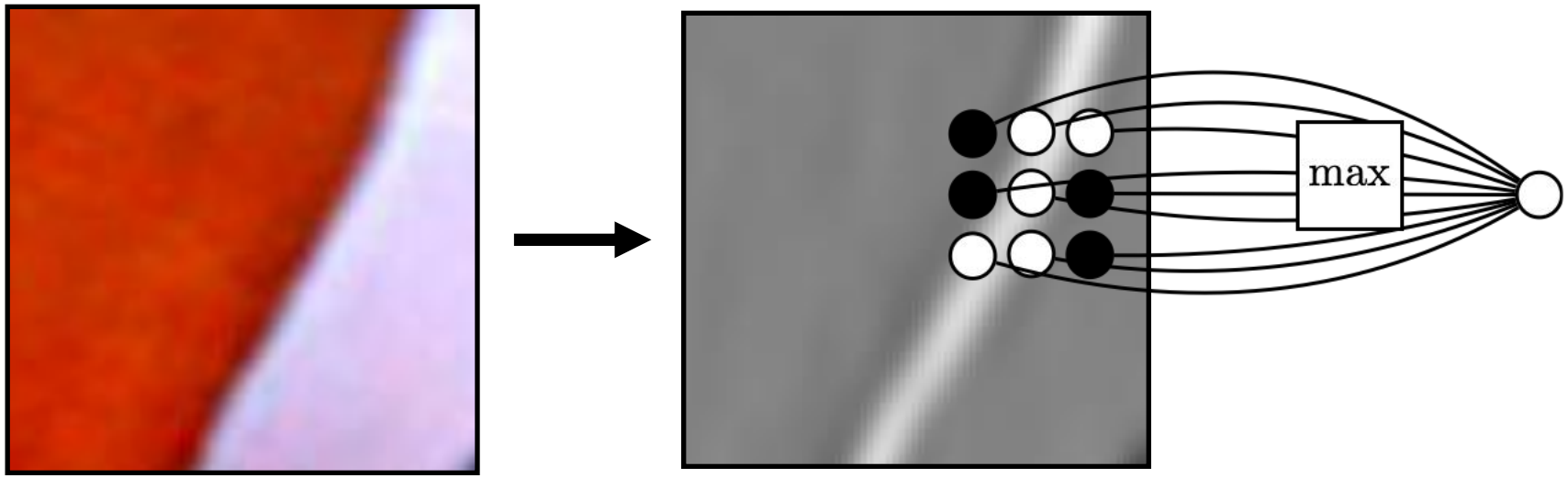
$$z_k = \max_{j \in \mathcal{N}(j)} g(y_j)$$

Mean pooling

$$z_k = \frac{1}{|\mathcal{N}|} \sum_{j \in \mathcal{N}(j)} g(y_j)$$

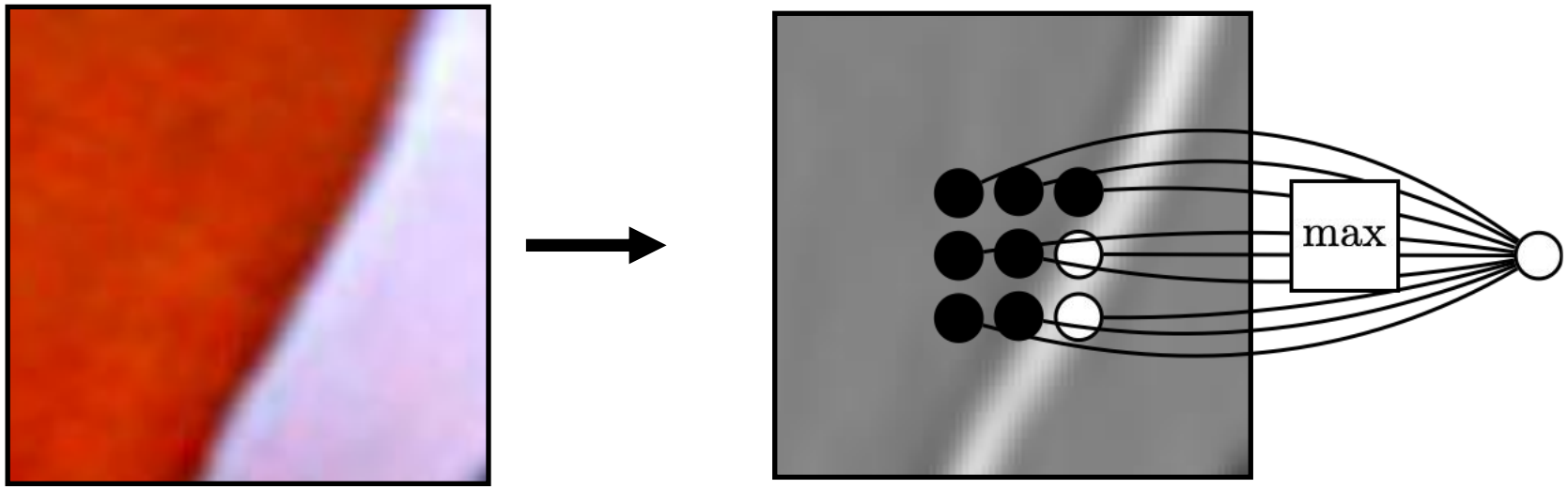
Pooling – Why?

Pooling across spatial locations achieves stability w.r.t. small translations:



Pooling – Why?

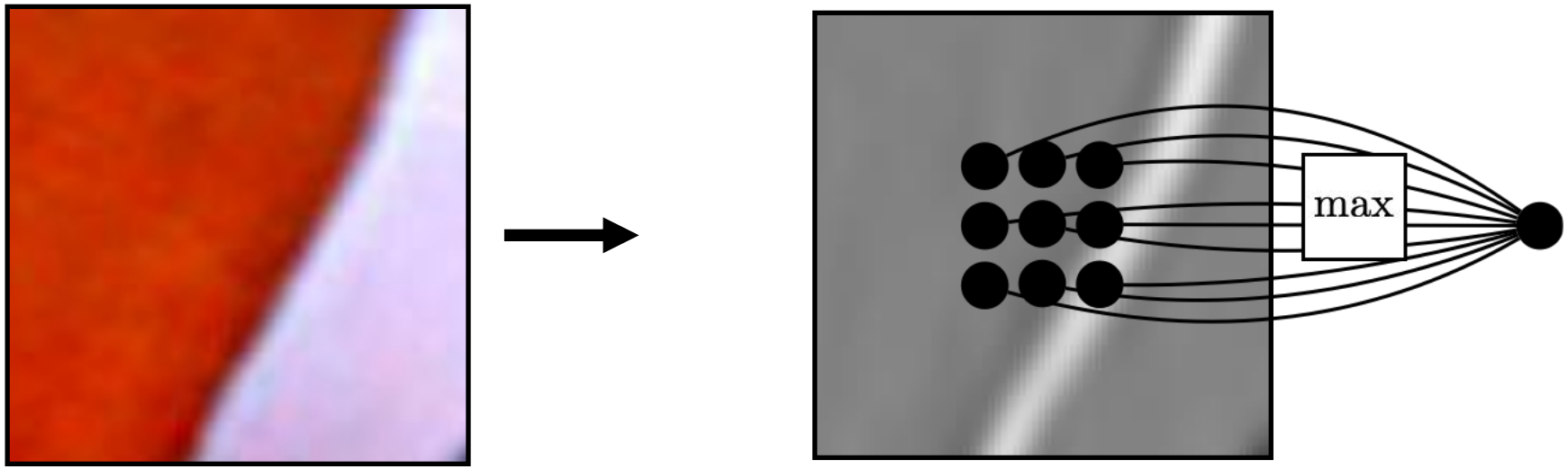
Pooling across spatial locations achieves stability w.r.t. small translations:



large response regardless of exact position of edge

Pooling – Why?

Pooling across spatial locations achieves stability w.r.t. small translations:

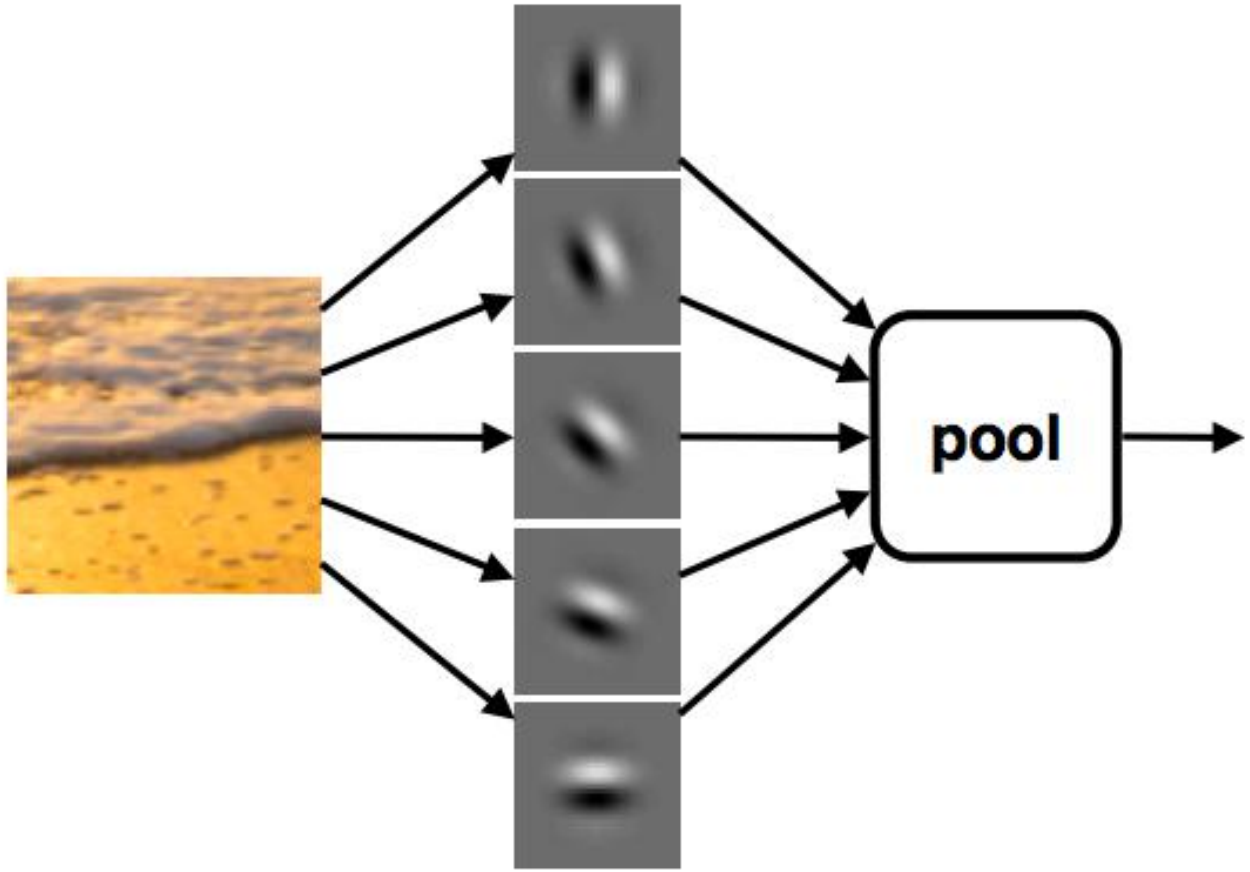


CNNs are stable w.r.t. diffeomorphisms

["Unreasonable effectiveness of Deep Features as a Perceptual Metric", Zhang et al. 201

Pooling – Why?

