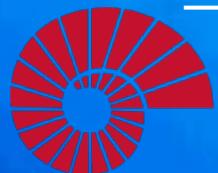


# COMP547

## DEEP UNSUPERVISED LEARNING

Lecture #3 – Neural Networks Basics II:  
Sequential Processing with NNs



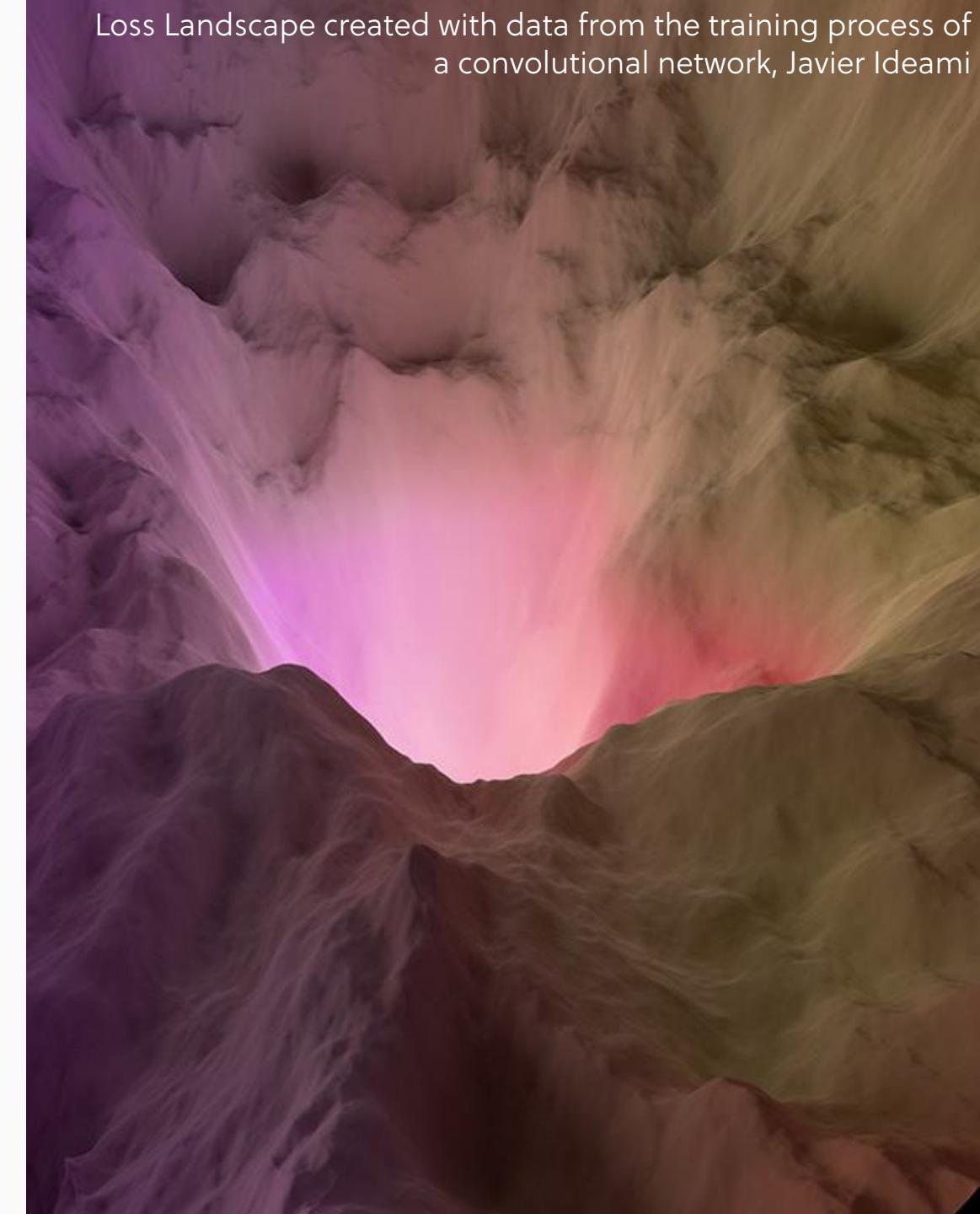
KOÇ  
UNIVERSITY

Aykut Erdem // Koç University // Spring 2026

# Previously on COMP547

- deep learning
- computation in a neural net
- optimization
- backpropagation
- training tricks
- convolutional neural networks

Loss Landscape created with data from the training process of  
a convolutional network, Javier Ideami



# Lecture overview

- sequence modeling
- recurrent neural networks (RNNs)
- how to train RNNs
- long short-term memory (LSTM)
- gated recurrent unit (GRU)
- sequence to sequence modeling

**Disclaimer:** Much of the material and slides for this lecture were borrowed from

- Bill Freeman, Antonio Torralba and Phillip Isola's MIT 6.869 class
- Fei-Fei Li, Andrej Karpathy and Justin Johnson's CS231n class
- Arun Mallya's tutorial on Recurrent Neural Networks

# Sequences



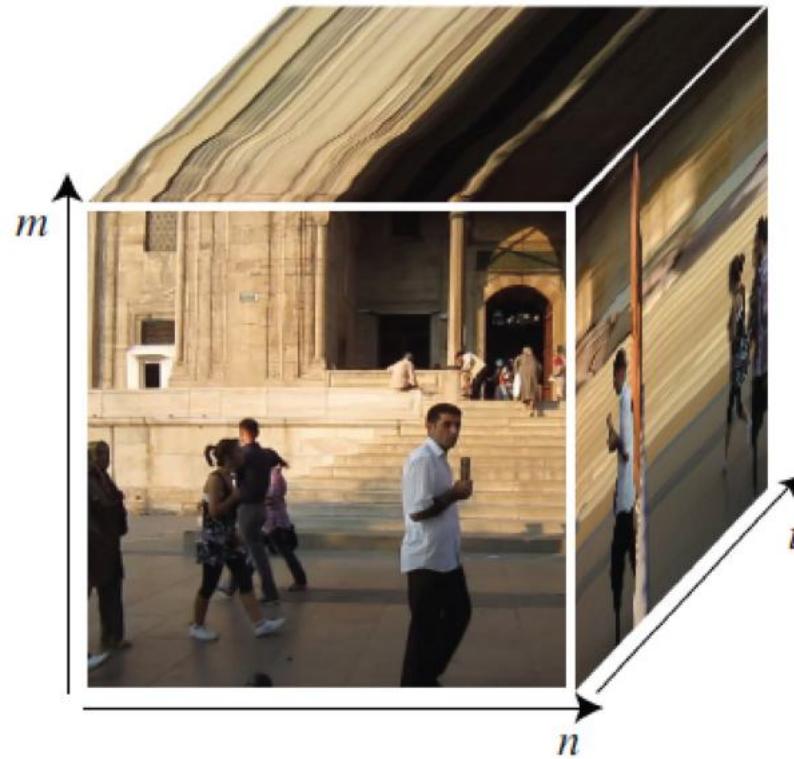
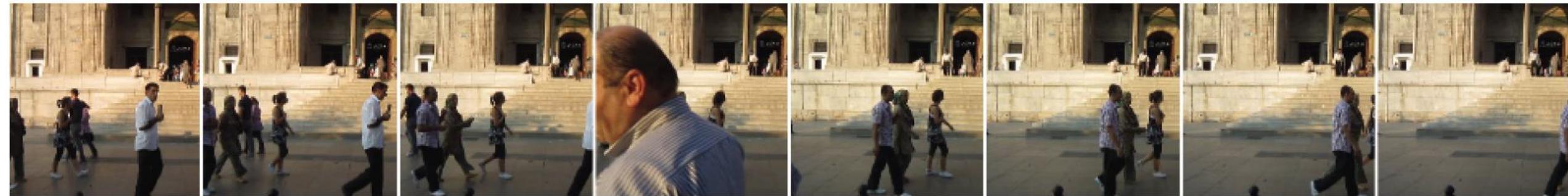
→  
time

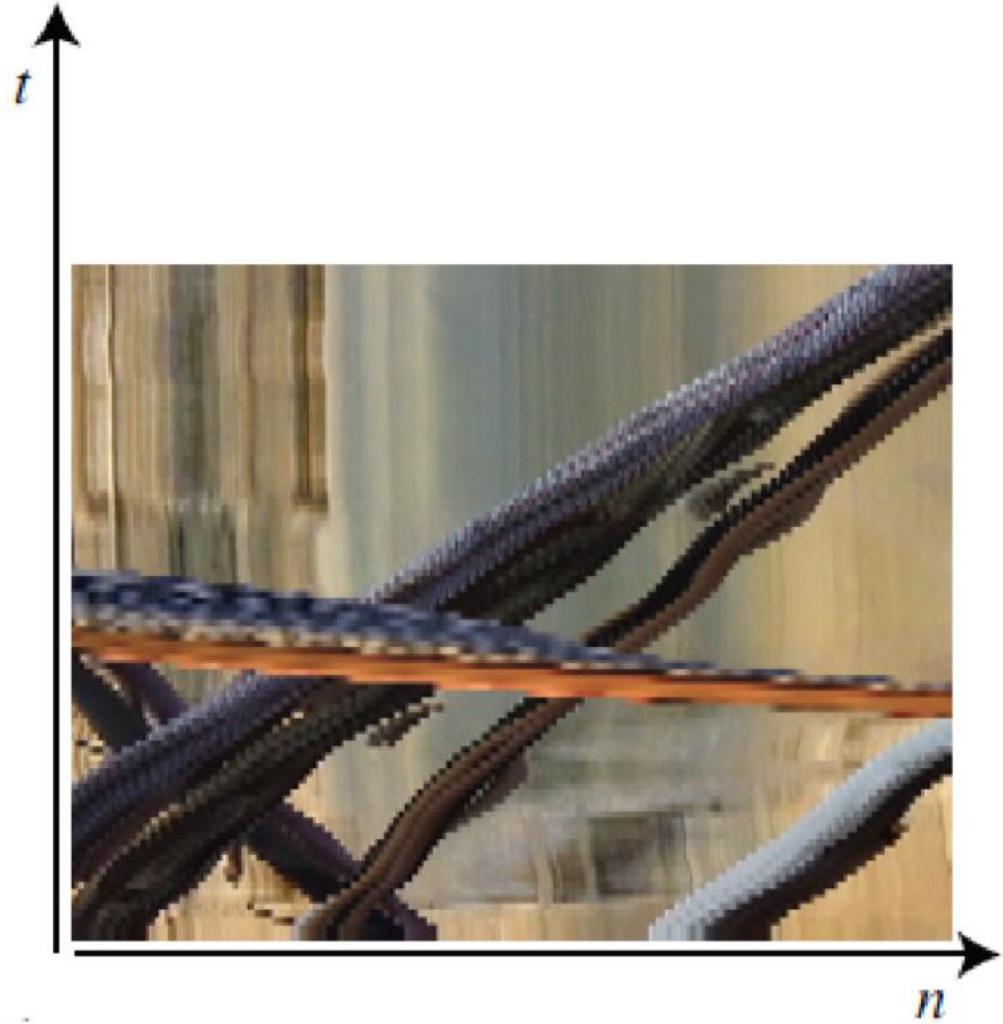
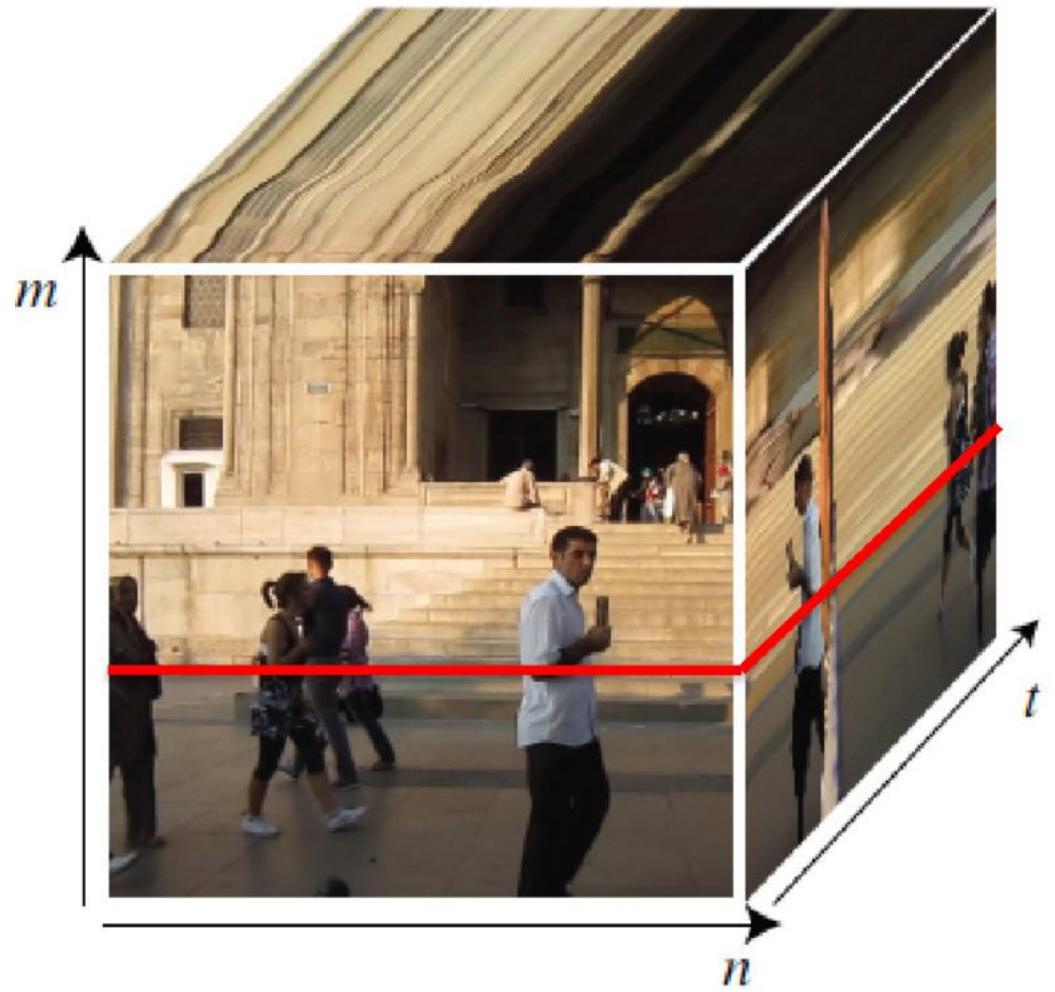
"An", "evening", "stroll", "through", "a", "city", "square"

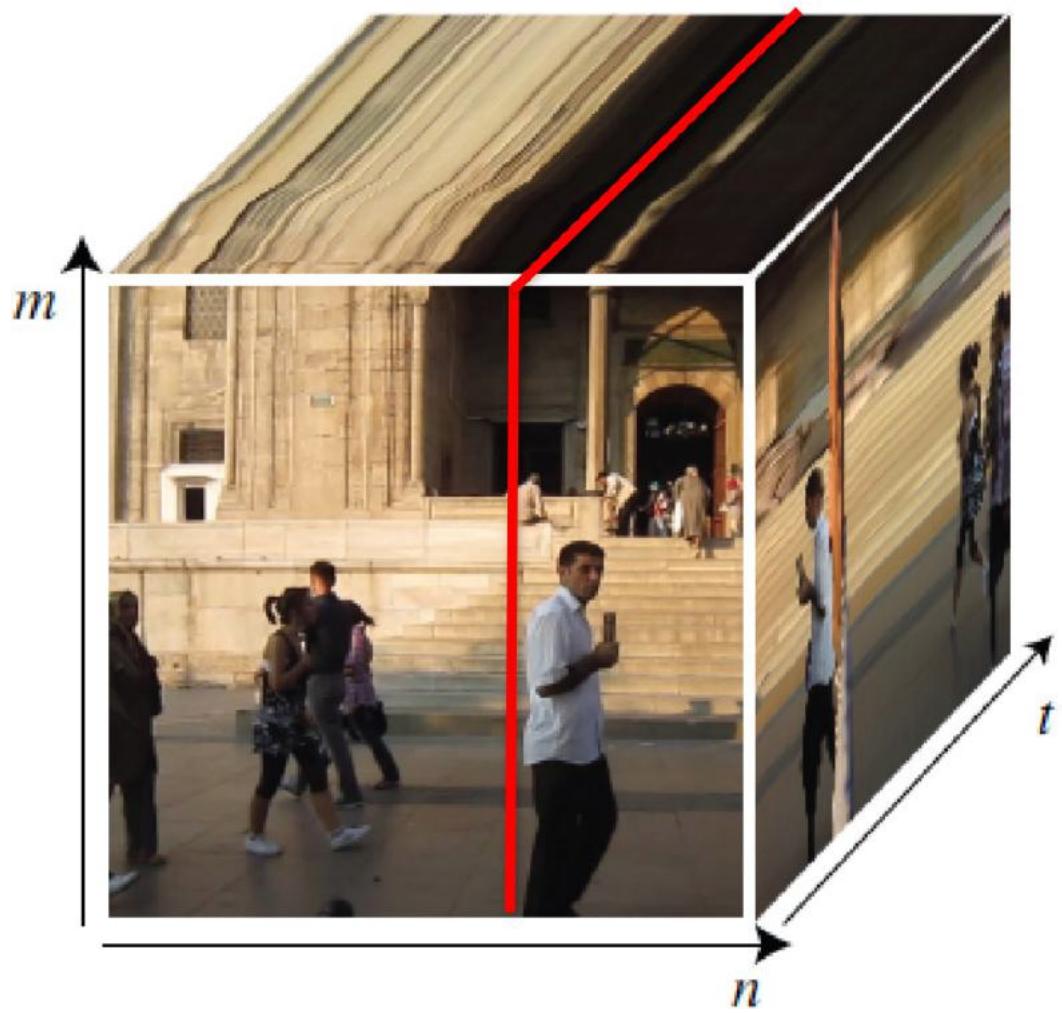
→  
time

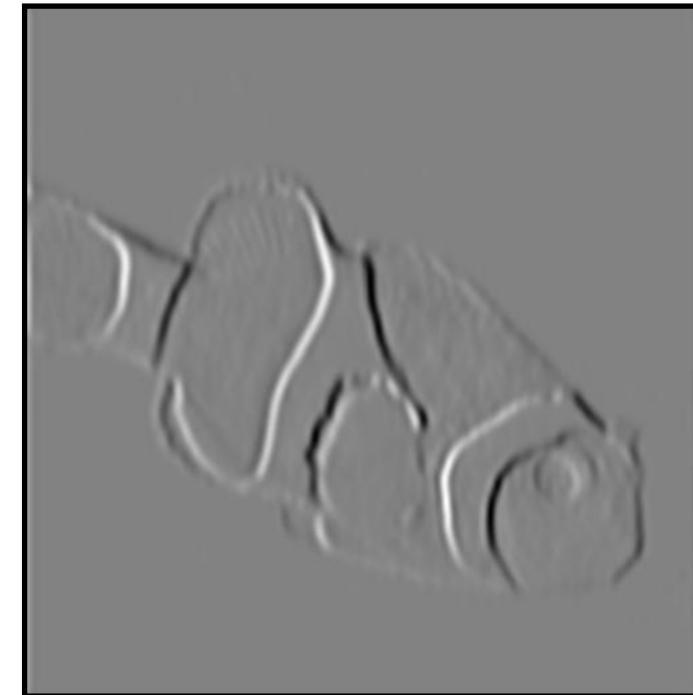
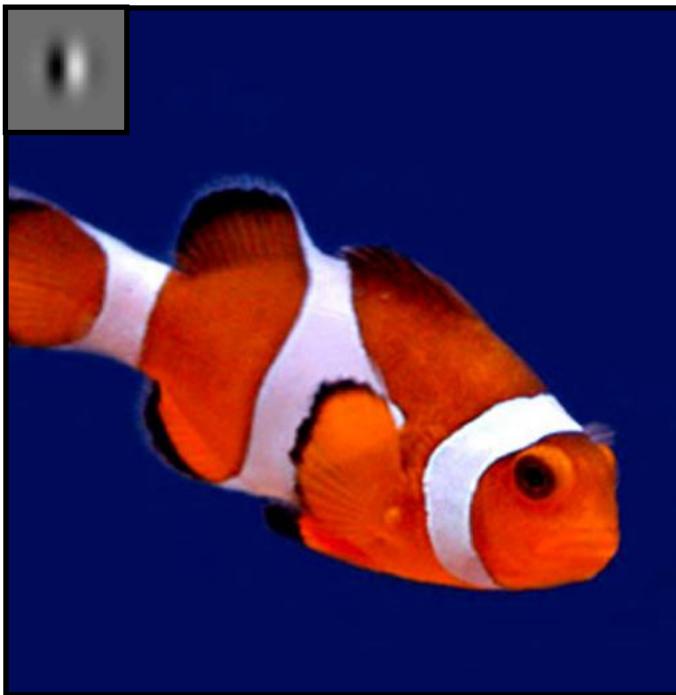


