

COMP201

Computer Systems & Programming

Lecture #22 – More Cache Memories



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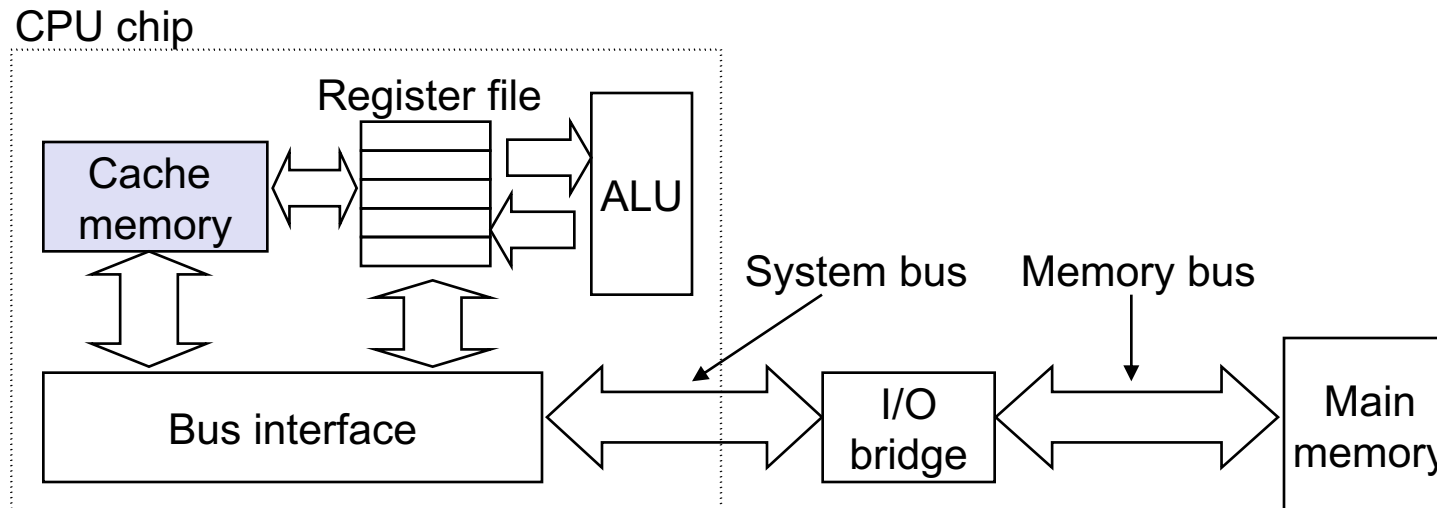
Aykut Erdem // Koç University // Spring 2025

Recap

- Cache basics
- Principle of locality
- Cache memory organization and operation

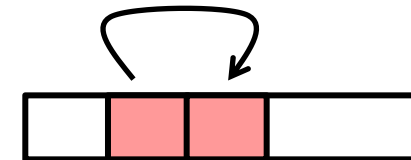
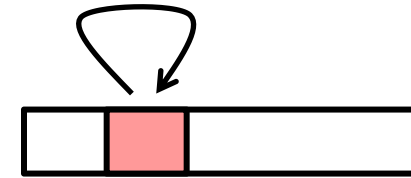
Recap: Cache Memories

- Cache memories are small, fast SRAM-based memories managed automatically in hardware
 - Hold frequently accessed blocks of main memory
- CPU looks first for data in cache
- Typical system structure:



Recap: Why Caches Work

- **Principle of Locality:** Programs tend to use data and instructions with addresses near or equal to those they have used recently
- **Temporal locality:**
 - Recently referenced items are likely to be referenced again in the near future
- **Spatial locality:**
 - Items with nearby addresses tend to be referenced close together in time



Recap: Good Locality Example

- Does this function have good locality with respect to array `a`?

```
int sum_array_rows(int a[M][N])
{
    int i, j, sum = 0;

    for (i = 0; i < M; i++)
        for (j = 0; j < N; j++)
            sum += a[i][j];
    return sum;
}
```

M = 3,
N = 4

a[0][0]	a[0][1]	a[0][2]	a[0][3]
a[1][0]	a[1][1]	a[1][2]	a[1][3]
a[2][0]	a[2][1]	a[2][2]	a[2][3]

Access Pattern:
stride = 1

1)	a[0][0]
2)	a[0][1]
3)	a[0][2]
4)	a[0][3]
5)	a[1][0]
6)	a[1][1]
7)	a[1][2]
8)	a[1][3]
9)	a[2][0]
10)	a[2][1]
11)	a[2][2]
12)	a[2][3]

Layout in Memory

a	a	a	a	a	a	a	a	a	a	a	a
[0]	[0]	[0]	[0]	[1]	[1]	[1]	[1]	[2]	[2]	[2]	[2]
[0]	[1]	[2]	[3]	[0]	[1]	[2]	[3]	[0]	[1]	[2]	[3]

Note: 76 is just one possible starting address of array `a`

↑ 76 ↑ 92 ↑ 108

Recap: Bad Locality Example

- Does this function have good locality with respect to array *a*?

```
int sum_array_cols(int a[M][N])
{
    int i, j, sum = 0;

    for (j = 0; j < N; j++)
        for (i = 0; i < M; i++)
            sum += a[i][j];
    return sum;
}
```

M = 3,
N = 4

a[0][0]	a[0][1]	a[0][2]	a[0][3]
a[1][0]	a[1][1]	a[1][2]	a[1][3]
a[2][0]	a[2][1]	a[2][2]	a[2][3]

Access Pattern:
stride = 4

1)	a[0][0]
2)	a[1][0]
3)	a[2][0]
4)	a[0][1]
5)	a[1][1]
6)	a[2][1]
7)	a[0][2]
8)	a[1][2]
9)	a[2][2]
10)	a[0][3]
11)	a[1][3]
12)	a[2][3]

Layout in Memory

a	a	a	a	a	a	a	a	a	a	a	a
[0]	[0]	[0]	[0]	[1]	[1]	[1]	[1]	[2]	[2]	[2]	[2]
[0]	[1]	[2]	[3]	[0]	[1]	[2]	[3]	[0]	[1]	[2]	[3]

Note: 76 is just one possible starting address of array *a*

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Plan for Today

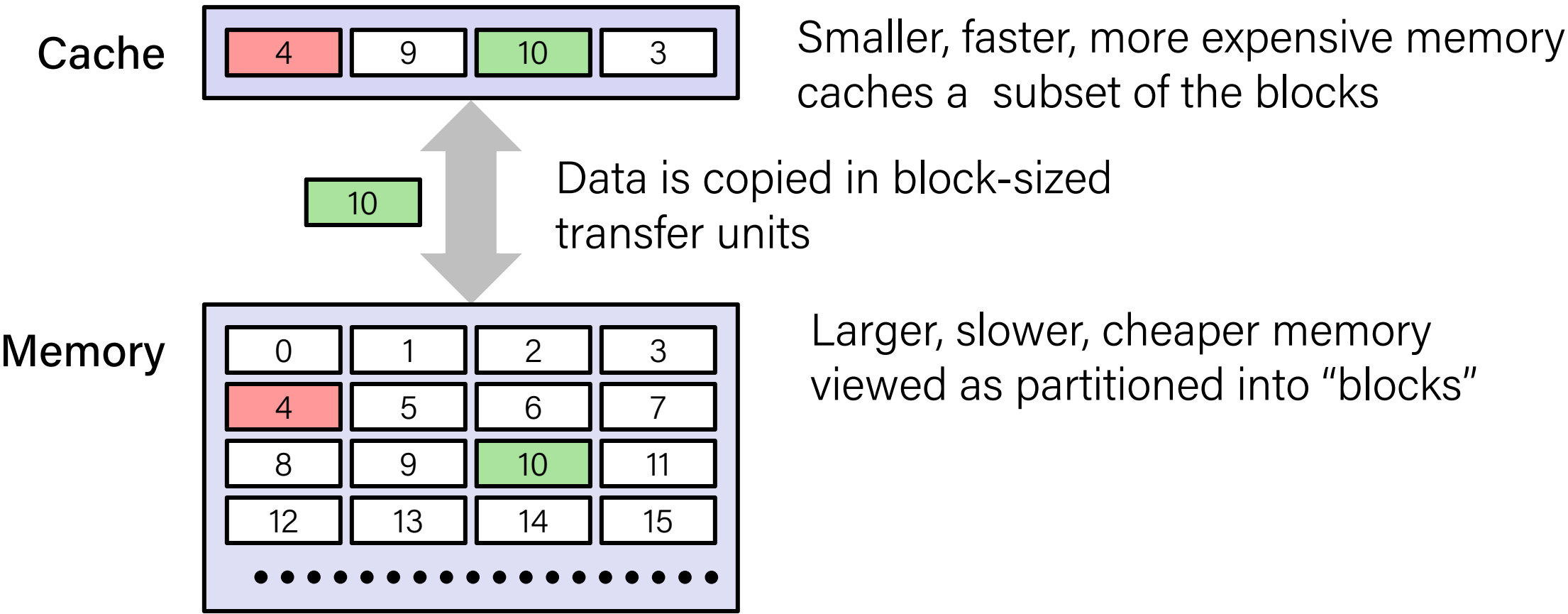
- Cache memory organization and operation
- Memory Mountain

Disclaimer: Slides for this lecture were borrowed from
—Randal E. Bryant and David R. O'Hallaron's CMU 15-213 class
—Porter Jones' UW CSE 351 class

Lecture Plan

- Cache memory organization and operation
- Memory Mountain

Cache Organization



Cache Performance Metrics

- Huge difference between a cache hit and a cache miss
 - Could be 100x speed difference between accessing cache and main memory (measured in *clock cycles*)
- Miss Rate (MR)
 - Fraction of memory references not found in cache (misses / accesses)
= 1 - Hit Rate
- Hit Time (HT)
 - Time to deliver a block in the cache to the processor
 - Includes time to determine whether the block is in the cache
- Miss Penalty (MP)
 - Additional time required because of a miss

Can we have more than one cache?

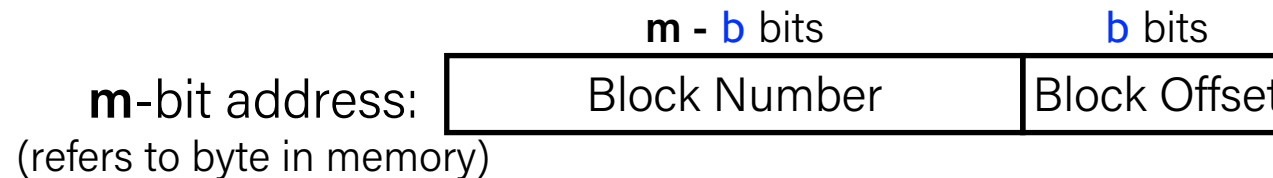
- Why would we want to do that?
 - Avoid going to memory!
 - Typical performance numbers:
 - Miss Rate
 - L1 MR = 3-10%
 - L2 MR = Quite small (*e.g.* $< 1\%$), depending on parameters, etc.
 - Hit Time
 - L1 HT = 4 clock cycles
 - L2 HT = 10 clock cycles
 - Miss Penalty
 - $P = 50\text{-}200$ cycles for missing in L2 & going to main memory
 - Trend: increasing!
- (1) Optimize L1 for high HT**
(2) Optimize L2 for low MR

Cache Organization

- **Block Size (B):** unit of transfer between cache and main memory
 - Given in bytes and always a power of 2 (*e.g.* 64 bytes)
 - Blocks consist of adjacent bytes (differ in address by 1)
 - Spatial locality!

Cache Organization

- **Block Size (B):** unit of transfer between cache and main memory
 - Given in bytes and always a power of 2 (*e.g.* 64 bytes)
 - Blocks consist of adjacent bytes (differ in address by 1)
 - Spatial locality!
- Offset field
 - Low-order $\log_2(B) = b$ bits of address tell you which byte within a block
 - $(\text{address}) \bmod 2^n = n$ lowest bits of address
 - $(\text{address}) \bmod (\# \text{ of bytes in a block})$



Question

- If we have 6-bit addresses and block size $B = 4$ bytes, which block and byte does `0x15` refer to?

	Block Num	Block Offset
A.	1	1
B.	1	5
C.	5	1
D.	5	5
E.	We're lost...	

Recap:



If we have 6-bit addresses and block size $B = 4$ bytes, which block and byte does 0x15 refer to?

① Start presenting to display the poll results on this slide.

Question

- If we have 6-bit addresses and block size $B = 4$ bytes, which block and byte does **0x15** refer to?

	Block Num	Block Offset
A.	1	1
B.	1	5
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D.	5	5
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0x ¹ ⁵
Address: 0b 0 1 0 1 / 0 1

Offset width = $\log_2(B) = \log_2(4) = 2$ bits

Cache Organization

- **Cache Size (C):** amount of *data* the cache can store
 - Cache can only hold so much data (subset of next level)
 - Given in bytes (C) or number of blocks (C/B)
 - Example: $C = 32 \text{ KiB} = 512 \text{ blocks}$ if using 64-byte blocks
- Where should data go in the cache?
 - We need a mapping from memory addresses to specific locations in the cache to make checking the cache for an address **fast**
- What is a data structure that provides fast lookup?
 - Hash table!

