# COMP547

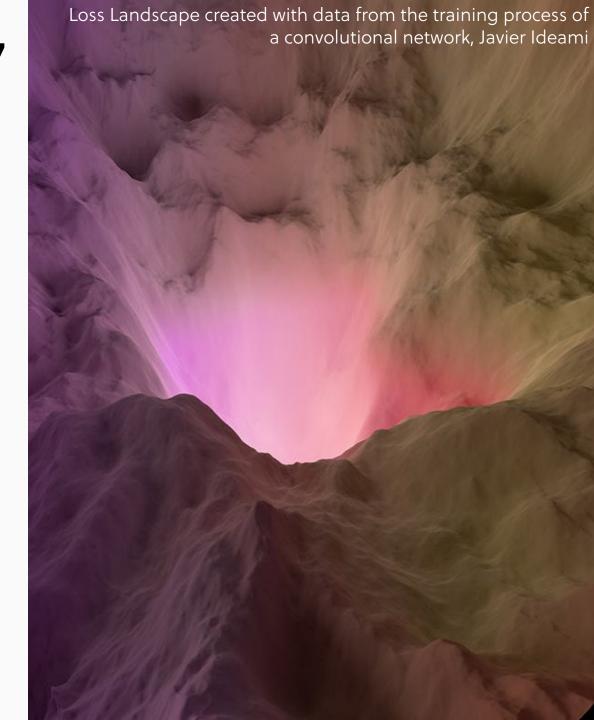
DEEP UNSUPERVISED LEARNING
Lecture #3 – Neural Networks Basics II:
Sequential Processing with NNs



Aykut Erdem // Koç University // Spring 2025

## Previously on COMP547

- deep learning
- computation in a neural net
- optimization
- backpropagation
- training tricks
- convolutional neural networks



#### Lecture overview

- sequence modeling
- recurrent neural networks (RNNs)
- how to train RNNs
- long short-term memory (LSTM)
- gated recurrent unit (GRU)
- sequence to sequence modeling

Disclaimer: Much of the material and slides for this lecture were borrowed from

- —Bill Freeman, Antonio Torralba and Phillip Isola's MIT 6.869 class
- —Fei-Fei Li, Andrej Karpathy and Justin Johnson's CS231n class
- —Arun Mallya's tutorial on Recurrent Neural Networks

#### Sequences



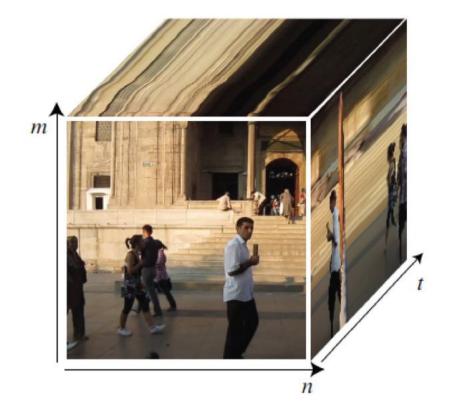
time

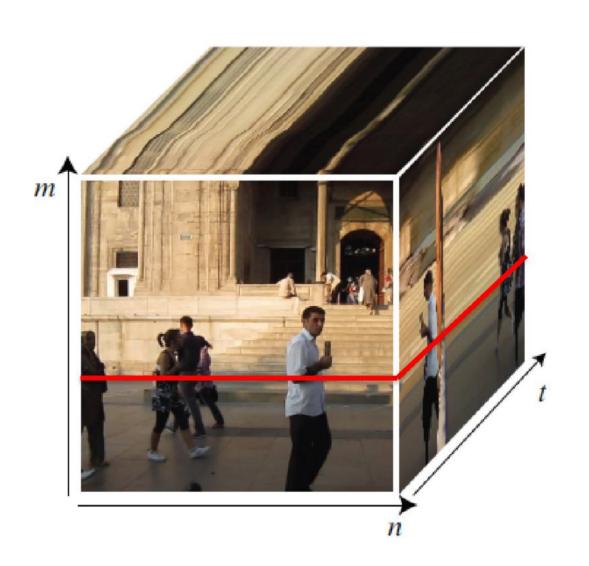
"An", "evening", "stroll", "through", "a", "city", "square"

time



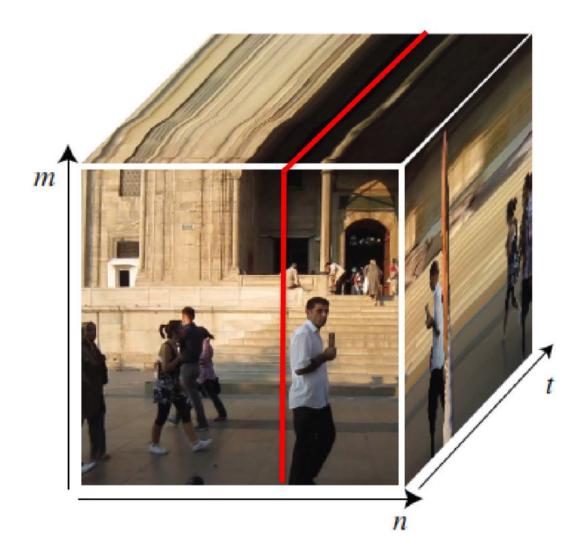




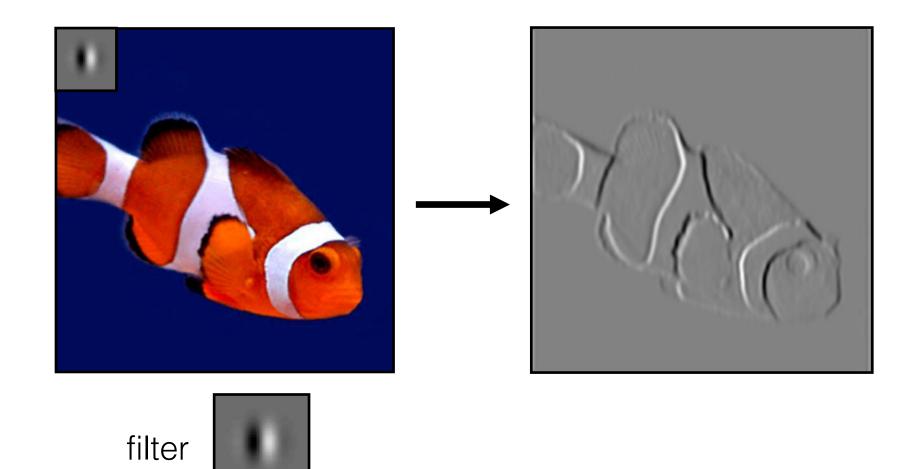




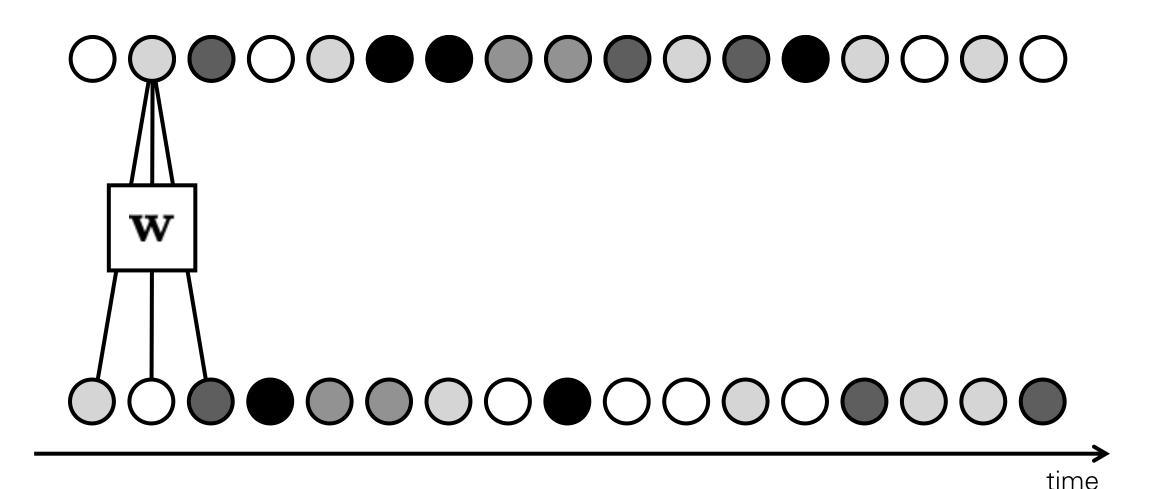
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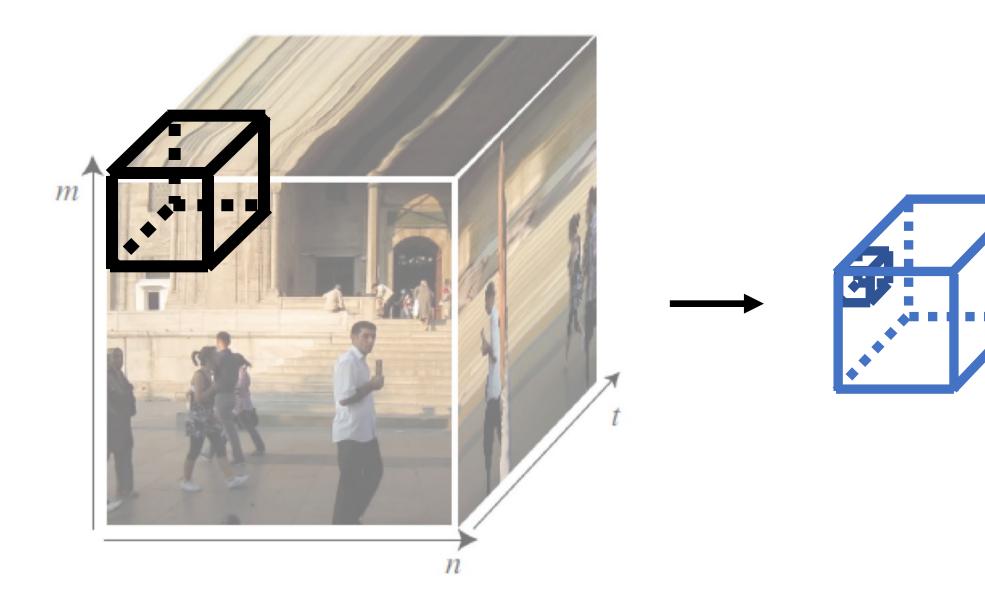


