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COMP201

Computer Systems & Programming

Lecture #21 – Cache Memories

KOÇ UNIVERSITY

Aykut Erdem // Koç University // Spring 2021

Image: Professor Farnsworth (Futurama)

Good news, everyone!

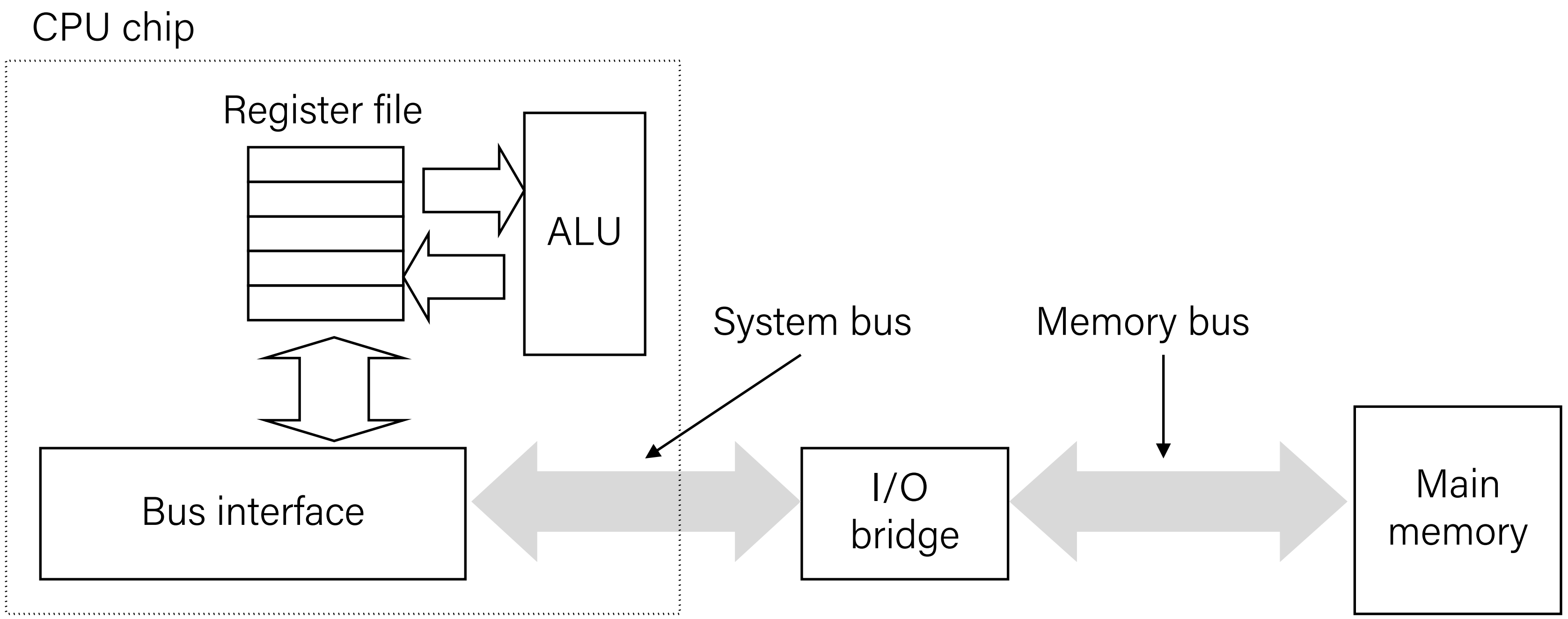
- Assigment 5 out
  (due May 13, 23:59)
- No lectures on May 10-12

Recap

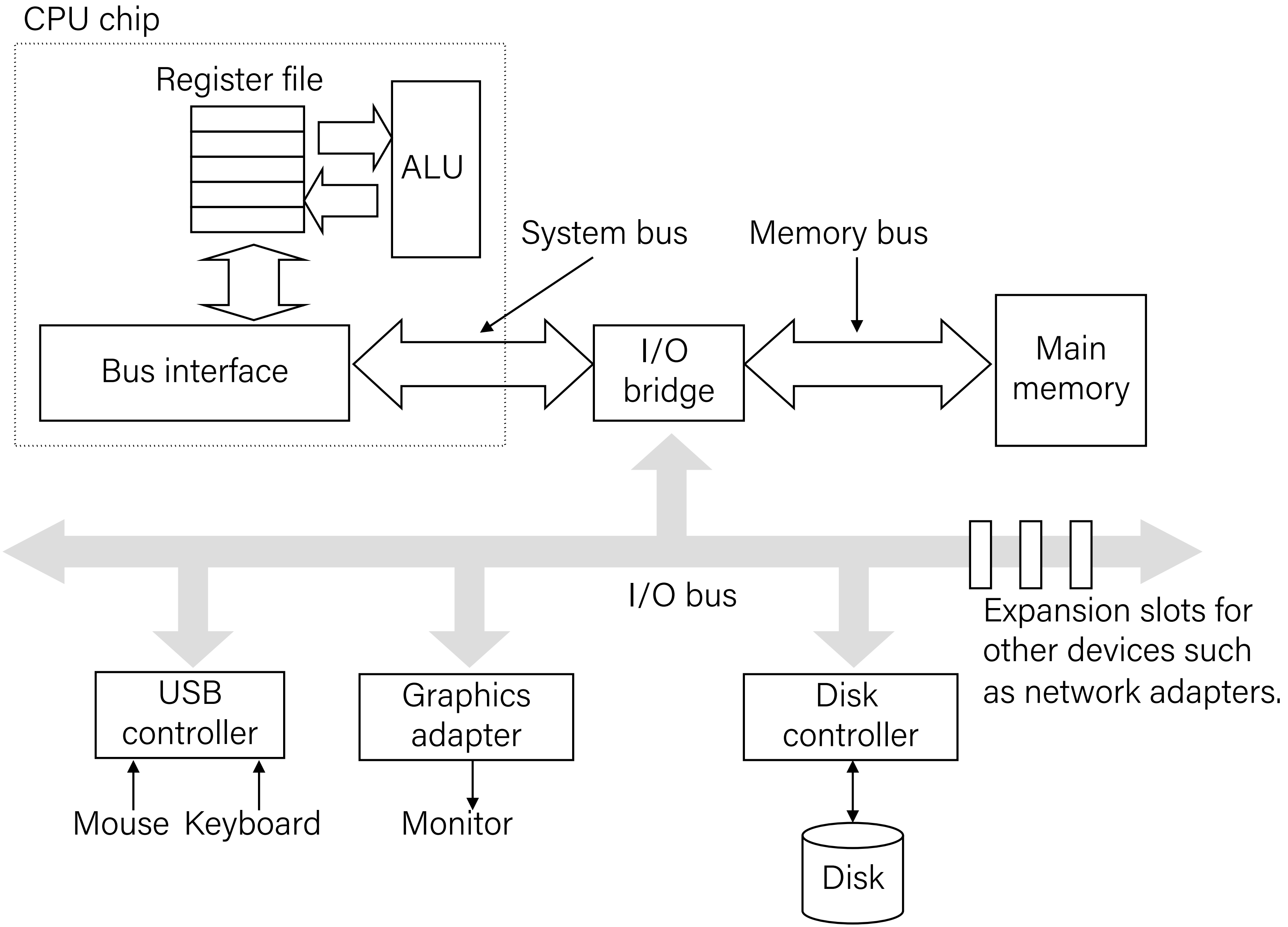
- Storage technologies and trends
- Locality of reference
- Caching in the memory hierarchy

Recap: Traditional Bus Structure Connecting CPU and Memory

- A bus is a collection of parallel wires that carry address, data, and control signals.
- Buses are typically shared by multiple devices.



Recap: I/O Bus


CPU chip
Register file
ALU
System bus
Memory bus
Bus interface
I/O bridge
Main memory
I/O bus
Expansion slots for other devices such as network adapters.
USB controller
Graphics adapter
Disk controller
Mouse
Keyboard
Monitor
Disk

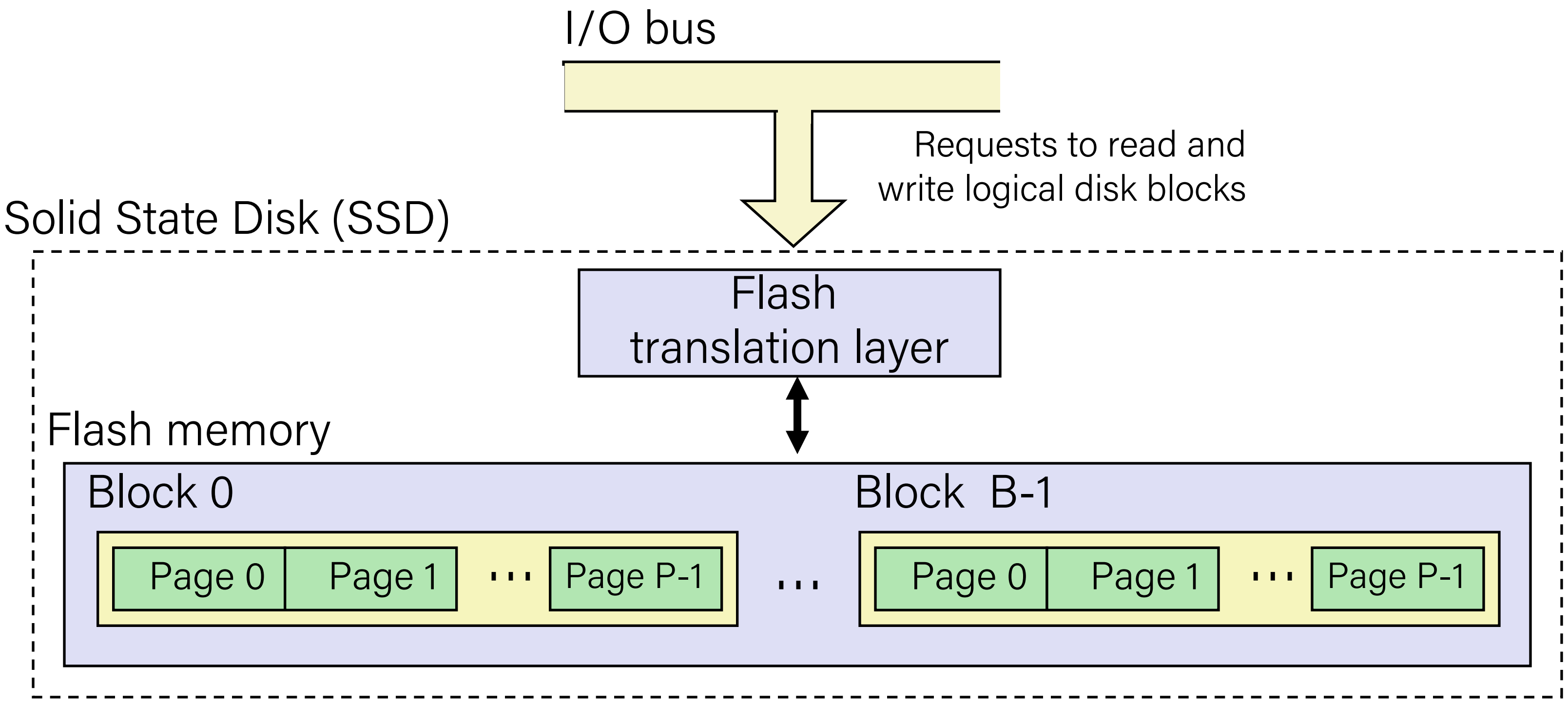
Recap: Disk Access Time

- Average time to access some target sector approximated by:
  - Taccess = Tavg seek + Tavg rotation + Tavg transfer
- Seek time (Tavg seek)
  - Time to position heads over cylinder containing target sector.
  - Typical Tavg seek is 3—9 ms
- Rotational latency (Tavg rotation)
  - Time waiting for first bit of target sector to pass under r/w head.
  - Tavg rotation = 1/2 × 1/RPMs x 60 sec/1 min
  - Typical Tavg rotation = 7200 RPMs
- Transfer time (Tavg transfer)
  - Time to read the bits in the target sector.
  - Tavg transfer = 1/RPM × 1/(avg # sectors/track) × 60 secs/1 min.

Recap: Disk Access Time Example

- Given:
  - Rotational rate = 7,200 RPM
  - Average seek time = 9 ms.
  - Avg # sectors/track = 400.
- Derived:
  - Tavg rotation = 1/2 × (60 secs/7200 RPM) × 1000 ms/sec = 4 ms.
  - Tavg transfer = 60/7200 RPM × 1/400 secs/track × 1000 ms/sec = 0.02 ms
  - Taccess = 9 ms + 4 ms + 0.02 ms
- Important points:
  - Access time dominated by seek time and rotational latency.
  - First bit in a sector is the most expensive, the rest are free.
  - SRAM access time is about 4 ns/doubleword, DRAM about 60 ns
    - Disk is about 40,000 times slower than SRAM,
    - 2,500 times slower then DRAM.

