

# COMP547

## DEEP UNSUPERVISED LEARNING

Lecture #2 – Neural Networks Basics and  
Spatial Processing with CNNs



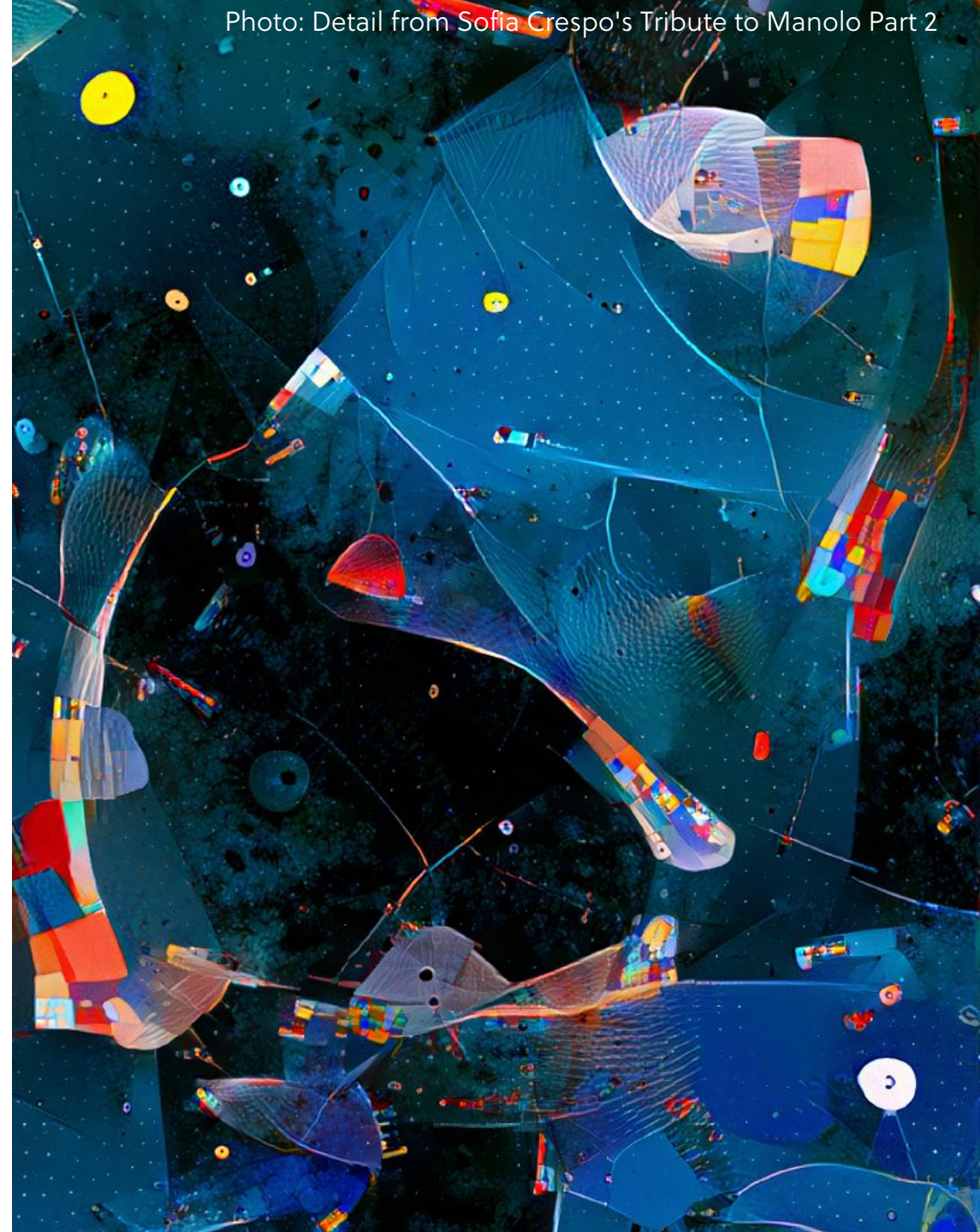
**KOÇ  
UNIVERSITY**

Aykut Erdem // Koç University // Spring 2021

# Previously on COMP547

- course logistics
- course topics
- what is deep unsupervised learning

Photo: Detail from Sofia Crespo's Tribute to Manolo Part 2



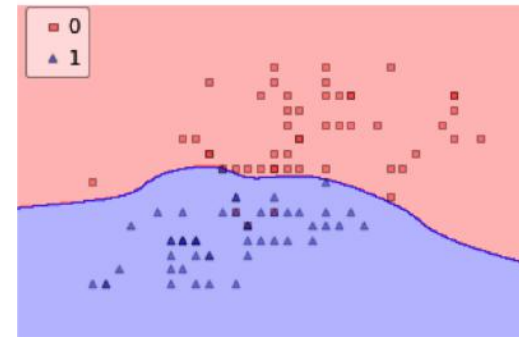
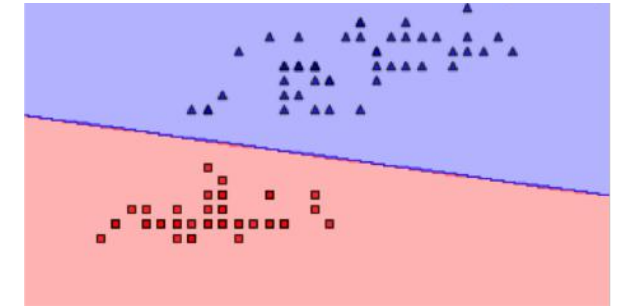
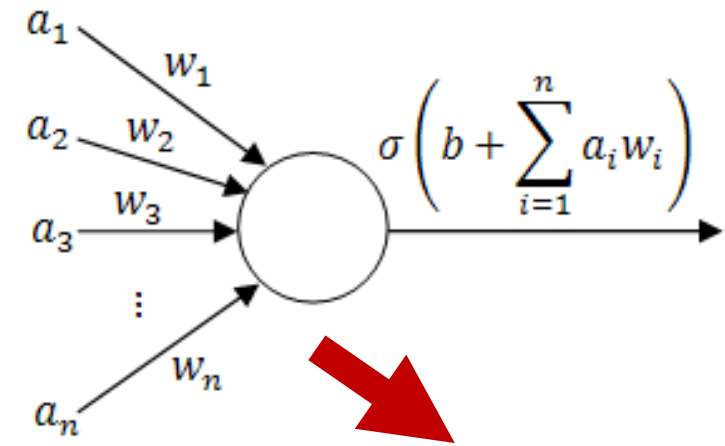
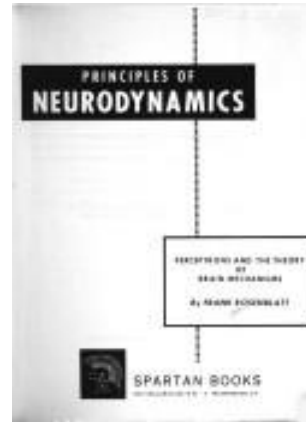
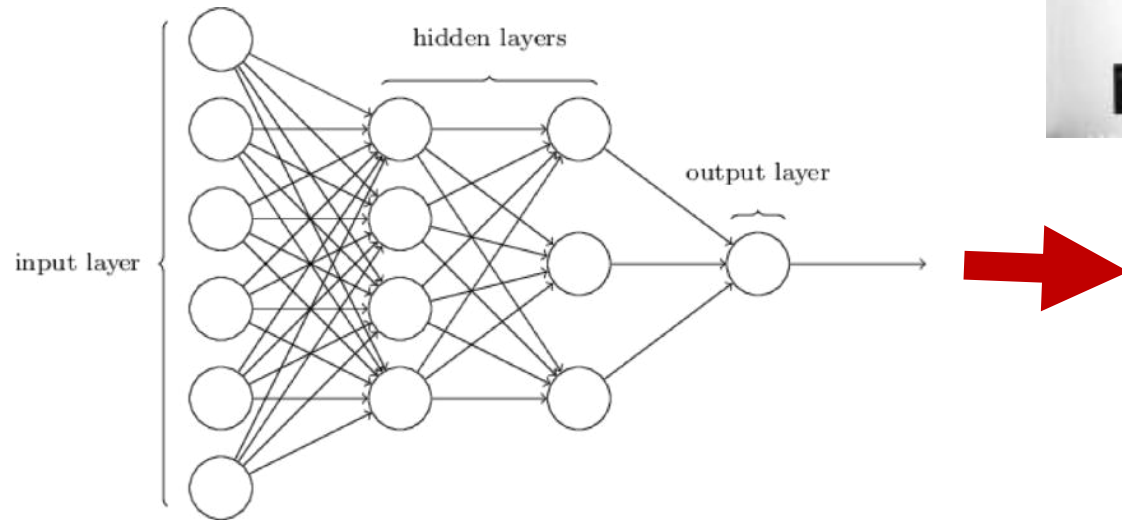


# Lecture overview

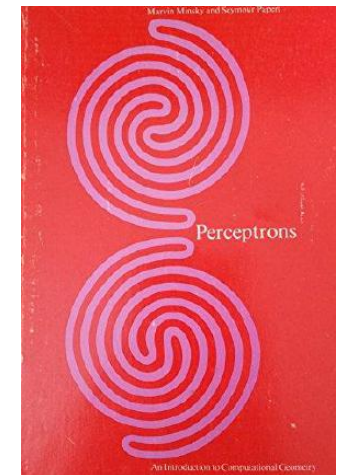
- deep learning
  - computation in a neural net
  - optimization
  - backpropagation
  - training tricks
  - convolutional neural networks
- 
- **Disclaimer:** Much of the material and slides for this lecture were borrowed from
    - Costis Daskalakis and Aleksander Madry's MIT 6.883 class
    - Bill Freeman, Antonio Torralba and Phillip Isola's MIT 6.869 class

# Humble beginnings

- Perceptron [Rosenblatt '58]

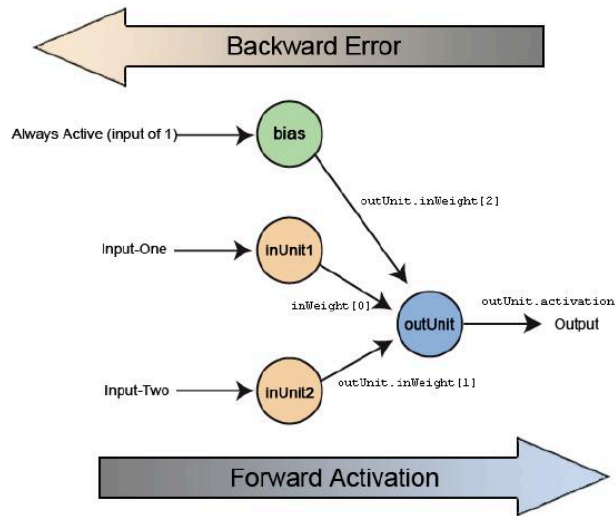


- Criticism of Perceptrons (XOR affair) [Minsky Papert '69]
  - Effectively causes a "deep learning winter"



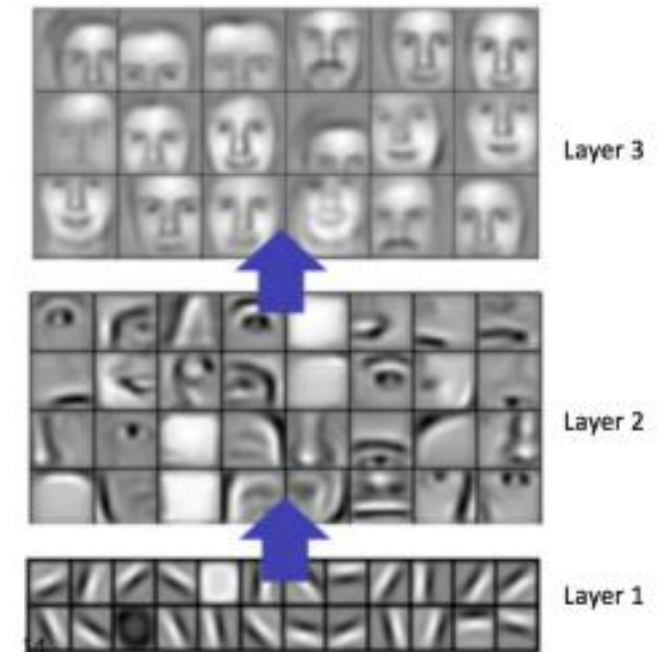
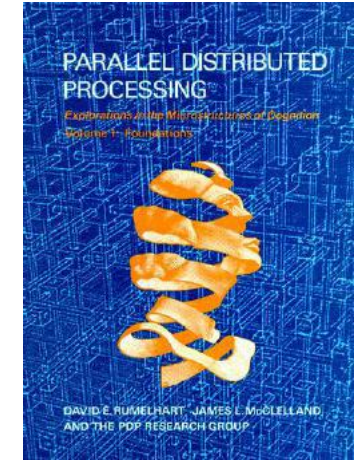
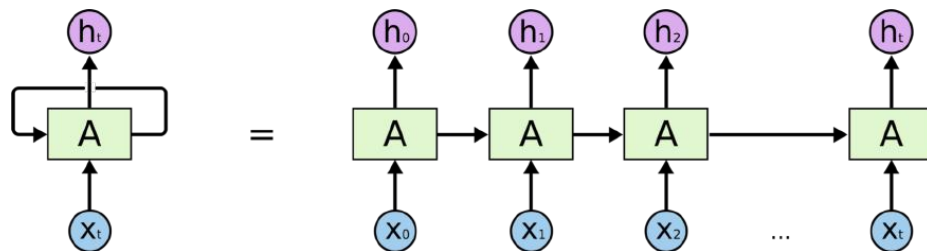
# (Early) Spring

- Back-propagation [Rumelhart et al. '86, LeCun '85, Parker '85]



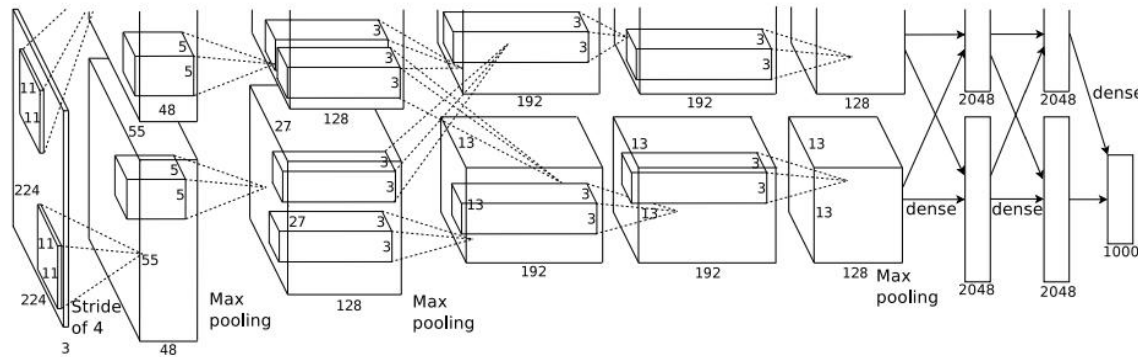
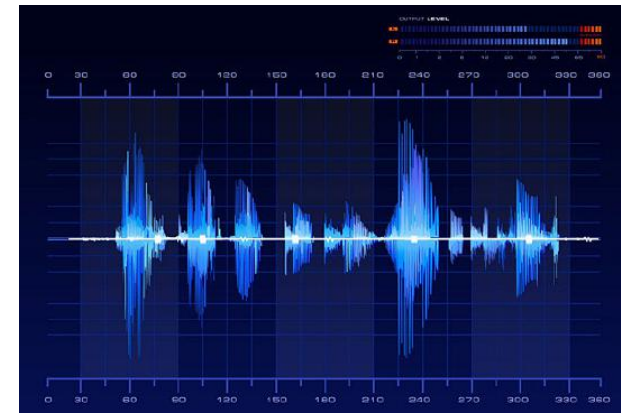
- Convolutional layers [LeCun et al. '90]

- Recurrent Neural Networks/Long Short-Term Memory (LSTM) [Hochreiter Schmidhuber '97]

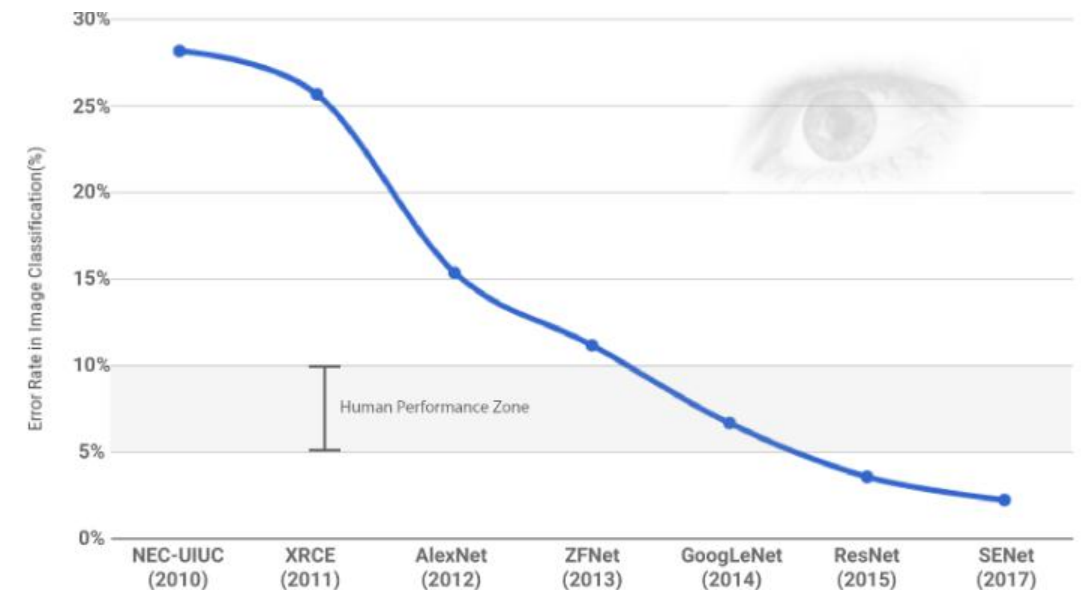


# Summer

- 2006: First big success: speech recognition
- 2012: Breakthrough in computer vision: AlexNet [Krizhevsky et al. '12]



- 2015: Deep learning-based vision models outperform humans





# What enabled this success?

- Better architectures (e.g., ReLUs) and regularization techniques (e.g. Dropout)

IMAGENET

- Sufficiently large datasets



- Enough computational power



# Deep learning

- Modeling the visual world is incredibly complicated. We need high capacity models.
- In the past, we didn't have enough data to fit these models. But now we do!
- We want a class of **high capacity models** that are **easy to optimize**.

**Deep neural networks!**





Classification  
units



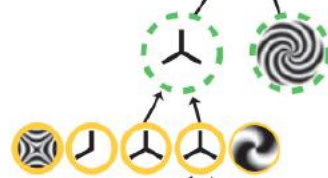
PIT/AIT



V4/PIT



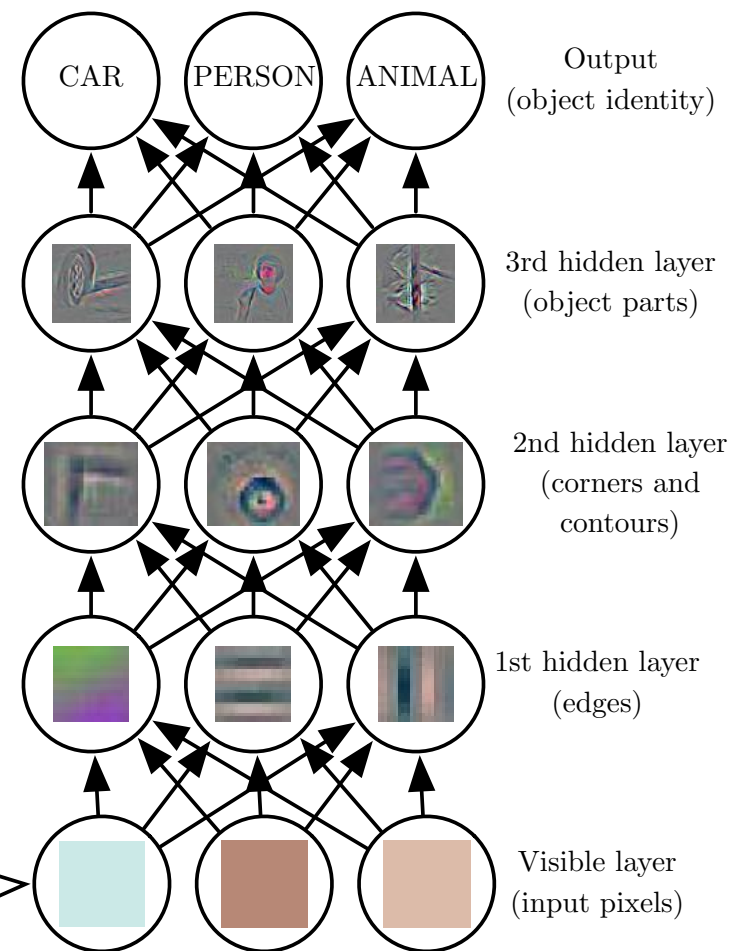
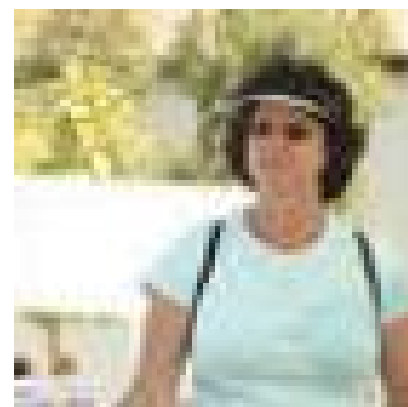
V2/V4