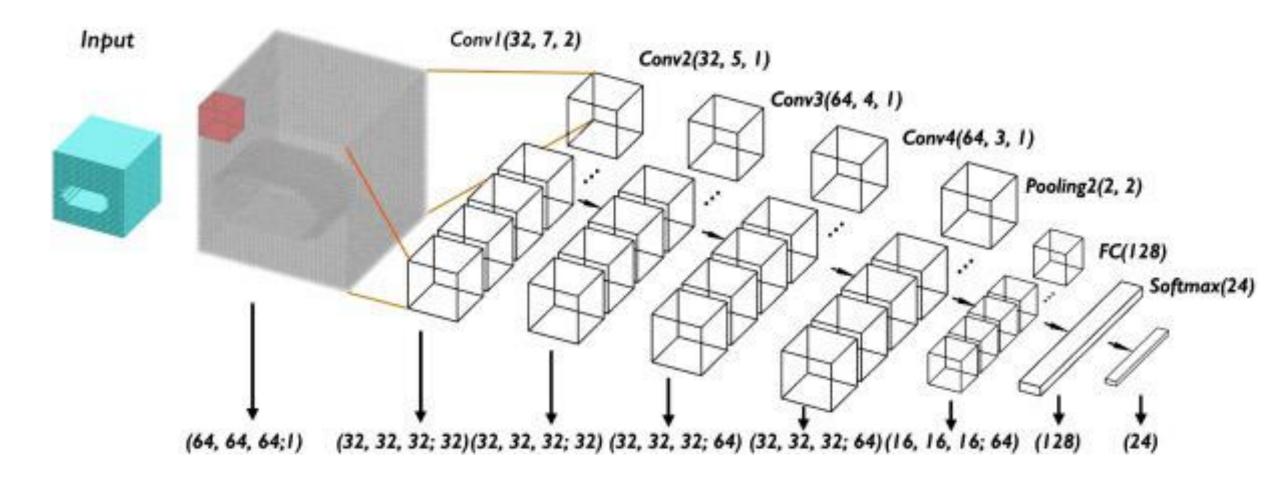




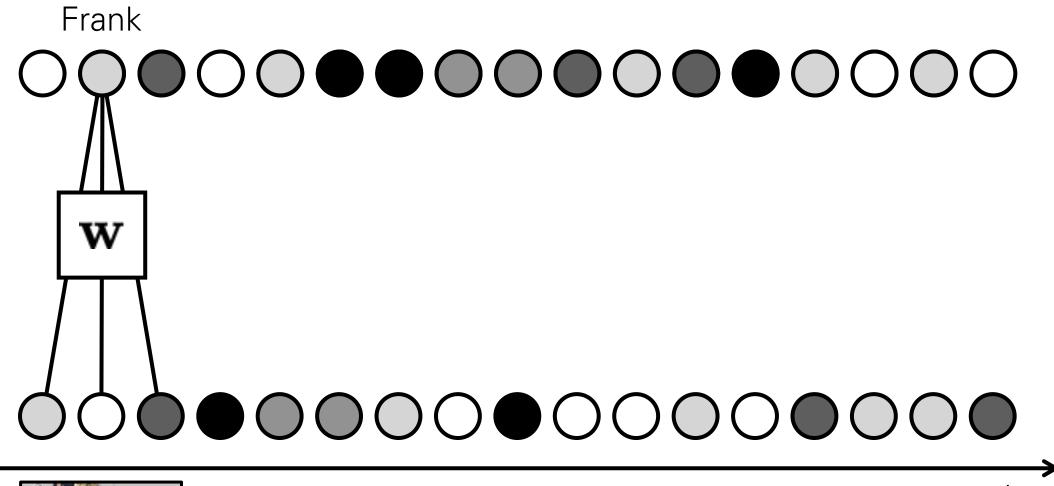
#### Convolutions in time



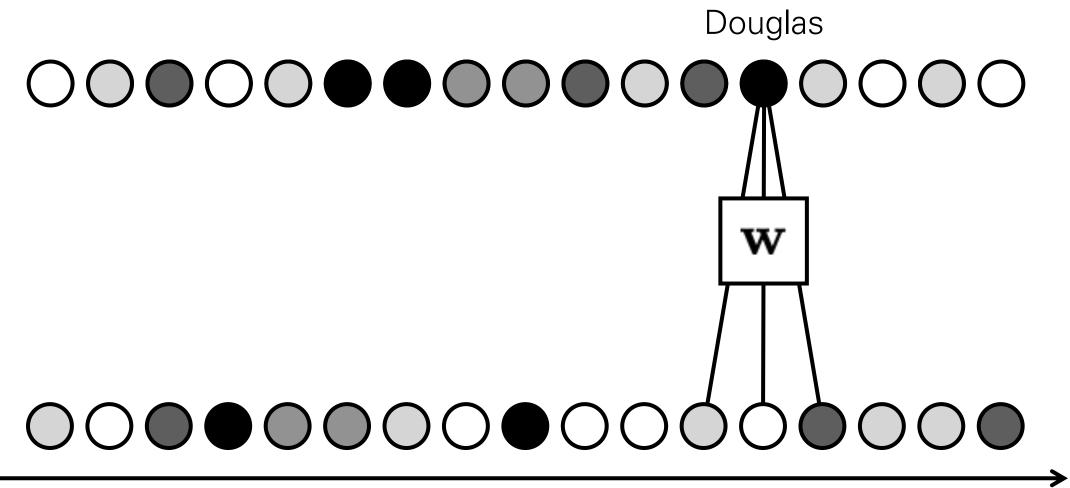




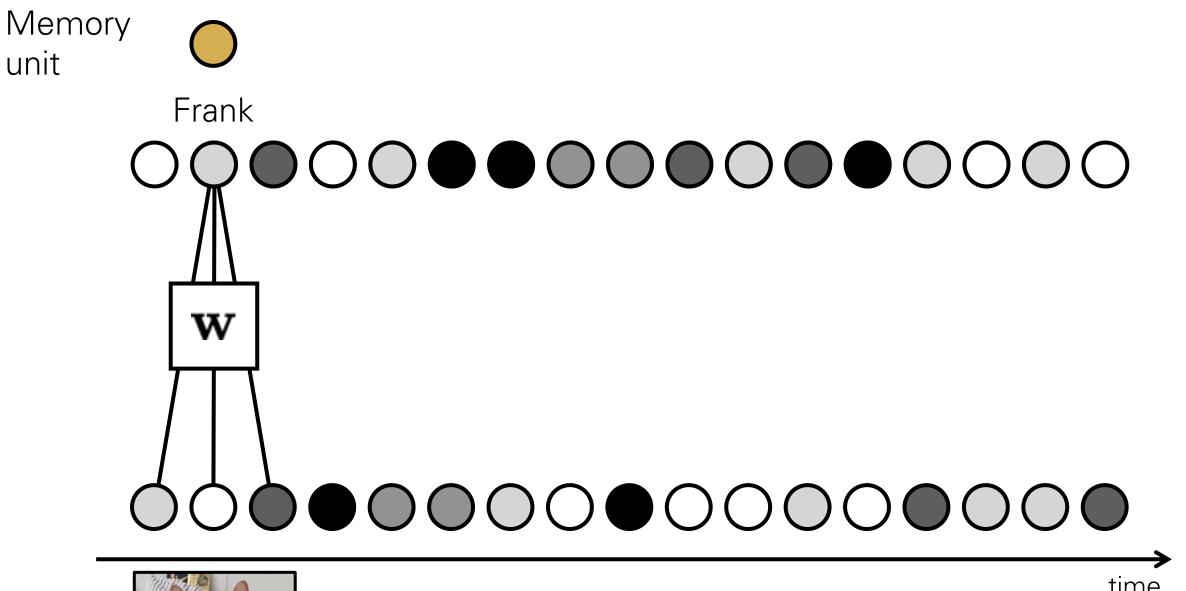
[fig from FeatureNet: Machining feature recognition based on 3D Convolution Neural Network]

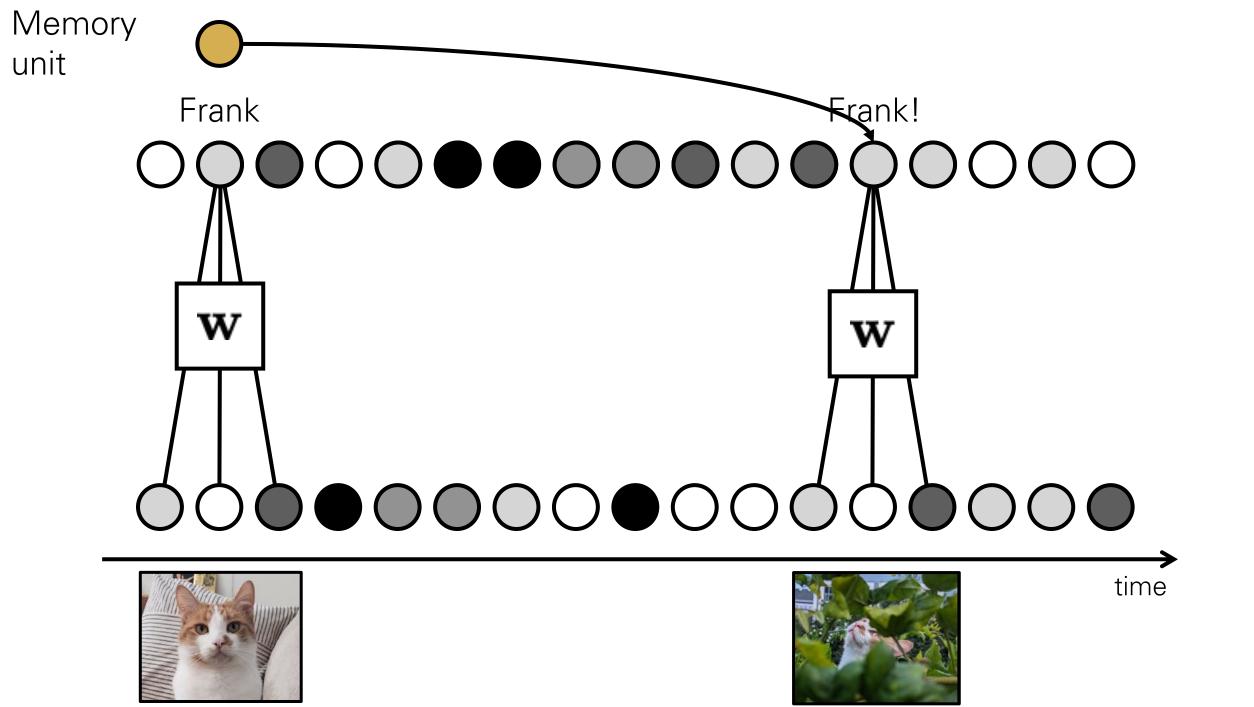


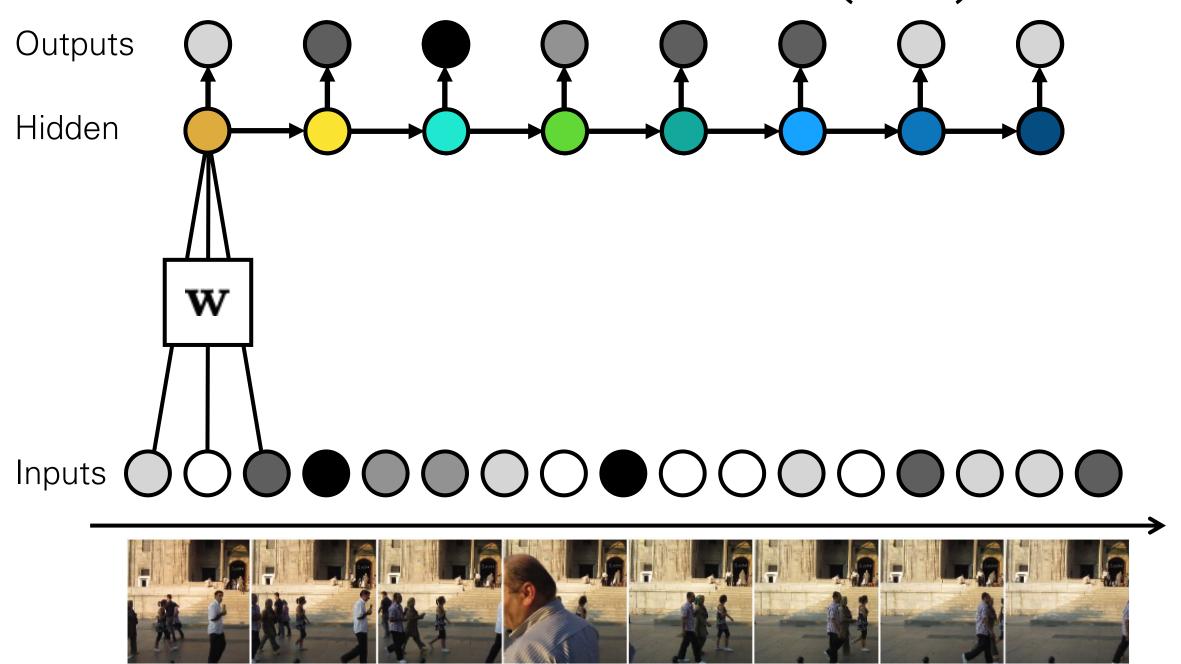








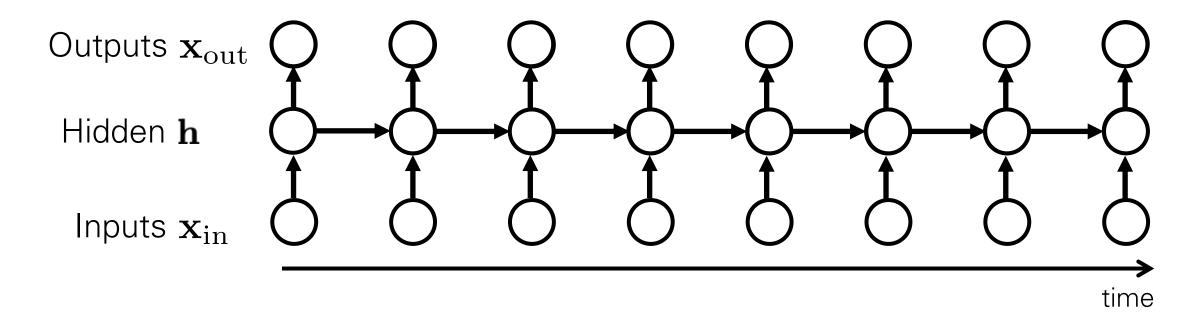


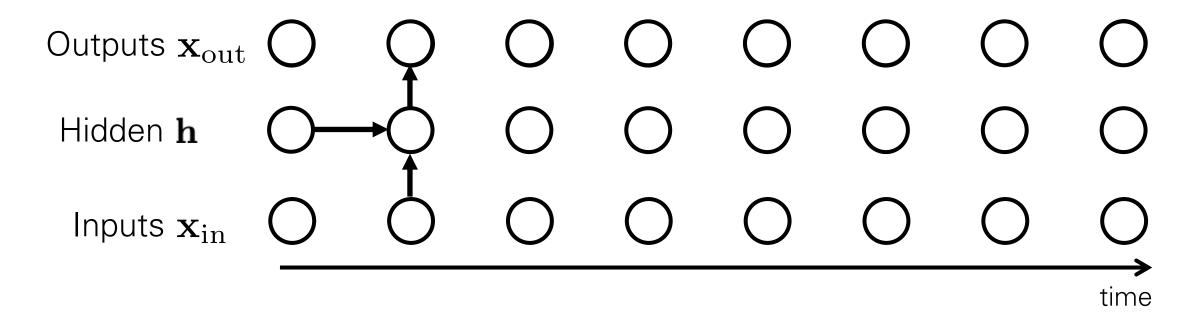


#### To model sequences, we need

- 1. to deal with variable length sequences
- 2. to maintain sequence order
- 3. to keep track of long-term dependencies
- 4. to share parameters across the sequence