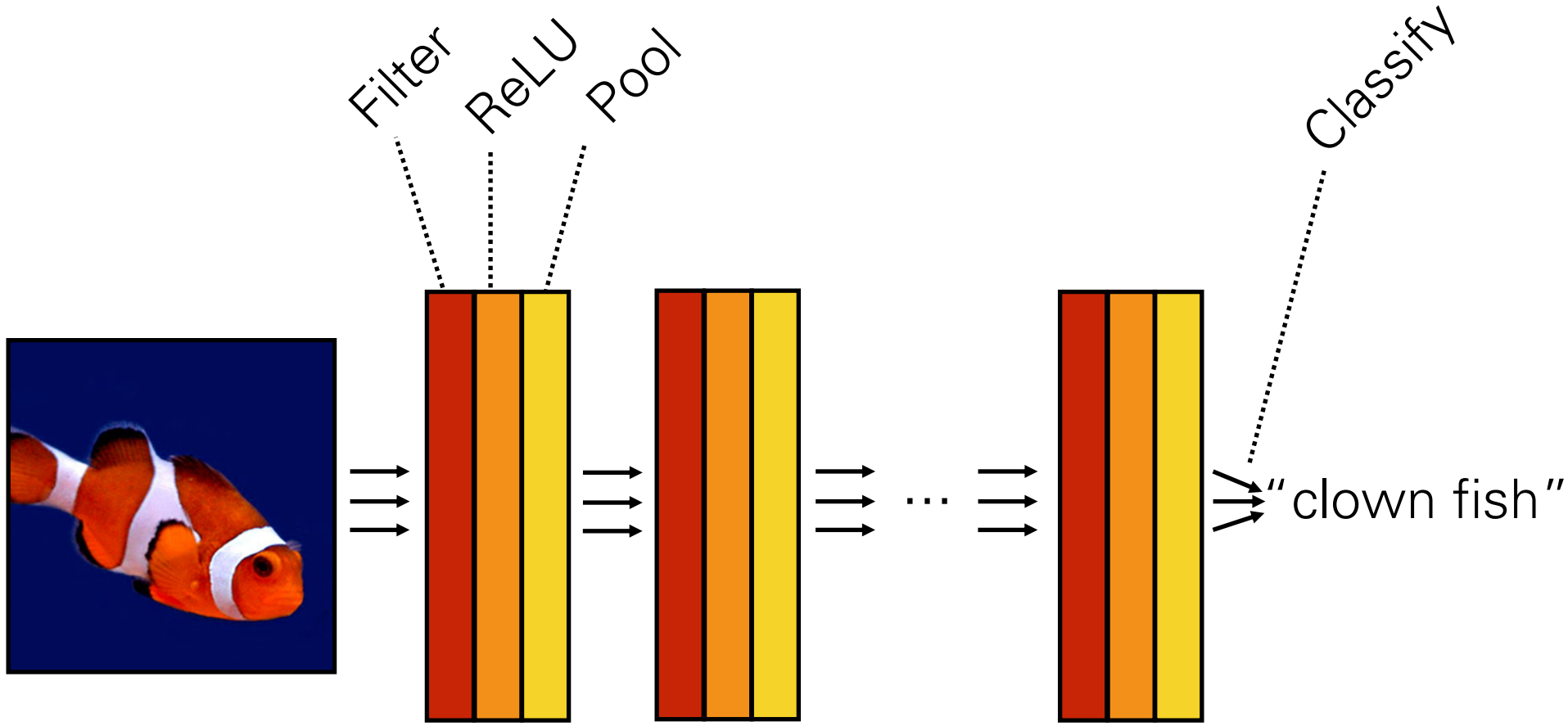
Pooling across feature channels (filter outputs) can achieve other kinds of invariances:



large response for any edge, regardless of its orientation

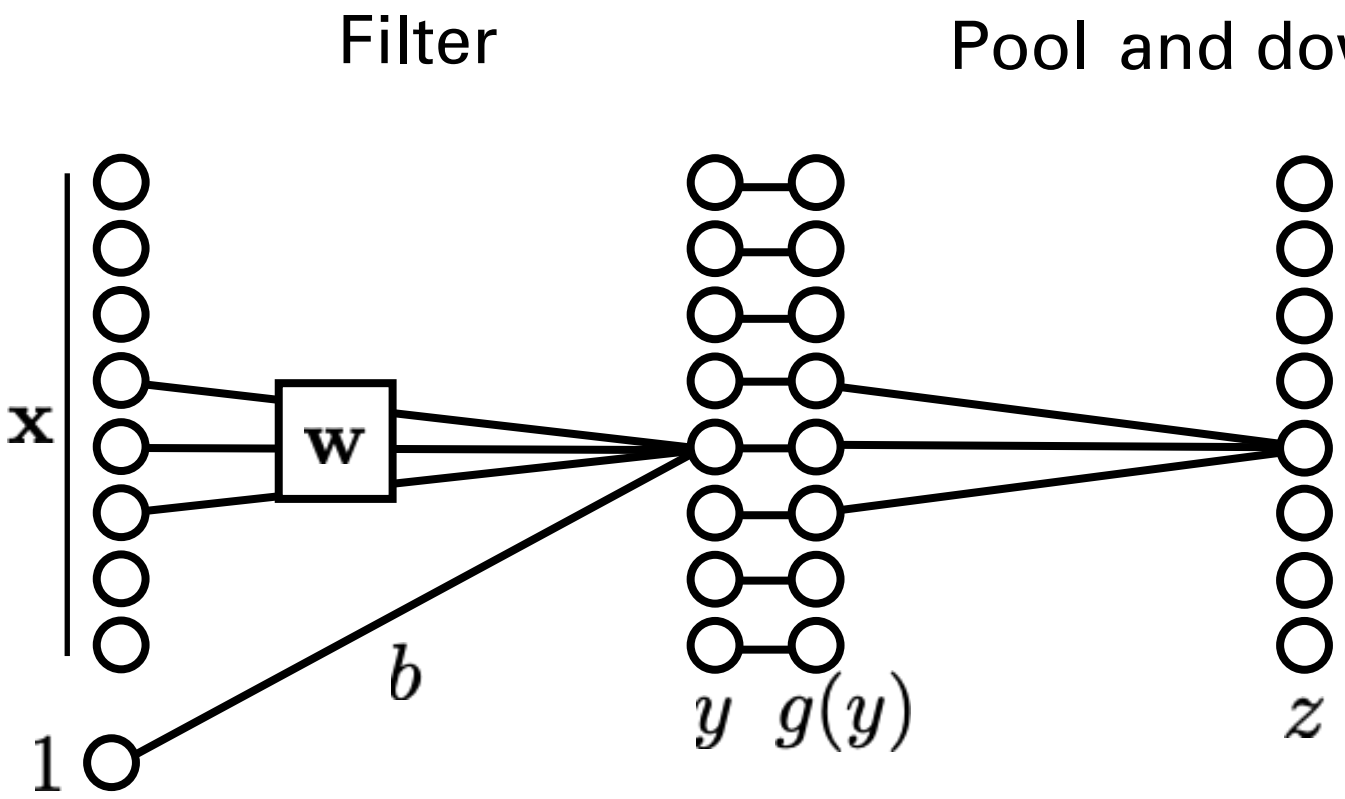
[Derived from slide by Andrea Vedaldi]