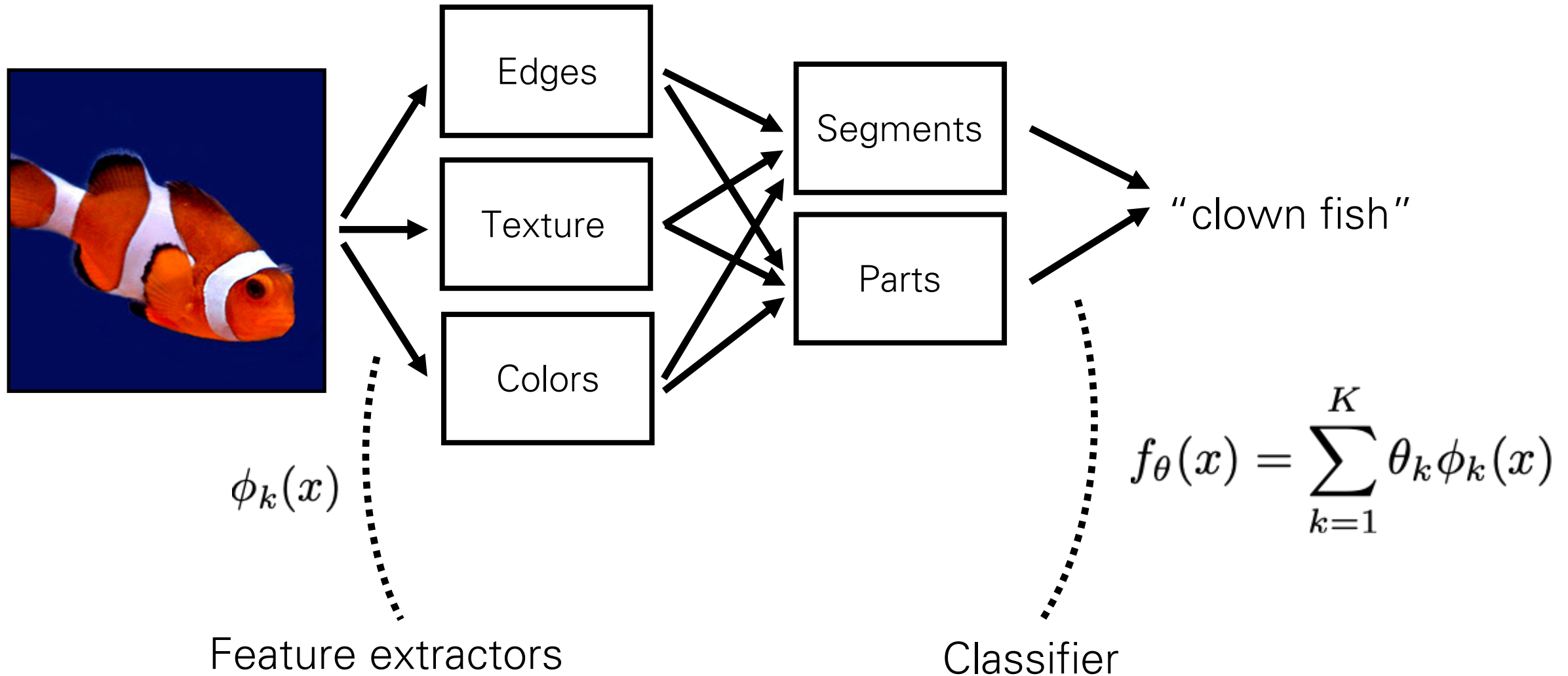
V1/V2



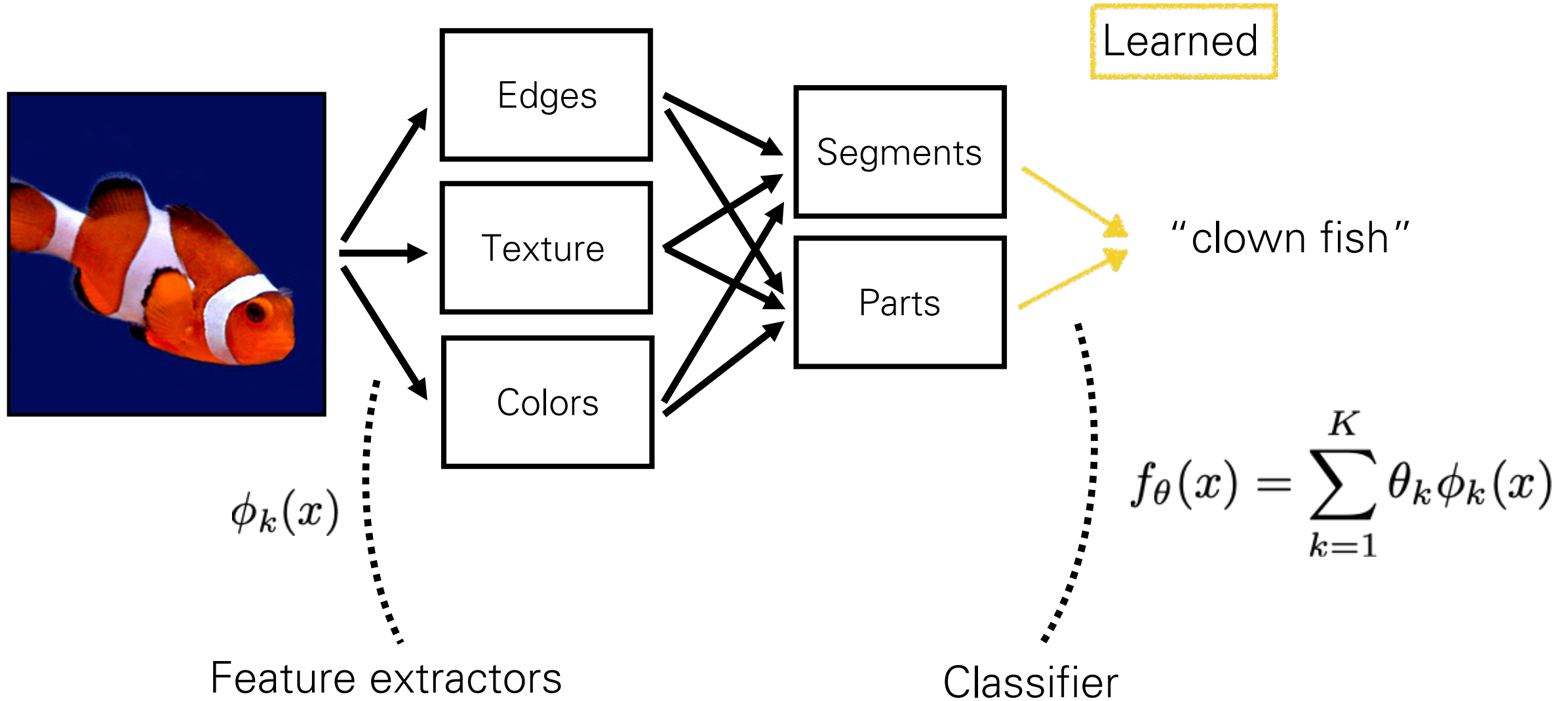
Serre, 2014



# Object recognition

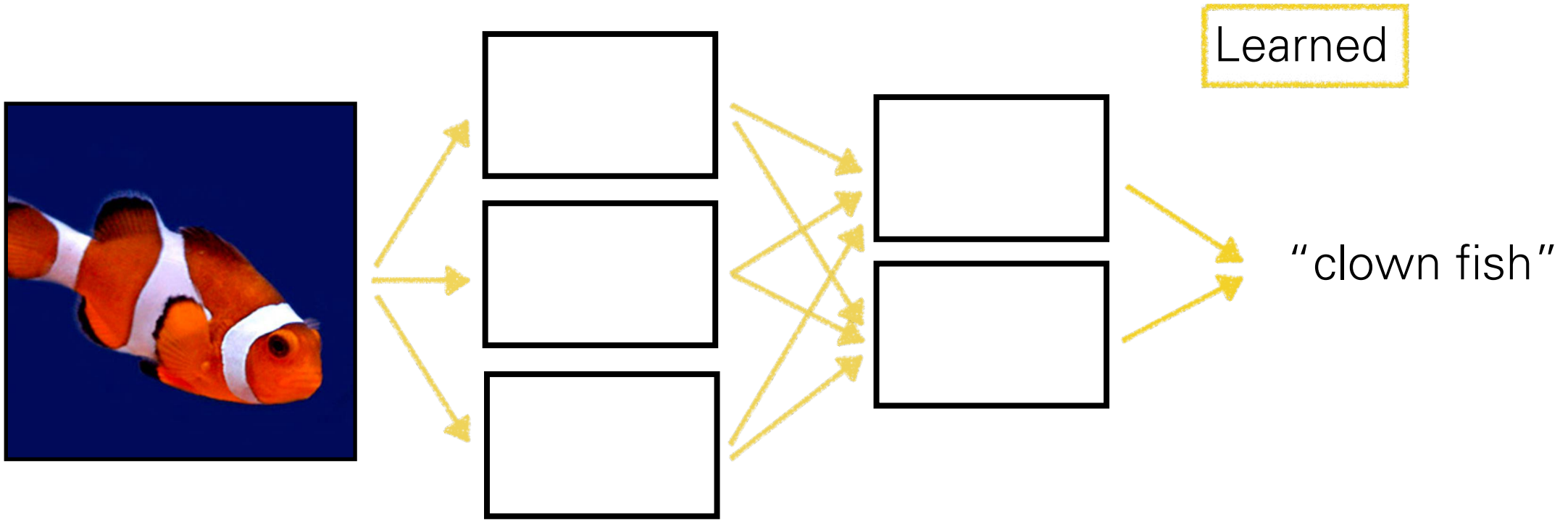


# Object recognition

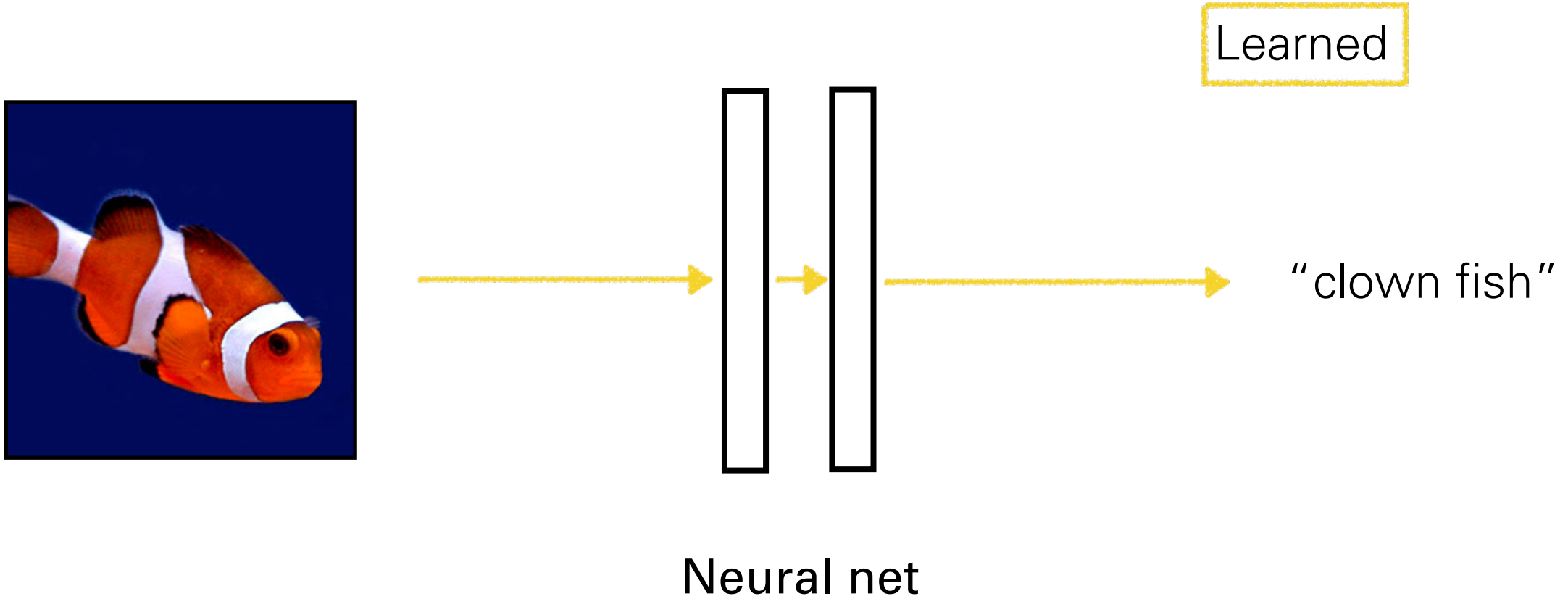




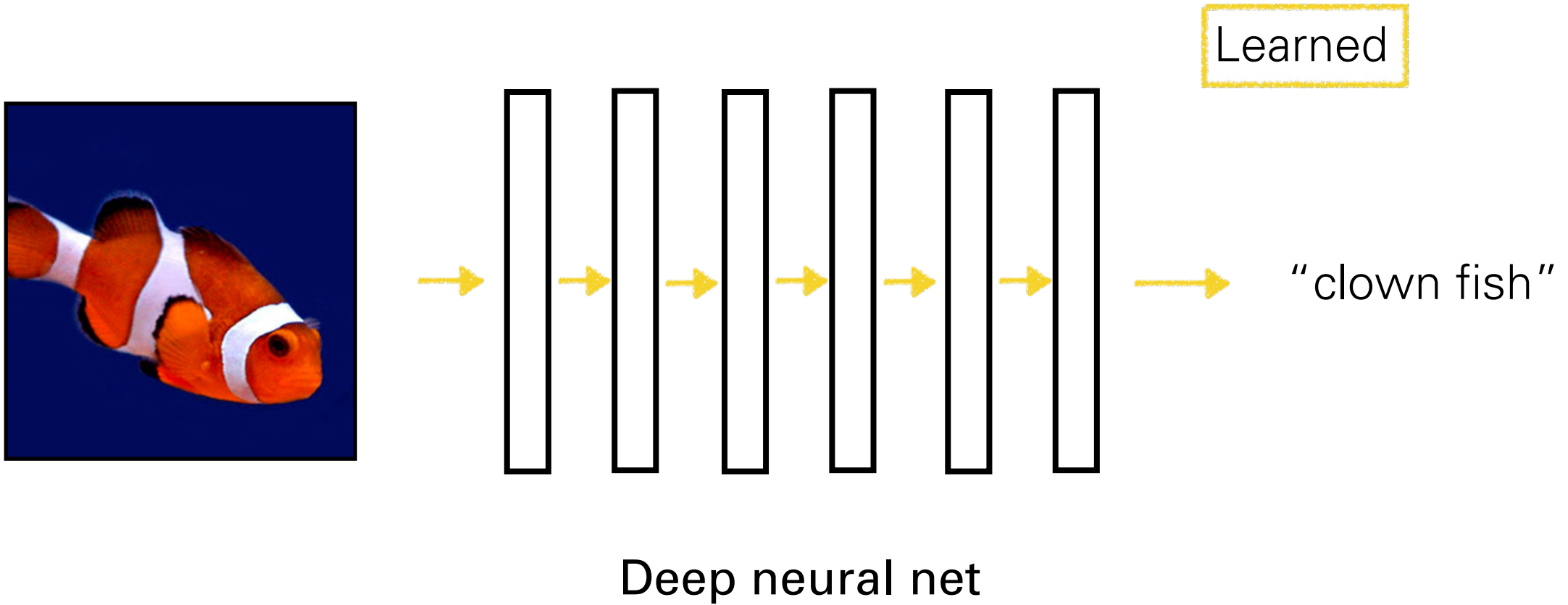
# Object recognition



# Object recognition



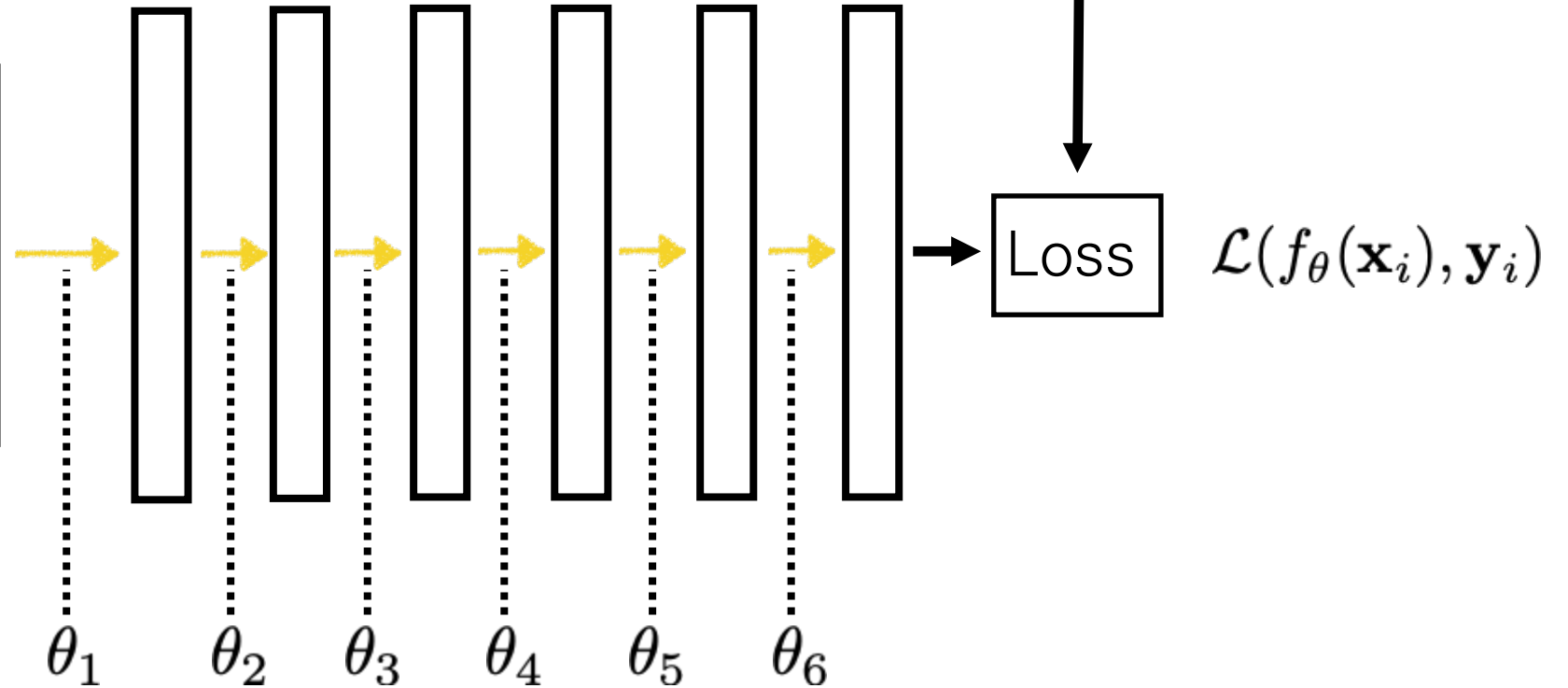
# Object recognition





# Deep learning

$y_i$   
"clown fish"

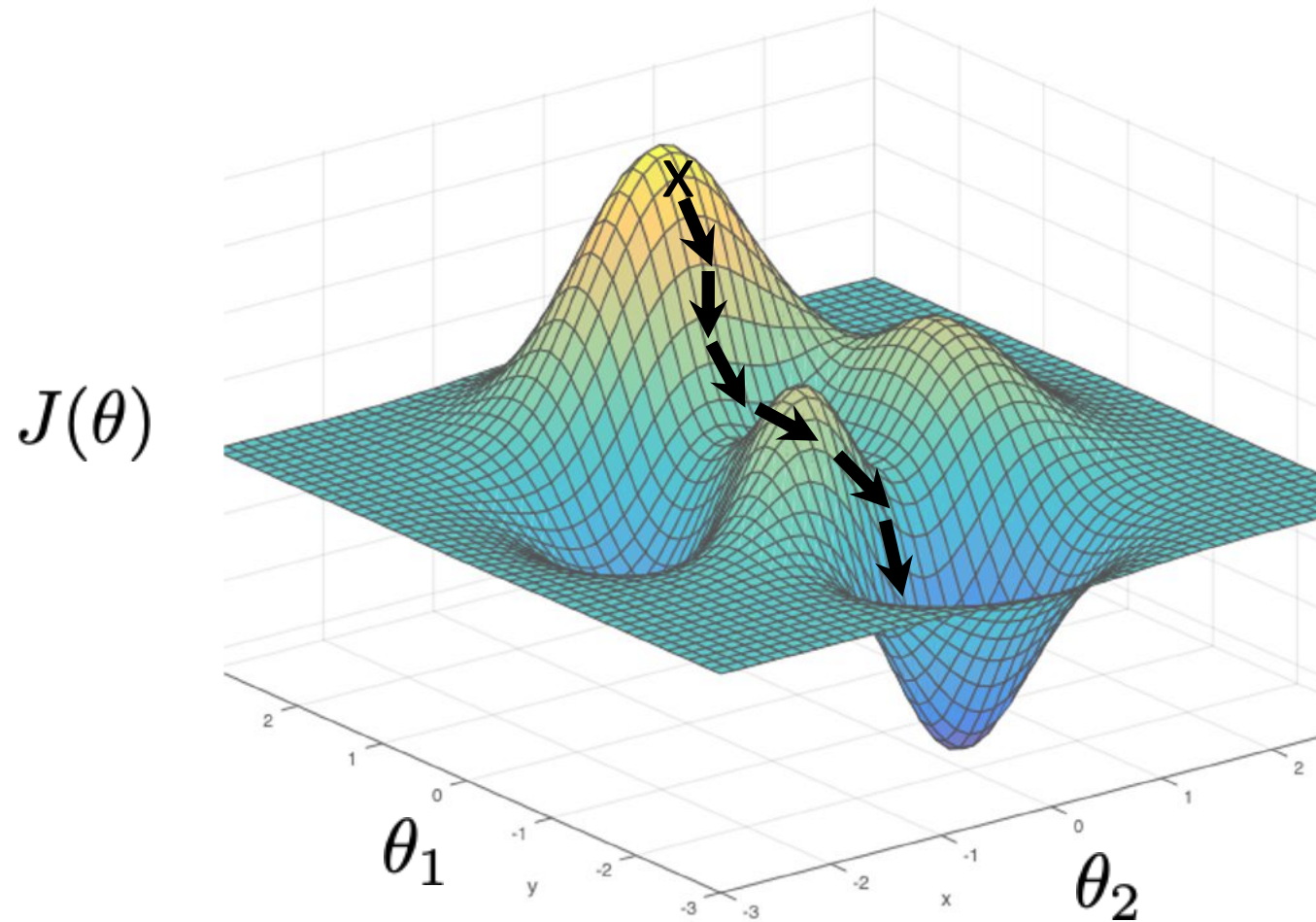


$$\theta^* = \arg \min_{\theta} \sum_{i=1}^N \mathcal{L}(f_{\theta}(\mathbf{x}_i), y_i)$$

# Gradient descent

$$\theta^* = \arg \min_{\theta} \underbrace{\sum_{i=1}^N \mathcal{L}(f_{\theta}(\mathbf{x}_i), \mathbf{y}_i)}_{J(\theta)}$$

# Gradient descent



$$\theta^* = \arg \min_{\theta} J(\theta)$$



# Gradient descent

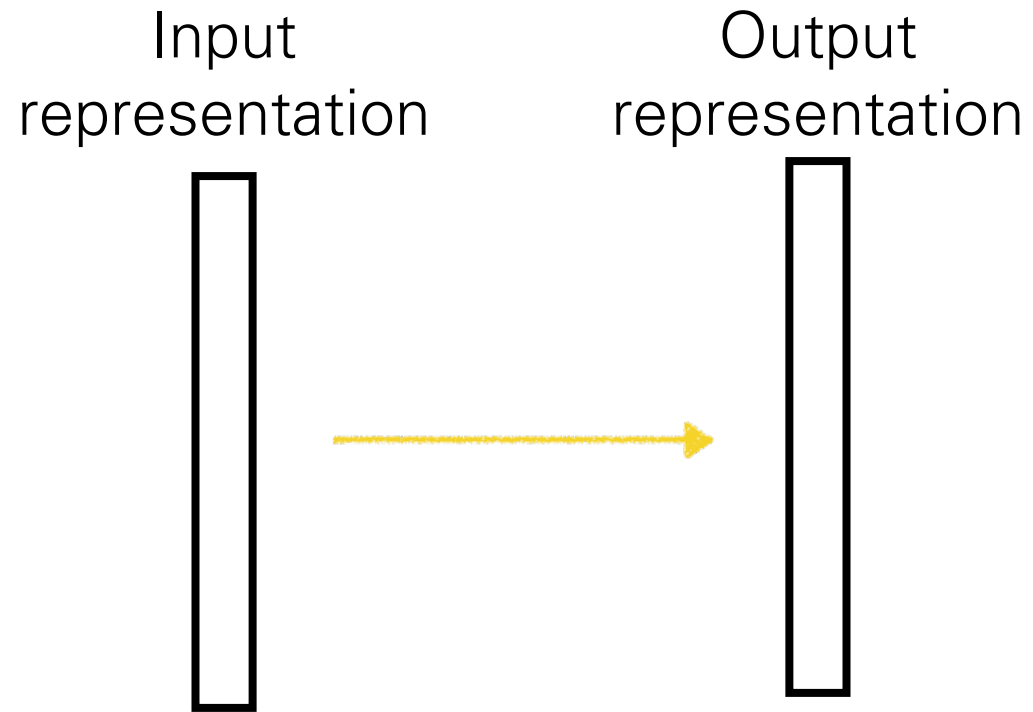
$$\theta^* = \arg \min_{\theta} \underbrace{\sum_{i=1}^N \mathcal{L}(f_{\theta}(\mathbf{x}_i), \mathbf{y}_i)}_{J(\theta)}$$

One iteration of gradient descent:

$$\theta^{t+1} = \theta^t - \eta_t \left. \frac{\partial J(\theta)}{\partial \theta} \right|_{\theta=\theta^t}$$

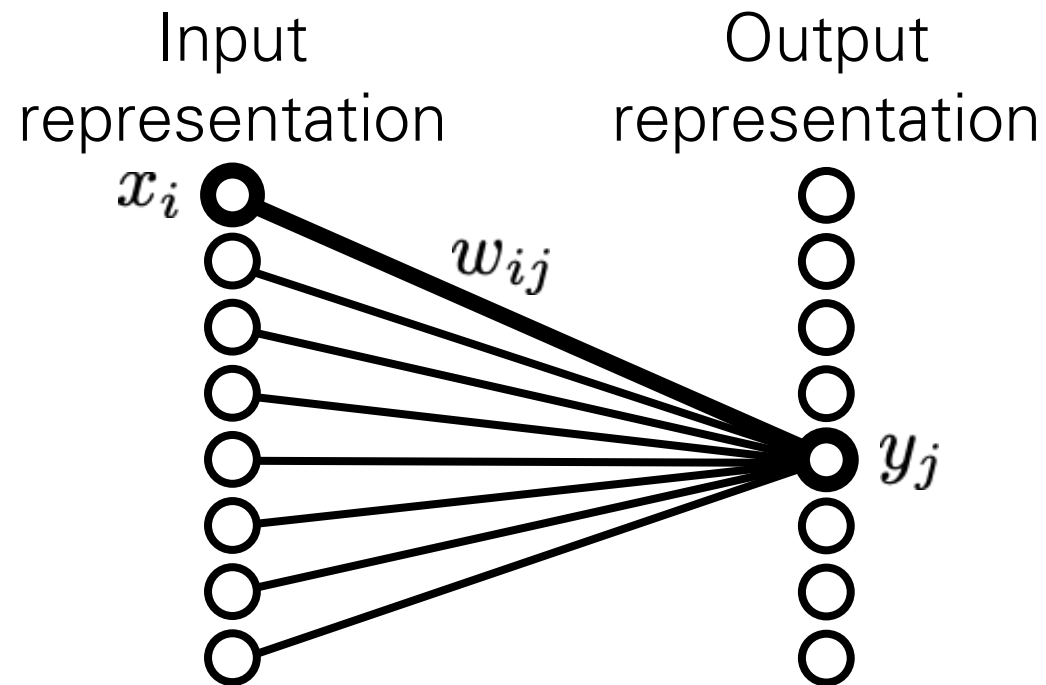
learning rate

# Computation in a neural net



# Computation in a neural net

Linear layer

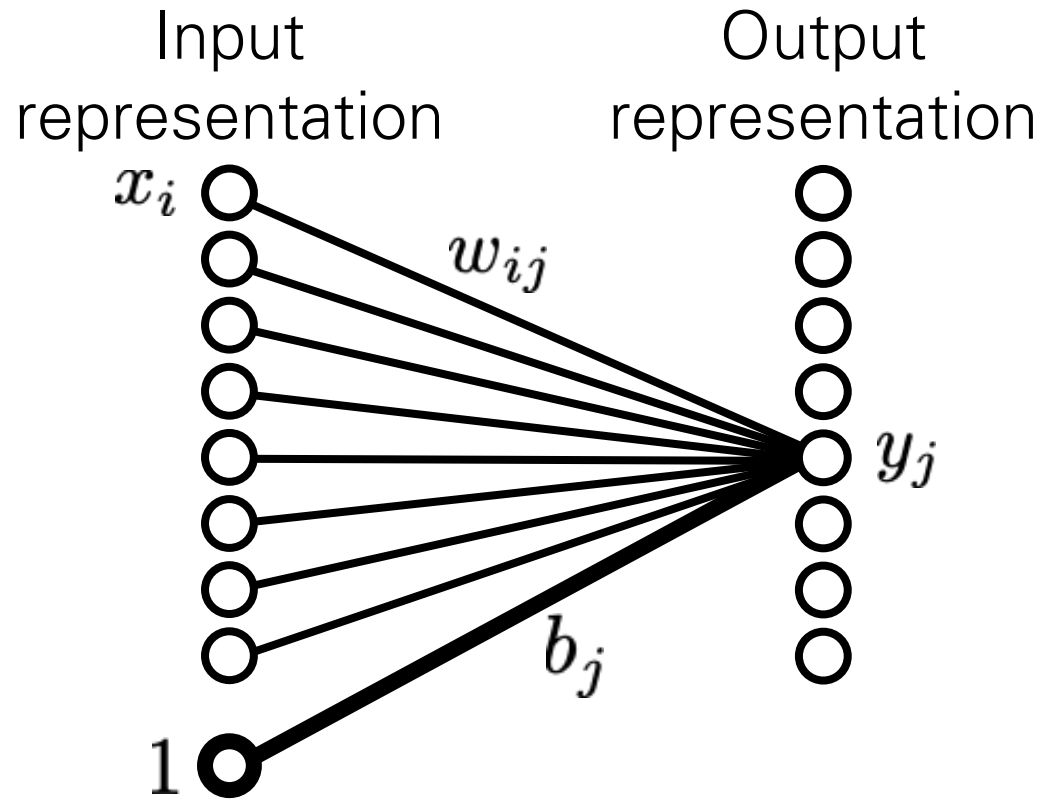


$$y_j = \sum_i w_{ij} x_i$$



# Computation in a neural net

## Linear layer



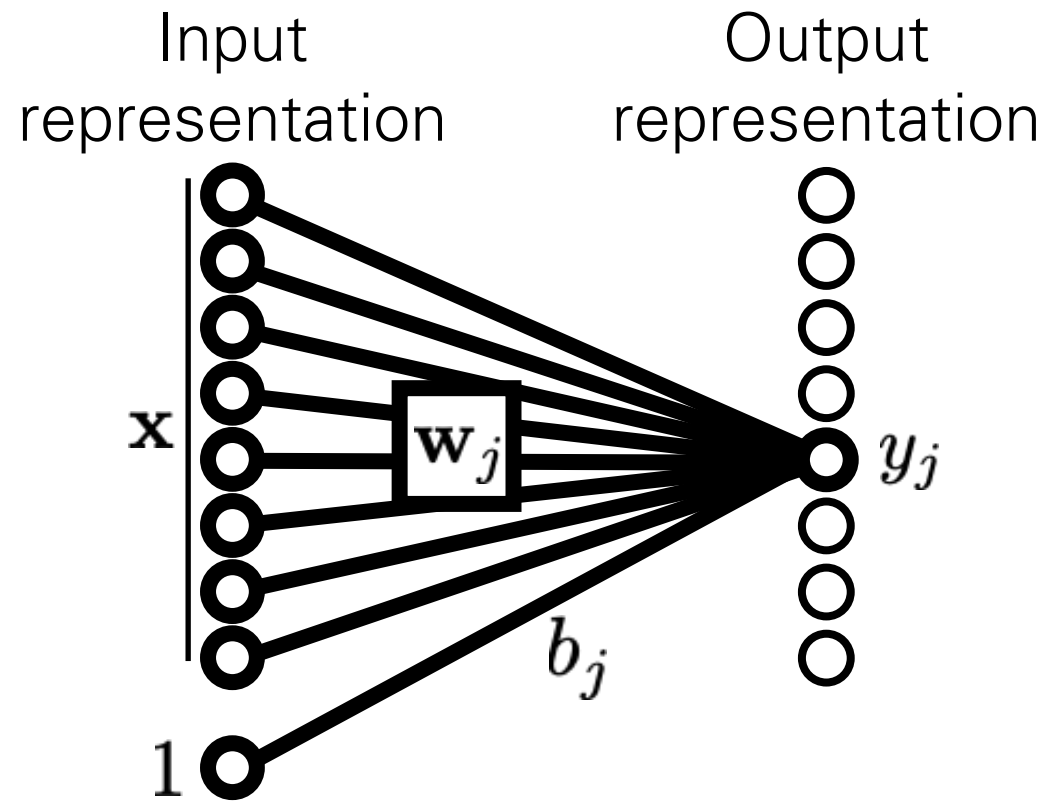
$$y_j = \sum_i w_{ij} x_i + b_j$$

weights

bias

# Computation in a neural net

## Linear layer



$$y_j = \mathbf{x}^T \mathbf{w}_j + b_j$$

weights

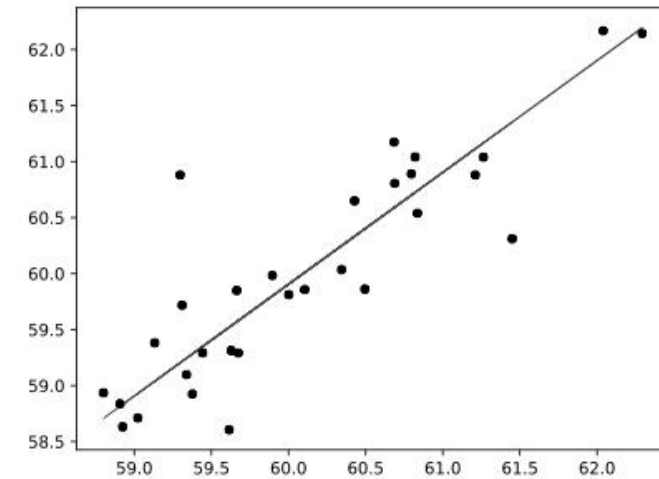
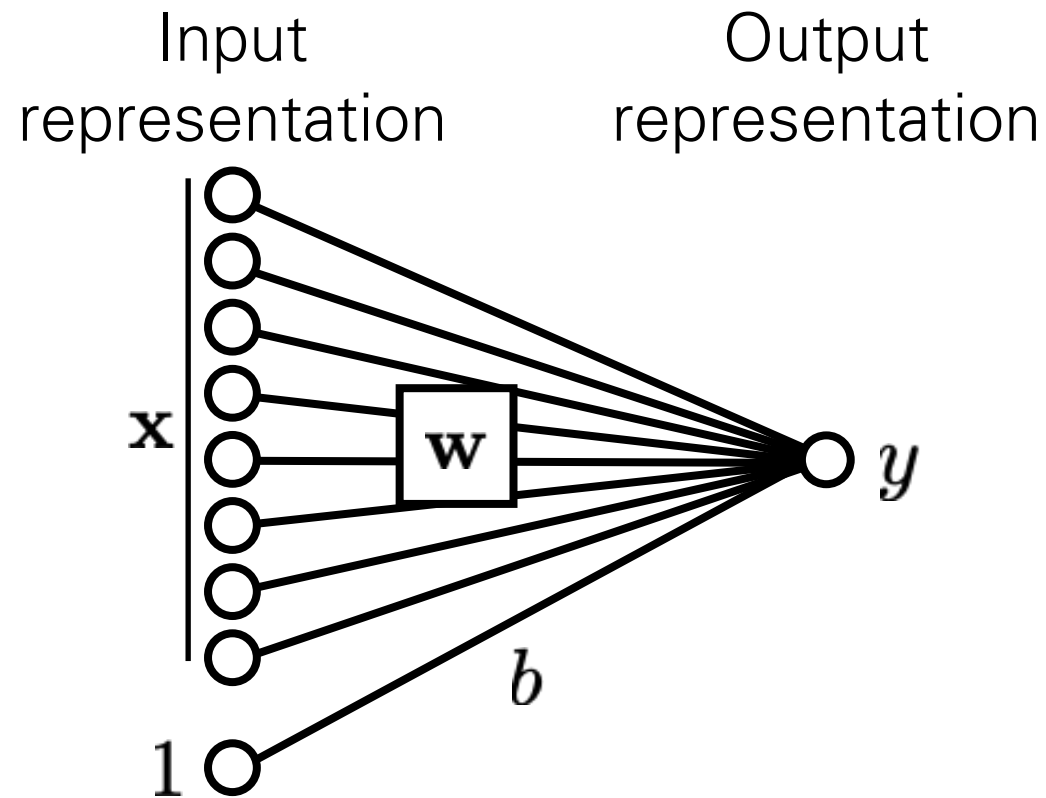
bias