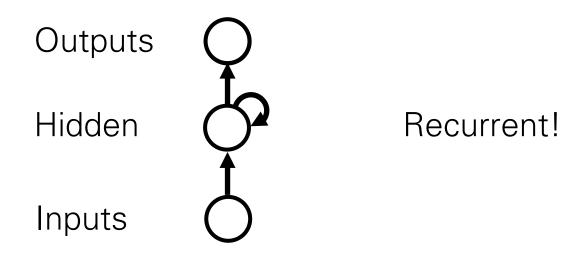
## Recurrent Neural Networks



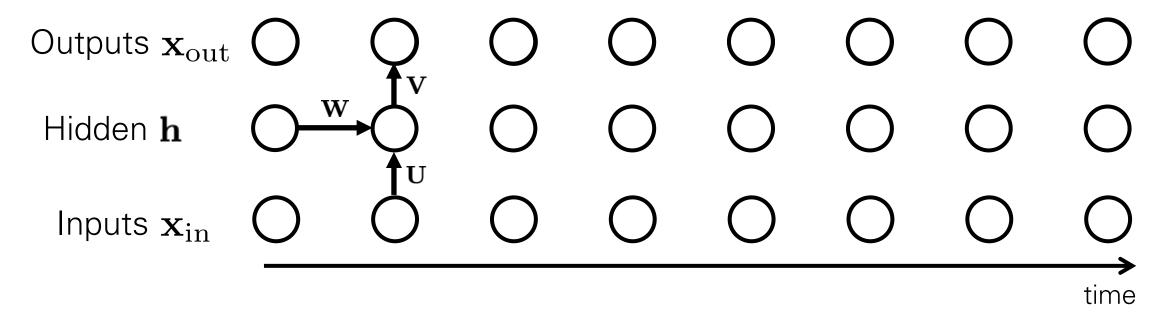


$$\mathbf{h}_t = f\left(\mathbf{h}_{t-1}, \mathbf{x}_{\text{in}}[t]\right)$$

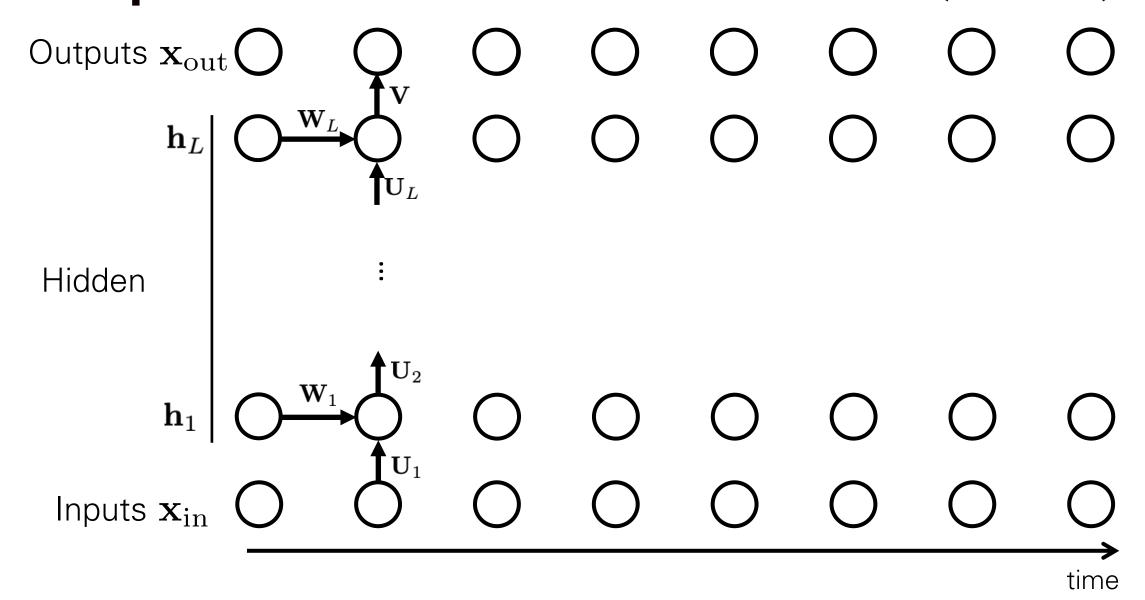
$$\mathbf{x}_{\mathrm{out}}\left[t\right] = g\left(\mathbf{h}_{t}\right)$$



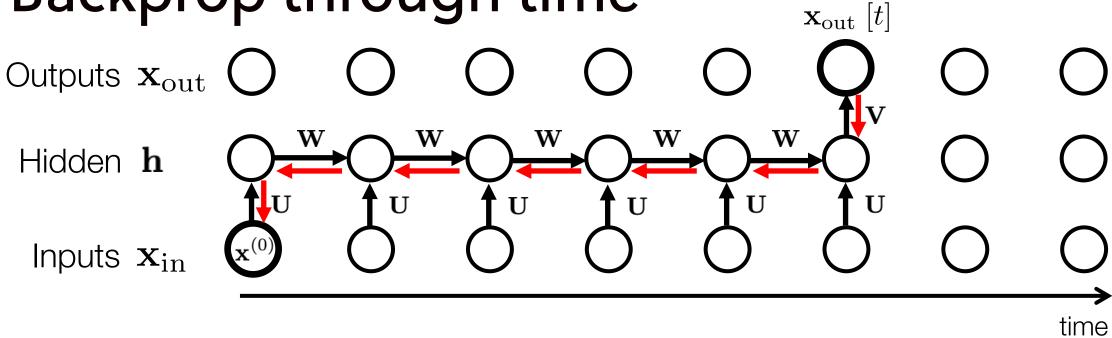
$$\mathbf{h}_{t} = f\left(\mathbf{h}_{t-1}, \mathbf{x}_{\text{in}}[t]\right)$$
$$\mathbf{x}_{\text{out}} [t] = g\left(\mathbf{h}_{t}\right)$$



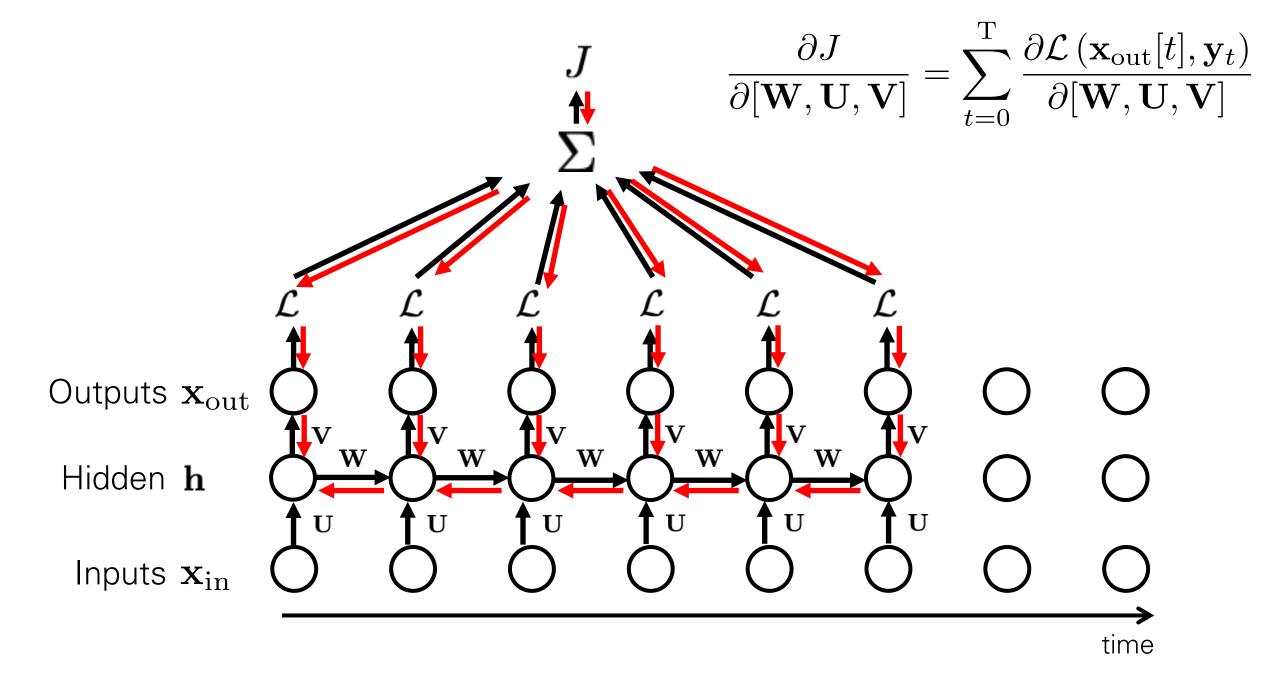
$$\mathbf{h}_{t} = \sigma_{1} \left( \mathbf{W} \mathbf{h}_{t-1} + \mathbf{U} \mathbf{x}_{\text{in}} \left[ t \right] + \mathbf{b} \right)$$
$$\mathbf{x}_{\text{out}} \left[ t \right] = \sigma_{2} \left( \mathbf{V} \mathbf{h}_{t} + \mathbf{c} \right)$$



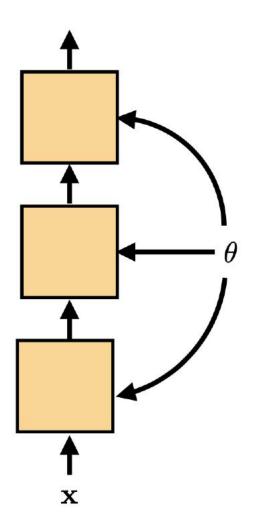
# Backprop through time

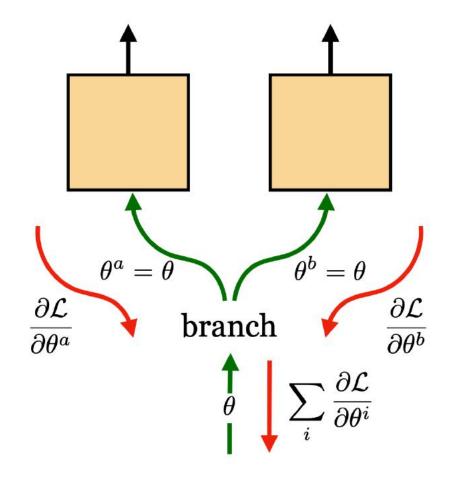


$$\frac{\partial \mathbf{x}_{\text{out}}[t]}{\partial \mathbf{x}_{\text{in}}[0]} = \frac{\partial \mathbf{x}_{\text{out}}[t]}{\partial \mathbf{h}_{T}} \frac{\partial \mathbf{h}_{T}}{\partial \mathbf{h}_{T-1}} \cdots \frac{\partial \mathbf{h}_{1}}{\partial \mathbf{h}_{0}} \frac{\partial \mathbf{h}_{0}}{\partial \mathbf{x}_{\text{in}}[0]}$$