Review: Hash Tables for Fast Lookup

Insert:

5

27

34

102

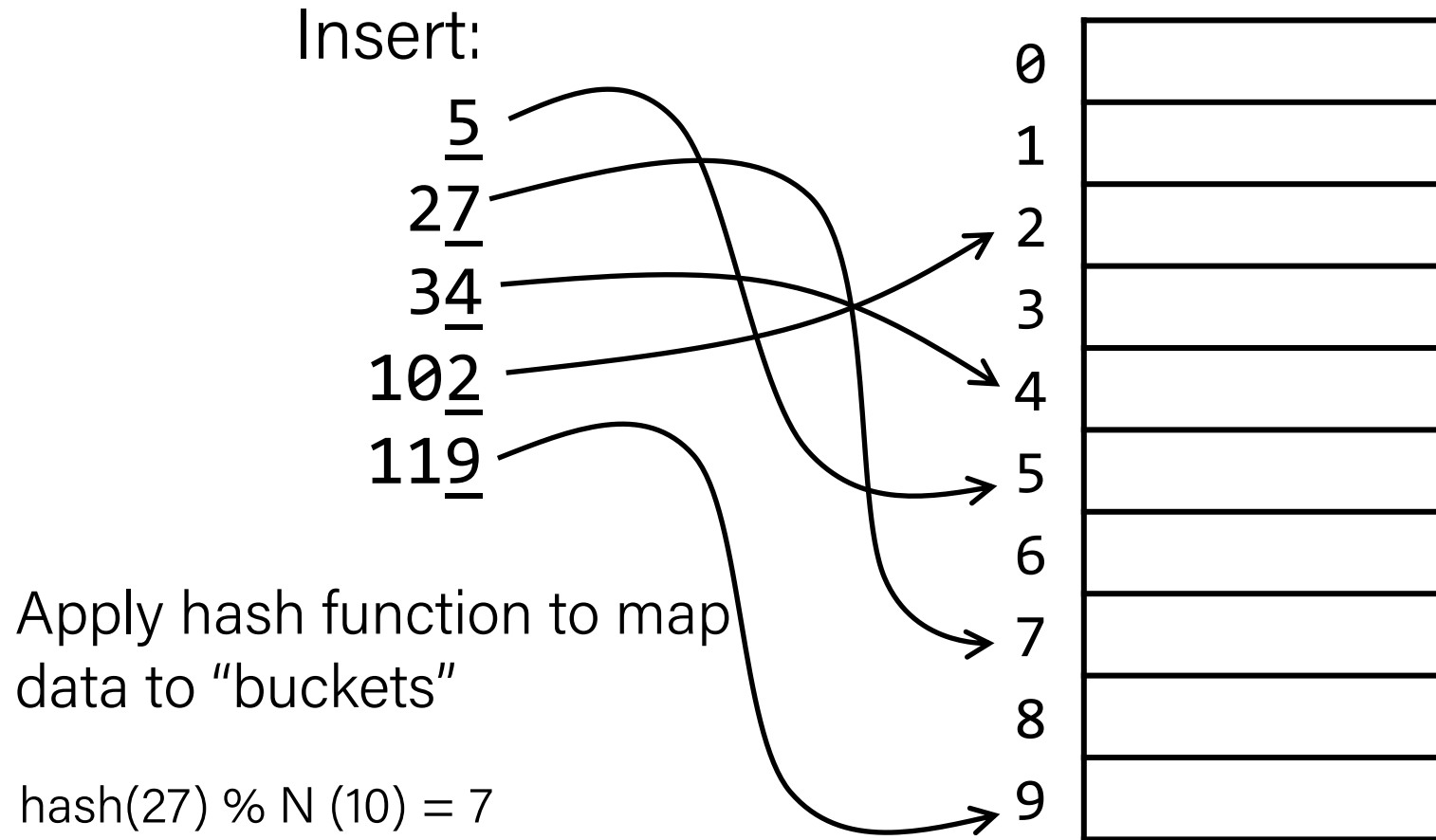
119

Apply hash function to map
data to "buckets"

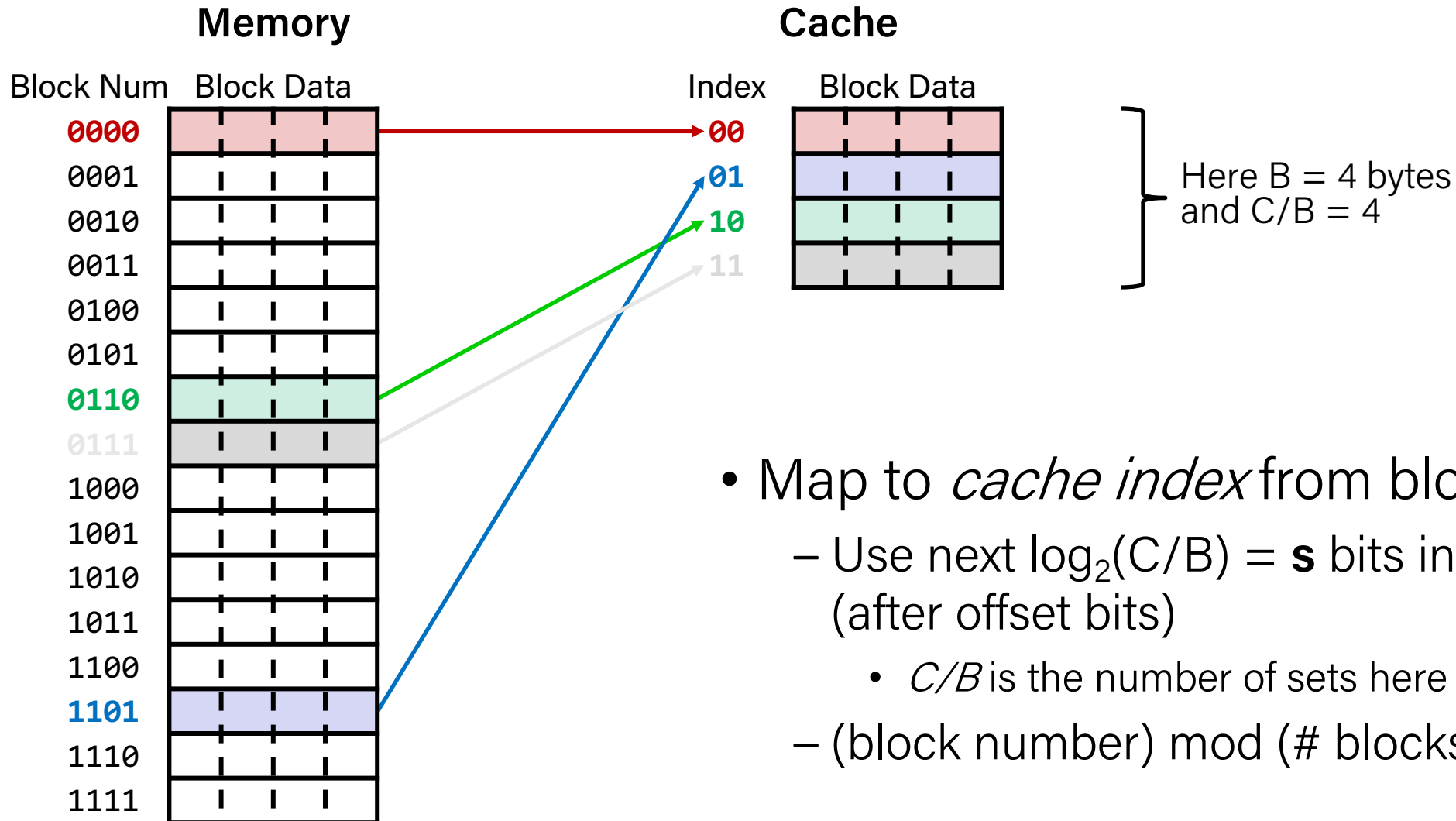
$$\text{hash}(27) \% N(10) = 7$$

0	
1	
2	
3	
4	
5	
6	
7	
8	
9	

Review: Hash Tables for Fast Lookup

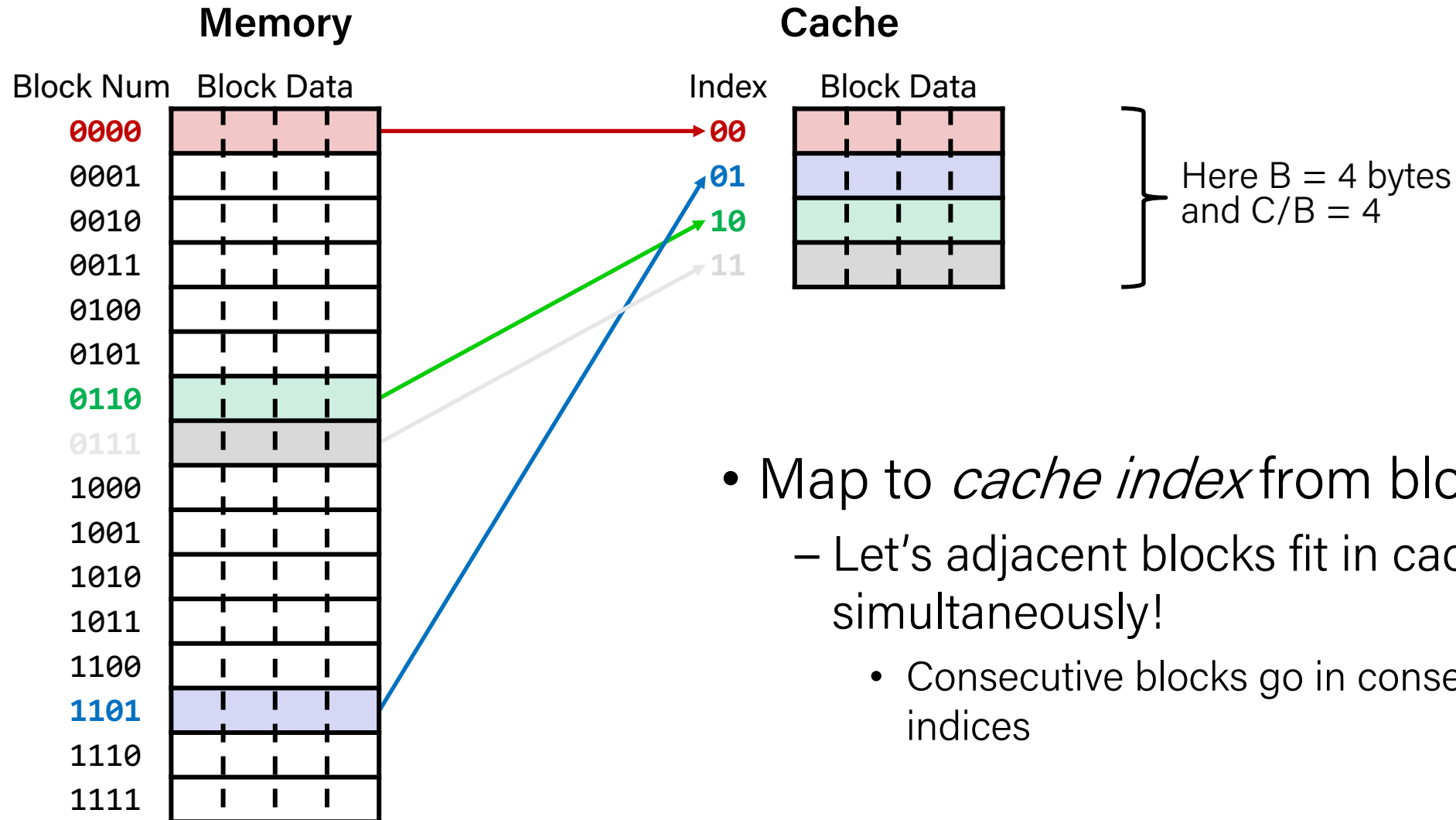


Place Data in Cache by Hashing Address



- Map to *cache index* from block number
 - Use next $\log_2(C/B) = s$ bits in the address (after offset bits)
 - C/B is the number of sets here
 - (block number) mod (# blocks in cache)

Place Data in Cache by Hashing Address



Practice Question

- 6-bit addresses, block size $B = 4$ bytes, and our cache holds $S = 4$ blocks.
- A request for address **0x2A** results in a cache miss. Which set index does this block get loaded into and which 3 other addresses are loaded along with it?

Practice Question

- 6-bit addresses, block size $B = 4$ bytes, and our cache holds $S = 4$ blocks.
 $C = S \times B = 16$ bytes $b = \log_2(4) = 2$ bits $s = \log_2(4) = 2$ bits
- A request for address **0x2A** results in a cache miss. Which set index does this block get loaded into and which 3 other addresses are loaded along with it?