→  
time

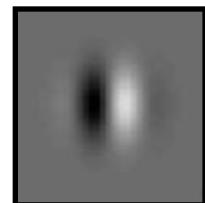




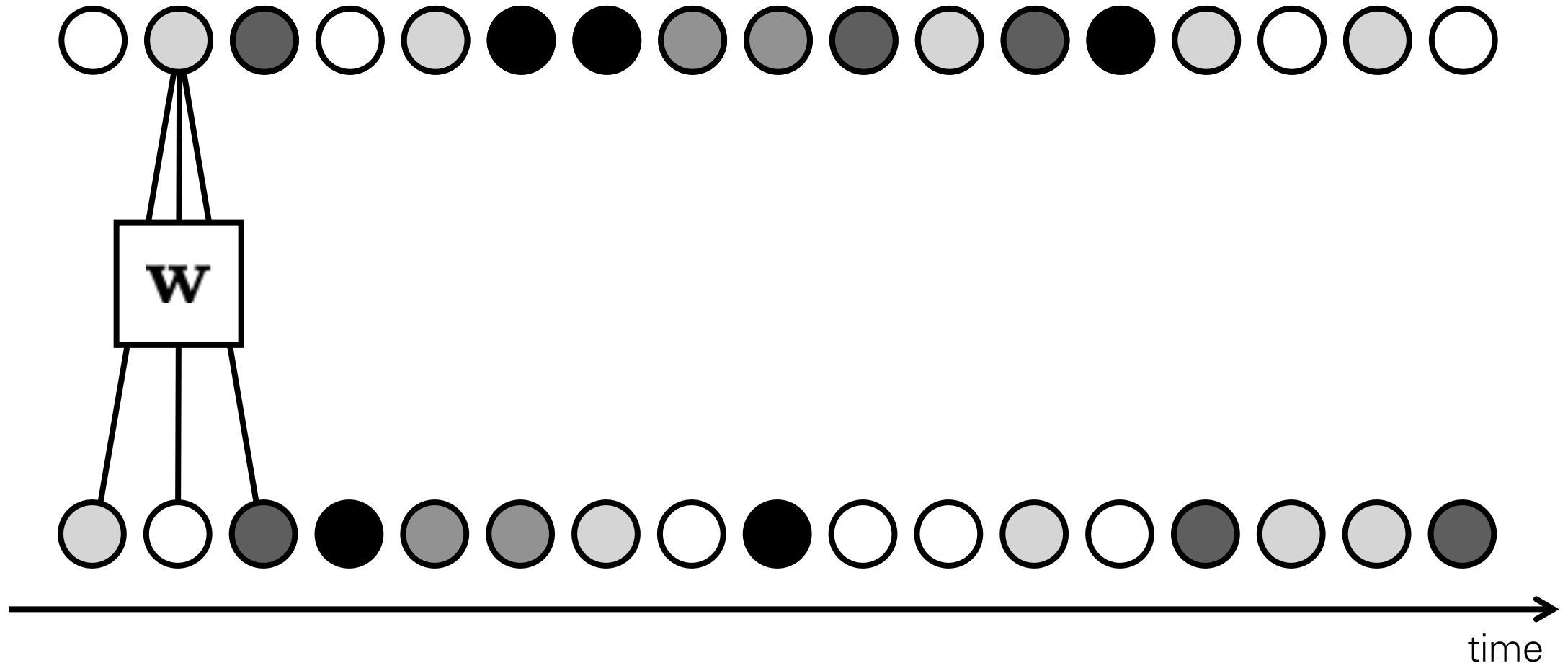


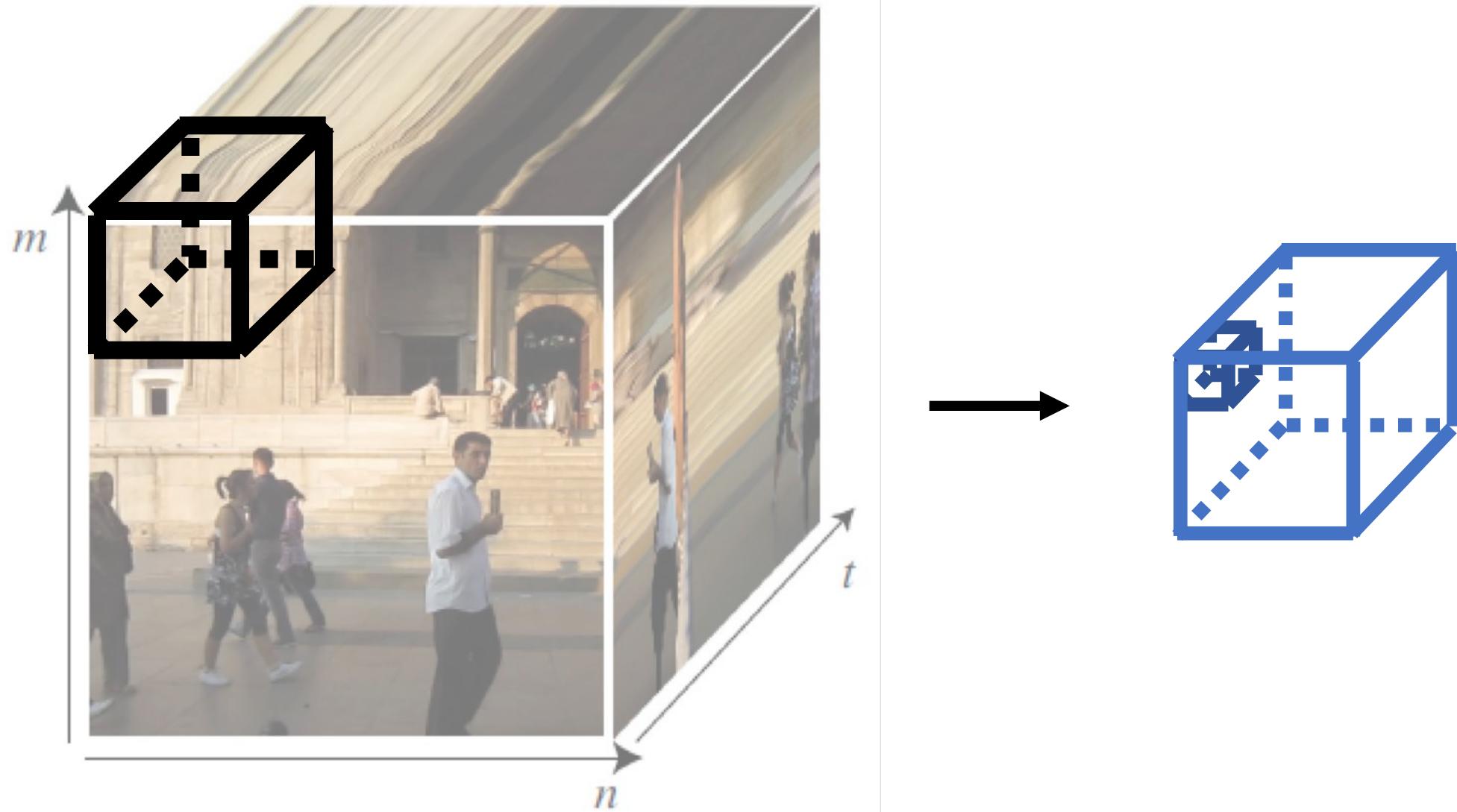


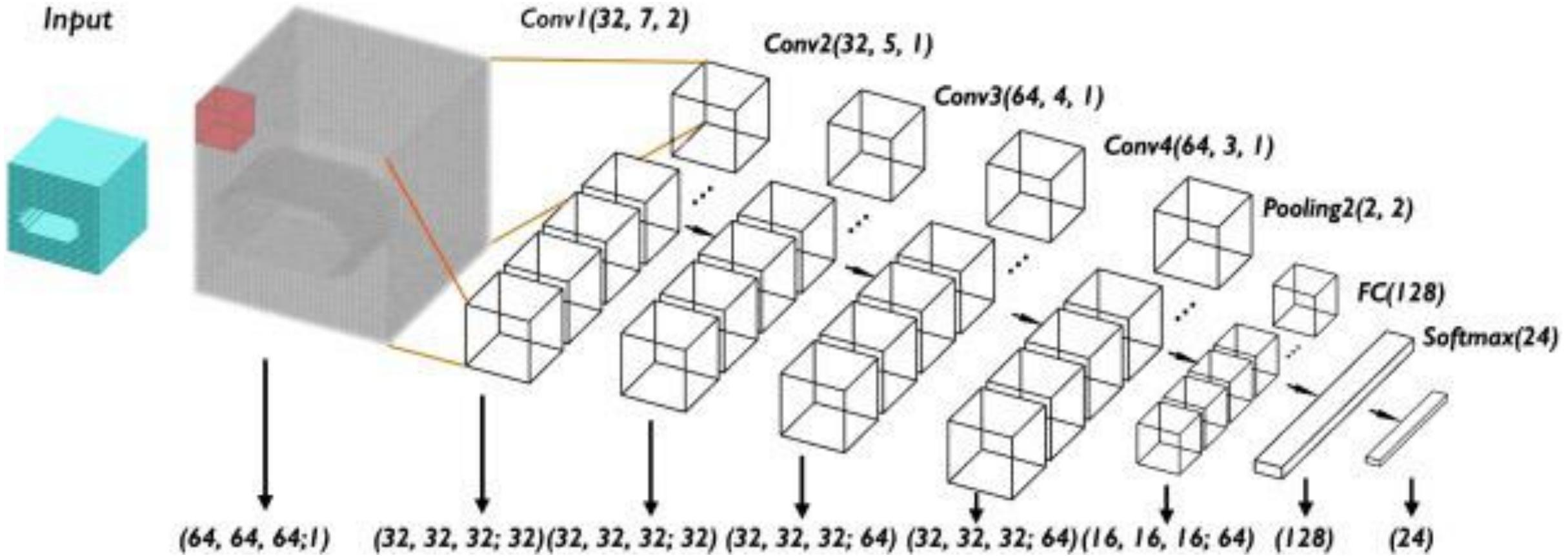
filter



# Convolutions in time

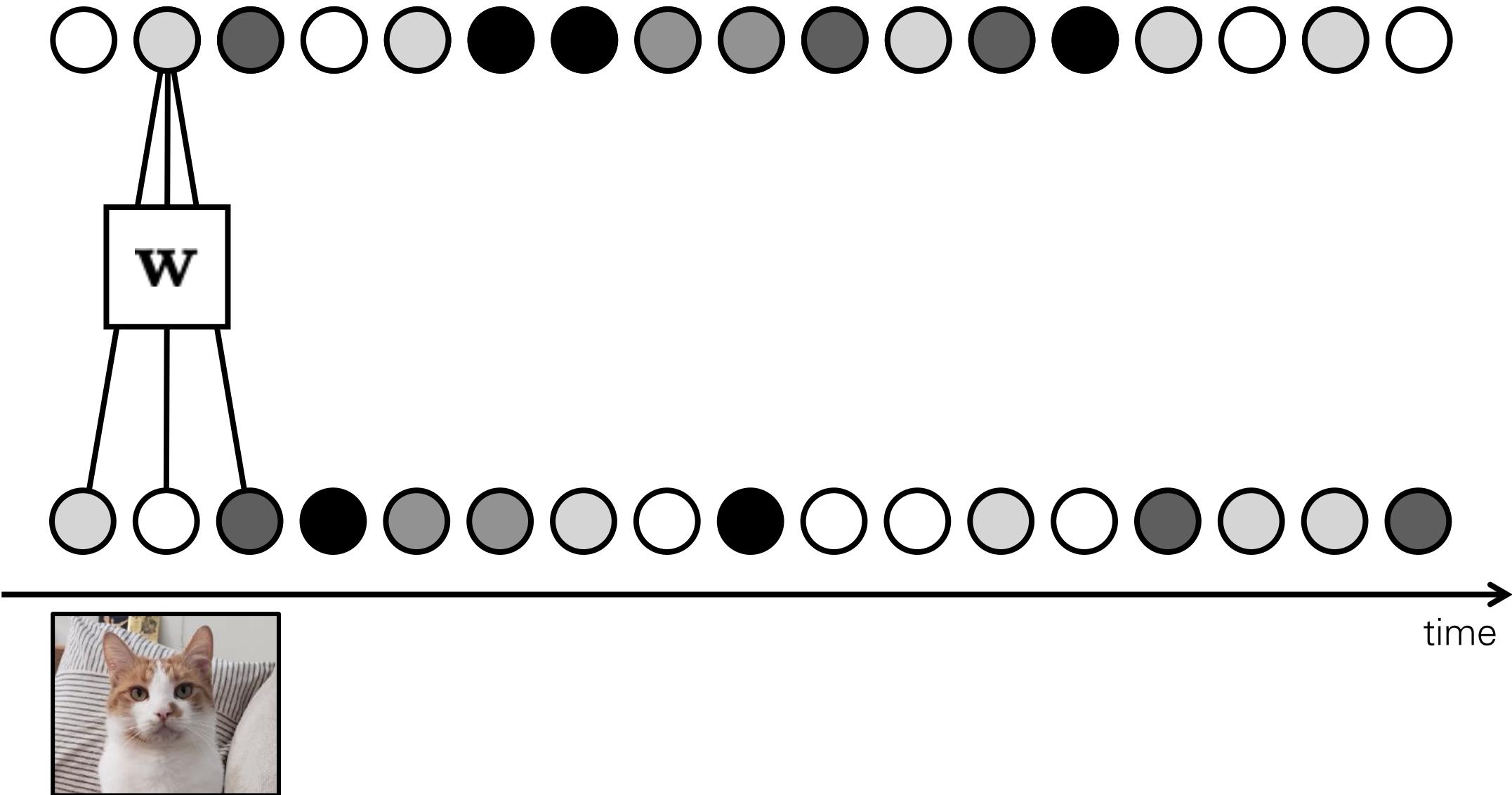




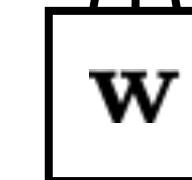
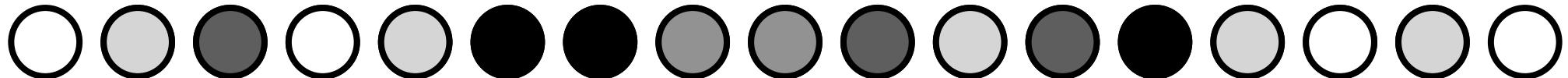


[fig from FeatureNet: Machining feature recognition based on 3D Convolution Neural Network]

Frank



Douglas



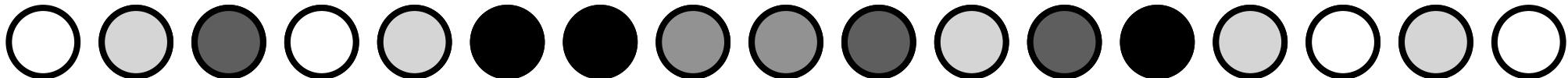
time →



Memory  
unit



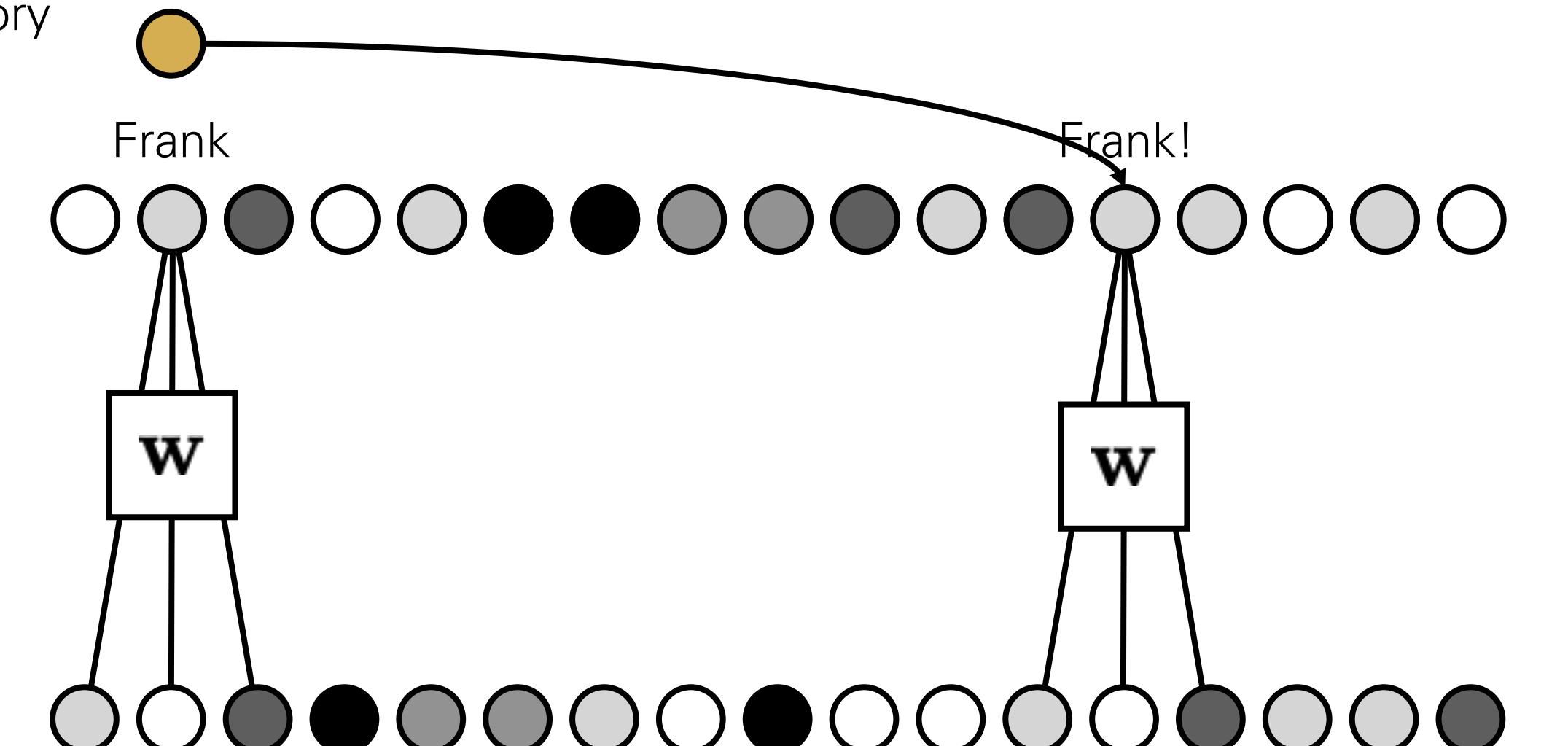
Frank



time →

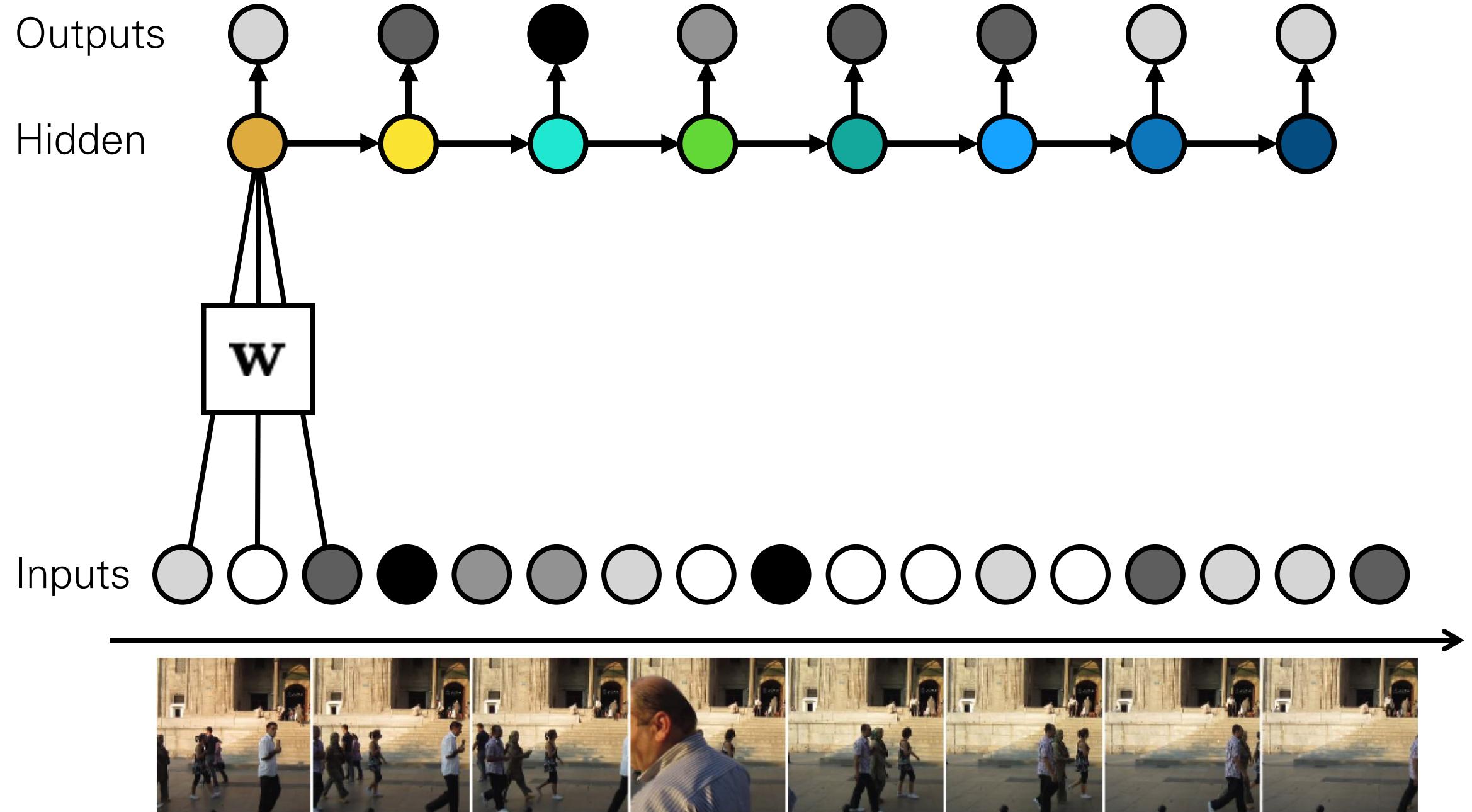


Memory  
unit



time →

# Recurrent Neural Networks (RNNs)

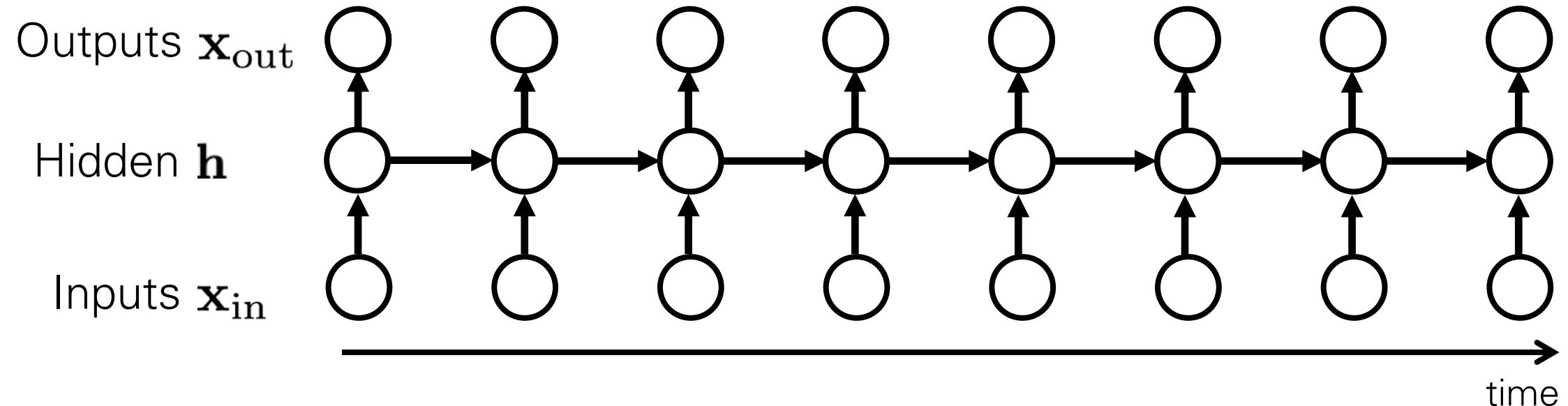


# To model sequences, we need

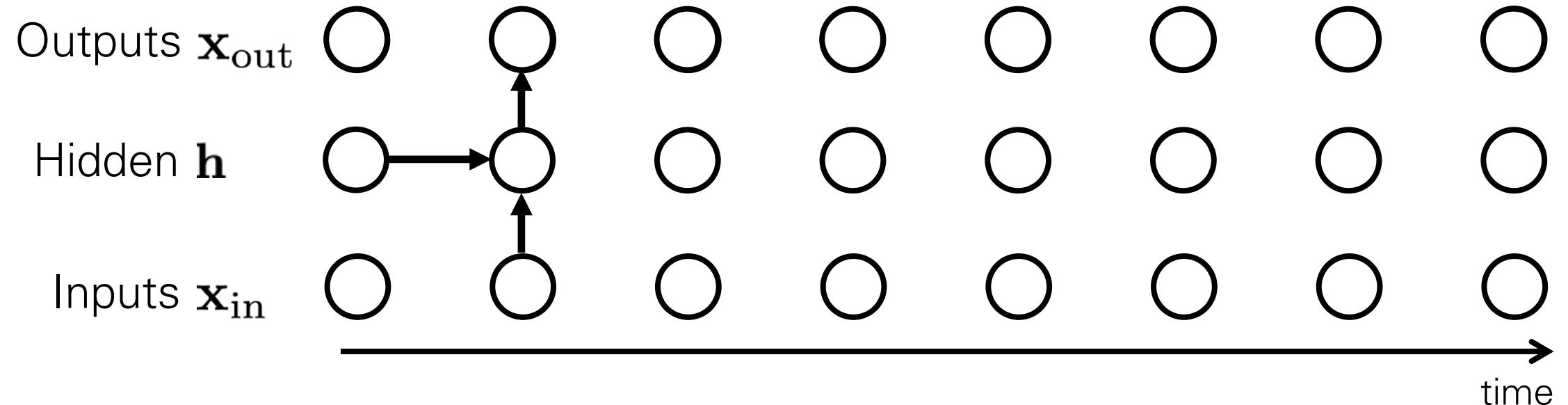
1. to deal with **variable length** sequences
2. to maintain **sequence order**
3. to keep track of **long-term dependencies**
4. to **share parameters** across the sequence

# Recurrent Neural Networks

# Recurrent Neural Networks (RNNs)



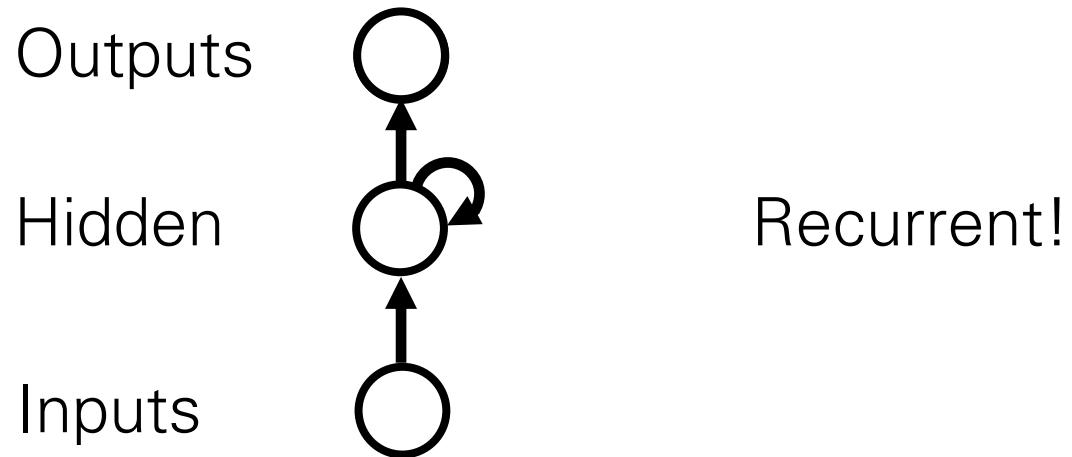
# Recurrent Neural Networks (RNNs)



$$\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_{\text{in}}[t])$$

$$\mathbf{x}_{\text{out}}[t] = g(\mathbf{h}_t)$$

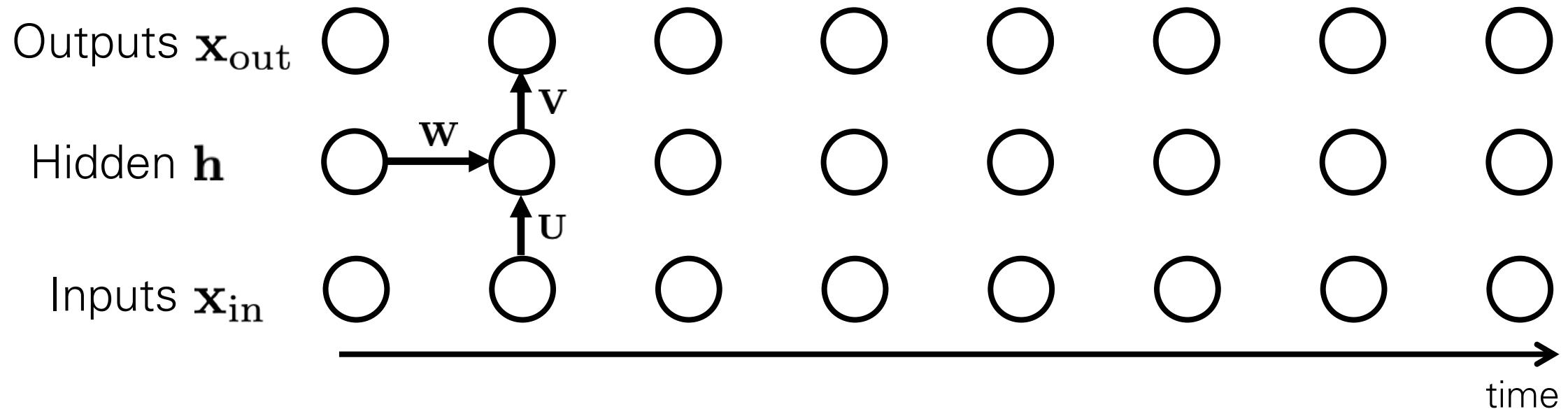
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$$\mathbf{h}_t = f(\mathbf{h}_{t-1}, \mathbf{x}_{\text{in}}[t])$$

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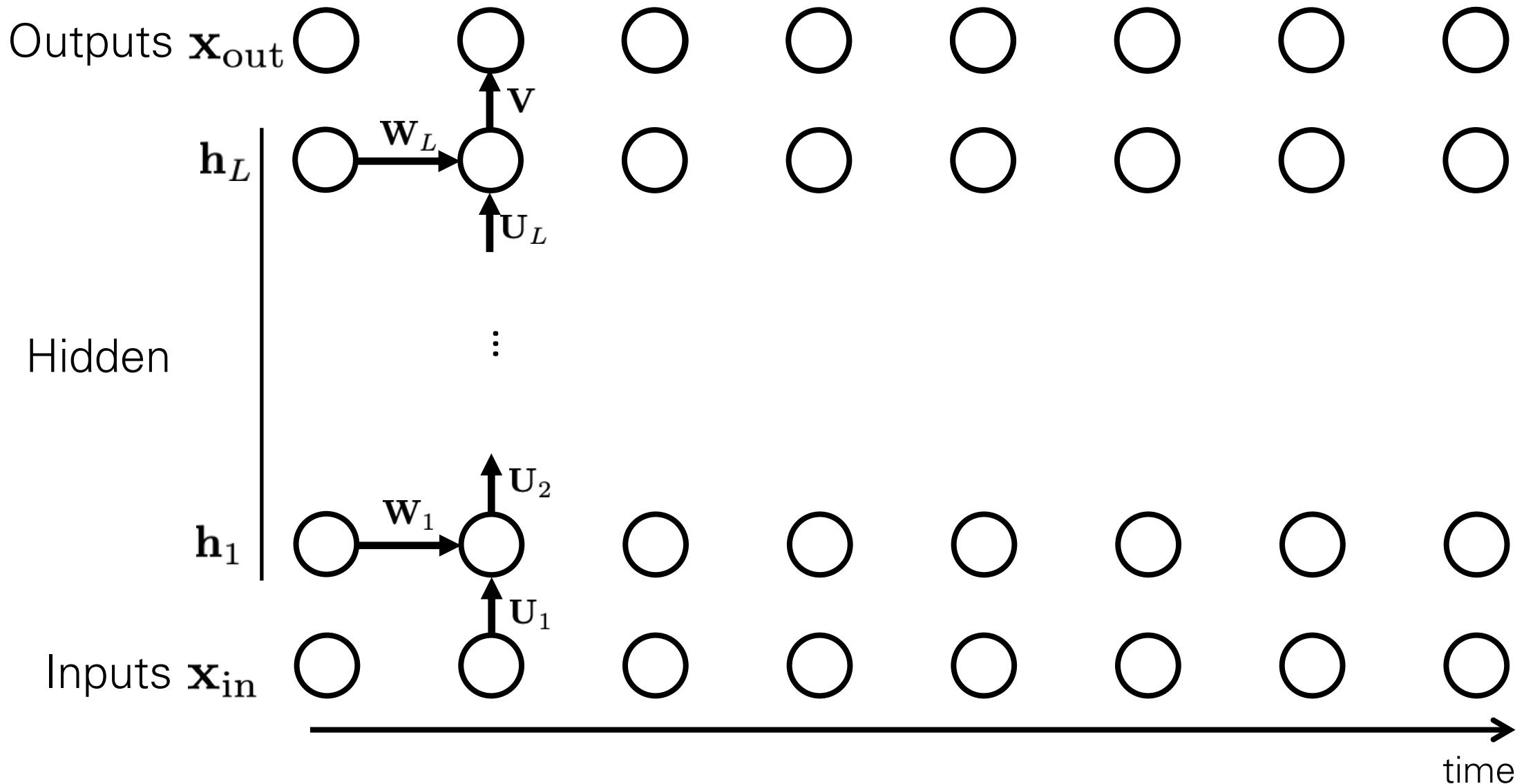
# Recurrent Neural Networks (RNNs)