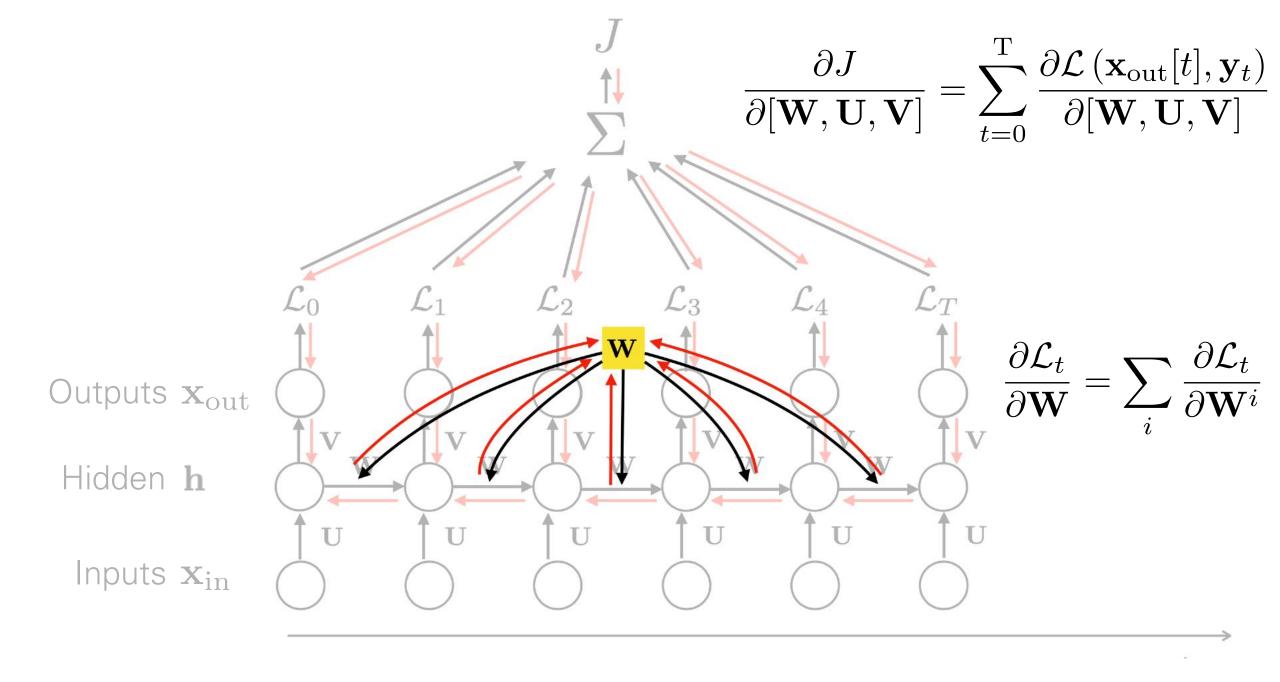


#### Parameter Sharing





Parameter sharing —> sum gradients

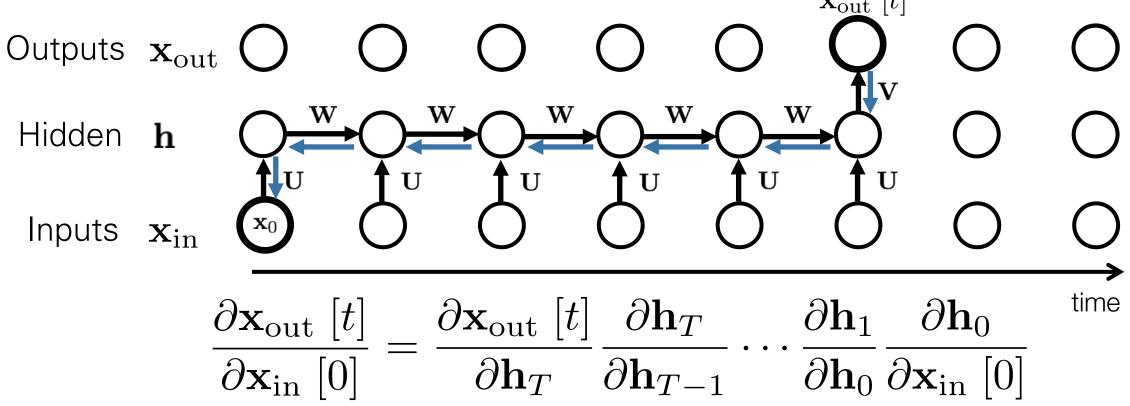


#### The problem of long-range dependencies

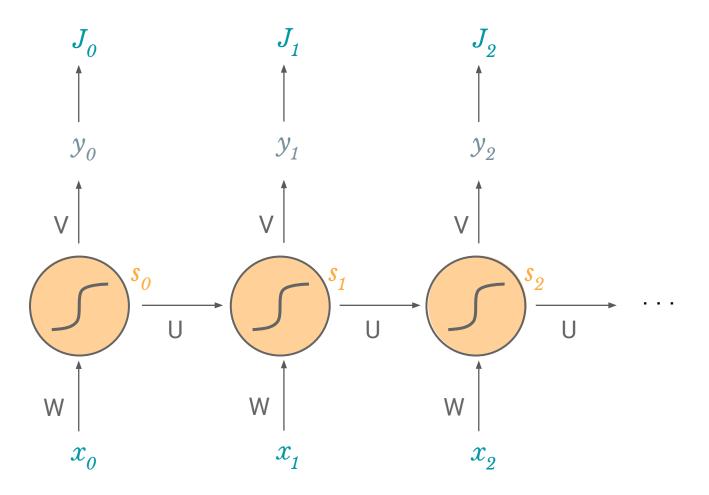
Why not remember everything?

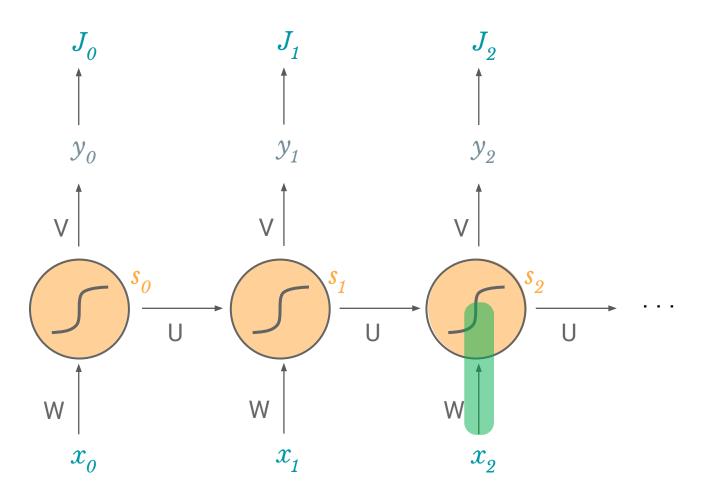
- Memory size grows with t
- This kind of memory is **nonparametric**: there is no finite set of parameters we can use to model it
- RNNs make a Markov assumption the future hidden state only depends on the immediately preceding hidden state
- By putting the right info into the hidden state, RNNs can model dependencies that are arbitrarily far apart

The problem of long-range dependencies

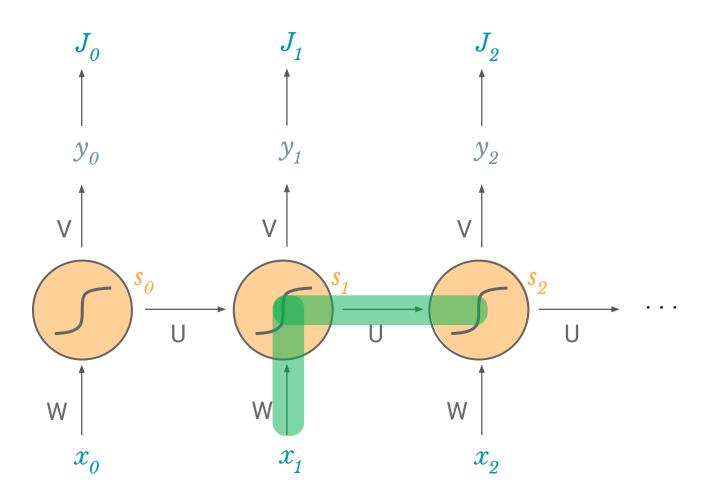


- Capturing long-range dependences requires propagating information through a long chain of dependences.
- Old observations are forgotten
- Stochastic gradients become high variance (noisy), and gradients may vanish or explode

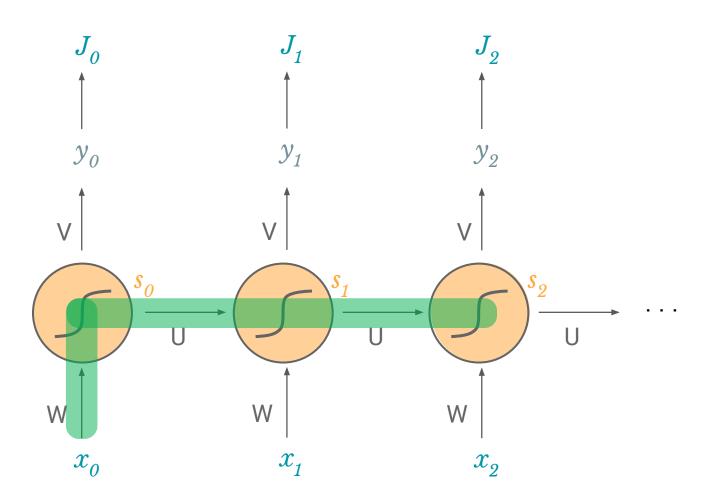




$$\frac{\partial s_2}{\partial W}$$

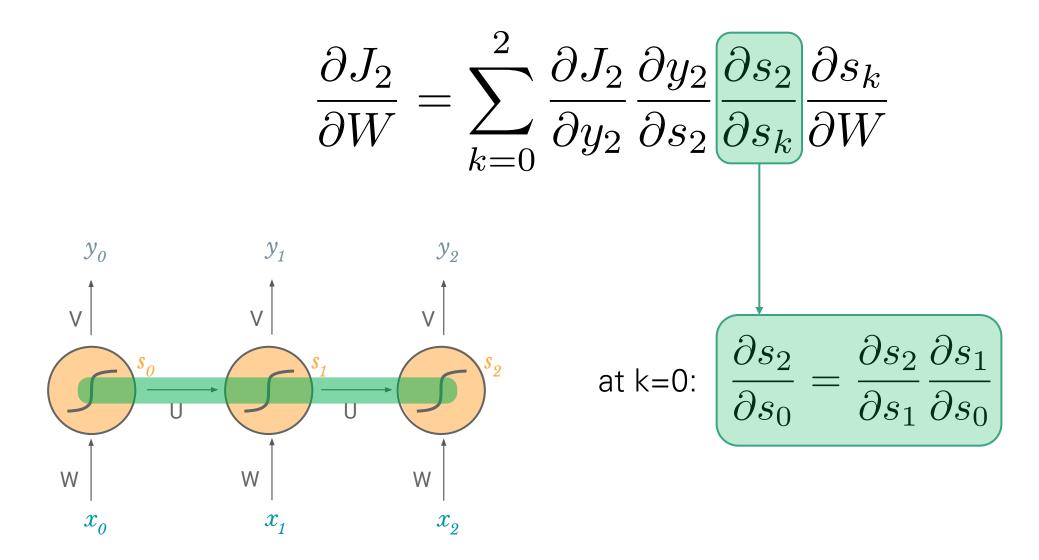


$$\frac{\partial s_2}{\partial W} + \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial W}$$



$$\frac{\partial s_2}{\partial W} + \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial W} + \frac{\partial s_2}{\partial s_0} \frac{\partial s_0}{\partial W}$$

#### Vanishing Gradient Problem

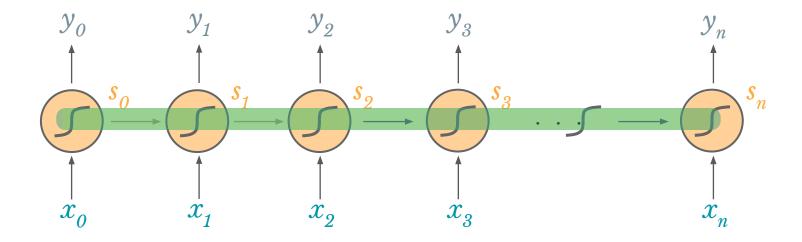


#### Vanishing Gradient Problem

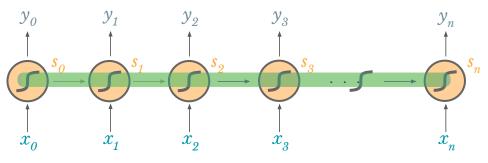
$$\frac{\partial J_n}{\partial W} = \sum_{k=0}^{n} \frac{\partial J_n}{\partial y_n} \frac{\partial y_n}{\partial s_n} \frac{\partial s_n}{\partial s_k} \frac{\partial s_k}{\partial W}$$

$$\frac{\partial s_n}{\partial s_{n-1}} \frac{\partial s_{n-1}}{\partial s_{n-2}} \cdots \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial s_0}$$

as the gap between timesteps gets bigger, this product gets longer and longer!



#### Vanishing Gradient Problem



what are each of these terms?

what are each of these terms? 
$$\frac{\partial s_n}{\partial s_{n-1}} \frac{\partial s_n}{\partial s_{n-2}} \cdots \frac{\partial s_3}{\partial s_2} \frac{\partial s_2}{\partial s_1} \frac{\partial s_1}{\partial s_0}$$

$$\frac{\partial s_n}{\partial s_{n-1}} = W^T \operatorname{diag} \left[ f'(W_{s_{j-1}+Ux_j}) \right]$$

$$W =$$
sampled from standard normal distribution = mostly < 1

$$f = \text{tanh or sigmoid so } f' < 1$$

we're multiplying a lot of small numbers together.

#### Vanishing Gradient Problem

we're multiplying a lot of small numbers together.

#### so what?

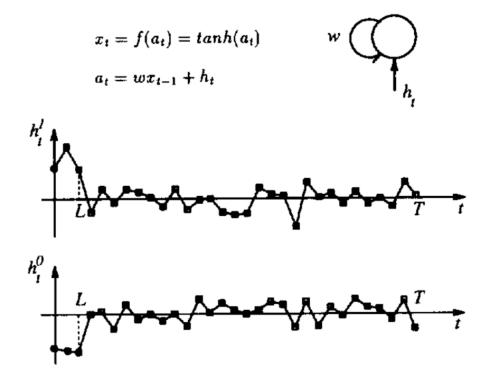
errors due to further back timesteps have increasingly smaller gradients.