0x 2 A
Address: 0b 1 0 | 1 0 | 1 0
 index offset
 └───┘
 block number

addresses w/block number 1010

0b101000 = 0x28

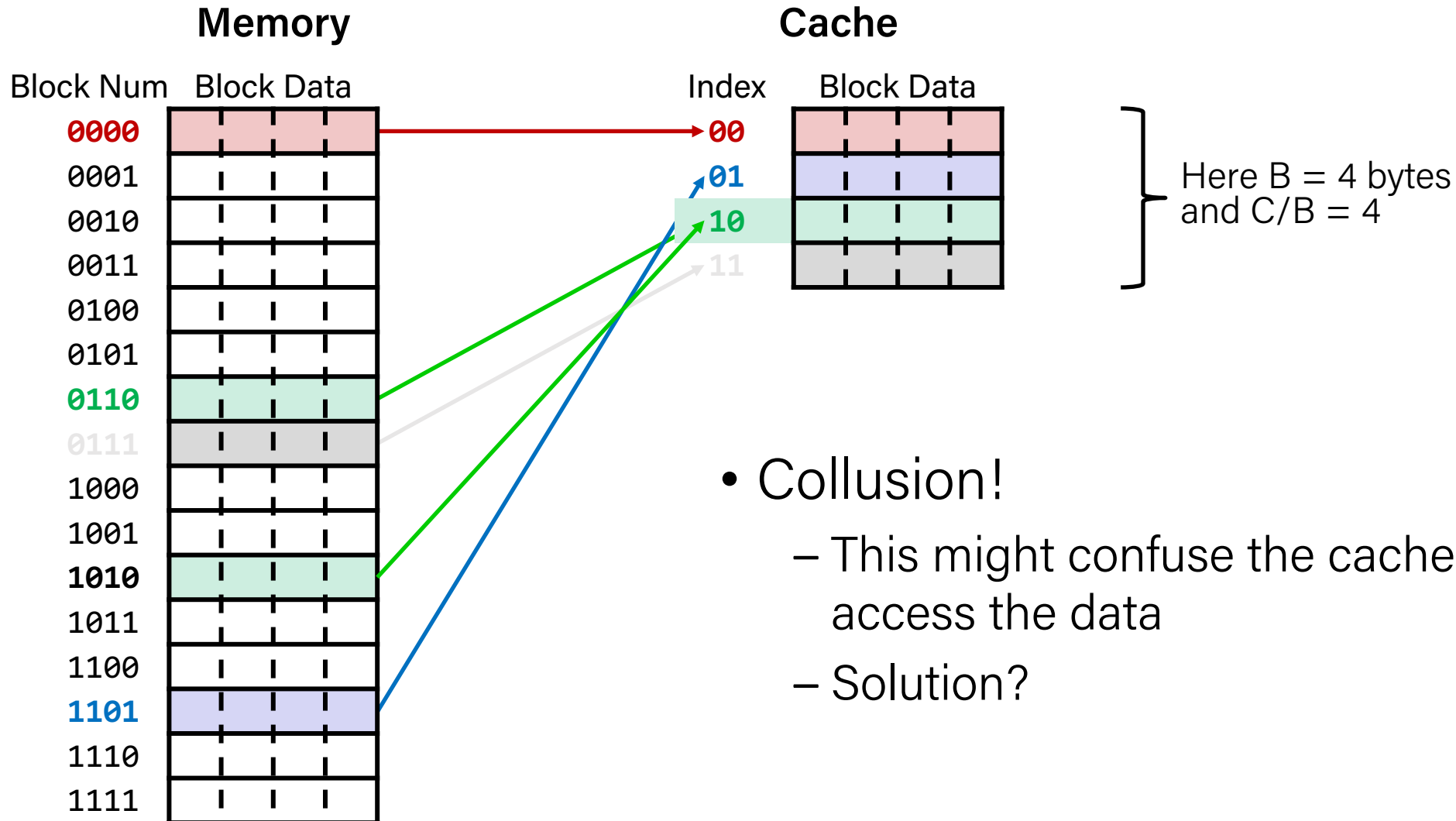
0b101001 = 0x29

0b101010 = 0x2A

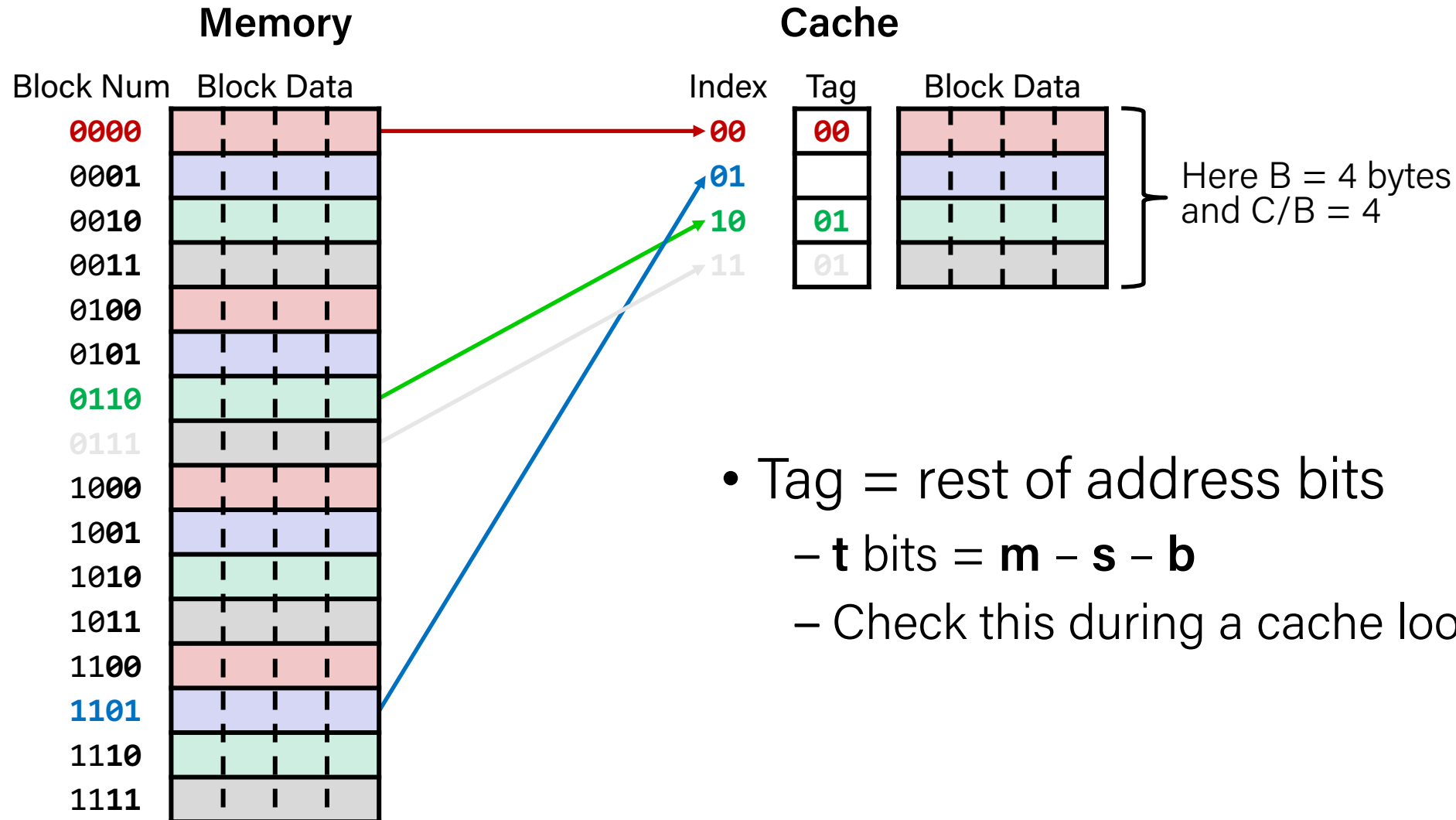
0b101011 = 0x2B

**These are loaded
into cache!**

Place Data in Cache by Hashing Address



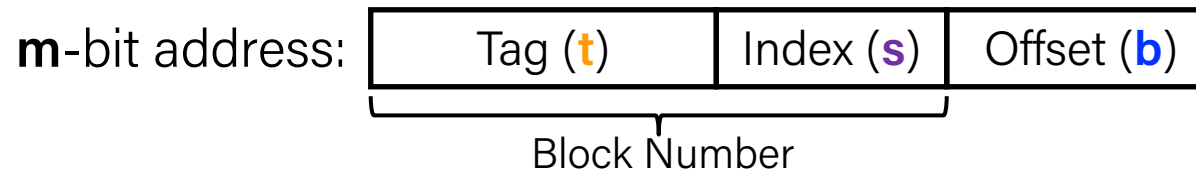
Tags Differentiate Blocks in Same Index



Checking for a Requested Address

- CPU sends address request for chunk of data
 - Address and requested data are not the same thing!
 - Analogy: your friend \neq their phone number

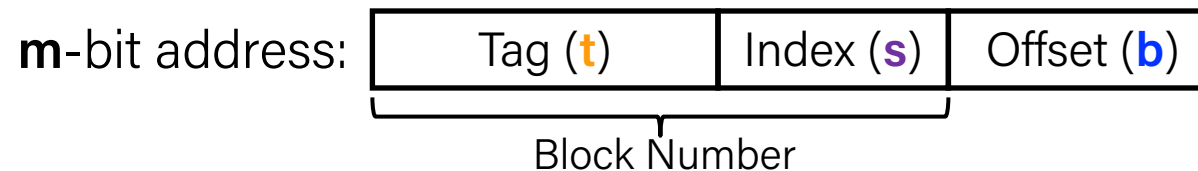
- TIO address breakdown:



- **Index** field tells you where to look in cache
 - **Tag** field lets you check that data is the block you want
 - **Offset** field selects specified start byte within block
- **Note:** **t** and **s** sizes will change based on hash function

Checking for a Requested Address Example

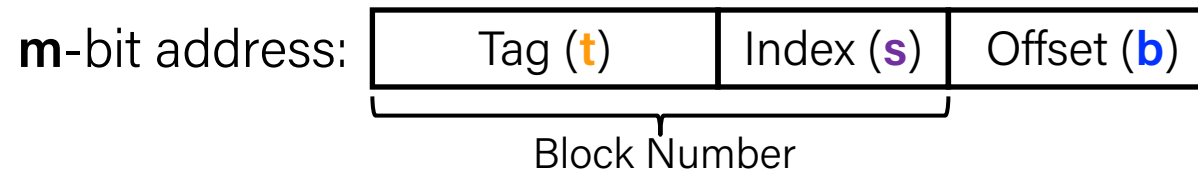
- Using 8-bit addresses.
- Cache Params: block size (B) = 4 bytes, cache size (C) = 32 bytes (which means number of sets is $C/B = 8$ sets).
 - Offset bits (b) = $\log_2(B) = 2$ bits
 - Index bits (s) = $\log_2(\text{number of sets}) = 3$ bits
 - Tag bits (t) = Rest of the bits in the address = $8 - 2 - 3 = 3$ bits



- What are the fields for address 0xBA?
 - Tag bits (unique id for block):
 - Index bits (cache set block maps to):
 - Offset bits (byte offset within block):

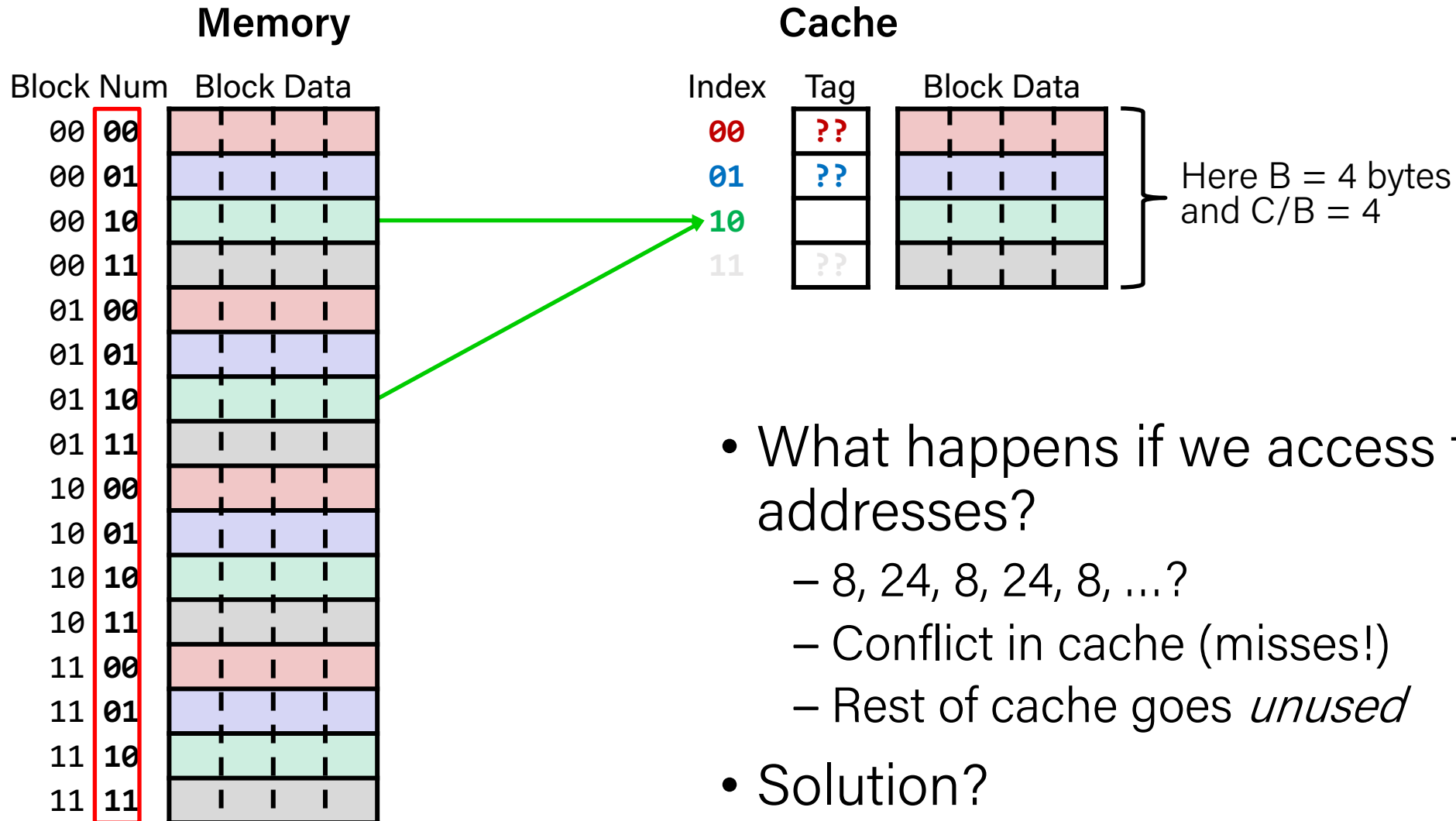
Checking for a Requested Address Example