Computation in a neural net

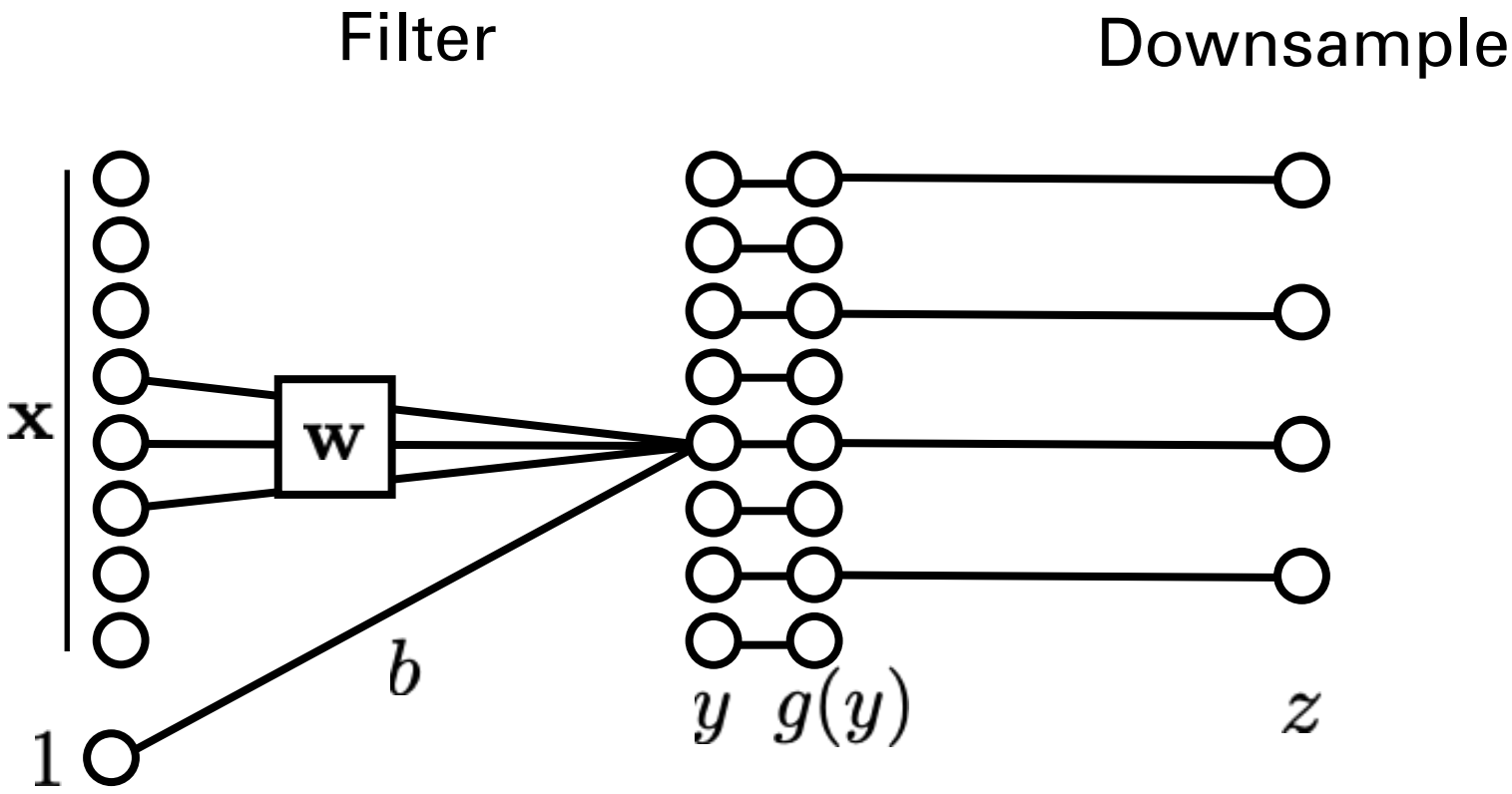


$$f(\mathbf{x}) = f_L(\ldots f_2(f_1(\mathbf{x})))$$

Downsampling

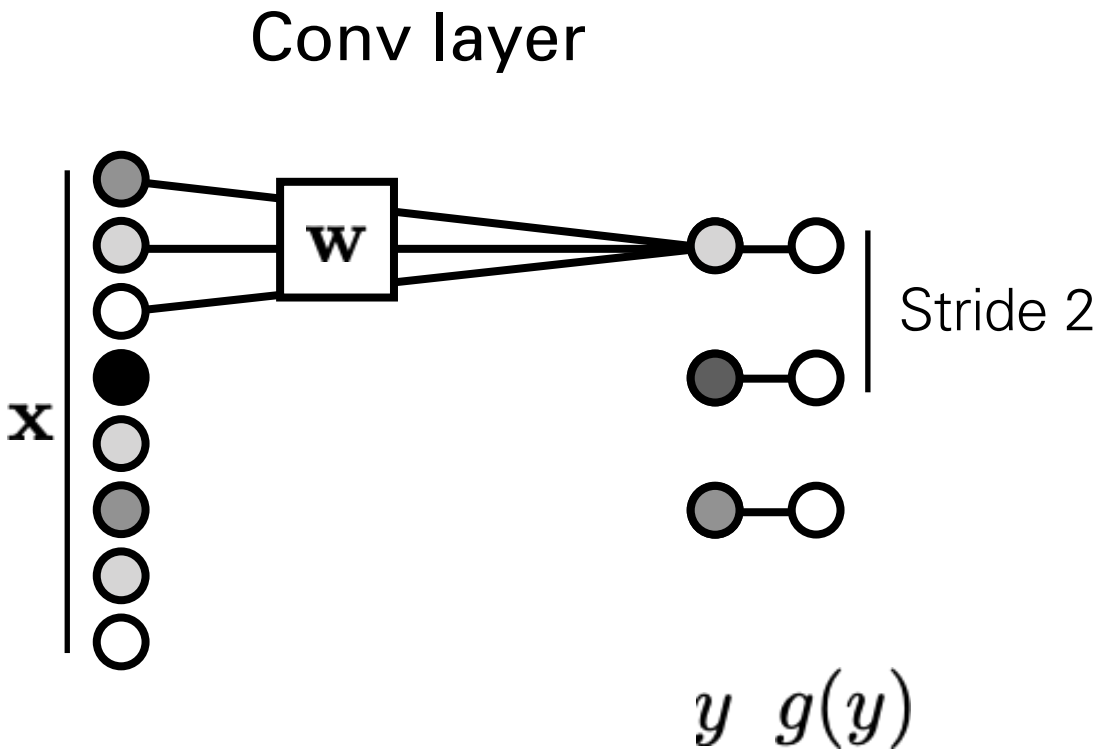


Downsampling



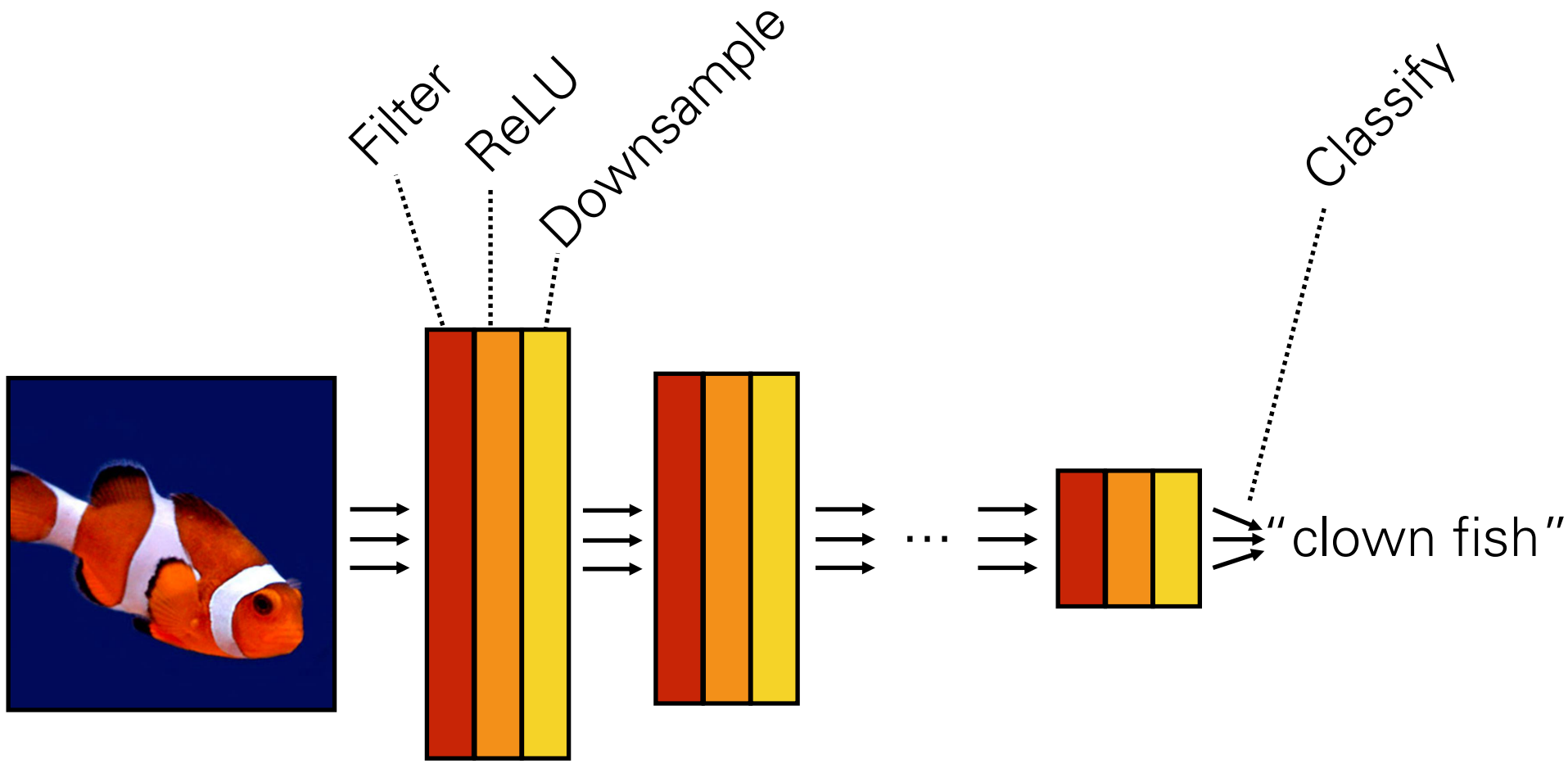
$$\mathbb{R}^{H^{(l)} \times W^{(l)} \times C^{(l)}} \rightarrow \mathbb{R}^{H^{(l+1)} \times W^{(l+1)} \times C^{(l+1)}}$$

Strided convolution



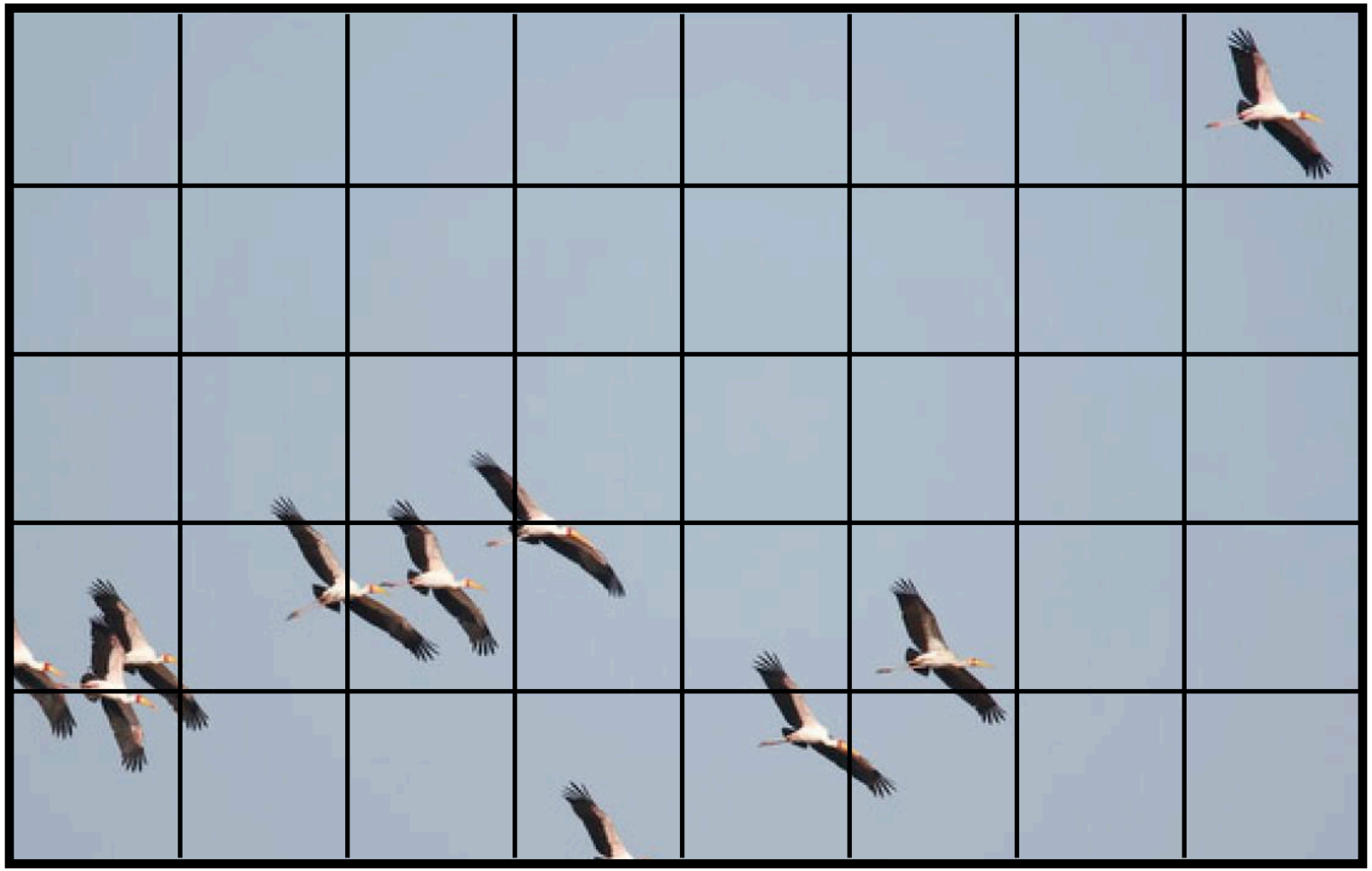
Strided convolutions combine convolution and downsampling into a single operation.

Computation in a neural net

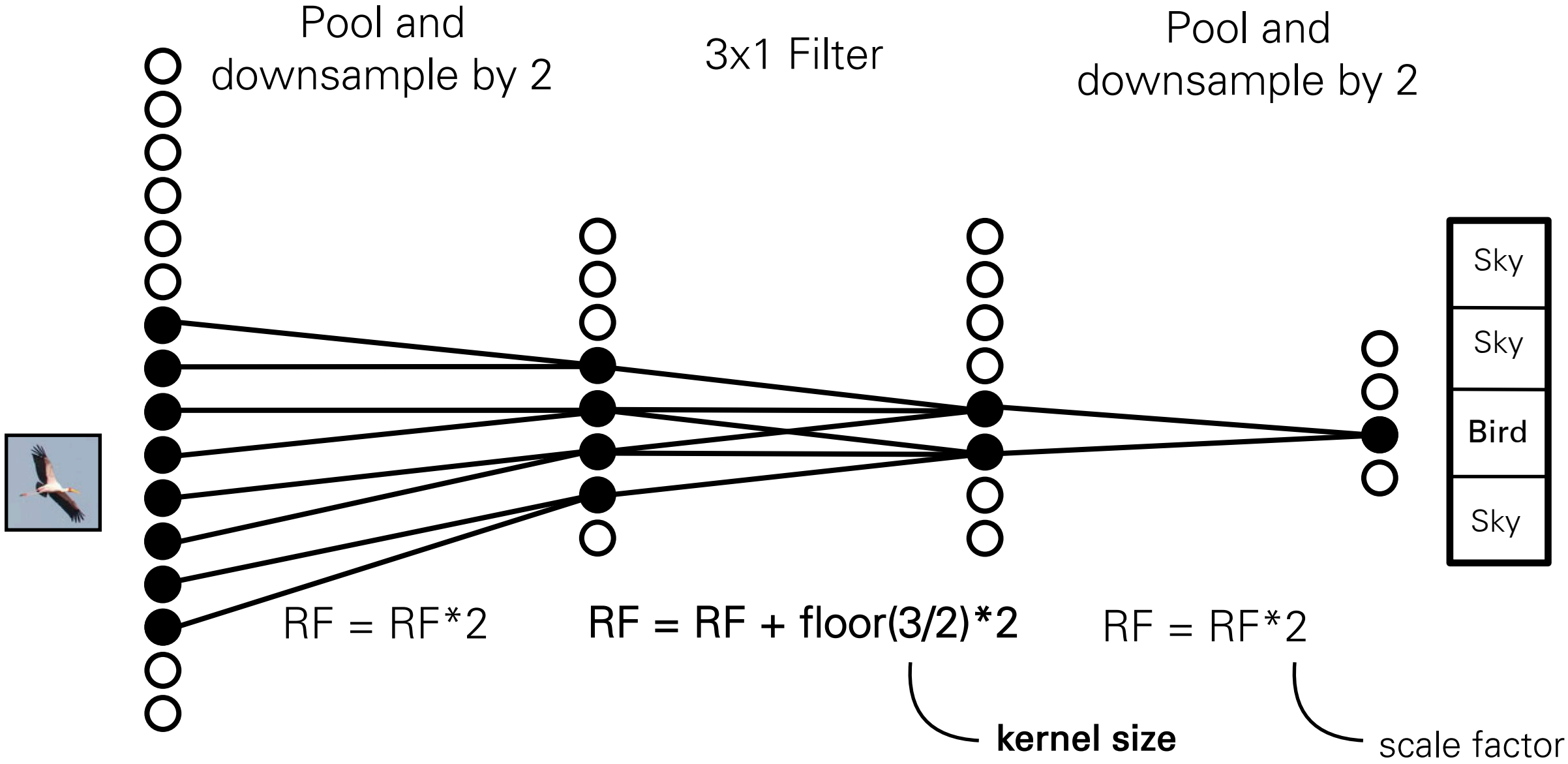


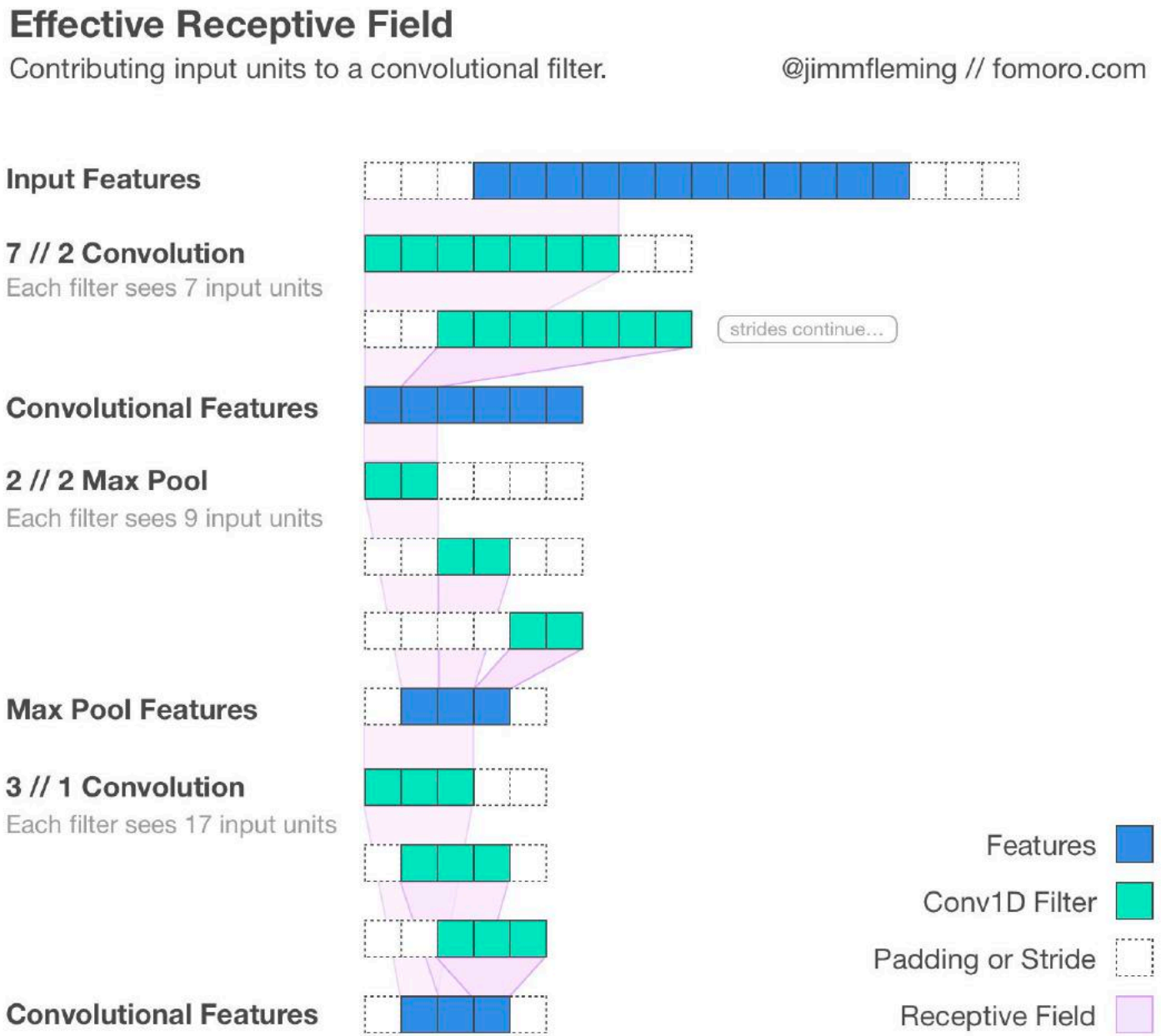
$$f(\mathbf{x}) = f_L(\dots f_2(f_1(\mathbf{x})))$$