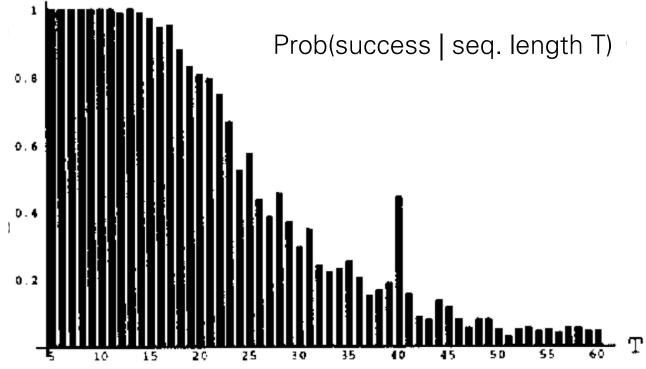
#### so what?

parameters become biased to capture shorter-term dependencies.

## A Toy Example

- 2 categories of sequences
- Can the single tanh unit learn to store for T time steps 1 bit of information given by the sign of initial input?





#### Vanishing Gradient Problem

"In France, I had a great time and I learnt some of the \_\_\_\_ language."

our parameters are not trained to capture long-term dependencies, so the word we predict will mostly depend on the previous few words, not much earlier ones

## Long-Term Dependencies



• The RNN gradient is a product of Jacobian matrices, each associated with a step in the forward computation. To store information robustly in a finite-dimensional state, the dynamics must be contractive [Bengio et al 1994].

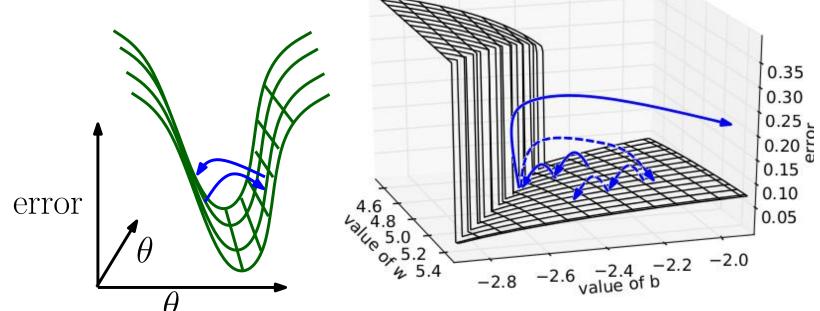
$$L = L(s_T(s_{T-1}(\dots s_{t+1}(s_t, \dots))))$$

$$\frac{\partial L}{\partial s_t} = \frac{\partial L}{\partial s_T} \frac{\partial s_T}{\partial s_{T-1}} \dots \frac{\partial s_{t+1}}{\partial s_t}$$

- Problems:
  - sing. values of Jacobians > 1 → gradients explode
  - or sing. values < → gradients shrink & vanish</li>
  - or random → variance grows exponentially

## **Gradient Norm Clipping**

$$\begin{array}{l} \hat{\mathbf{g}} \leftarrow \frac{\partial error}{\partial \theta} \\ \text{if } ||\hat{\mathbf{g}}|| \geq threshold \text{ then} \\ \hat{\mathbf{g}} \leftarrow \frac{threshold}{\|\hat{\mathbf{g}}\|} \hat{\mathbf{g}} \\ \text{end if} \end{array}$$

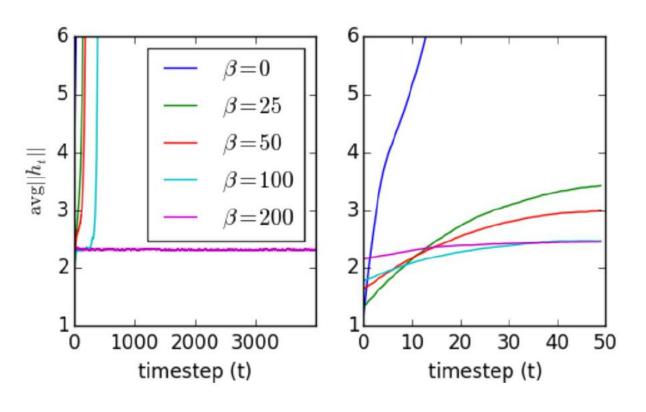


#### Regularization: Norm-stabilizer

 Stabilize the activations of RNNs by penalizing the squared distance between successive hidden states' norms

$$\beta \frac{1}{T} \sum_{t=1}^{T} (\|h_t\|_2 - \|h_{t-1}\|_2)^2$$

 Enforce the norms of the hidden layer activations approximately constant across time



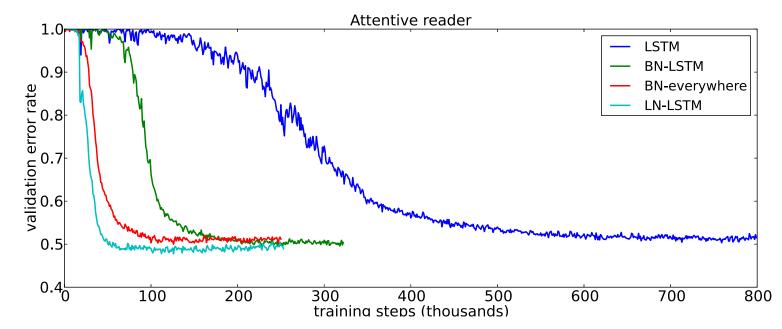
# Regularization: Layer Normalization

- Similar to batch normalization
- Computes the normalization statistics separately at each time step
- Effective for stabilizing the hidden state dynamics in RNNs
- Reduces training time

$$\mathbf{h}^{t} = f \left[ \frac{\mathbf{g}}{\sigma^{t}} \odot \left( \mathbf{a}^{t} - \mu^{t} \right) + \mathbf{b} \right]$$

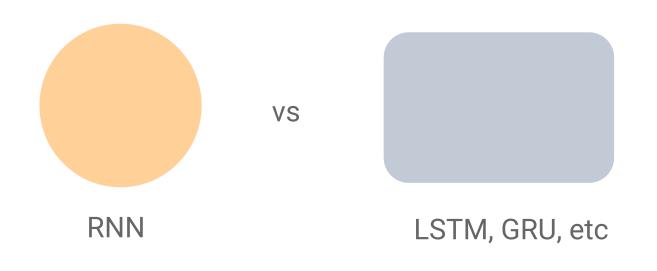
$$\mu^t = \frac{1}{H} \sum_{i=1}^H a_i^t$$

$$\sigma^t = \sqrt{\frac{1}{H} \sum_{i=1}^{H} \left(a_i^t - \mu^t\right)^2}$$



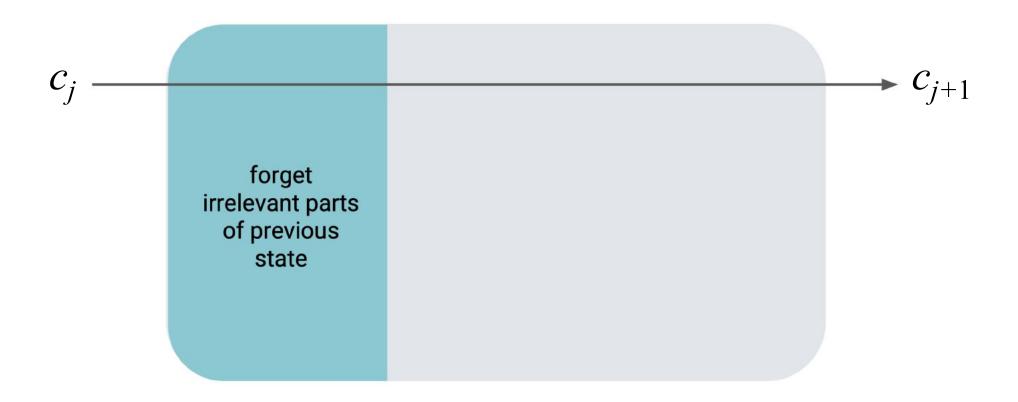
#### **Gated Cells**

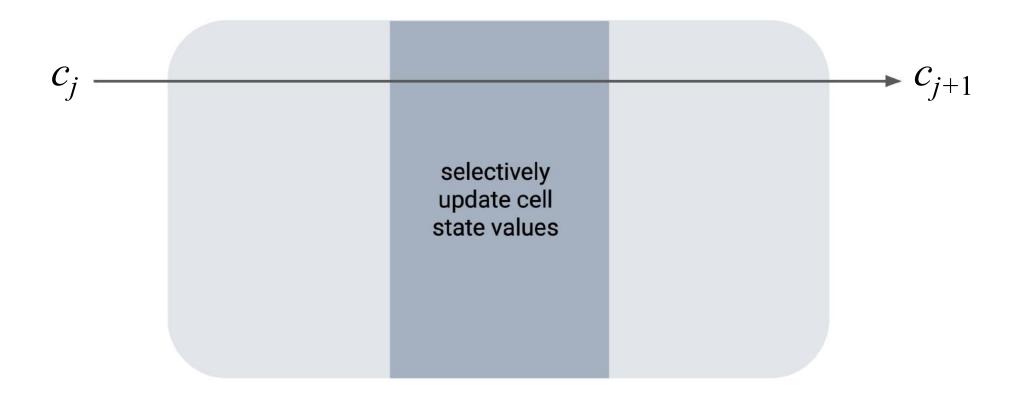
 rather each node being just a simple RNN cell, make each node a more complex unit with gates controlling what information is passed through

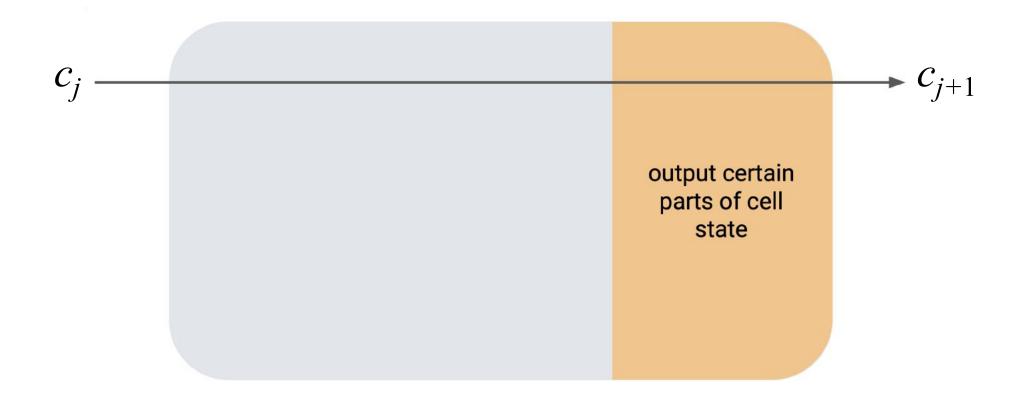


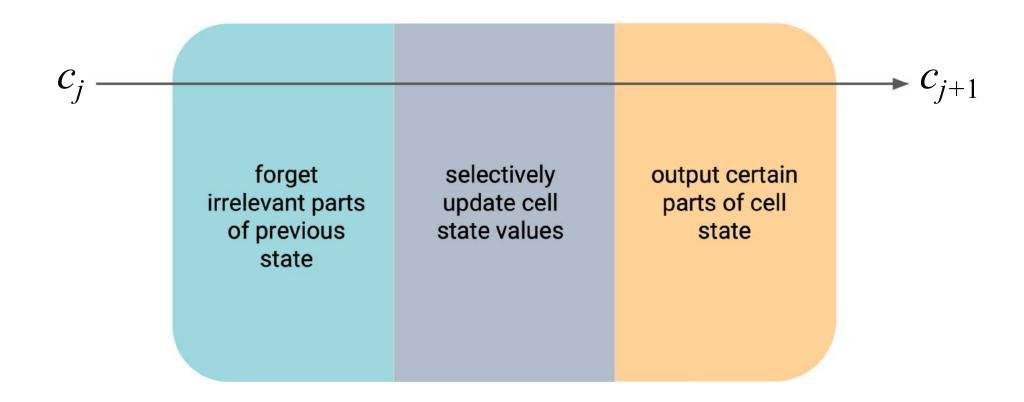
Long short term memory cells are able to keep track of information throughout many timesteps.