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 - Index bits (s) = $\log_2(\text{number of sets}) = 3$ bits
 - Tag bits (t) = Rest of the bits in the address = $8 - 2 - 3 = 3$ bits



- What are the fields for address 0xBA?
 - Tag bits (unique id for block): 0x5 101 110 10
 - Index bits (cache set block maps to): 0x6 5 6 2
 - Offset bits (byte offset within block): 0x2

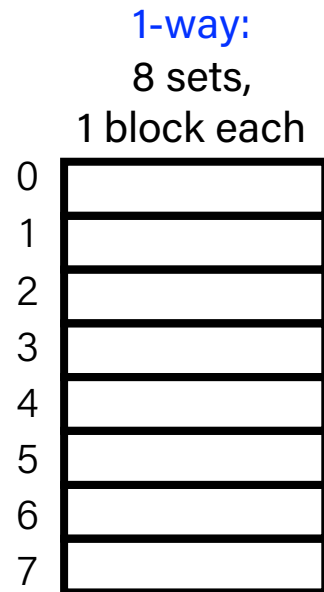
Direct-Mapped Cache Problem



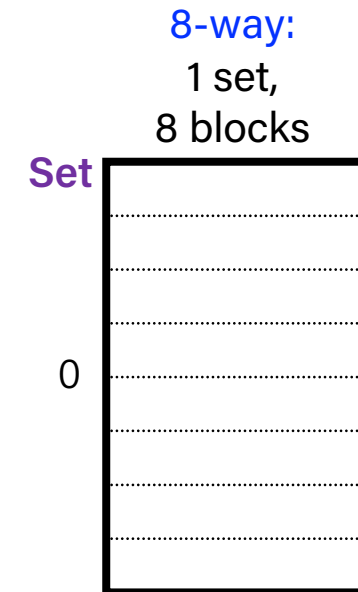
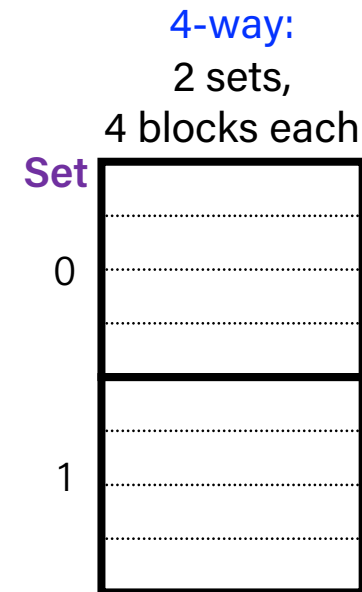
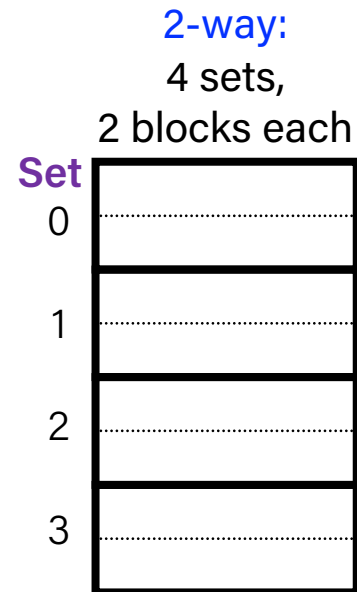
- What happens if we access the following addresses?
 - 8, 24, 8, 24, 8, ...?
 - Conflict in cache (misses!)
 - Rest of cache goes *unused*
- Solution?

Associativity

- What if we could store data in any place in the cache?
 - More complicated hardware = more power consumed, slower
- So we *combine* the two ideas:
 - Each address maps to exactly one **set**
 - Each set can store block in more than one **way**



direct-mapped



fully associative

Cache Puzzle

- Based on the following behavior, which of the following block sizes is NOT possible for our cache?
 - Cache starts *empty*, also known as a **cold cache**
 - Access (addr: hit/miss) stream:
 - (14: miss), (15: hit), (16: miss)
- A. 4 bytes
- B. 8 bytes
- C. 16 bytes
- D. 32 bytes
- E. We're lost...



Based on the following behavior, which of the following block size is NOT possible for our cache?

① Start presenting to display the poll results on this slide.

Cache Puzzle

- Based on the following behavior, which of the following block size is NOT possible for our cache?

- Cache starts *empty*, also known as a **cold cache**
- Access (addr: hit/miss) stream:
 - (14: miss), (15: hit), (16: miss)

hit: block is already in cache!
miss: block is not in cache,
 pulls block from memory
 and puts it in cache

A. 4 bytes

B. 8 bytes

C. 16 bytes

D. 32 bytes

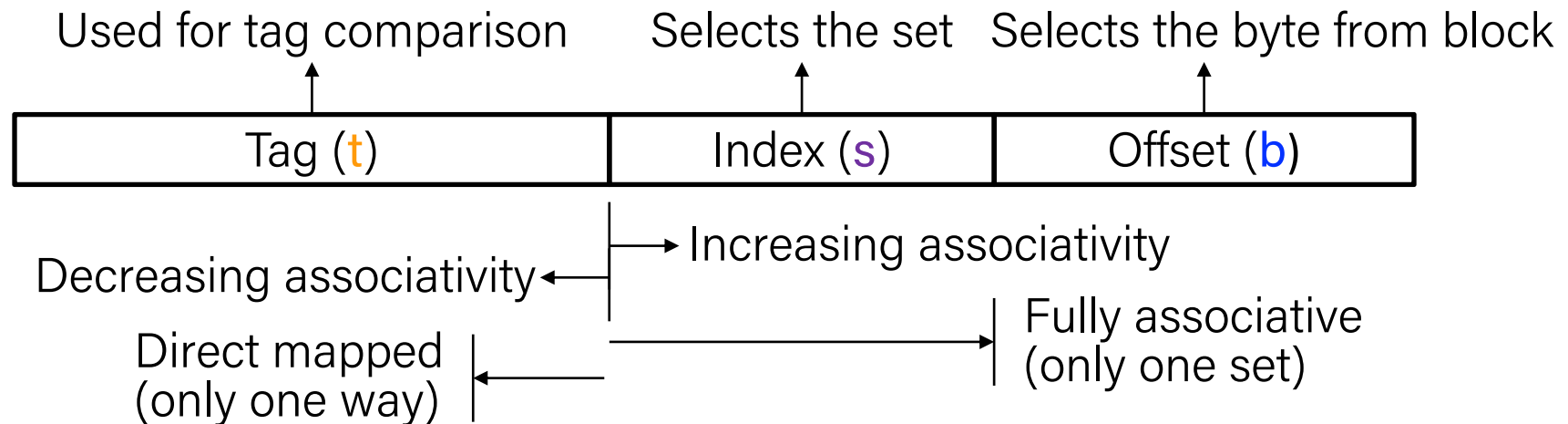
E. We're lost...

- ① Pulls block /w 14 into cache ② 15 is in the same block at 14 ③ 16 is not in block w/ 14 and 15

	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16		
Mem																			
K = 4													X ✓				X		
K = 8																	X ✓		X
K = 16															X ✓		X		
K = 32															X ✓ ✓				

Cache Organization

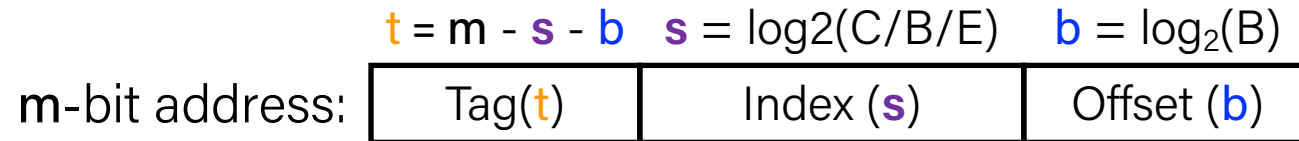
- **Associativity (E):** # of ways for each set
 - Such a cache is called an "*E-way set associative cache*"
 - We now index into cache *sets*, of which there are $S = C/B/E$
 - Use lowest $\log_2(C/B/E) = \mathbf{s}$ bits of block address
 - Direct-mapped: $E = 1$, so $\mathbf{s} = \log_2(C/B)$ as we saw previously
 - Fully associative: $E = C/B$, so $\mathbf{s} = 0$ bits



Example Placement

block size:	16 bytes
capacity:	8 blocks
address:	16 bits

- Where would data from address `0x1833` be placed?
 - Binary: `0b 0001 1000 0011 0011`



$s = ?$
Direct-mapped

Set	Tag	Data
0		
1		
2		
3		
4		
5		
6		
7		

$s = ?$
2-way set associative

Set	Tag	Data
0		
1		
2		
3		

$s = ?$
4-way set associative

Set	Tag	Data
0		
1		

Example Placement

block size:	16 bytes
capacity:	8 blocks
address:	16 bits

- Where would data from address 0x1833 be placed?

– Binary: 0b 0001 1000 0011 0011

m = 4
m = 2
m = 1

$$t = m - s - b \quad s = \log_2(C/B/E) \quad b = \log_2(B)$$

m-bit address:

Tag(<i>t</i>)	Index (<i>s</i>)	Offset (<i>b</i>)
-----------------	--------------------	---------------------

s = $\log_2(8)=3$ bits
Direct-mapped

Set	Tag	Data
0		
1		
2		
3		✓
4		
5		
6		
7		

s = $\log_2(8/2)=2$ bits
2-way set associative

Set	Tag	Data
0		
1		
2		
3		✓
		✓

s = $\log_2(8/4)=1$ bit
4-way set associative

Set	Tag	Data
0		
		✓
1		✓
		✓
		✓

Block Placement

- *Any* empty block in the correct set may be used to store block
- If there are no empty blocks, which one should we replace?
 - No choice for direct-mapped caches
 - Caches typically use something close to **least recently used (LRU)** (hardware usually implements "*not most recently used*")

Direct-mapped

Set	Tag	Data
0		
1		
2		
3		✓
4		
5		
6		
7		

2-way set associative

Set	Tag	Data
0		
1		
2		
3		✓
		✓

4-way set associative

Set	Tag	Data
0		
1		✓
		✓
		✓
		✓

Question

- We have a cache of size 2 KB with block size of 128 bytes. If our cache has 2 sets, what is its associativity?
 - A. 2
 - B. 4
 - C. 8
 - D. 16
- If addresses are 16 bits wide, how wide is the Tag field?