Receptive fields



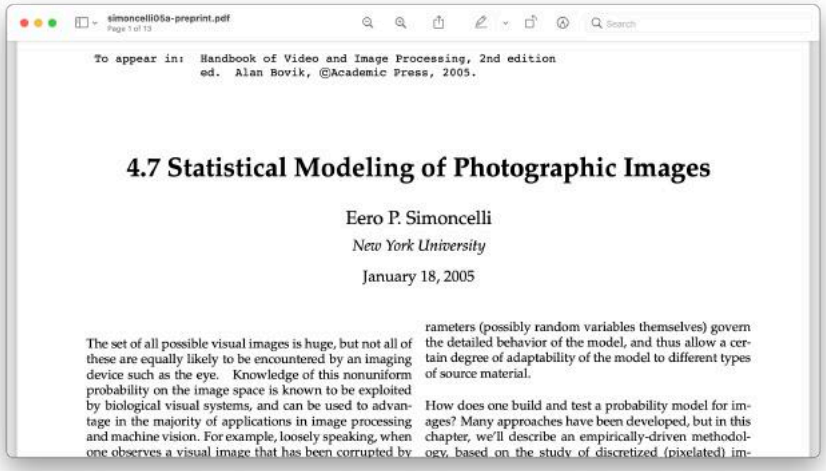
Receptive fields





[http://fomoro.com/tools/receptive-fields/index.html]

Why CNNs?



To appear in: Handbook of Video and Image Processing, 2nd edition
ed. Alan Bovik, @Academic Press, 2005.

4.7 Statistical Modeling of Photographic Images

Eero P. Simoncelli
New York University

January 18, 2005

The set of all possible visual images is huge, but not all of these are equally likely to be encountered by an imaging device such as the eye. Knowledge of this nonuniform probability on the image space is known to be exploited by biological visual systems, and can be used to advantage in the majority of applications in image processing and machine vision. For example, loosely speaking, when one observes a visual image that has been corrupted by

rameters (possibly random variables themselves) govern the detailed behavior of the model, and thus allow a certain degree of adaptability of the model to different types of source material.

How does one build and test a probability model for images? Many approaches have been developed, but in this chapter, we'll describe an empirically-driven methodology, based on the study of discretized (pixelated) im-

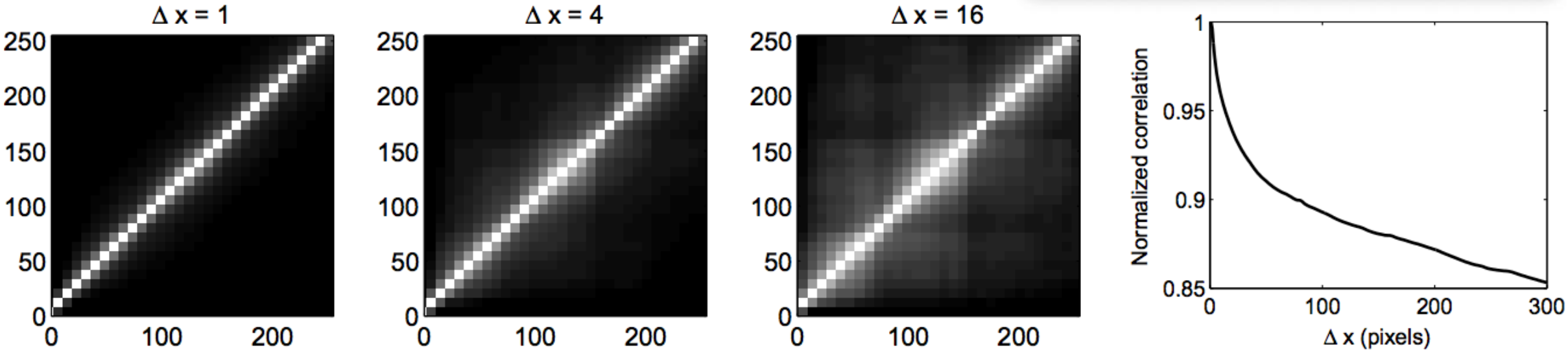


Fig. 1. (a) Scatterplots of pairs of pixels at three different spatial displacements, averaged over five examples images. (b) Autocorrelation function. Photographs are of New York City street scenes, taken with a Canon 10D digital camera, and processed in RAW linear sensor mode (producing pixel intensities are in roughly proportional to light intensity). Correlations were computed on the logs of these sensor intensity values [41].

[http://6.869.csail.mit.edu/fa18/notes/simoncelli2005.pdf]

Why CNNs?

Statistical dependences between pixels decay as a power law of distance between the pixels.

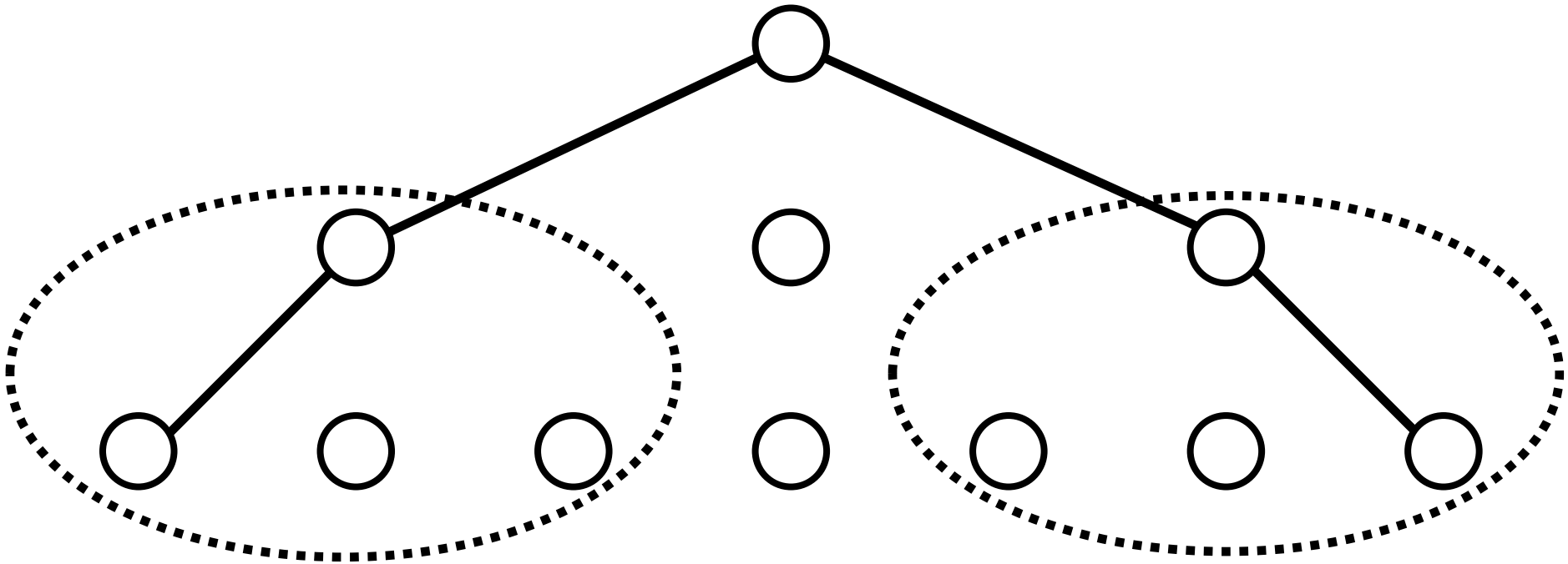
It is therefore often sufficient to model local dependences only. —> **Convolution**

More generally, we should allocate parameters that model dependences in proportion to the strength of those dependences. —> **Multiscale, hierarchical representations**

[For more discussion, see "Why does Deep and Cheap Learning Work So Well?", Lin et al. 2017]

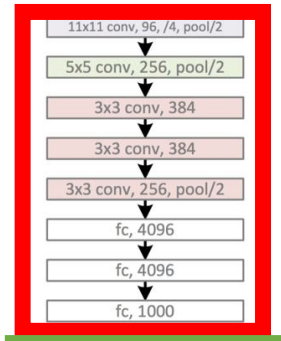
Why CNNs?

Capturing long-range dependences:



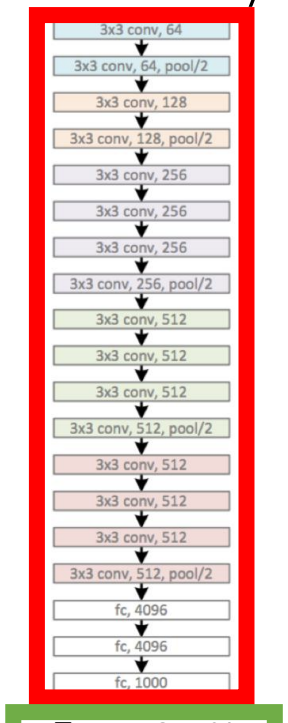
Deep Neural Networks for Visual Recognition