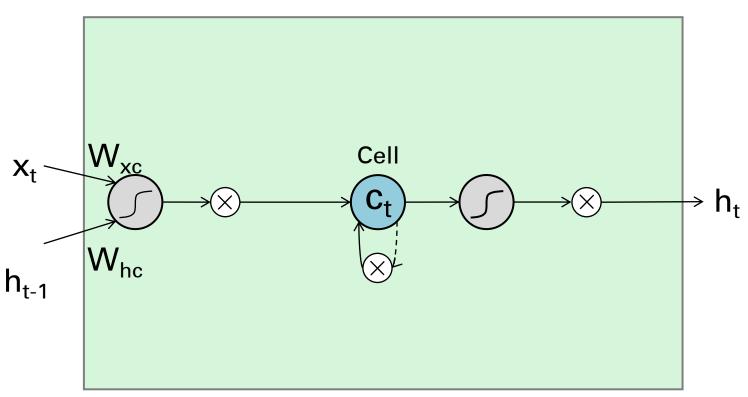








#### The LSTM Idea

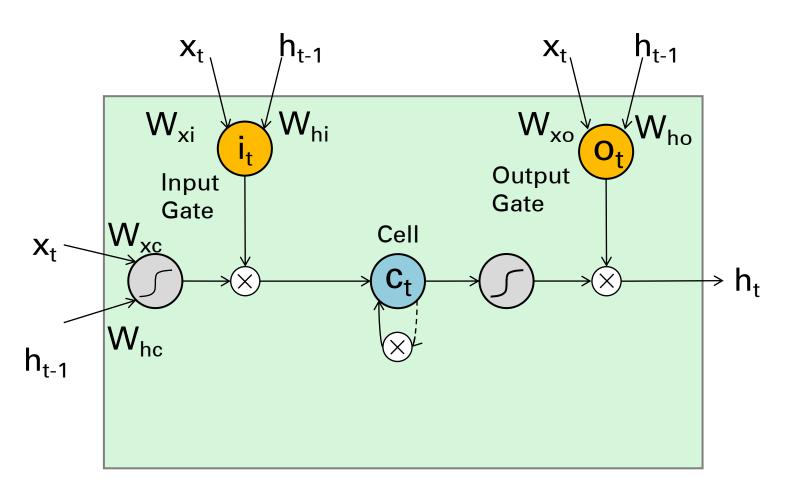


$$c_{t} = c_{t-1} + \tanh W \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix}$$

$$h_{t} = \tanh c_{t}$$

<sup>\*</sup> Dashed line indicates time-lag

#### The Original LSTM Cell

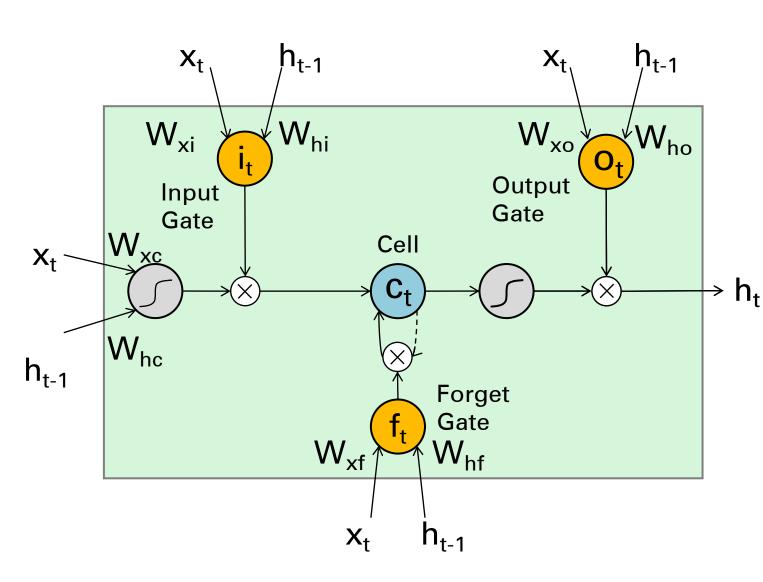


$$c_{t} = c_{t-1} + i_{t} \otimes \tanh W \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix}$$

$$h_t = o_t \otimes \tanh c_t$$

$$i_{t} = \sigma \left( W_{i} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{i} \right)$$

Similarly for ot

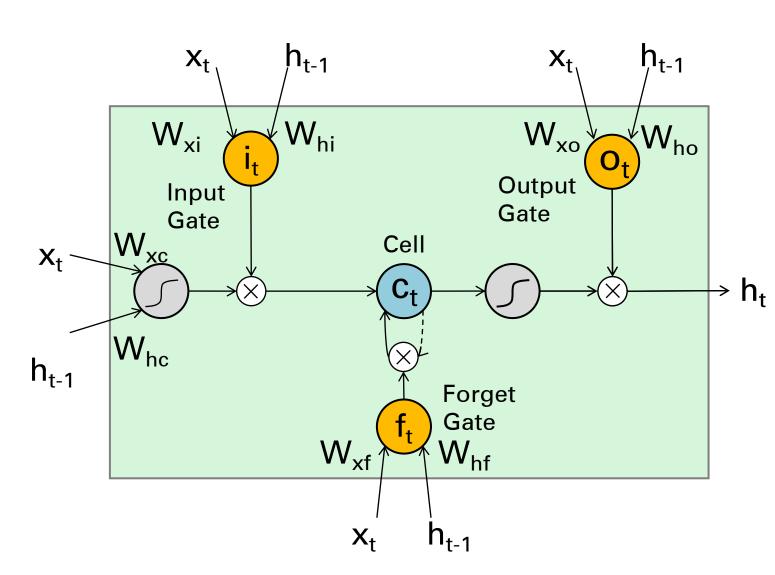


$$i_{t} = \sigma \left( W_{i} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{i} \right)$$

$$c_{t} = f_{t} \otimes c_{t-1} + i_{t} \otimes \tanh W \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix}$$

$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h_t = o_t \otimes \tanh c_t$$



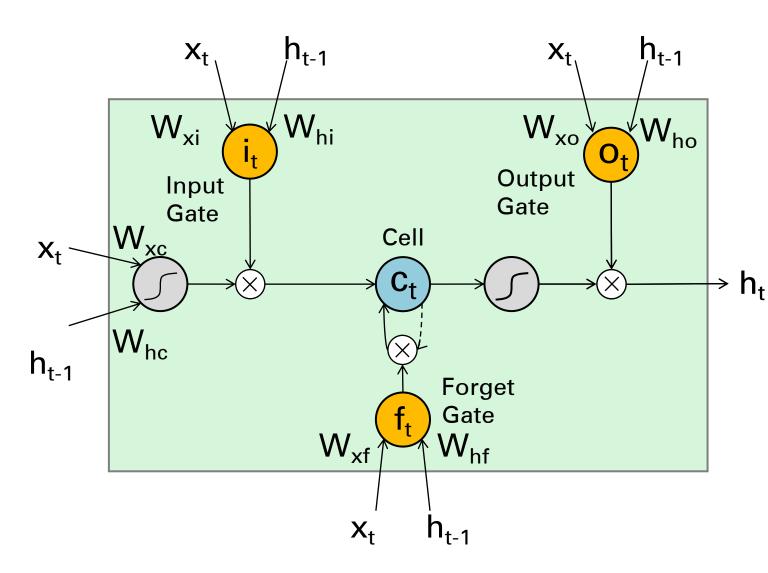
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$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h_t = o_t \otimes \tanh c_t$$

**forget gate** decides what information is going to be thrown away from the cell state



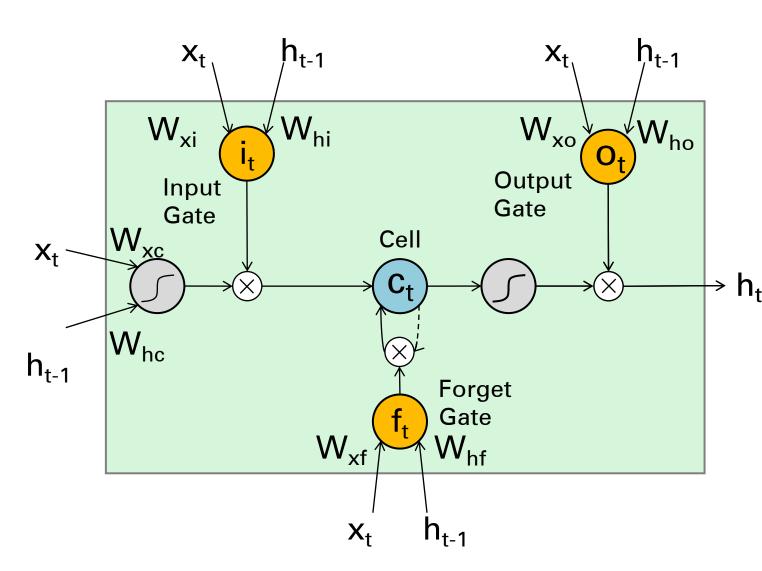
$$i_{t} = \sigma \left( W_{i} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{i} \right)$$

$$c_{t} = f_{t} \otimes c_{t-1} + \begin{vmatrix} i_{t} \otimes \tanh W \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix}$$

$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h_t = o_t \otimes \tanh c_t$$

input gate and a tanh layer decides what information is going to be stored in the cell state



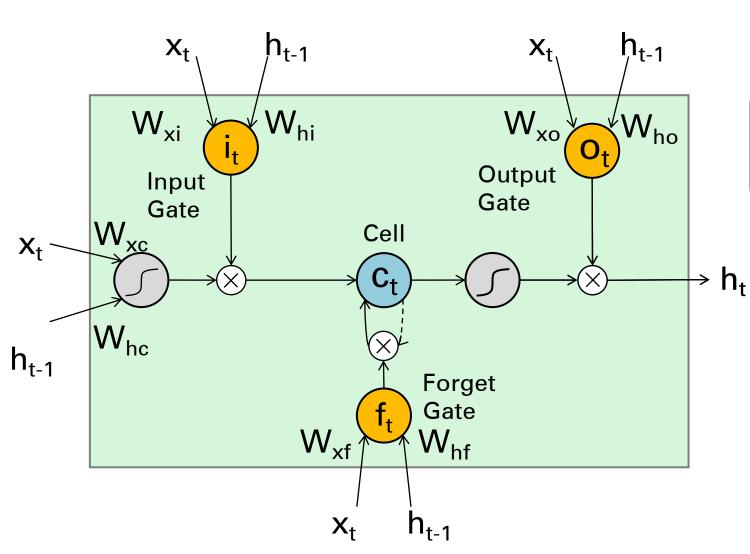
$$i_{t} = \sigma \left( W_{i} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{i} \right)$$

$$c_{t} = f_{t} \otimes c_{t-1} + i_{t} \otimes \tanh W \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix}$$

$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h_t = o_t \otimes \tanh c_t$$

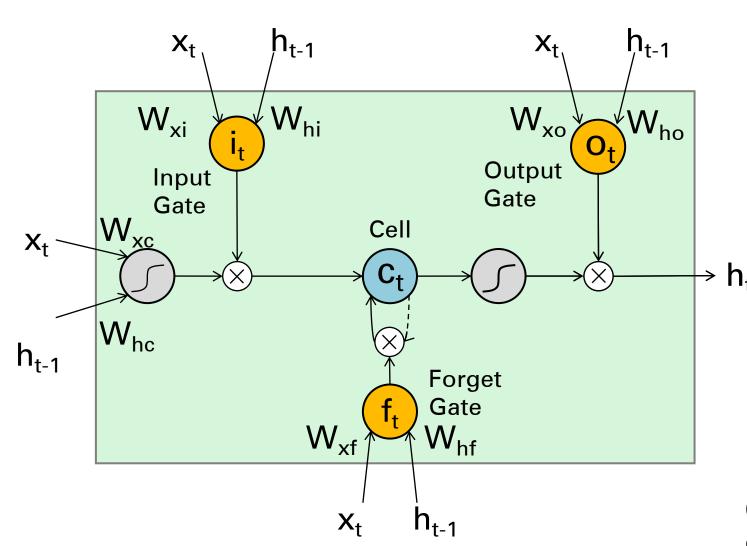
Update the old cell state with the new one.



$$i_{t} = \sigma \left( W_{i} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{i} \right)$$

$$c_{t} = f_{t} \otimes c_{t-1} + i_{t} \otimes \tanh W \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix}$$

inpu t gate	forget gate	behavior
0	1	remember the previous value
1	1	add to the previous value
0	0	erase the value
1	0	overwrite the value



$$i_{t} = \sigma \left( W_{i} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{i} \right)$$

$$c_{t} = f_{t} \otimes c_{t-1} + i_{t} \otimes \tanh W \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix}$$

$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

$$h_{t} = o_{t} \otimes \tanh c_{t}$$

$$o_{i} = \sigma \left( W_{o} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{o} \right)$$

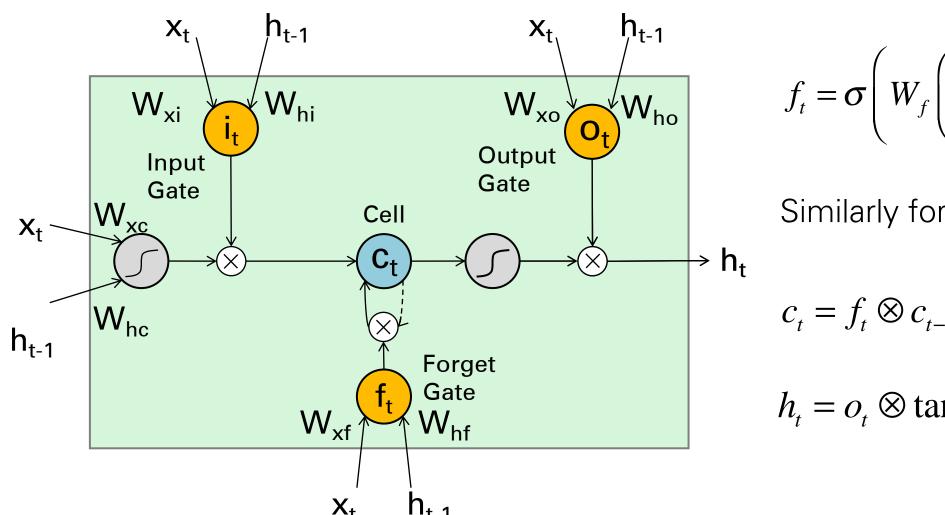
Output gate decides what is going to be outputted. The final output is based on cell state and output of sigmoid gate.