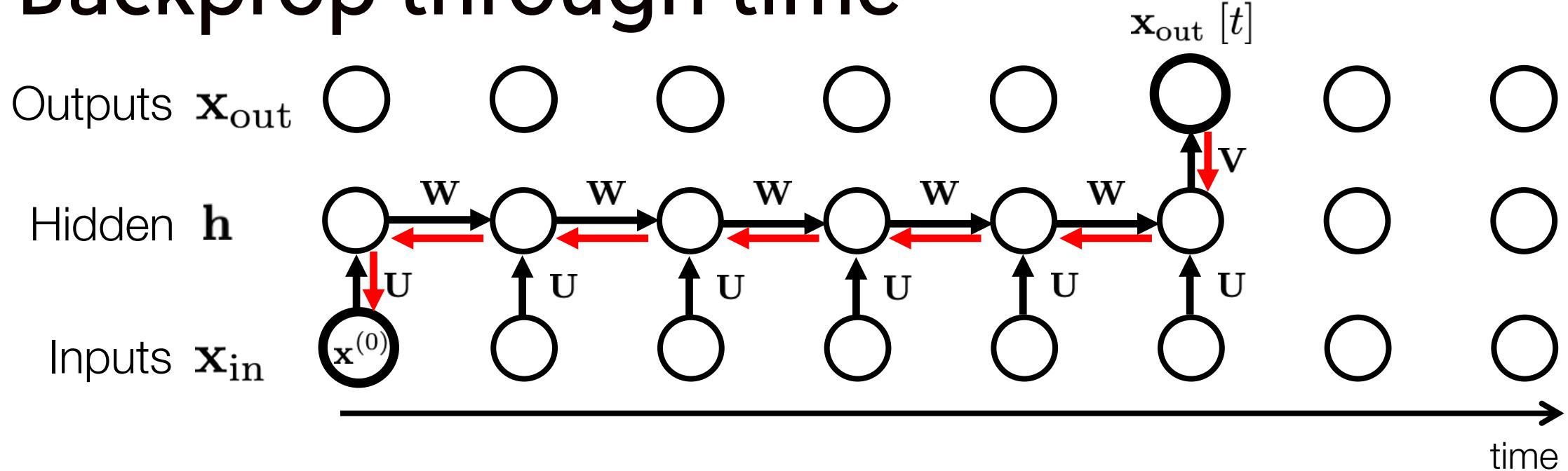
$$\mathbf{h}_t = \sigma_1 (\mathbf{W}\mathbf{h}_{t-1} + \mathbf{U}\mathbf{x}_{\text{in}}[t] + \mathbf{b})$$

$$\mathbf{x}_{\text{out}}[t] = \sigma_2 (\mathbf{V}\mathbf{h}_t + \mathbf{c})$$

# Deep Recurrent Neural Networks (RNNs)

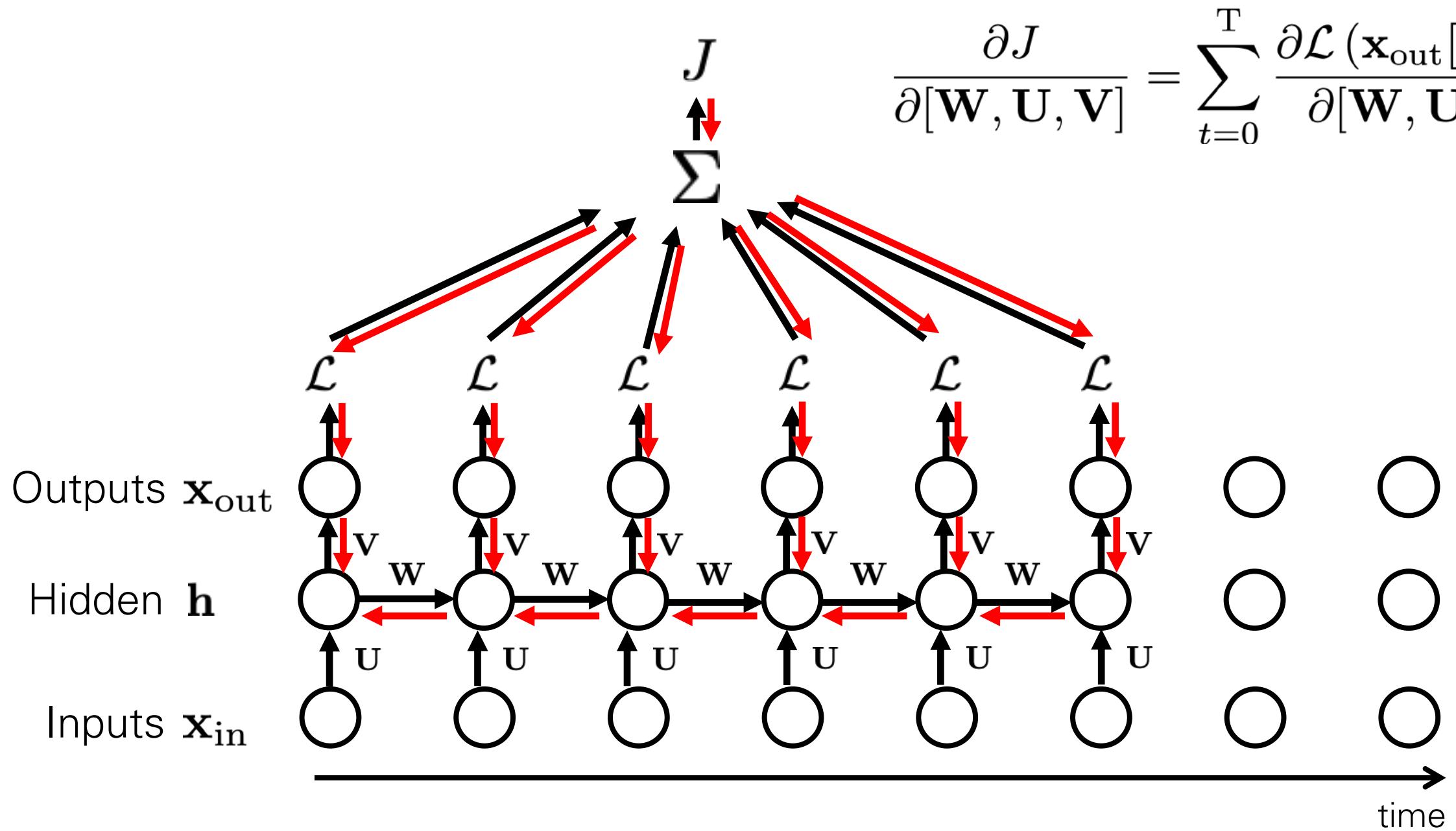


# Backprop through time

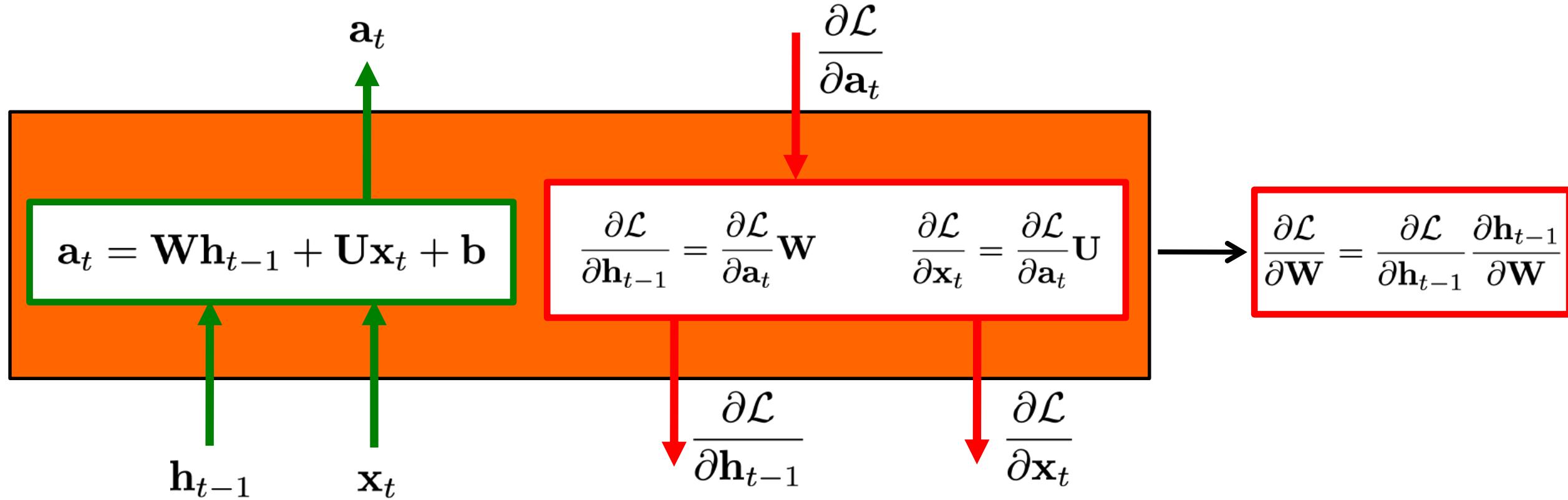


$$\frac{\partial \mathbf{x}_{\text{out}}[t]}{\partial \mathbf{x}_{\text{in}}[0]} = \frac{\partial \mathbf{x}_{\text{out}}[t]}{\partial \mathbf{h}_T} \frac{\partial \mathbf{h}_T}{\partial \mathbf{h}_{T-1}} \cdots \frac{\partial \mathbf{h}_1}{\partial \mathbf{h}_0} \frac{\partial \mathbf{h}_0}{\partial \mathbf{x}_{\text{in}}[0]}$$

$$\frac{\partial J}{\partial [\mathbf{W}, \mathbf{U}, \mathbf{V}]} = \sum_{t=0}^T \frac{\partial \mathcal{L}(\mathbf{x}_{\text{out}}[t], \mathbf{y}_t)}{\partial [\mathbf{W}, \mathbf{U}, \mathbf{V}]}$$

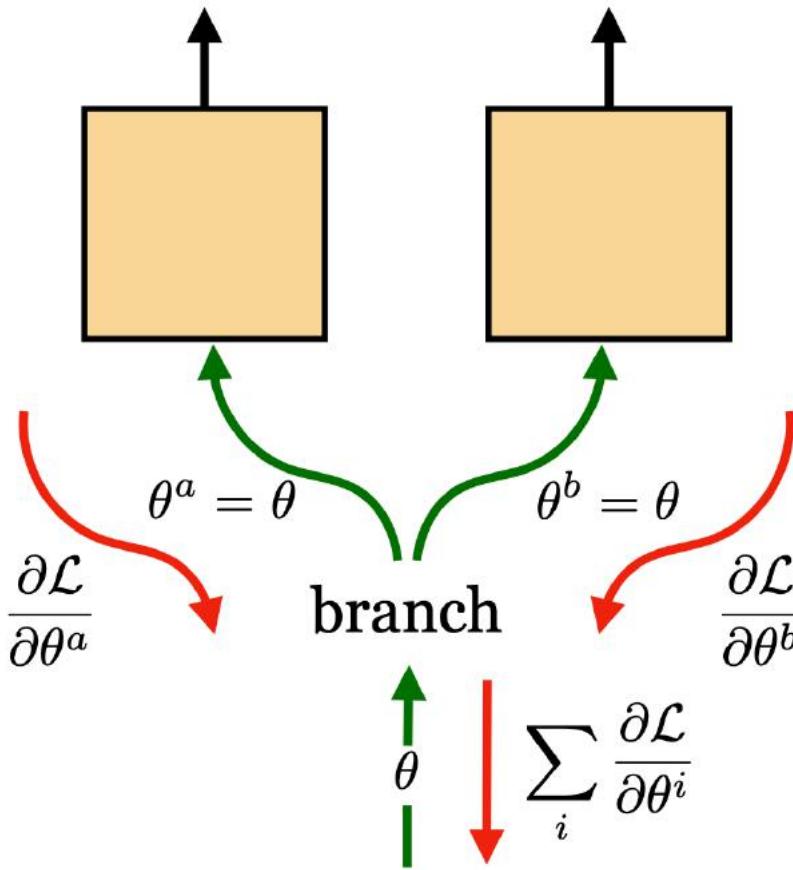
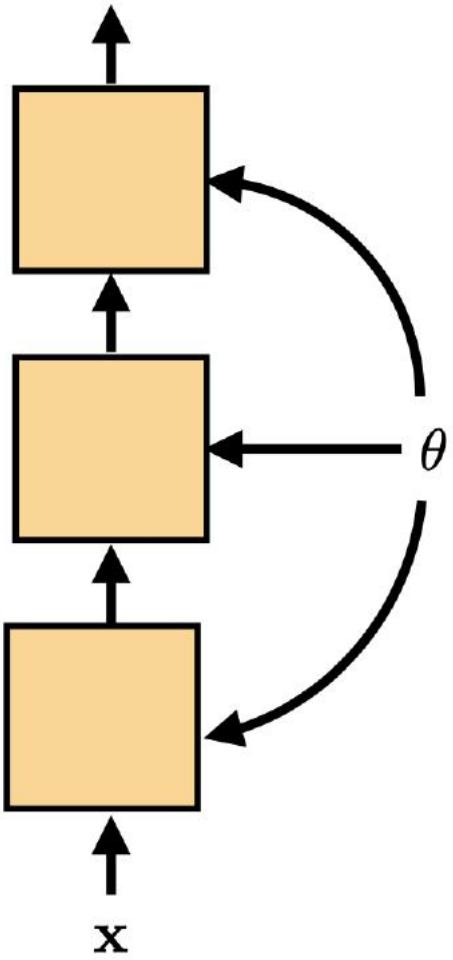


# Recurrent Linear Layer

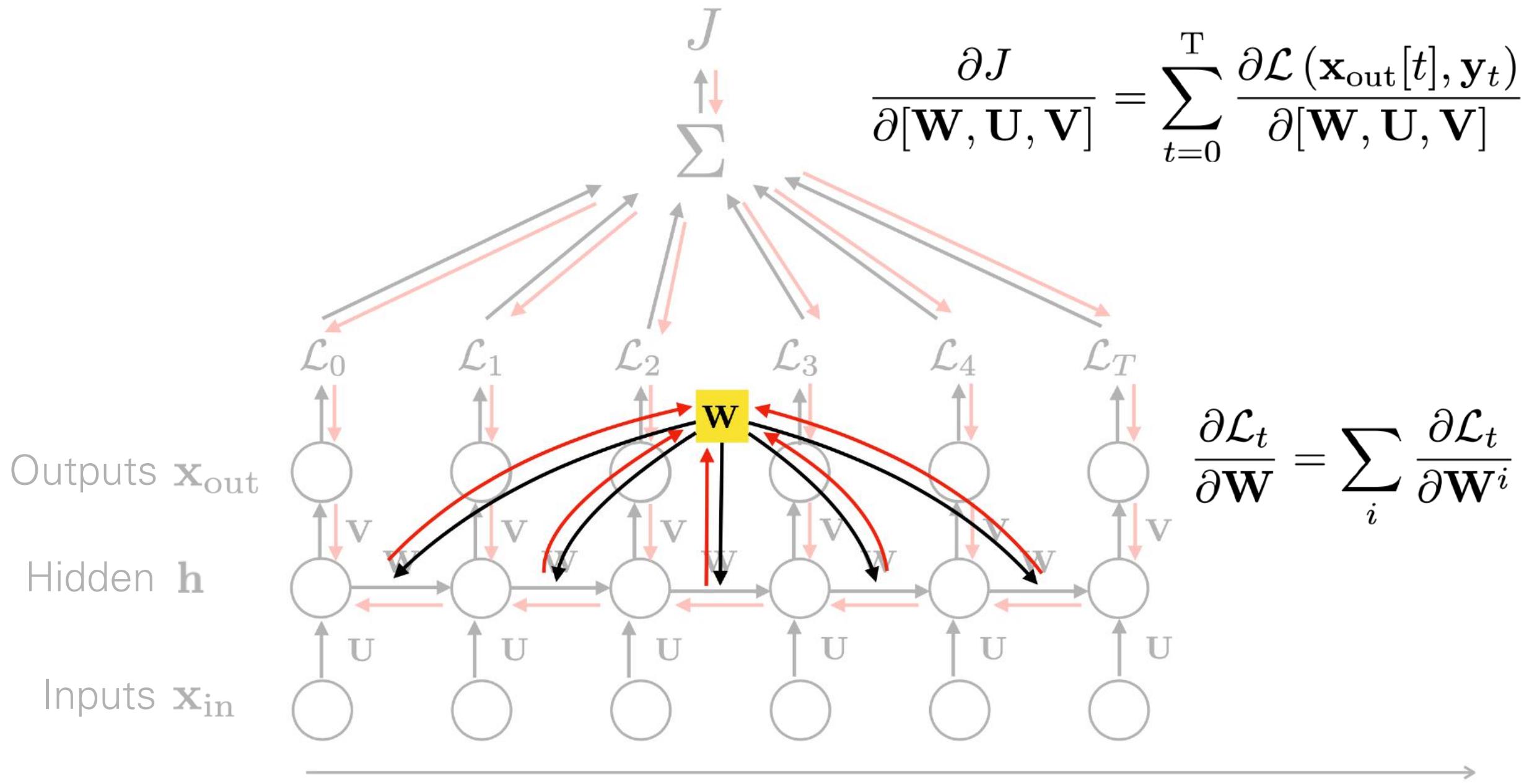


$$\frac{\partial J}{\partial \mathbf{W}} = \sum_{t=0}^T \frac{\partial \mathcal{L}(\hat{\mathbf{y}}_t, \mathbf{y}_t)}{\partial \mathbf{W}}$$

# Parameter Sharing



Parameter sharing —> sum gradients

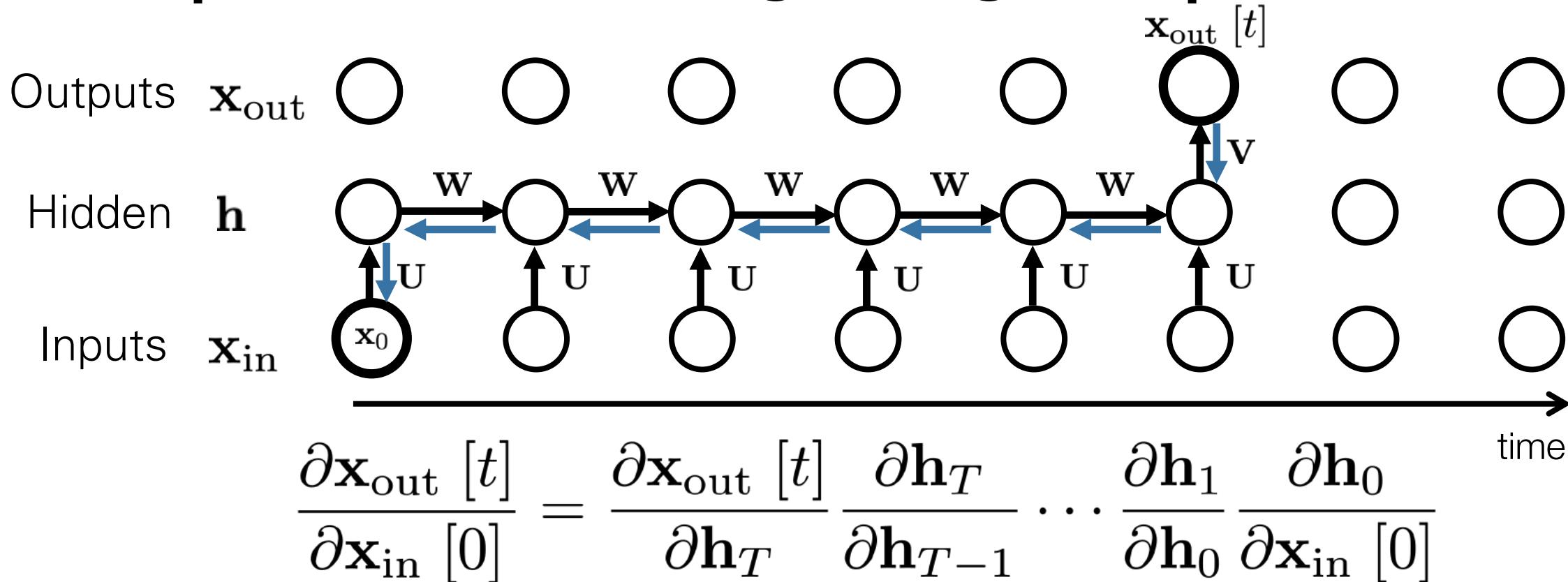


# The problem of long-range dependencies

Why not remember everything?

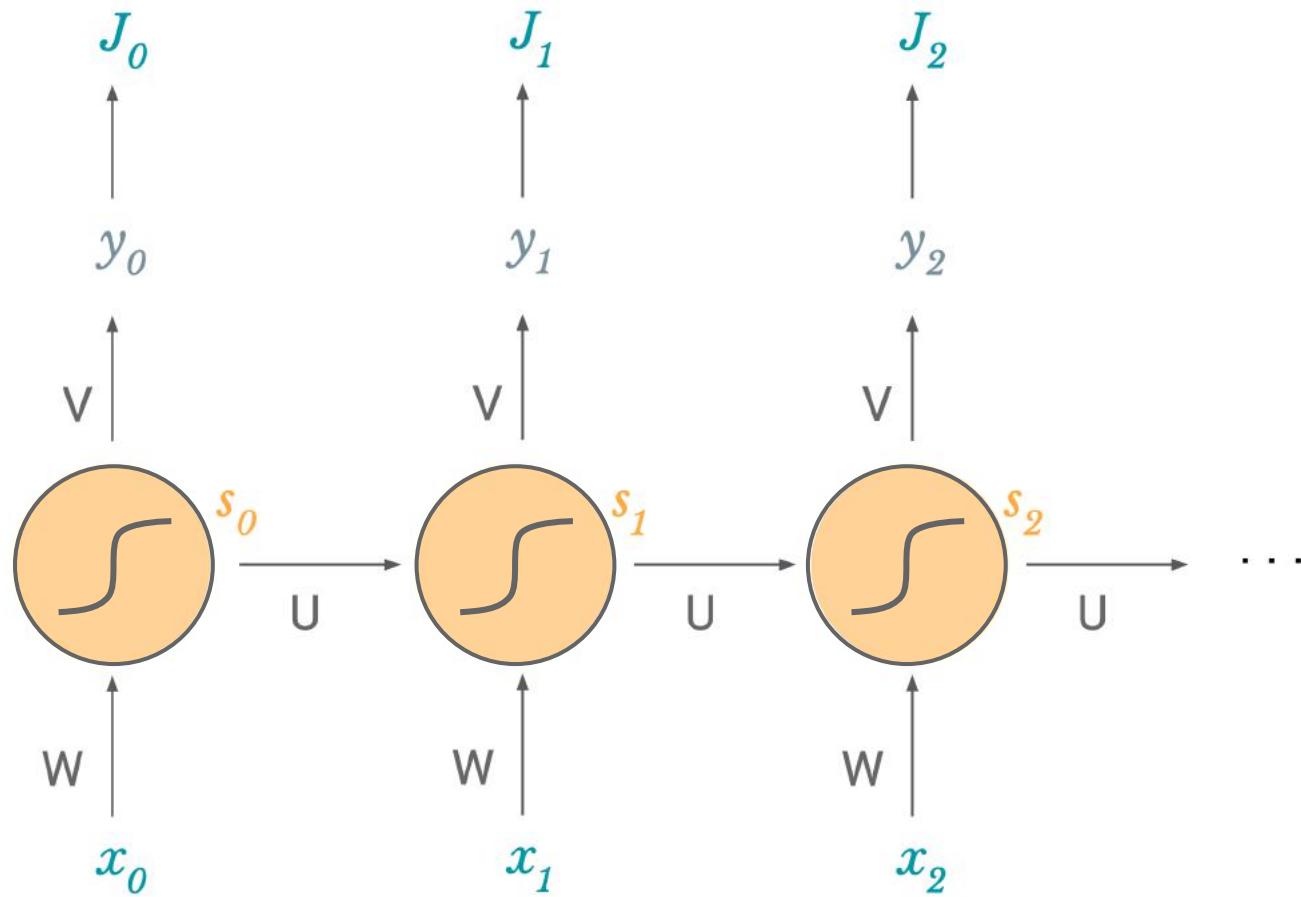
- Memory size grows with  $t$
- This kind of memory is **nonparametric**: there is no finite set of parameters we can use to model it
- RNNs make a Markov assumption — the future hidden state only depends on the immediately preceding hidden state
- By putting the right info into the hidden state, RNNs can model dependencies that are arbitrarily far apart

# The problem of long-range dependencies

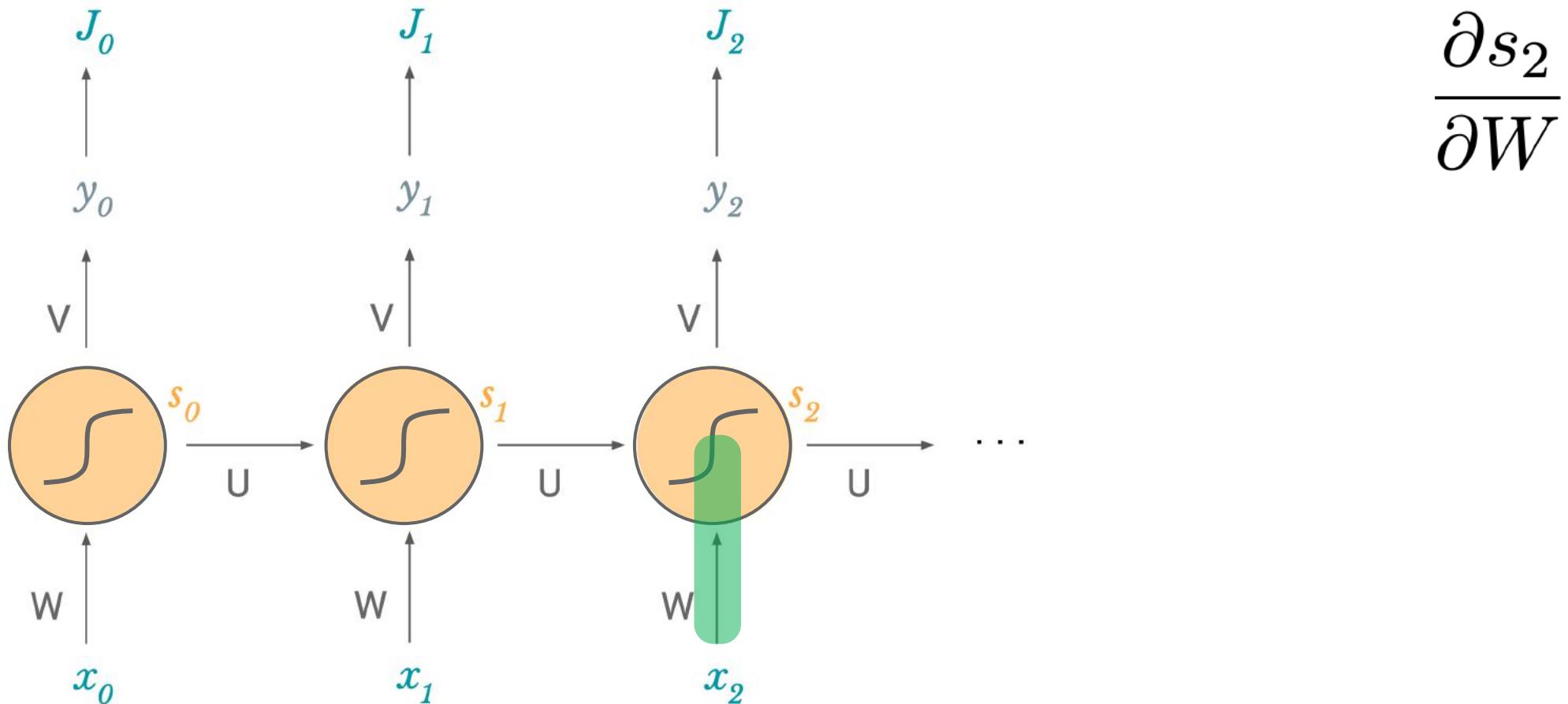


- Capturing long-range dependences requires propagating information through a long chain of dependences.
- Old observations are forgotten
- Stochastic gradients become high variance (noisy), and gradients may **vanish or explode**