Question

$$(C = 2 \times 2^{10} \text{ bytes})$$

$$(B = 2^7 \text{ bytes})$$

- We have a cache of size 2 KB with block size of 128 bytes. If our cache has 2 sets, what is its associativity?

$$(S = 2)$$

A. 2

$$\text{num blocks} = C / K = 2^{11} / 2^7 = 2^4 = 16 \text{ blocks}$$

B. 4

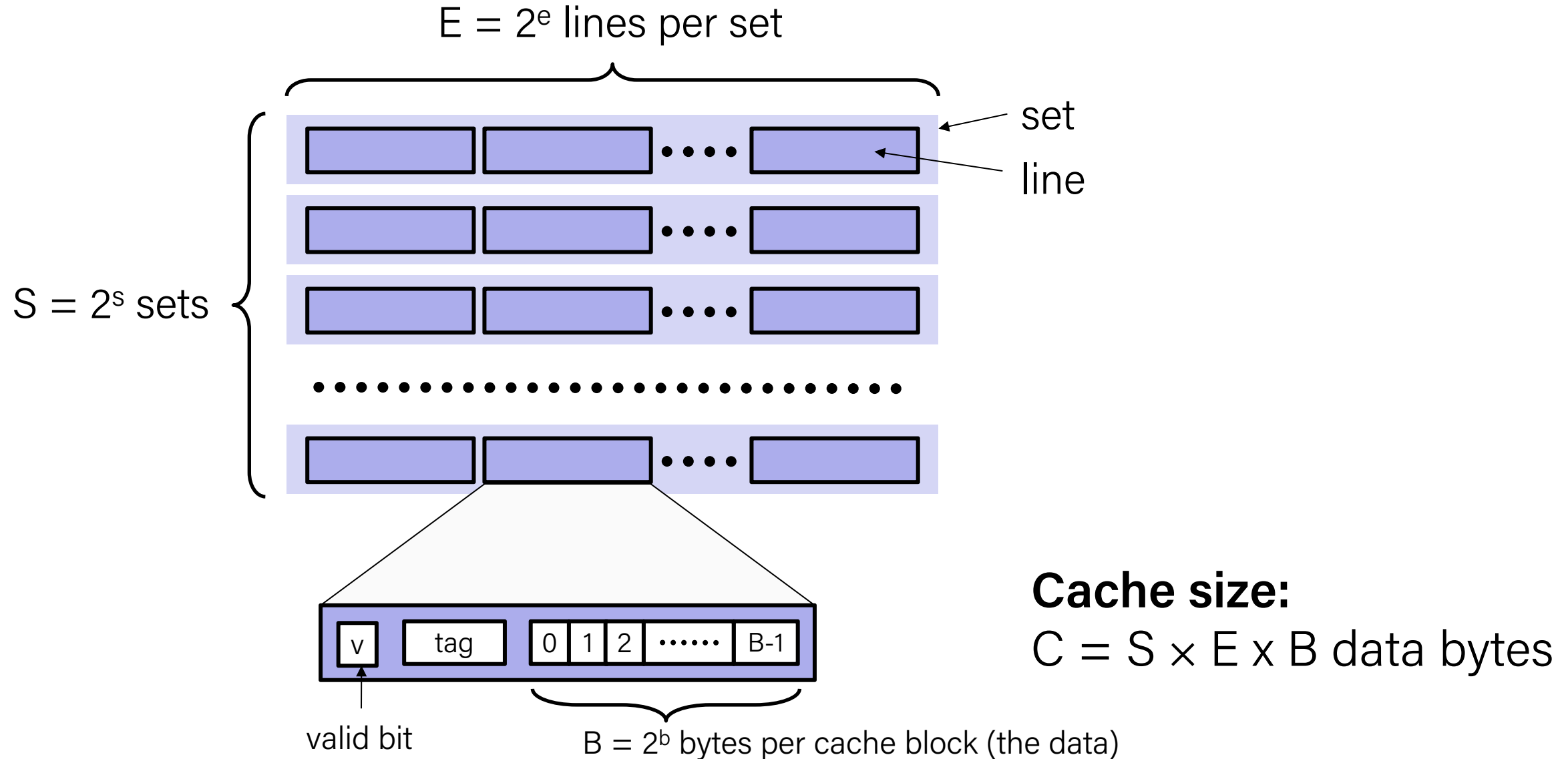
C. 8

$$\text{blocks per set} = E = 16 / 2 = 8$$

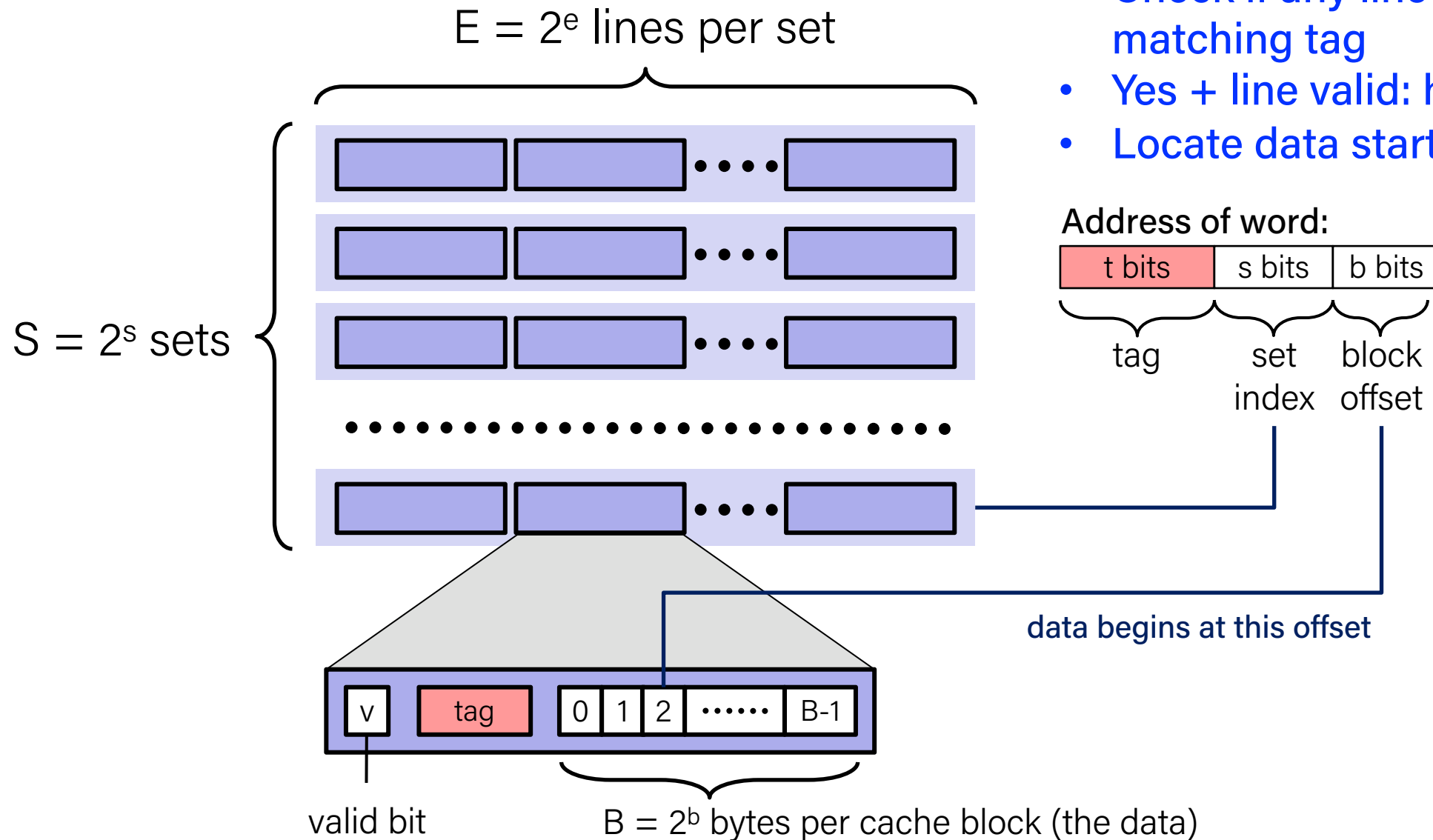
D. 16

- If addresses are 16 bits wide, how wide is the Tag field? $= 16 - 7 - 1 = 8$

General Cache Organization (S, E, B)



Cache Read

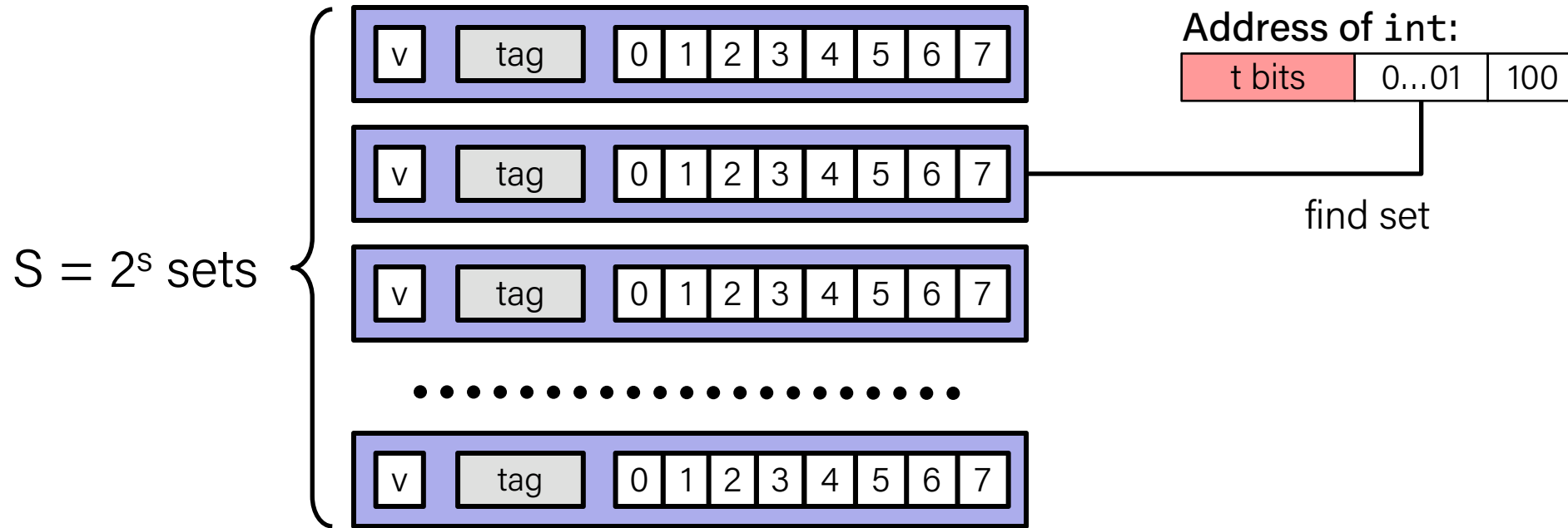


- Locate set
- Check if any line in set has matching tag
- Yes + line valid: hit
- Locate data starting at offset

Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

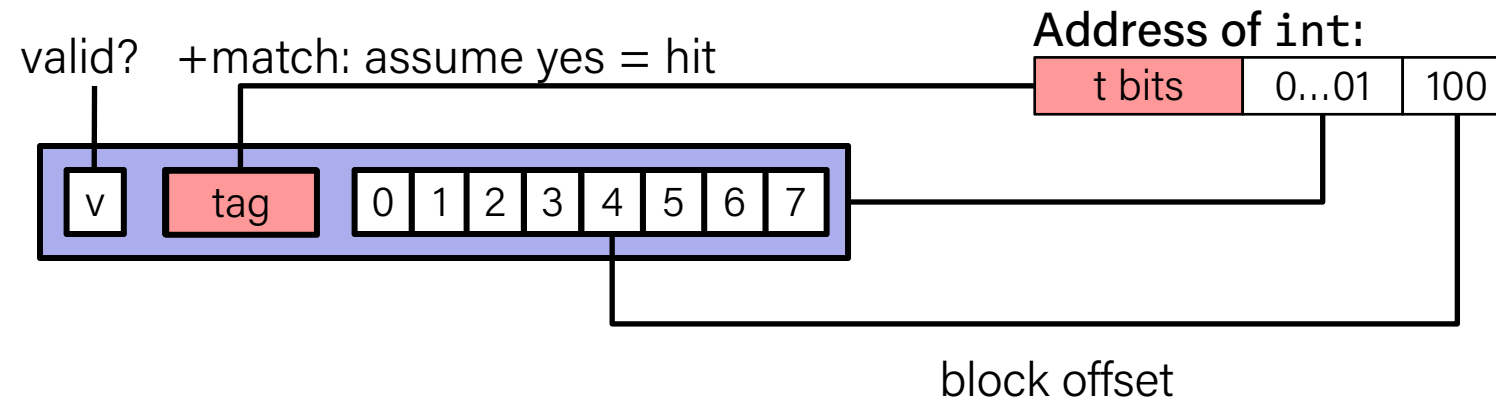
Assume: cache block size 8 bytes



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

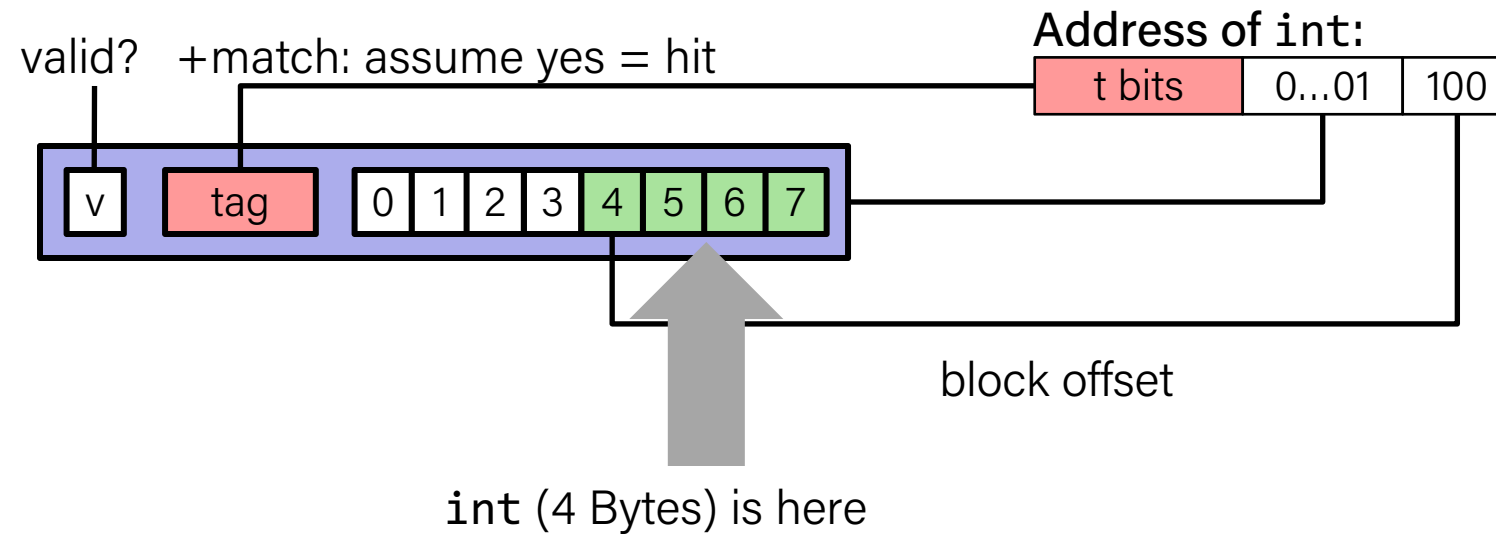
Assume: cache block size 8 bytes



Example: Direct Mapped Cache (E = 1)