Recap: Solid State Disks (SSDs)



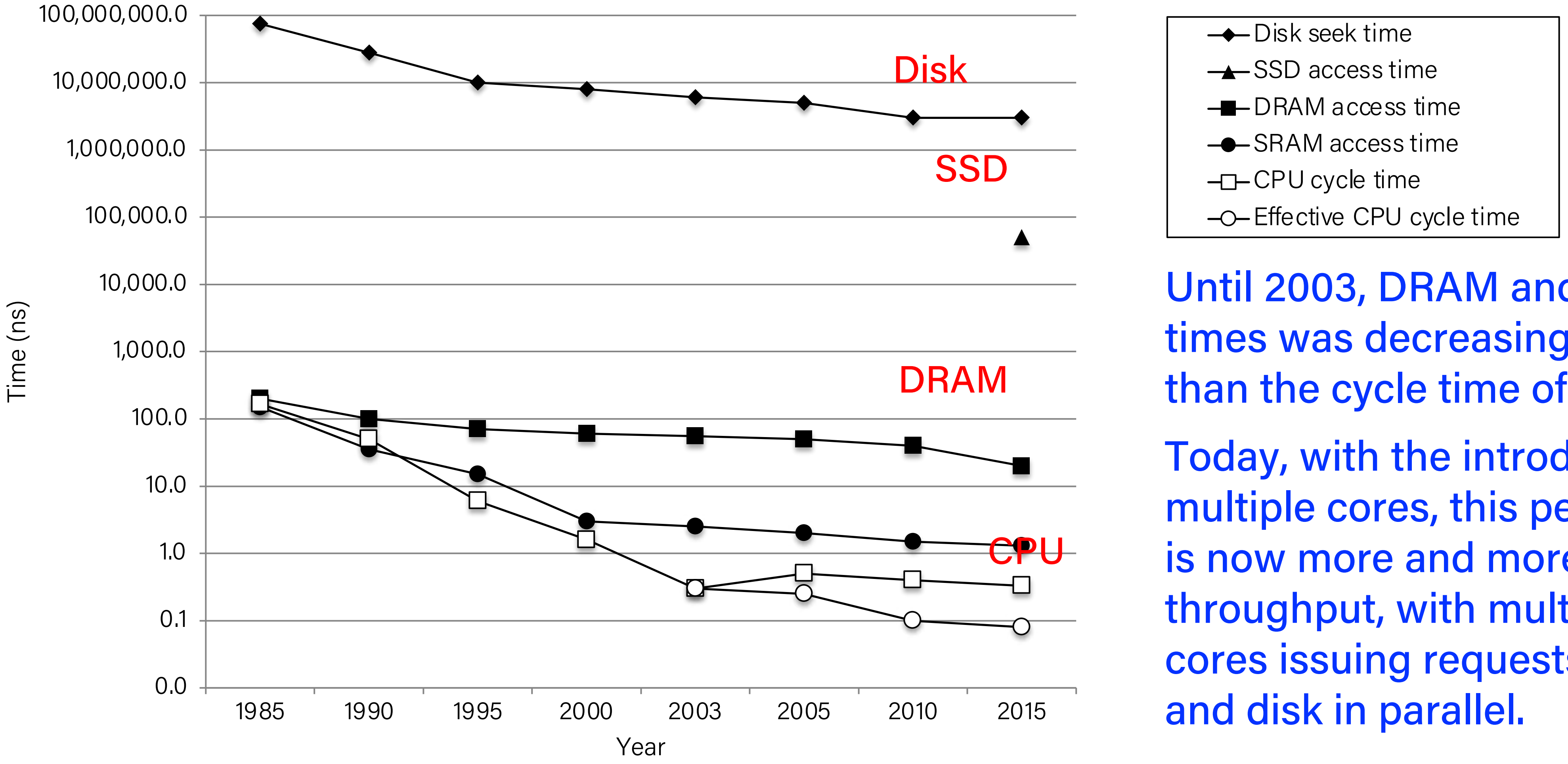
- Pages: 512KB to 4KB, Blocks: 32 to 128 pages
- Data read/written in units of pages.
- Page can be written only after its block has been erased
- A block wears out after about 100,000 repeated writes.

Recap: SSD Tradeoffs vs Rotating Disks

- **Advantages**
  - No moving parts → faster, less power, more rugged
- **Disadvantages**
  - Have the potential to wear out
    - Mitigated by “wear leveling logic” in flash translation layer
    - E.g. Intel SSD 730 guarantees 128 petabyte (128 x 1015 bytes) of writes before they wear out
  - In 2015, about 30 times more expensive per byte
- **Applications**
  - MP3 players, smart phones, laptops
  - Beginning to appear in desktops and servers

Recap: The CPU-Memory Gap

- The gap widens between DRAM, disk, and CPU speeds.



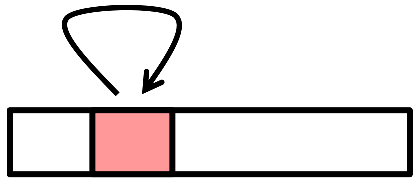
Until 2003, DRAM and disk access times was decreasing more slowly than the cycle time of a processor.

Today, with the introduction of multiple cores, this performance gap is now more and more a function of throughput, with multiple processor cores issuing requests to the DRAM and disk in parallel.

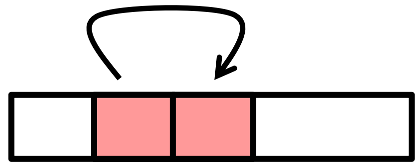
Recap: Locality

- **Principle of Locality:** Programs tend to use data and instructions with addresses near or equal to those they have used recently

- **Temporal locality:**
  - Recently referenced items are likely to be referenced again in the near future



- **Spatial locality:**
  - Items with nearby addresses tend to be referenced close together in time



Recap: Locality Example

```
sum = 0;
for (i = 0; i < n; i++)
   sum += a[i];
return sum;
```

- Data references
  - Reference array elements in succession (stride-1 reference pattern). — Spatial locality
  - Reference variable sum each iteration. — Temporal locality
- Instruction references
  - Reference instructions in sequence. — Spatial locality
  - Cycle through loop repeatedly. — Temporal locality

Plan for Today

- Cache memory organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

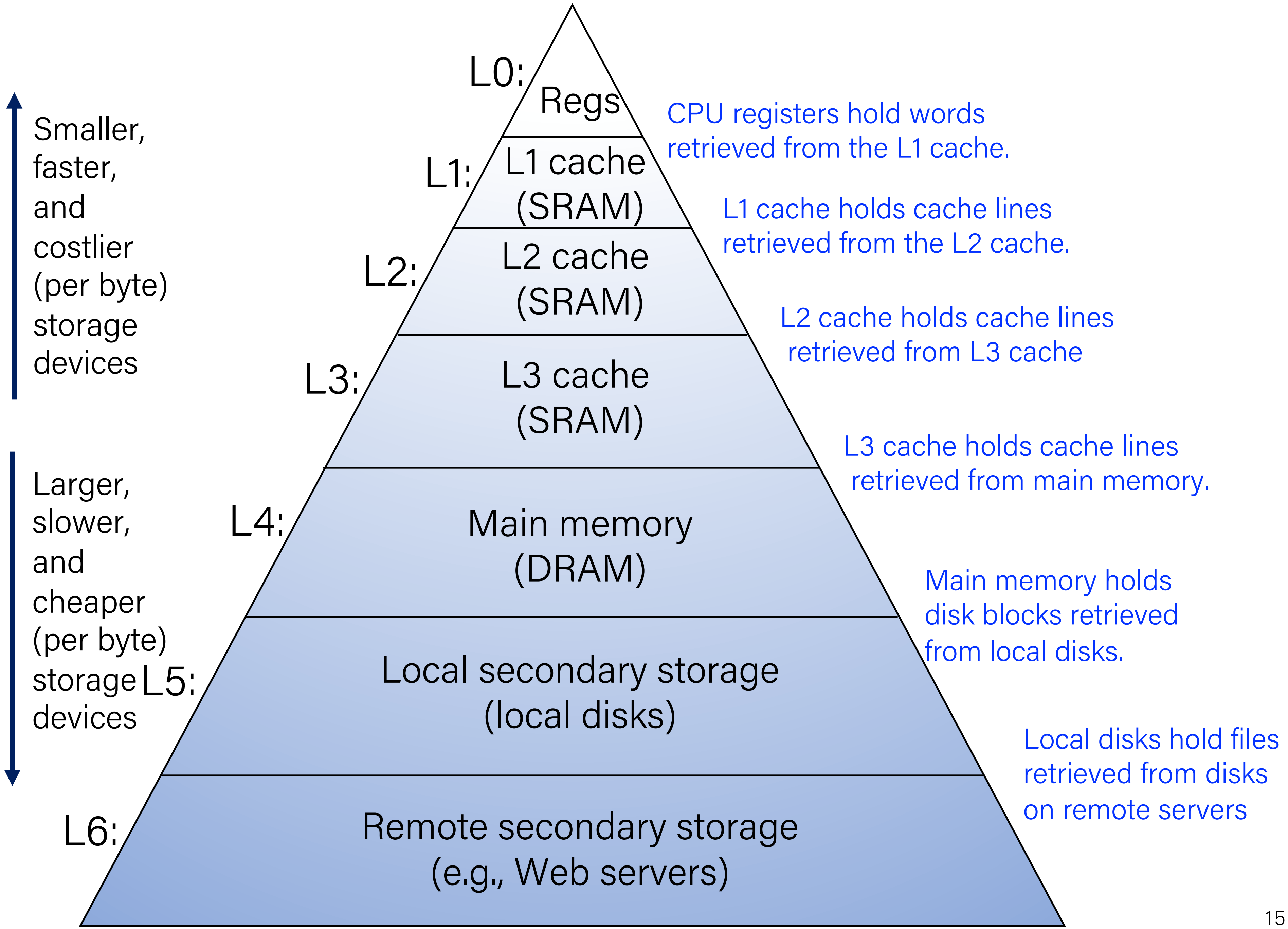
Disclaimer: Slides for this lecture were borrowed from

—Randal E. Bryant and David R. O'Hallaroni's CMU 15-213 class

Lecture Plan

- Cache memory organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

Example Memory Hierarchy

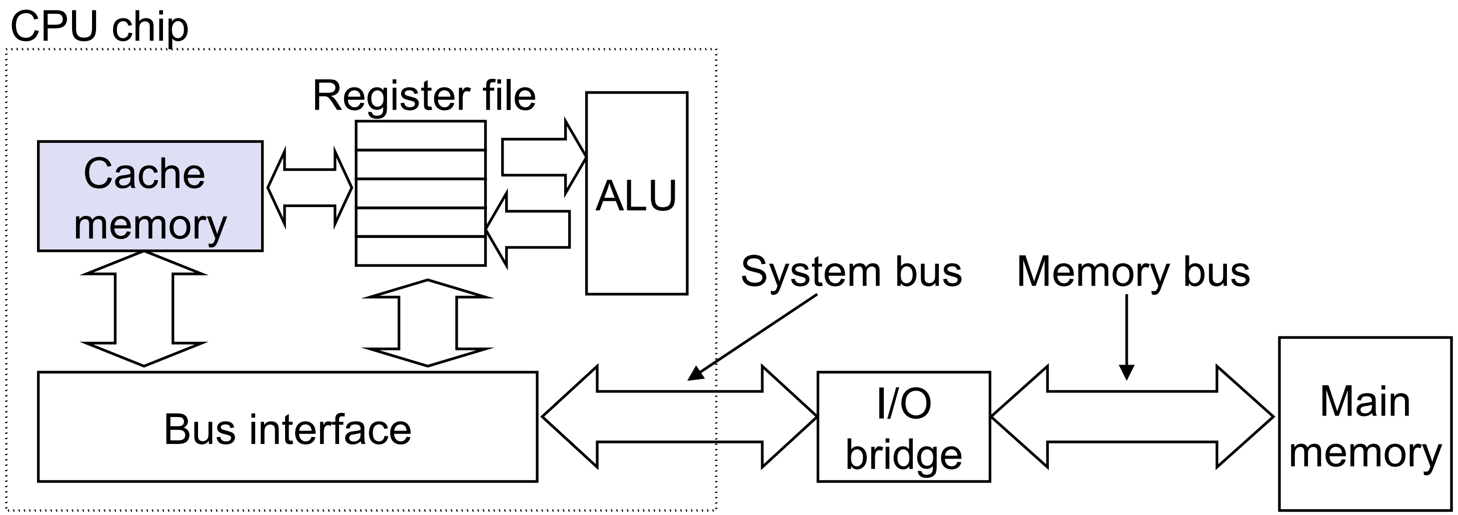


Examples of Caching in the Mem. Hierarchy

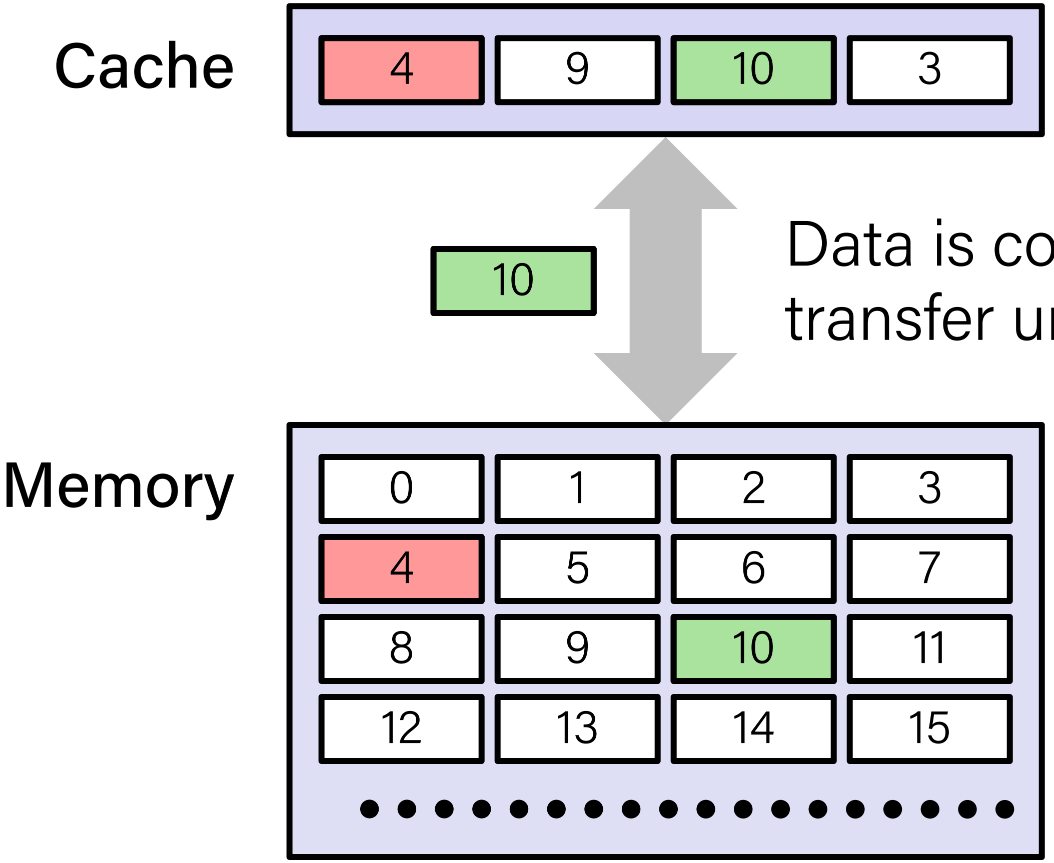
Cache Type	What is Cached?	Where is it Cached?	Latency (cycles)	Managed By
Registers	4-8 bytes words	CPU core	0	Compiler
TLB	Address translations	On-Chip TLB	0	Hardware MMU
L1 cache	64-byte blocks	On-Chip L1	4	Hardware
L2 cache	64-byte blocks	On-Chip L2	10	Hardware
Virtual Memory	4-KB pages	Main memory	100	Hardware + OS
Buffer cache	Parts of files	Main memory	100	OS
Disk cache	Disk sectors	Disk controller	100,000	Disk firmware
Network buffer cache	Parts of files	Local disk	10,000,000	NFS client
Browser cache	Web pages	Local disk	10,000,000	Web browser
Web cache	Web pages	Remote server disks	1,000,000,000	Web proxy server