$$\theta = \{\mathbf{W}, \mathbf{b}\}$$

parameters of the model

# Example: linear regression with a neural net

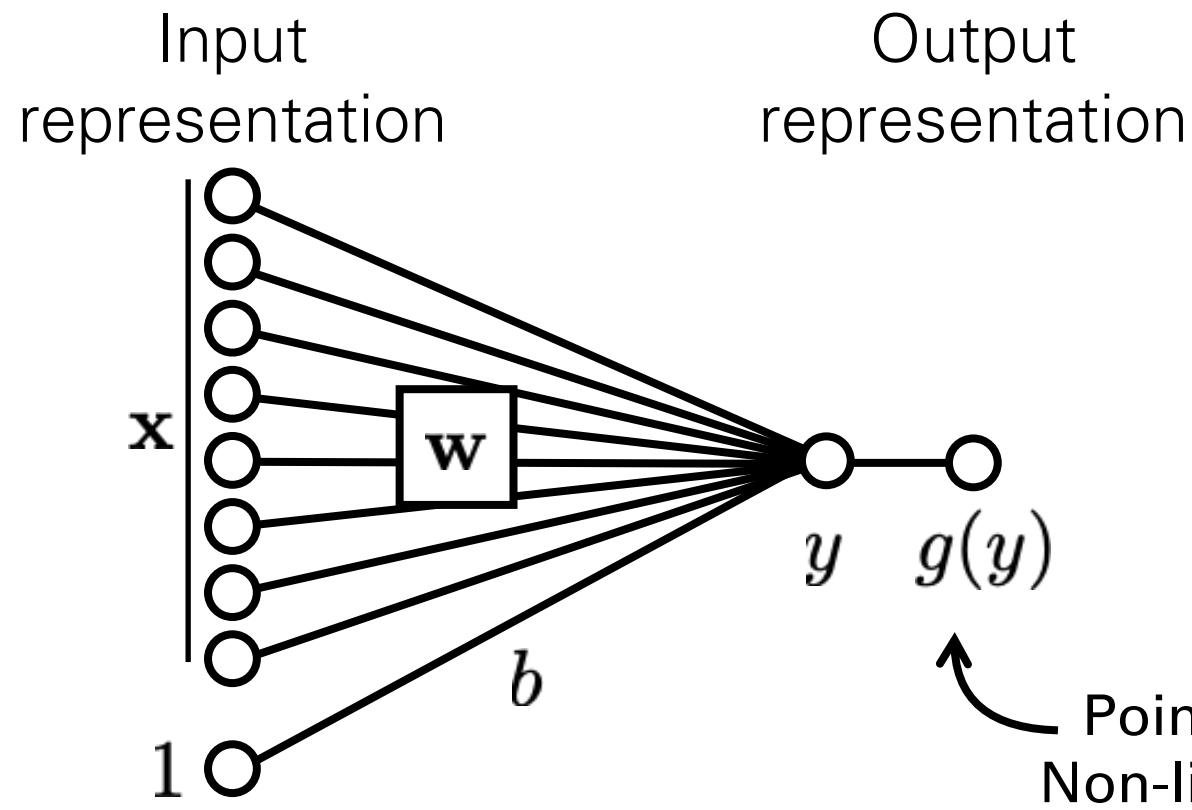
Linear layer



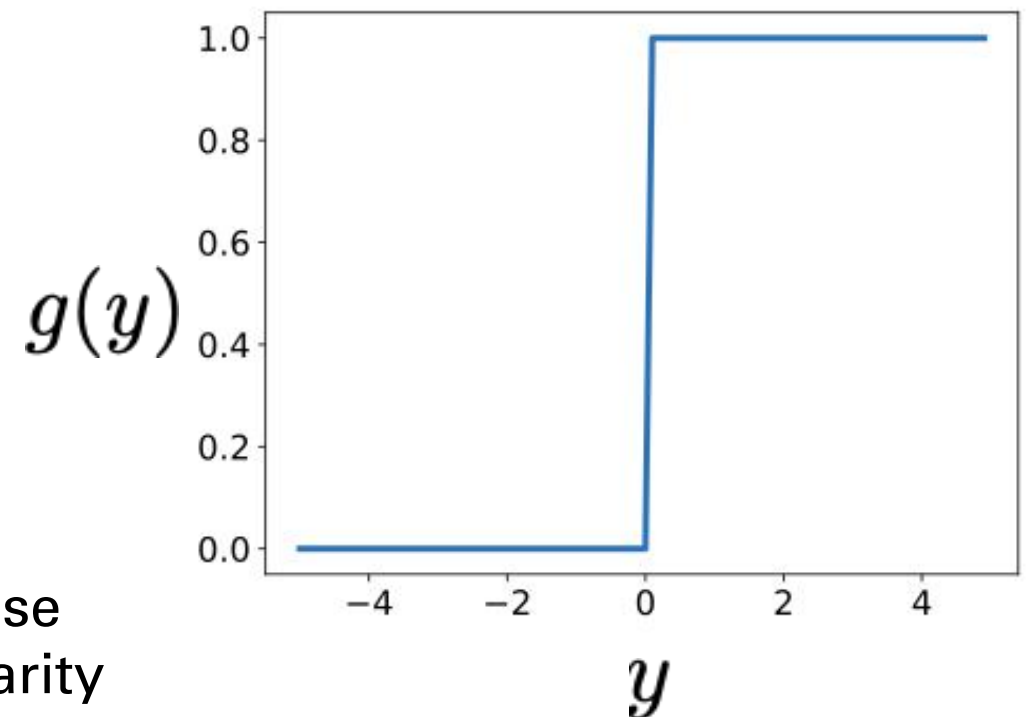
$$f_{\mathbf{w},b}(\mathbf{x}) = \mathbf{x}^T \mathbf{w} + b$$

# Computation in a neural net

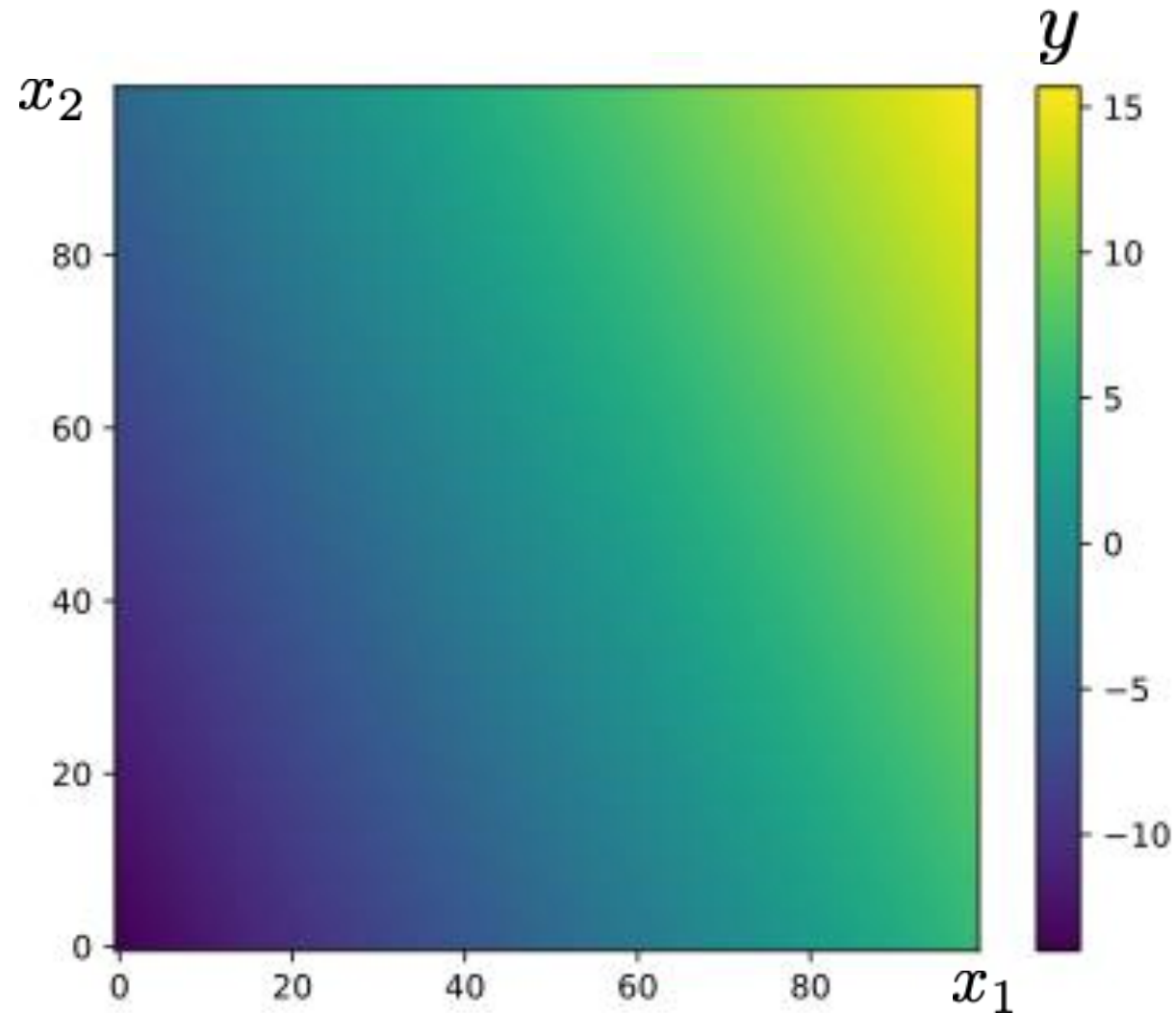
“Perceptron”



$$g(y) = \begin{cases} 1, & \text{if } y > 0 \\ 0, & \text{otherwise} \end{cases}$$



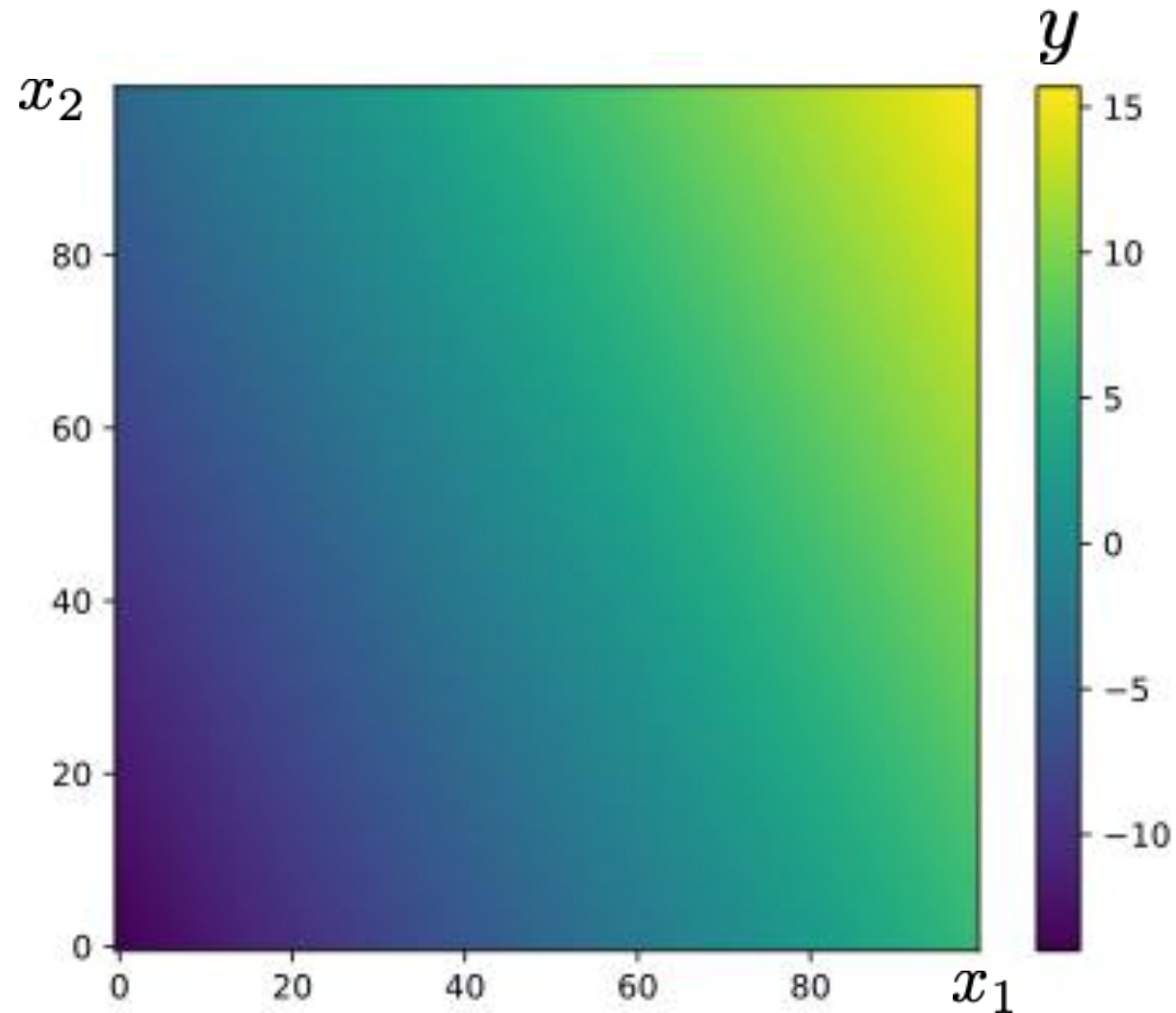
# Example: linear classification with a perceptron



$$y = \mathbf{x}^T \mathbf{w} + b$$



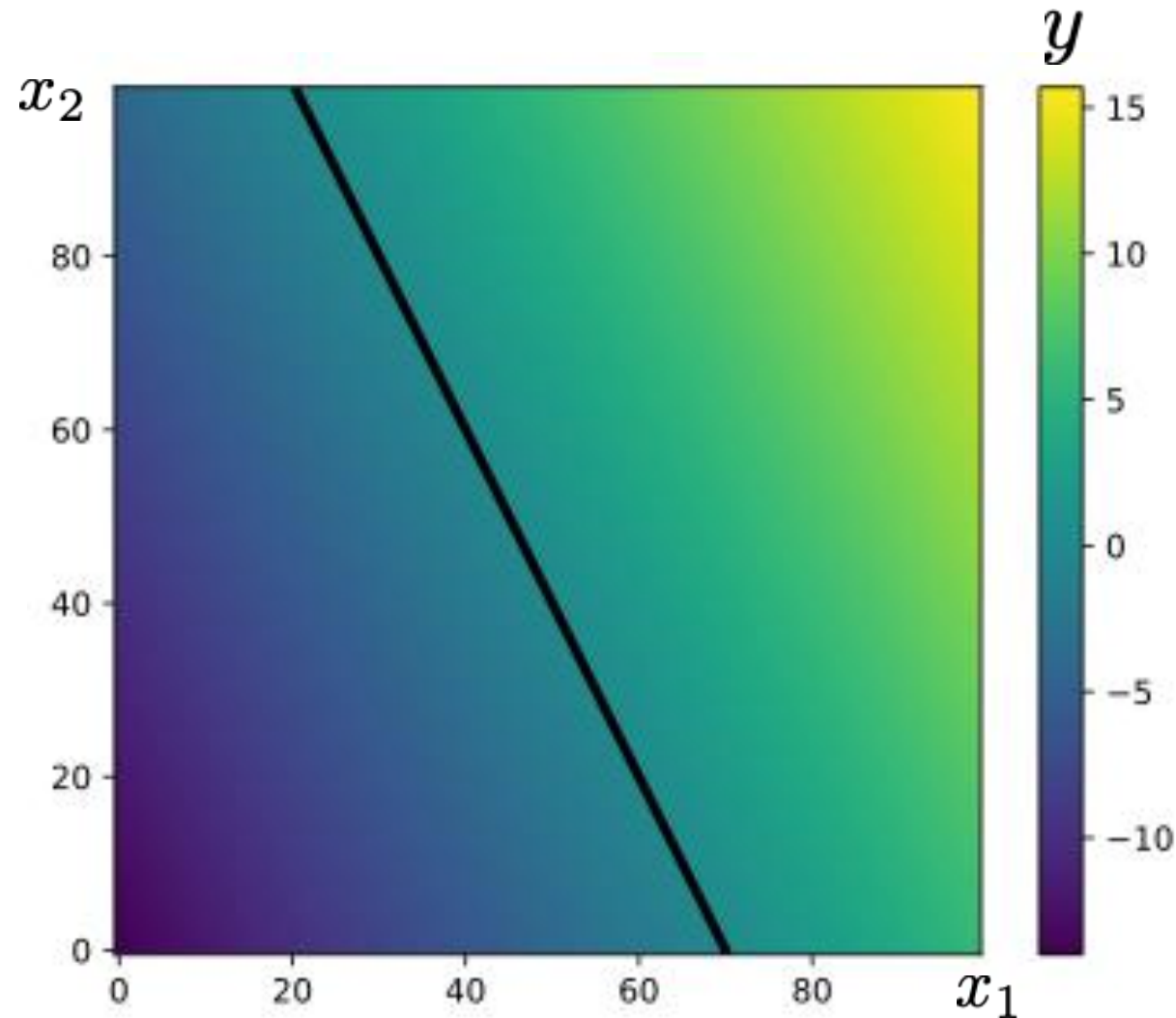
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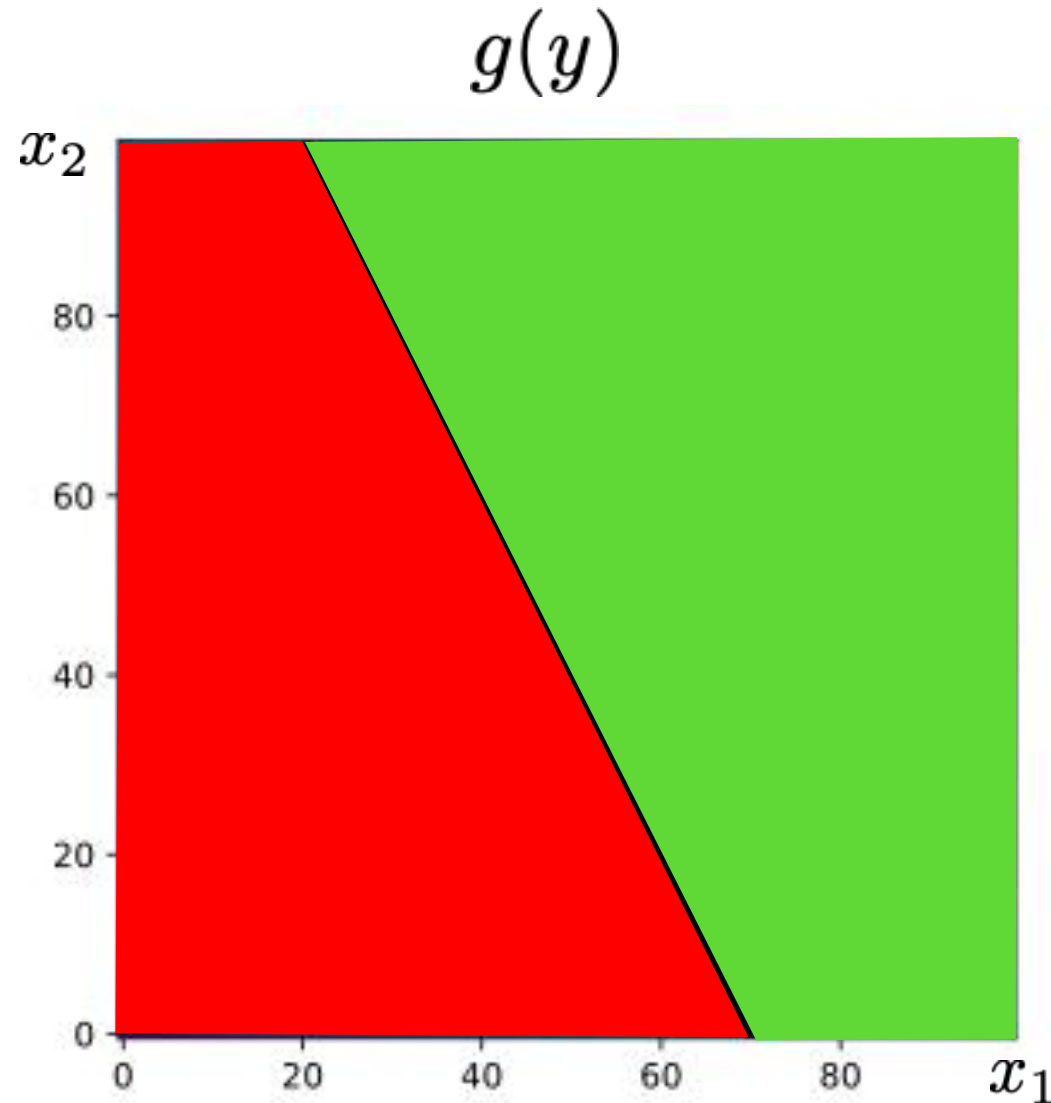
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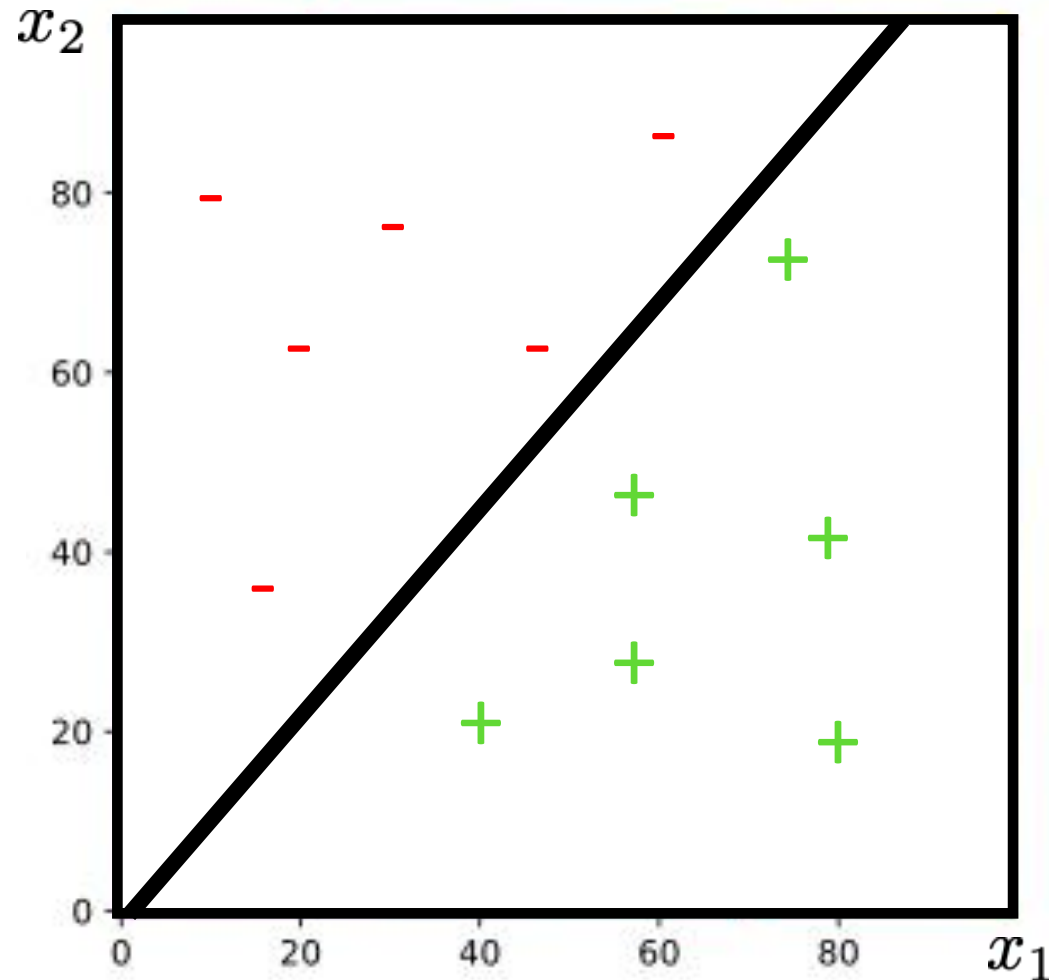
# Example: linear classification with a perceptron



$$y = \mathbf{x}^T \mathbf{w} + b$$

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# Example: linear classification with a perceptron