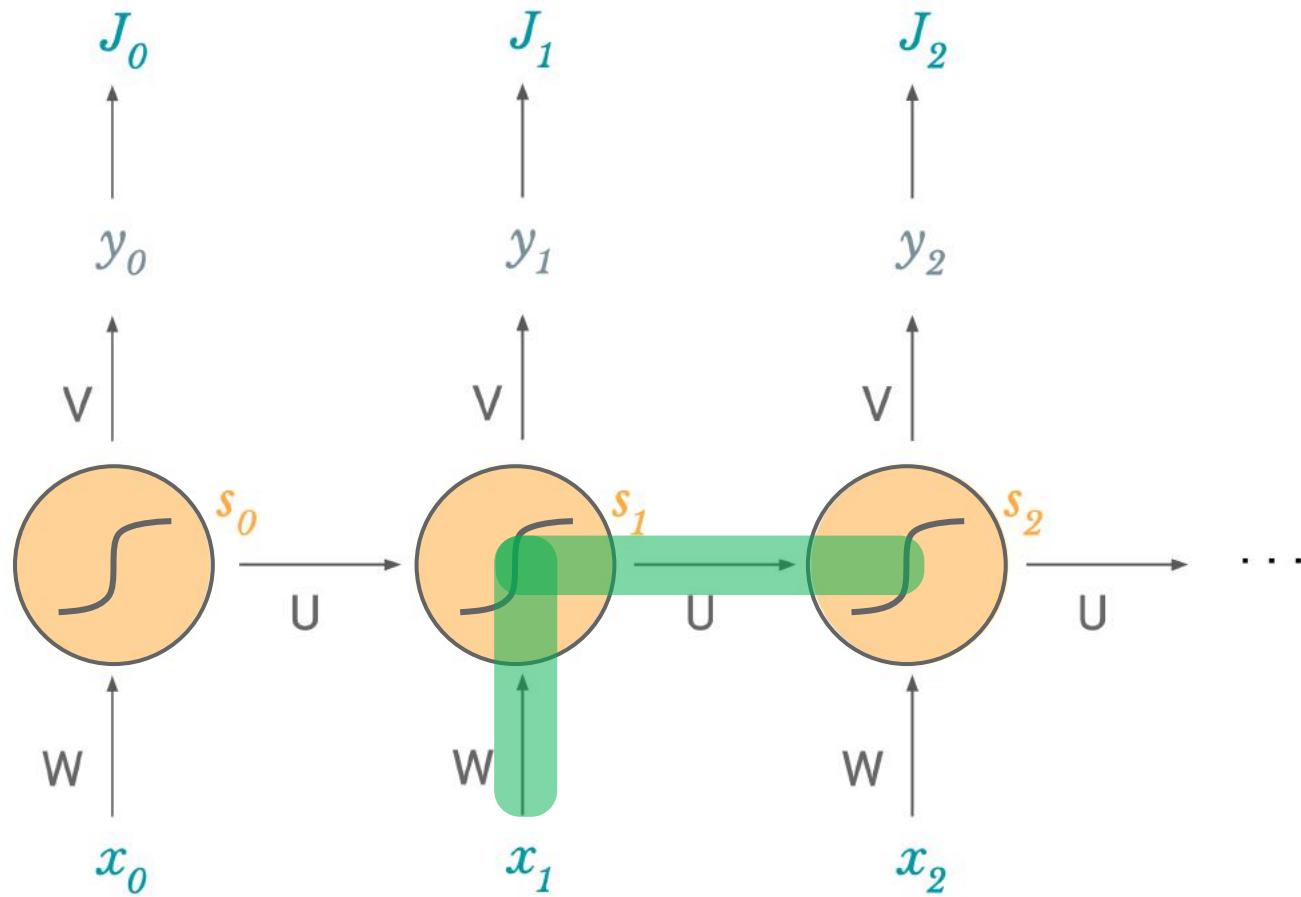
# Let's try it out for W with the chain rule:



# Let's try it out for W with the chain rule:

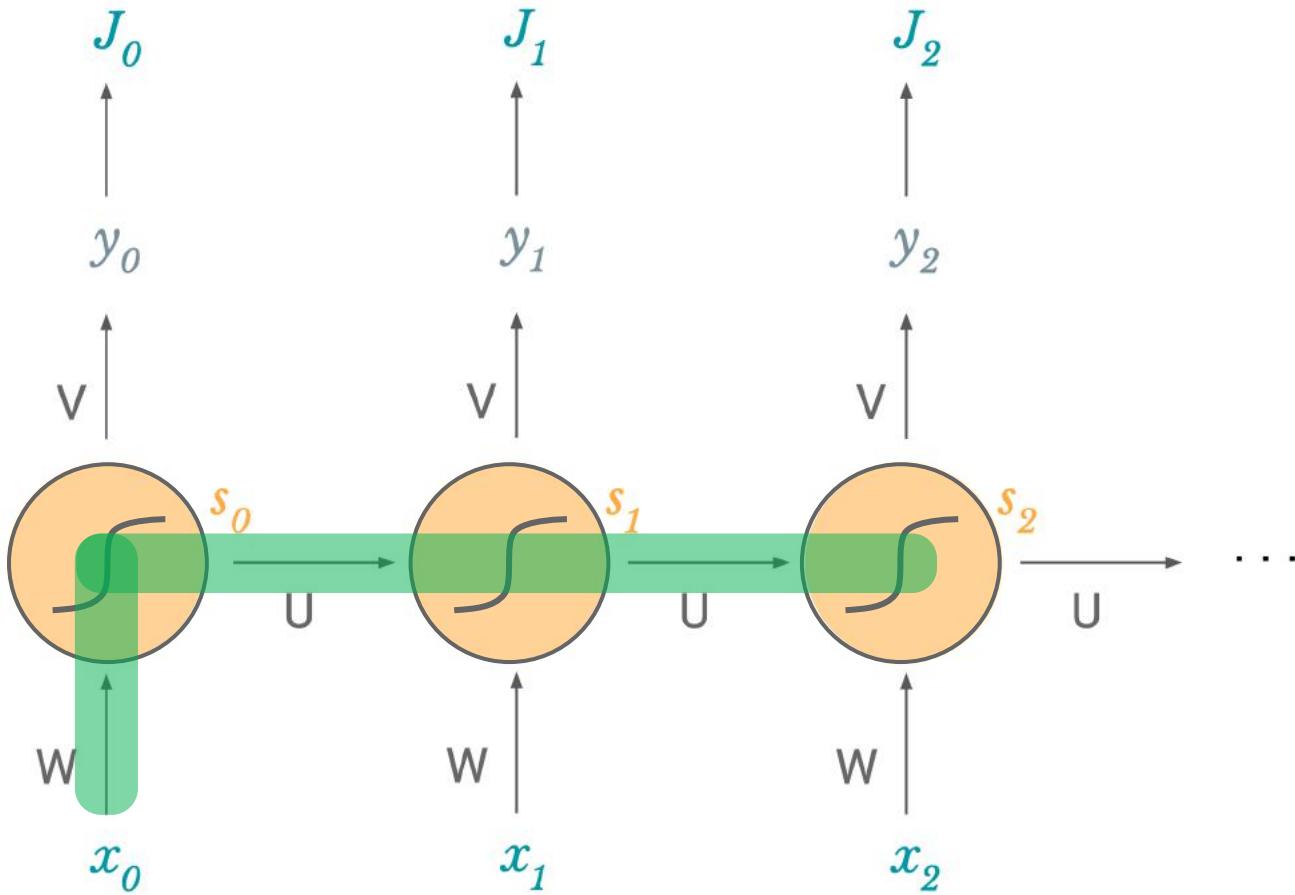


# Let's try it out for W with the chain rule:



$$\frac{\partial s_2}{\partial W} + \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial W}$$

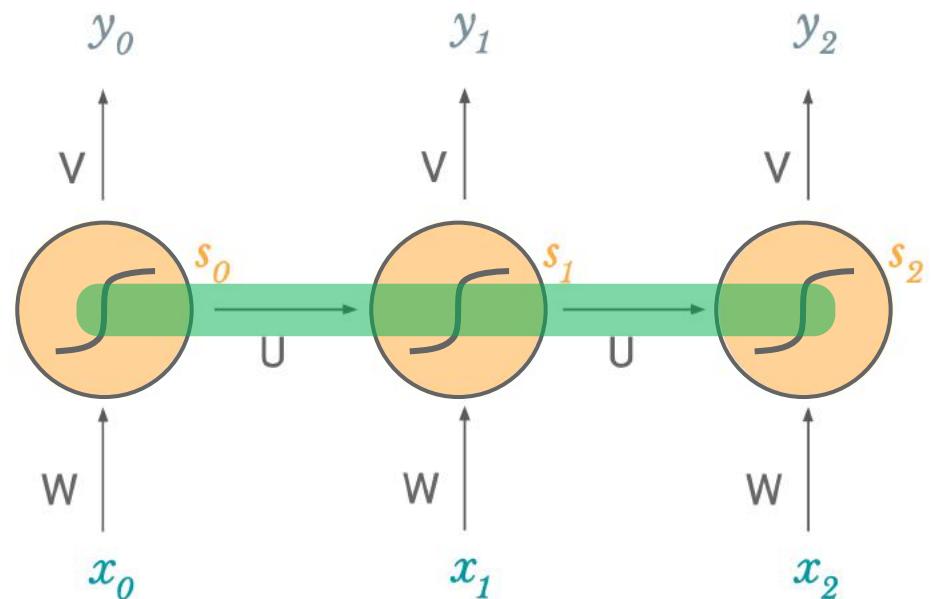
# Let's try it out for W with the chain rule:



$$\begin{aligned} & \frac{\partial s_2}{\partial W} \\ & + \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial W} \\ & + \frac{\partial s_2}{\partial s_0} \frac{\partial s_0}{\partial W} \end{aligned}$$

# Vanishing Gradient Problem

$$\frac{\partial J_2}{\partial W} = \sum_{k=0}^2 \frac{\partial J_2}{\partial y_2} \frac{\partial y_2}{\partial s_2} \frac{\partial s_2}{\partial s_k} \frac{\partial s_k}{\partial W}$$



at k=0:

$$\frac{\partial s_2}{\partial s_0} = \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial s_0}$$

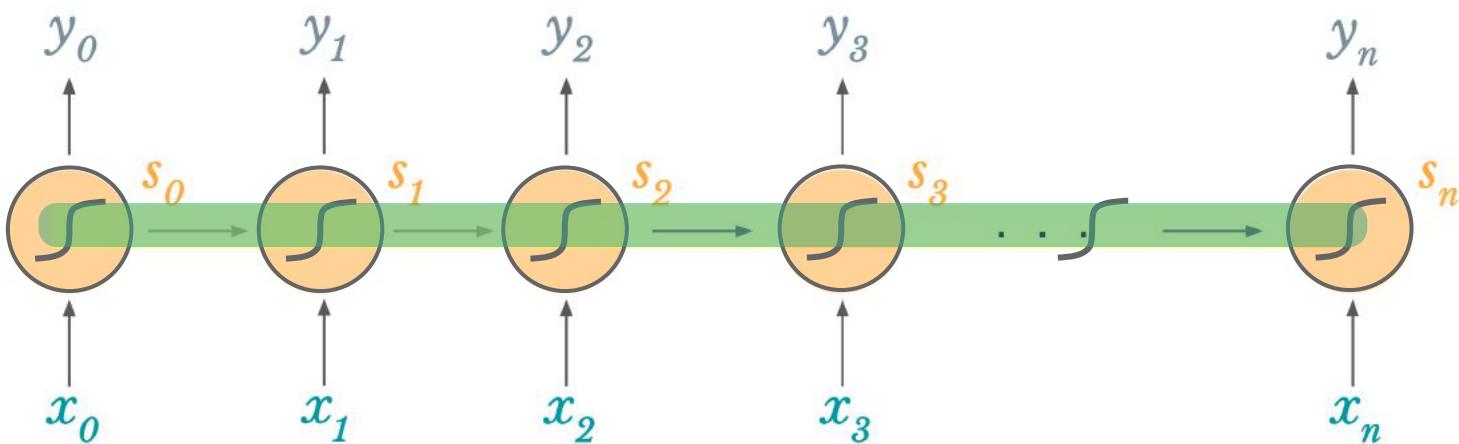
# Vanishing Gradient Problem

$$\frac{\partial J_n}{\partial W} = \sum_{k=0}^n \frac{\partial J_n}{\partial y_n} \frac{\partial y_n}{\partial s_n} \frac{\partial s_n}{\partial s_k} \frac{\partial s_k}{\partial W}$$

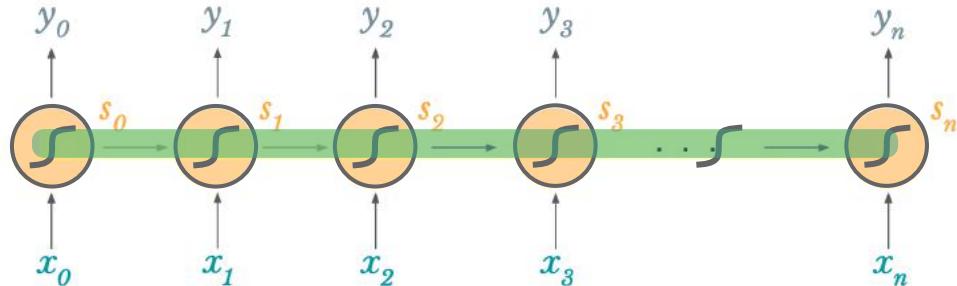
$$\frac{\partial s_n}{\partial s_{n-1}} \frac{\partial s_{n-1}}{\partial s_{n-2}} \dots \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial s_0}$$



as the gap between timesteps gets bigger, this product gets longer and longer!



# Vanishing Gradient Problem



what are each of these terms?

$$\frac{\partial s_n}{\partial s_{n-1}} = W^T \text{diag} [f'(W_{s_{j-1}} + Ux_j)]$$

$W$  = sampled from standard  
normal distribution = mostly  $< 1$

$f = \tanh$  or sigmoid so  $f' < 1$

$$\frac{\partial s_n}{\partial s_{n-1}} \frac{\partial s_{n-1}}{\partial s_{n-2}} \dots \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial s_0}$$

**we're multiplying a lot of small numbers together.**

# Vanishing Gradient Problem

we're multiplying a lot of **small numbers** together.

**so what?**

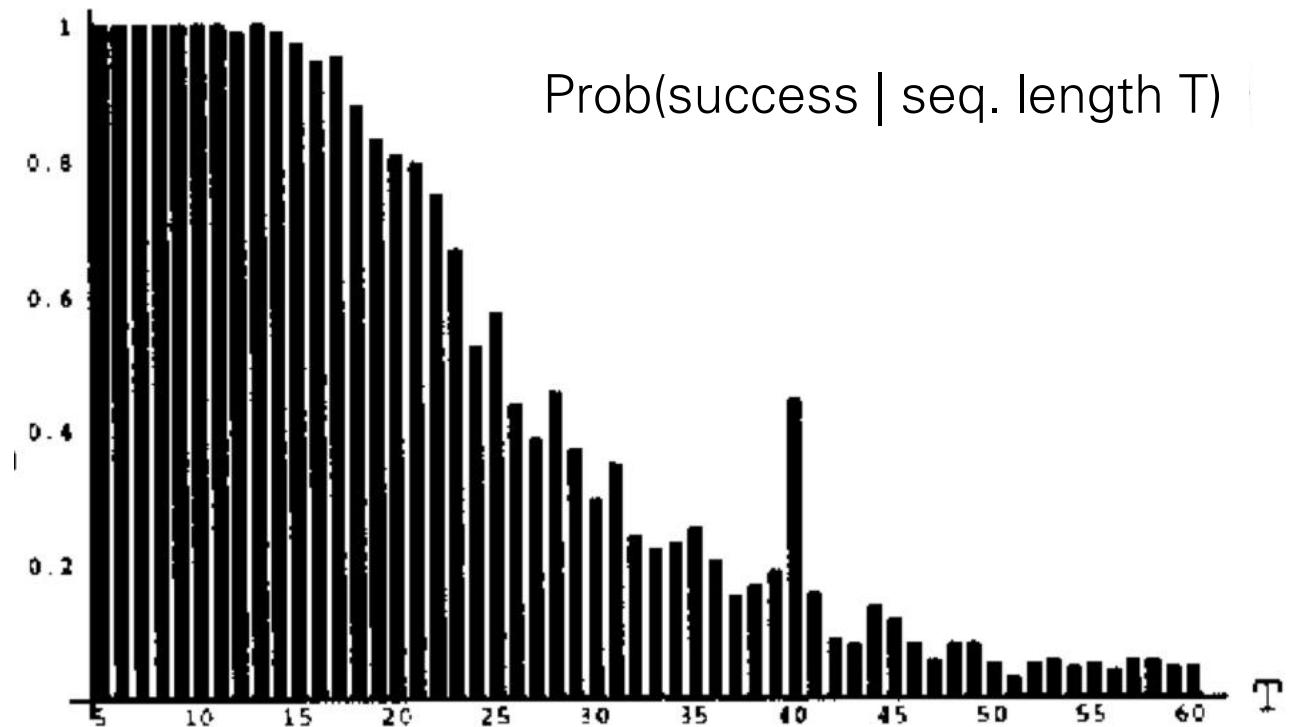
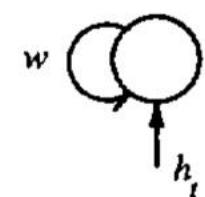
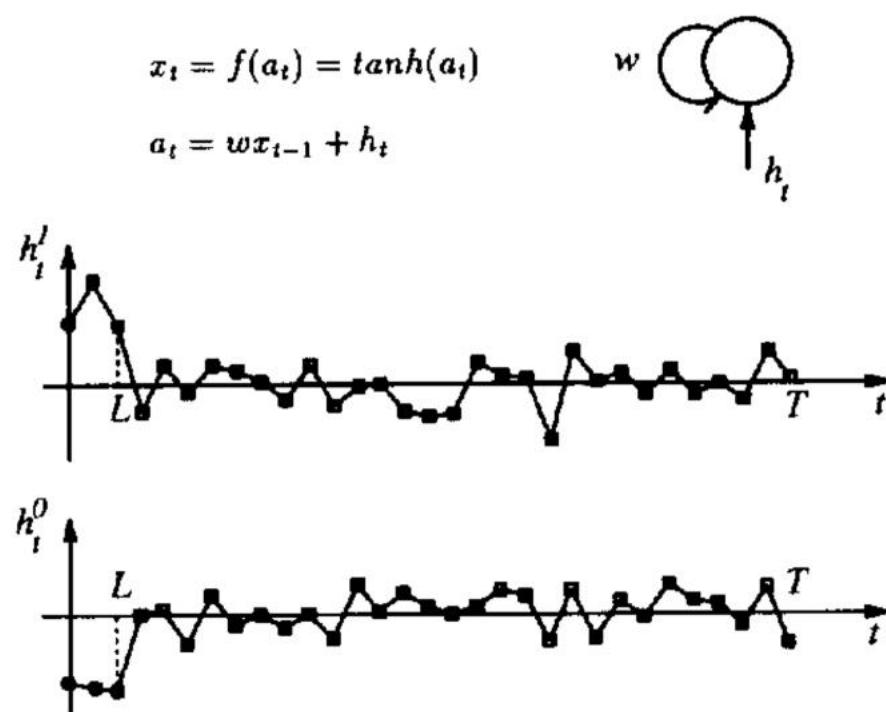
errors due to further back timesteps have increasingly  
**smaller gradients**.

**so what?**

parameters become biased to **capture shorter-term**  
dependencies.

# A Toy Example

- 2 categories of sequences
- Can the single tanh unit learn to store for  $T$  time steps 1 bit of information given by the sign of initial input?



# Vanishing Gradient Problem

“In France, I had a great time and I learnt some  
of the \_\_\_\_\_ language.”



our parameters are not trained to capture long-term  
dependencies, so the word we predict will mostly depend on  
the previous few words, not much earlier ones

# Long-Term Dependencies



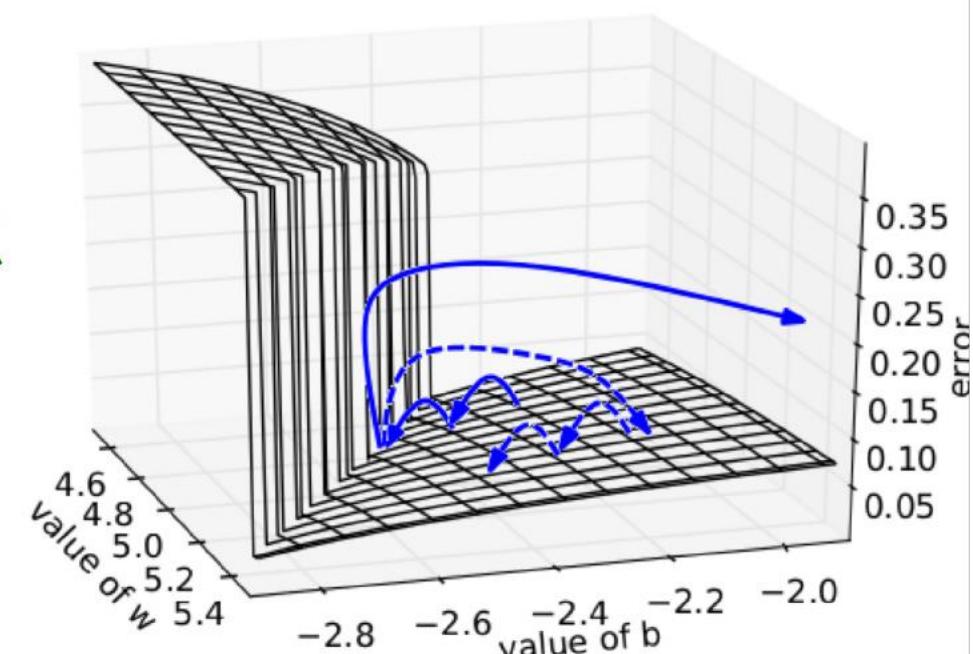
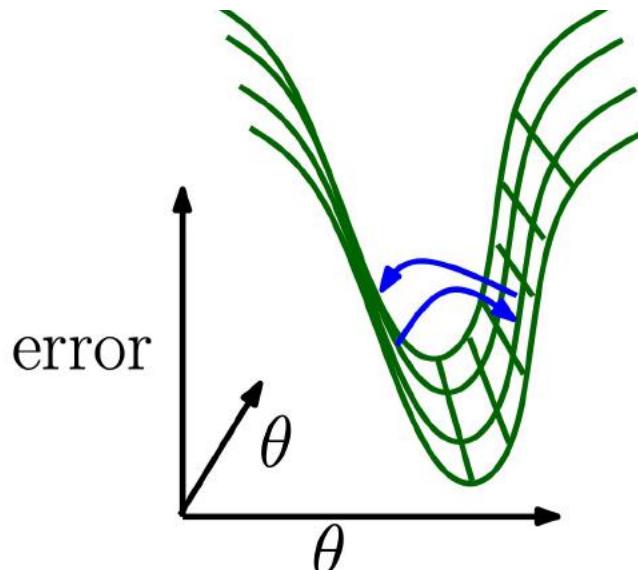
- The RNN gradient is a product of Jacobian matrices, each associated with a step in the forward computation. To store information robustly in a finite-dimensional state, the dynamics must be contractive [Bengio et al 1994].

$$L = L(s_T(s_{T-1}(\dots s_{t+1}(s_t, \dots))))$$
$$\frac{\partial L}{\partial s_t} = \frac{\partial L}{\partial s_T} \frac{\partial s_T}{\partial s_{T-1}} \dots \frac{\partial s_{t+1}}{\partial s_t}$$

- Problems:
  - sing. values of Jacobians  $> 1 \rightarrow$  **gradients explode**
  - or sing. values  $<$   $\rightarrow$  **gradients shrink & vanish**
  - or random  $\rightarrow$  **variance grows exponentially**

# Gradient Norm Clipping

```
 $\hat{\mathbf{g}} \leftarrow \frac{\partial \text{error}}{\partial \theta}$ 
if  $\|\hat{\mathbf{g}}\| \geq \text{threshold}$  then
     $\hat{\mathbf{g}} \leftarrow \frac{\text{threshold}}{\|\hat{\mathbf{g}}\|} \hat{\mathbf{g}}$ 
end if
```



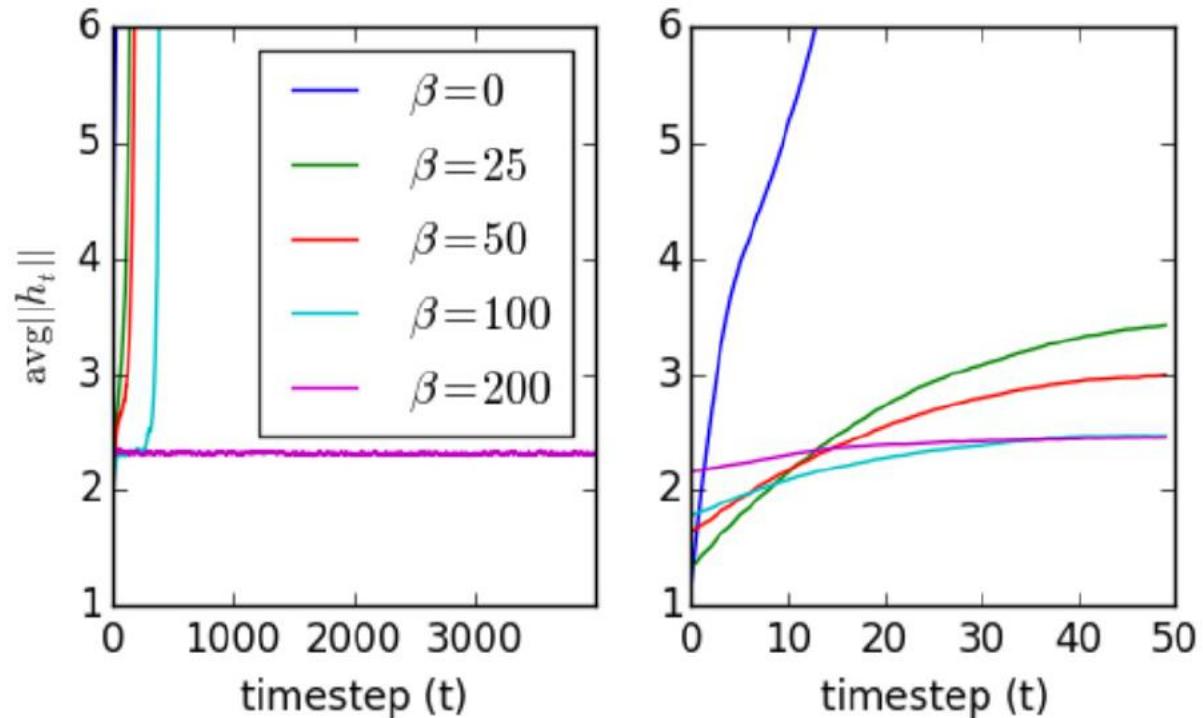
[Recurrent neural network regularization. Zaremba et al., arXiv 2014.](#)

# Regularization: Norm-stabilizer

- Stabilize the activations of RNNs by penalizing the squared distance between successive hidden states' norms

$$\beta \frac{1}{T} \sum_{t=1}^T (\|h_t\|_2 - \|h_{t-1}\|_2)^2$$

- Enforce the norms of the hidden layer activations approximately constant across time