Cache Memories

- Cache memories are small, fast SRAM-based memories managed automatically in hardware
  - Hold frequently accessed blocks of main memory
- CPU looks first for data in cache
- Typical system structure:



General Cache Concepts

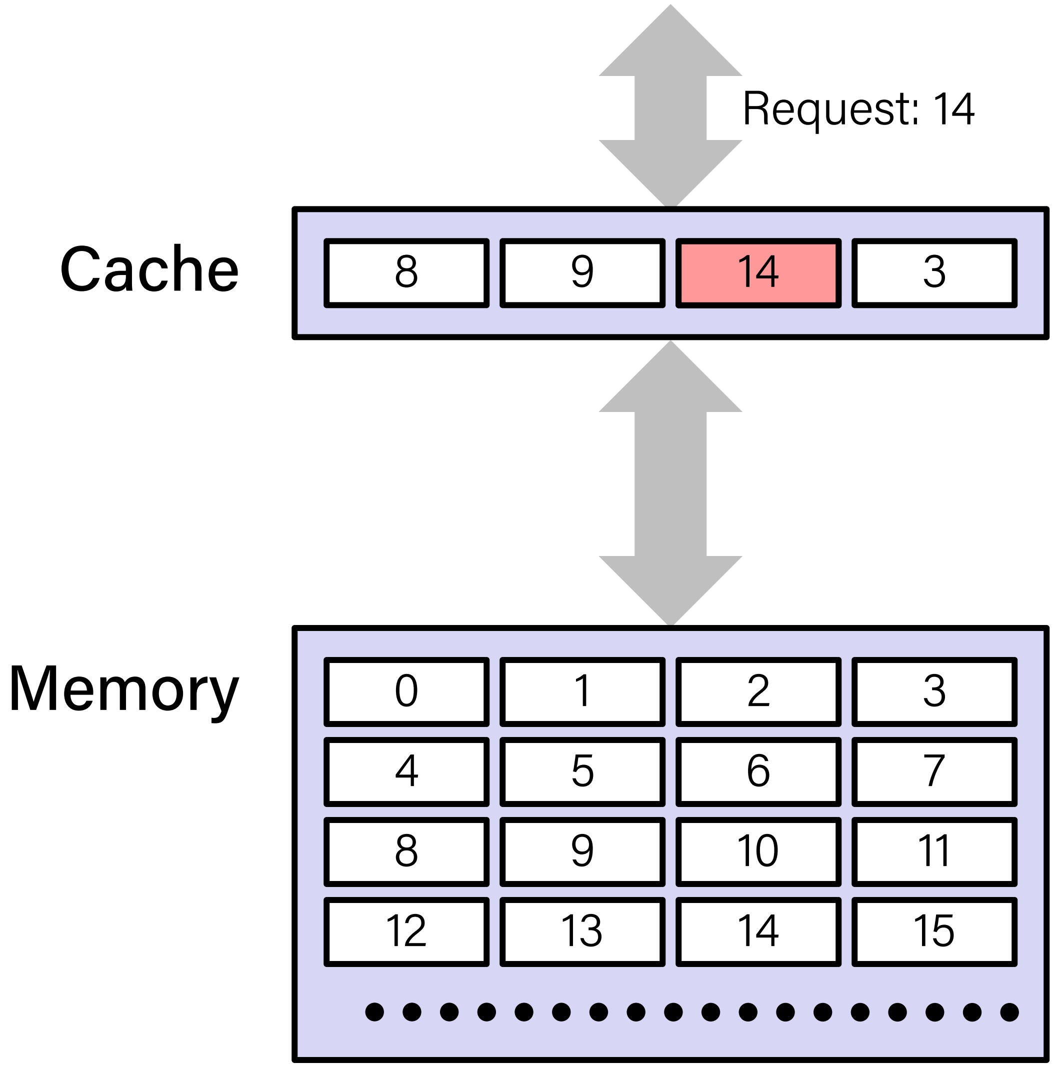


Smaller, faster, more expensive memory caches a subset of the blocks

Data is copied in block-sized transfer units

Larger, slower, cheaper memory viewed as partitioned into “blocks”

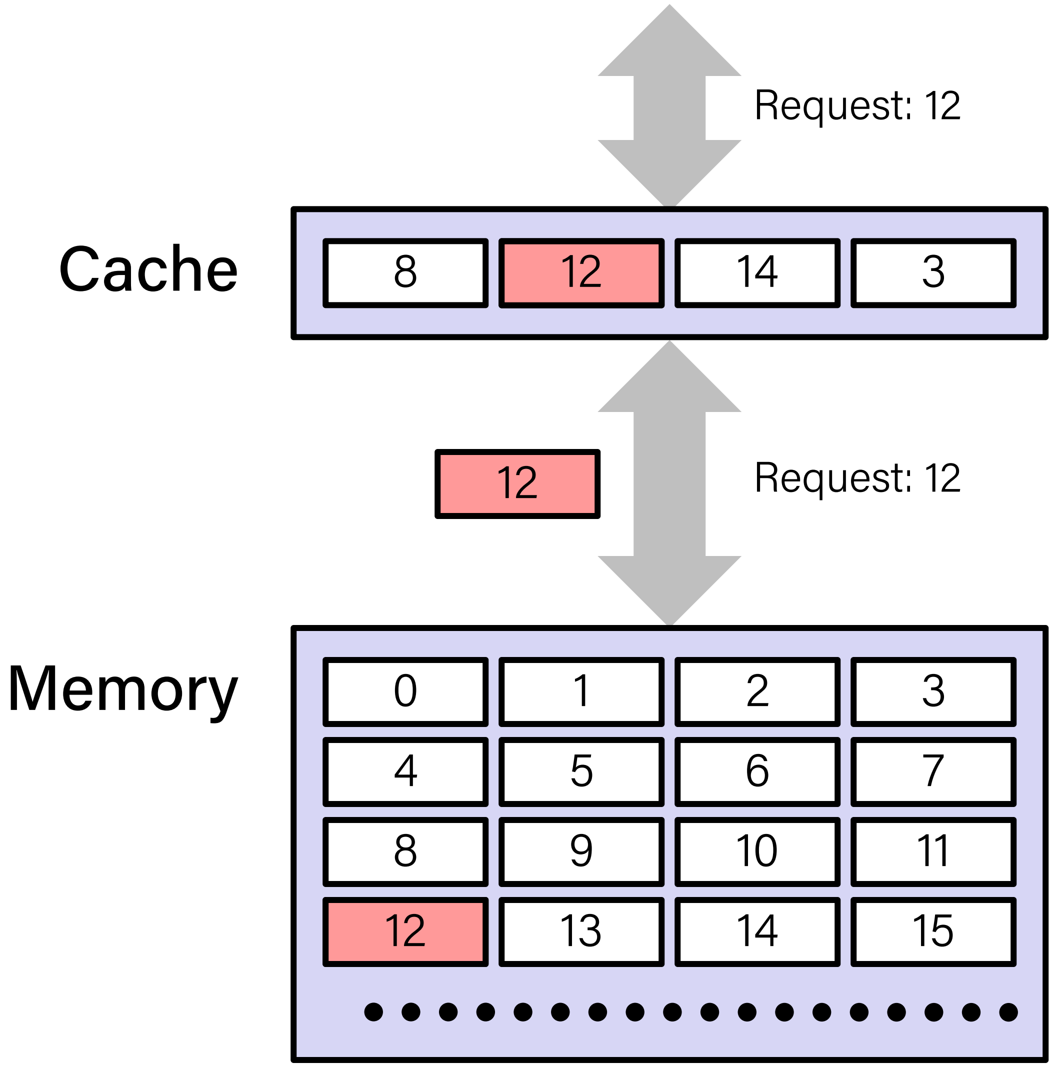
General Cache Concepts: Hit



Data in block b is needed

Block b is in cache: **Hit!**

General Cache Concepts: Miss



Data in block b is needed

Block b is not in cache: **Miss!**

Block b is fetched from memory

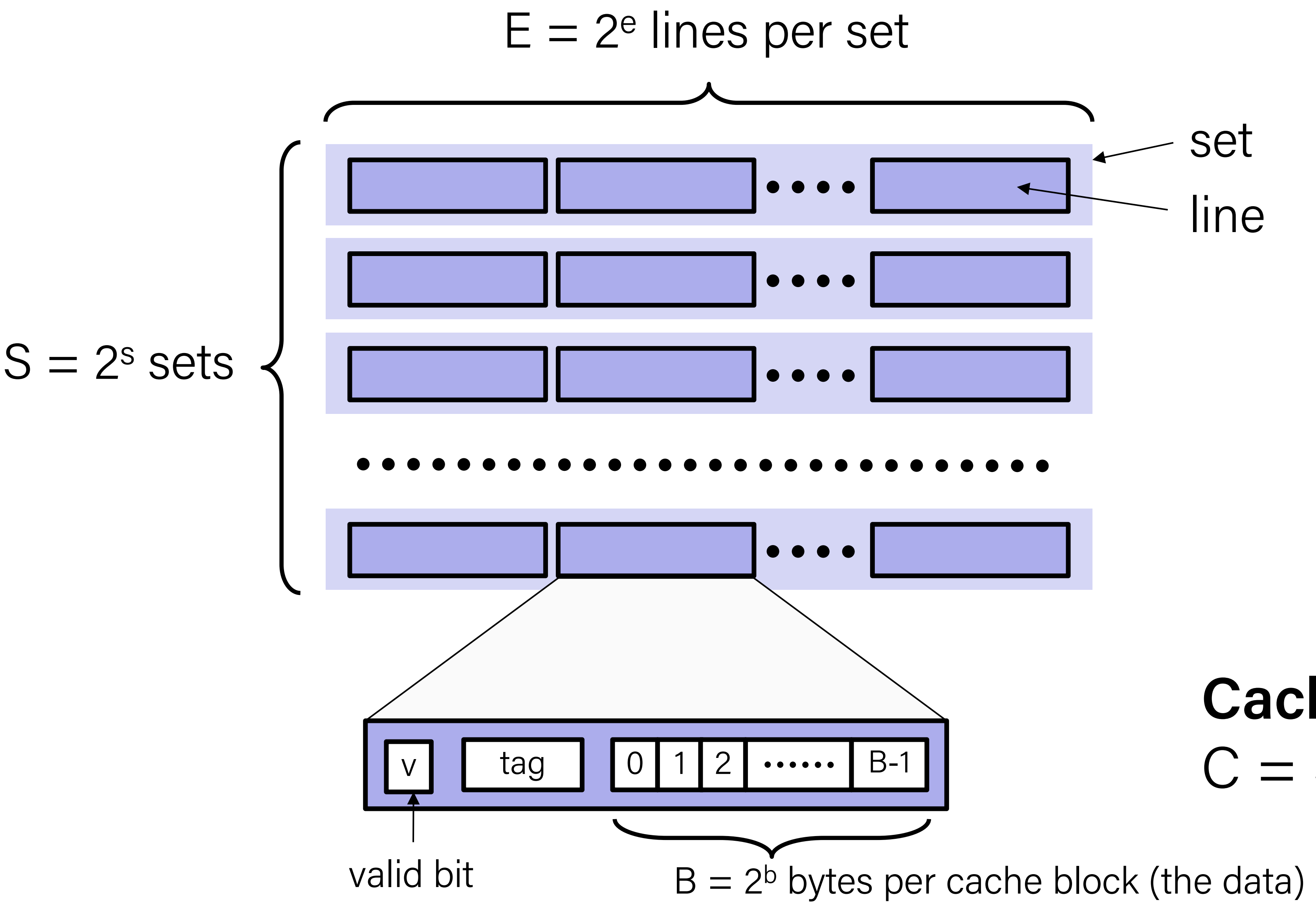
Block b is stored in cache

- **Placement policy:** determines where b goes
- **Replacement policy:** determines which block gets evicted (victim)

Types of Cache Misses

- **Cold (compulsory) miss**
  - Cold misses occur because the cache is empty.
- **Conflict miss**
  - Most caches limit blocks at level k+1 to a small subset (sometimes a singleton) of the block positions at level k.
    - E.g. Block i at level k+1 must be placed in block (i mod 4) at level k.
  - Conflict misses occur when the level k cache is large enough, but multiple data objects all map to the same level k block.
    - E.g. Referencing blocks 0, 8, 0, 8, 0, 8, ... would miss every time.
- **Capacity miss**
  - Occurs when the set of active cache blocks (working set) is larger than the cache.