2012: AlexNet
5 conv. layers



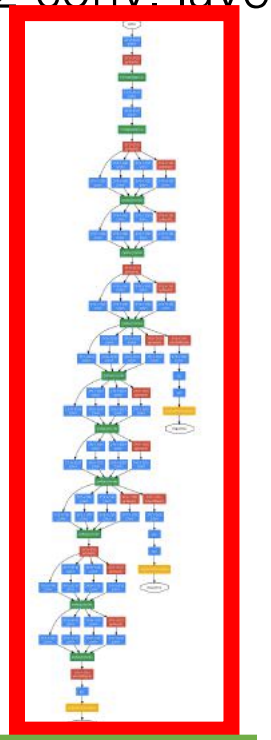
Error: 15.3%

2014: VGG
16 conv. layers



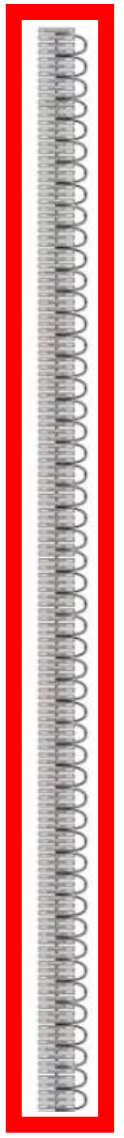
Error: 8.5%

2015: GoogLeNet
22 conv. layers



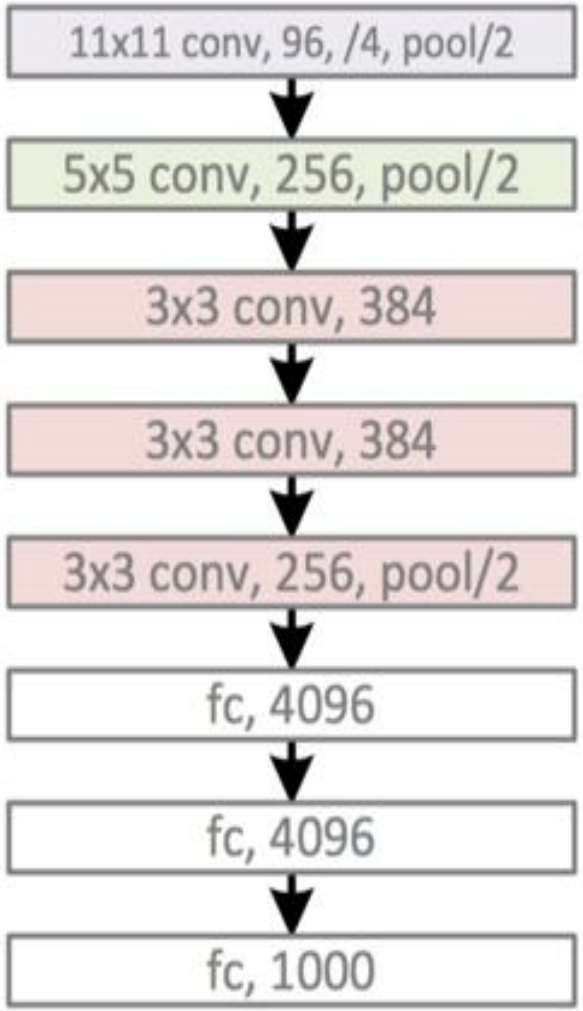
Error: 7.8%

2016: ResNet
>100 conv. layers



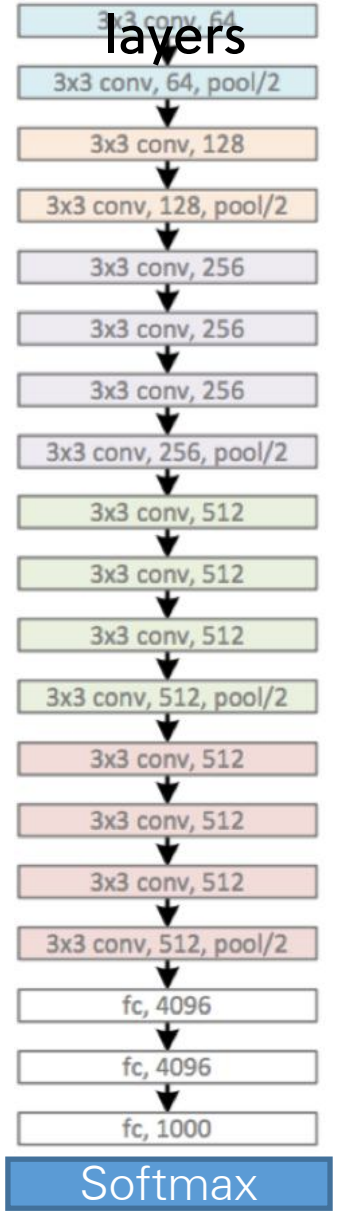
Error: 4.4%

2012: AlexNet
5 conv. layers



Error: 15.3%

2014: VGG
16 conv.
layers

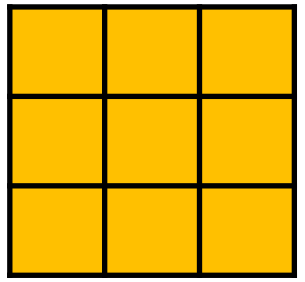


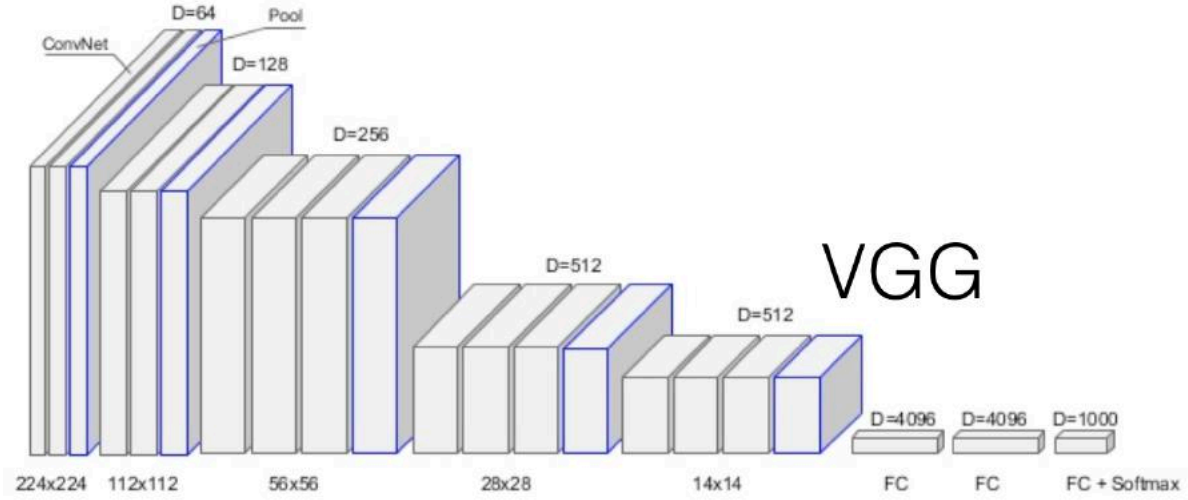
Error: 8.5%

VERY DEEP CONVOLUTIONAL NETWORKS FOR LARGE-SCALE IMAGE RECOGNITION

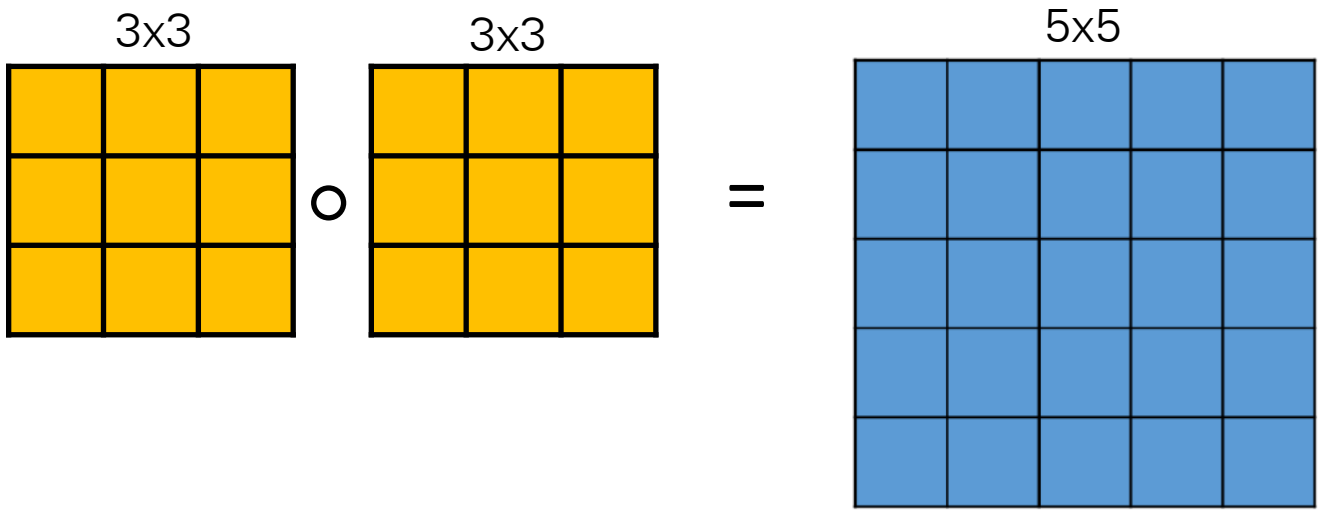
https://arxiv.org/pdf/1409.1556.pdf

Small convolutional kernels: 3x3
ReLu non-linearities
>100 million parameters.

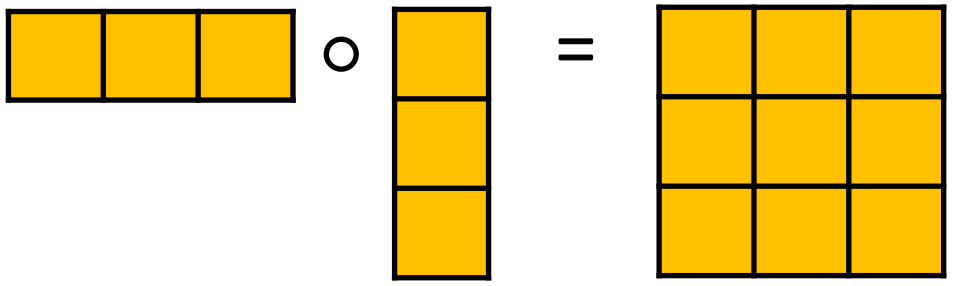




Chaining convolutions

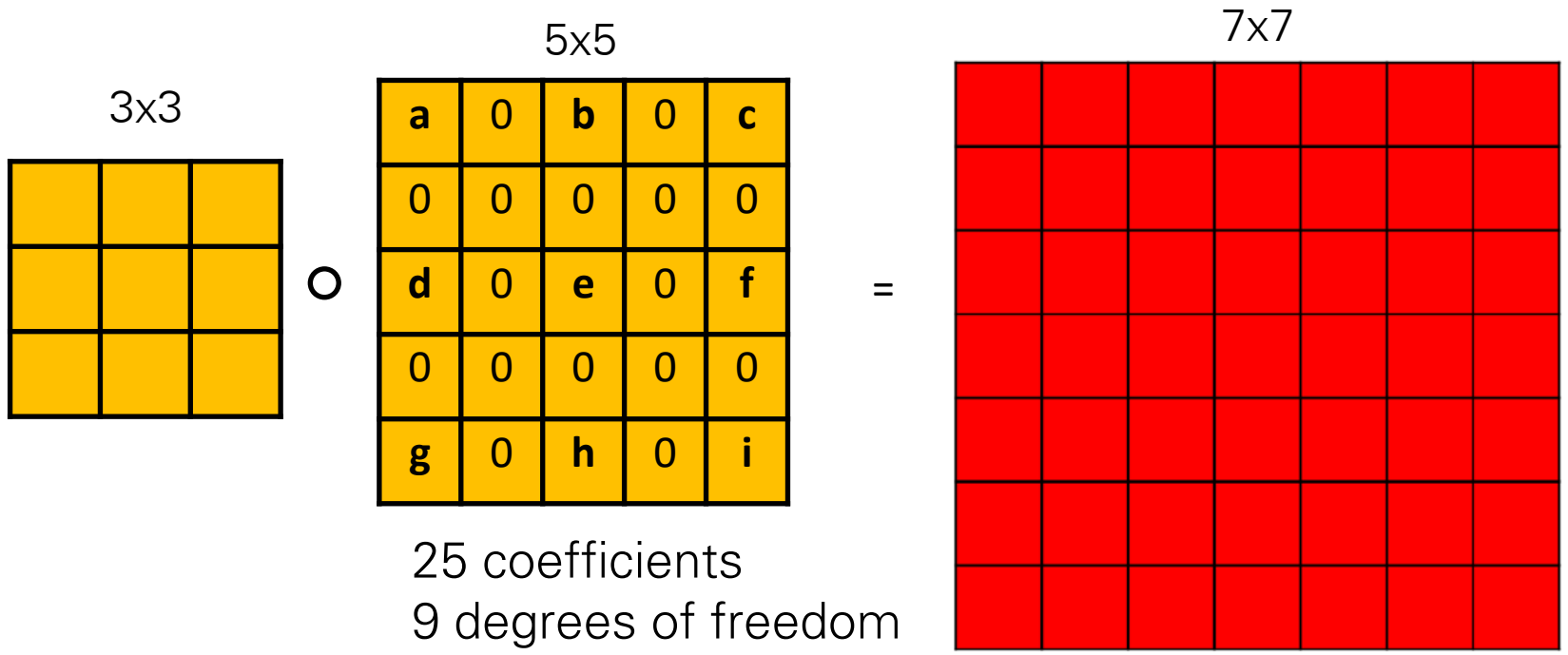


25 coefficients, but only
18 degrees of freedom



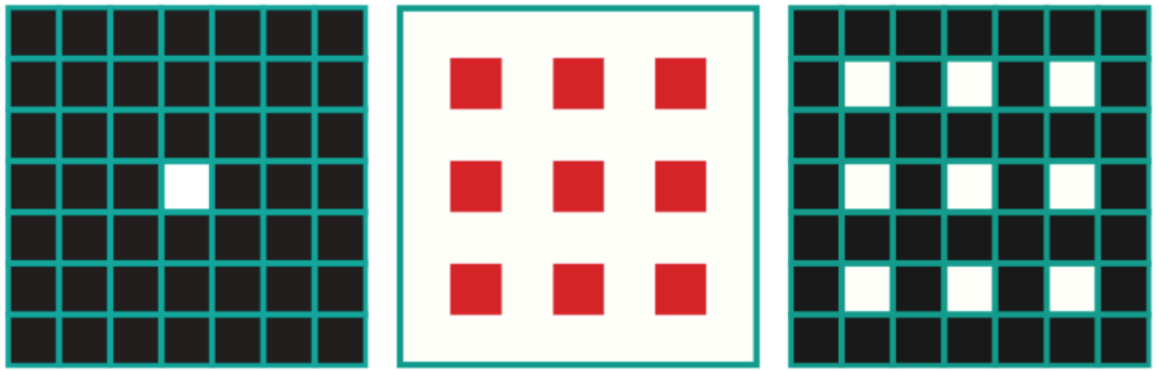
9 coefficients, but only
6 degrees of freedom.
Only separable filters… would this be enough?