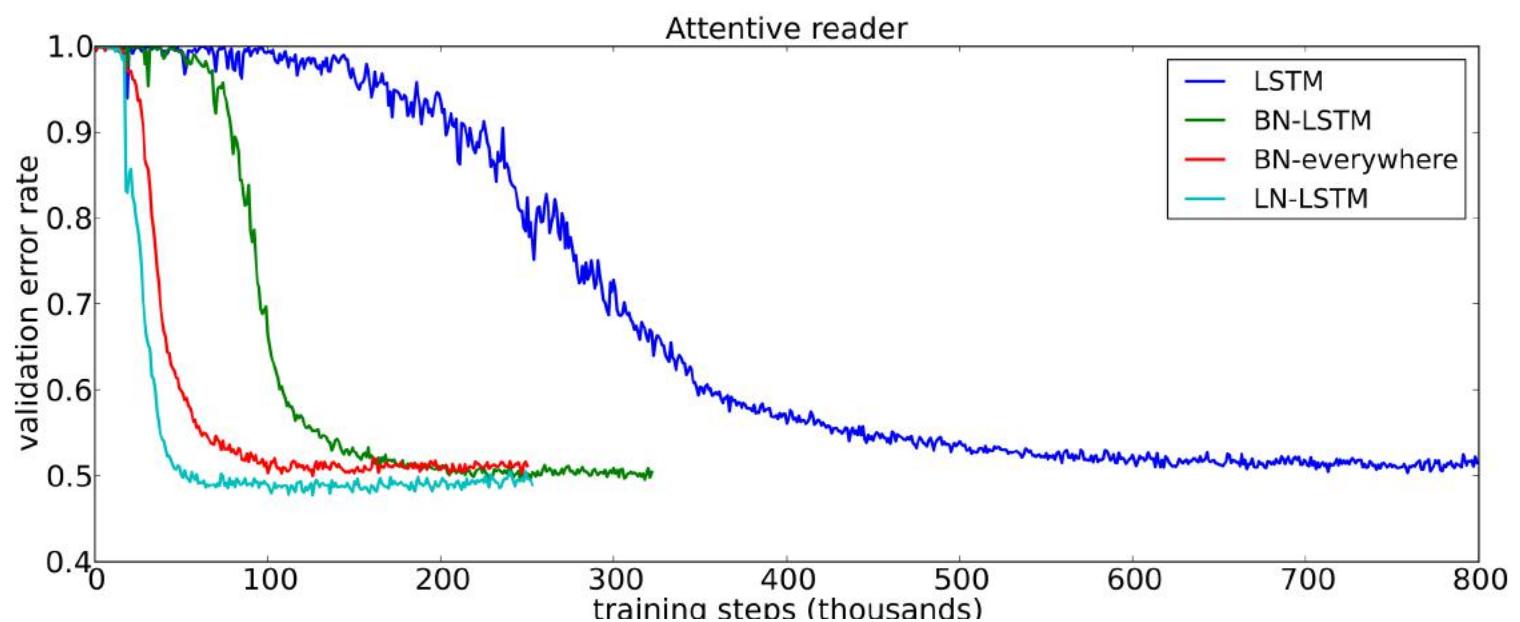
# Regularization: Layer Normalization

- Similar to batch normalization
- Computes the normalization statistics separately at each time step
- Effective for stabilizing the hidden state dynamics in RNNs
- Reduces training time

$$\mathbf{h}^t = f \left[ \frac{\mathbf{g}}{\sigma^t} \odot (\mathbf{a}^t - \mu^t) + \mathbf{b} \right]$$

$$\mu^t = \frac{1}{H} \sum_{i=1}^H a_i^t$$

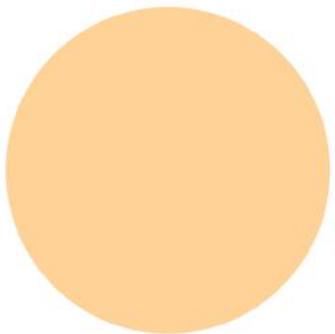
$$\sigma^t = \sqrt{\frac{1}{H} \sum_{i=1}^H (a_i^t - \mu^t)^2}$$



Layer Normalization [Ba, Kiros & Hinton, 2016]

# Gated Cells

- rather than each node being just a simple RNN cell, make each node a more **complex unit with gates** controlling what information is passed through



RNN

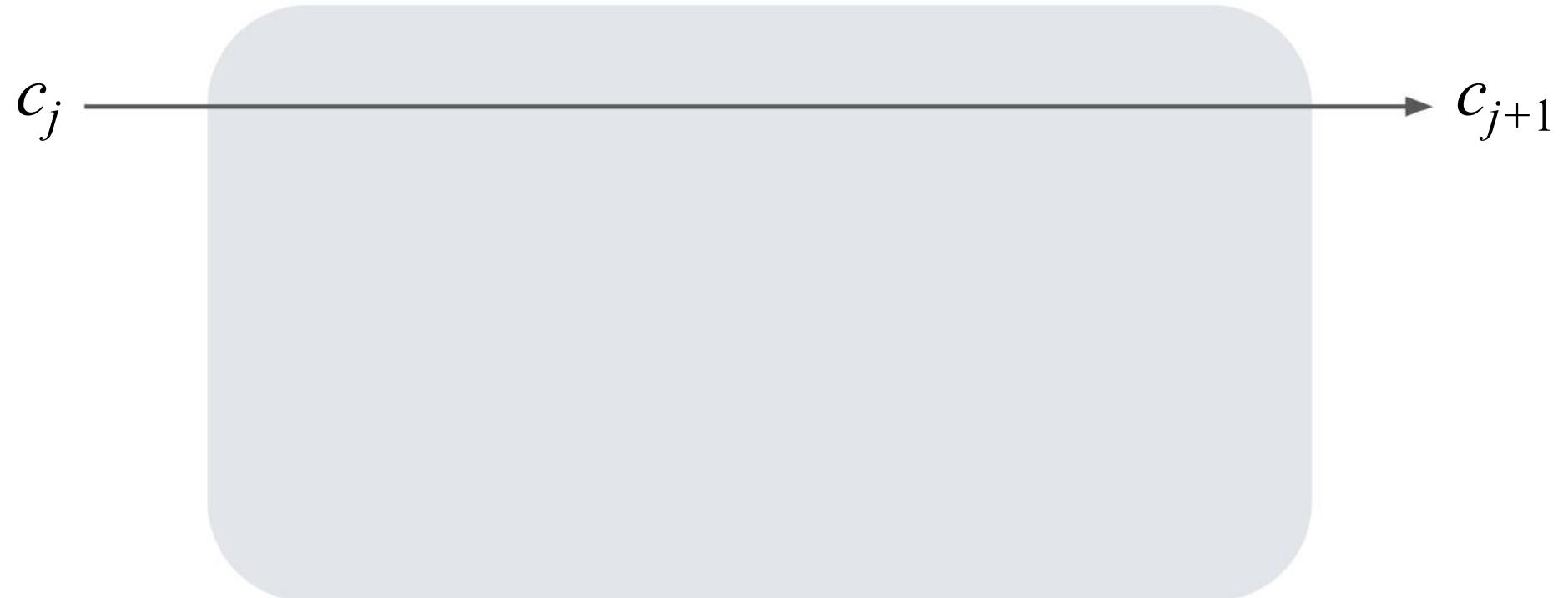
vs



LSTM, GRU, etc

**Long short term memory** cells are able to keep track of information throughout many timesteps.

# Long Short-Term Memory (LSTM)



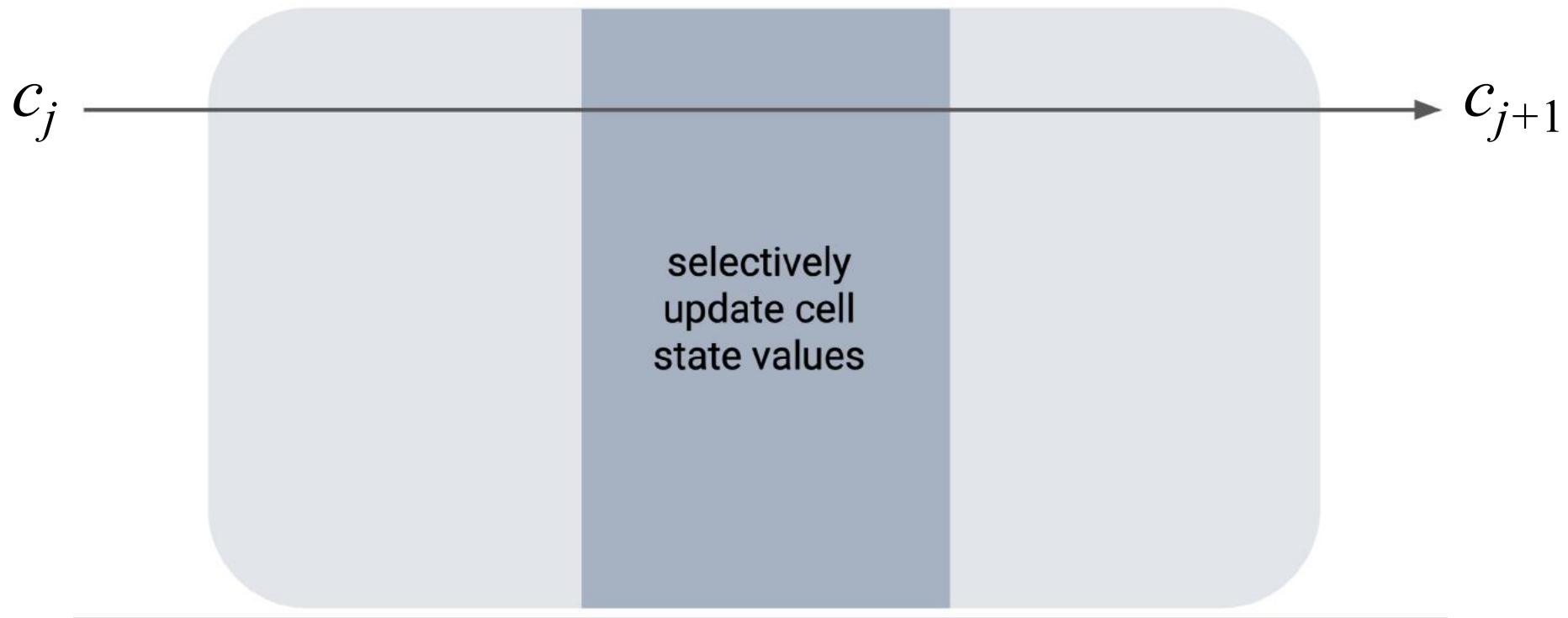
[Long Short-Term Memory \[Hochreiter et al., 1997\]](#)

# Long Short-Term Memory (LSTM)



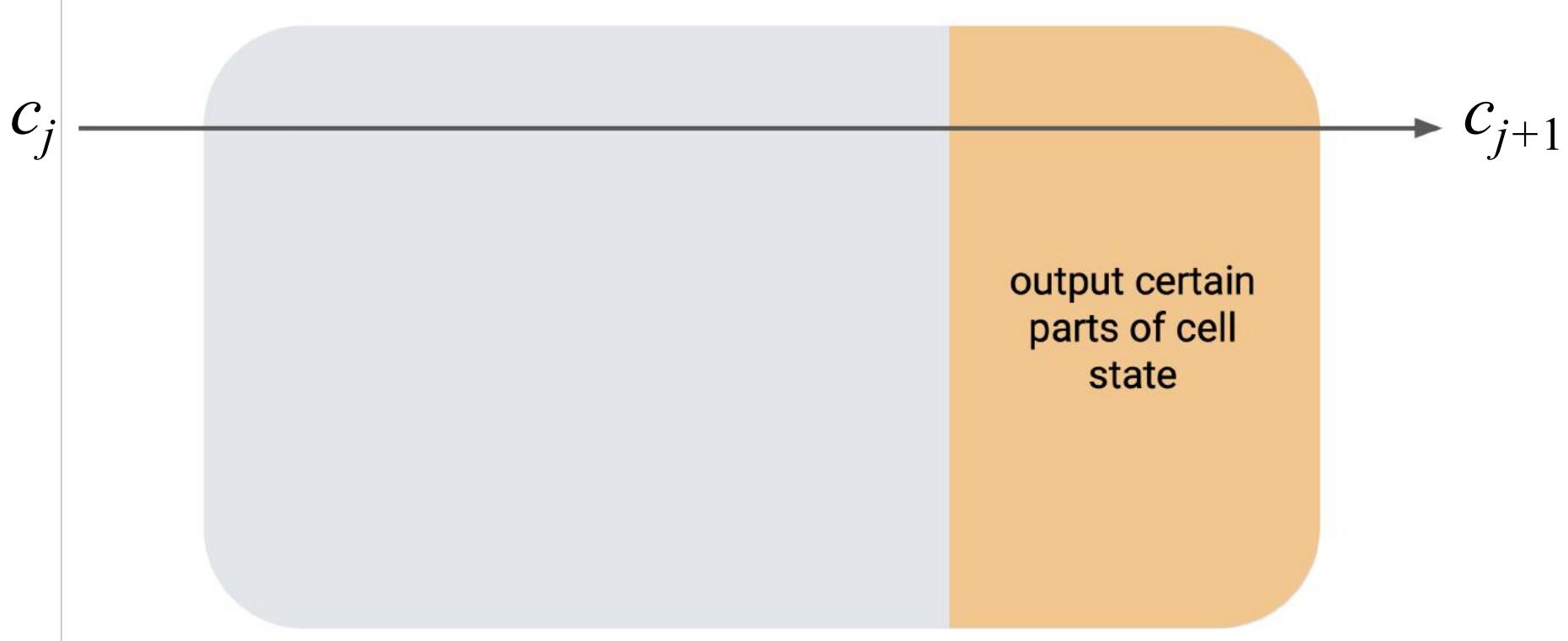
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# Long Short-Term Memory (LSTM)



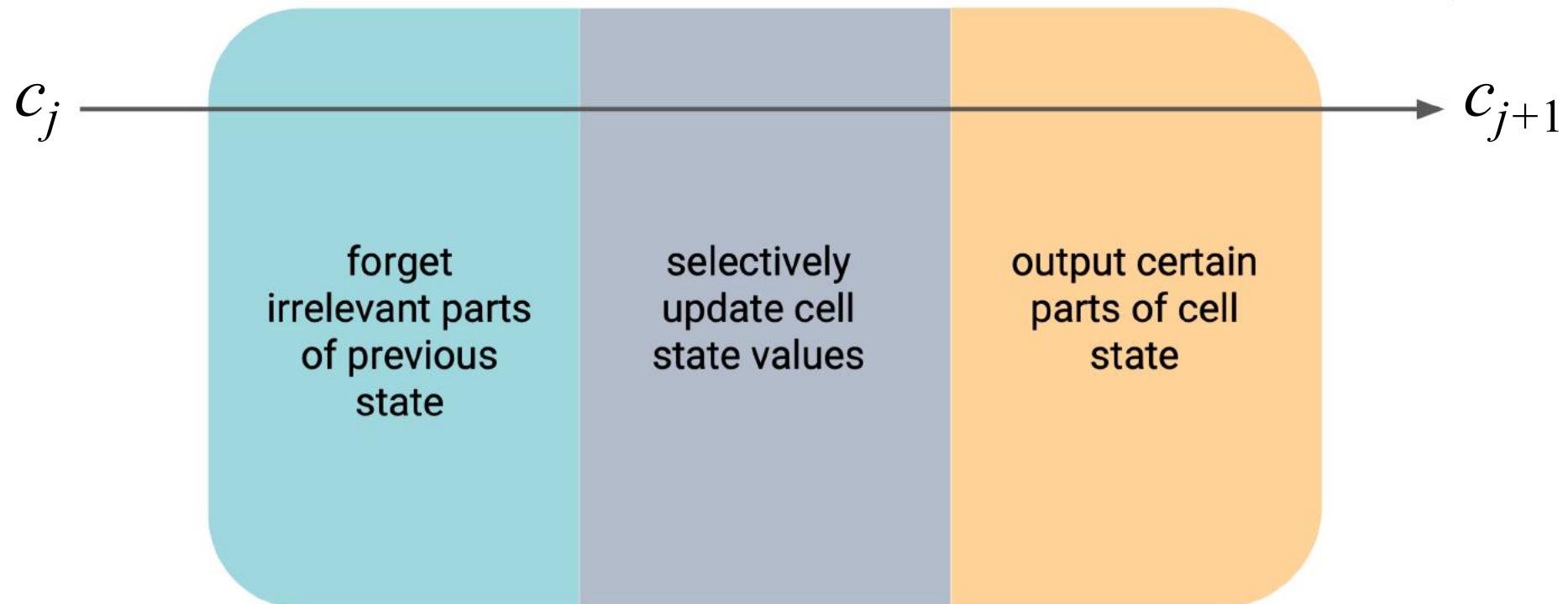
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# Long Short-Term Memory (LSTM)



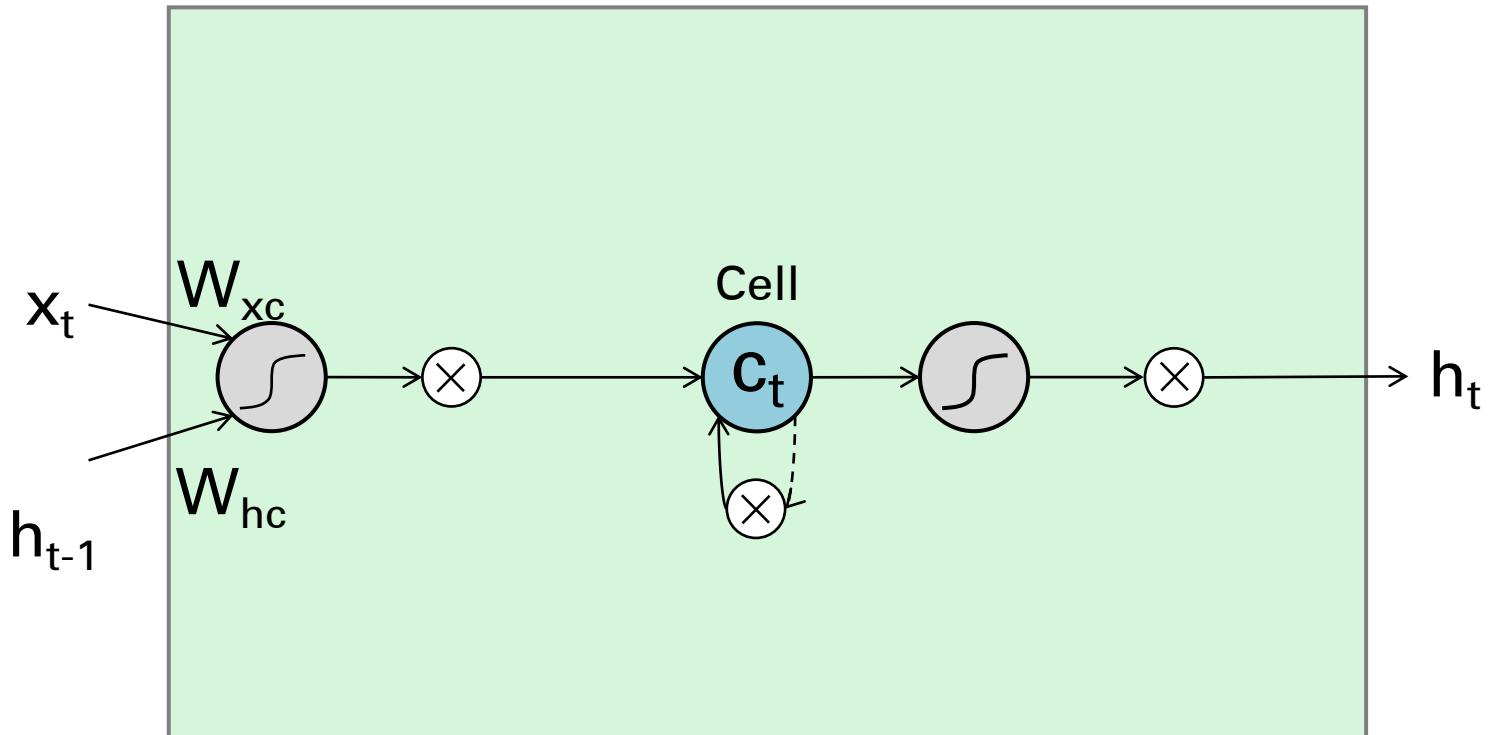
[Long Short-Term Memory \[Hochreiter et al., 1997\]](#)

# Long Short-Term Memory (LSTM)



[Long Short-Term Memory \[Hochreiter et al., 1997\]](#)

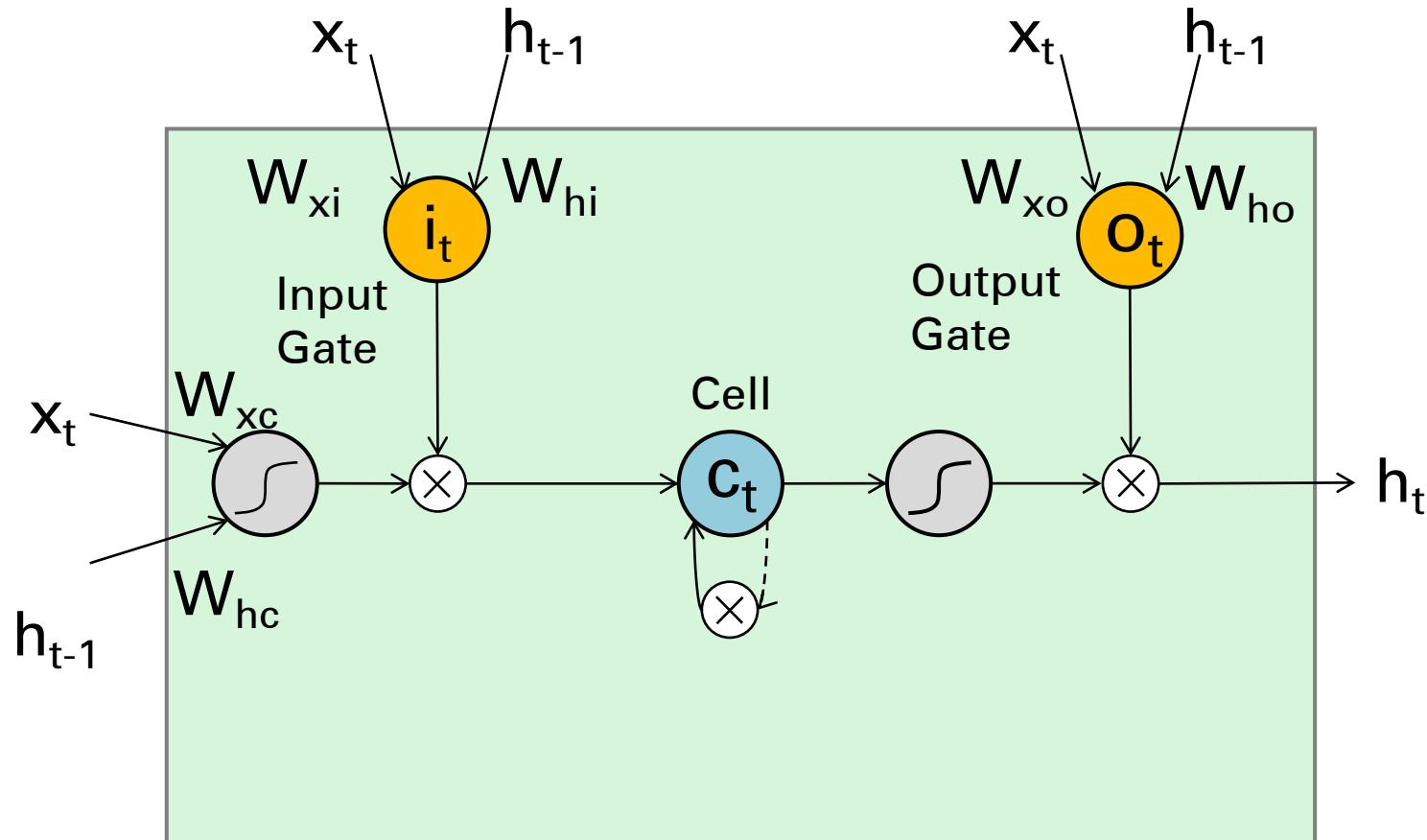
# The LSTM Idea



\* Dashed line indicates time-lag

$$c_t = c_{t-1} + \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$
$$h_t = \tanh c_t$$

# The Original LSTM Cell



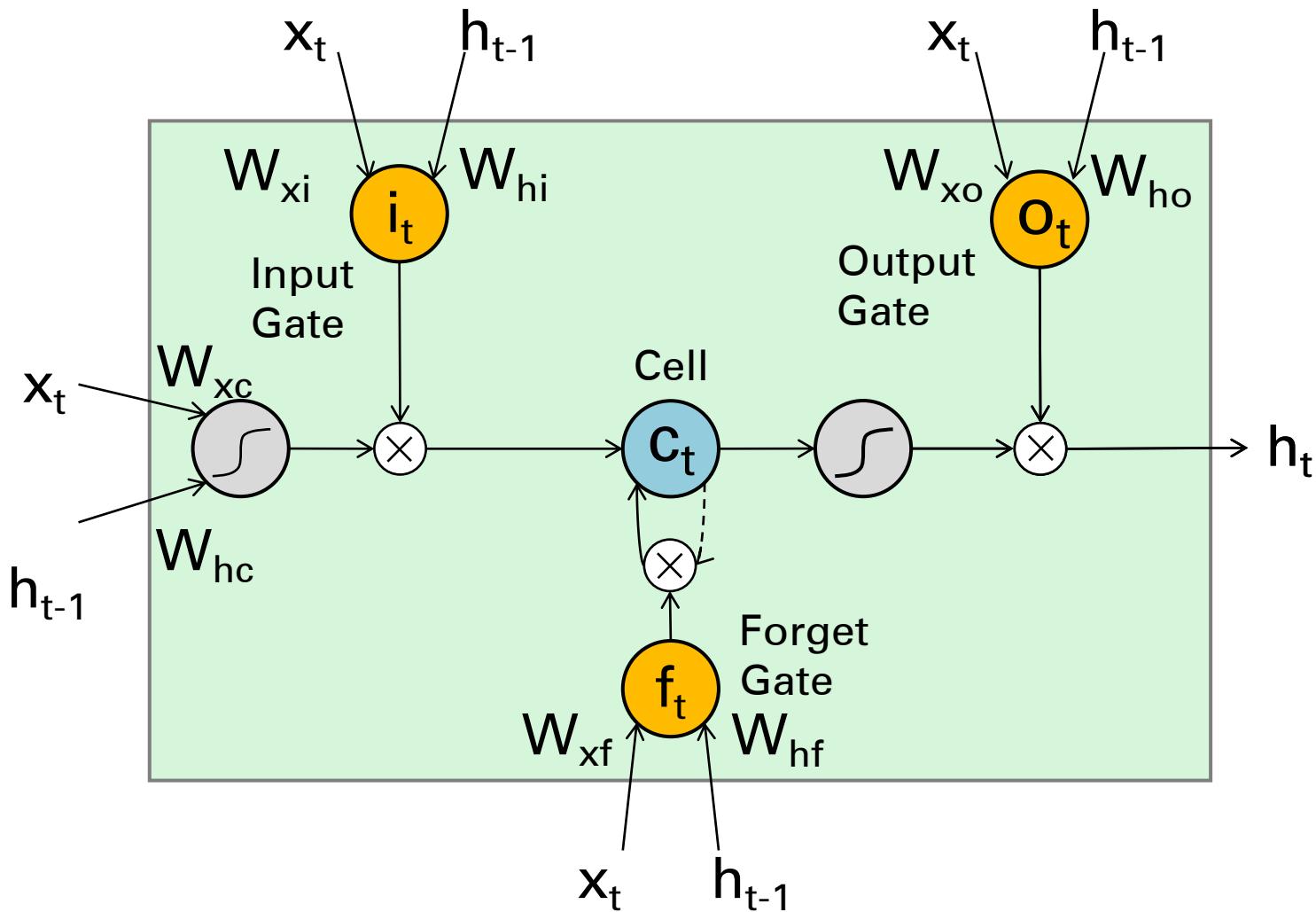
$$c_t = c_{t-1} + i_t \otimes \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

$$h_t = o_t \otimes \tanh c_t$$

$$i_t = \sigma \left( W_i \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_i \right)$$