General Cache Organization (S, E, B)

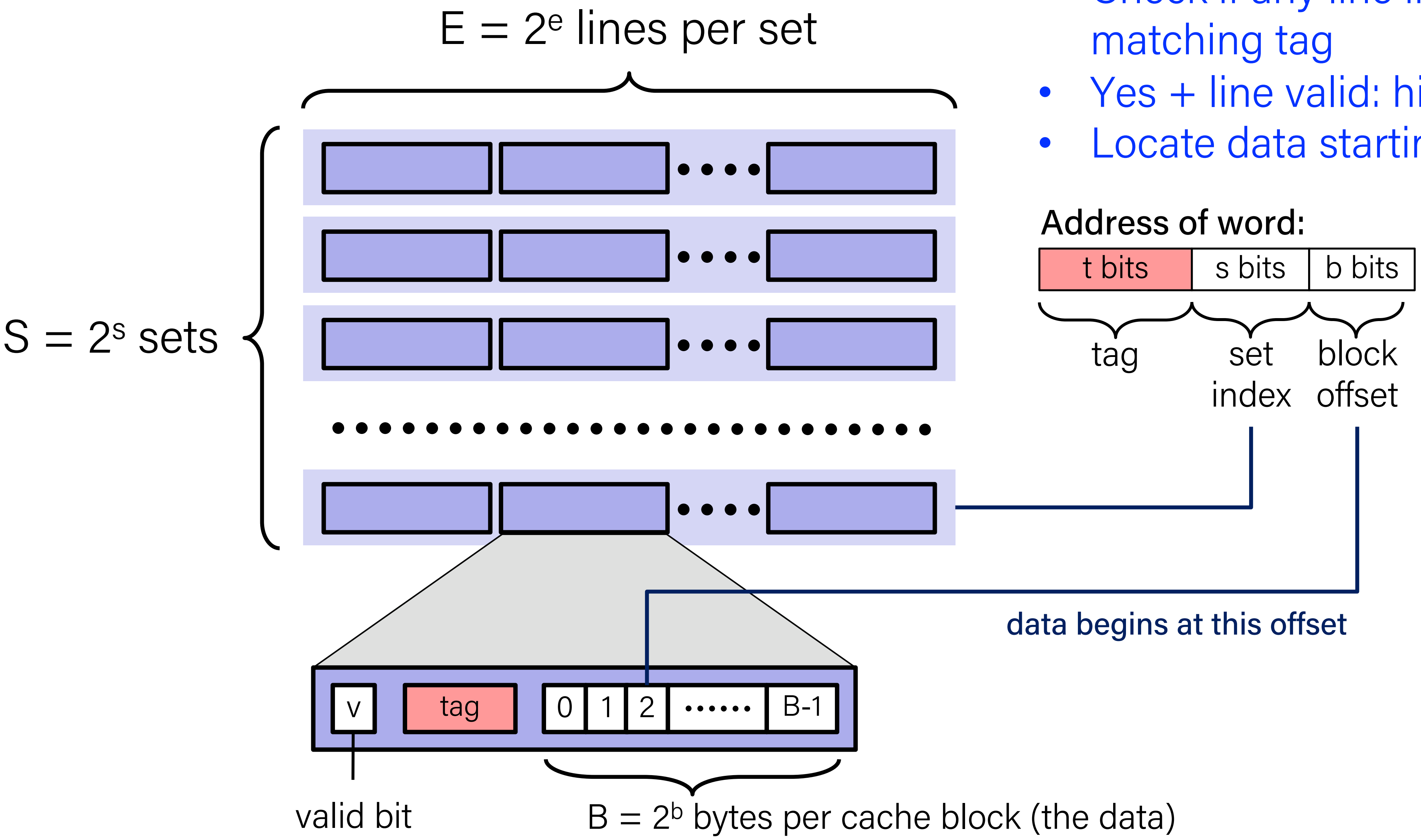


Cache size:

$C = S \times E \times B$ data bytes

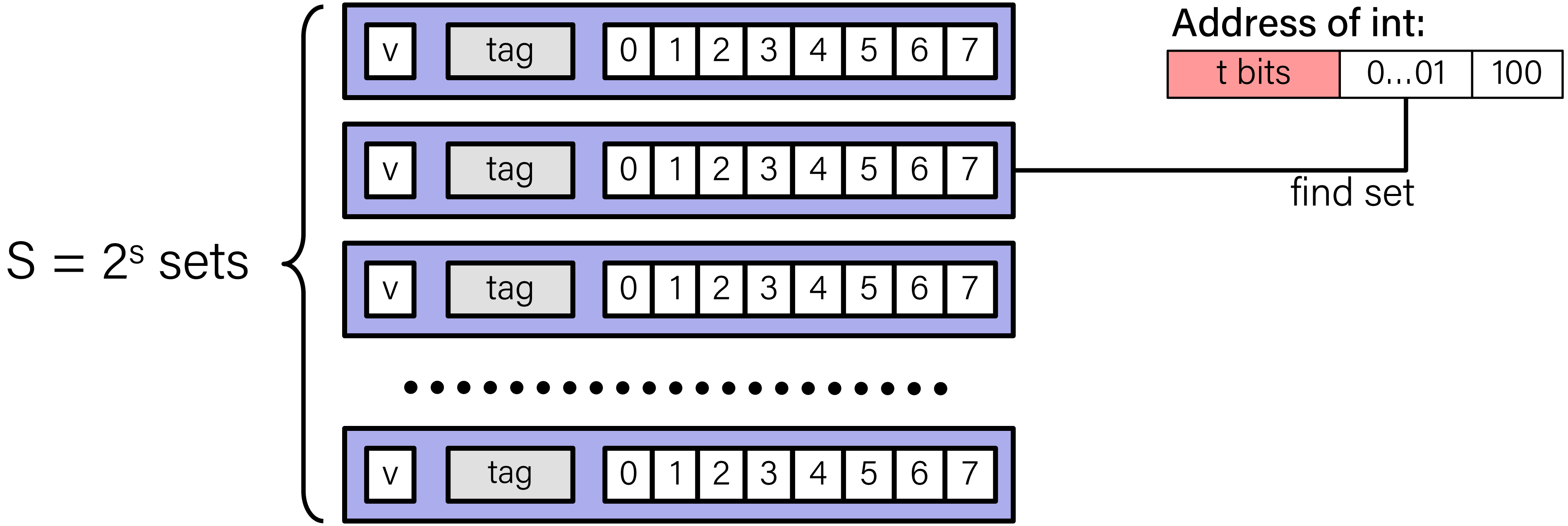
Cache Read

- Locate set
- Check if any line in set has matching tag
- Yes + line valid: hit
- Locate data starting at offset



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set
Assume: cache block size 8 bytes



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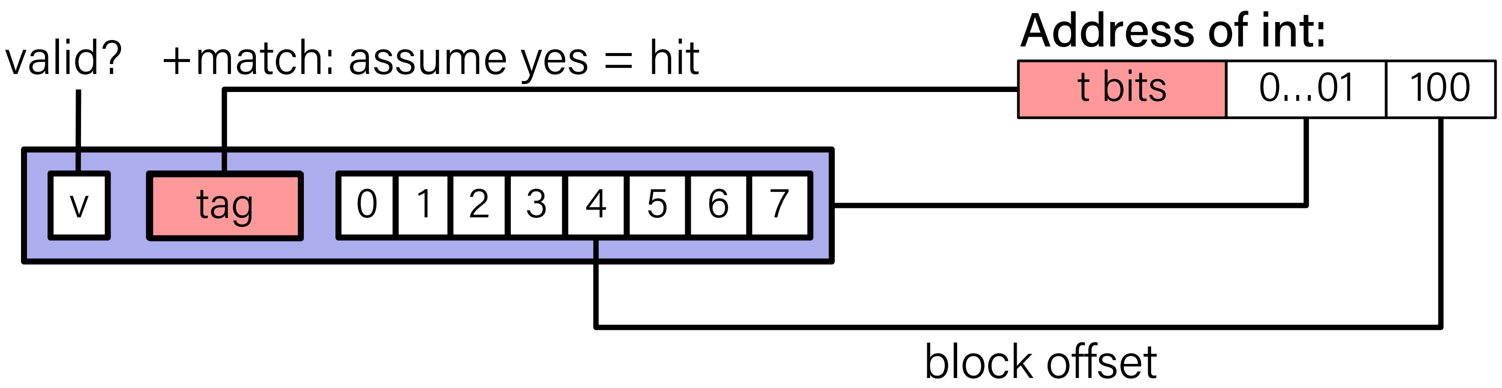

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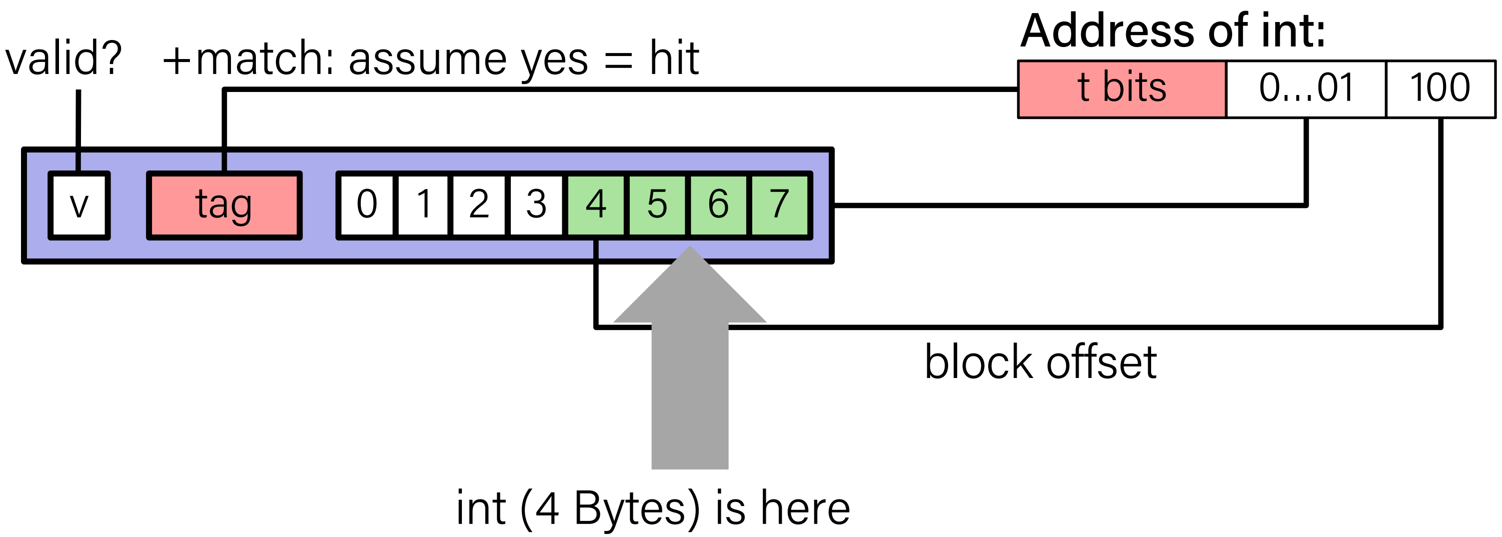


If tag doesn't match: old line is evicted and replaced

Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

M=16 bytes (4-bit addresses), B=2 bytes/block, S=4 sets, E=1 Blocks/set

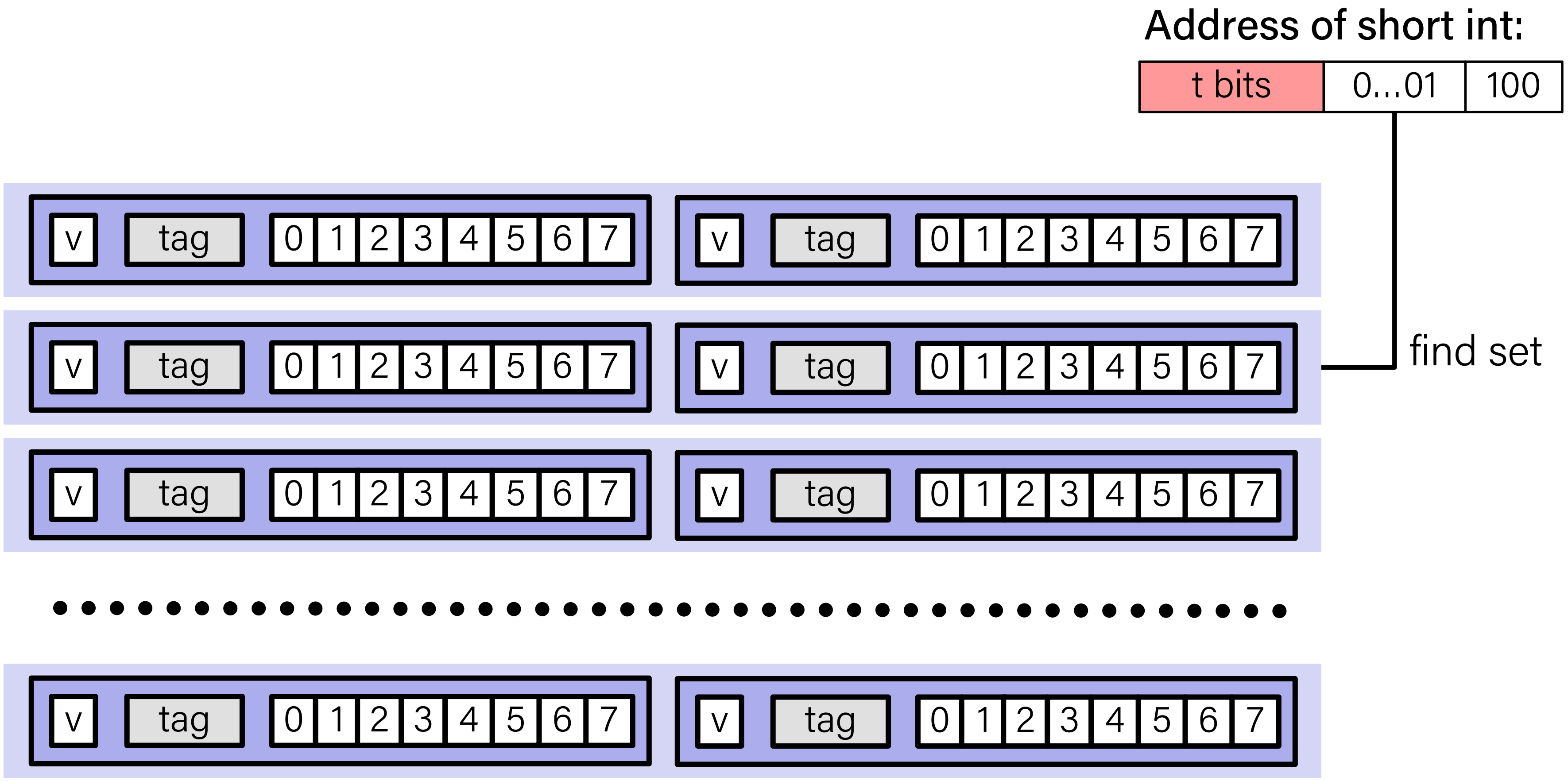
	v	Tag	Block
Set 0	1	0	M[0-1]
Set 1			
Set 2			
Set 3	1	0	M[6-7]