Dilated convolutions



25 coefficients
9 degrees of freedom

49 coefficients
18 degrees of freedom



(a) Input (b) Dilation 2 (c) Output

[https://arxiv.org/pdf/1511.07122.pdf]

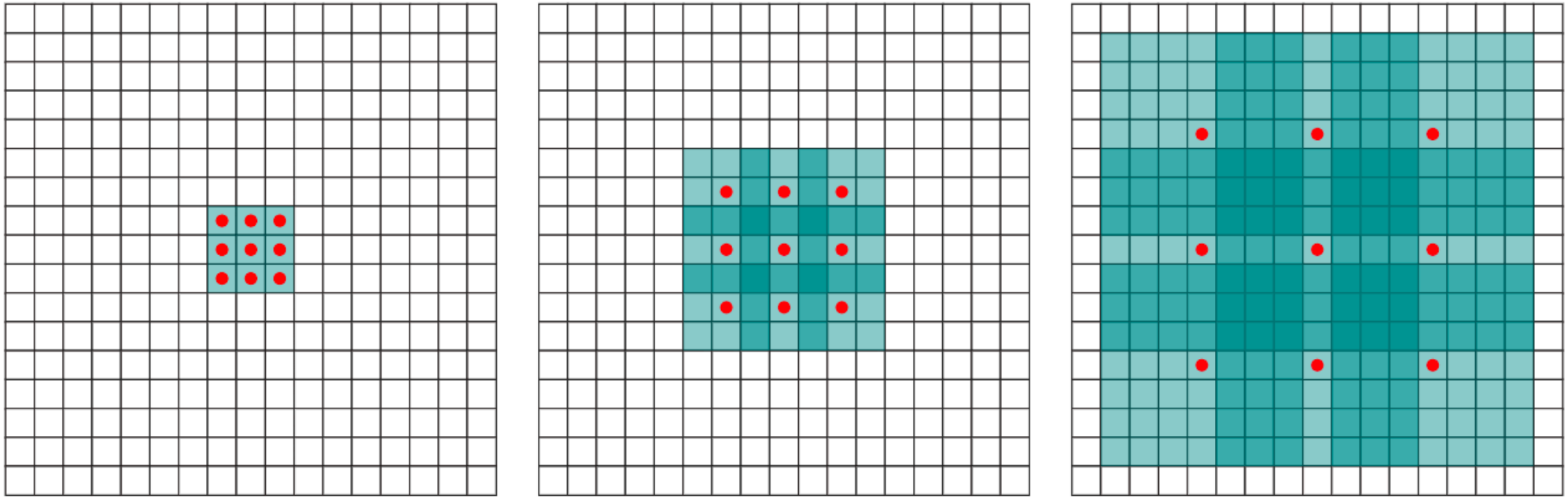
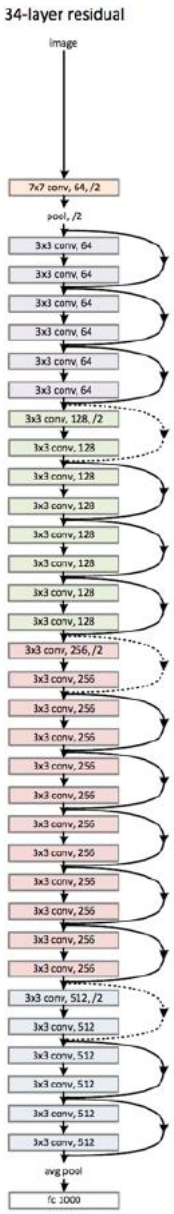


Figure 1: Systematic dilation supports exponential expansion of the receptive field without loss of resolution or coverage. (a) F_1 is produced from F_0 by a 1-dilated convolution; each element in F_1 has a receptive field of 3×3. (b) F_2 is produced from F_1 by a 2-dilated convolution; each element in F_2 has a receptive field of 7×7. (c) F_3 is produced from F_2 by a 4-dilated convolution; each element in F_3 has a receptive field of 15×15. The number of parameters associated with each layer is identical. The receptive field grows exponentially while the number of parameters grows linearly.

[https://arxiv.org/pdf/1511.07122.pdf]

2016: ResNet
>100 conv. layers

Error: 4.4%



Deep Residual Learning for Image Recognition

https://arxiv.org/pdf/1512.03385.pdf

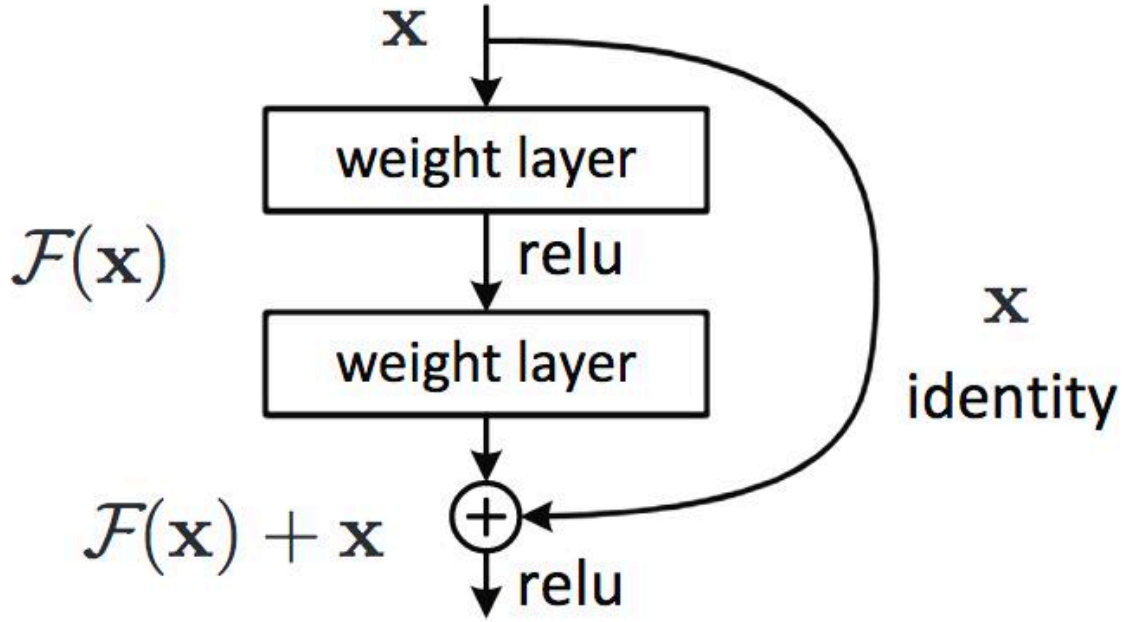
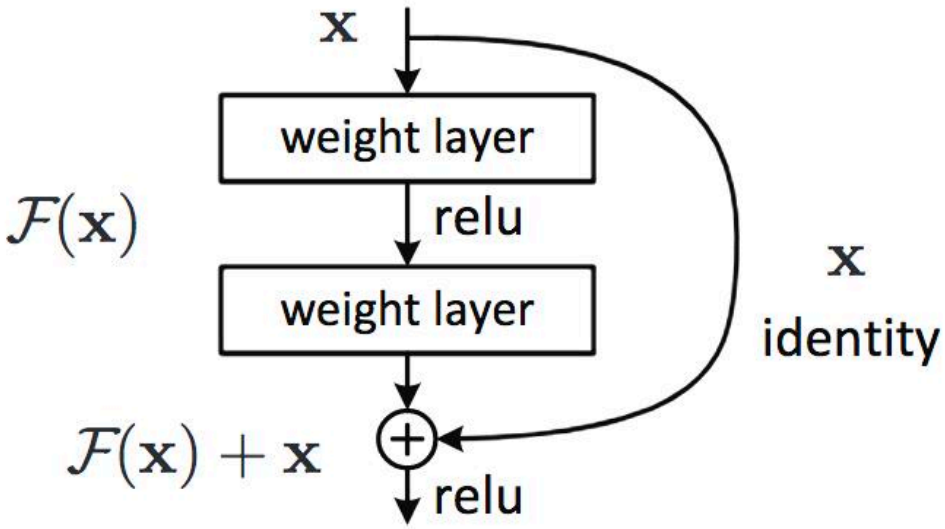
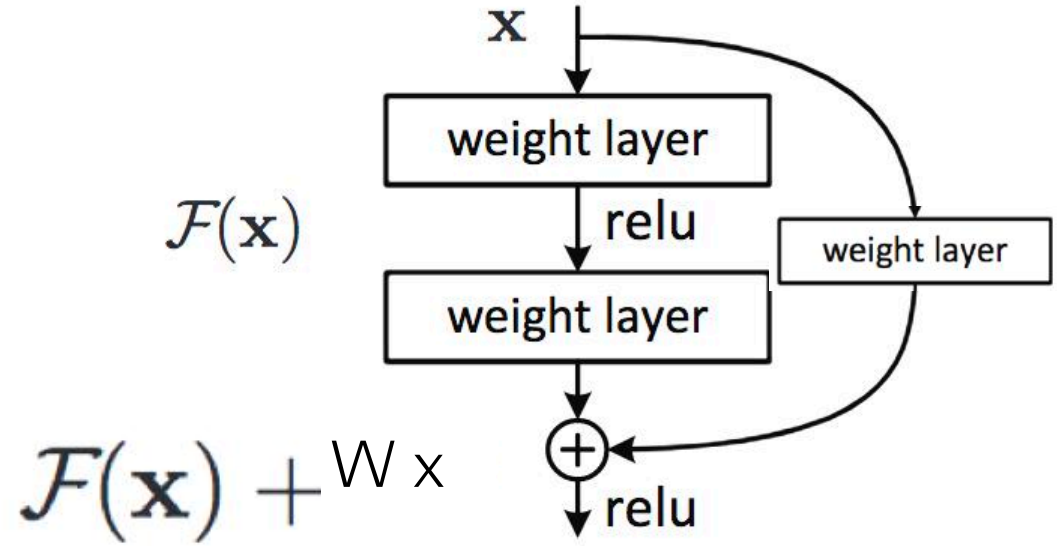


Figure 2. Residual learning: a building block.

If output has same size as input:

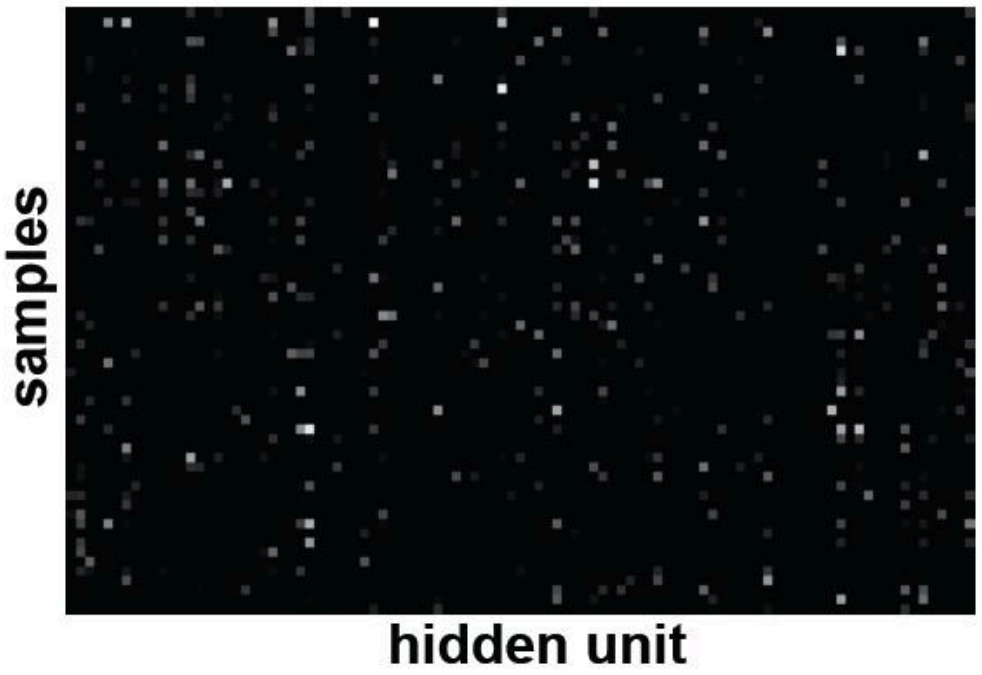


If output has a different size:



Other good things to know

- Check gradients numerically by finite differences
- Visualize hidden activations — should be uncorrelated and high variance

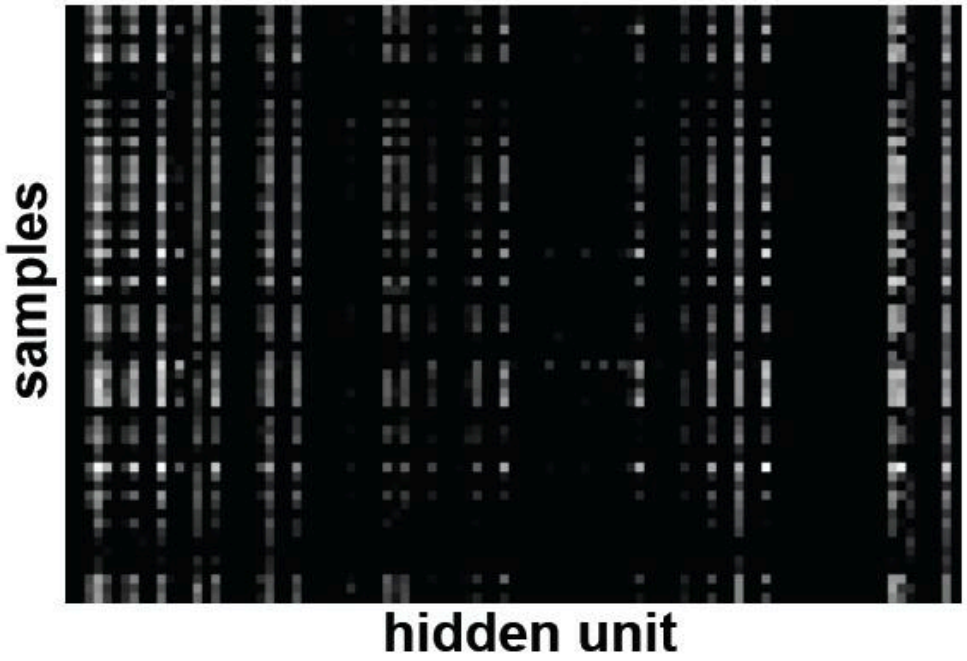


Good training: hidden units are sparse across samples and across features.