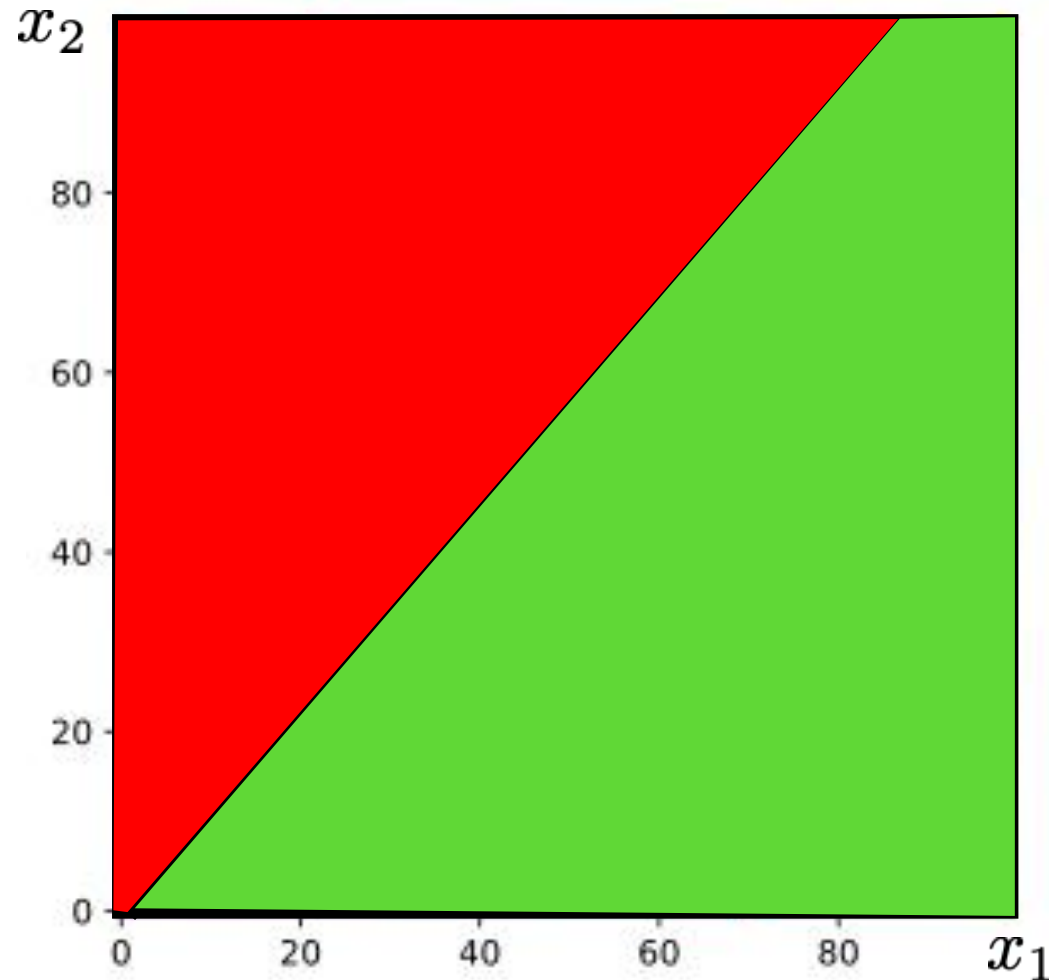


$$\hat{y} = \mathbf{x}^T \mathbf{w} + b$$

$$g(\hat{y}) = \begin{cases} 1, & \text{if } \hat{y} > 0 \\ 0, & \text{otherwise} \end{cases}$$

$$\mathbf{w}^*, b^* = \arg \min_{\mathbf{w}, b} \mathcal{L}(g(\hat{y}), y_i)$$

# Example: linear classification with a perceptron



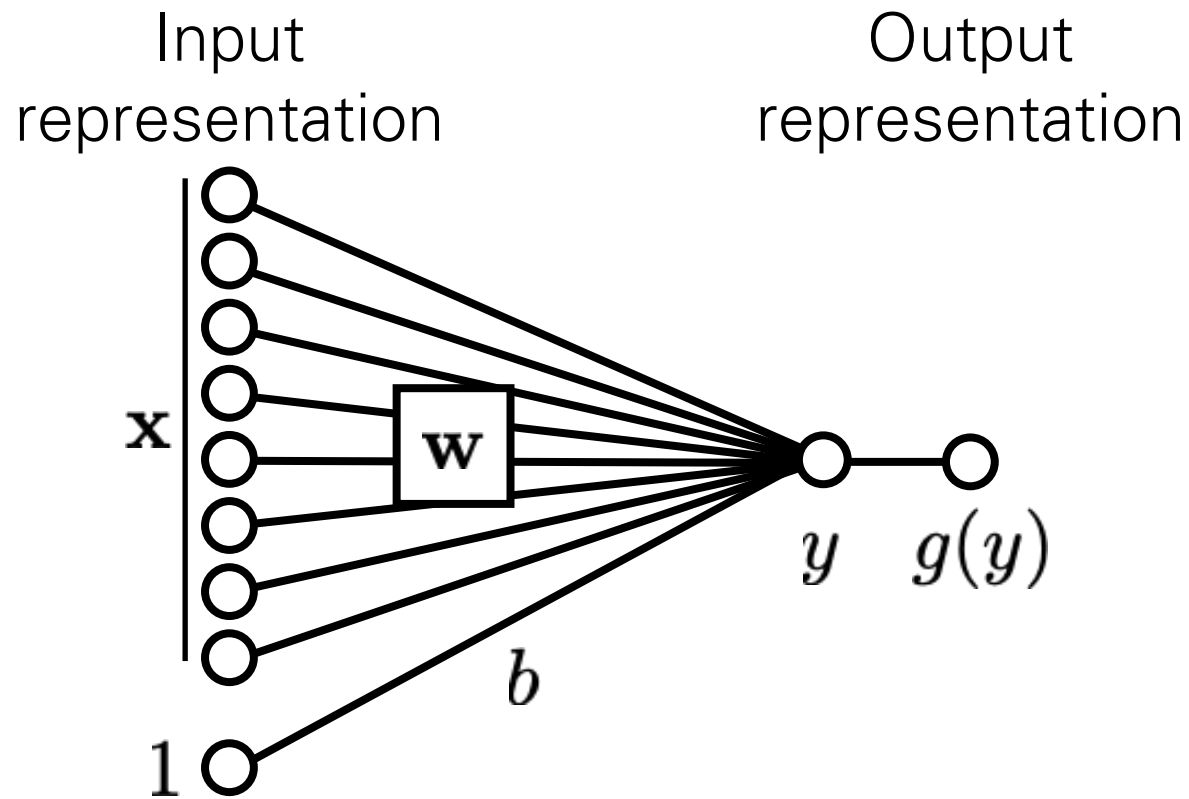
$$\hat{y} = \mathbf{x}^T \mathbf{w} + b$$

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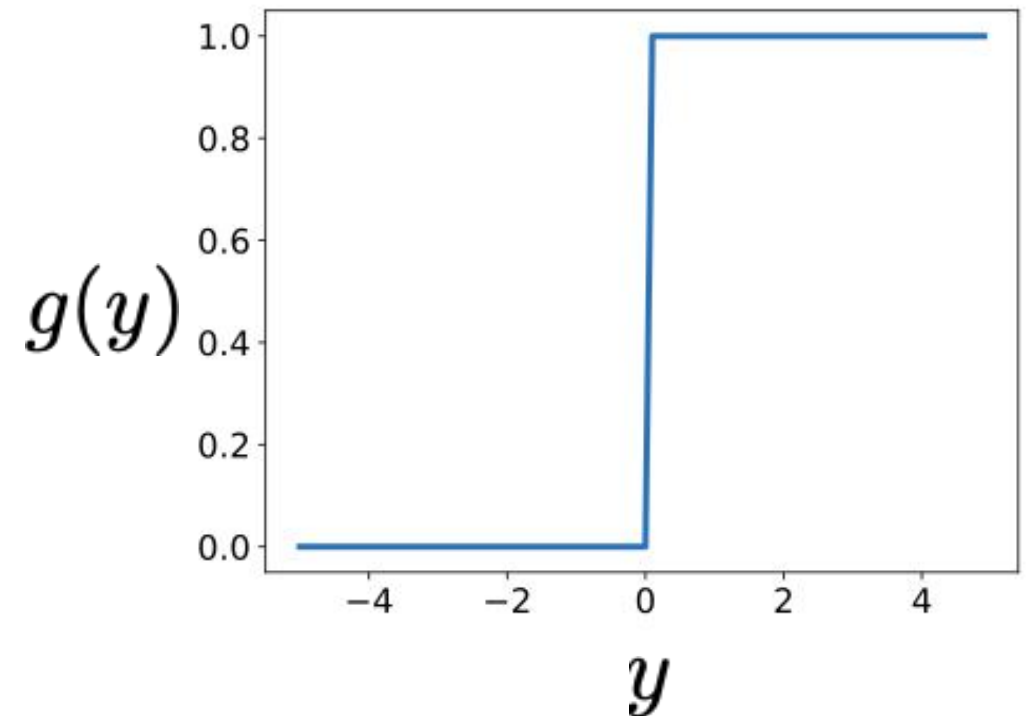
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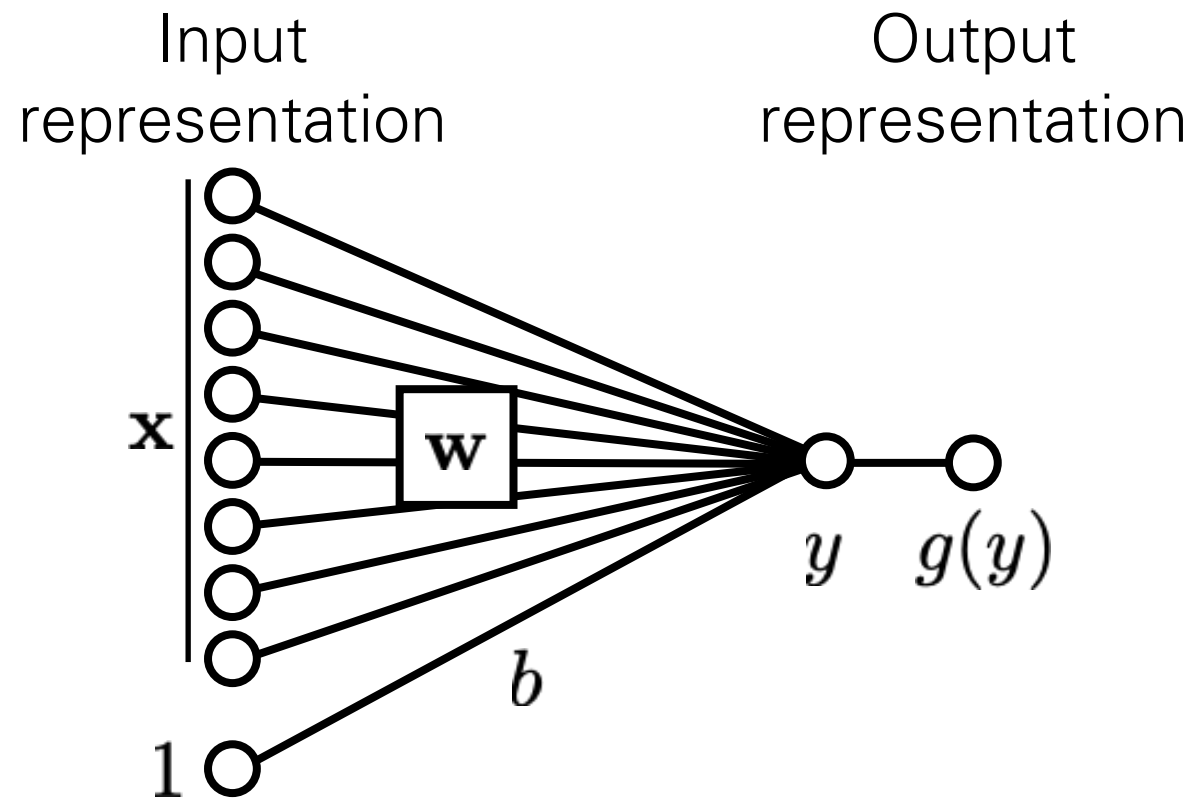
# Computation in a neural net



$$g(y) = \begin{cases} 1, & \text{if } y > 0 \\ 0, & \text{otherwise} \end{cases}$$

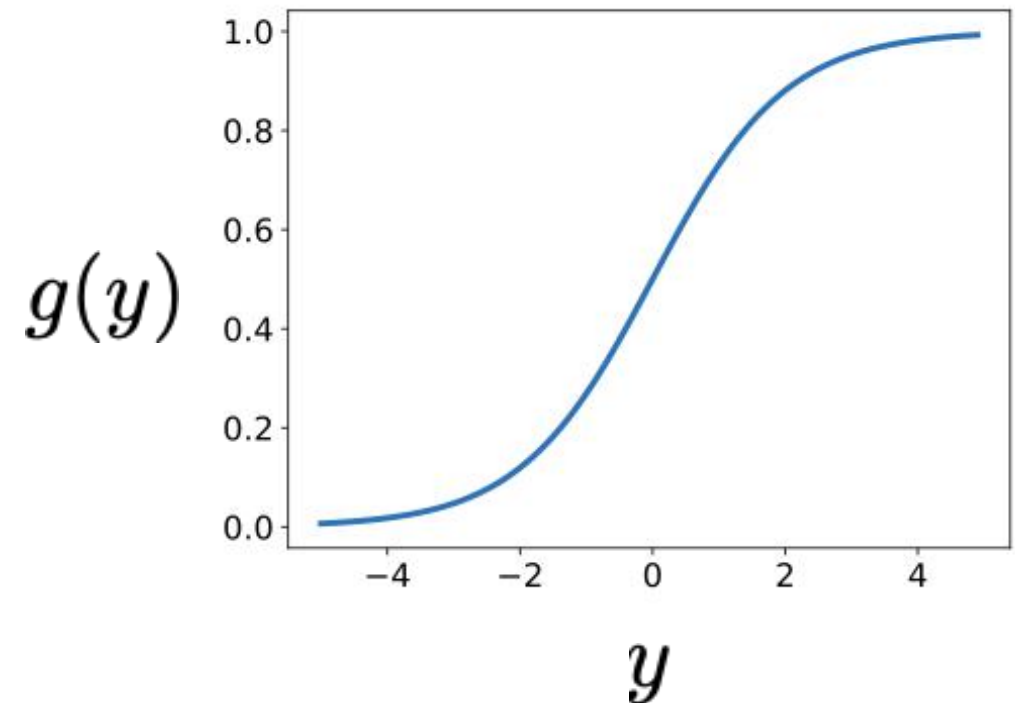


# Computation in a neural net – nonlinearity



Sigmoid

$$g(y) = \frac{1}{1 + e^{-y}}$$

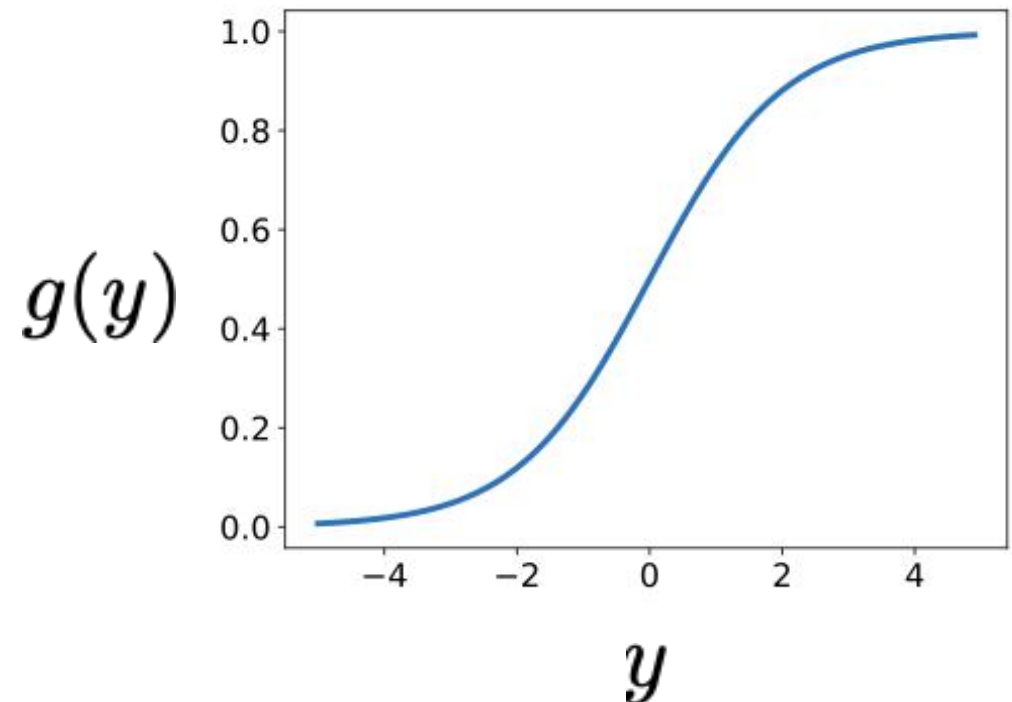


# Computation in a neural net – nonlinearity

- Interpretation as firing rate of neuron
- Bounded between [0,1]
- Saturation for large +/- inputs
- Gradients go to zero
- Outputs centered at 0.5  
(poor conditioning)
- Not used in practice

Sigmoid

$$g(y) = \frac{1}{1 + e^{-y}}$$



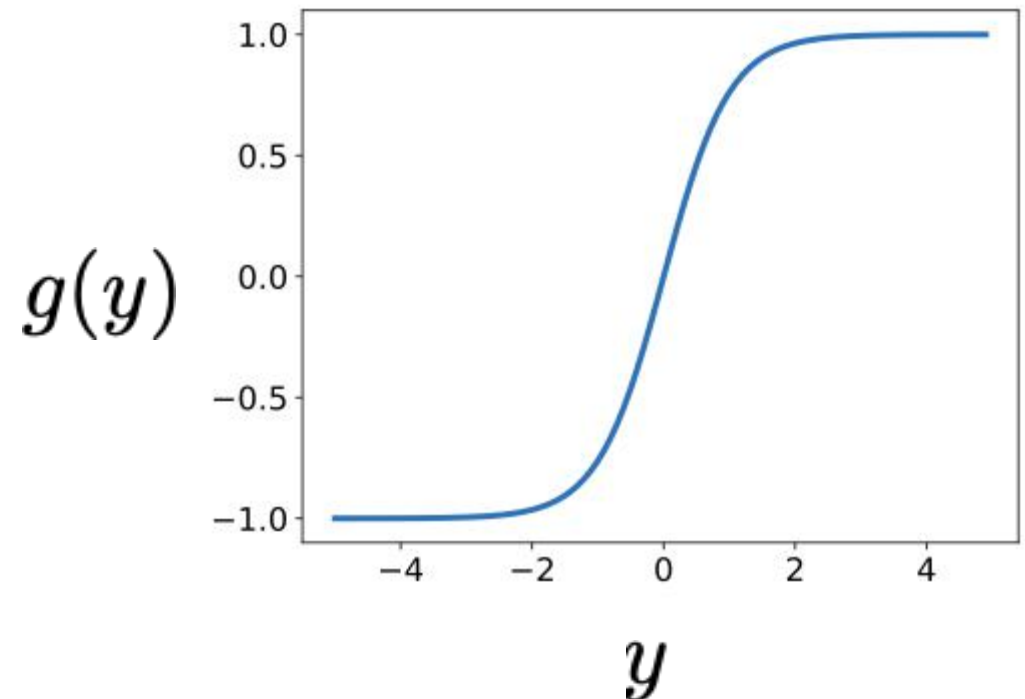
# Computation in a neural net – nonlinearity

- Bounded between  $[-1, +1]$
- Saturation for large +/- inputs
- Gradients go to zero
- Outputs centered at 0
- Preferable to sigmoid

$$\tanh(x) = 2 \text{ sigmoid}(2x) - 1$$

Tanh

$$g(y) = \frac{e^y - e^{-y}}{e^y + e^{-y}}$$



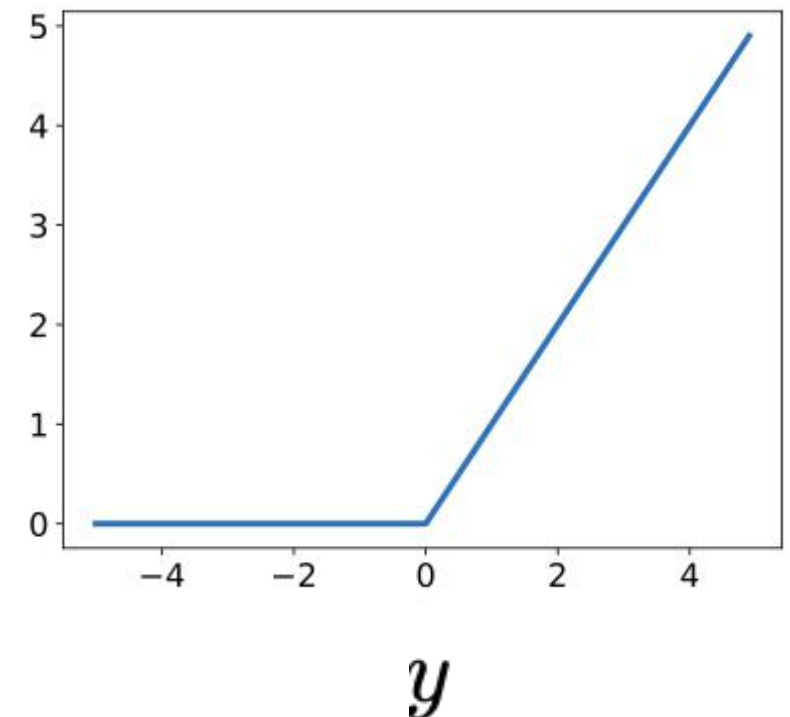
# Computation in a neural net – nonlinearity

- Unbounded output (on positive side)
- Efficient to implement:  $\frac{\partial g}{\partial y} = \begin{cases} 0, & \text{if } y < 0 \\ 1, & \text{if } y \geq 0 \end{cases}$
- Also seems to help convergence  
(see 6x speedup vs tanh in [Krizhevsky et al.])
- Drawback: if strongly in negative region, unit is dead forever (no gradient).
- Default choice: widely used in current models.

Rectified linear unit (ReLU)

$$g(y) = \max(0, y)$$

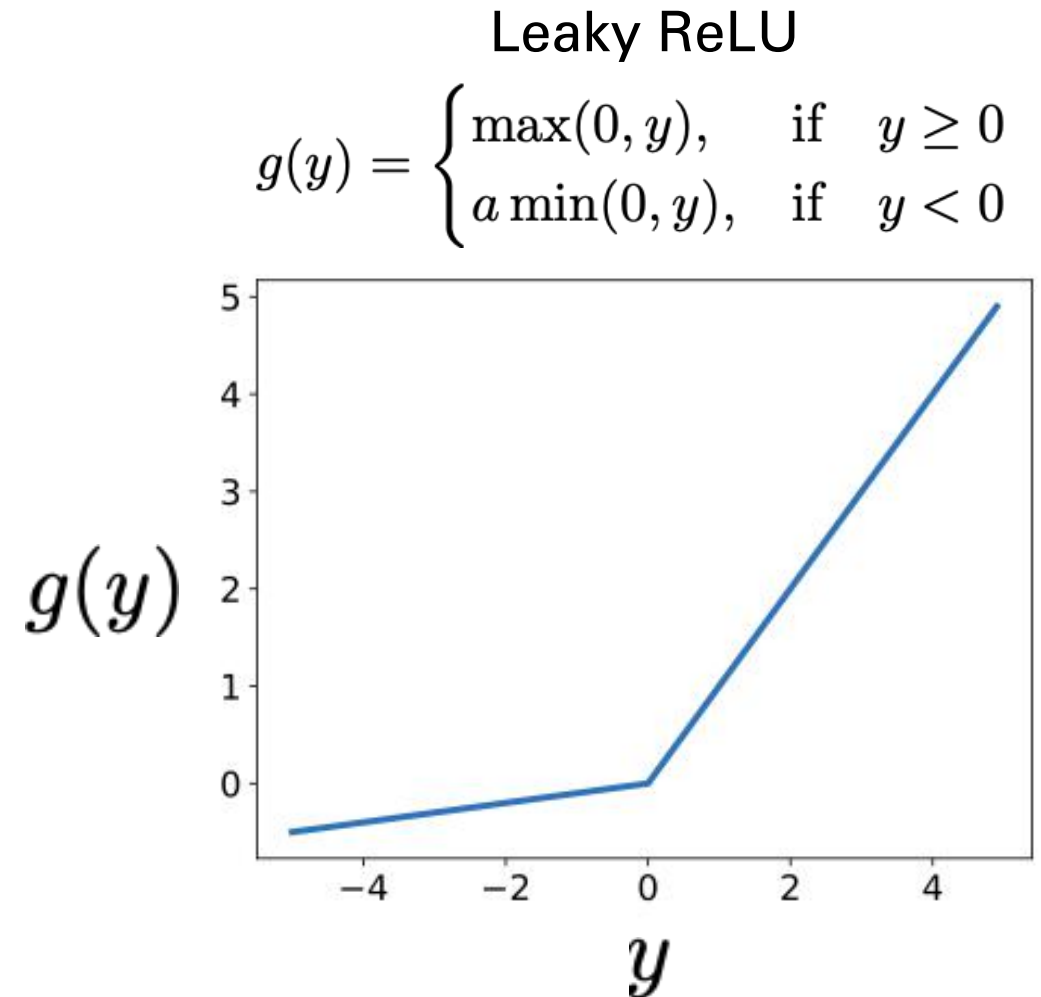
$g(y)$



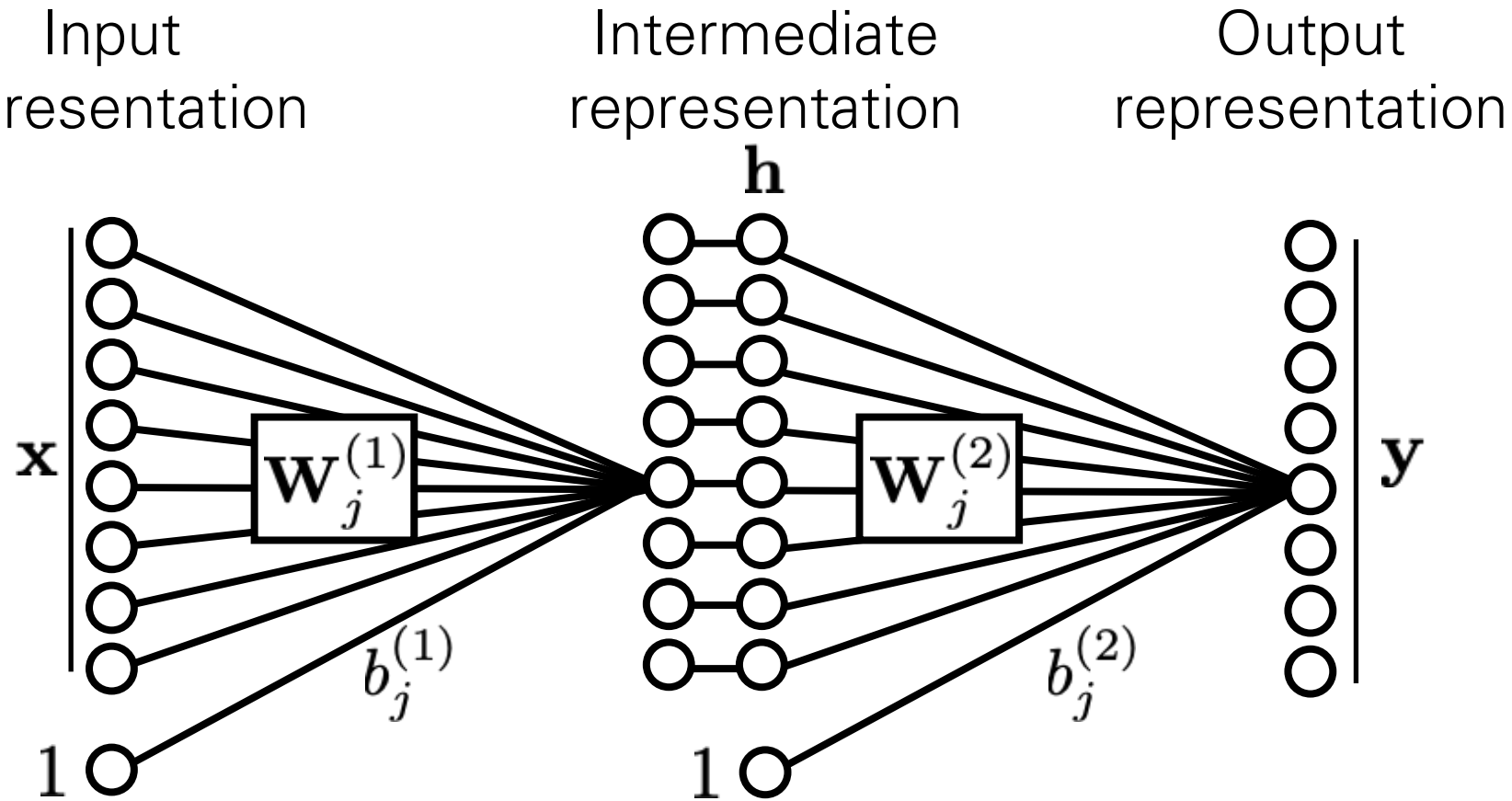


# Computation in a neural net – nonlinearity

- where  $\alpha$  is small (e.g. 0.02)
- Efficient to implement:  $\frac{\partial g}{\partial y} = \begin{cases} -a, & \text{if } y < 0 \\ 1, & \text{if } y \geq 0 \end{cases}$
- Also known as probabilistic ReLU (PReLU)
- Has non-zero gradients everywhere (unlike ReLU)
- $\alpha$  can also be learned (see Kaiming He et al. 2015).