Address trace (reads, one byte per read):

0	[$0\underline{00}0_2$],	miss
1	[$0\underline{00}1_2$],	hit
7	[$0\underline{11}1_2$],	miss
8	[$1\underline{00}0_2$],	miss
0	[$0\underline{00}0_2$]	miss

E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set
Assume: cache block size 8 bytes



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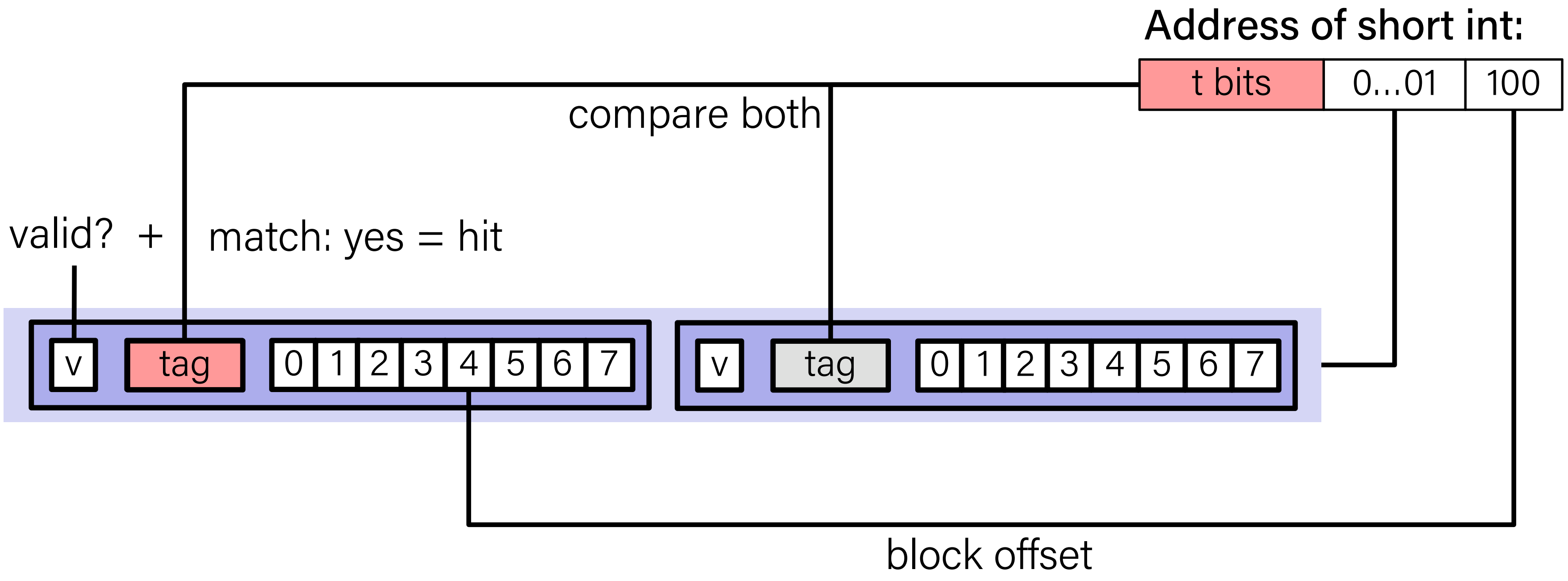

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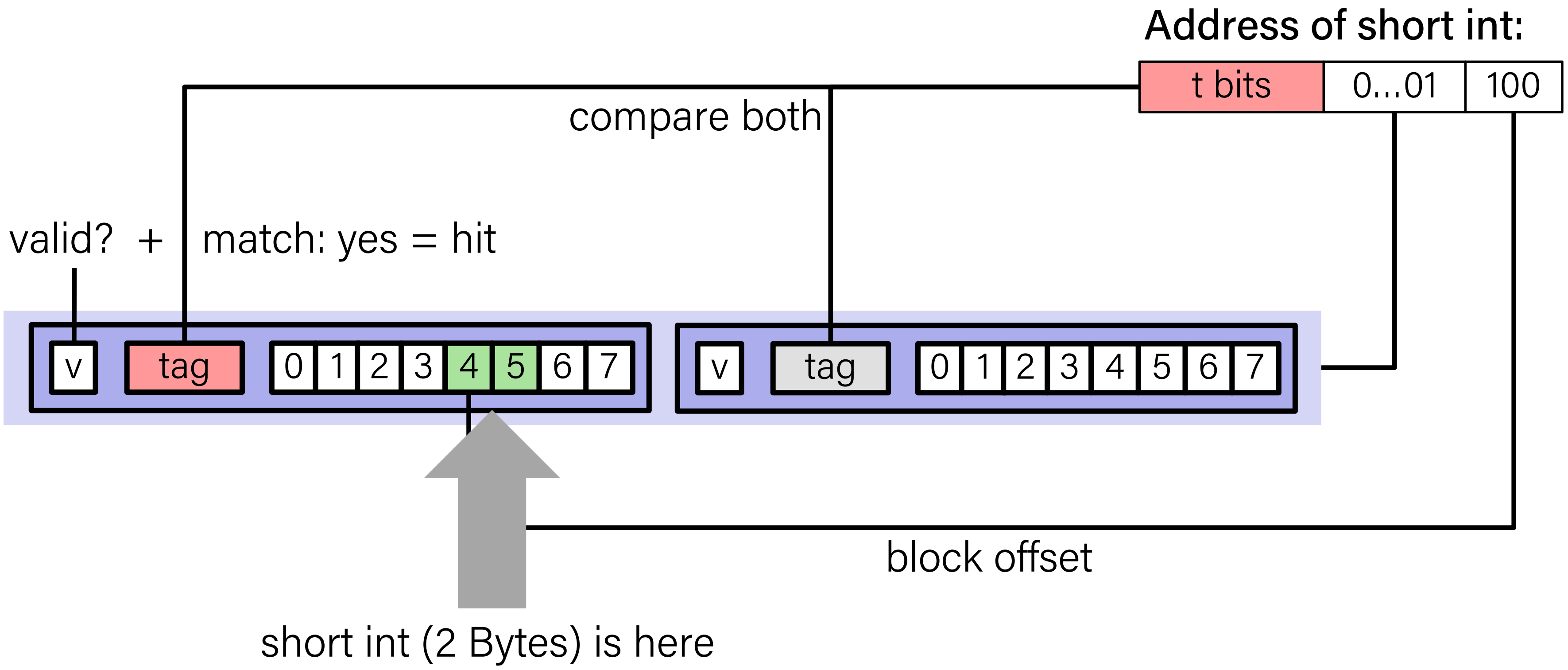


No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), …

2-Way Set Associative Cache Simulation

t=2	s=1	b=1
xx	x	x

M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 blocks/set

Address trace (reads, one byte per read):

0	$[00\underline{0}0_2]$,	miss
1	$[00\underline{0}1_2]$,	hit
7	$[01\underline{1}1_2]$,	miss
8	$[10\underline{0}0_2]$,	miss
0	$[00\underline{0}0_2]$	hit

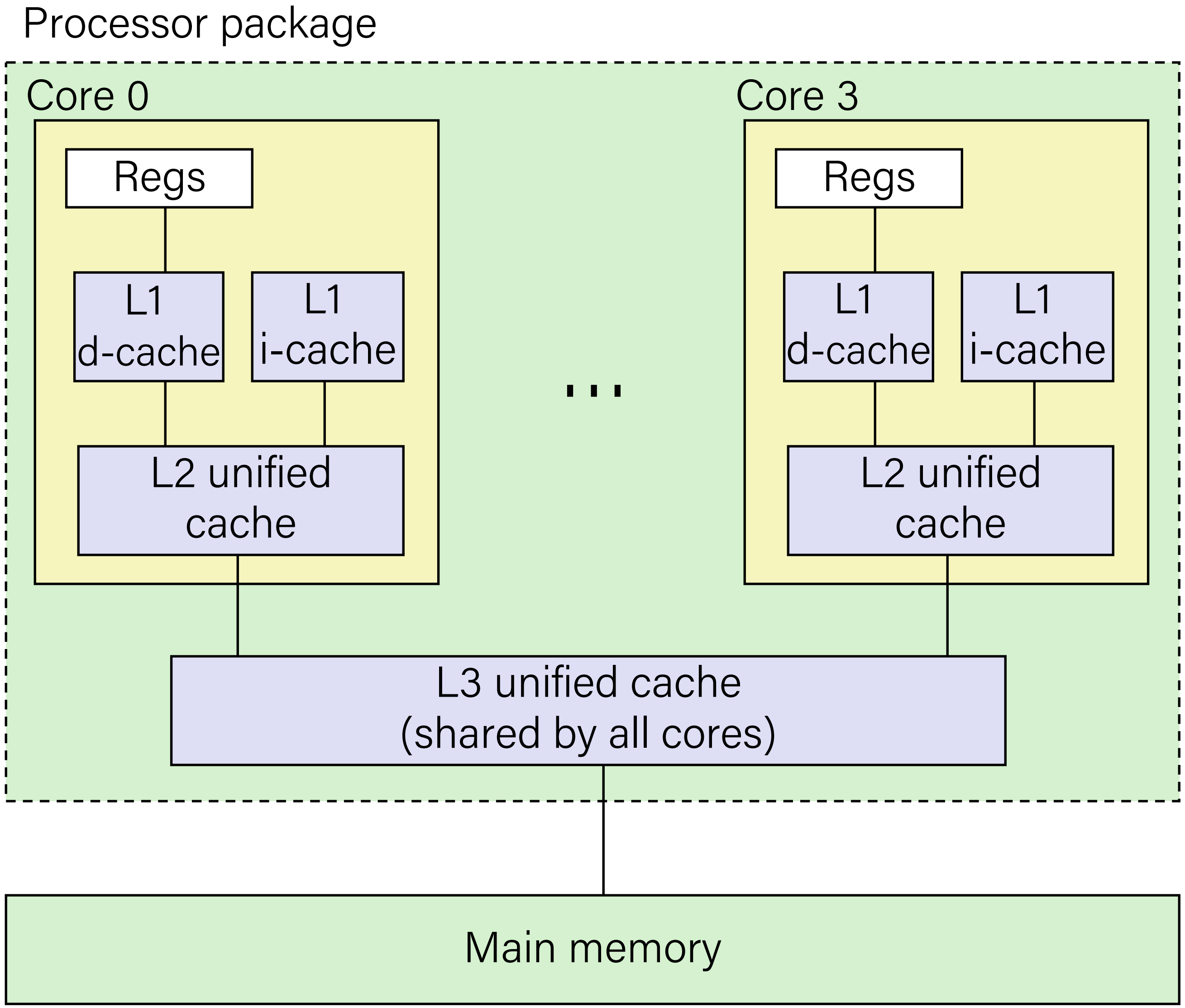
	v	Tag	Block
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	0		

Question Break

What about writes?

- **Multiple copies of data exist:**
  - L1, L2, L3, Main Memory, Disk
- **What to do on a write-hit?**
  - Write-through (write immediately to memory)
  - Write-back (defer write to memory until replacement of line)
    - Need a dirty bit (line different from memory or not)
- **What to do on a write-miss?**
  - Write-allocate (load into cache, update line in cache)
    - Good if more writes to the location follow
  - No-write-allocate (writes straight to memory, does not load into cache)
- **Typical**
  - Write-through + No-write-allocate
  - **Write-back + Write-allocate**

Intel Core i7 Cache Hierarchy



L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 10 cycles

L3 unified cache:
8 MB, 16-way,
Access: 40-75 cycles

Block size: 64 bytes for all caches.

Cache Performance Metrics

- **Miss Rate**
  - Fraction of memory references not found in cache (misses / accesses)
    = 1 – hit rate
  - Typical numbers (in percentages):
    - 3-10% for L1
    - can be quite small (e.g., < 1%) for L2, depending on size, etc.
- **Hit Time**
  - Time to deliver a line in the cache to the processor
    - includes time to determine whether the line is in the cache
  - Typical numbers:
    - 4 clock cycle for L1
    - 10 clock cycles for L2
- **Miss Penalty**
  - Additional time required because of a miss
    - typically 50-200 cycles for main memory (Trend: increasing!)