[Derived from slide by Marc'Aurelio Ranzato]

Other good things to know

- Check gradients numerically by finite differences
- Visualize hidden activations — should be uncorrelated and high variance

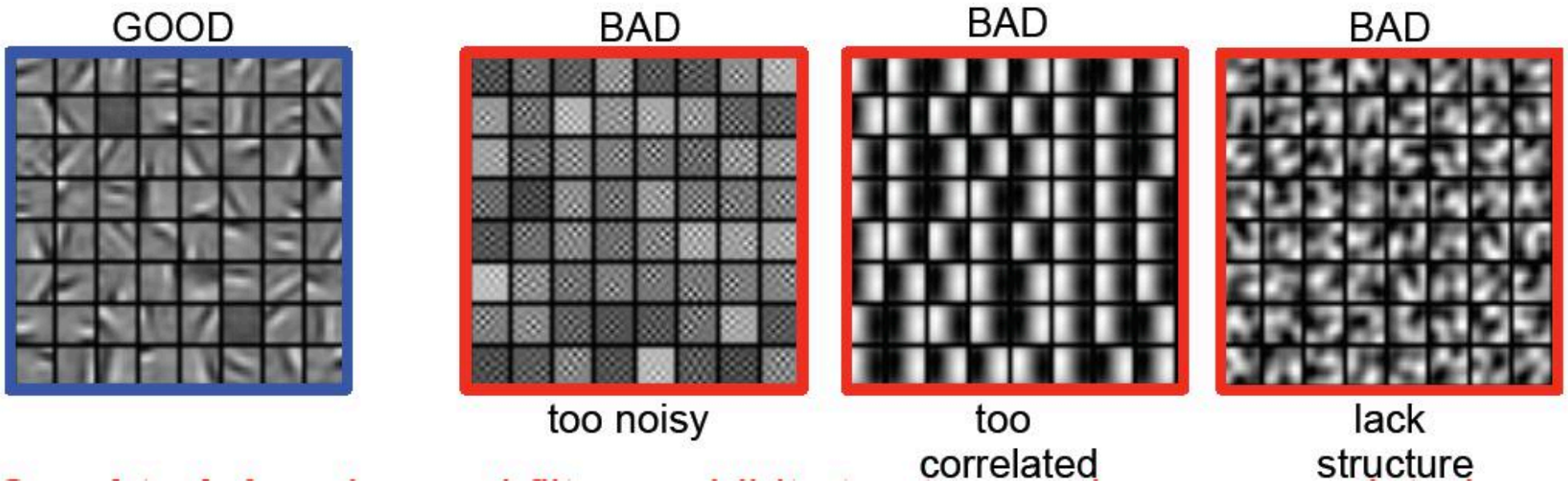


Bad training: many hidden units ignore the input and/or exhibit strong correlations.

[Derived from slide by Marc'Aurelio Ranzato]

Other good things to know

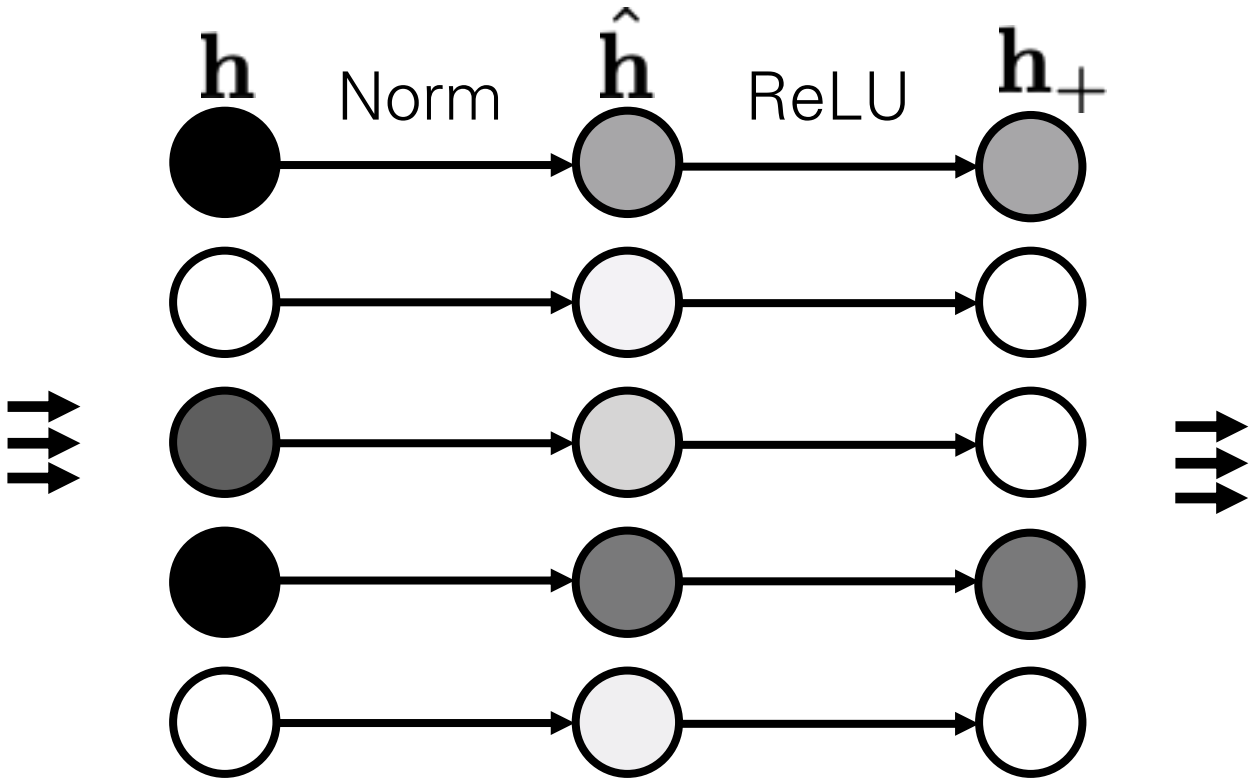
- Check gradients numerically by finite differences
- Visualize hidden activations — should be uncorrelated and high variance
- Visualize filters



Good training: learned filters exhibit structure and are uncorrelated.

[Derived from slide by Marc'Aurelio Ranzato]

Normalization layers



$$\hat{h}_k = \frac{h_k - \mathbb{E}[h_k]}{\sqrt{\mathbf{Var}[h_k]}}$$

Keep track of mean and variance of a unit (or a population of units) over time.

Standardize unit activations by subtracting mean and dividing by variance.

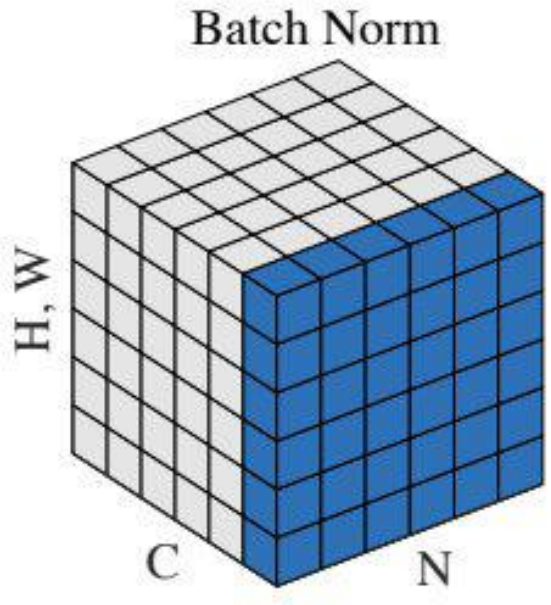
Squashes units into a **standard range**, avoiding overflow.

Also achieves **invariance** to mean and variance of the training signal.

Both these properties reduce the effective capacity of the model, i.e. regularize the model.

Normalization layers

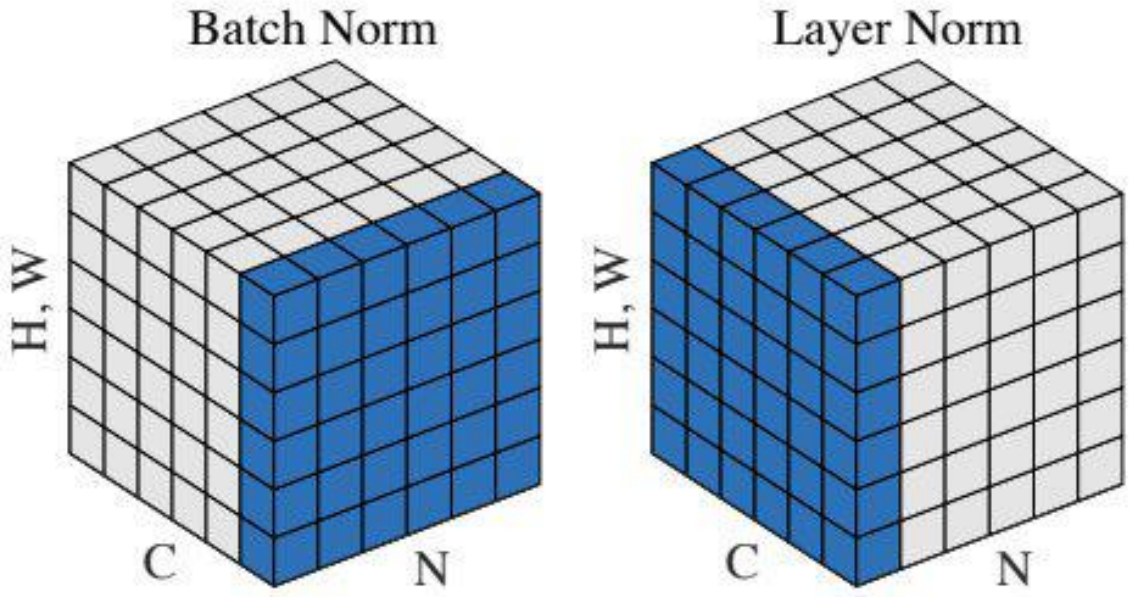
Normalization layers



Normalize w.r.t. a single hidden unit's pattern of activation over training examples (a batch of examples).

[Figure from Wu & He, arXiv 2018]

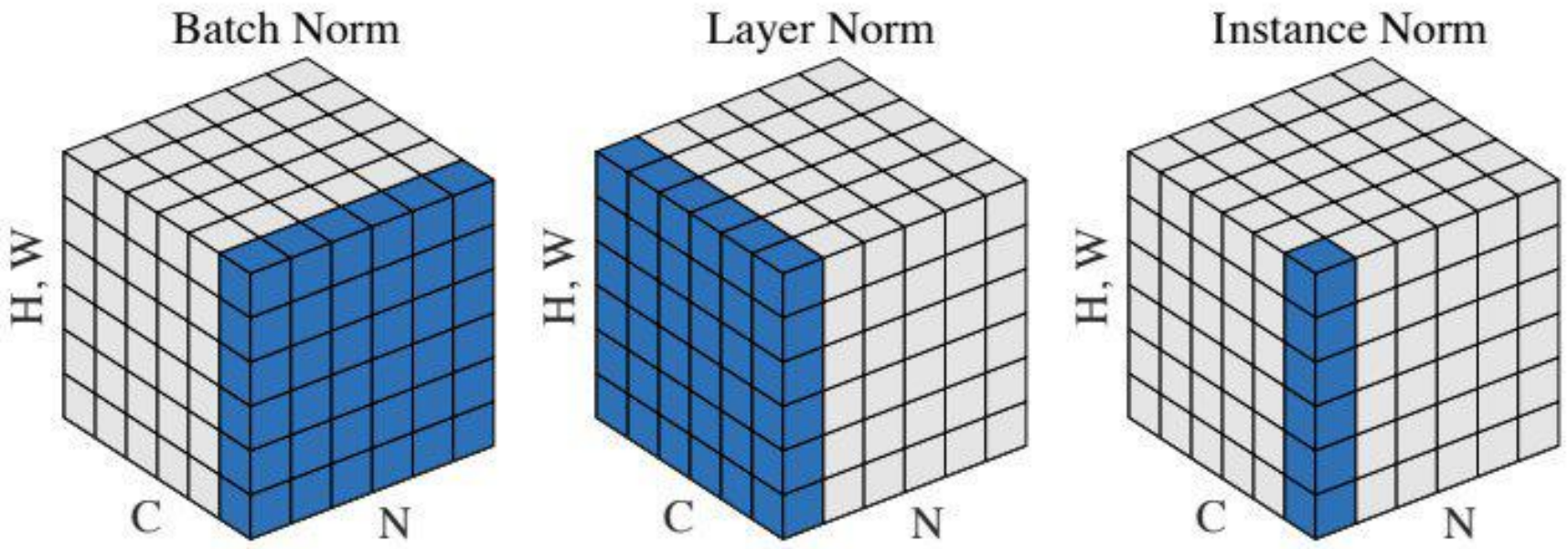
Normalization layers



Normalize w.r.t. the mean and variance of the activations of all the hidden units (neurons) on this layer (c).