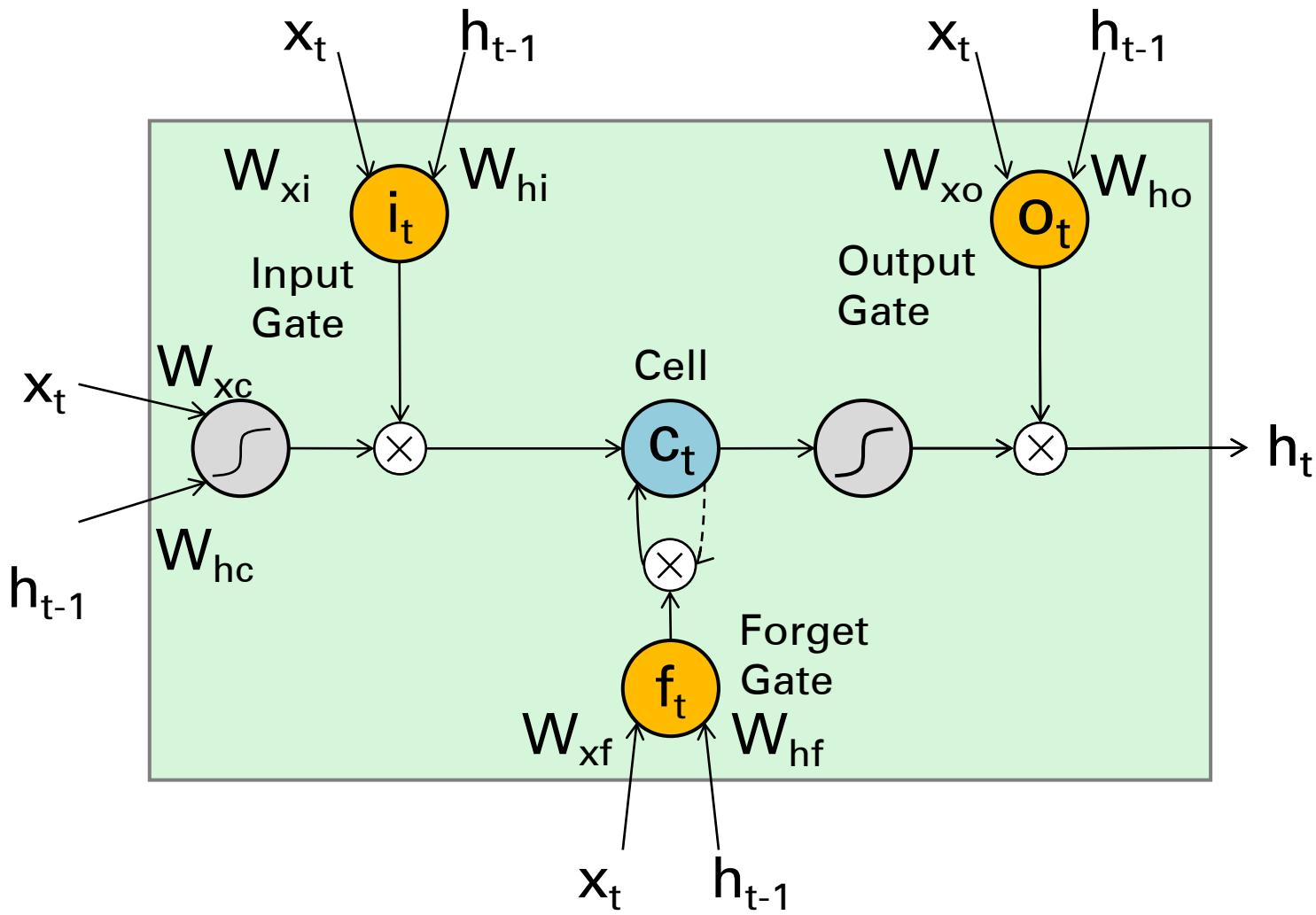
Similarly for  $o_t$

# The Popular LSTM Cell



$$i_t = \sigma \left( W_i \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_i \right)$$
$$c_t = f_t \otimes c_{t-1} + i_t \otimes \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$
$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$
$$h_t = o_t \otimes \tanh c_t$$

# The Popular LSTM Cell



$$i_t = \sigma \left( W_i \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_i \right)$$

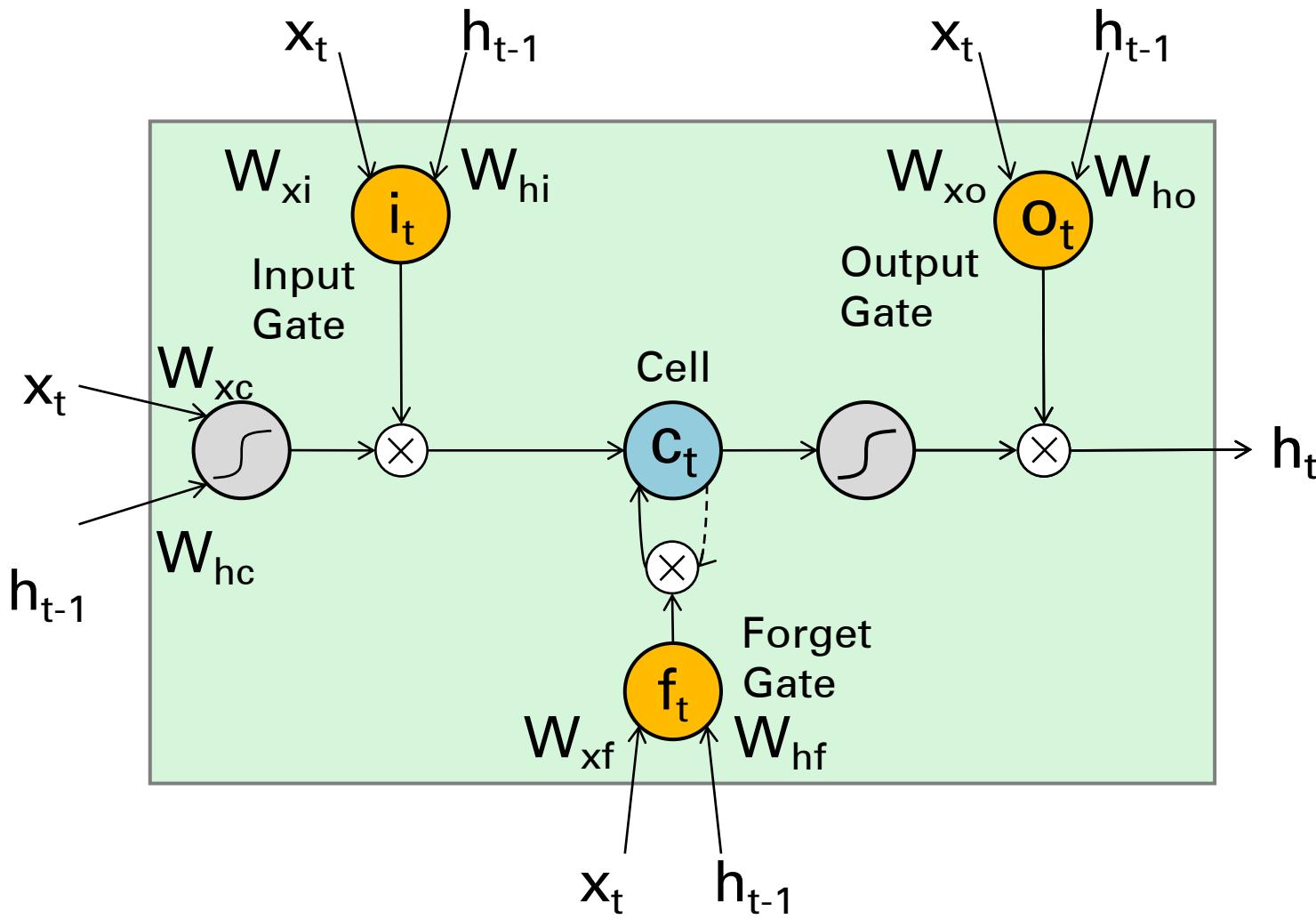
$$c_t = f_t \otimes c_{t-1} + i_t \otimes \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h_t = o_t \otimes \tanh c_t$$

**forget gate** decides what information is going to be thrown away from the cell state

# The Popular LSTM Cell



$$i_t = \sigma \left( W_i \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_i \right)$$

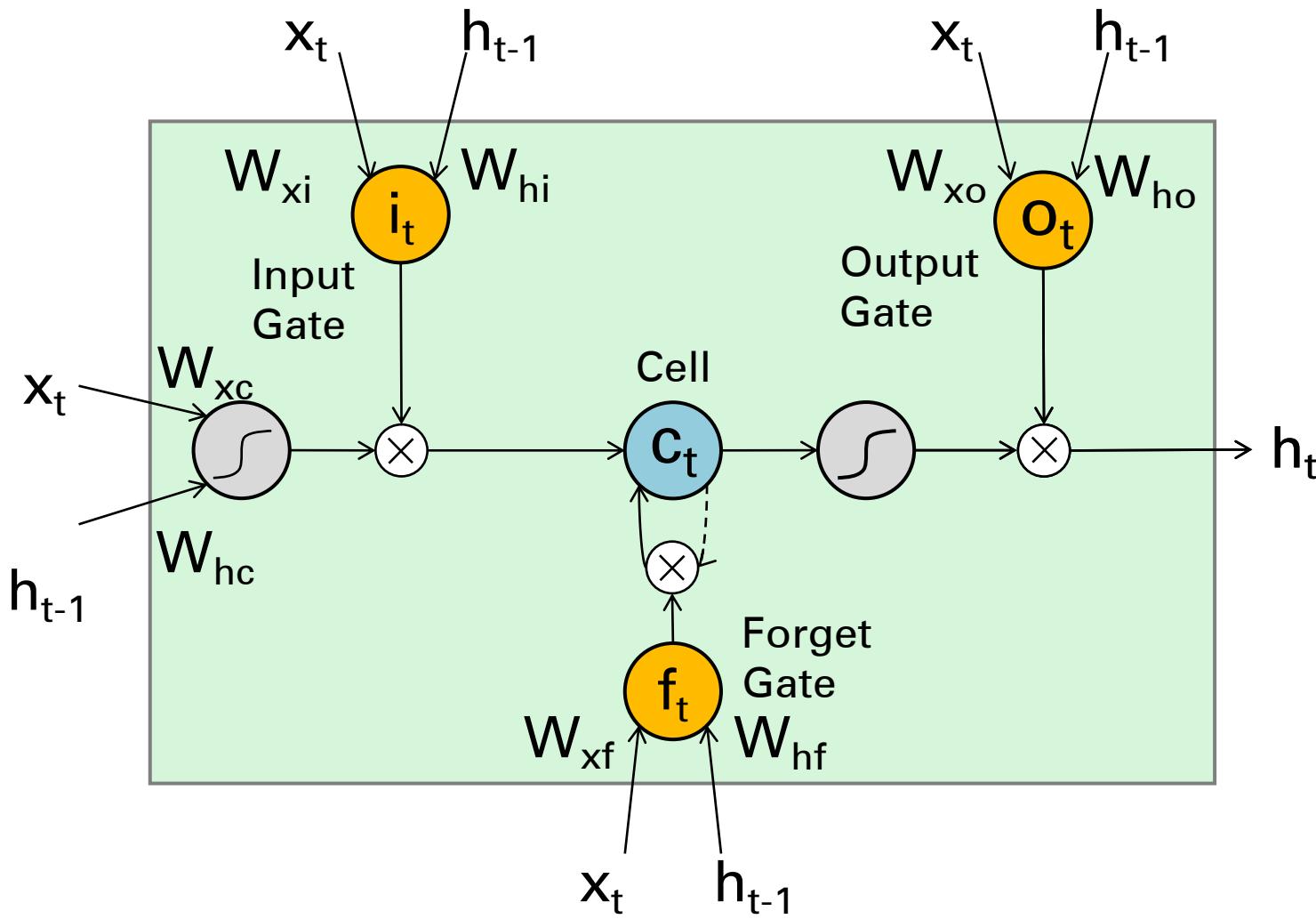
$$c_t = f_t \otimes c_{t-1} + i_t \otimes \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h_t = o_t \otimes \tanh c_t$$

**input gate** and **a tanh layer** decides what information is going to be stored in the cell state

# The Popular LSTM Cell



$$i_t = \sigma \left( W_i \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_i \right)$$

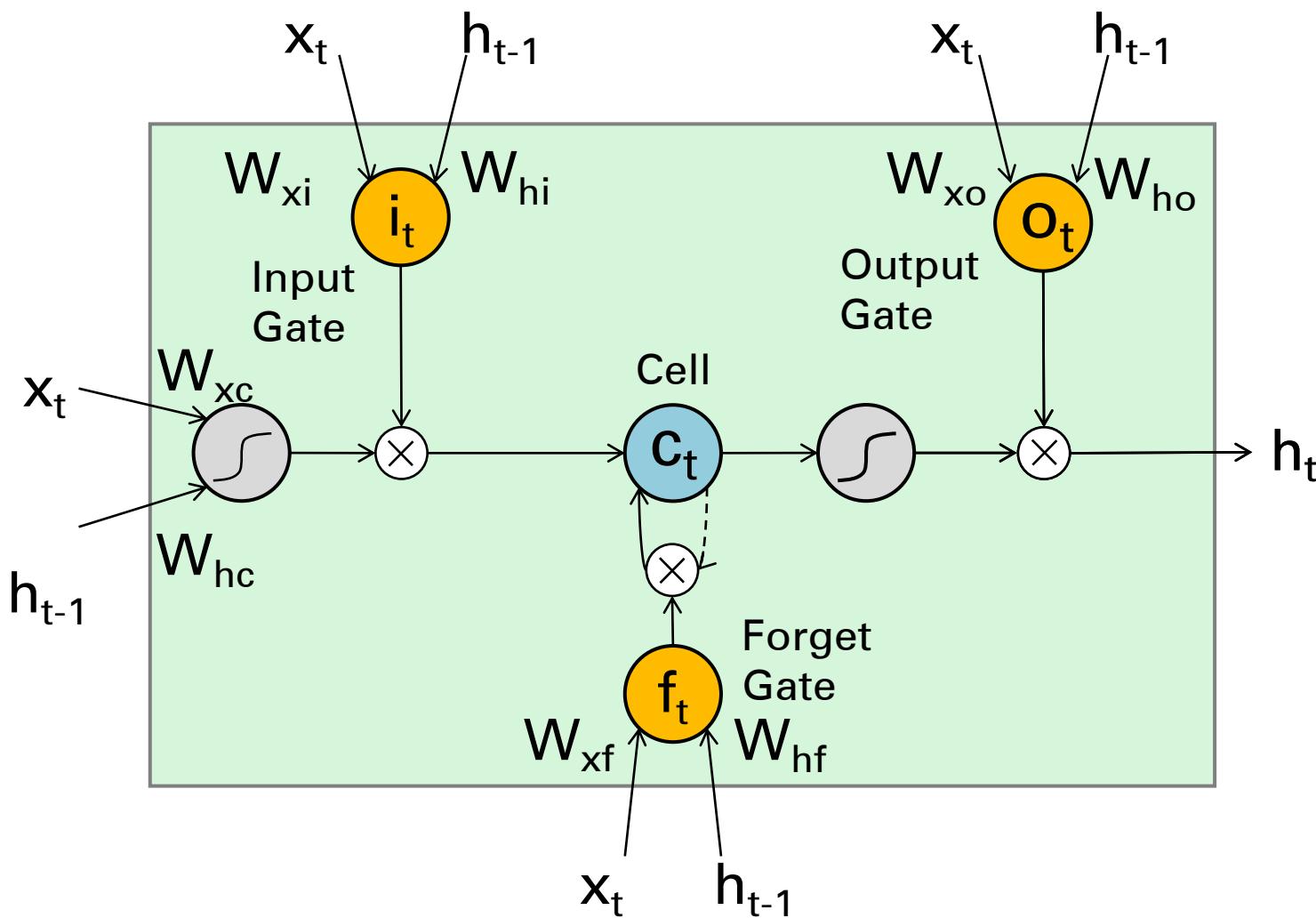
$$c_t = f_t \otimes c_{t-1} + i_t \otimes \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h_t = o_t \otimes \tanh c_t$$

Update the old cell state with the new one.

# The Popular LSTM Cell

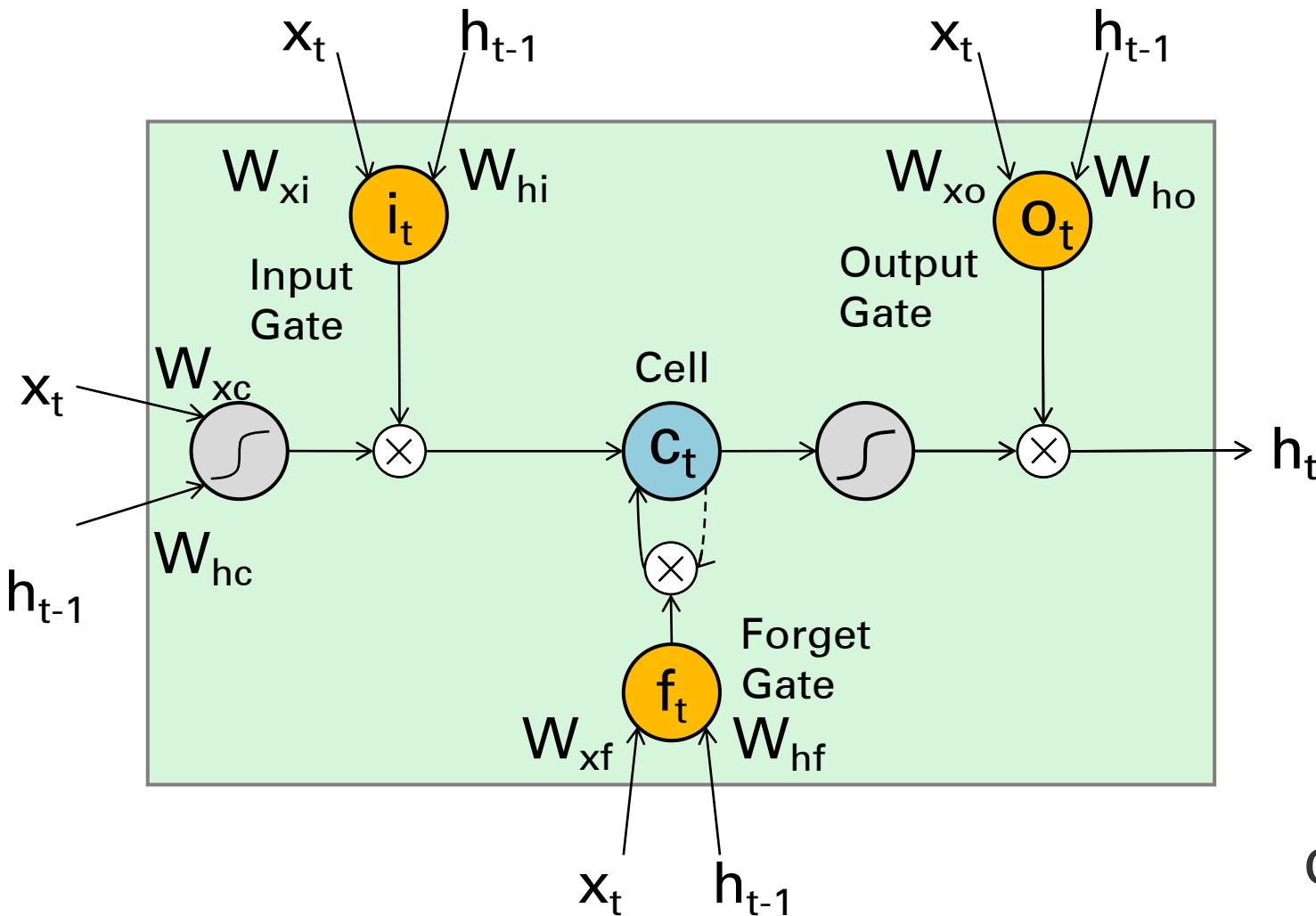


$$i_t = \sigma \left( W_i \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_i \right)$$

$$c_t = f_t \otimes c_{t-1} + i_t \otimes \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

input gate	forget gate	behavior
0	1	remember the previous value
1	1	add to the previous value
0	0	erase the value
1	0	overwrite the value

# The Popular LSTM Cell



$$i_t = \sigma \left( W_i \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_i \right)$$

$$c_t = f_t \otimes c_{t-1} + i_t \otimes \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h_t = o_t \otimes \tanh c_t$$

$$o_t = \sigma \left( W_o \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_o \right)$$

**Output gate** decides what is going to be outputted. The final output is based on cell state and output of sigmoid gate.

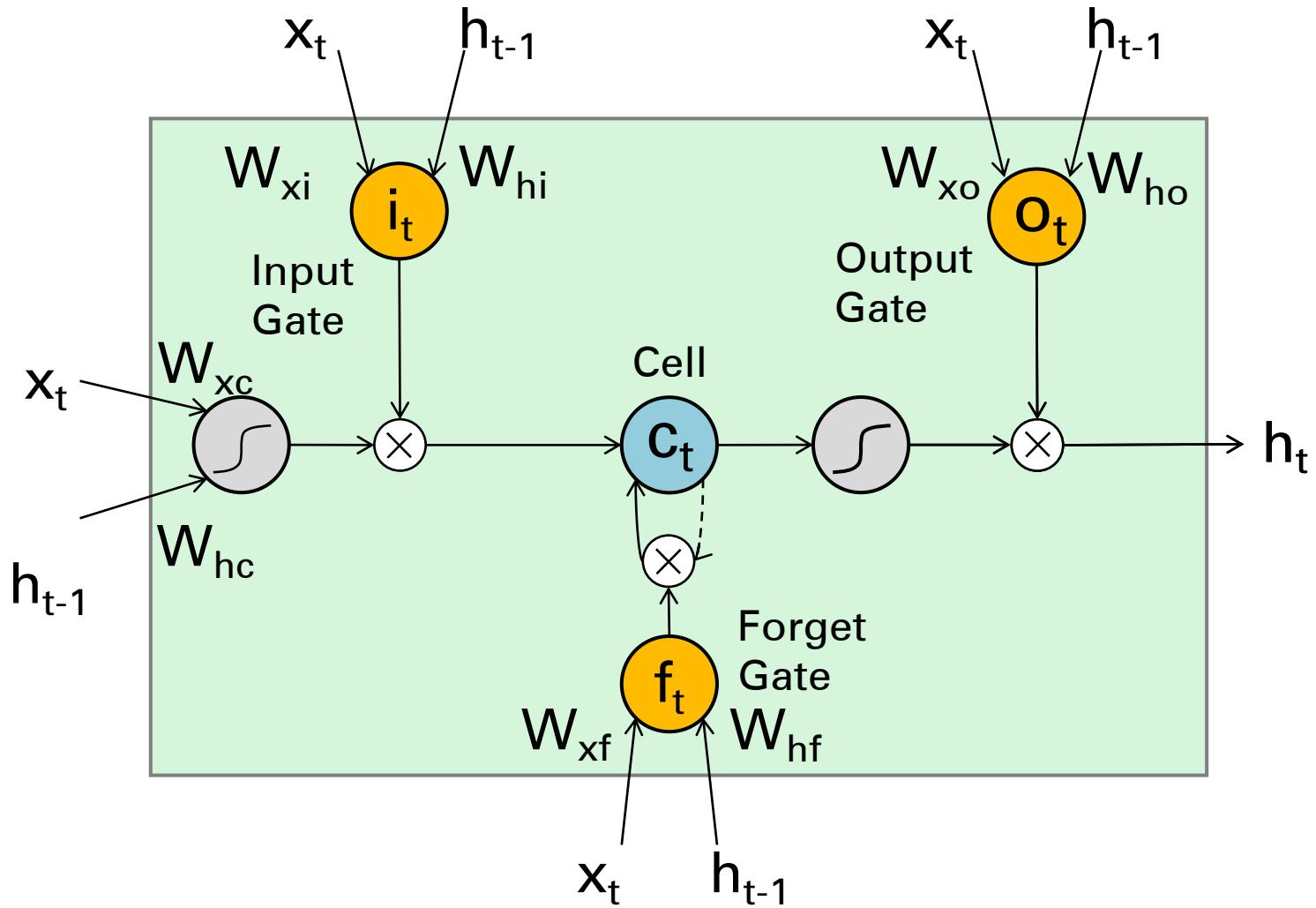
# LSTM – Forward/Backward

Illustrated LSTM Forward and Backward Pass

<http://arunmallya.github.io/writeups/nn/lstm/index.html>

# LSTM variants

# The Popular LSTM Cell



$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

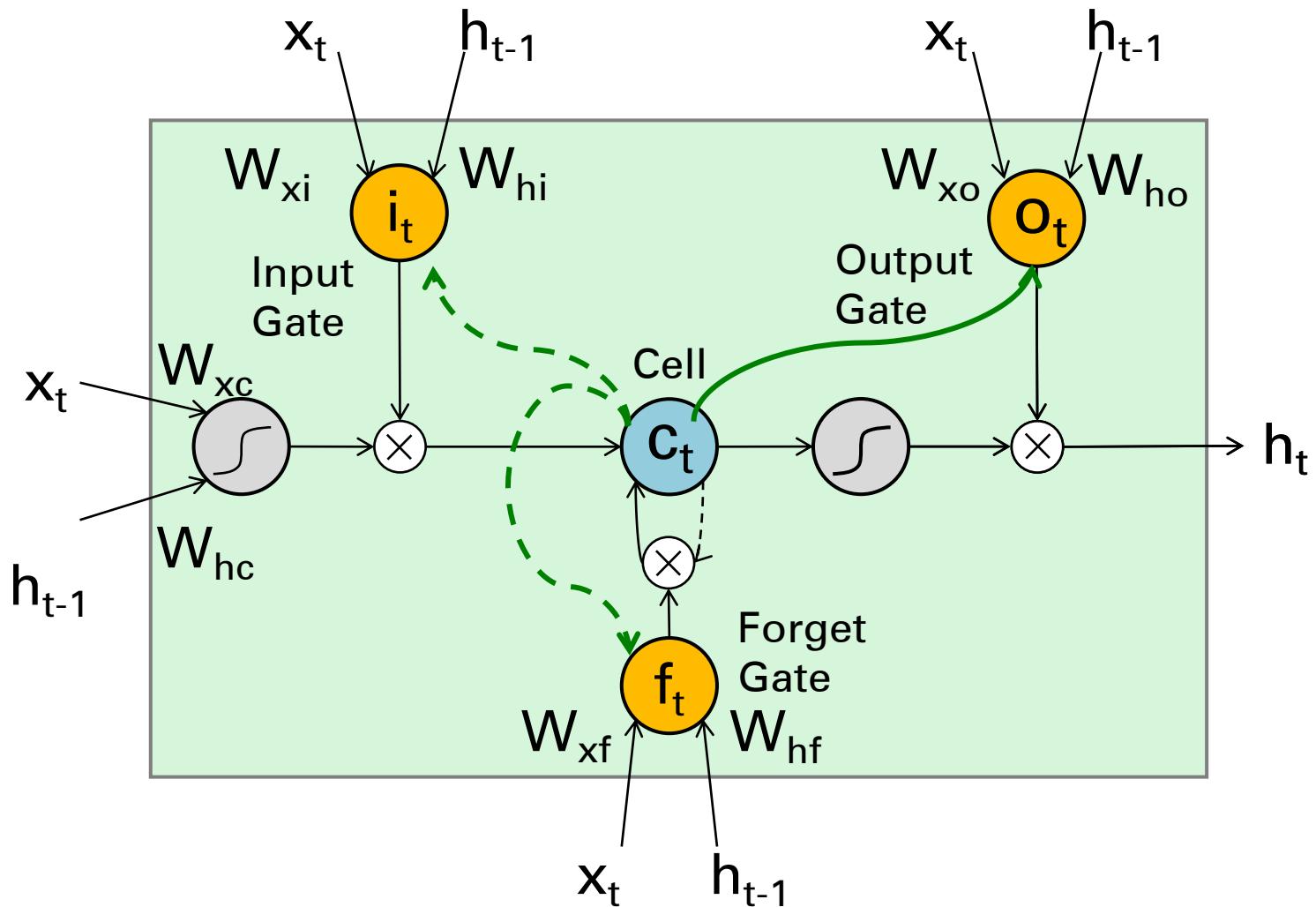
Similarly for  $i_t, o_t$

$$c_t = f_t \otimes c_{t-1} + i_t \otimes \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

$$h_t = o_t \otimes \tanh c_t$$

\* Dashed line indicates time-lag

# Extension I: Peephole LSTM



$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \\ C_{t-1} \end{pmatrix} + b_f \right)$$

Similarly for  $i_t, o_t$  (uses  $C_t$ )

$$c_t = f_t \otimes c_{t-1} + i_t \otimes \tanh W \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix}$$

$$h_t = o_t \otimes \tanh c_t$$

- Add **peephole connections**.
- All gate layers look at the cell state!