Let's think about those numbers

- Huge difference between a hit and a miss
  - Could be 100x, if just L1 and main memory

- Would you believe 99% hits is twice as good as 97%?
  - Consider:
    cache hit time of 1 cycle
    miss penalty of 100 cycles

  - Average access time:
  - 97% hits: 1 cycle + 0.03 * 100 cycles = 4 cycles
  - 99% hits: 1 cycle + 0.01 * 100 cycles = 2 cycles

- **This is why "miss rate" is used instead of "hit rate"**

Writing Cache Friendly Code

- Make the common case go fast
  - Focus on the inner loops of the core functions

- Minimize the misses in the inner loops
  - Repeated references to variables are good (temporal locality)
  - Stride-1 reference patterns are good (spatial locality)

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories

Lecture Plan

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

The Memory Mountain

- **Read throughput** (read bandwidth)
  - Number of bytes read from memory per second (MB/s)

- **Memory mountain:** Measured read throughput as a function of spatial and temporal locality.
  - Compact way to characterize memory system performance.

Memory Mountain Test Function

```c
long data[MAXELEMS];  /* Global array to traverse */

/* test - Iterate over first "elems" elements of
 *        array "data" with stride of "stride", using
 *        using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }
    return ((acc0 + acc1) + (acc2 + acc3));
}
```

mountain/mountain.c

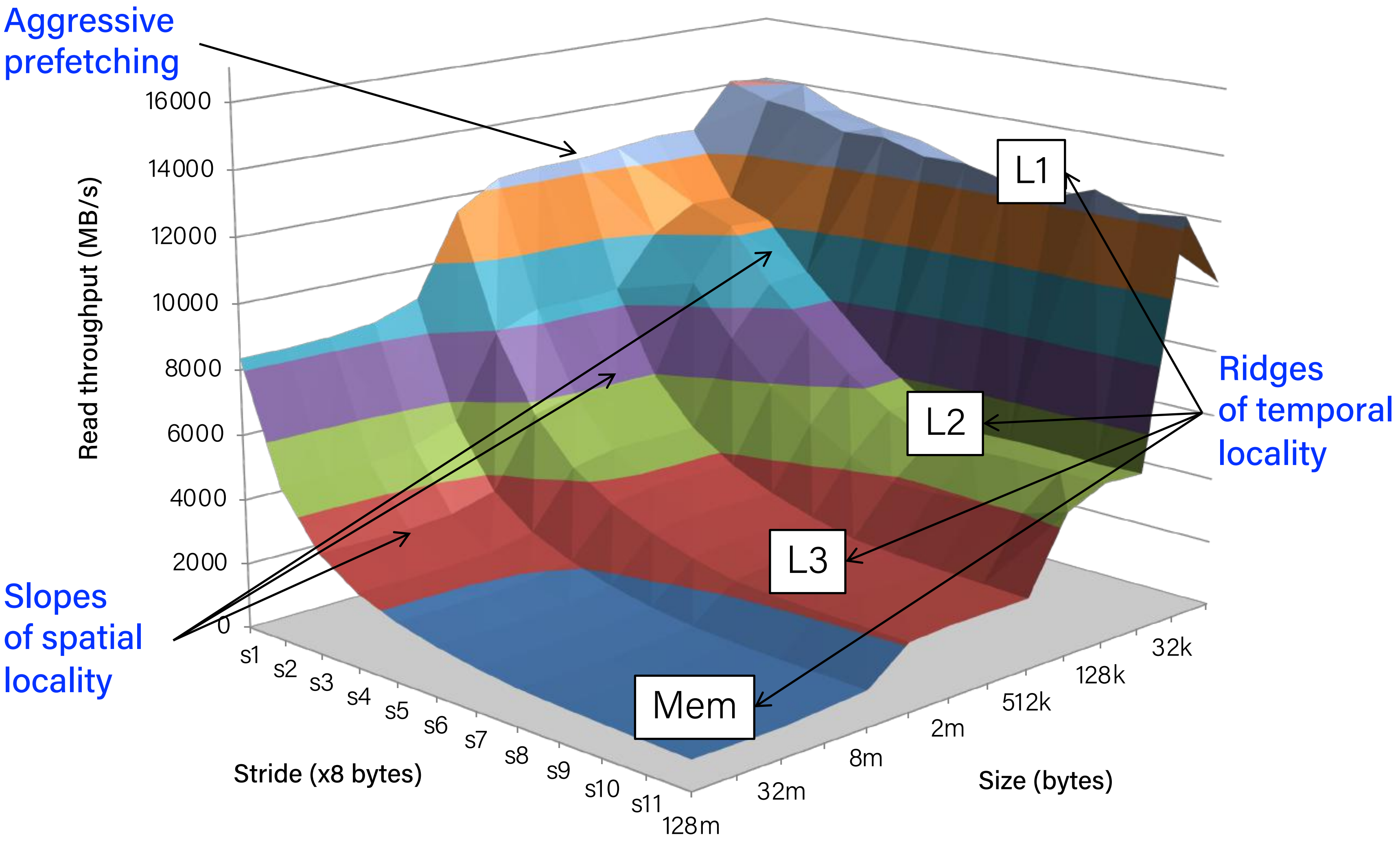
Call `test()` with many combinations of `elems` and `stride`.

For each elems and stride:

1. Call test() once to warm up the caches.

2. Call test() again and measure the read throughput(MB/s)

The Memory Mountain



Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

Lecture Plan

- Cache organization and operation
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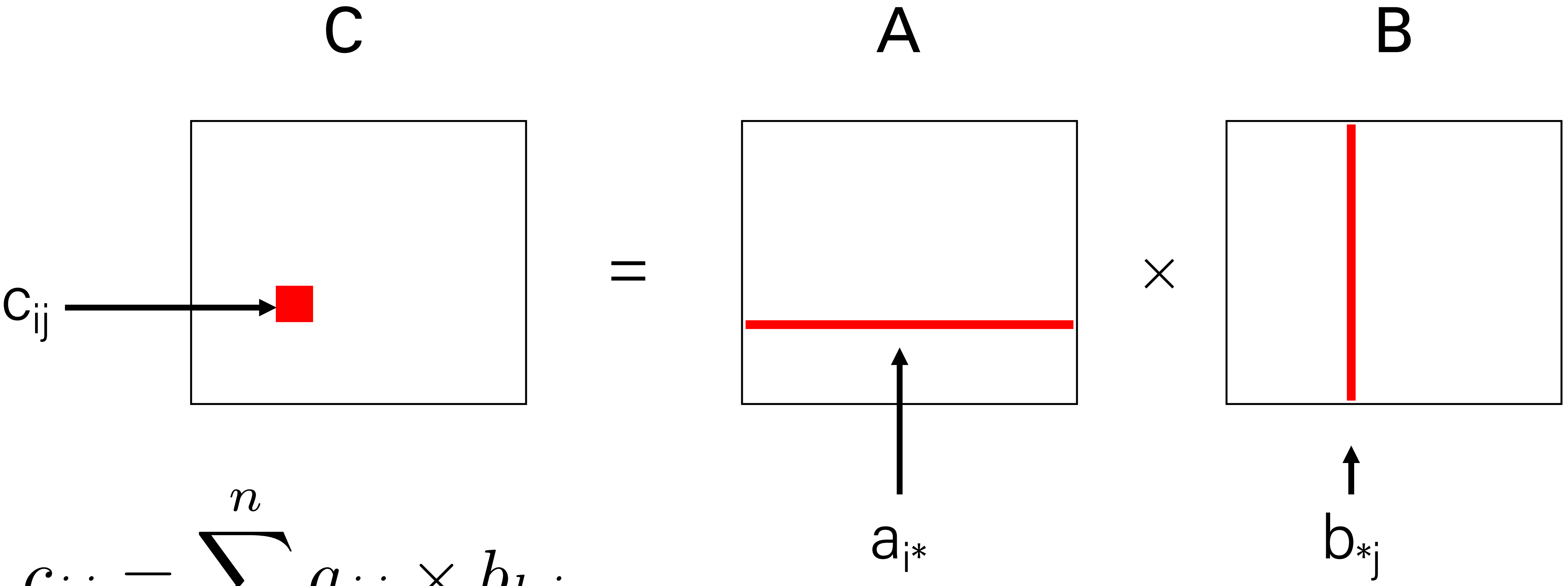
Matrix Multiplication Example



$$c_{ij} = \sum_{k=1}^{n} a_{ij} \times b_{kj}$$

Matrix Multiplication Example

- Description:
  - Multiply N x N matrices
  - Matrix elements are doubles (8 bytes)
  - $O(N^3)$ total operations
  - N reads per source element
  - N values summed per destination
    - but may be able to hold in register

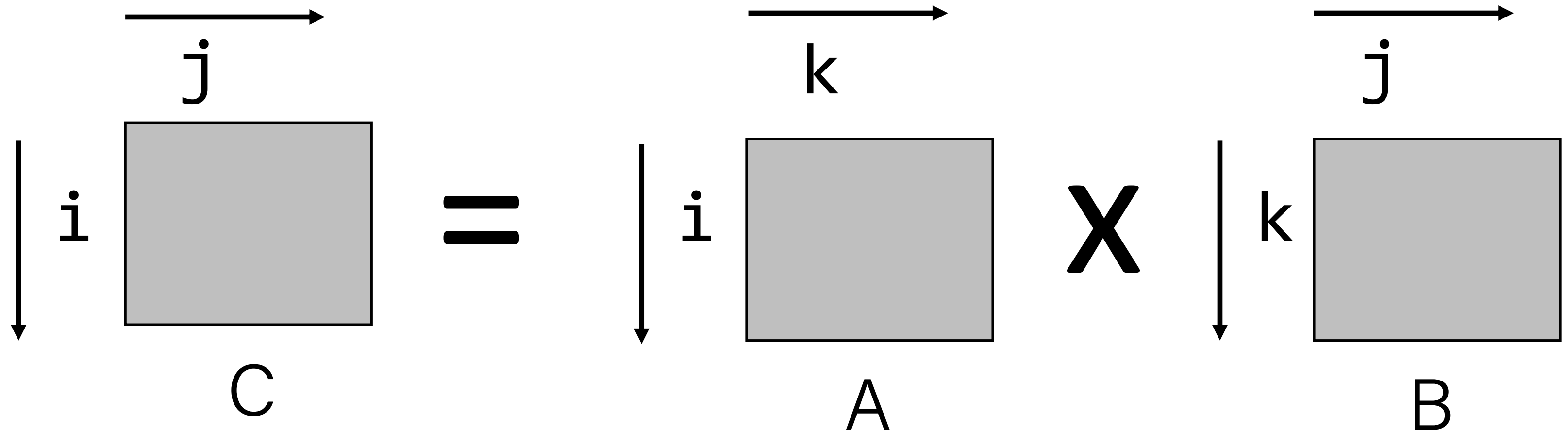
Variable `sum` held in register

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

`matmult/mm.c`

Miss Rate Analysis for Matrix Multiply

- Assume
  - Block size = 32B (big enough for four doubles)
  - Matrix dimension (N) is very large
    - Approximate 1/N as 0.0
  - Cache is not even big enough to hold multiple rows
- **Analysis Method:**
  - Look at access pattern of inner loop



Layout of C Arrays in Memory (review)

- C arrays allocated in row-major order
  - each row in contiguous memory locations

- Stepping through columns in one row:

```
for (i = 0; i < N; i++)
  sum += a[0][i];
```

  - accesses successive elements
  - if block size (B) > sizeof(aij) bytes, exploit spatial locality

  miss rate = sizeof(aij) / B

- Stepping through rows in one column:

```
for (i = 0; i < n; i++)
  sum += a[i][0];
```

  - accesses distant elements
  - no spatial locality!

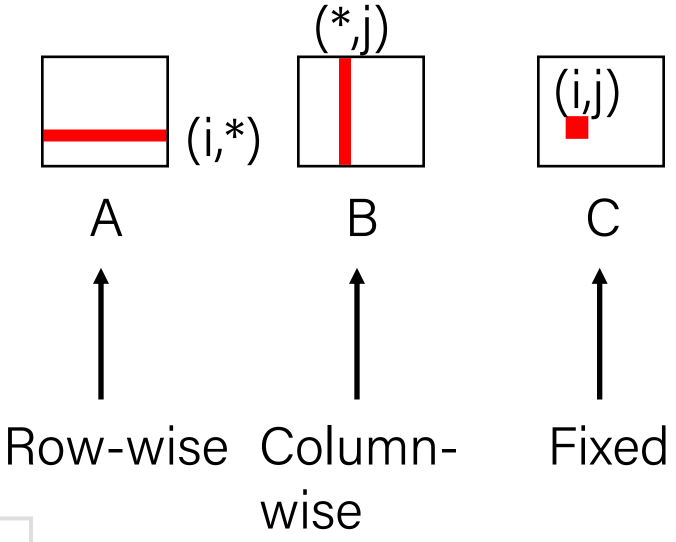
  miss rate = 1 (i.e. 100%)

Matrix Multiplication (ijk)

```
/* ijk */
for (i=0; i<n; i++)  {
  for (j=0; j<n; j++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum;
  }
}
```

`matmult/mm.c`

Inner loop:



Misses per inner loop iteration:

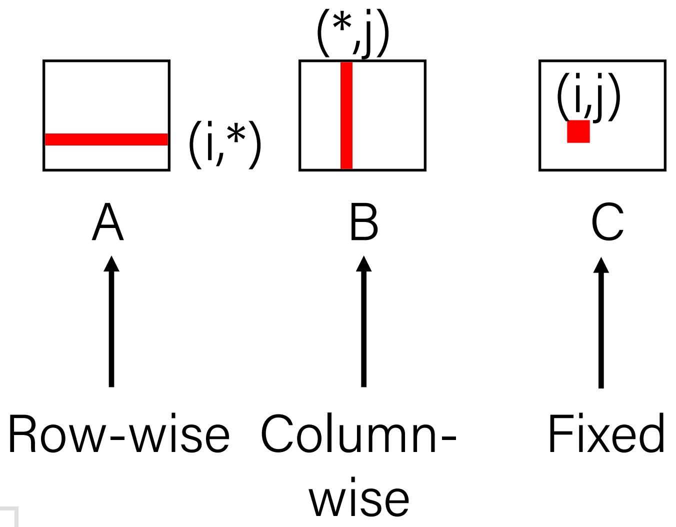
A	B	C
0.25	1.0	0.0

Matrix Multiplication (jik)

```
/* jik */
for (j=0; j<n; j++) {
  for (i=0; i<n; i++) {
    sum = 0.0;
    for (k=0; k<n; k++)
      sum += a[i][k] * b[k][j];
    c[i][j] = sum
  }
}
```