[Figure from Wu & He, arXiv 2018]

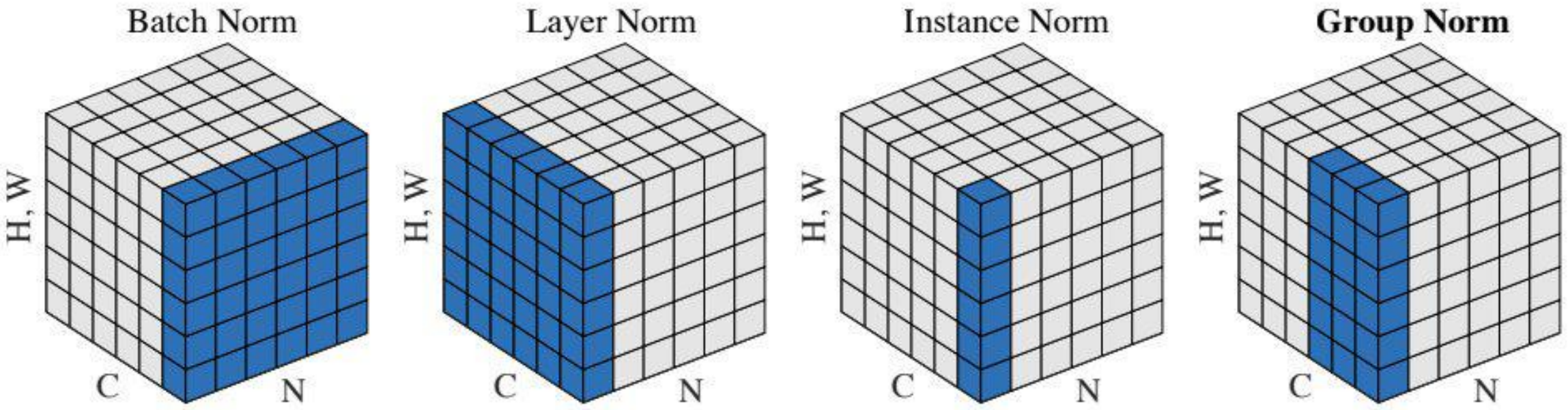
Normalization layers



Normalize w.r.t. the mean and variance of the activations of all the hidden units (neurons) on this layer (c) that process this particular location (h,w) in the image.

[Figure from Wu & He, arXiv 2018]

Normalization layers

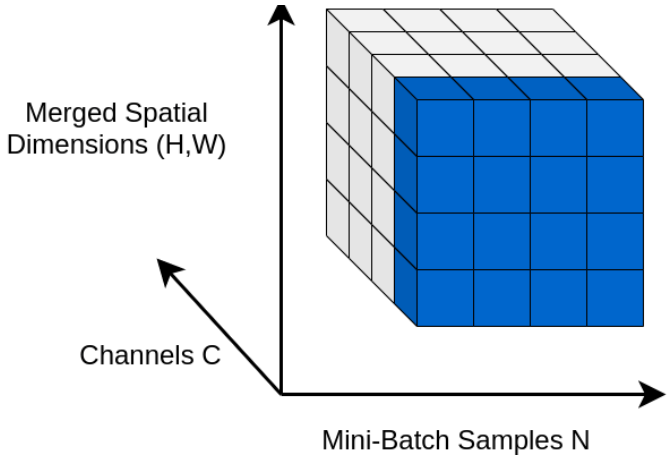


Might as well…

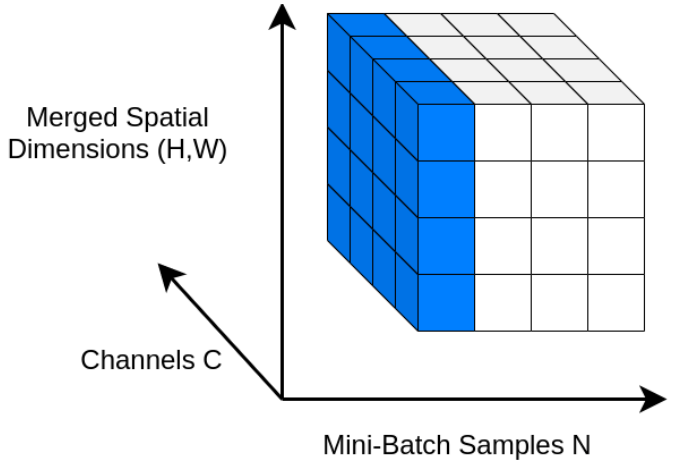
[Figure from Wu & He, arXiv 2018]

Normalization layers

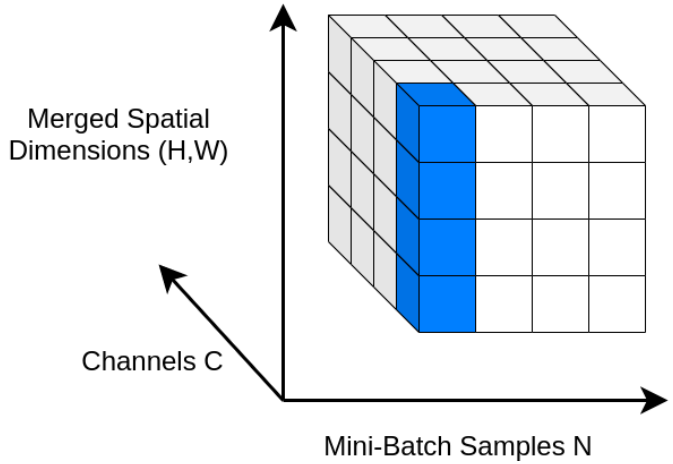
Batch Normalization (2015)



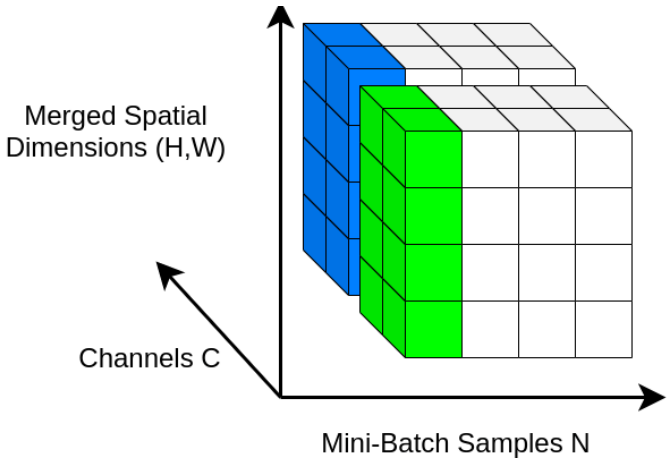
Layer Normalization (2016)



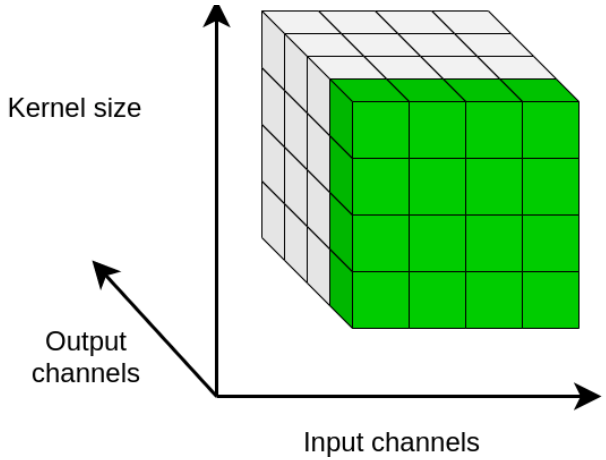
Instance Normalization (2016)



Group Normalization (2018)



Weight Standardization (2019)



[https://theaisummer.com/normalization]

No normalization layers

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CHARACTERIZING SIGNAL PROPAGATION TO CLOSE THE PERFORMANCE GAP IN UNNORMALIZED RESNETS

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ABSTRACT

Batch Normalization is a key component in almost all state-of-the-art image classifiers, but it also introduces practical challenges: it breaks the independence between training examples within a batch, can incur compute and memory overhead, and often results in unexpected bugs. Building on recent theoretical analyses of deep ResNets at initialization, we propose a simple set of analysis tools to characterize signal propagation on the forward pass, and leverage these tools to design highly performant ResNets without activation normalization layers. Crucial to our success is an adapted version of the recently proposed Weight Standardization. Our analysis tools show how this technique preserves the signal in networks with ReLU or Swish activation functions by ensuring that the per-channel activation means do not grow with depth. Across a range of FLOP budgets, our networks attain performance competitive with the state-of-the-art EfficientNets on ImageNet.

1 INTRODUCTION

BatchNorm has become a core computational primitive in deep learning (Ioffe & Szegedy, 2015), and it is used in almost all state-of-the-art image classifiers (Tan & Le, 2019; Wei et al., 2020). A number of different benefits of BatchNorm have been identified. It smoothens the loss landscape (Santurkar et al., 2018), which allows training with larger learning rates (Bjorck et al., 2018), and the noise arising from the minibatch estimates of the batch statistics introduces implicit regularization (Luo et al., 2019). Crucially, recent theoretical work (Balduzzi et al., 2017; De & Smith, 2020) has demonstrated that BatchNorm ensures good signal propagation at initialization in deep residual networks with identity skip connections (He et al., 2016b;a), and this benefit has enabled practitioners to train deep ResNets with hundreds or even thousands of layers (Zhang et al., 2019).

However, BatchNorm also has many disadvantages. Its behavior is strongly dependent on the batch

High-Performance Large-Scale Image Recognition Without Normalization

Andrew Brock [1] **Soham De** [1] **Samuel L. Smith** [1] **Karen Simonyan** [1]

Abstract

Batch normalization is a key component of most image classification models, but it has many undesirable properties stemming from its dependence on the batch size and interactions between examples. Although recent work has succeeded in training deep ResNets without normalization layers, these models do not match the test accuracies of the best batch-normalized networks, and are often unstable for large learning rates or strong data augmentations. In this work, we develop an adaptive gradient clipping technique which overcomes these instabilities, and design a significantly improved class of Normalizer-Free ResNets. Our smaller models match the test accuracy of an EfficientNet-B7 on ImageNet while being up to 8.7× faster to train, and our largest models attain a new state-of-the-art top-1 accuracy of 86.5%. In addition, Normalizer-Free models attain significantly better performance than their batch-normalized counterparts when fine-tuning on ImageNet after large-scale pre-training on a dataset of 300 million labeled images, with our best models obtaining an accuracy of 89.2%.[2]

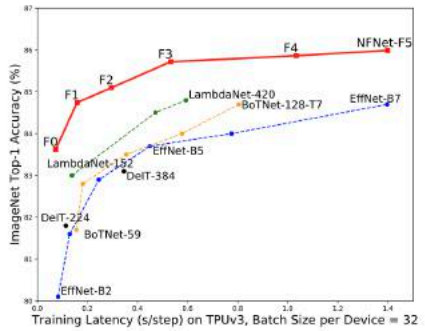

Figure 1. **ImageNet Validation Accuracy vs Training Latency.** All numbers are single-model, single crop. Our NFNet-F1 model achieves comparable accuracy to an EffNet-B7 while being 8.7× faster to train. Our NFNet-F5 model has similar training latency to EffNet-B7, but achieves a state-of-the-art 86.0% top-1 accuracy on ImageNet. We further improve on this using Sharpness Aware Minimization (Foret et al., 2021) to achieve 86.5% top-1 accuracy.

1. Introduction

The vast majority of recent models in computer vision are variants of deep residual networks (He et al., 2016b;a), trained with batch normalization (Ioffe & Szegedy, 2015).

However, batch normalization has three significant practical disadvantages. First, it is a surprisingly expensive computational primitive, which incurs memory overhead (Rota Bulò et al., 2018), and significantly increases the time required to evaluate the gradient in some networks (Gitman & Ginsburg, 2017). Second, it introduces a discrepancy between the behaviour of the model during training and at inference time (Summers & Dinneen, 2019; Singh & Shrivastava, 2019).

Next Lecture:
Sequential Processing with RNNs