#### LSTM - Forward/Backward

Illustrated LSTM Forward and Backward Pass

http://arunmallya.github.io/writeups/nn/lstm/index.html

# LSTM variants



$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \end{pmatrix} + b_f \right)$$

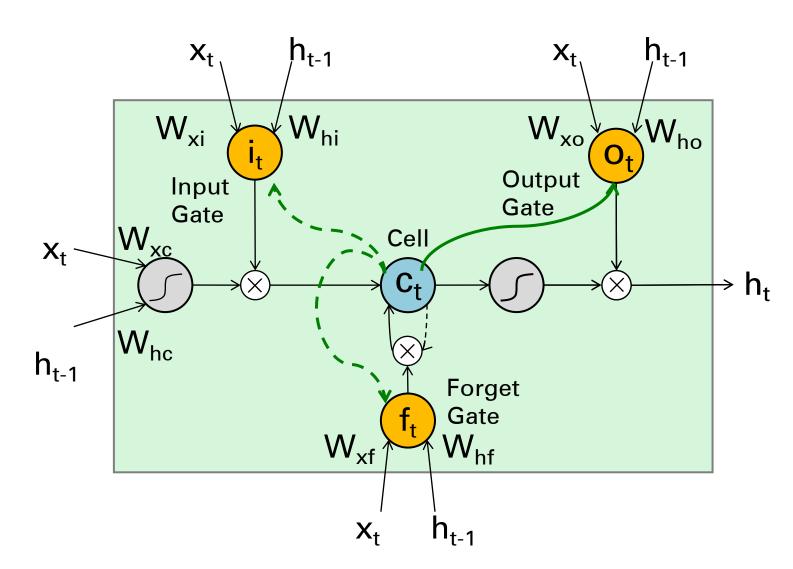
Similarly for i<sub>t</sub>, o<sub>t</sub>

$$c_{t} = f_{t} \otimes c_{t-1} + i_{t} \otimes \tanh W \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix}$$

$$h_t = o_t \otimes \tanh c_t$$

<sup>\*</sup> Dashed line indicates time-lag

#### **Extension I: Peephole LSTM**



$$f_t = \sigma \left( W_f \begin{pmatrix} x_t \\ h_{t-1} \\ C_{t-1} \end{pmatrix} + b_f \right)$$

Similarly for i<sub>t</sub>, o<sub>t</sub> (uses c<sub>t</sub>)

$$c_{t} = f_{t} \otimes c_{t-1} + i_{t} \otimes \tanh W \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix}$$

$$h_t = o_t \otimes \tanh c_t$$

- Add peephole connections.
  - All gate layers look at the cell state!

<sup>\*</sup> Dashed line indicates time-lag

#### Other minor variants

Coupled Input and Forget Gate

$$f_t = 1 - i_t$$

• Full Gate Recurrence

$$f_t = \sigma \left( \begin{array}{c} \begin{pmatrix} x_t \\ h_{t-1} \\ \vdots \\ i_{t-1} \\ f_{t-1} \\ O_{t-1} \end{array} \right) + b_f$$

#### LSTM: A Search Space Odyssey

- Tested the following variants, using Peephole LSTM as standard:
  - 1. No Input Gate (NIG)
  - 2. No Forget Gate (NFG)
  - 3. No Output Gate (NOG)
  - 4. No Input Activation Function (NIAF)
  - No Output Activation Function (NOAF)
  - 6. No Peepholes (NP)
  - 7. Coupled Input and Forget Gate (CIFG)
  - 8. Full Gate Recurrence (FGR)
- On the tasks of:
  - Timit Speech Recognition: Audio frame to 1 of 61 phonemes
  - IAM Online Handwriting Recognition: Sketch to characters
  - JSB Chorales: Next-step music frame prediction

#### LSTM: A Search Space Odyssey

- The standard LSTM performed reasonably well on multiple datasets and none of the modifications significantly improved the performance
- Coupling gates and removing peephole connections simplified the LSTM without hurting performance much
- The forget gate and output activation are crucial

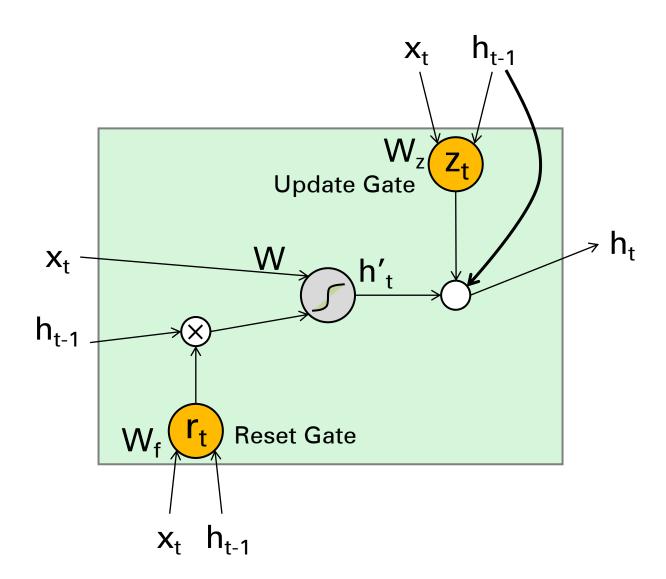
 Found interaction between learning rate and network size to be minimal – indicates calibration can be done using a small network first

# Gated Recurrent Unit

#### Gated Recurrent Unit (GRU)

- A very simplified version of the LSTM
  - Merges forget and input gate into a single 'update' gate
  - Merges cell and hidden state
- Has fewer parameters than an LSTM and has been shown to outperform LSTM on some tasks

<u>Learning Phrase Representations using RNN Encoder-Decoder for Statistical Machine Translation</u>
[Cho et al.,14]

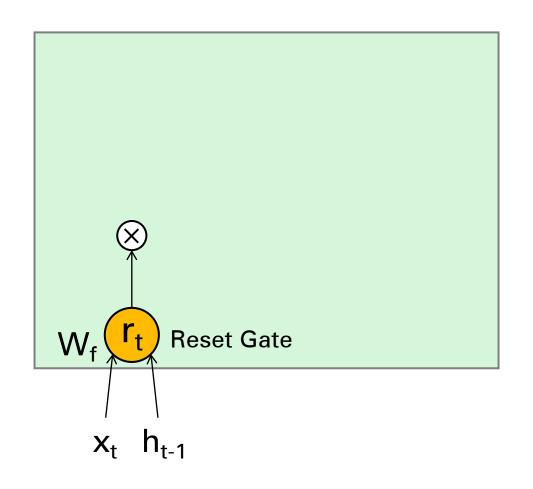


$$r_{t} = \sigma \left( W_{r} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{f} \right)$$

$$h'_{t} = \tanh W \begin{pmatrix} x_{t} \\ r_{t} \otimes h_{t-1} \end{pmatrix}$$

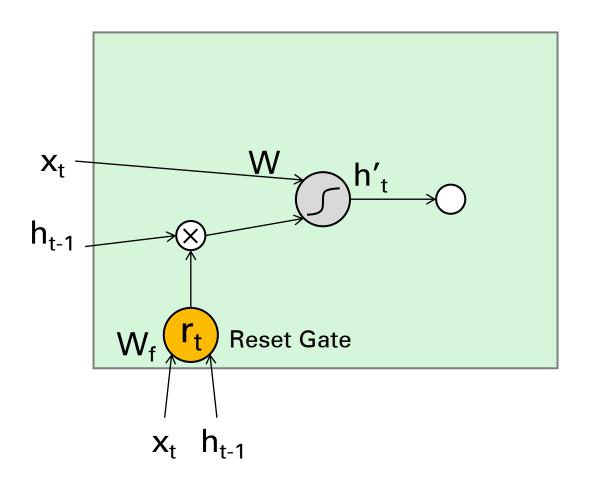
$$z_{t} = \sigma \left( W_{z} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{z} \right)$$

$$h_{t} = (1 - z_{t}) \otimes h_{t-1} + z_{t} \otimes h'_{t}$$



$$r_{t} = \sigma \left( W_{r} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{f} \right)$$

computes a **reset gate** based on current input and hidden state

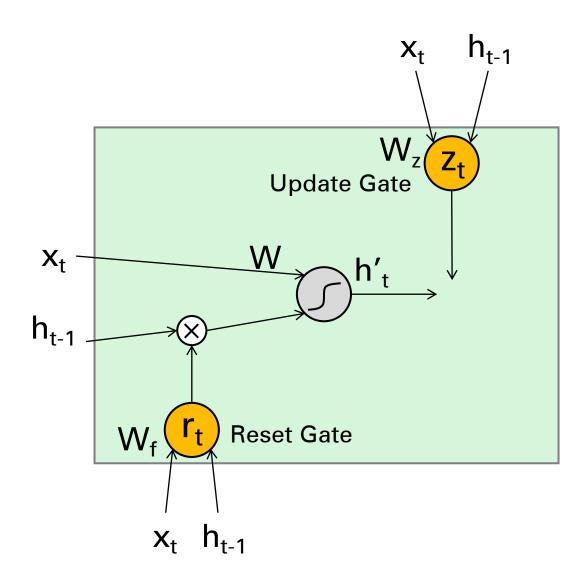


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$$h'_{t} = \tanh W \begin{pmatrix} x_{t} \\ r_{t} \otimes h_{t-1} \end{pmatrix}$$

computes the **hidden state** based on current input and hidden state

if reset gate unit is ~0, then this ignores previous memory and only stores the new input information

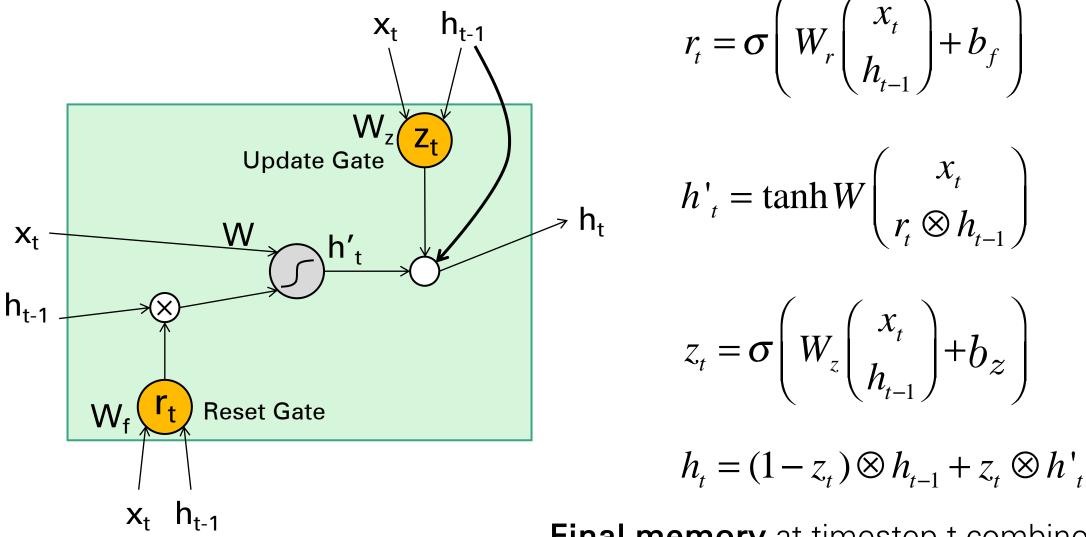


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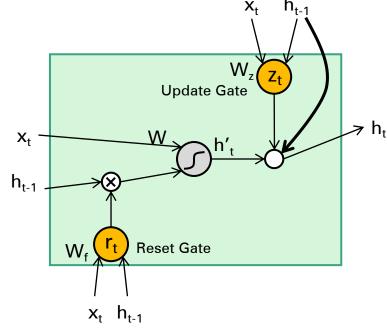
$$z_{t} = \sigma \left( W_{z} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{z} \right)$$

computes an **update gate** again based on current input and hidden state



**Final memory** at timestep t combines both current and previous timesteps

#### **GRU** Intuition



$$r_{t} = \sigma \left( W_{r} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{f} \right)$$

$$h'_{t} = \tanh W \begin{pmatrix} x_{t} \\ r_{t} \otimes h_{t-1} \end{pmatrix}$$

$$z_{t} = \sigma \left( W_{z} \begin{pmatrix} x_{t} \\ h_{t-1} \end{pmatrix} + b_{f} \right)$$
$$h_{t} = (1 - z_{t}) \otimes h_{t-1} + z_{t} \otimes h'_{t}$$

- If reset is close to 0, ignore previous hidden state
  - Allows model to drop information that is irrelevant in the future
- Update gate z controls how much of past state should matter now.
  - If z close to 1, then we can copy information in that unit through many time steps! Less vanishing gradient!
- Units with short-term dependencies often have reset gates very active

#### LSTMs and GRUs

#### Good

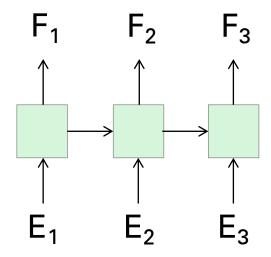
• Careful initialization and optimization of vanilla RNNs can enable them to learn long(ish) dependencies, but gated additive cells, like the LSTM and GRU, often just work.