`matmult/mm.c`

Inner loop:



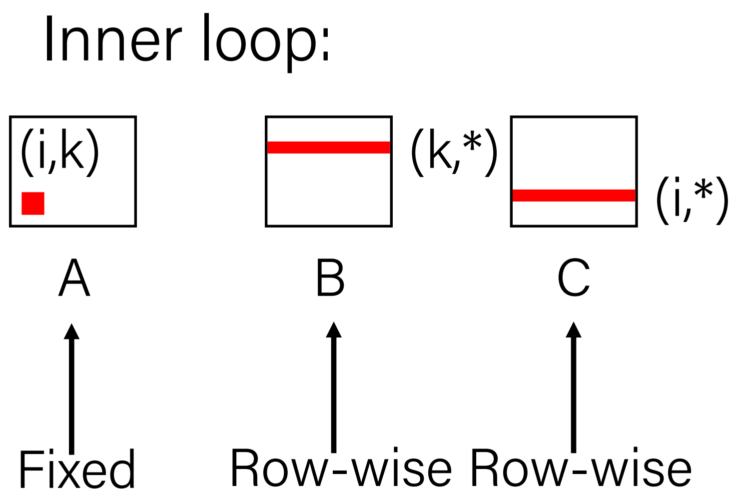
Misses per inner loop iteration:

A	B	C
0.25	1.0	0.0

Matrix Multiplication (kij)

```
/* ikj */
for (k=0; k<n; k++) {
  for (i=0; i<n; i++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

matmult/mm.c



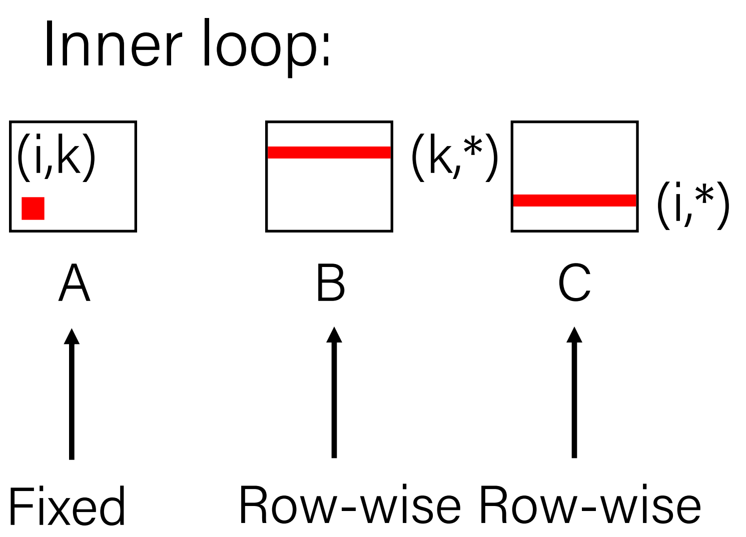
Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (ikj)

```
/* ikj */
for (i=0; i<n; i++) {
  for (k=0; k<n; k++) {
    r = a[i][k];
    for (j=0; j<n; j++)
      c[i][j] += r * b[k][j];
  }
}
```

`matmult/mm.c`



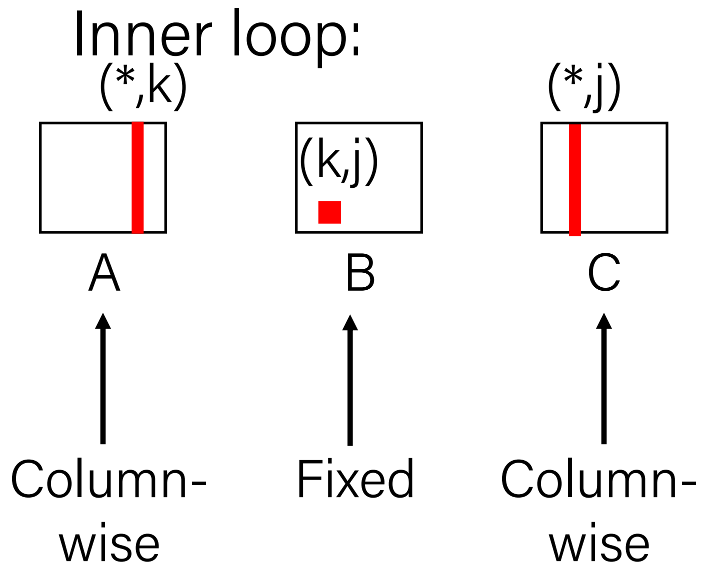
Misses per inner loop iteration:

A	B	C
0.0	0.25	0.25

Matrix Multiplication (jki)

```
/* jki */
for (j=0; j<n; j++) {
  for (k=0; k<n; k++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

`matmult/mm.c`



Misses per inner loop iteration:

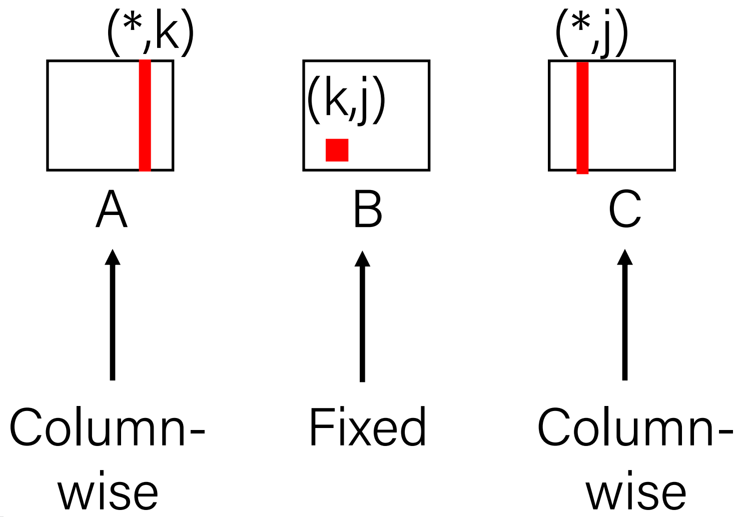
A	B	C
1.0	0.0	1.0

Matrix Multiplication (kji)

```
/* kji */
for (k=0; k<n; k++) {
  for (j=0; j<n; j++) {
    r = b[k][j];
    for (i=0; i<n; i++)
      c[i][j] += a[i][k] * r;
  }
}
```

matmult/mm.c

Inner loop:



Misses per inner loop iteration:

A	B	C
1.0	0.0	1.0

Summary of Matrix Multiplication

```
for (i=0; i<n; i++) {
  for (j=0; j<n; j++) {
   sum = 0.0;
   for (k=0; k<n; k++)
     sum += a[i][k] * b[k][j];
   c[i][j] = sum;
 }
}
```

ijk (& jik):

- 2 loads, 0 stores
- misses/iter = 1.25

```
for (k=0; k<n; k++) {
 for (i=0; i<n; i++) {
  r = a[i][k];
  for (j=0; j<n; j++)
   c[i][j] += r * b[k][j];
 }
}
```

kij (& ikj):

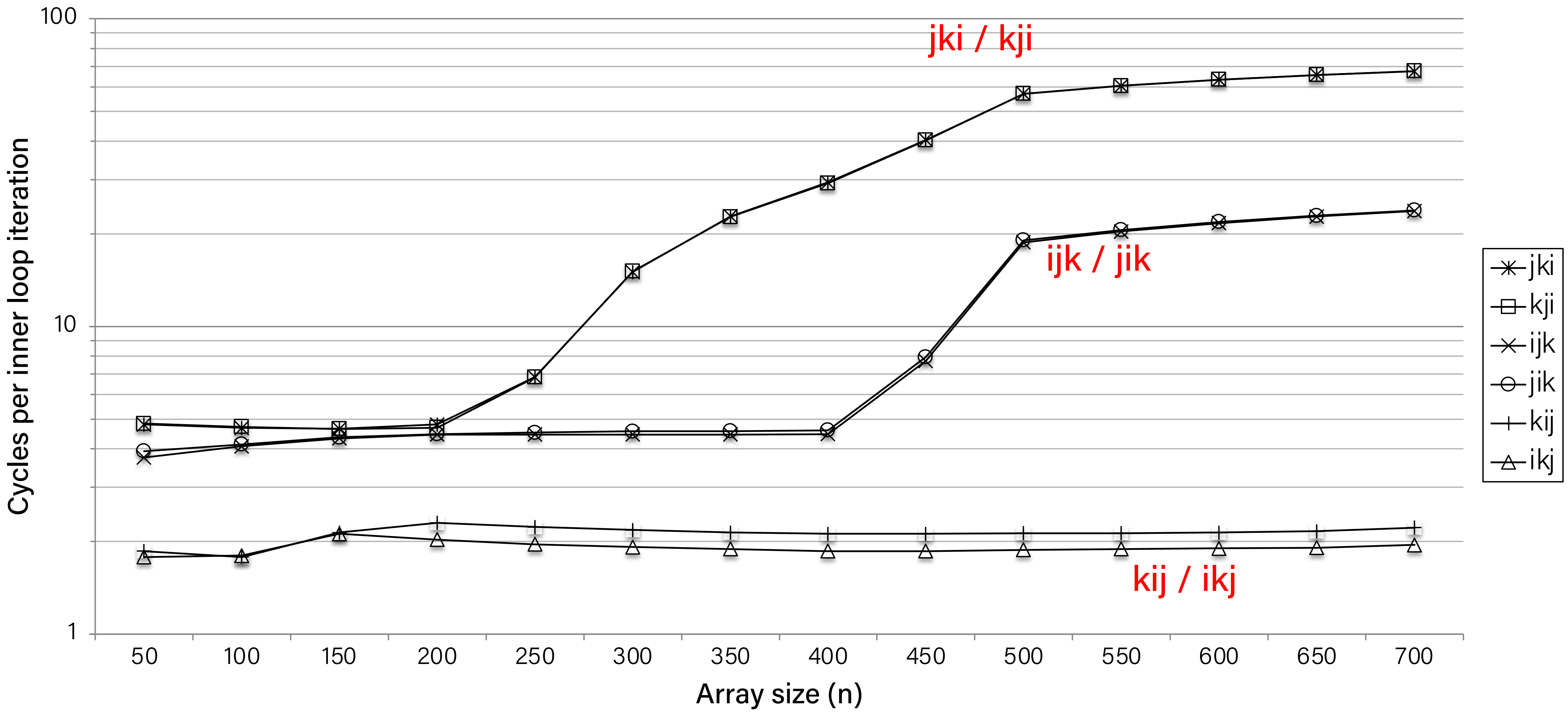
- 2 loads, 1 store
- misses/iter = 0.5

```
for (j=0; j<n; j++) {
 for (k=0; k<n; k++) {
   r = b[k][j];
   for (i=0; i<n; i++)
    c[i][j] += a[i][k] * r;
 }
}
```

jki (& kji):

- 2 loads, 1 store
- misses/iter = 2.0

Core i7 Matrix Multiply Performance



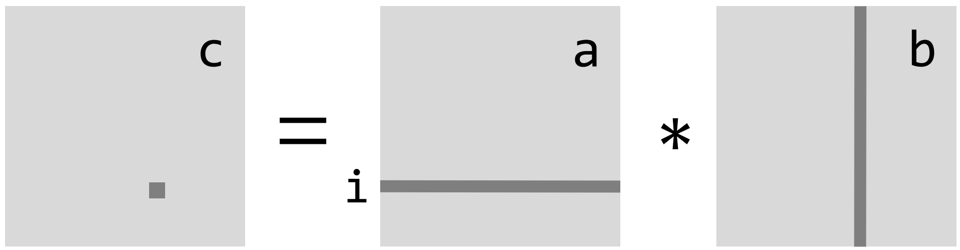
Question Break

Lecture Plan

- Cache organization and operation
- Performance impact of caches
  - The memory mountain
  - Rearranging loops to improve spatial locality
  - Using blocking to improve temporal locality

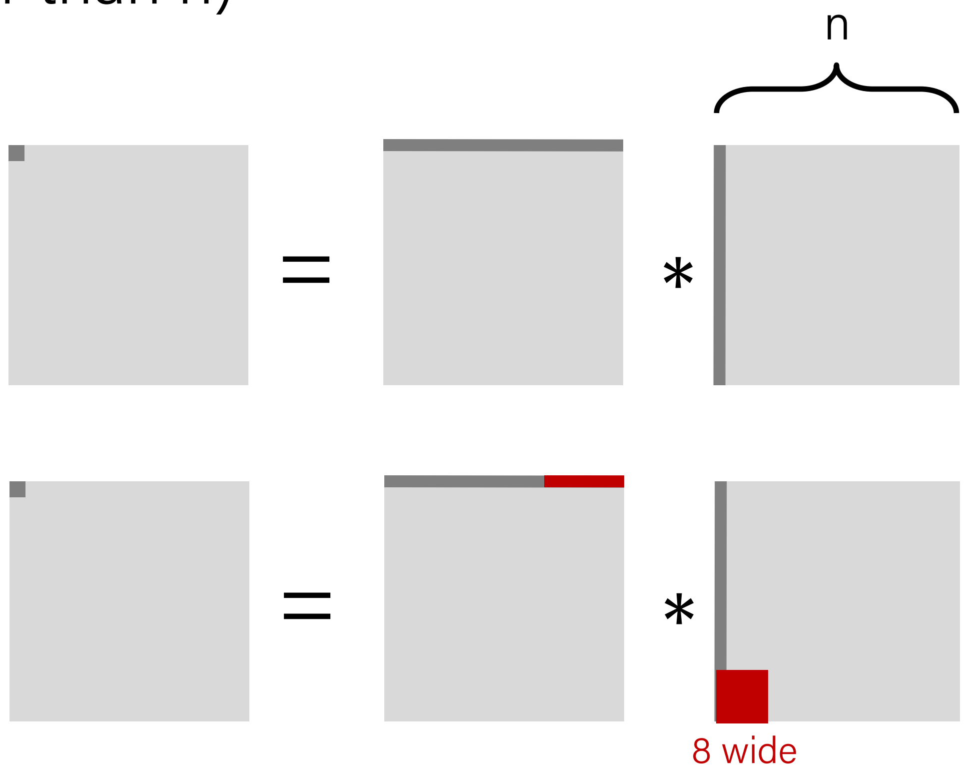
Example: Matrix Multiplication

```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i++)
      for (j = 0; j < n; j++)
           for (k = 0; k < n; k++)
             c[i*n + j] += a[i*n + k] * b[k*n + j];
}
```