Direct mapped: One line per set

Assume: cache block size 8 bytes



If tag doesn't match: old line is evicted and replaced

Direct-Mapped Cache Simulation

t=1	s=2	b=1
x	xx	x

M=16 bytes (4-bit addresses), B=2 bytes/block,
S=4 sets, E=1 Blocks/set

	v	Tag	Block
Set 0	1	0	M[0-1]
Set 1			
Set 2			
Set 3	1	0	M[6-7]

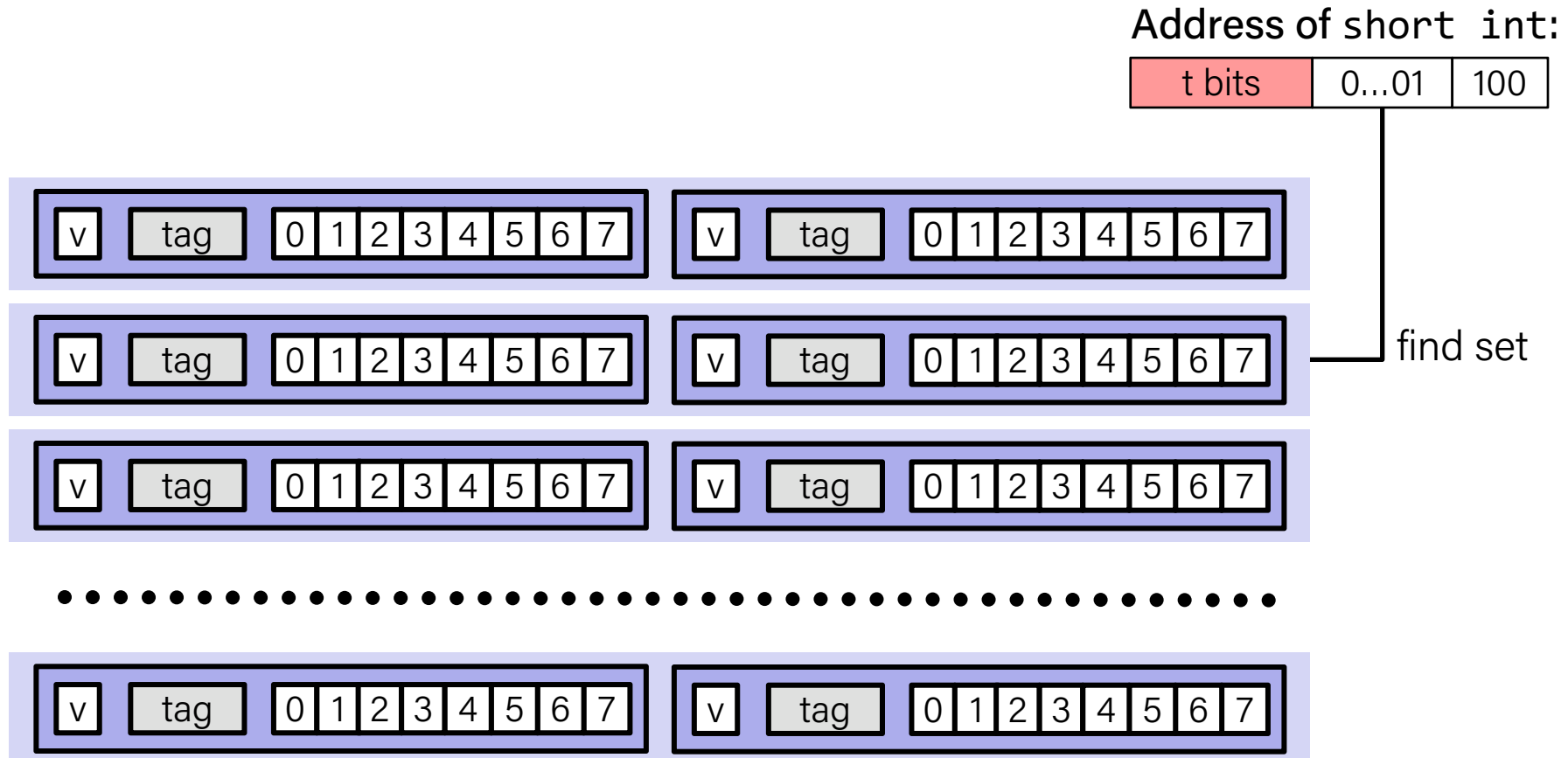
Address trace (reads, one byte per read):

0	[<u>0000</u> ₂],	miss
1	[<u>0001</u> ₂],	hit
7	[<u>0111</u> ₂],	miss
8	[<u>1000</u> ₂],	miss
0	[<u>0000</u> ₂]	miss

E-way Set Associative Cache (Here: $E = 2$)

$E = 2$: Two lines per set

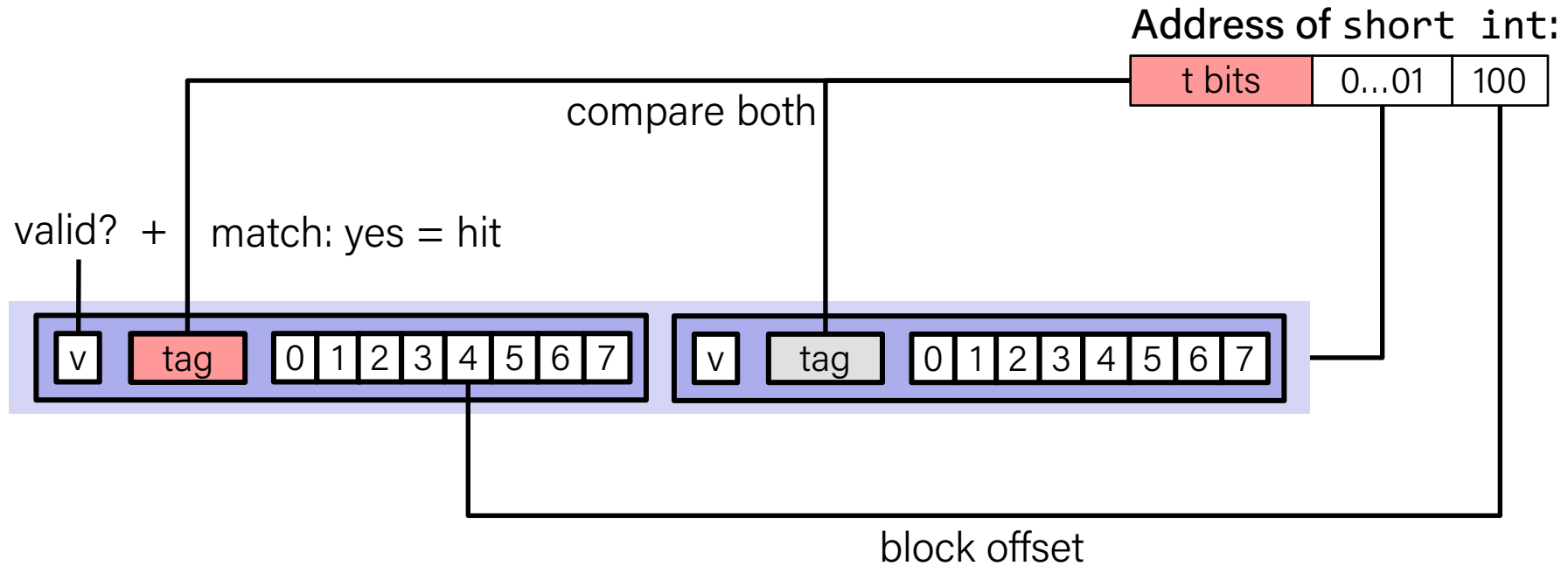
Assume: cache block size 8 bytes



E-way Set Associative Cache (Here: E = 2)

E = 2: Two lines per set

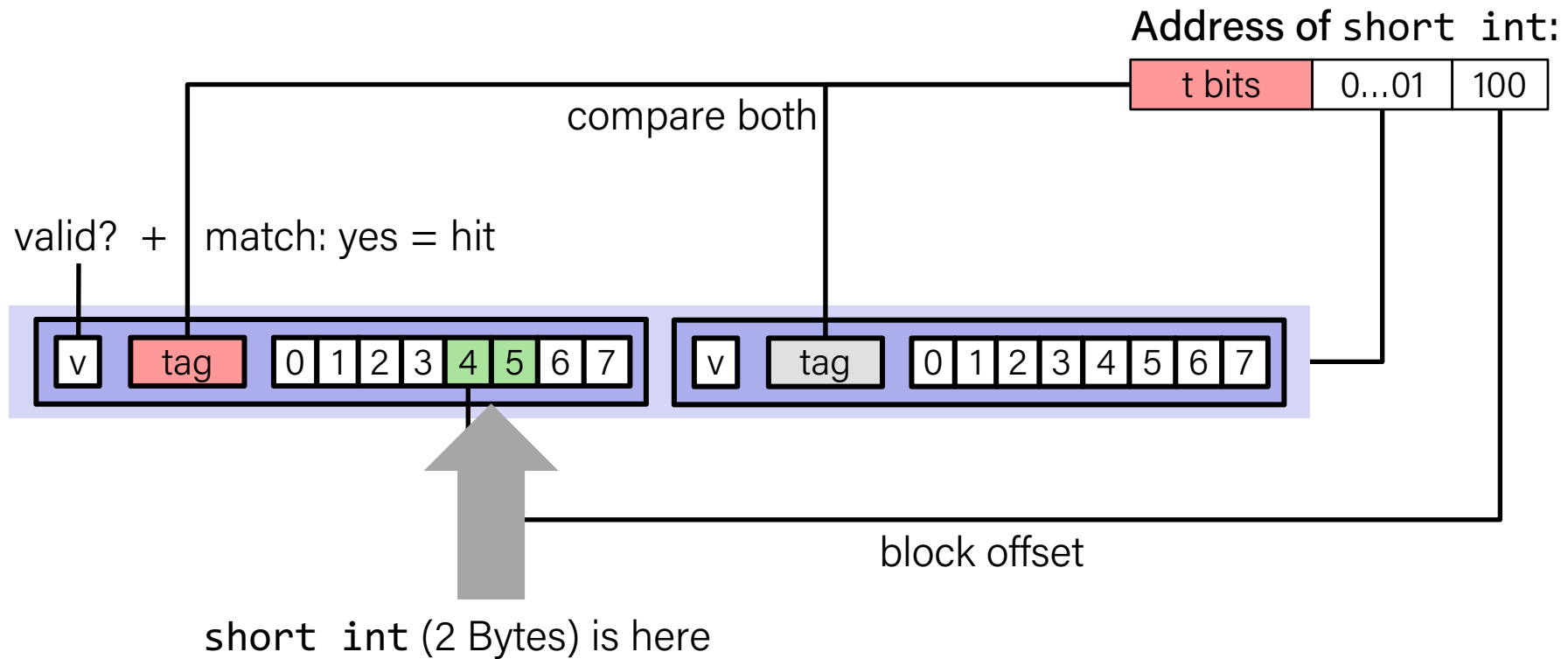
Assume: cache block size 8 bytes



E-way Set Associative Cache (Here: $E = 2$)

$E = 2$: Two lines per set

Assume: cache block size 8 bytes



No match:

- One line in set is selected for eviction and replacement
- Replacement policies: random, least recently used (LRU), ...