#### Bad

 LSTMs and GRUs have considerably more parameters and computation per memory cell than a vanilla RNN, as such they have less memory capacity per parameter\*

#### Is RNNs enough?

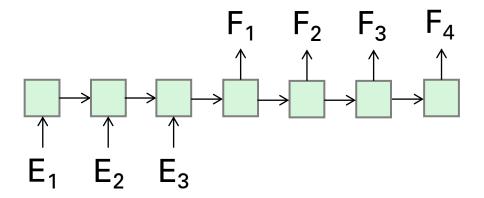
- Consider the problem of translation of English to French
- E.g. What is your name → Comment tu t'appelle
- Is the below architecture suitable for this problem?



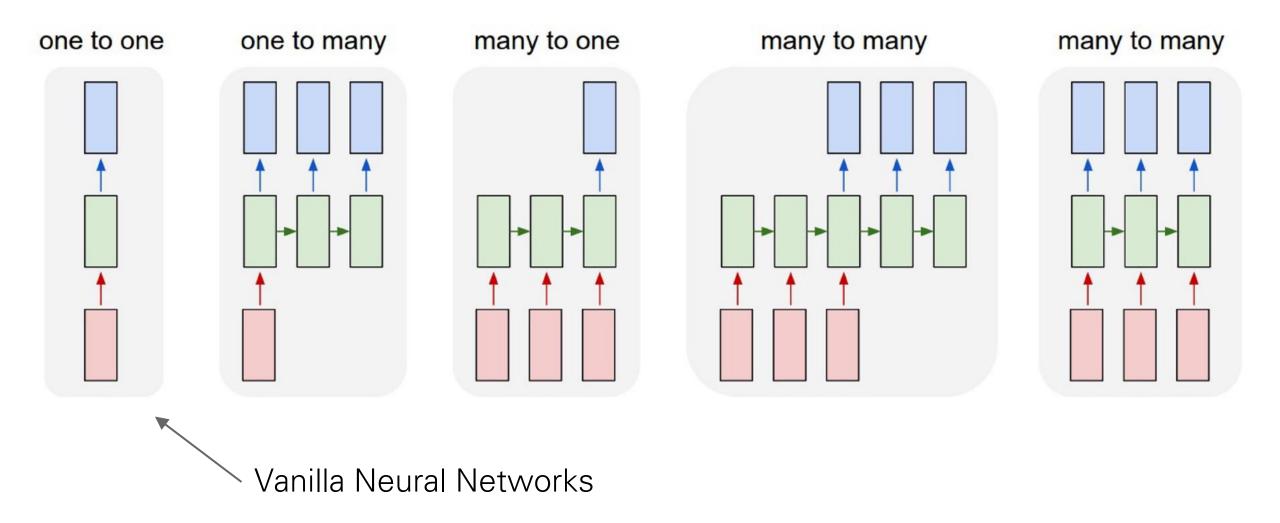
 No, sentences might be of different length and words might not align. Need to see entire sentence before translating

#### Encoder-decoder seq2seq model

- Consider the problem of translation of English to French
- E.g. What is your name → Comment tu t'appelle
- Sentences might be of different length and words might not align.
   Need to see entire sentence before translating



Input-Output nature depends on the structure of the problem at hand



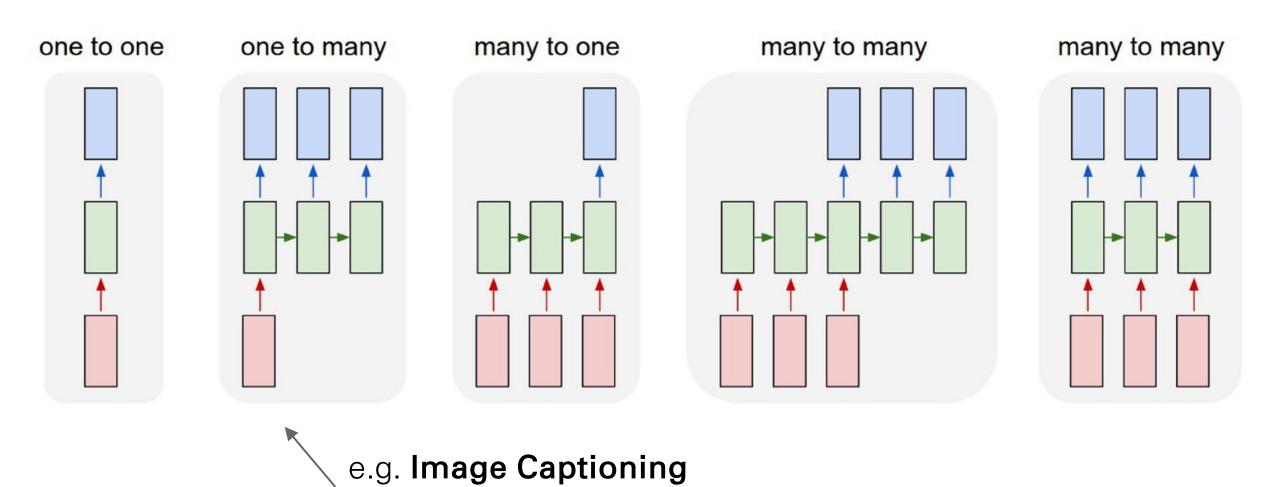
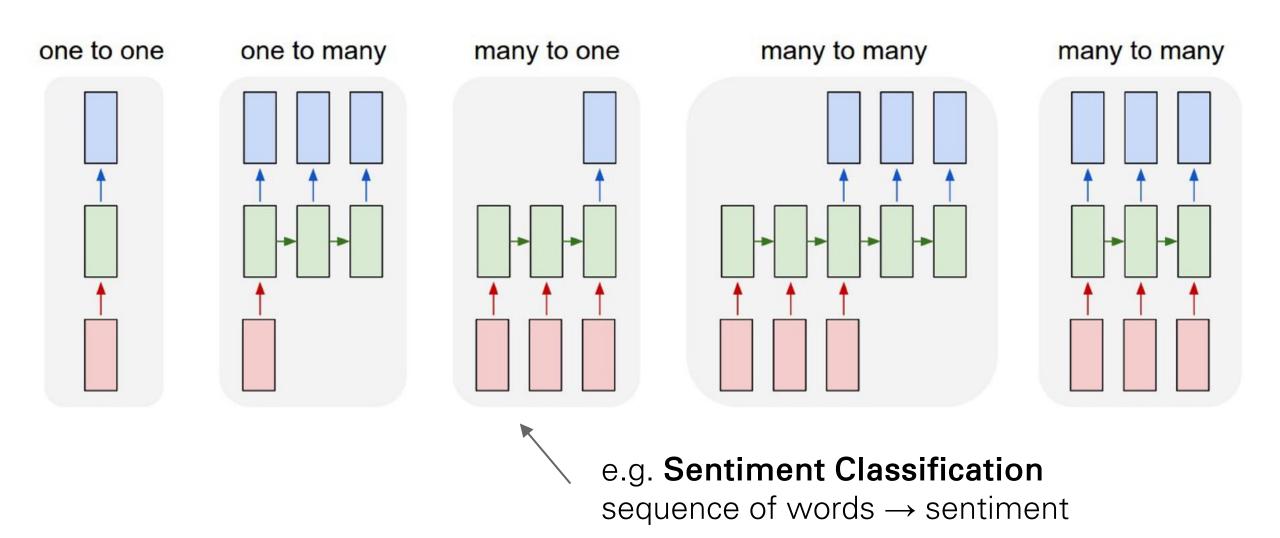
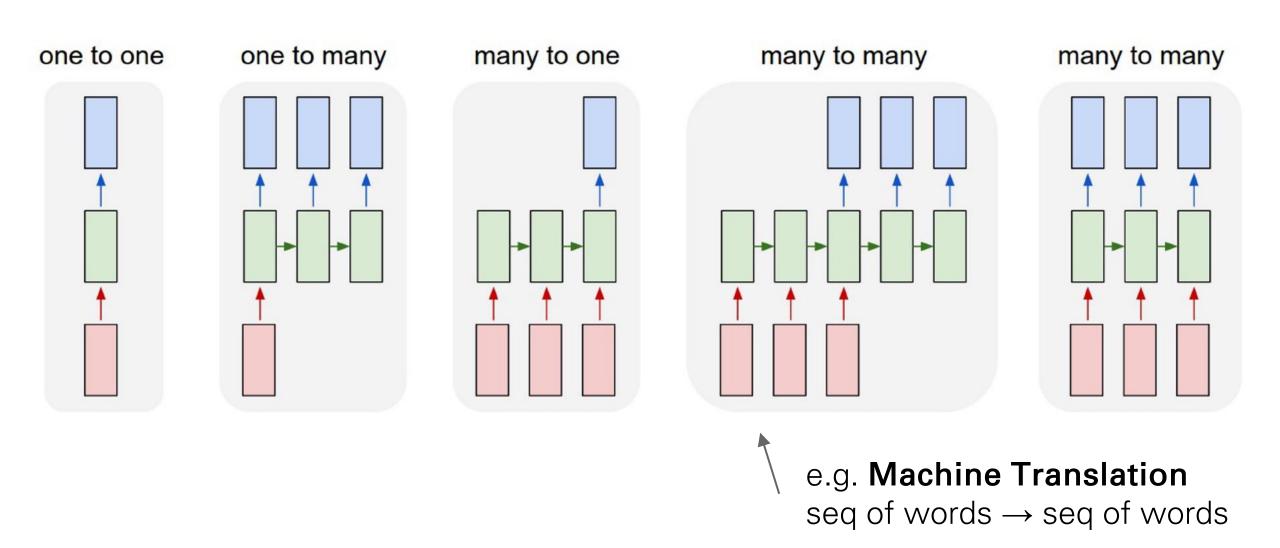
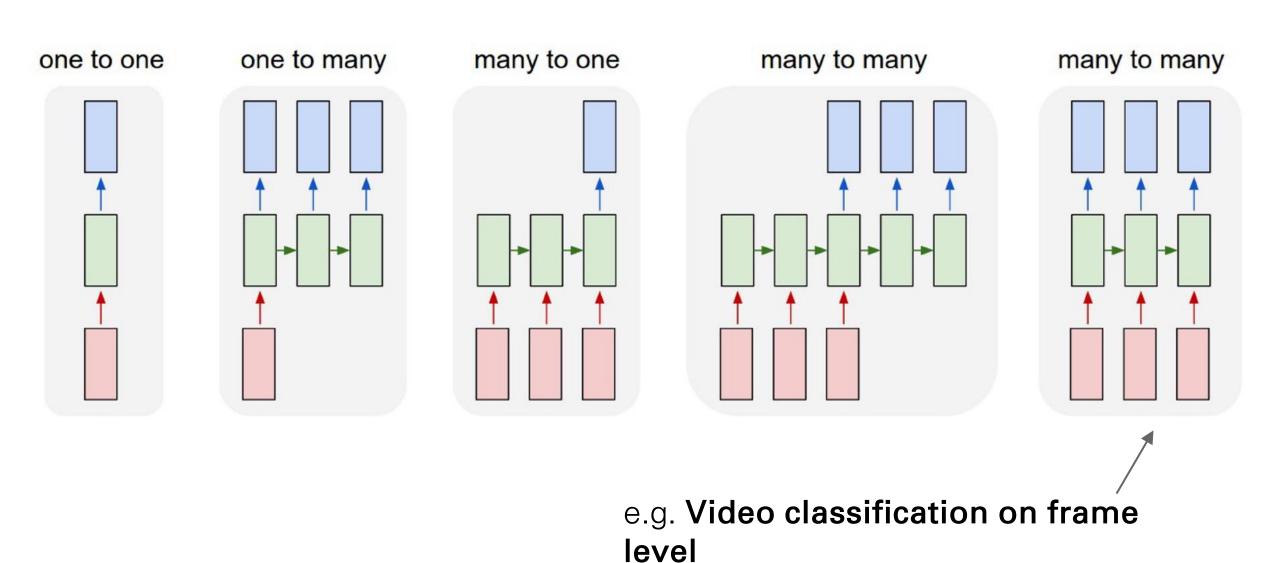


image → sequence of words

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# Next Lecture: Attention and Transformers