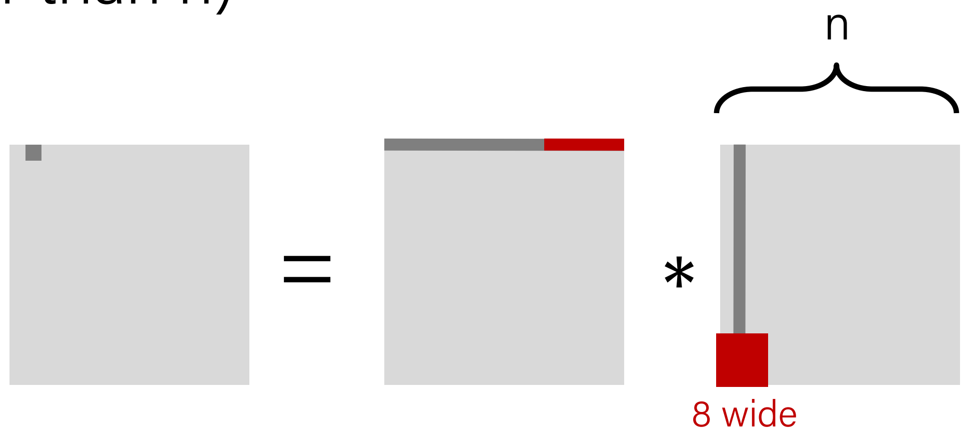
Cache Miss Analysis

- Assume
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)
- **First iteration:**
  - n/8 + n = 9n/8 misses
  - Afterwards in cache: (schematic)



Cache Miss Analysis

- Assume
  - Matrix elements are doubles
  - Cache block = 8 doubles
  - Cache size C << n (much smaller than n)



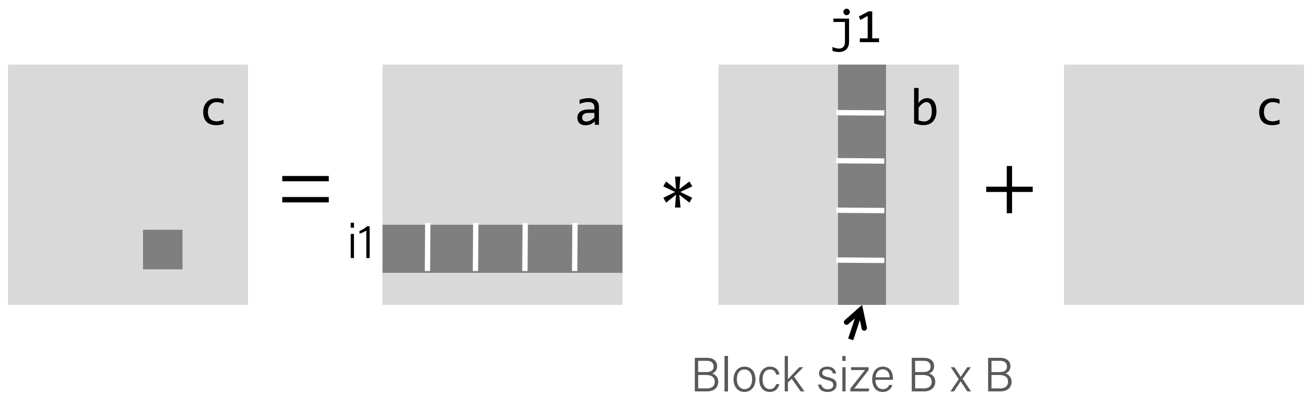
- **Second iteration:**
  - Again:
    n/8 + n = 9n/8 misses

- **Total misses:**
  - $9n/8 * n^2 = (9/8) * n^3$

Blocked Matrix Multiplication

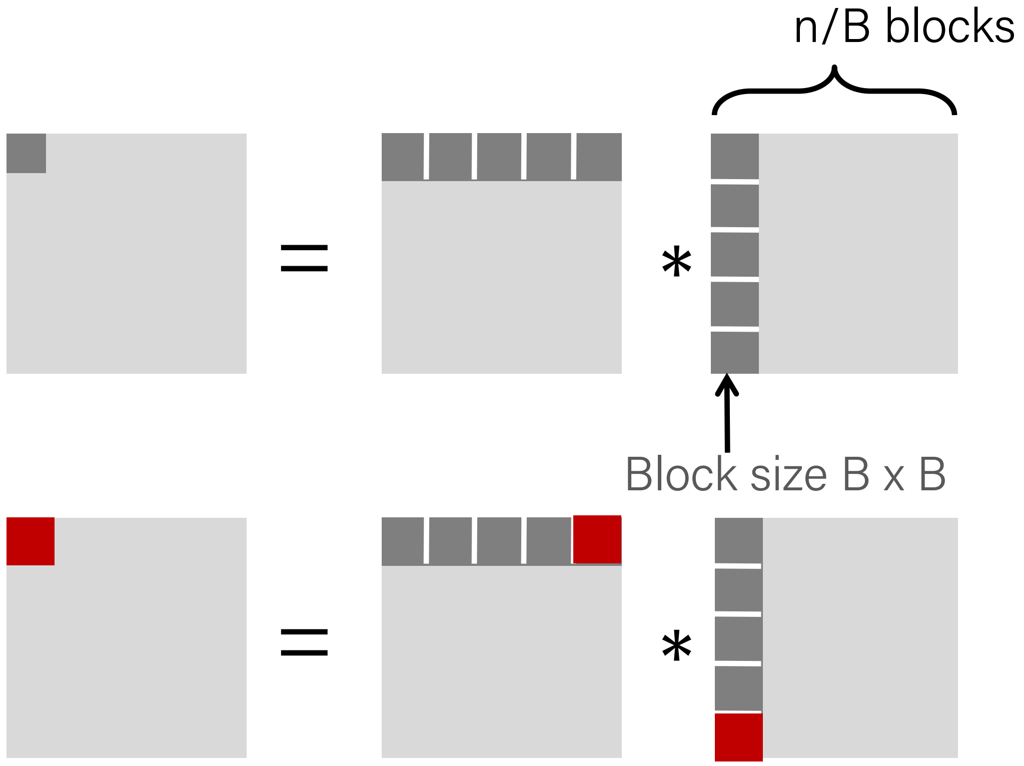
```
c = (double *) calloc(sizeof(double), n*n);

/* Multiply n x n matrices a and b  */
void mmm(double *a, double *b, double *c, int n) {
    int i, j, k;
    for (i = 0; i < n; i+=B)
        for (j = 0; j < n; j+=B)
            for (k = 0; k < n; k+=B)
                /* B x B mini matrix multiplications */
                 for (i1 = i; i1 < i+B; i++)
                     for (j1 = j; j1 < j+B; j++)
                         for (k1 = k; k1 < k+B; k++)
                             c[i1*n+j1] += a[i1*n + k1]*b[k1*n + j1];
}
```

matmult/bmm.c



Cache Miss Analysis

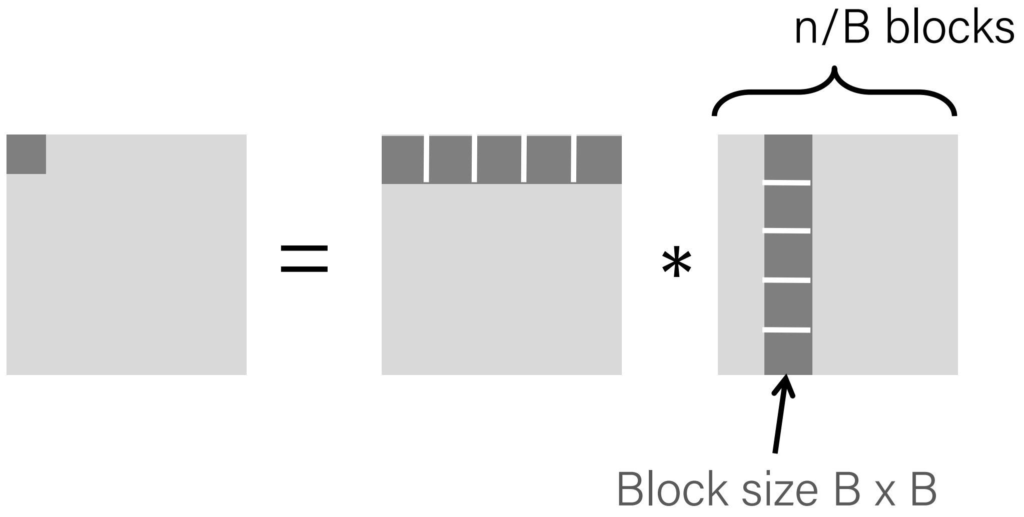
- Assume
  - Cache block = 8 doubles
  - Cache size $C \ll n$ (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$
- **First (block) iteration:**
  - $B^2/8$ misses for each block
  - $2n/B * B^2/8 = nB/4$ (omitting matrix c)
  - Afterwards in cache (schematic)



Cache Miss Analysis

- Assume:
  - Cache block = 8 doubles
  - Cache size $C << n$ (much smaller than n)
  - Three blocks ■ fit into cache: $3B^2 < C$

- **Second (block) iteration**:
  - Same as first iteration
  - $2n/B * B^2/8 = nB/4$



- Total misses:
  - $nB/4 * (n/B)^2 = n^3/(4B)$

Blocking Summary

- **No blocking:** (9/8) * n^3
- **Blocking:** 1/(4B) * n^3

- **Suggest largest possible block size B, but limit $3B^2 < C$!**

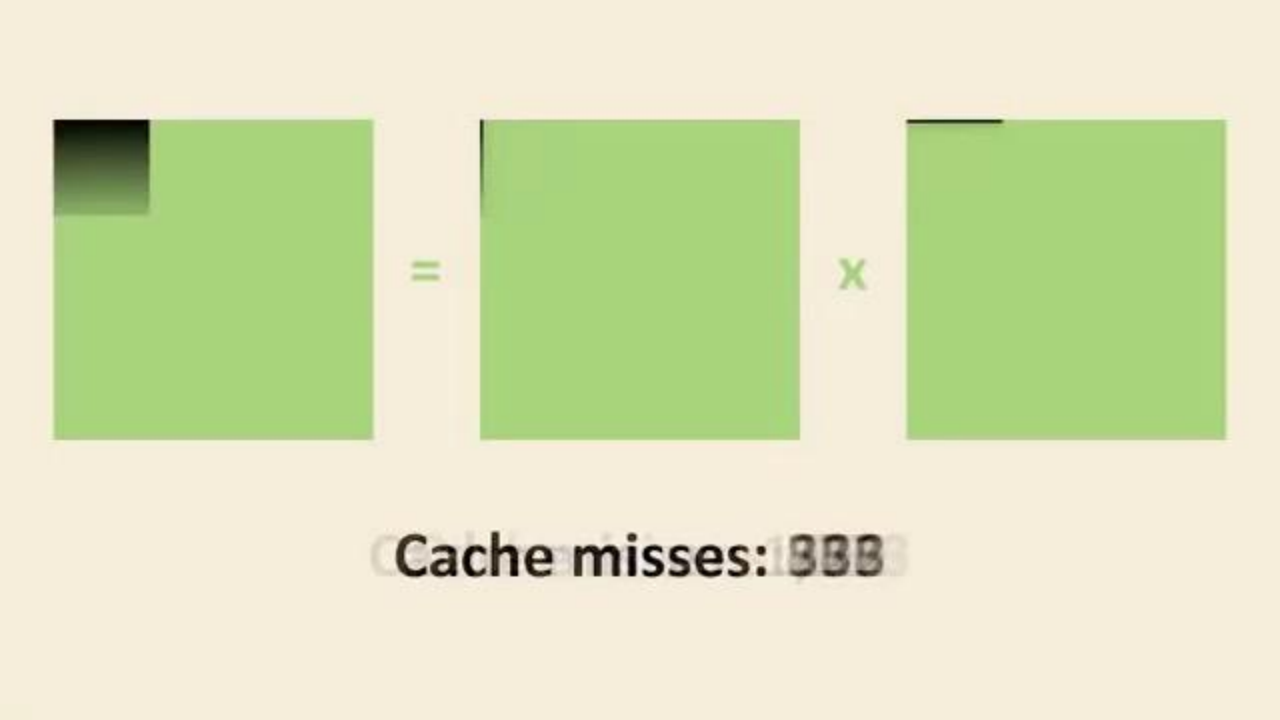
- **Reason for dramatic difference:**
  - Matrix multiplication has inherent temporal locality:
    - Input data: $3n^2$, computation $2n^3$
    - Every array elements used O(n) times!
  - But program has to be written properly

Naïve vs. Blocked Matrix Multiplication

Naïve Multiplication

≈ 1,020,000 cache misses

Blocked Multiplication



≈ 90,000 cache misses

Video credit: Vasily Volkov

Recap

- Cache memories can have significant performance impact
- You can write your programs to exploit this!
  - Focus on the inner loops, where bulk of computations and memory accesses occur.
  - Try to maximize spatial locality by reading data objects with sequentially with stride 1.
  - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

Next time: Debugging and Design