2-Way Set Associative Cache Simulation

t=2 s=1 b=1

XX	X	X
----	---	---

M=16 byte addresses, B=2 bytes/block,
S=2 sets, E=2 blocks/set

	v	Tag	Block
Set 0	1	00	M[0-1]
	1	10	M[8-9]
Set 1	1	01	M[6-7]
	0		

Address trace (reads, one byte per read):

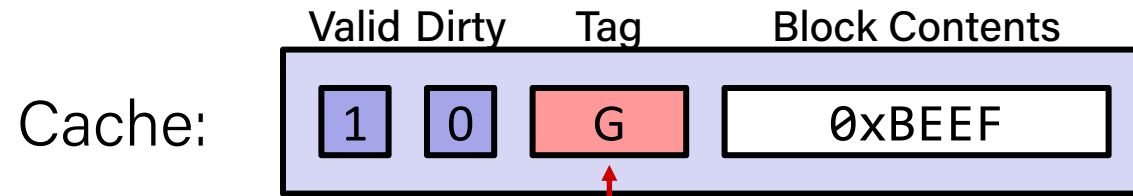
0	[00 <u>0</u> 0 ₂],	miss
1	[00 <u>0</u> 1 ₂],	hit
7	[01 <u>1</u> 1 ₂],	miss
8	[10 <u>0</u> 0 ₂],	miss
0	[00 <u>0</u> 0 ₂]	hit

What about writes?

- Multiple copies of data exist:
 - L1, L2, L3, Main Memory, Disk
- What to do on a write-hit?
 - Write-through (write immediately to memory)
 - Write-back (defer write to memory until replacement of line)
 - Need a dirty bit (line different from memory or not)
- What to do on a write-miss?
 - Write-allocate (load into cache, update line in cache)
 - Good if more writes to the location follow
 - No-write-allocate (writes straight to memory, does not load into cache)
- Typical
 - Write-through + No-write-allocate
 - Write-back + Write-allocate

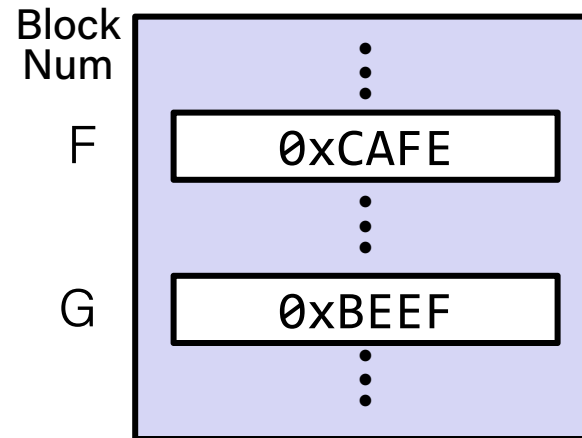
Write-back, Write Allocate Example

Note: While unrealistic, this example assumes that all requests have offset 0 and are for a block's worth of data.



There is only one set in this tiny cache,
so the tag is the entire block number!

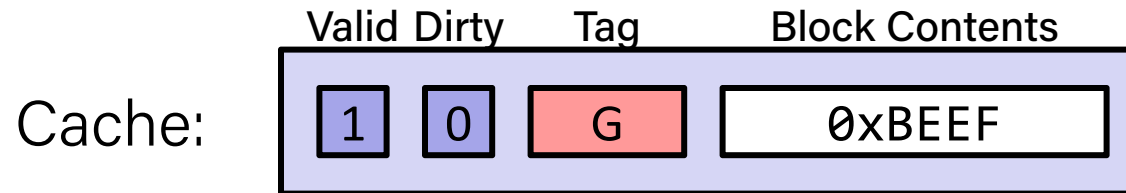
Memory:



Write-back, Write Allocate Example

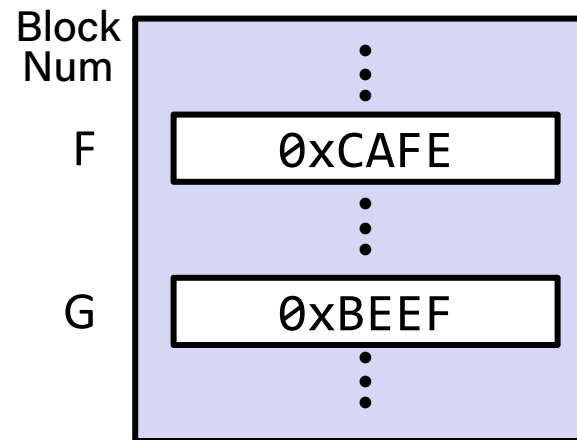
1) `mov $0xFACE, (F)` Not valid x86, just using block num instead of full byte address to keep the example simple

Write Miss!



Step 1: Bring F into cache

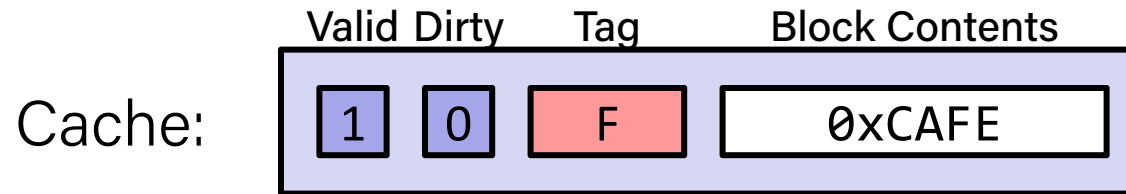
Memory:



Write-back, Write Allocate Example

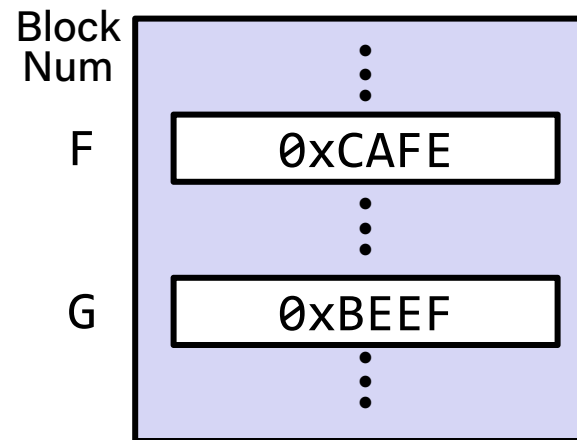
(1) `mov $0xFACE, (F)`

Write Miss



Step 1: Bring F into cache

Memory:

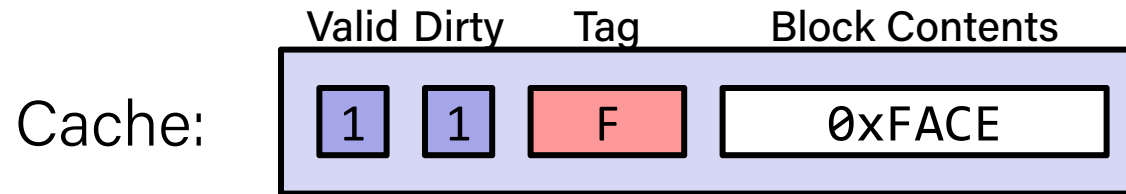


Step 2: Write 0xFACE to cache only and set the dirty bit

Write-back, Write Allocate Example

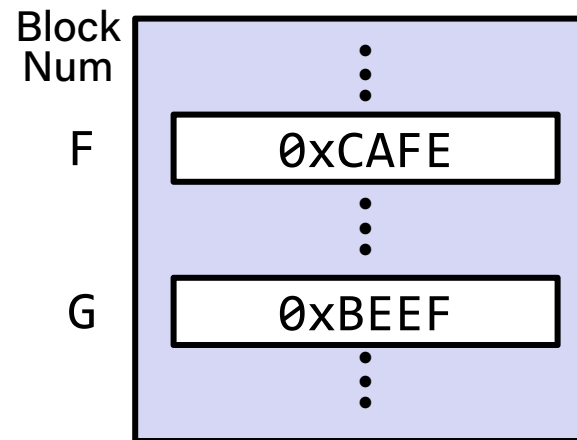
(1) `mov $0xFACE, (F)`

Write Miss



Step 1: Bring F into cache

Memory:



Step 2: Write 0xFACE to cache only and set the dirty bit

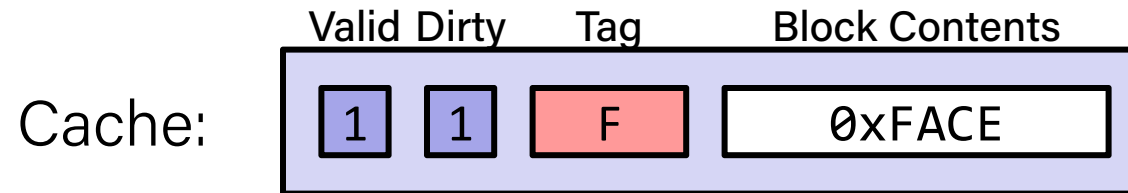
Write-back, Write Allocate Example

(1) `mov $0xFACE, (F)`

Write Miss

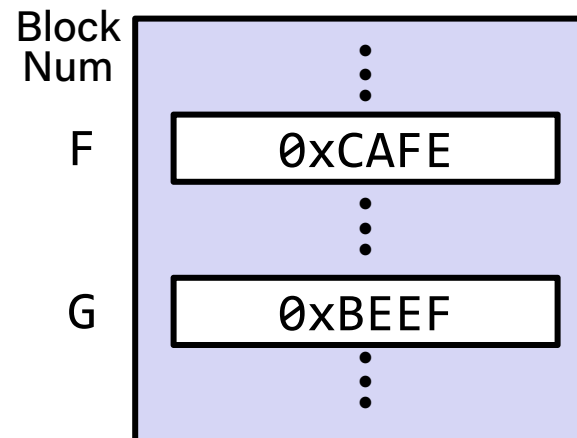
(2) `mov $0xFEED, (F)`

Write Hit!



Step: Write 0xFEED to cache only (and set the dirty bit)

Memory:



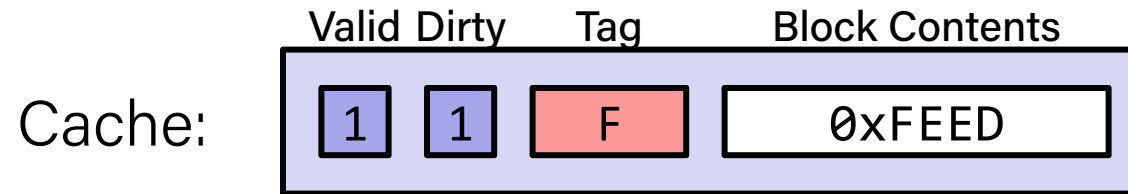
Write-back, Write Allocate Example

(1) `mov $0xFACE, (F)`

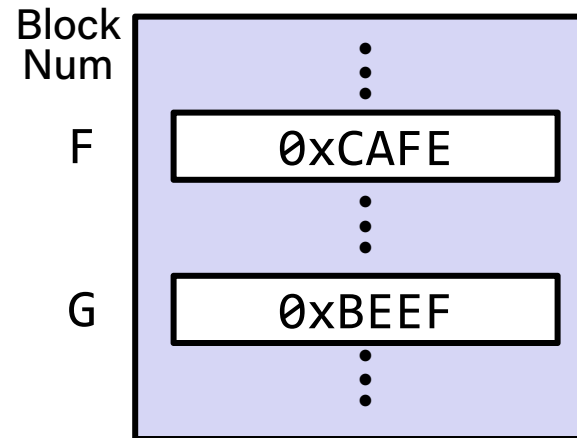
Write Miss

(2) `mov $0xFEED, (F)`

Write Hit!



Memory:

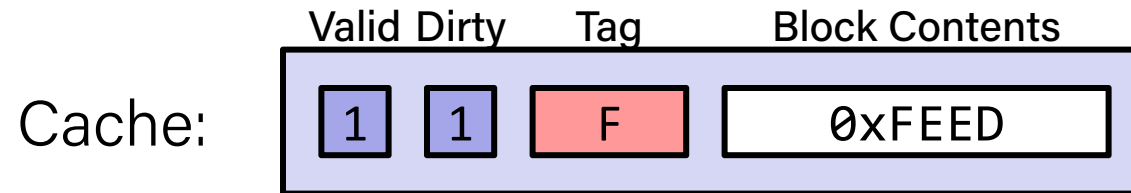


Write-back, Write Allocate Example

(1) `mov $0xFACE, (F)`
Write Miss

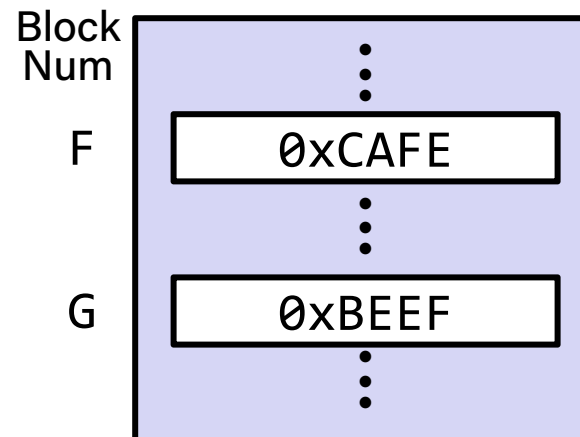
(2) `mov $0xFEED, (F)`
Write Hit!