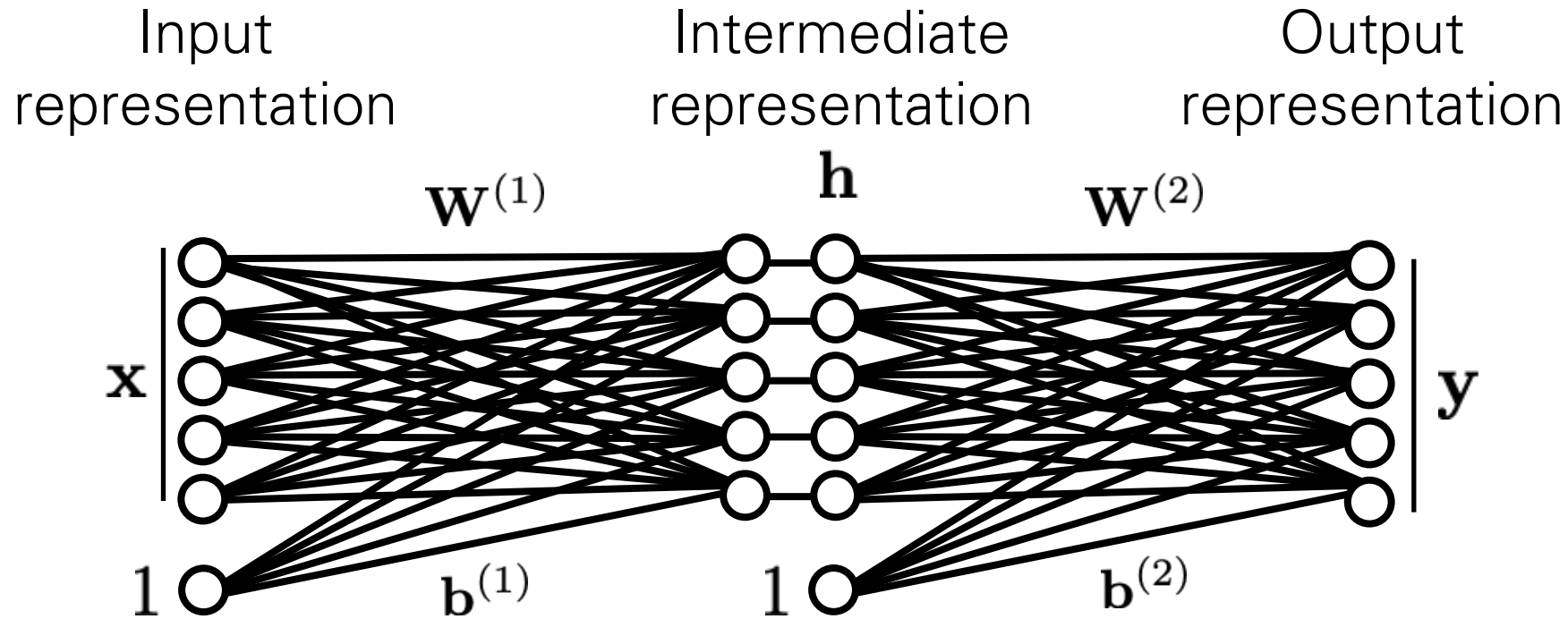


# Stacking layers



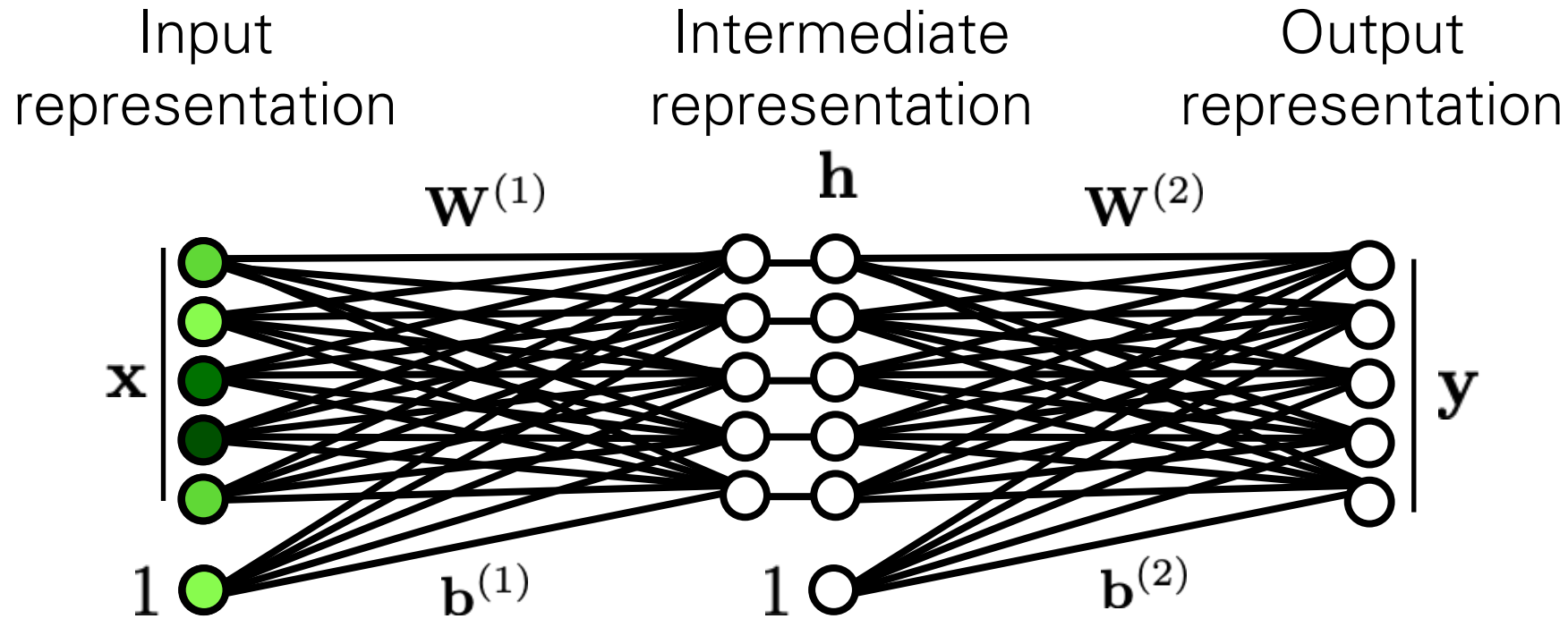
$\mathbf{h}$  = "hidden units"

# Stacking layers



$$\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(L)}\}$$

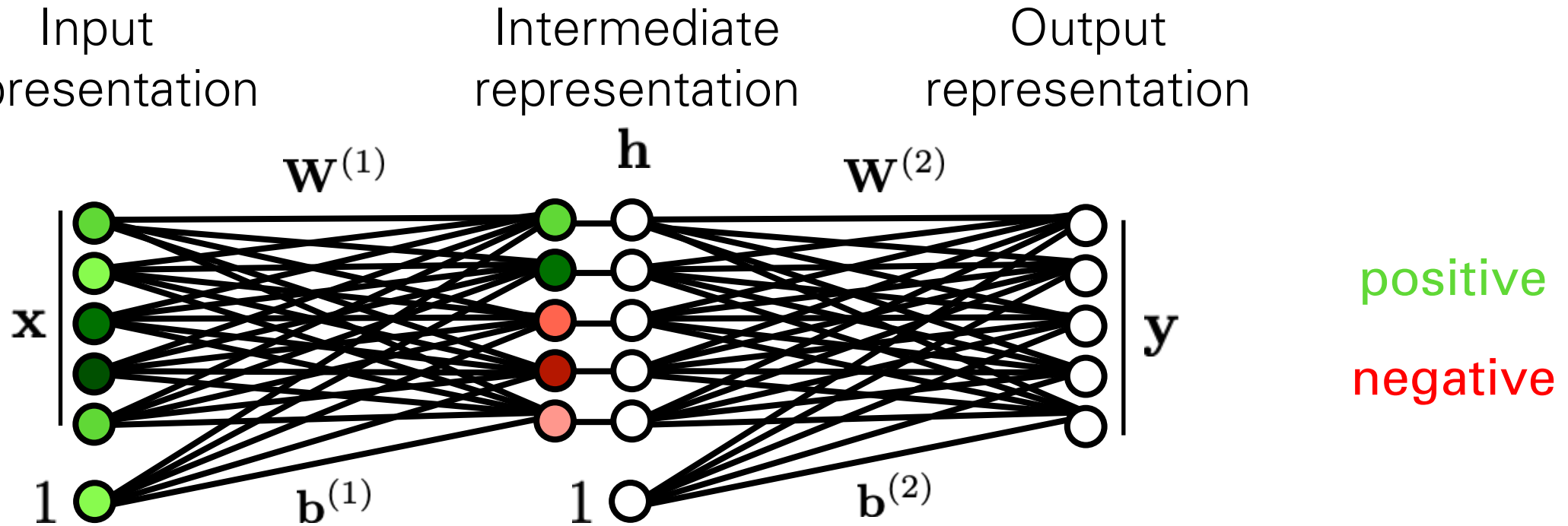
# Stacking layers



positive  
negative

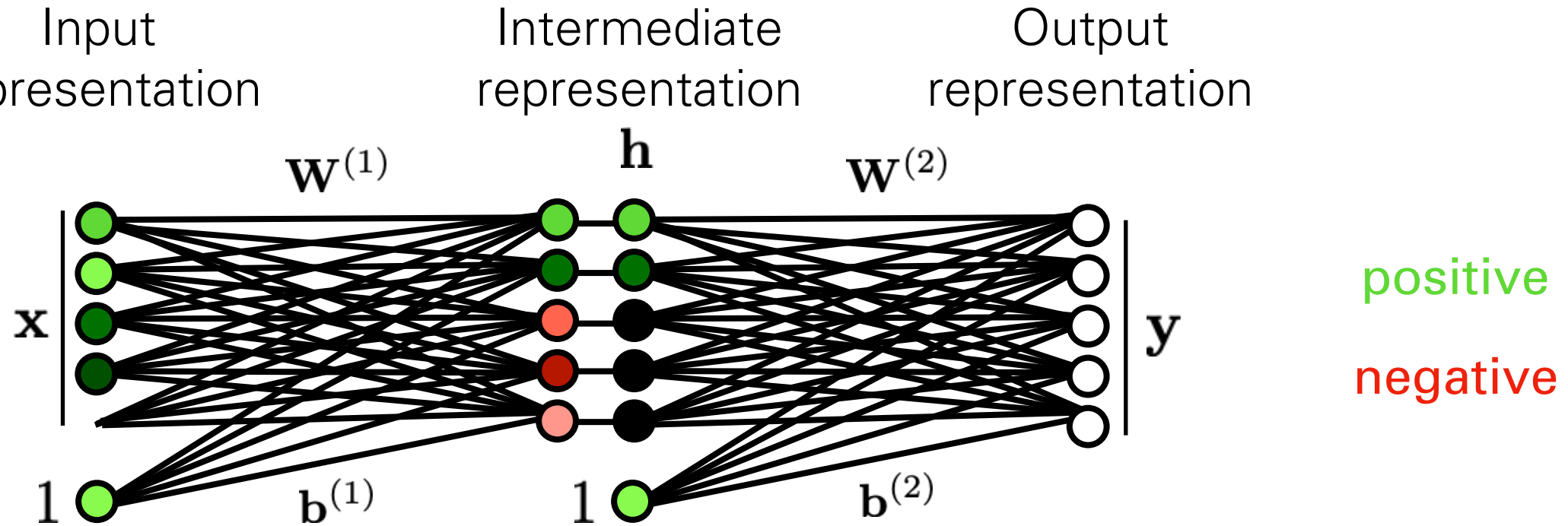
$$\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(L)}\}$$

# Stacking layers



$$\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(L)}\}$$

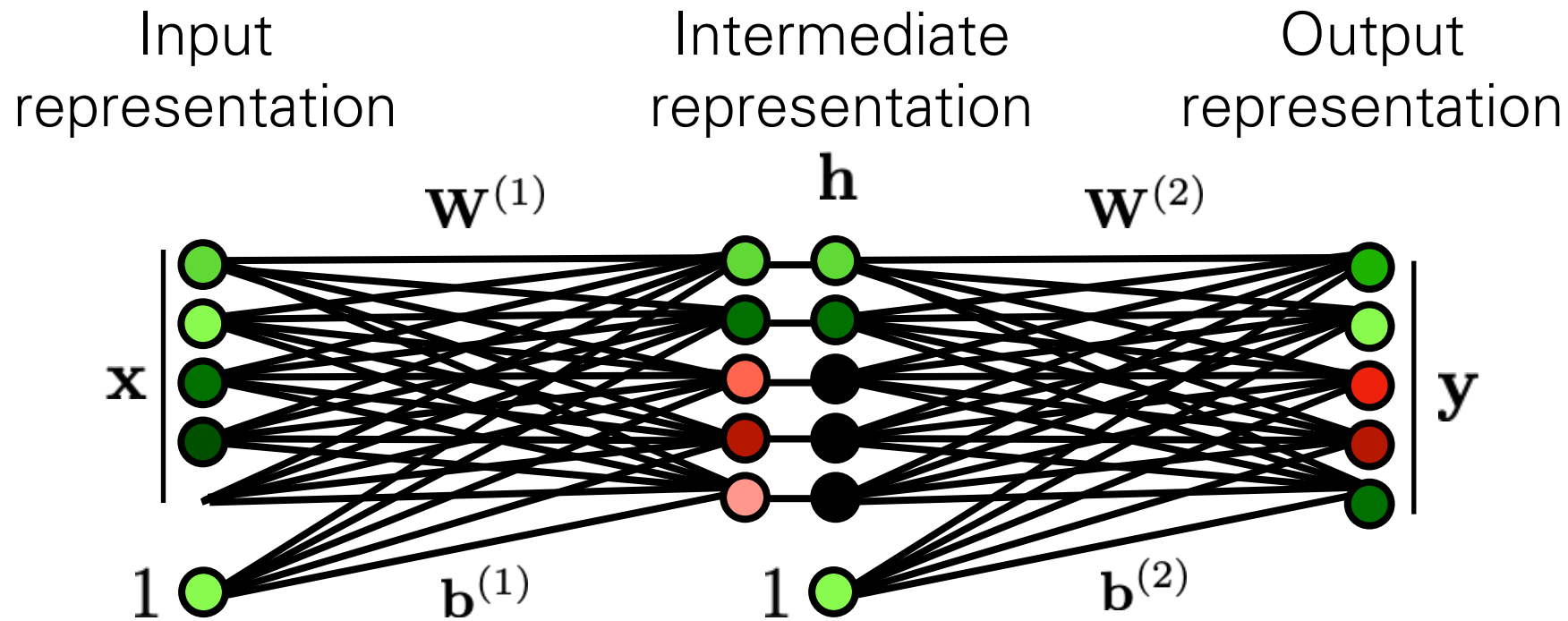
# Stacking layers



$$\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(L)}\}$$



# Stacking layers



positive  
negative

$$\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(L)}\}$$

# Representational power

- 1 layer? Linear decision surface.
- 2+ layers? In theory, can represent any function. Assuming non-trivial non-linearity.
  - Bengio 2009,  
<http://www.iro.umontreal.ca/~bengioy/papers/ftml.pdf>
  - Bengio, Courville, Goodfellow book  
<http://www.deeplearningbook.org/contents/mlp.html>
  - Simple proof by M. Neilsen  
<http://neuralnetworksanddeeplearning.com/chap4.html>
  - D. Mackay book  
<http://www.inference.phy.cam.ac.uk/mackay/itprnn/ps/482.491.pdf>
- But issue is efficiency: very wide two layers vs narrow deep model? In practice, more layers helps.

## DATA

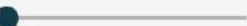
Which dataset do you want to use?



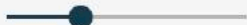
Ratio of training to test data: 50%



Noise: 0



Batch size: 10



REGENERATE

## FEATURES

Which properties do you want to feed in?

$X_1$



$X_2$



$X_1^2$



$X_2^2$



$X_1 X_2$



$\sin(X_1)$



$\sin(X_2)$



+

-

2 HIDDEN LAYERS

+

-

4 neurons

+

-

2 neurons

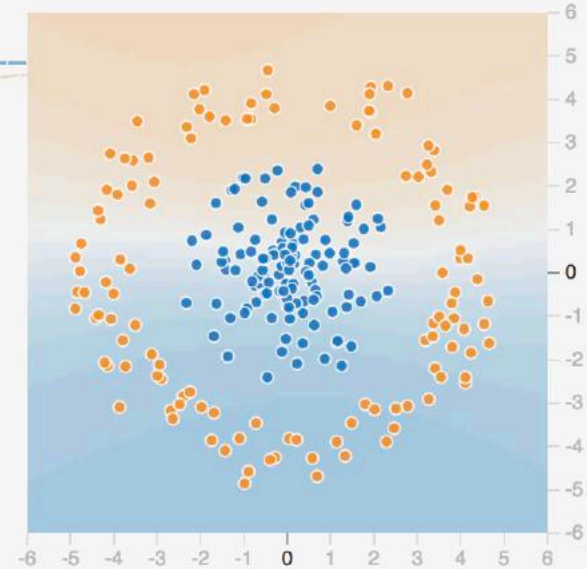
The outputs are mixed with varying **weights**, shown by the thickness of the lines.

This is the output from one **neuron**. Hover to see it larger.

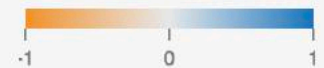
## OUTPUT

Test loss 0.540

Training loss 0.555



Colors shows data, neuron and weight values.

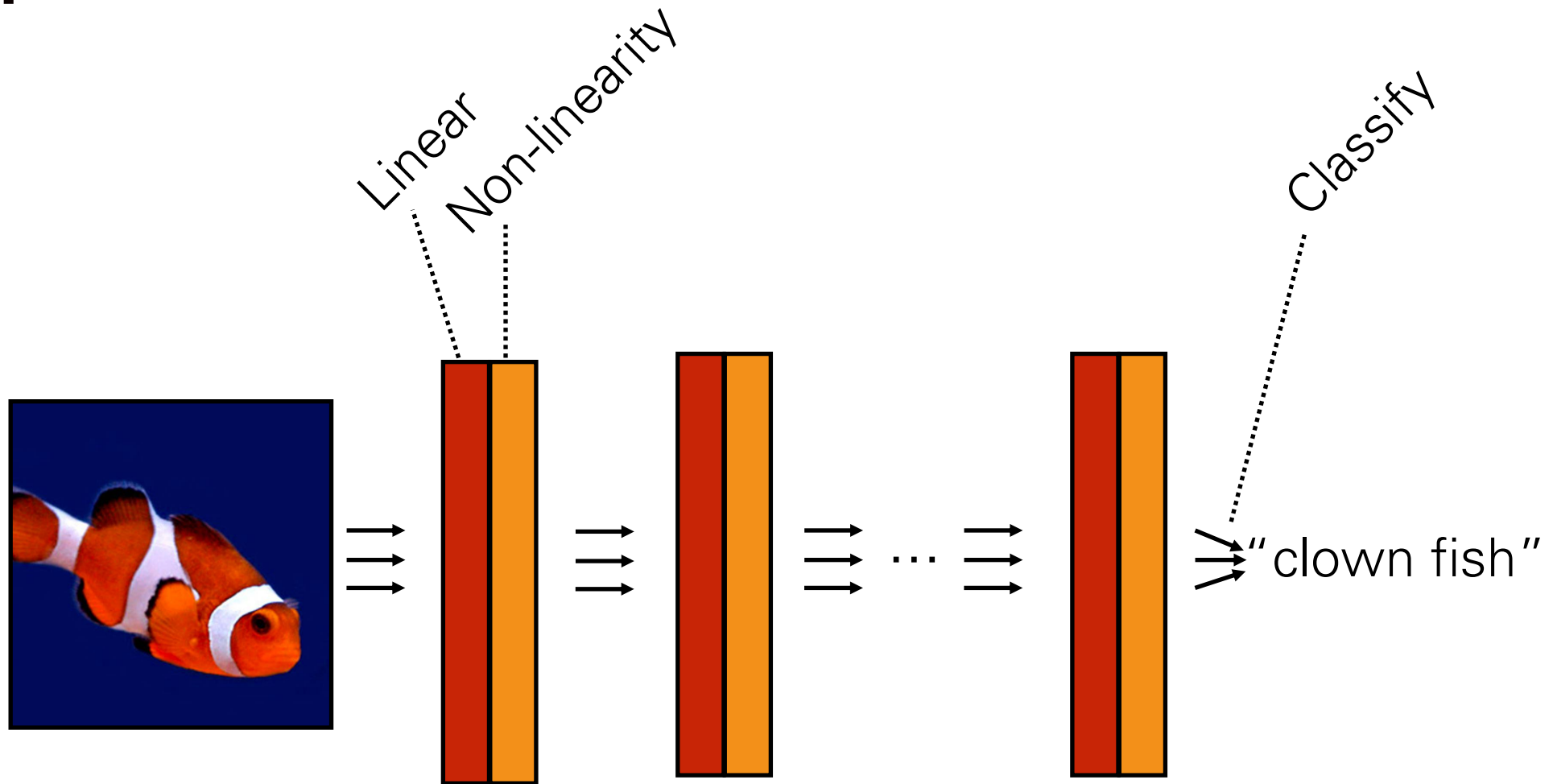


☐ Show test data

☐ Discretize output

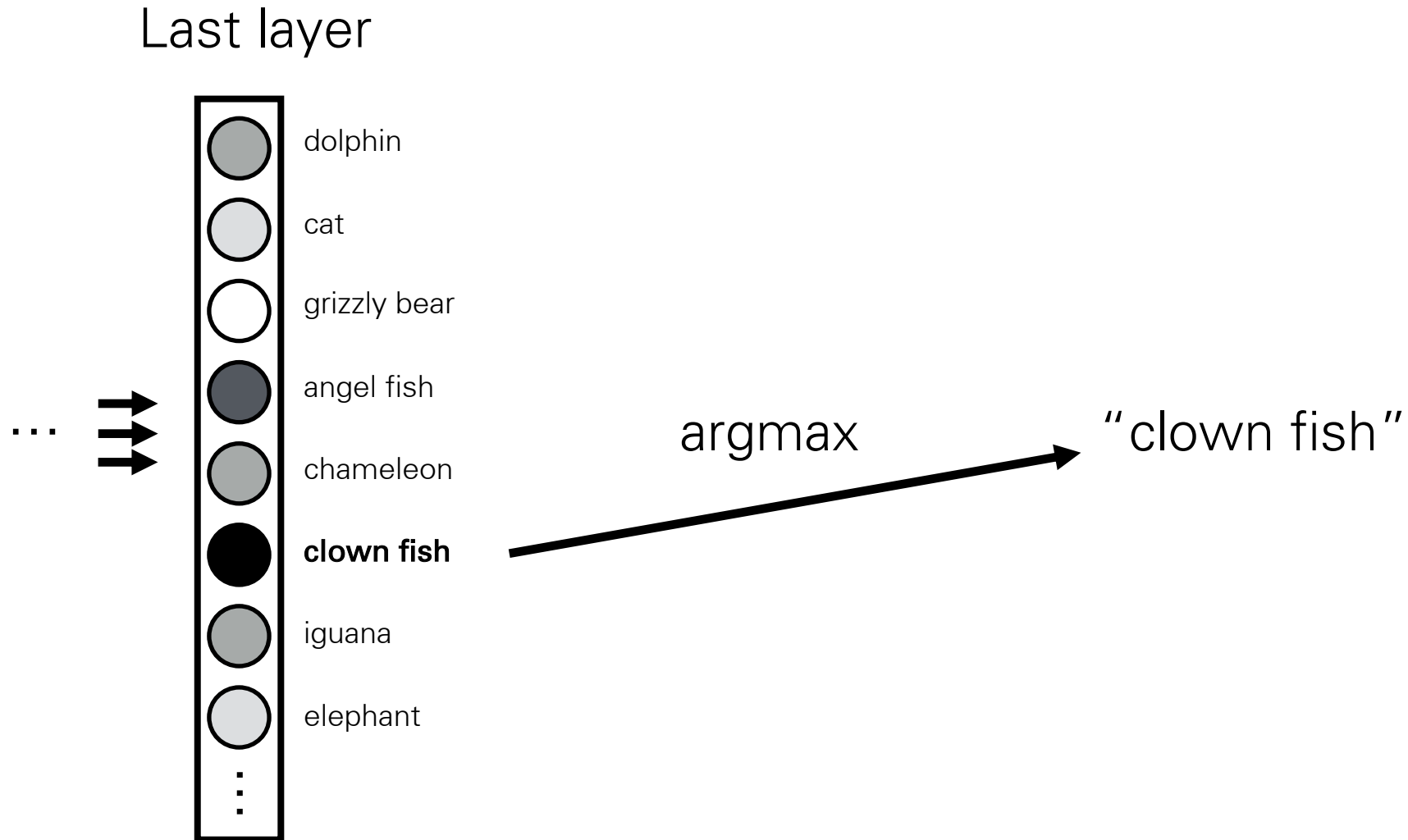
[<http://playground.tensorflow.org>]