\* Dashed line indicates time-lag

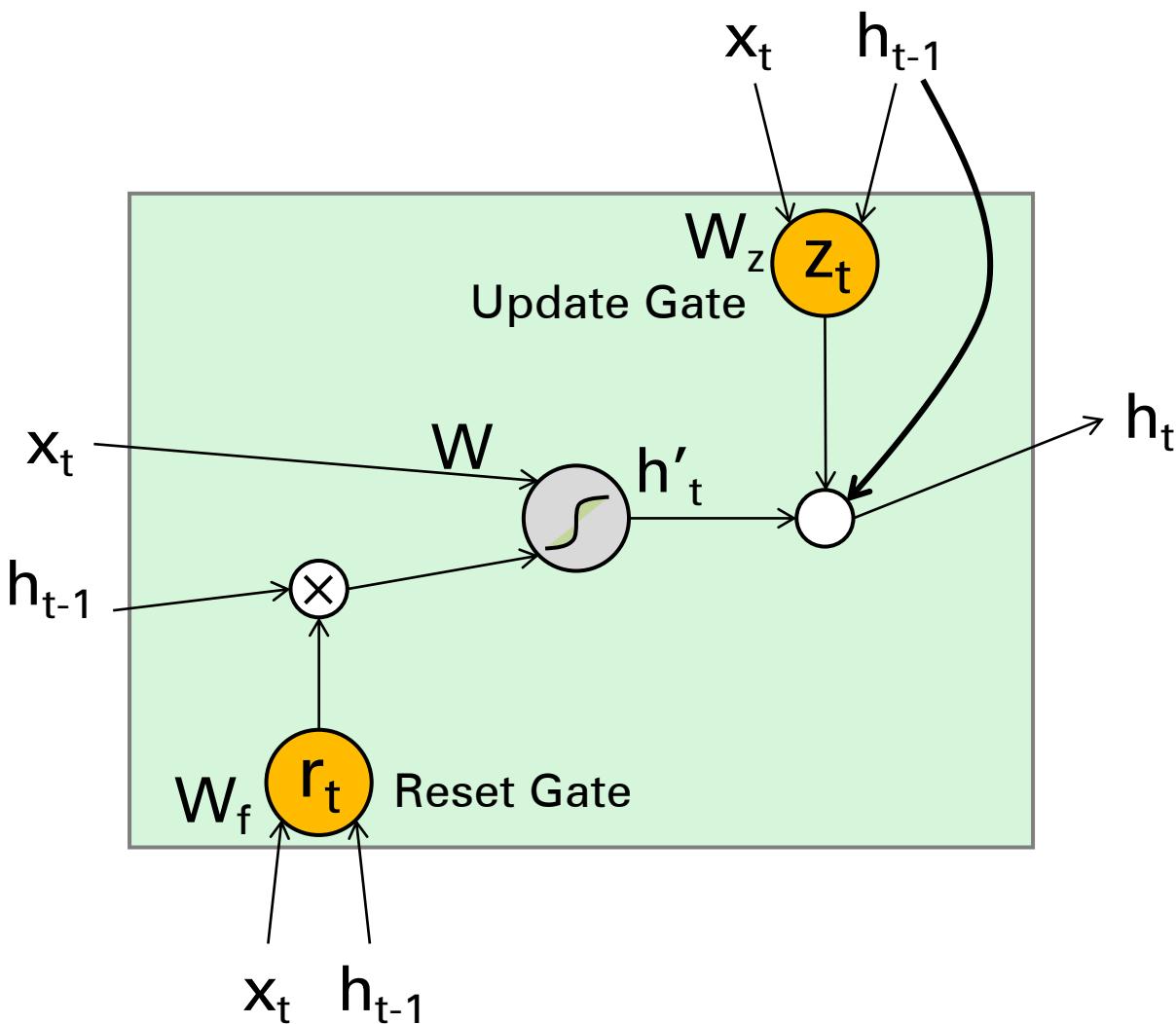
# Gated Recurrent Unit

# Gated Recurrent Unit (GRU)

- A very simplified version of the LSTM
  - Merges forget and input gate into a single ‘update’ gate
  - Merges cell and hidden state
- Has fewer parameters than an LSTM and has been shown to outperform LSTM on some tasks

[Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation \[Cho et al.,14\]](#)

# GRU



$$r_t = \sigma \left( W_r \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

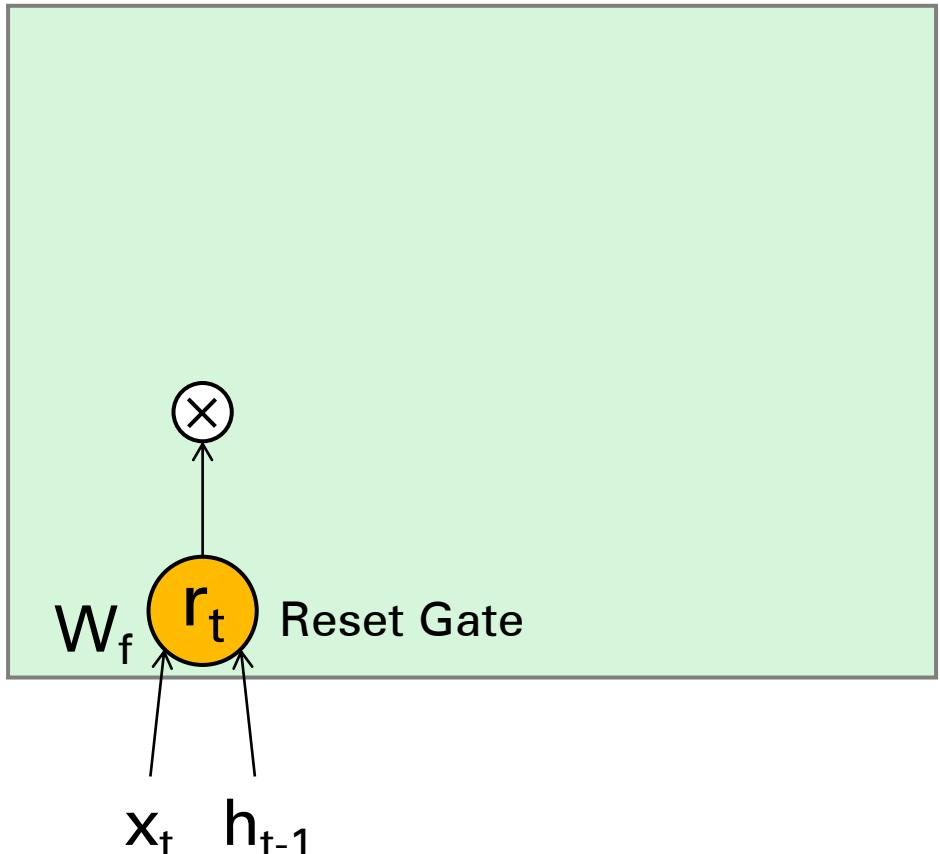
$$h'_t = \tanh W \begin{pmatrix} x_t \\ r_t \otimes h_{t-1} \end{pmatrix}$$

$$z_t = \sigma \left( W_z \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_z \right)$$

$$h_t = (1 - z_t) \otimes h_{t-1} + z_t \otimes h'_t$$

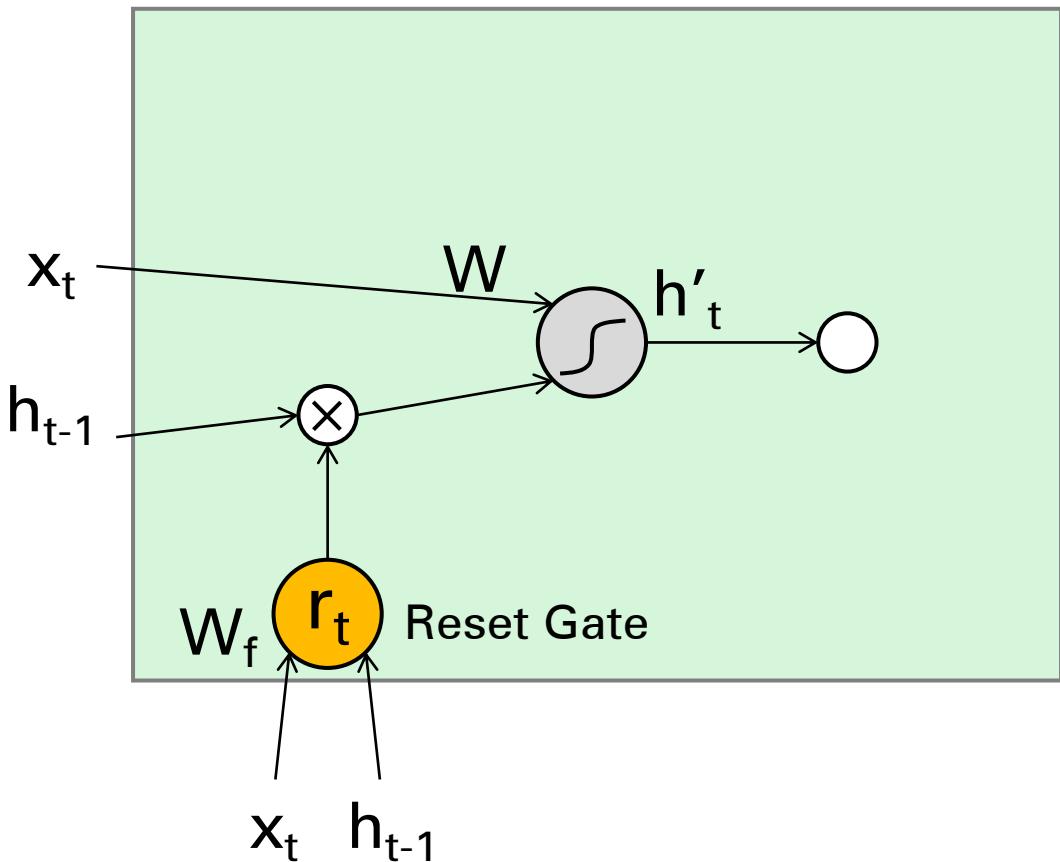
# GRU

$$r_t = \sigma \left( W_r \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$



computes a **reset gate** based on current input and hidden state

# GRU



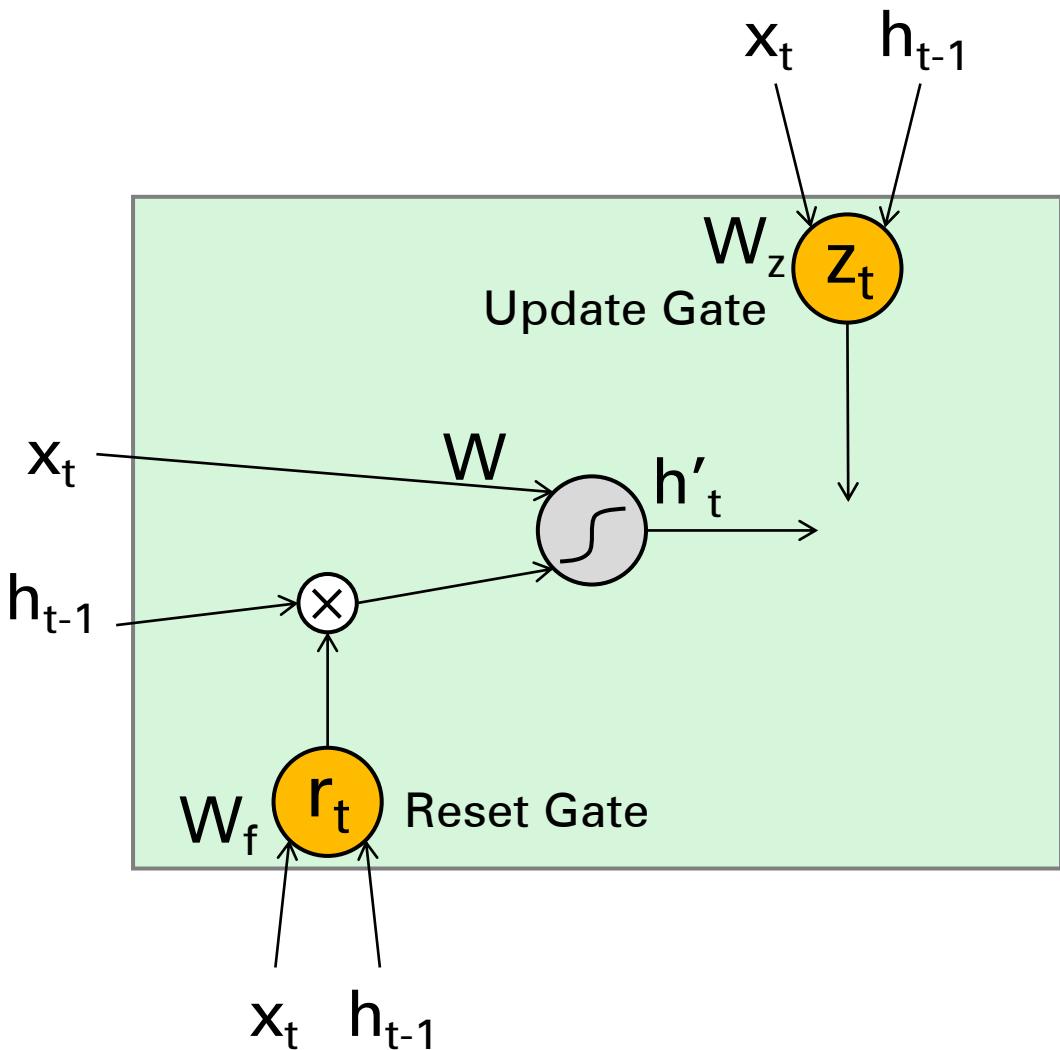
$$r_t = \sigma \left( W_r \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h'_t = \tanh W \begin{pmatrix} x_t \\ r_t \otimes h_{t-1} \end{pmatrix}$$

computes the **hidden state** based on current input and hidden state

if reset gate unit is  $\sim 0$ , then this ignores previous memory and only stores the new input information

# GRU



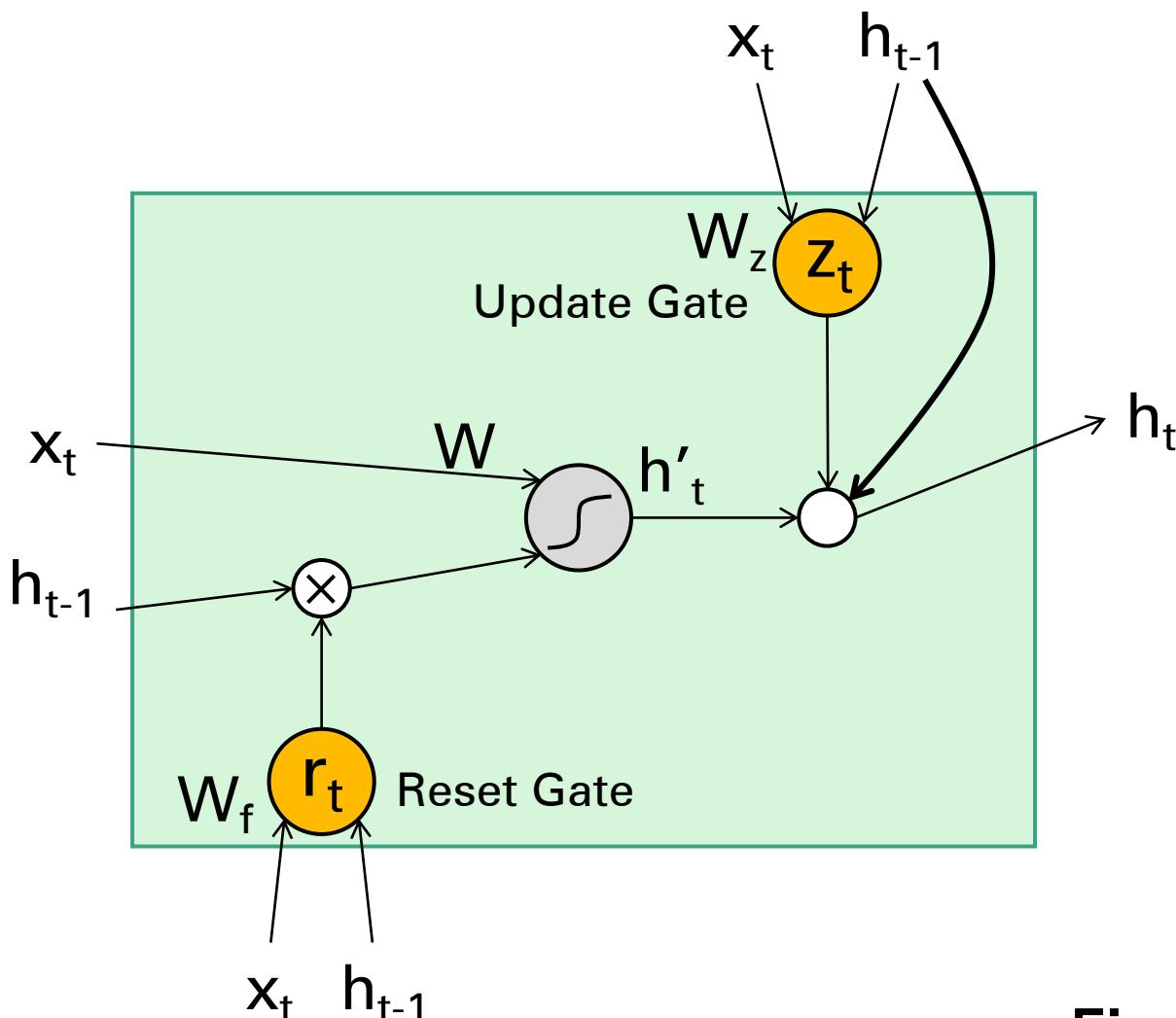
$$r_t = \sigma \left( W_r \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h'_t = \tanh W \begin{pmatrix} x_t \\ r_t \otimes h_{t-1} \end{pmatrix}$$

$$z_t = \sigma \left( W_z \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_z \right)$$

computes an **update gate** again based on current input and hidden state

# GRU



$$r_t = \sigma \left( W_r \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

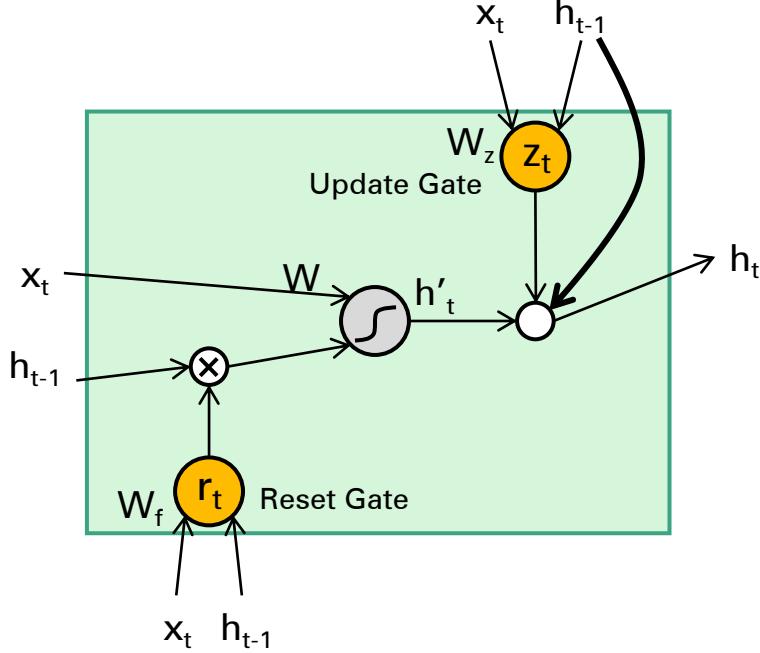
$$h'_t = \tanh W \begin{pmatrix} x_t \\ r_t \otimes h_{t-1} \end{pmatrix}$$

$$z_t = \sigma \left( W_z \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_z \right)$$

$$h_t = (1 - z_t) \otimes h_{t-1} + z_t \otimes h'_t$$

**Final memory** at timestep  $t$  combines both current and previous timesteps

# GRU Intuition



$$r_t = \sigma \left( W_r \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_r \right)$$

$$h'_t = \tanh W \begin{pmatrix} x_t \\ r_t \otimes h_{t-1} \end{pmatrix}$$

$$z_t = \sigma \left( W_z \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_z \right)$$

$$h_t = (1 - z_t) \otimes h_{t-1} + z_t \otimes h'_t$$

- If reset is close to 0, ignore previous hidden state
  - Allows model to drop information that is irrelevant in the future
- Update gate  $z$  controls how much of past state should matter now.
  - If  $z$  close to 1, then we can copy information in that unit through many time steps! **Less vanishing gradient!**
- Units with short-term dependencies often have reset gates very active

# LSTMs and GRUs

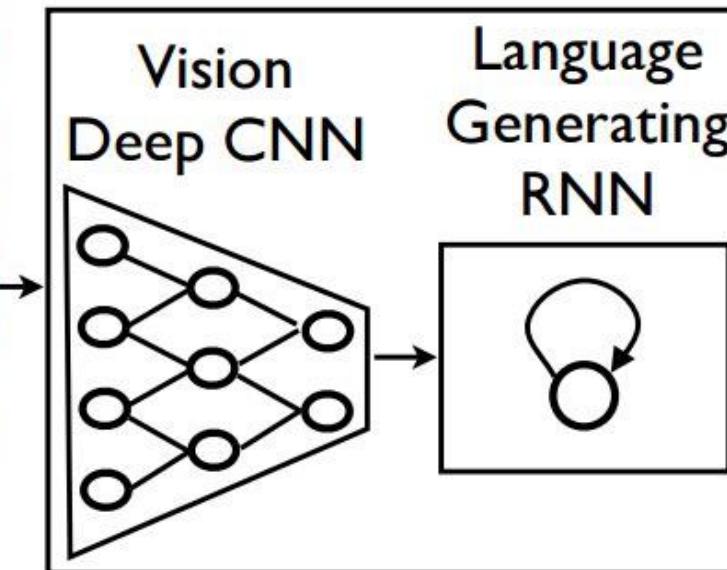
## Good

- Careful initialization and optimization of vanilla RNNs can enable them to learn long(ish) dependencies, but gated additive cells, like the LSTM and GRU, often just work.

## Bad

- LSTMs and GRUs have considerably more parameters and computation per memory cell than a vanilla RNN, as such they have less memory capacity per parameter\*

# Image Captioning



**A group of people shopping at an outdoor market.**

**There are many vegetables at the fruit stand.**

[Example above from: Vinyals, Toshev, Bengio, Erhan, CVPR 2015, <https://arxiv.org/abs/1411.4555>]

# Recipe for deep learning in a new domain

1. Transform your data into numbers (e.g., a vector)
2. Transform your goal into a numerical measure (objective function)
3. #1 and #2 specify the “learning problem”
4. Use a generic optimizer (SGD) and an appropriate architecture (e.g., CNN or RNN) to solve the learning problem

# How to represent words as numbers?

One-hot vector

Training data

x	y
{  ,	"Fish"
{  ,	"Grizzly"
{  ,	"Chameleon"

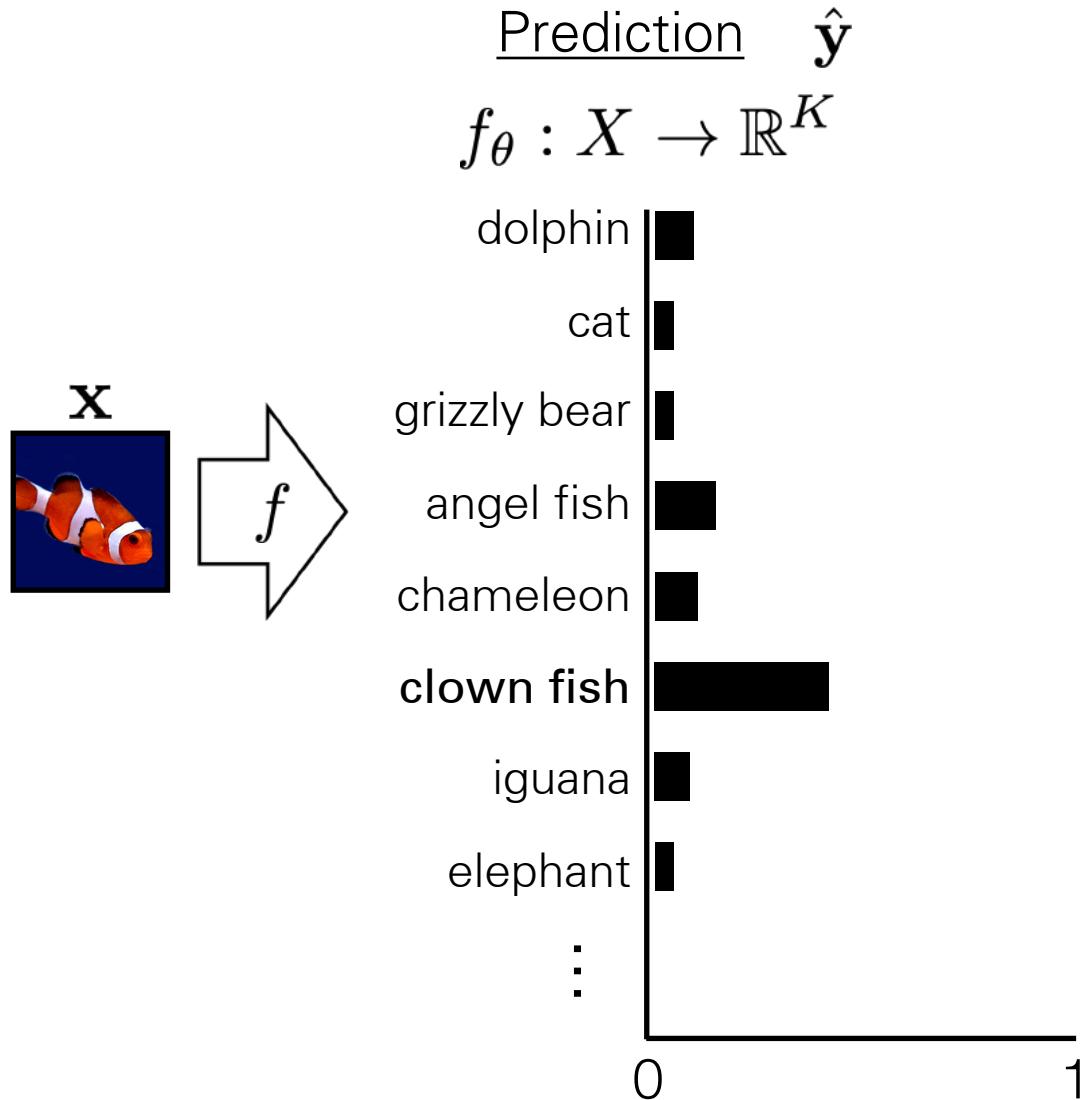
Training data

x	y
{  ,	1
{  ,	2
{  ,	3

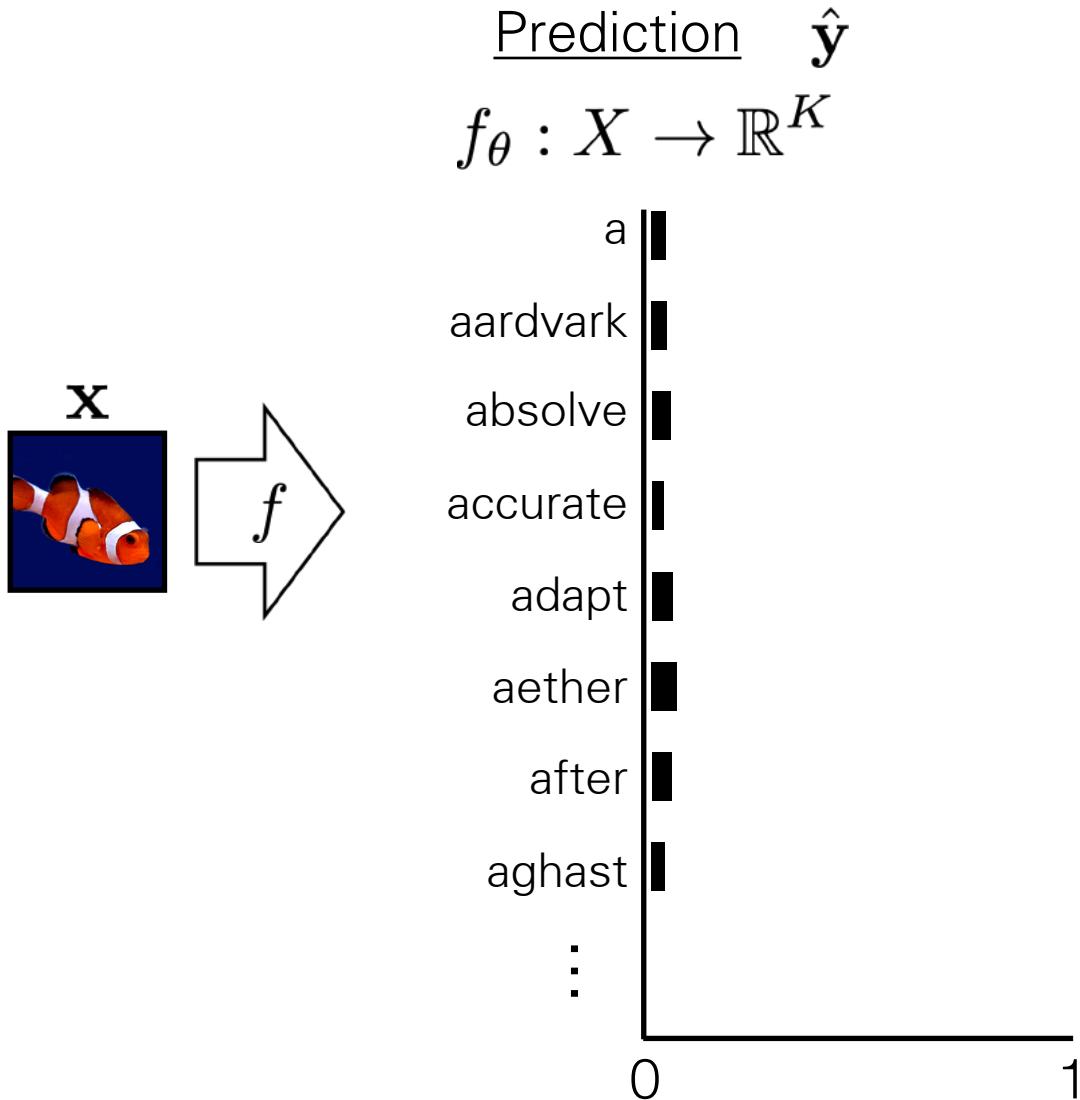
Training data

x	y
{  ,	[0,0,1]
{  ,	[0,1,0]
{  ,	[1,0,0]