(3) `mov (G), %ax`
Read Miss!



Step 1: Write F back to memory
since it is dirty

Memory:

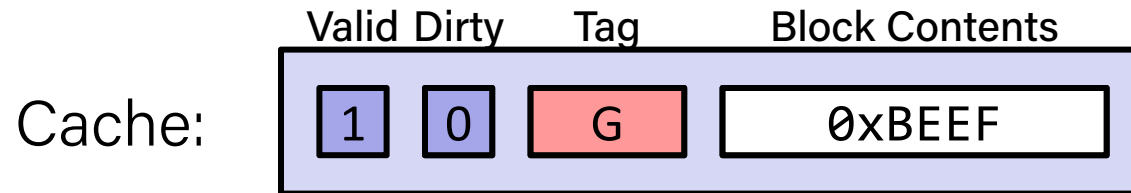


Write-back, Write Allocate Example

(1) `mov $0xFACE, (F)`
Write Miss

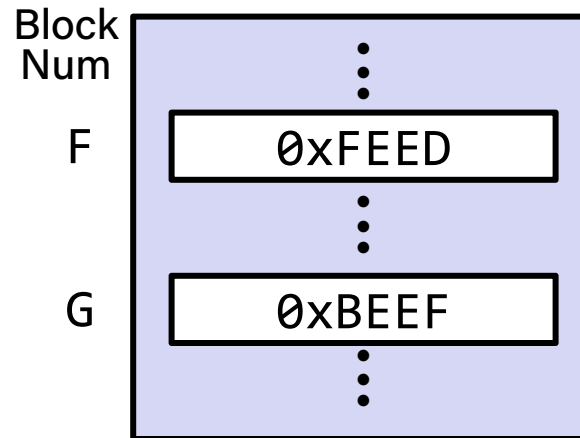
(2) `mov $0xFEED, (F)`
Write Hit!

(3) `mov (G), %ax`
Read Miss!



Step 1: Write F back to memory
since it is dirty

Memory:



Step 2: Bring G into the cache
so that we can copy it into %ax

Cache Simulator

<https://courses.cs.washington.edu/courses/cse351/cachesim>



Polling Question

- Which of the following cache statements is FALSE?
 - A. We can reduce compulsory misses by decreasing our block size
 - B. We can reduce conflict misses by increasing associativity
 - C. A write-back cache will save time for code with good temporal locality on writes
 - D. A write-through cache will always match data with the memory hierarchy level below it
 - E. We're lost...

Polling Question

- Which of the following cache statements is FALSE?

A. We can reduce compulsory misses by decreasing our block size

smaller block size pulls fewer bytes into cache on a miss

B. We can reduce conflict misses by increasing associativity

more options to place blocks before evictions occur

C. A write-back cache will save time for code with good temporal locality on writes

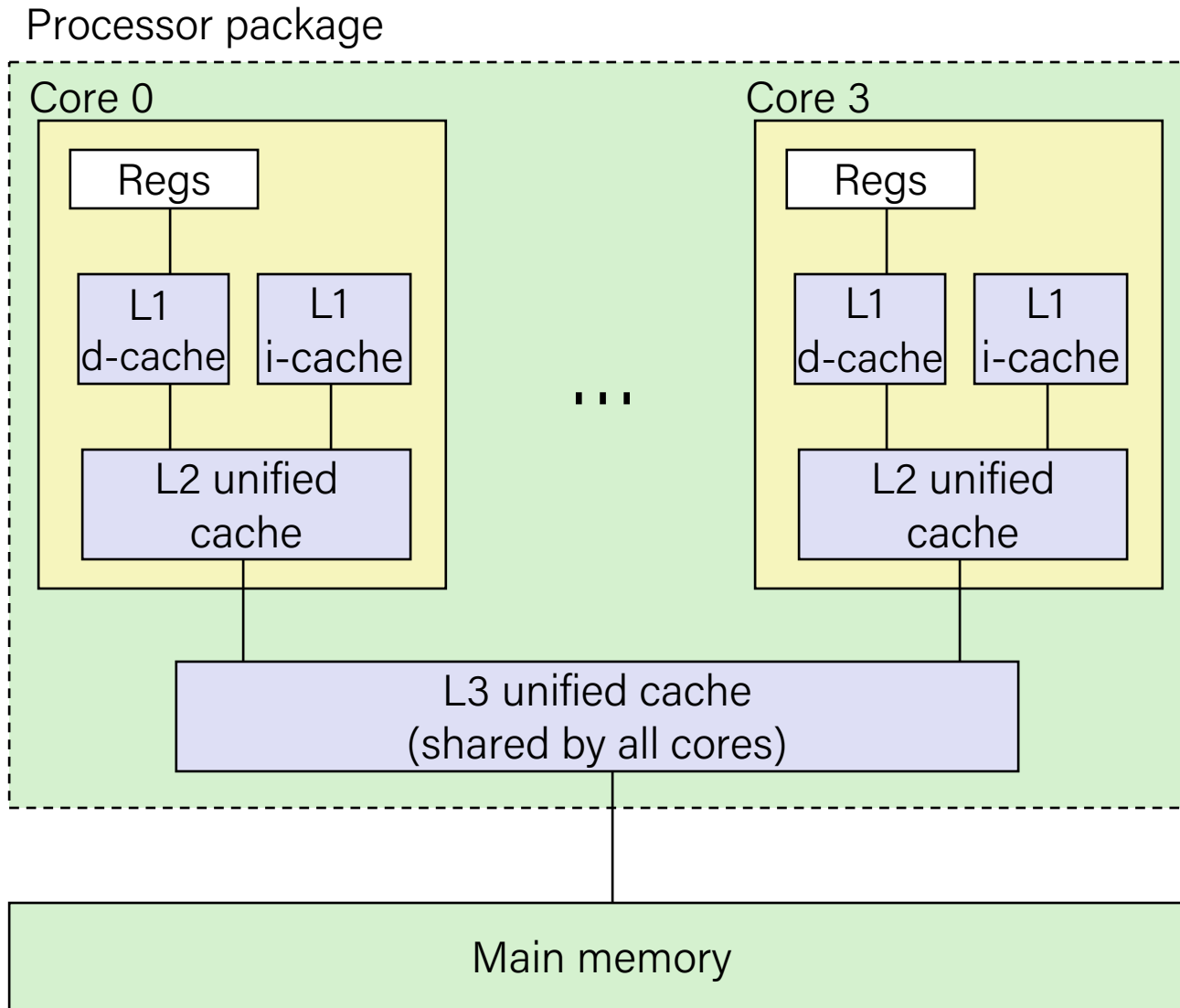
yes, its main goal is data consistency

D. A write-through cache will always match data with the memory hierarchy level below it

frequently-used blocks rarely get evicted, so fewer write-backs

E. We're lost...

Intel Core i7 Cache Hierarchy



L1 i-cache and d-cache:
32 KB, 8-way,
Access: 4 cycles

L2 unified cache:
256 KB, 8-way,
Access: 10 cycles

L3 unified cache:
8 MB, 16-way,
Access: 40-75 cycles

Block size: 64 bytes for all caches.

Lecture Plan

- Cache memory organization and operation
- The memory mountain

Writing Cache Friendly Code

- Make the common case go fast
 - Focus on the inner loops of the core functions
- Minimize the misses in the inner loops
 - Repeated references to variables are good ([temporal locality](#))
 - Stride-1 reference patterns are good ([spatial locality](#))

Key idea: Our qualitative notion of locality is quantified through our understanding of cache memories

The Memory Mountain

- **Read throughput** (read bandwidth)
 - Number of bytes read from memory per second (MB/s)
- **Memory mountain:** Measured read throughput as a function of spatial and temporal locality.
 - Compact way to characterize memory system performance.

Memory Mountain Test Function

```
long data[MAXELEMS]; /* Global array to traverse */

/* test - Iterate over first "elems" elements of
 *      array "data" with stride of "stride", using
 *      using 4x4 loop unrolling.
 */
int test(int elems, int stride) {
    long i, sx2=stride*2, sx3=stride*3, sx4=stride*4;
    long acc0 = 0, acc1 = 0, acc2 = 0, acc3 = 0;
    long length = elems, limit = length - sx4;

    /* Combine 4 elements at a time */
    for (i = 0; i < limit; i += sx4) {
        acc0 = acc0 + data[i];
        acc1 = acc1 + data[i+stride];
        acc2 = acc2 + data[i+sx2];
        acc3 = acc3 + data[i+sx3];
    }

    /* Finish any remaining elements */
    for (; i < length; i++) {
        acc0 = acc0 + data[i];
    }

    return ((acc0 + acc1) + (acc2 + acc3));
}
```

Call test() with many combinations of elems and stride.

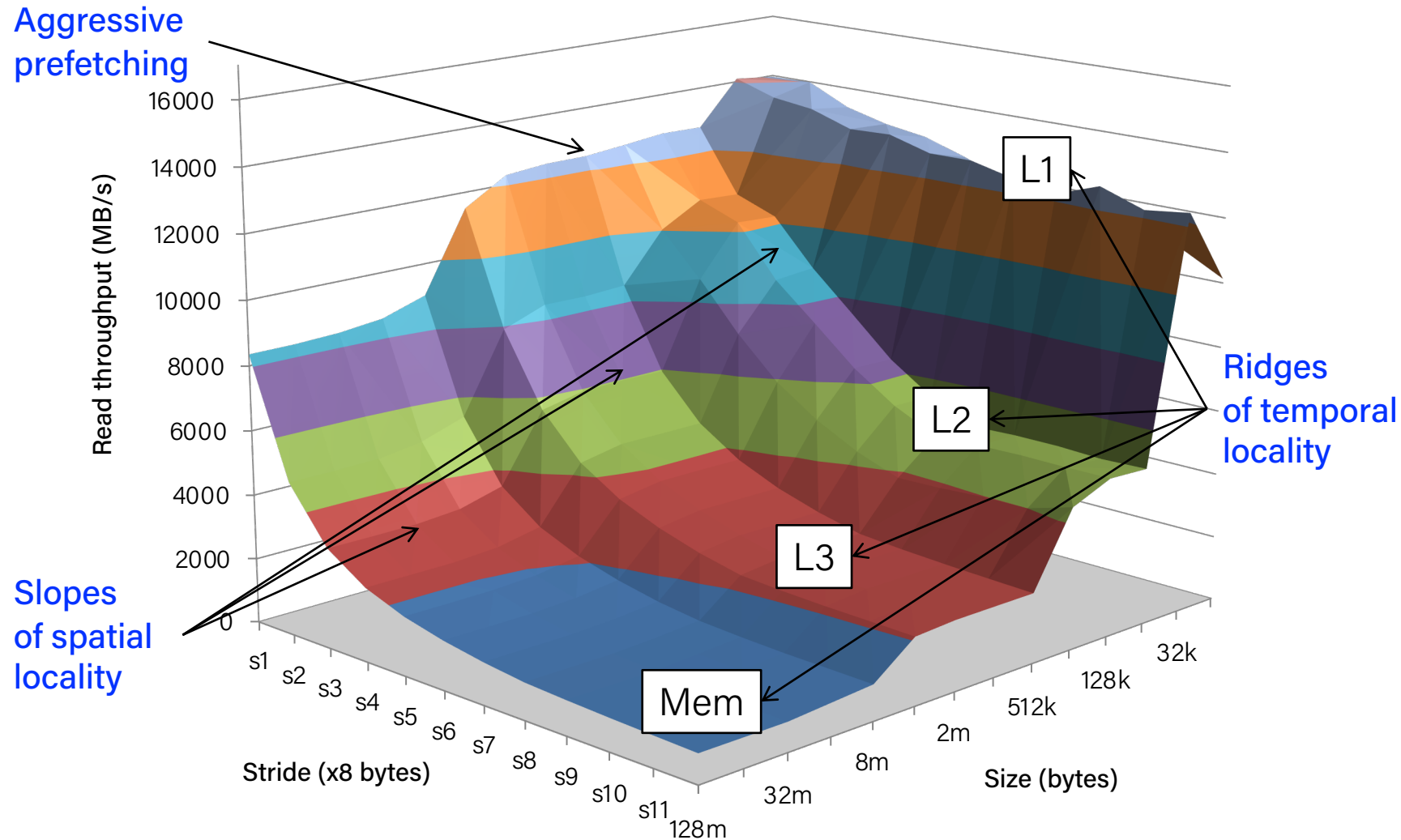
For each elems and stride:

1. Call test() once to warm up the caches.
2. Call test() again and measure the read throughput(MB/s)

mountain/mountain.c

The Memory Mountain

Aggressive
prefetching



Core i7 Haswell
2.1 GHz
32 KB L1 d-cache
256 KB L2 cache
8 MB L3 cache
64 B block size

Recap

- Cache memories can have significant performance impact
- You can write your programs to exploit this!
 - Focus on the inner loops, where bulk of computations and memory accesses occur.
 - Try to maximize spatial locality by reading data objects with sequentially with stride 1.
 - Try to maximize temporal locality by using a data object as often as possible once it's read from memory.

Next time: *Optimization*