# Deep nets

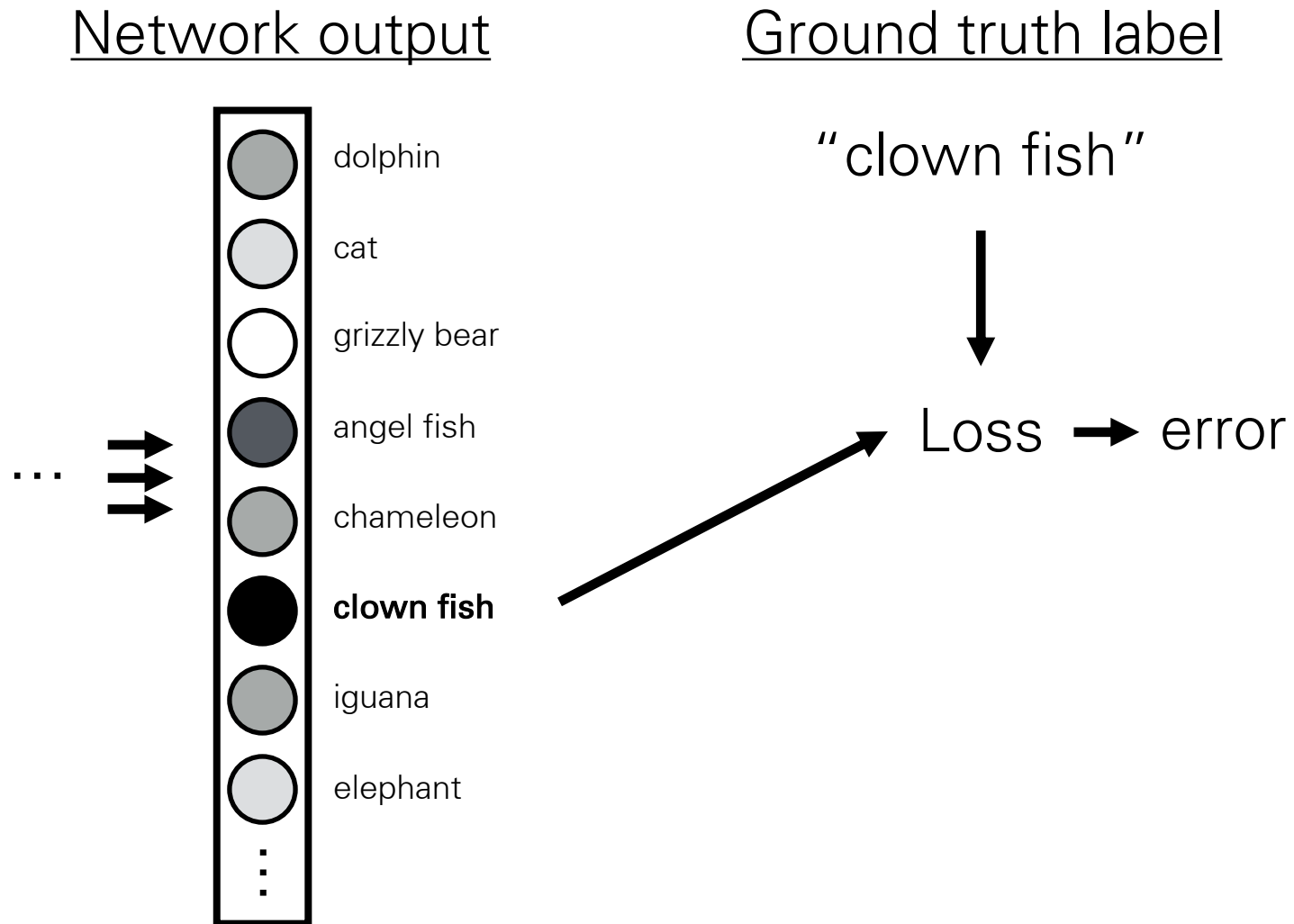


$$f(\mathbf{x}) = f_L(\dots f_2(f_1(\mathbf{x})))$$

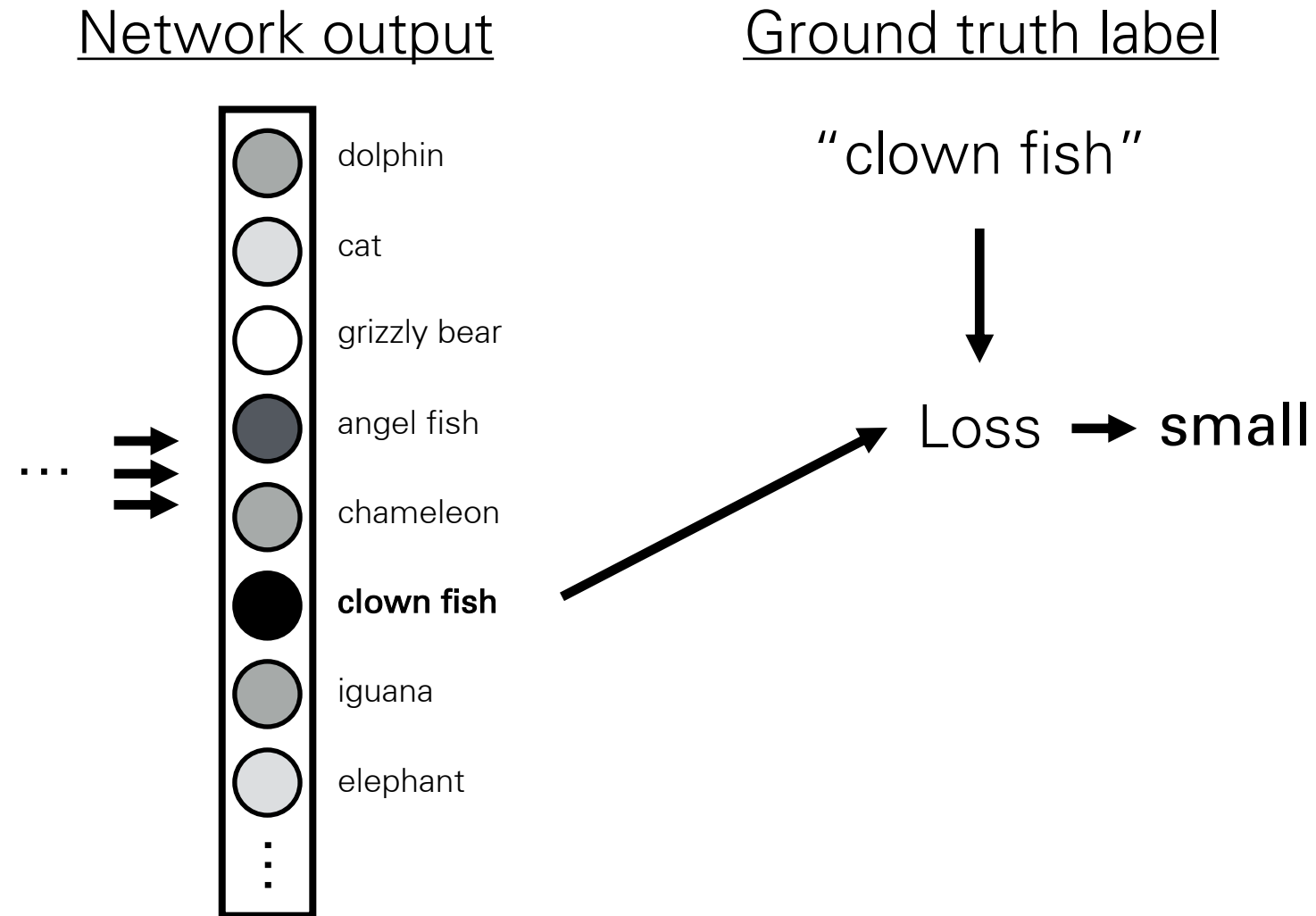
# Classifier layer



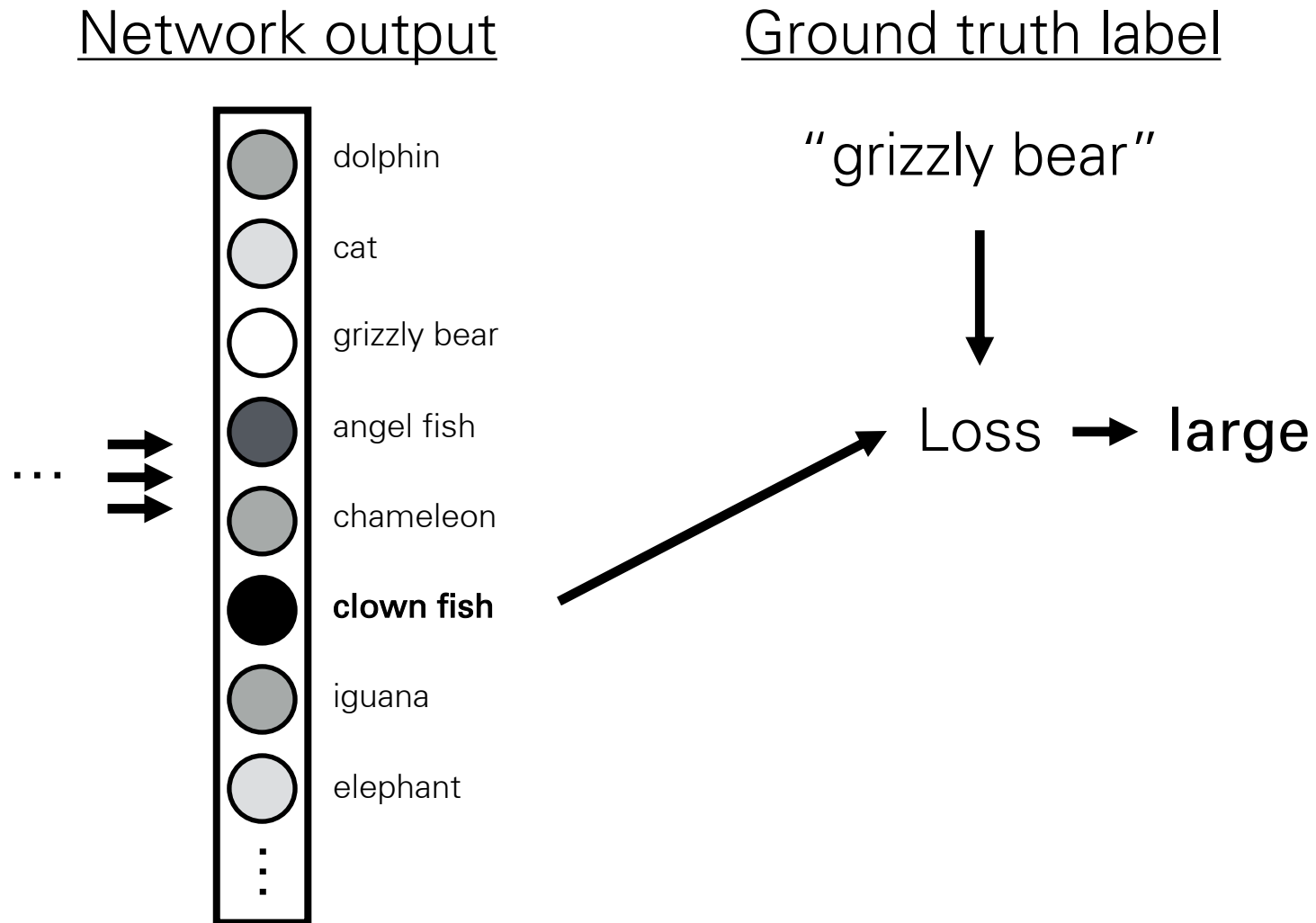
# Loss function



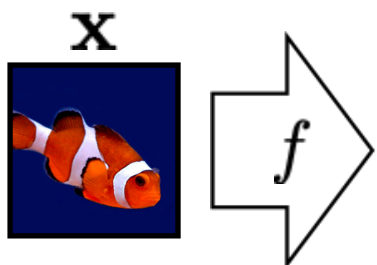
# Loss function



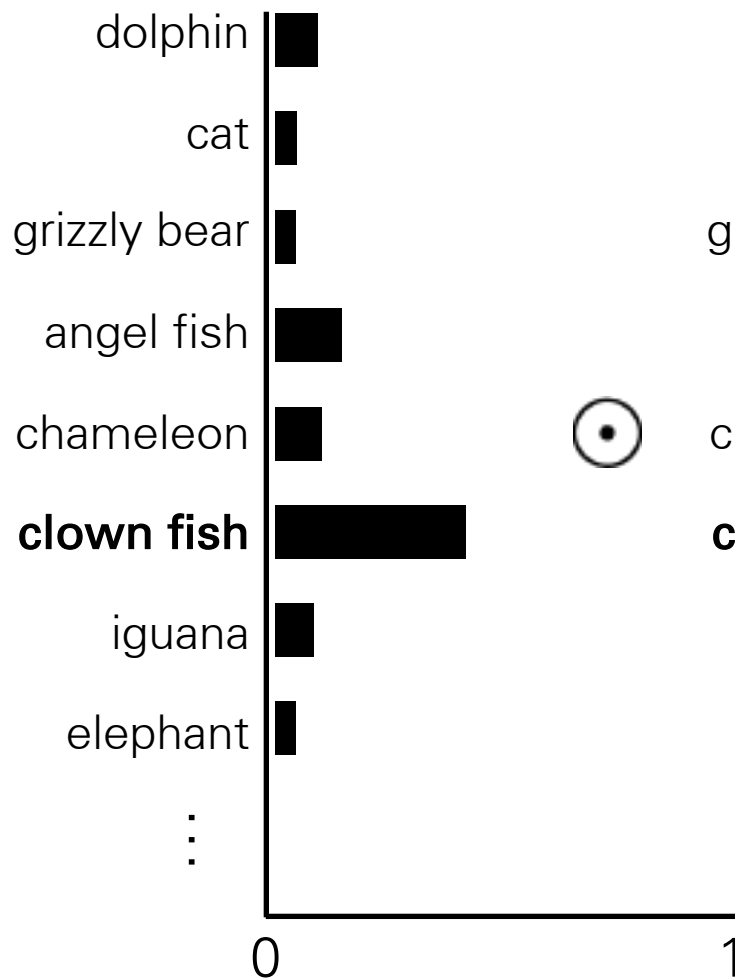
# Loss function



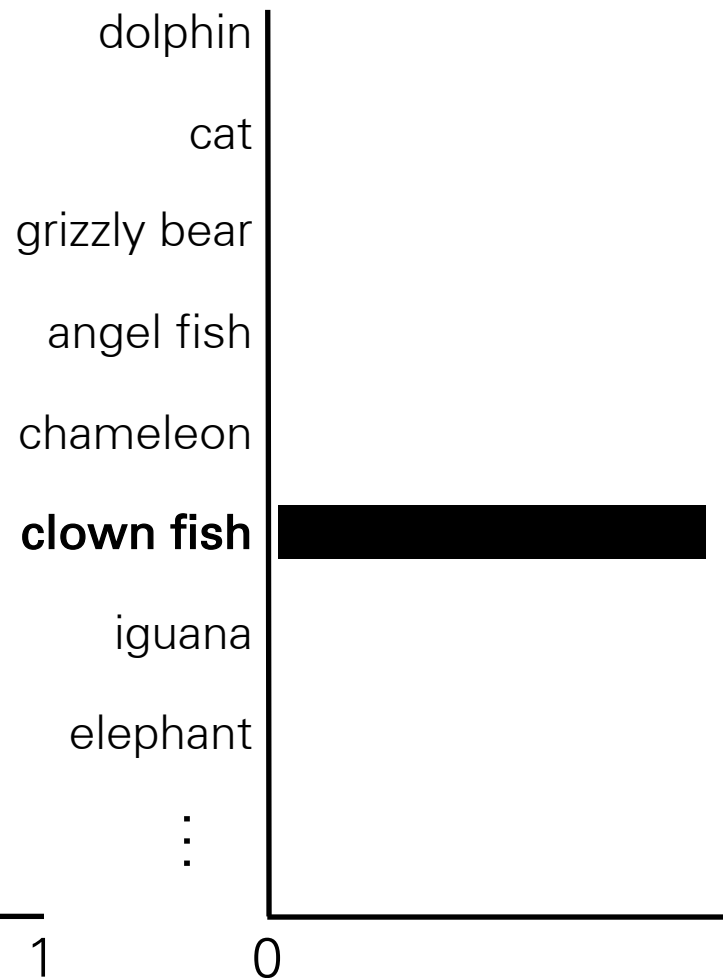




Prediction  $\hat{y}$   
 $f_{\theta} : X \rightarrow \mathbb{R}^K$

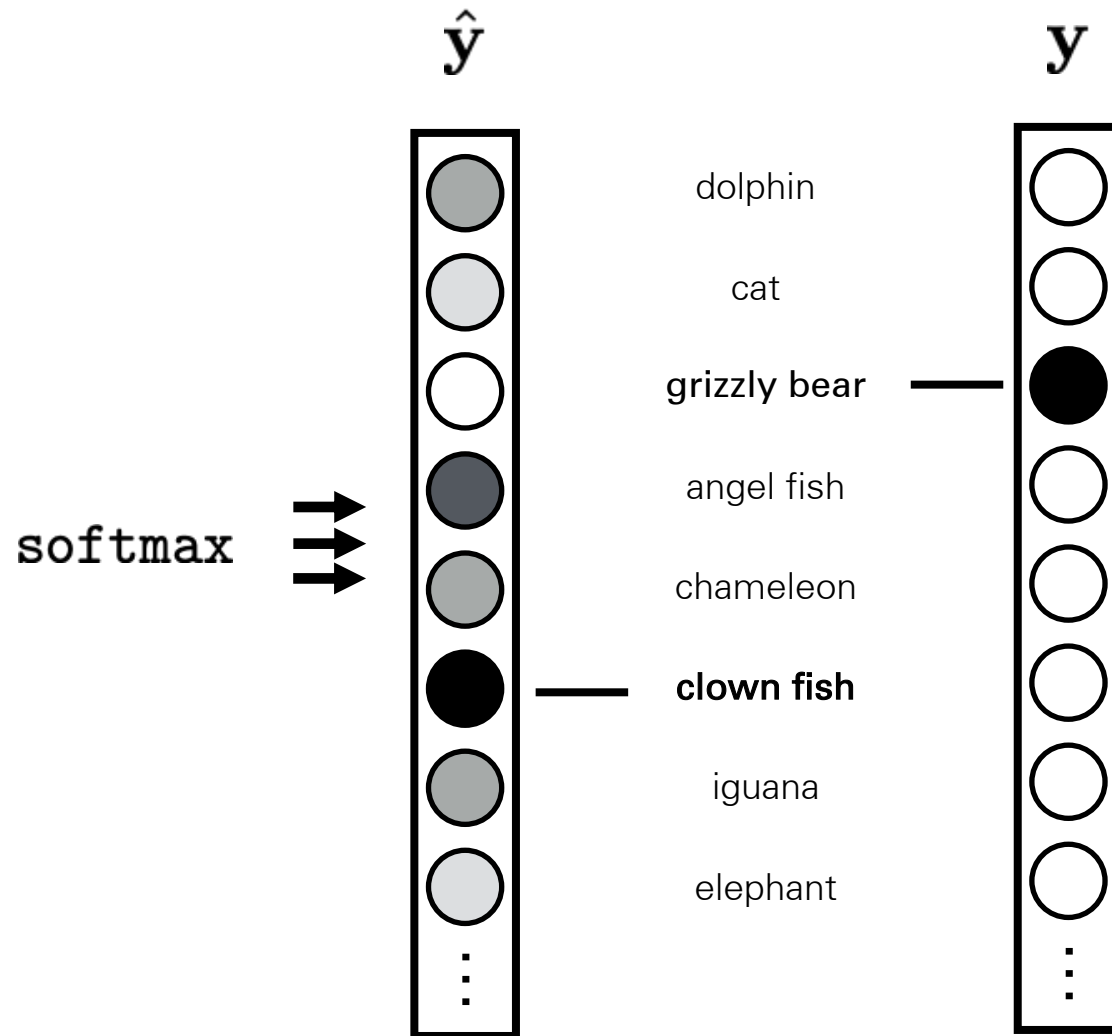


Ground truth label  $y$



Network output

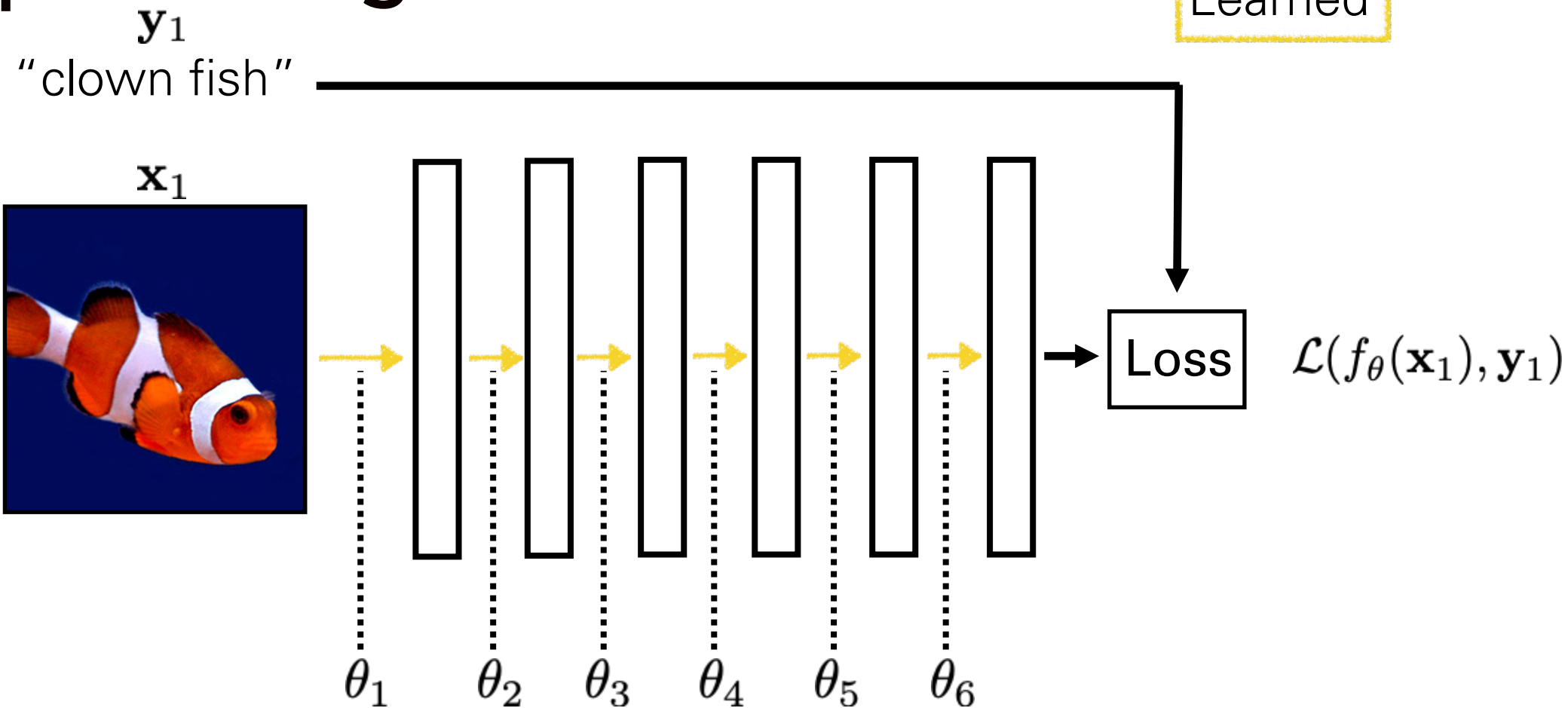
Ground truth label



Probability of the observed  
data under the model

$$H(\mathbf{y}, \hat{\mathbf{y}}) = - \sum_{k=1}^K y_k \log \hat{y}_k$$

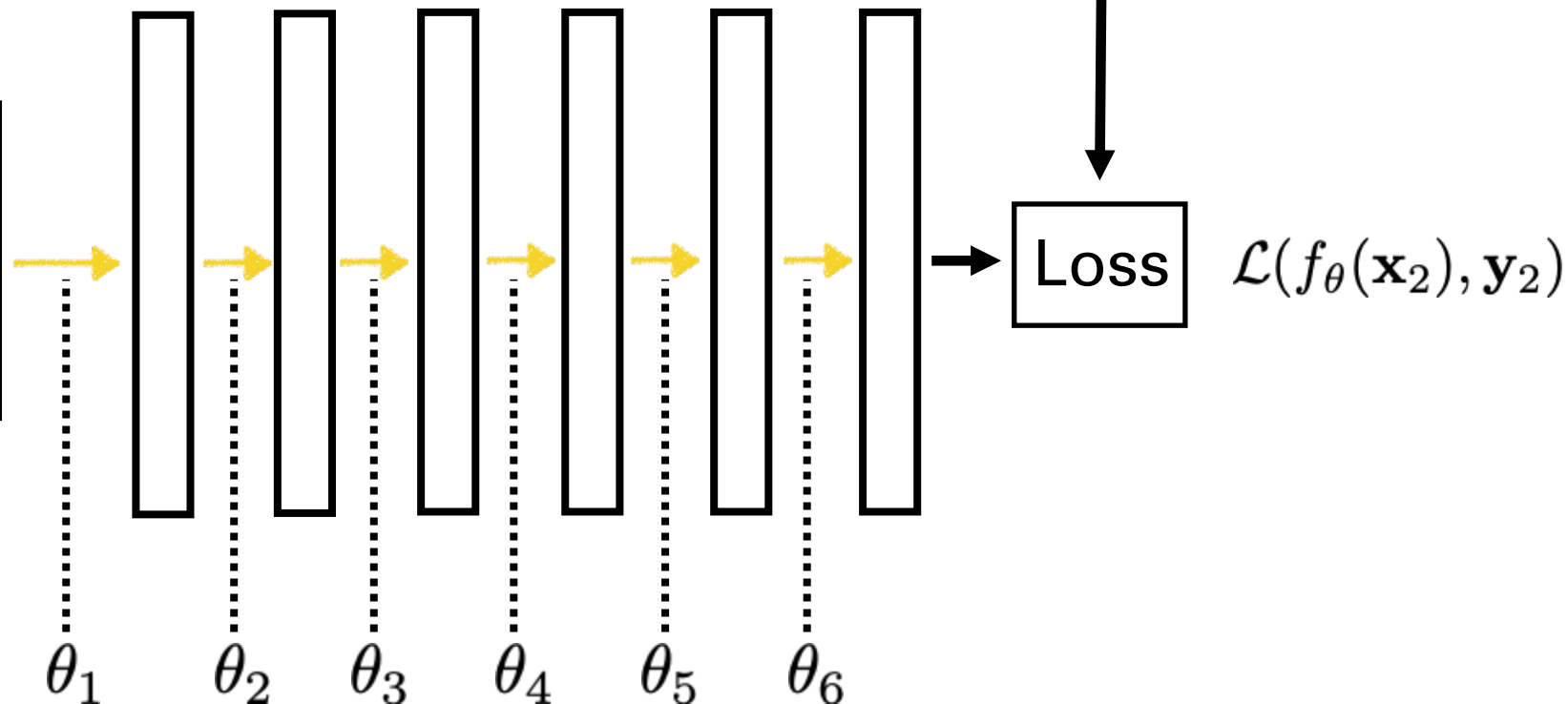
# Deep learning



$$\theta^* = \arg \min_{\theta} \sum_{i=1}^N \mathcal{L}(f_{\theta}(\mathbf{x}_i), \mathbf{y}_i)$$

# Deep learning

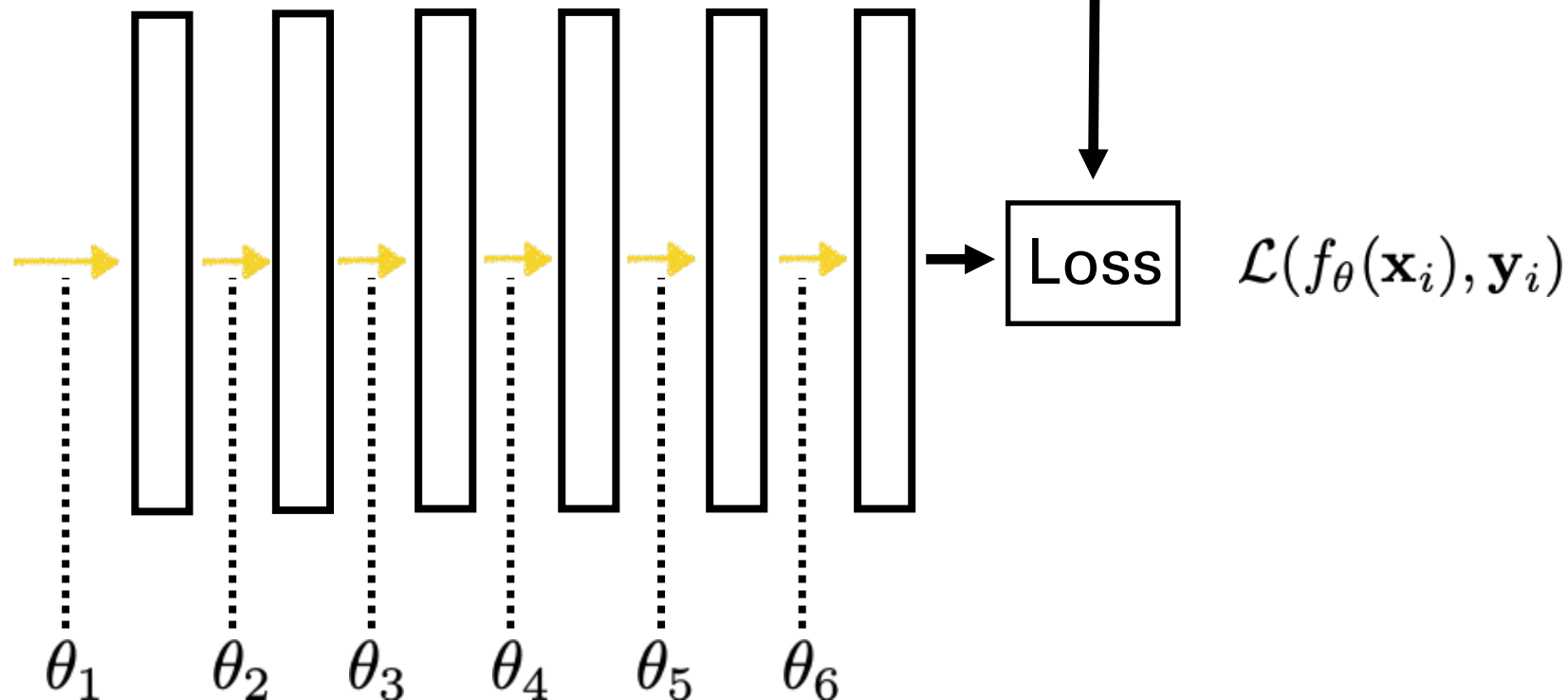
"grizzly bear"



$$\theta^* = \arg \min_{\theta} \sum_{i=1}^N \mathcal{L}(f_{\theta}(\mathbf{x}_i), \mathbf{y}_i)$$

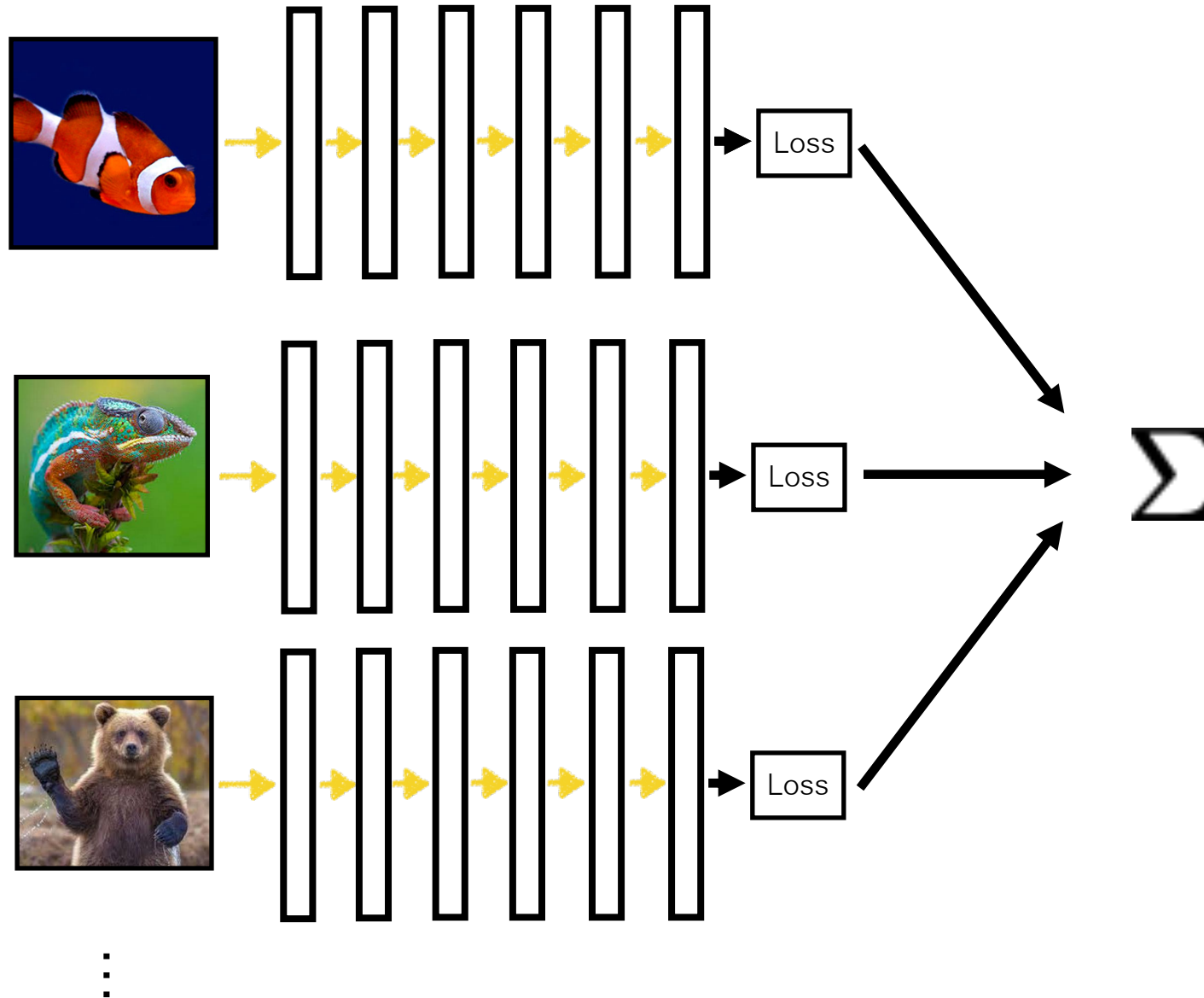
# Deep learning

$y_i$   
"chameleon"