# How to represent words as numbers?

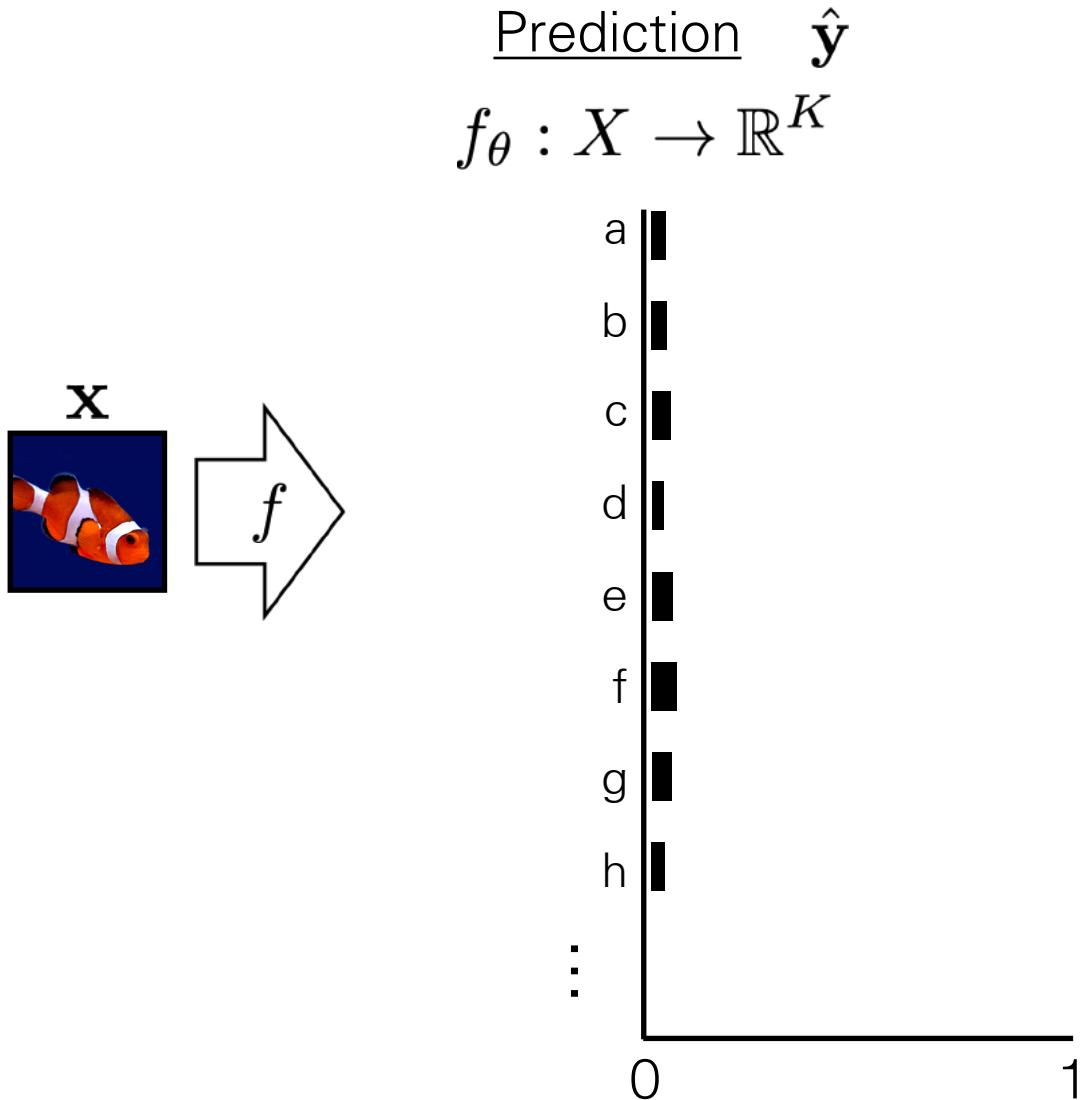


# How to represent words as numbers?

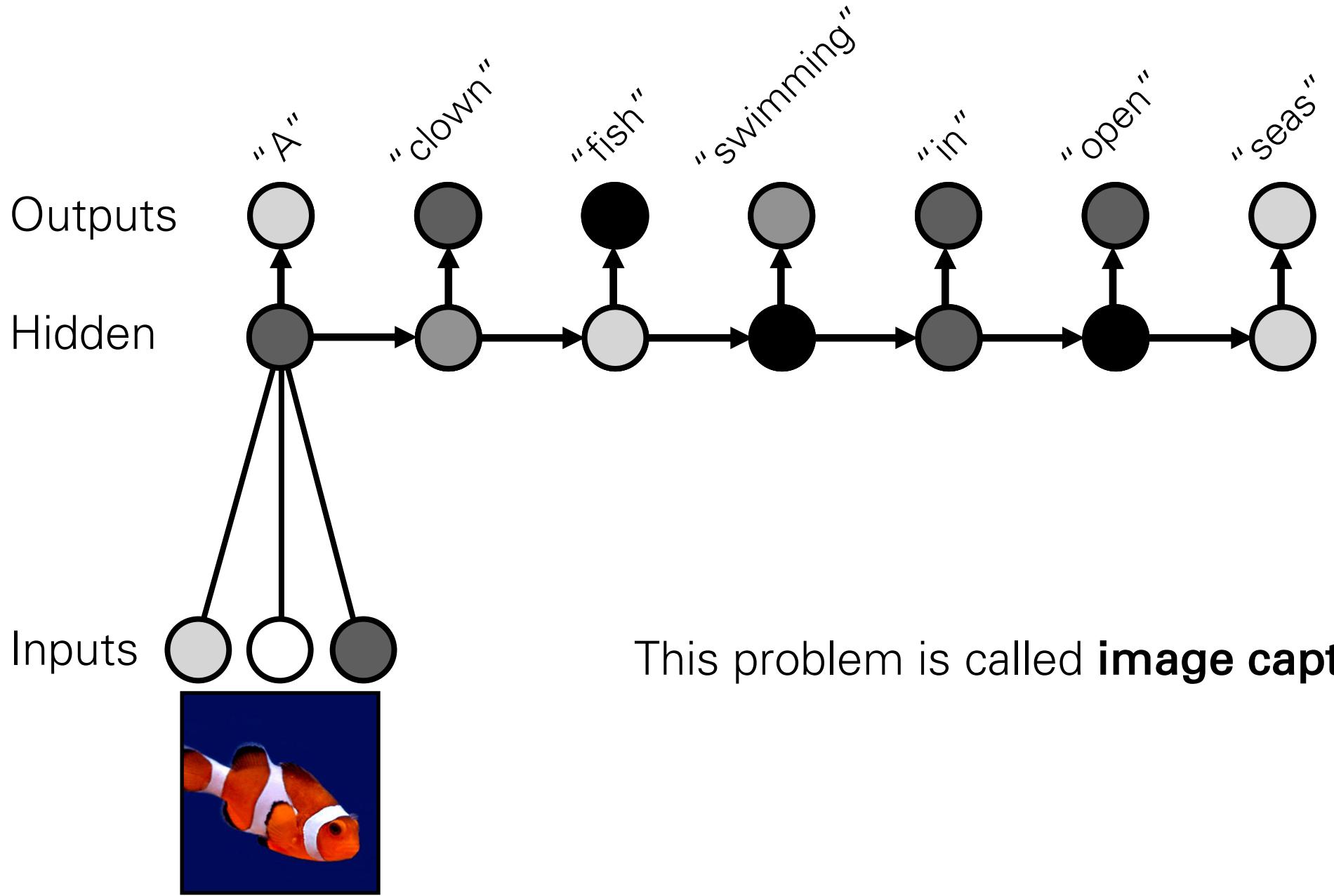


Rather than having just a handful of possible object classes, we can represent all words in a large vocabulary using a very large  $K$  (e.g.,  $K=100,000$ ).

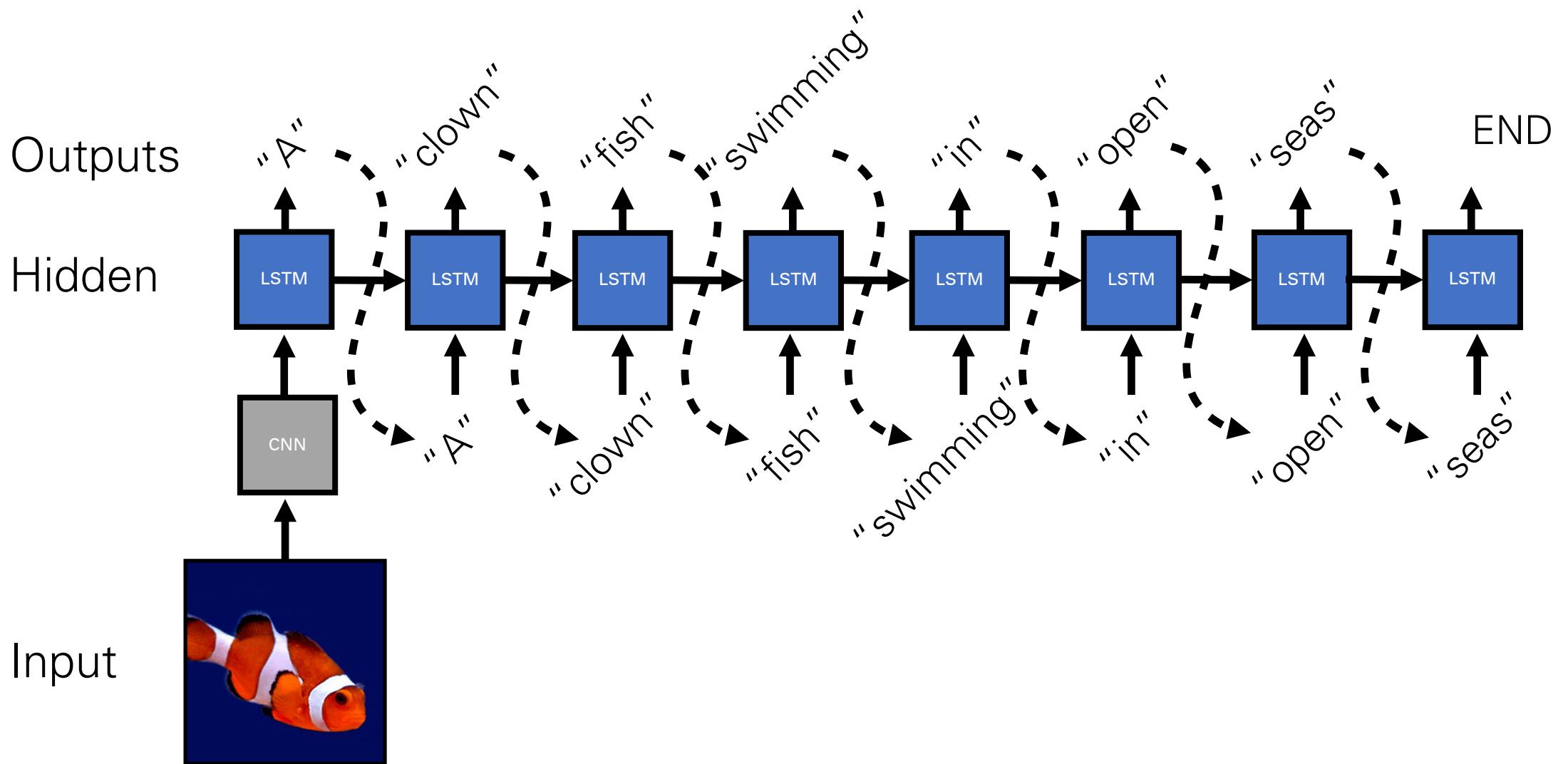
# How to represent words as numbers?



Or, represent each character as a class (e.g.,  $K=26$  for English letters), and represent words as a sequence of characters.



This problem is called **image captioning**

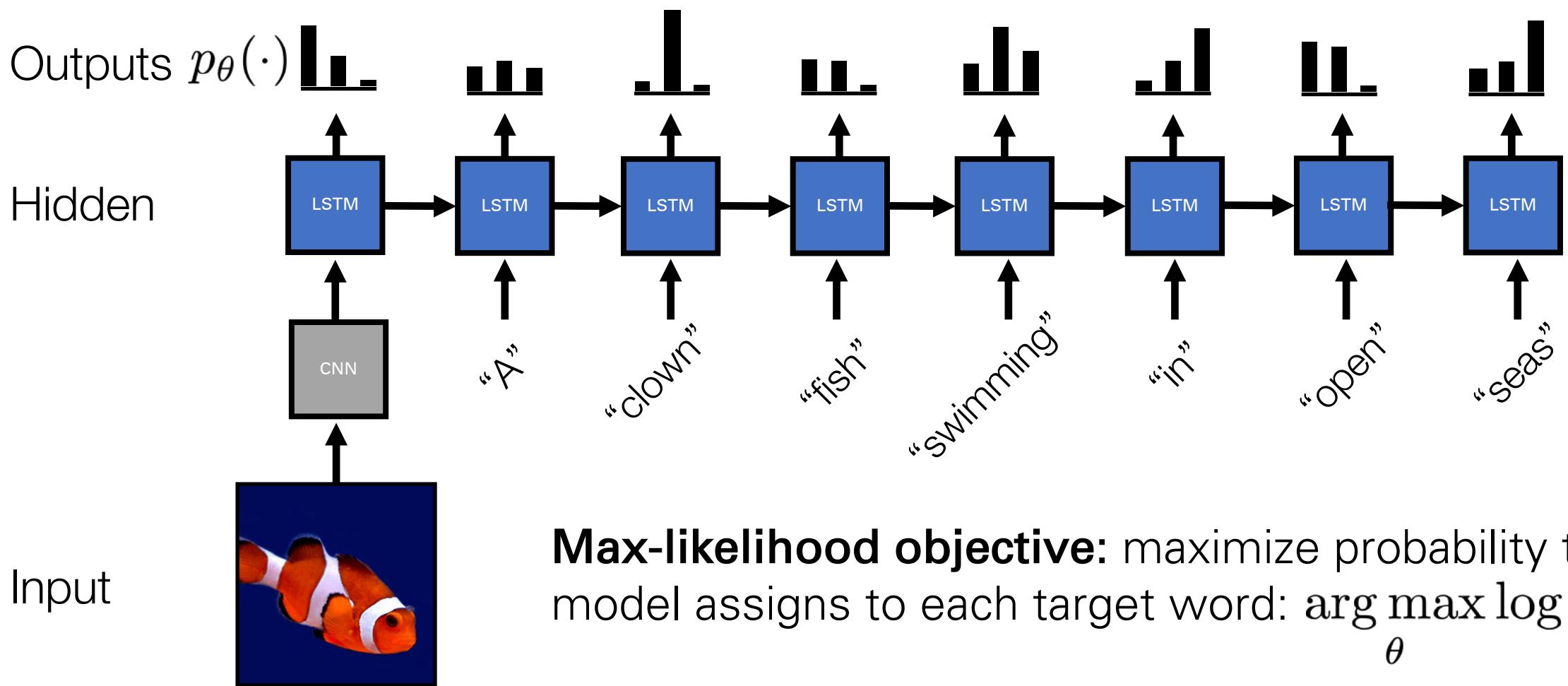


# Training

Targets  $y$

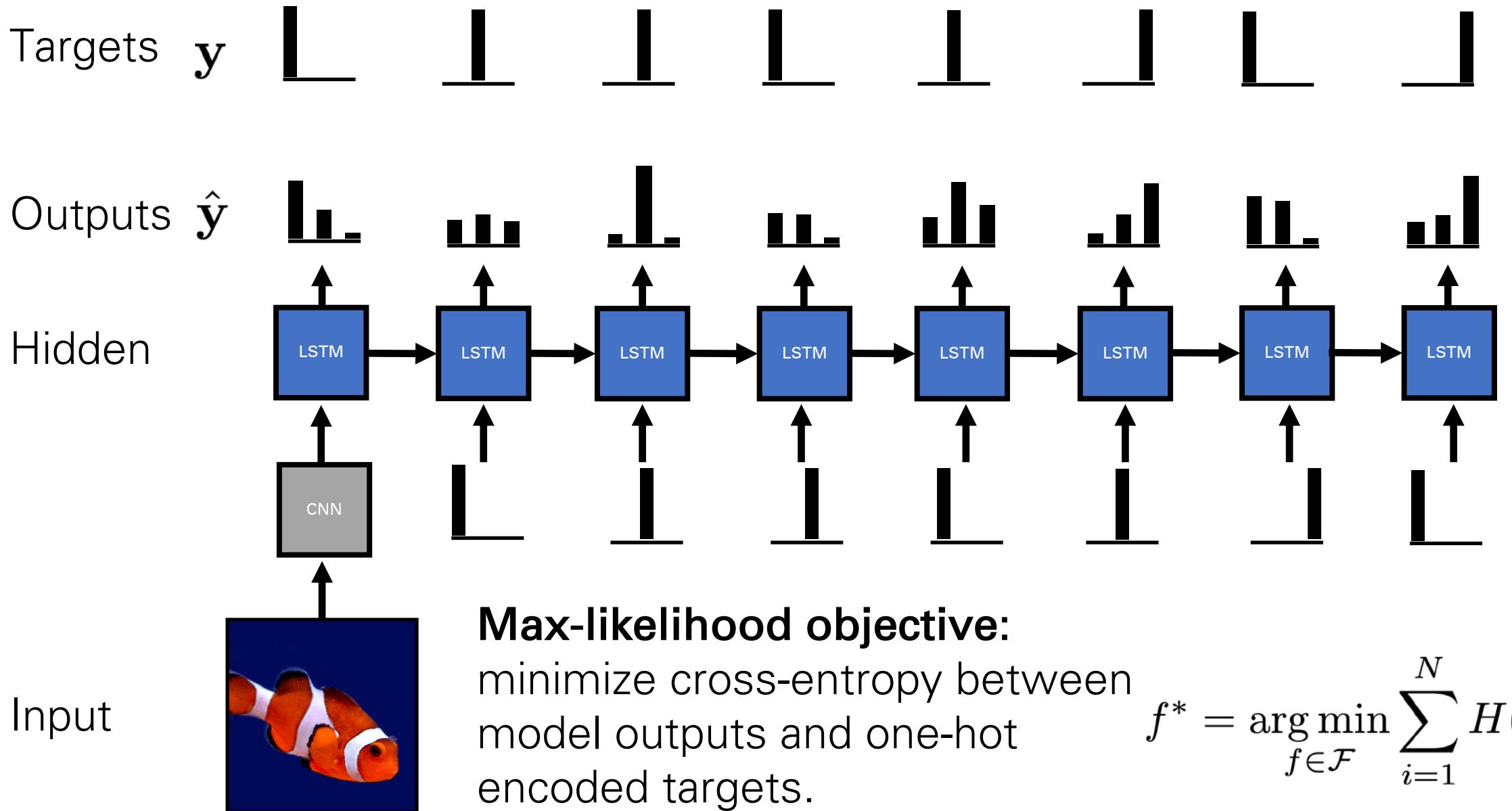
“A”, “clown”, “fish”, “Swimming”, “in”, “open”, “seas”, END

Outputs  $p_{\theta}(\cdot)$



**Max-likelihood objective:** maximize probability the model assigns to each target word:  $\arg \max_{\theta} \log p_{\theta}(y)$

# Training



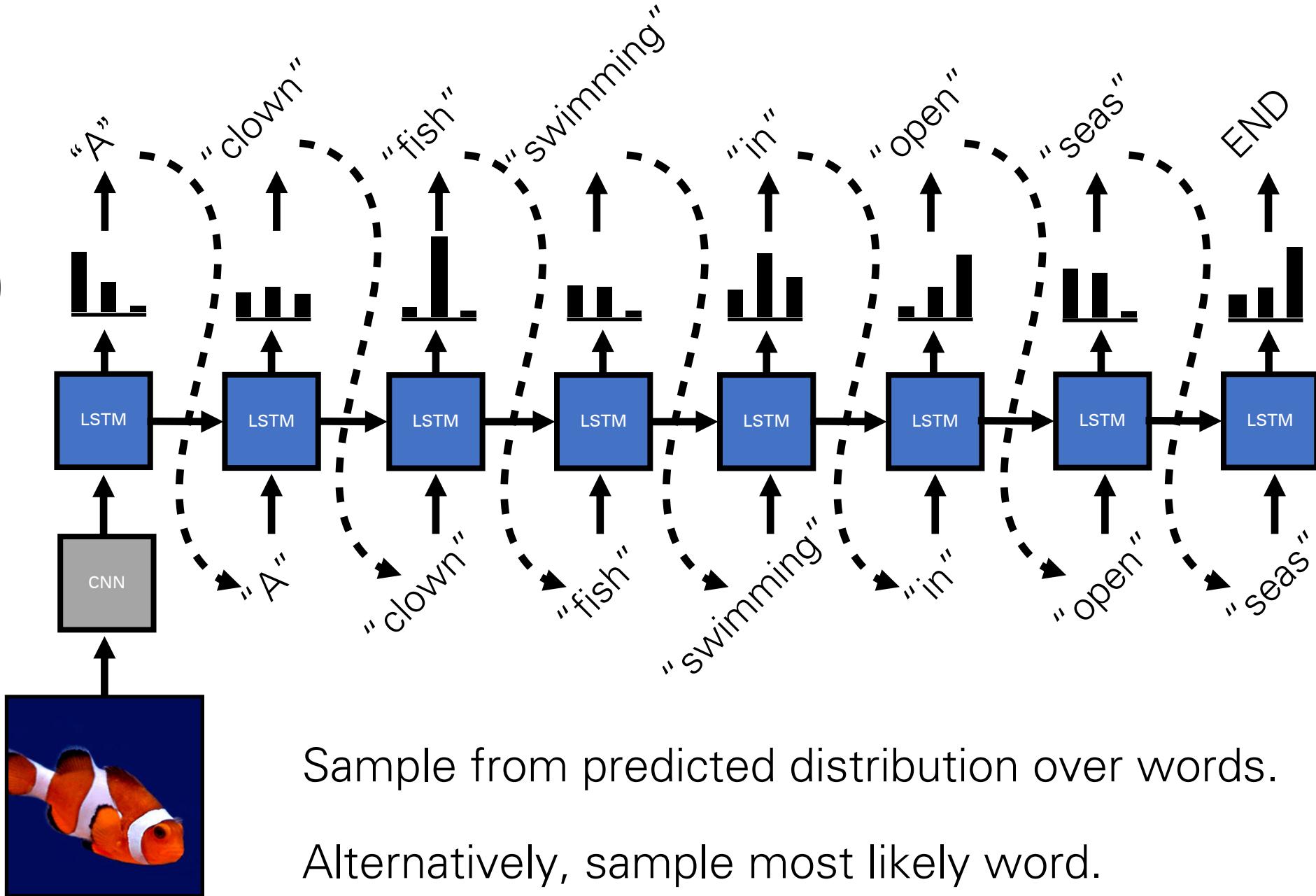
# Testing

Samples

Outputs  $p_\theta(\cdot)$

Hidden

Input



Sample from predicted distribution over words.

Alternatively, sample most likely word.

A person riding a motorcycle on a dirt road.



A group of young people playing a game of frisbee.



A herd of elephants walking across a dry grass field.



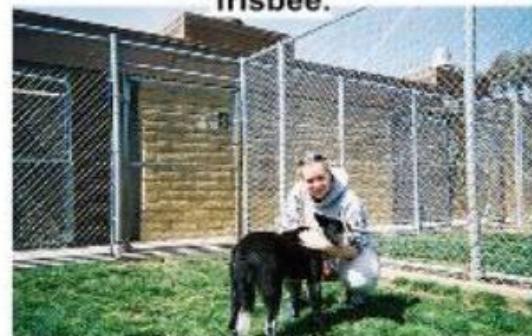
Two dogs play in the grass.



A skateboarder does a trick on a ramp.



A dog is jumping to catch a frisbee.



Two hockey players are fighting over the puck.



A little girl in a pink hat is blowing bubbles.



A refrigerator filled with lots of food and drinks.



A close up of a cat laying on a couch.



A red motorcycle parked on the side of the road.



A yellow school bus parked in a parking lot.



Describes without errors

Describes with minor errors

Somewhat related to the image

Unrelated to the image

# **Next Lecture:**

# **Attention and Transformers**