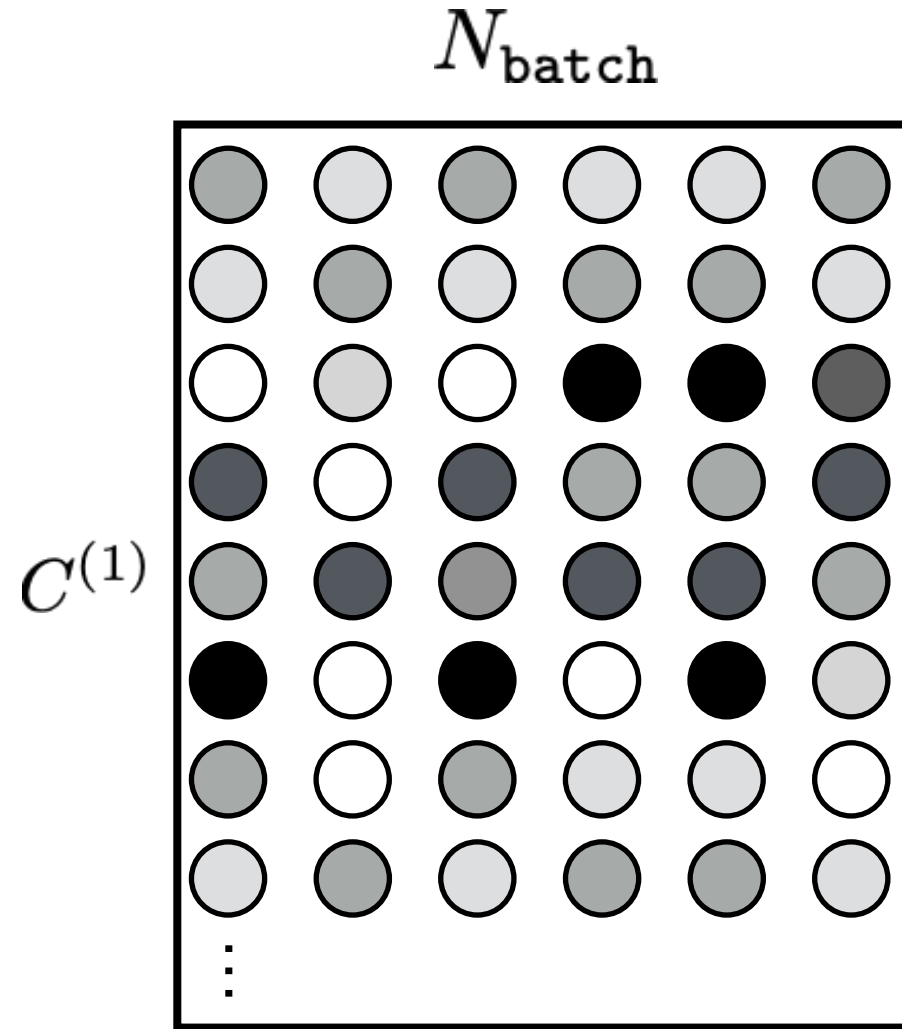
$$\theta^* = \arg \min_{\theta} \sum_{i=1}^N \mathcal{L}(f_{\theta}(\mathbf{x}_i), \mathbf{y}_i)$$

# Batch (parallel) processing



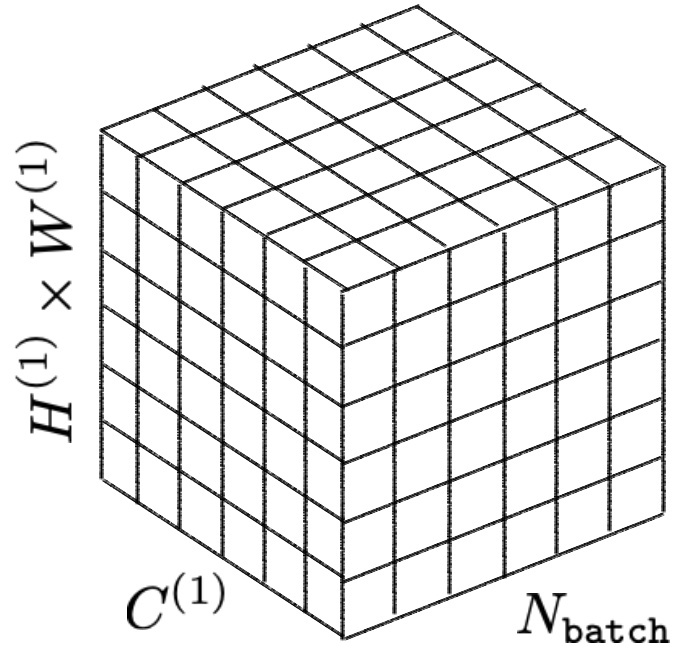
# Tensors

$$\mathbf{h}^{(1)} \in \mathbb{R}^{N_{\text{batch}} \times C^{(1)}}$$

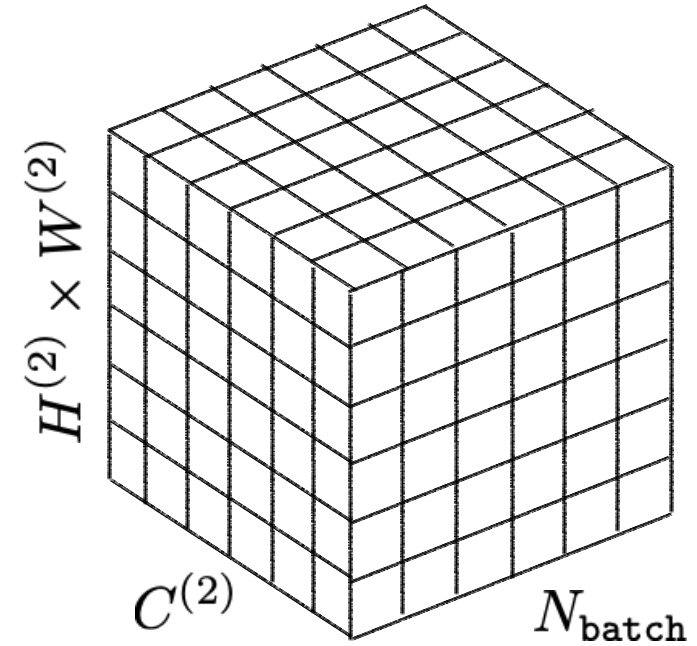


# "Tensor flow"

$$\mathbf{h}^{(1)} \in \mathbb{R}^{N_{\text{batch}} \times H^{(1)} \times W^{(1)} \times C^{(1)}}$$



$$\mathbf{h}^{(2)} \in \mathbb{R}^{N_{\text{batch}} \times H^{(2)} \times W^{(2)} \times C^{(2)}}$$





# Regularizing deep nets

Deep nets have millions of parameters!

On many datasets, it is easy to overfit — we may have more free parameters than data points to constrain them.

How can we regularize to prevent the network from overfitting?

1. Fewer neurons, fewer layers
2. Weight decay
3. Dropout
4. Normalization layers
5. ...

# Recall: regularized least squares

$$f_{\theta}(x) = \sum_{k=0}^K \theta_k x^k$$

$$R(\theta) = \lambda \|\theta\|_2^2$$



Only use polynomial terms if you really need them! Most terms should be zero

ridge regression, a.k.a., Tikhonov regularization

Probabilistic interpretation:  $R$  is a Gaussian **prior** over values of the parameters.

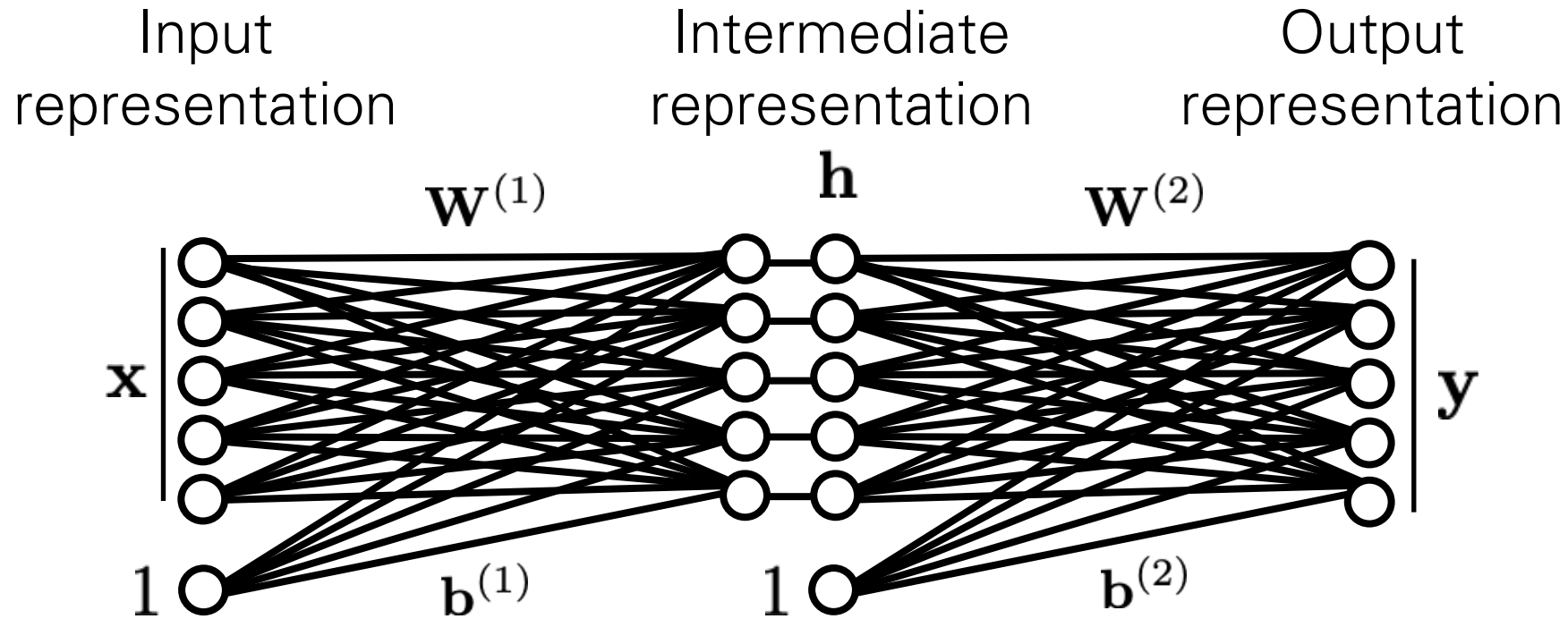
# Recall: regularized least squares

$$\theta^* = \arg \min_{\theta} \sum_{i=1}^N \mathcal{L}(f_{\theta}(\mathbf{x}_i), \mathbf{y}_i) + R(\theta)$$

$$R(\mathbf{W}) = \lambda \|\mathbf{W}\|_2^2 \quad \longleftarrow \quad \text{weight decay}$$

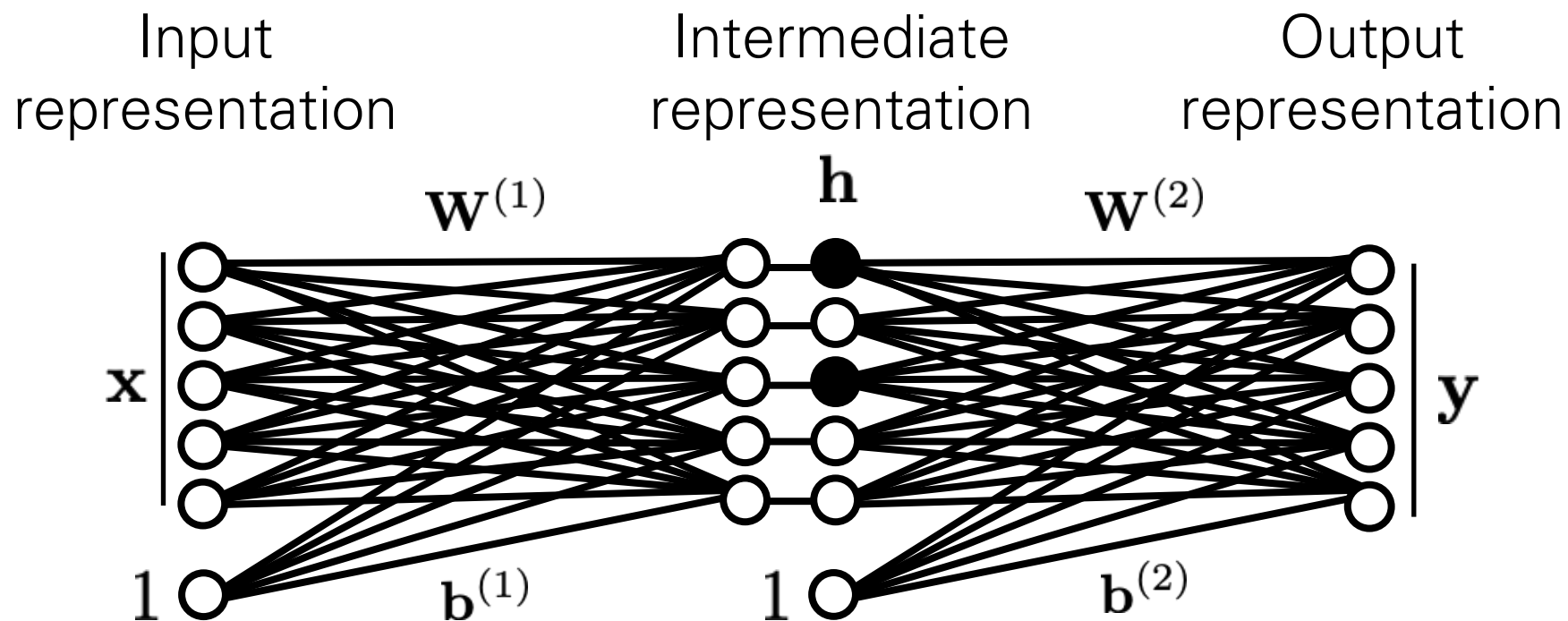
“We prefer to keep weights small.”

# Dropout



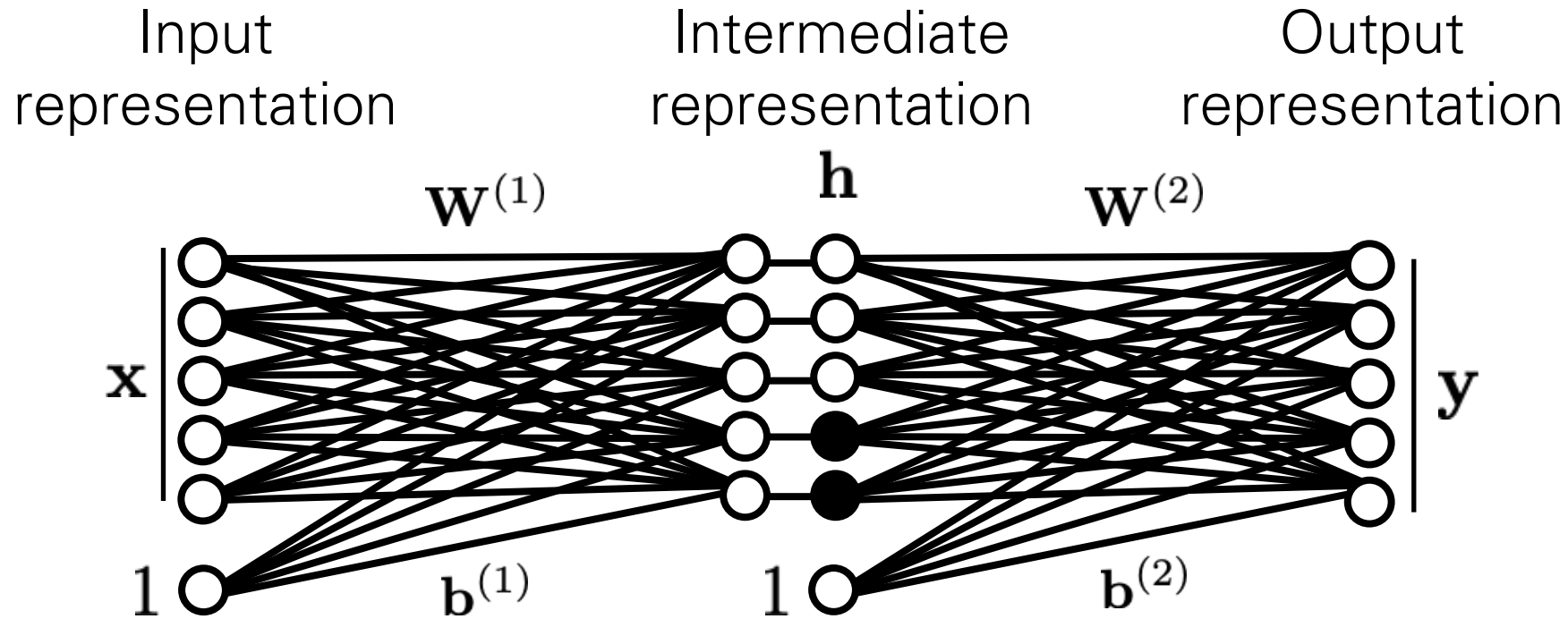
$$\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(L)}\}$$

# Dropout



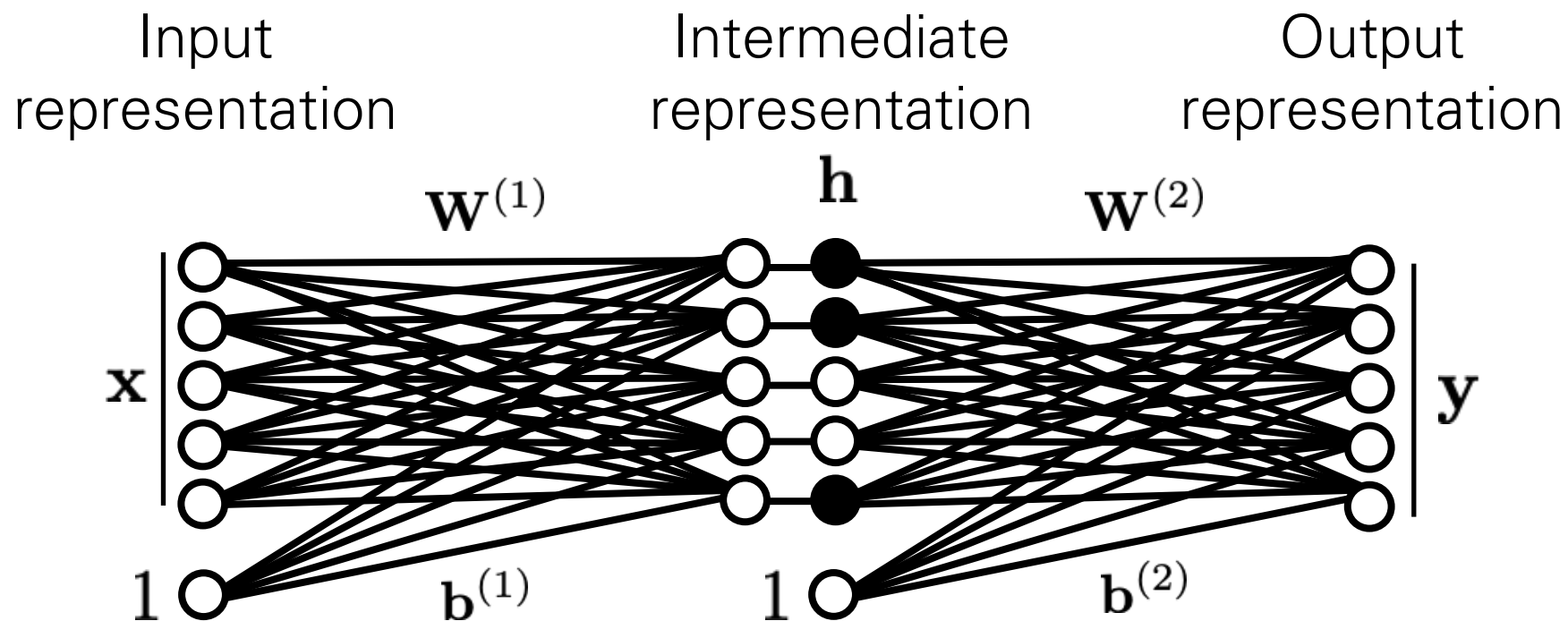
$$\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(L)}\}$$

# Dropout



$$\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(L)}\}$$

# Dropout



$$\theta = \{\mathbf{W}^{(1)}, \dots, \mathbf{W}^{(L)}, \mathbf{b}^{(1)}, \dots, \mathbf{b}^{(L)}\}$$

# Dropout

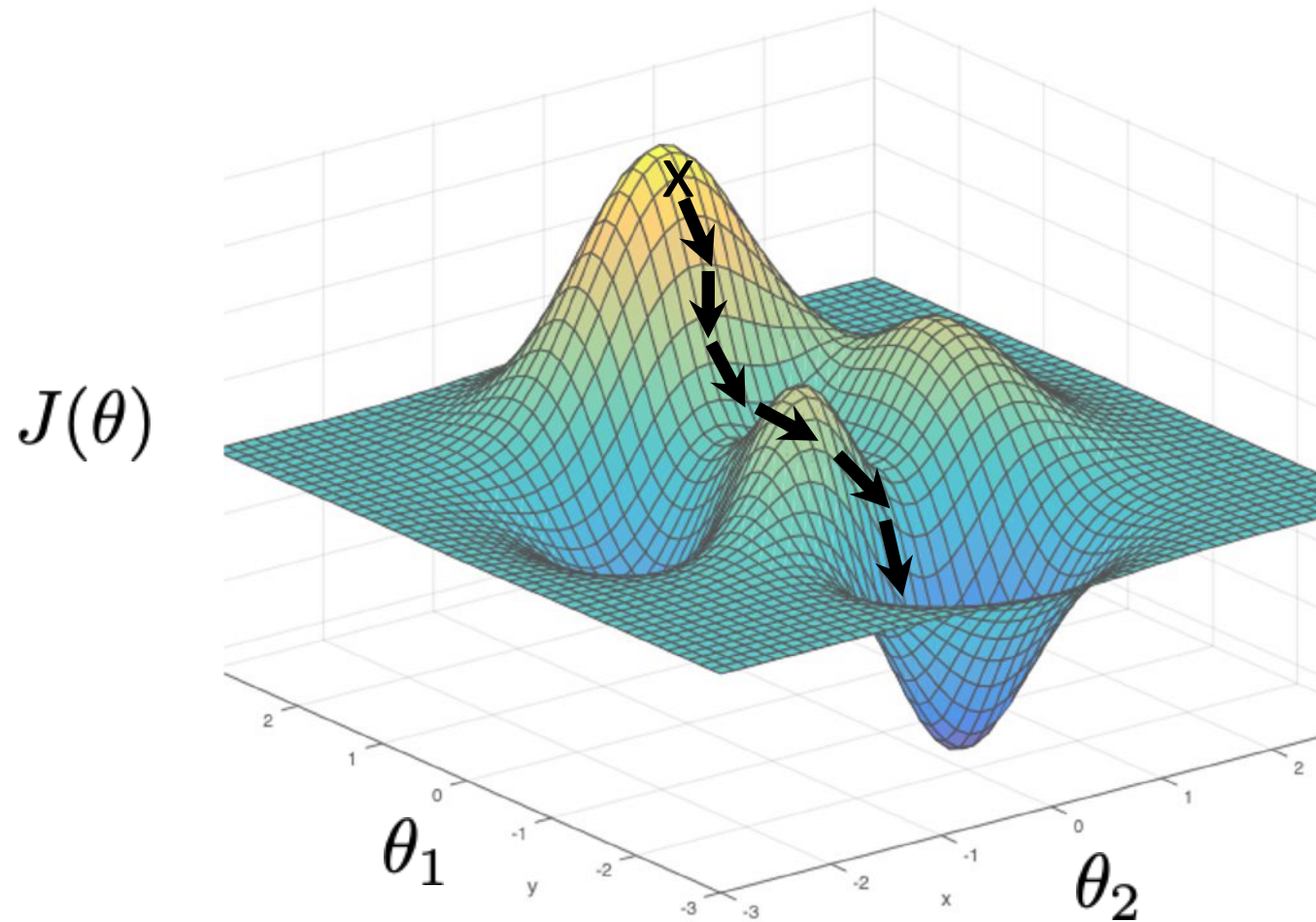
Randomly zero out hidden units.

Prevents network from relying too much on spurious correlations between different hidden units.

Can be understood as averaging over an exponential **ensemble** of subnetworks. This averaging smooths the function, thereby reducing the effective capacity of the network.



# Gradient descent



$$\theta^* = \arg \min_{\theta} J(\theta)$$

# Gradient descent

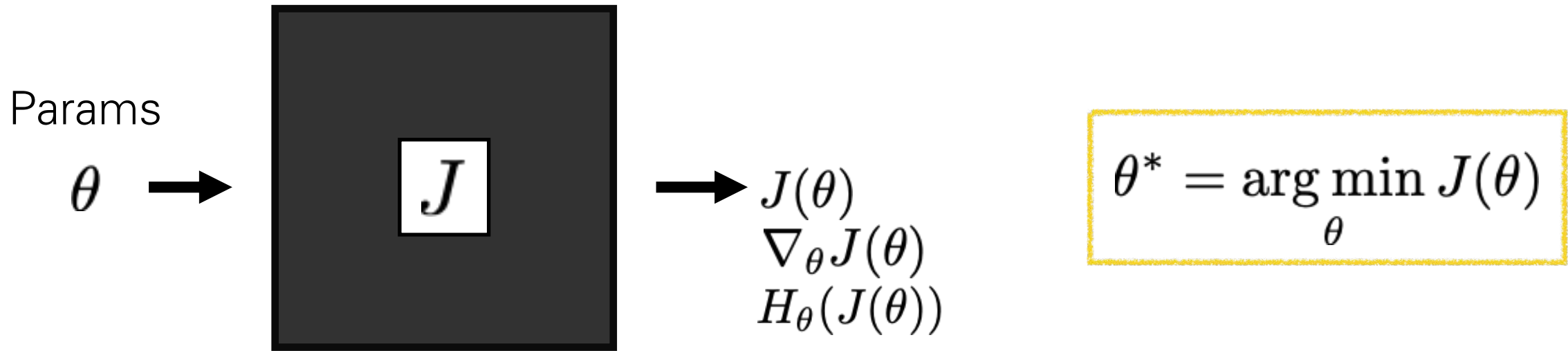
$$\theta^* = \arg \min_{\theta} \underbrace{\sum_{i=1}^N \mathcal{L}(f_{\theta}(\mathbf{x}_i), \mathbf{y}_i)}_{J(\theta)}$$

One iteration of gradient descent:

$$\theta^{t+1} = \theta^t - \eta_t \left. \frac{\partial J(\theta)}{\partial \theta} \right|_{\theta=\theta^t}$$

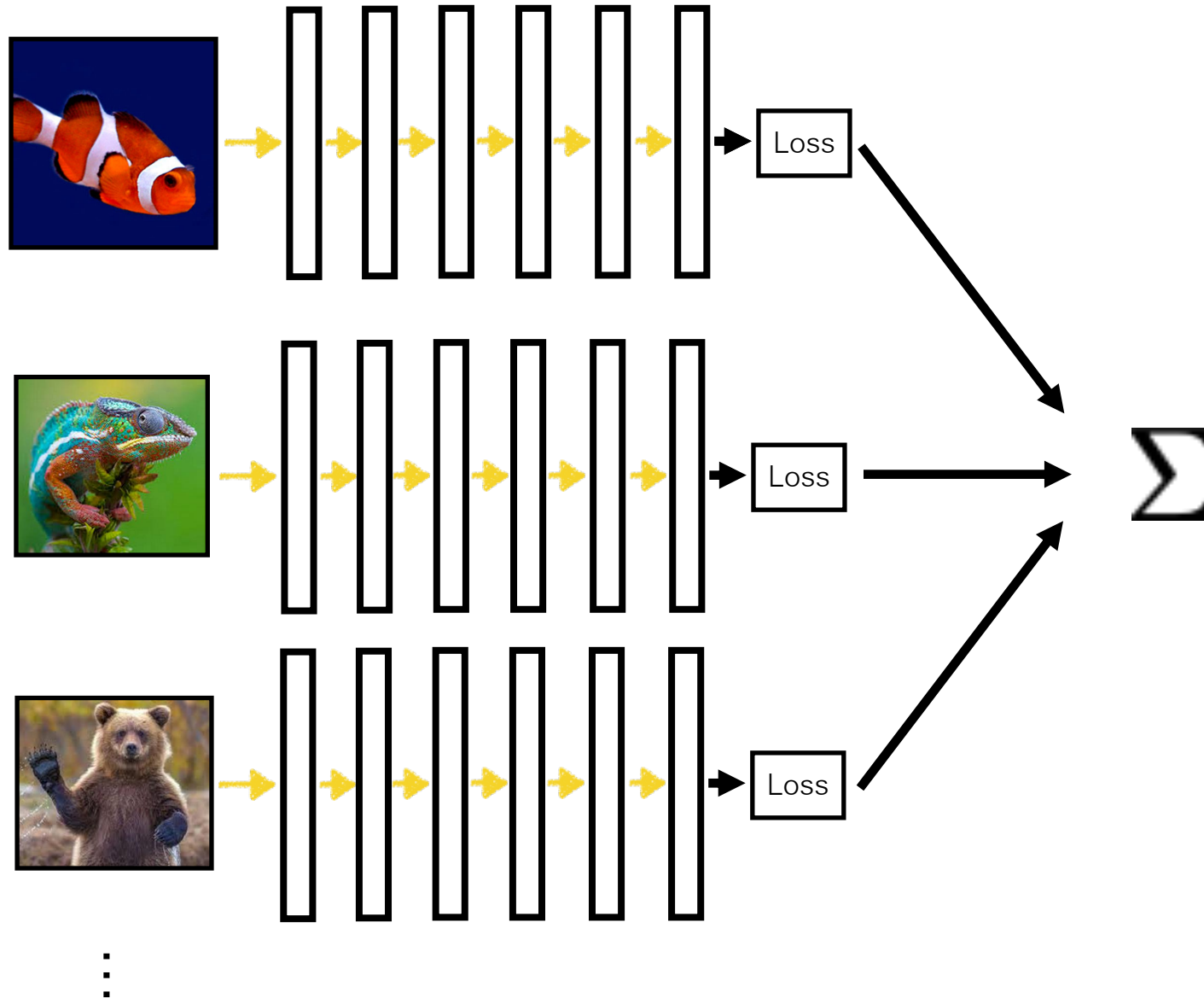
learning rate

# Optimization



- What's the knowledge we have about  $J$ ?
    - We can evaluate  $J(\theta)$
    - We can evaluate  $J(\theta)$  and  $\nabla_{\theta} J(\theta)$  Gradient
    - We can evaluate  $J(\theta)$ ,  $\nabla_{\theta} J(\theta)$ , and  $H_{\theta}(J(\theta))$  Hessian
- ← Black box optimization
- ← First order optimization
- ← Second order optimization

# Batch (parallel) processing



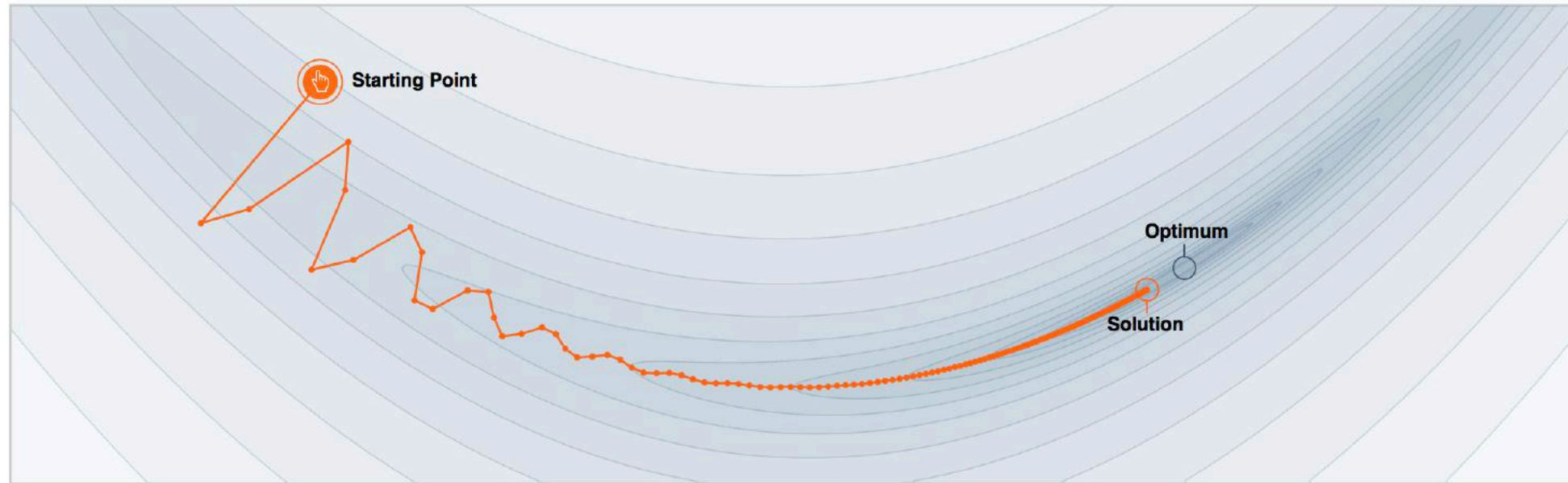
# Stochastic gradient descent (SGD)

- Want to minimize overall loss function  $J$ , which is sum of individual losses over each example.
- In Stochastic gradient descent, compute gradient on sub-set (batch) of data.
  - If batchsize=1 then  $\theta$  is updated after each example.
  - If batchsize=N (full set) then this is standard gradient descent.
- Gradient direction is noisy, relative to average over all examples (standard gradient descent).
- Advantages
  - Faster: approximate total gradient with small sample
  - Implicit regularizer
- Disadvantages
  - High variance, unstable updates

# Momentum

- Basic idea: like a ball rolling down a hill, we should build up speed so as to make faster progress when “on a roll”
- Can dampen oscillations in SGD updates
- Common in popular variants of SGD
  - Nesterov’s method
  - RMSProp
  - Adam

# Why Momentum Really Works



Step-size  $\alpha = 0.02$



Momentum  $\beta = 0.99$



We often think of Momentum as a means of dampening oscillations and speeding up the iterations, leading to faster convergence. But it has other interesting behavior. It allows a larger range of step-sizes to be used, and creates its own oscillations. What is going on?

GABRIEL GOH  
UC Davis

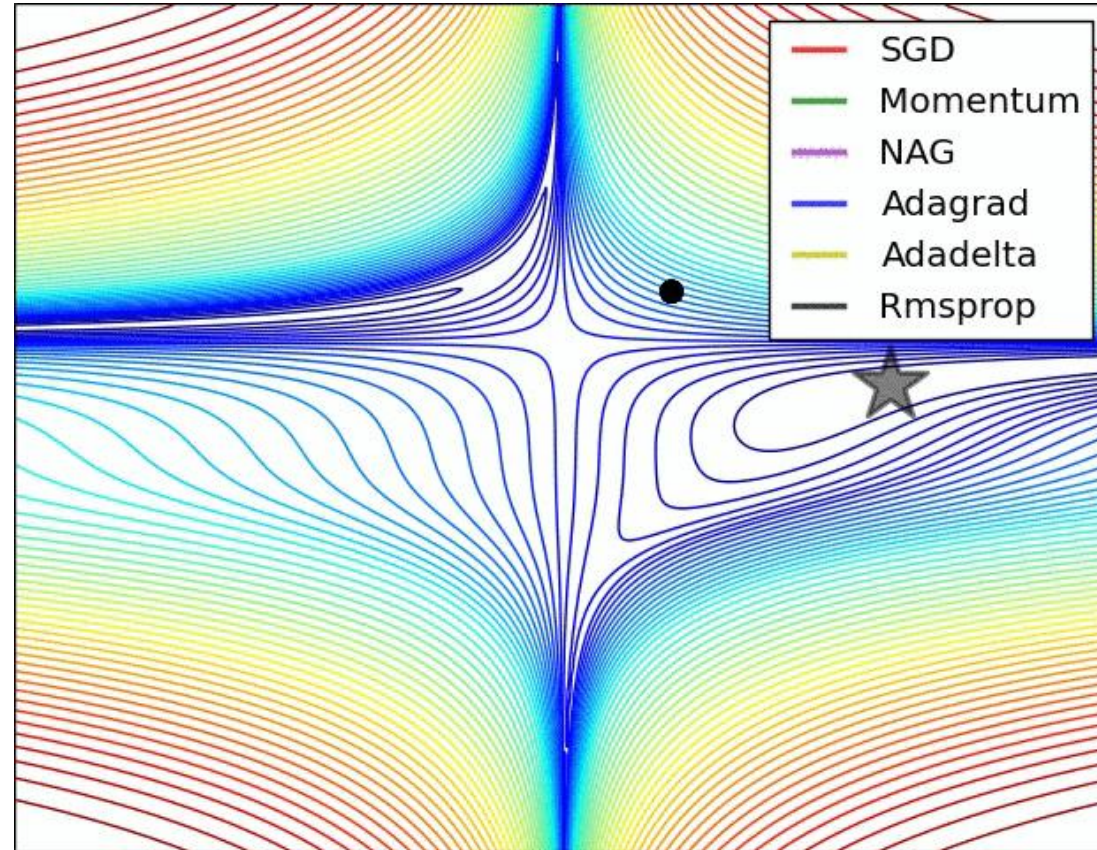
April. 4  
2017

Citation:  
Goh, 2017

[<https://distill.pub/2017/momentum/>]



# Comparison of gradient descent variants

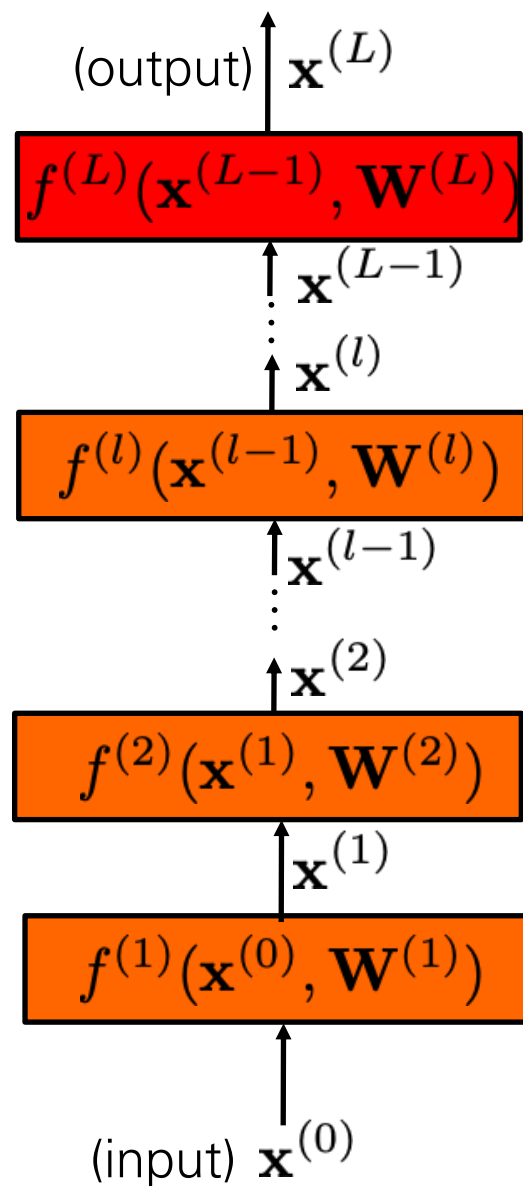


[<http://ruder.io/optimizing-gradient-descent/>]



# Forward pass

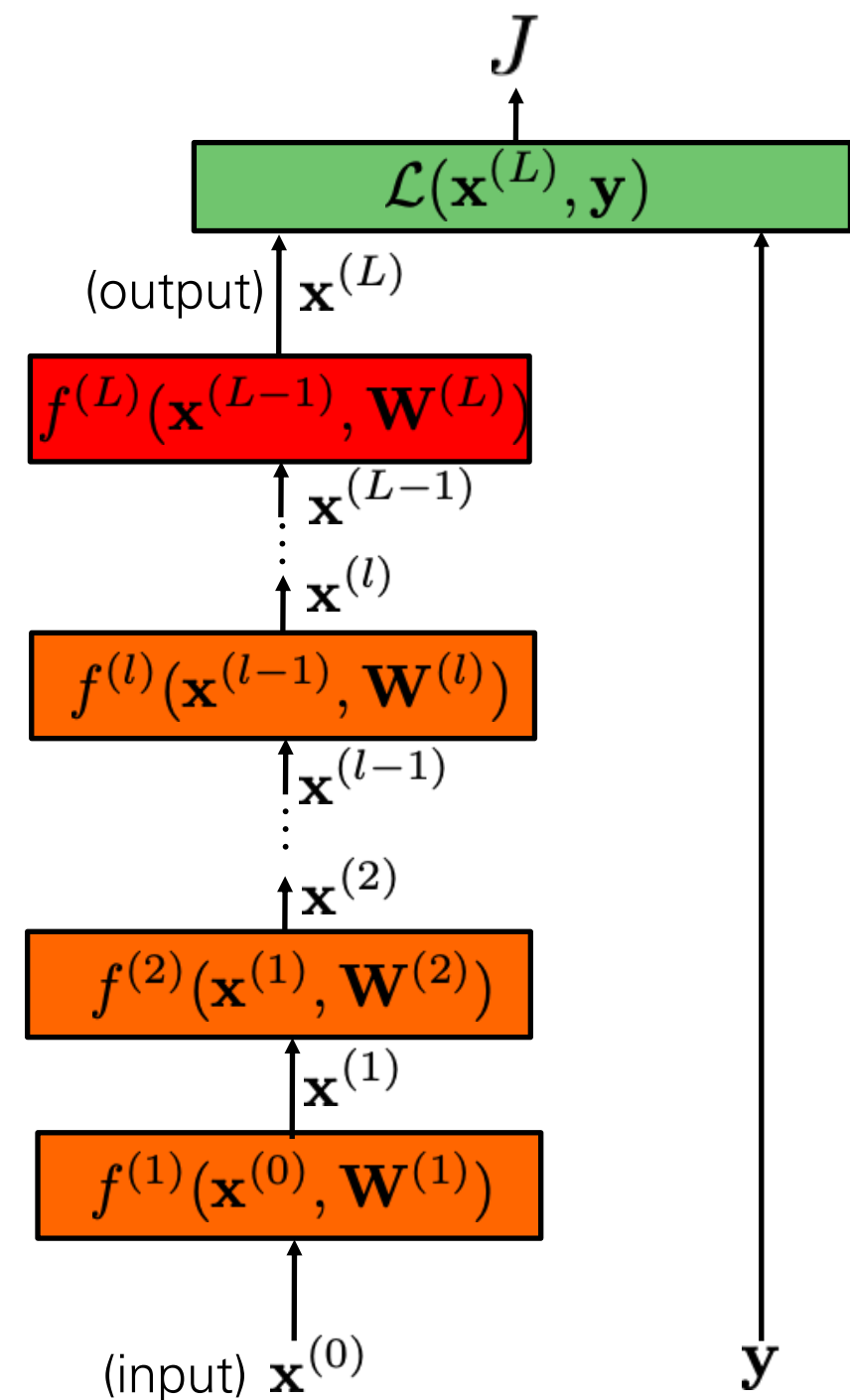
- Consider model with  $L$  layers. Layer  $l$  has vector of weights  $\mathbf{W}^{(l)}$
- Forward pass:** takes input  $\mathbf{x}^{(l-1)}$  and passes it through each layer  $f^{(l)}$ :  
$$\mathbf{x}^{(l)} = f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})$$
- Output of layer  $l$  is  $\mathbf{x}^{(l)}$ .
- Network output (top layer) is  $\mathbf{x}^{(L)}$ .



# Forward pass

- Consider model with  $L$  layers. Layer  $l$  has vector of weights  $\mathbf{W}^{(l)}$
- Forward pass:** takes input  $\mathbf{x}^{(l-1)}$  and passes it through each layer  $f^{(l)}$ :
$$\mathbf{x}^{(l)} = f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})$$
- Output of layer  $l$  is  $\mathbf{x}^{(l)}$ .
- Network output (top layer) is  $\mathbf{x}^{(L)}$ .
- Loss function  $\mathcal{L}$**  compares  $\mathbf{x}^{(L)}$  to  $\mathbf{y}$ .
- Overall energy is the sum of the cost over all training examples:

$$J = \sum_{i=1}^N \mathcal{L}(\mathbf{x}_i^{(L)}, \mathbf{y}_i)$$



# Gradient descent

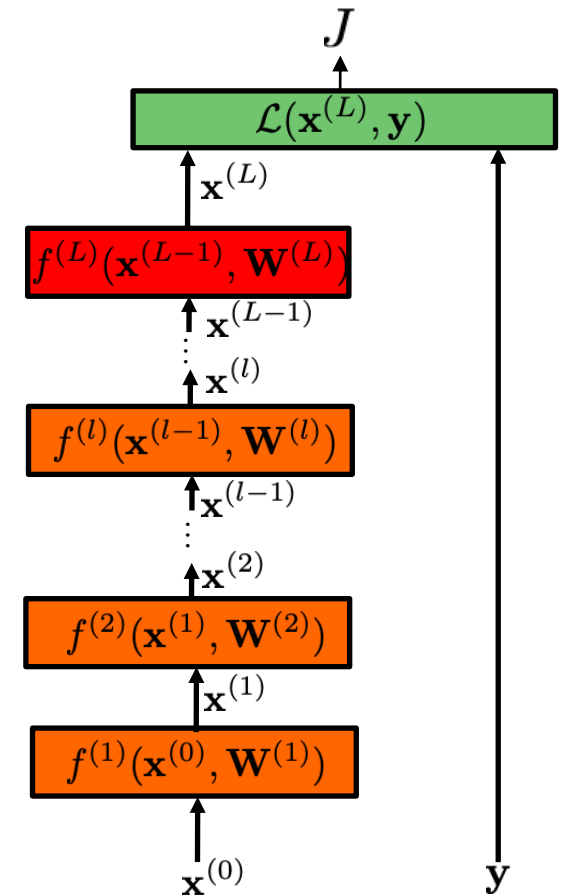
- We need to compute gradients of the cost with respect to model parameters  $\mathbf{W}^{(l)}$ .
- By design, each layer is differentiable with respect to its parameters and input.

# Computing gradients

To compute the gradients, we could start by writing the full energy  $J$  as a function of the network parameters.

$$J(\mathbf{W}) = \sum_{i=1} \mathcal{L}(f^{(L)}(\dots f^{(2)}(f^{(1)}(\mathbf{x}_i^{(0)}, \mathbf{W}^{(1)}), \mathbf{W}^{(2)}), \dots \mathbf{W}^{(L)}), \mathbf{y}_i)$$

And then compute the partial derivatives... instead, we can use the chain rule to derive a compact algorithm:  
**backpropagation**



# Computing gradients

The energy  $J$  is the sum of the losses associated to each training example  $\{\mathbf{x}_i^{(0)}, \mathbf{y}_i\}$

$$J(\mathbf{W}) = \sum_{i=1}^N \mathcal{L}(\mathbf{x}_i^{(L)}, \mathbf{y}_i; \mathbf{W})$$

Its gradient with respect to each of the network's parameters  $w$  is:

$$\frac{\partial J(\mathbf{W})}{\partial w} = \sum_{i=1}^N \frac{\partial \mathcal{L}(\mathbf{x}_i^{(L)}, \mathbf{y}_i; \mathbf{W})}{\partial w}$$

is how much  $J$  varies when the parameter  $w$  is varied.

# Computing gradients

We could write the loss function to get the gradients as:

$$\mathcal{L}(\mathbf{x}^{(L)}, \mathbf{y}; \mathbf{W}) = \mathcal{L}(f^{(L)}(\mathbf{x}^{(L-1)}, \mathbf{W}^{(L)}), \mathbf{y})$$


If we compute the gradient with respect to the parameters of the last layer (output layer)  $\mathbf{W}^{(L)}$ , using the **chain rule**:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(L)}} \cdot \frac{\partial \mathbf{x}^{(L)}}{\partial \mathbf{W}^{(L)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(L)}} \cdot \frac{\partial f^{(L)}(\mathbf{x}^{(L-1)}, \mathbf{W}^{(L)})}{\partial \mathbf{W}^{(L)}}$$

(How much the cost changes when we change  $\mathbf{W}^{(L)}$  is the product between how much the loss changes when we change the output of the last layer and how much the output changes when we change the layer parameters.)

# Computing gradients: loss layer

If we compute the gradient with respect to the parameters of the last layer (output layer)  $\mathbf{W}^{(L)}$ , using the chain rule:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(L)}} \cdot \frac{\partial \mathbf{x}^{(L)}}{\partial \mathbf{W}^{(L)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(L)}} \cdot \frac{\partial f^{(L)}(\mathbf{x}^{(L-1)}, \mathbf{W}^{(L)})}{\partial \mathbf{W}^{(L)}}$$


For example, for an Euclidean loss:

$$\mathcal{L}(\mathbf{x}^{(L)}, \mathbf{y}) = \frac{1}{2} \left\| \mathbf{x}^{(L)} - \mathbf{y} \right\|_2^2$$

Will depend on the layer structure and non-linearity.

The gradient is:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(L)}} = \mathbf{x}^{(L)} - \mathbf{y}$$

# Computing gradients: layer $l$

We could write the full loss function to get the gradients:

$$\mathcal{L}(\mathbf{x}^{(L)}, \mathbf{y}; \mathbf{W}) = \mathcal{L}(f^{(L)}(\dots f^{(2)}(f^{(1)}(\mathbf{x}^{(0)}, \mathbf{W}^{(1)}), \mathbf{W}^{(2)}), \dots \mathbf{W}^{(L)}), \mathbf{y})$$

If we compute the gradient with respect to  $w_i$ , using the chain rule:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(L)}} \cdot \frac{\partial \mathbf{x}^{(L)}}{\partial \mathbf{x}^{(L-1)}} \cdot \frac{\partial \mathbf{x}^{(L-1)}}{\partial \mathbf{x}^{(L-2)}} \cdots \frac{\partial \mathbf{x}^{(l+1)}}{\partial \mathbf{x}^{(l)}} \cdot \frac{\partial \mathbf{x}^{(l)}}{\partial \mathbf{W}^{(l)}}$$

$\frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}}$

$\frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{W}^{(l)}}$

And this can be  
computed iteratively!

This is easy.



# Backpropagation

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(L)}} = \underbrace{\frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(L)}} \cdot \frac{\partial \mathbf{x}^{(L)}}{\partial \mathbf{x}^{(L-1)}} \cdot \frac{\partial \mathbf{x}^{(L-1)}}{\partial \mathbf{x}^{(L-2)}} \cdots \frac{\partial \mathbf{x}^{(l+1)}}{\partial \mathbf{x}^{(l)}}}_{\frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}}} \cdot \frac{\partial \mathbf{x}^{(l)}}{\partial \mathbf{W}^{(l)}}$$

$\nearrow$   
 $\frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{W}^{(l)}}$

If we have the value of  $\frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}}$  we can compute the gradient at the layer below as:

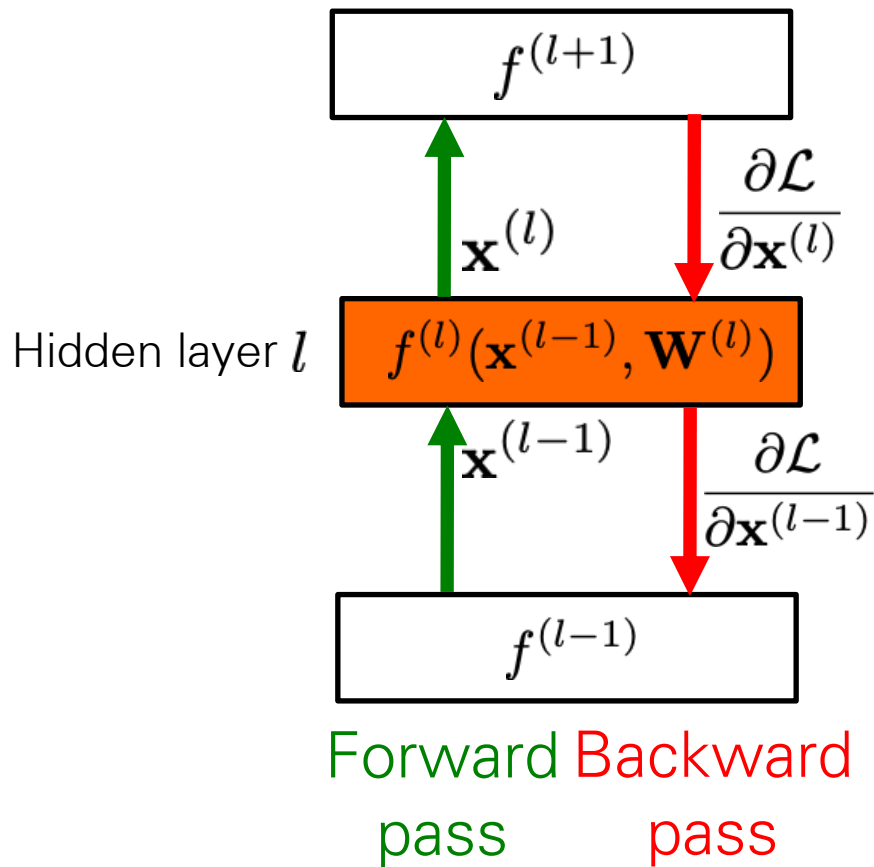
$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l-1)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}} \cdot \frac{\partial \mathbf{x}^{(l)}}{\partial \mathbf{x}^{(l-1)}}$$

$\nearrow$   
 Gradient  
layer l-1

$\nearrow$   
 Gradient  
layer l

$\nwarrow$   
 $\frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{x}^{(l-1)}}$

# Backpropagation — Goal: to update parameters of layer $l$



- Layer  $l$  has two inputs (during training)

$$\mathbf{x}^{(l-1)} \rightarrow \text{orange box}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}} \rightarrow \text{orange box}$$

- We compute the outputs

$$\text{orange box} \rightarrow \mathbf{x}^{(l)} = f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})$$

$$\text{orange box} \rightarrow \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l-1)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}} \cdot \frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{x}^{(l-1)}}$$

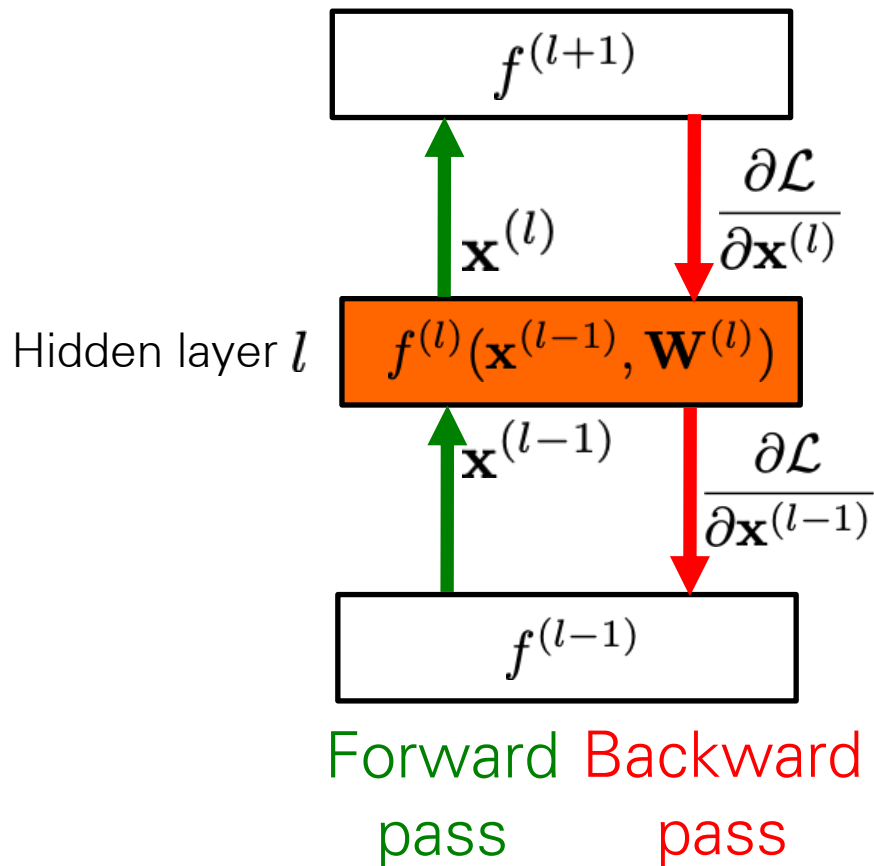
- To compute the output, we need:

$$\frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{x}^{(l-1)}}$$

- To compute the weight update, we need:

$$\frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{W}^{(l)}}$$

# Backpropagation — Goal: to update parameters of layer $l$



- Layer  $l$  has two inputs (during training)

$$\mathbf{x}^{(l-1)} \rightarrow \text{orange box}$$

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}} \rightarrow \text{orange box}$$

- We compute the outputs

$$\text{orange box} \rightarrow \mathbf{x}^{(l)} = f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})$$

$$\text{orange box} \rightarrow \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l-1)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}} \cdot \frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{x}^{(l-1)}}$$

- The weight update equation is:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(l)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}} \cdot \frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{W}^{(l)}}$$

$$\mathbf{W}^{(l)} \leftarrow \mathbf{W}^{(l)} + \eta \left( \frac{\partial J}{\partial \mathbf{W}^{(l)}} \right)^T \quad (\text{sum over all training examples to get } J)$$

# Backpropagation Summary

- Forward pass: for each training example, compute the outputs for all layers:

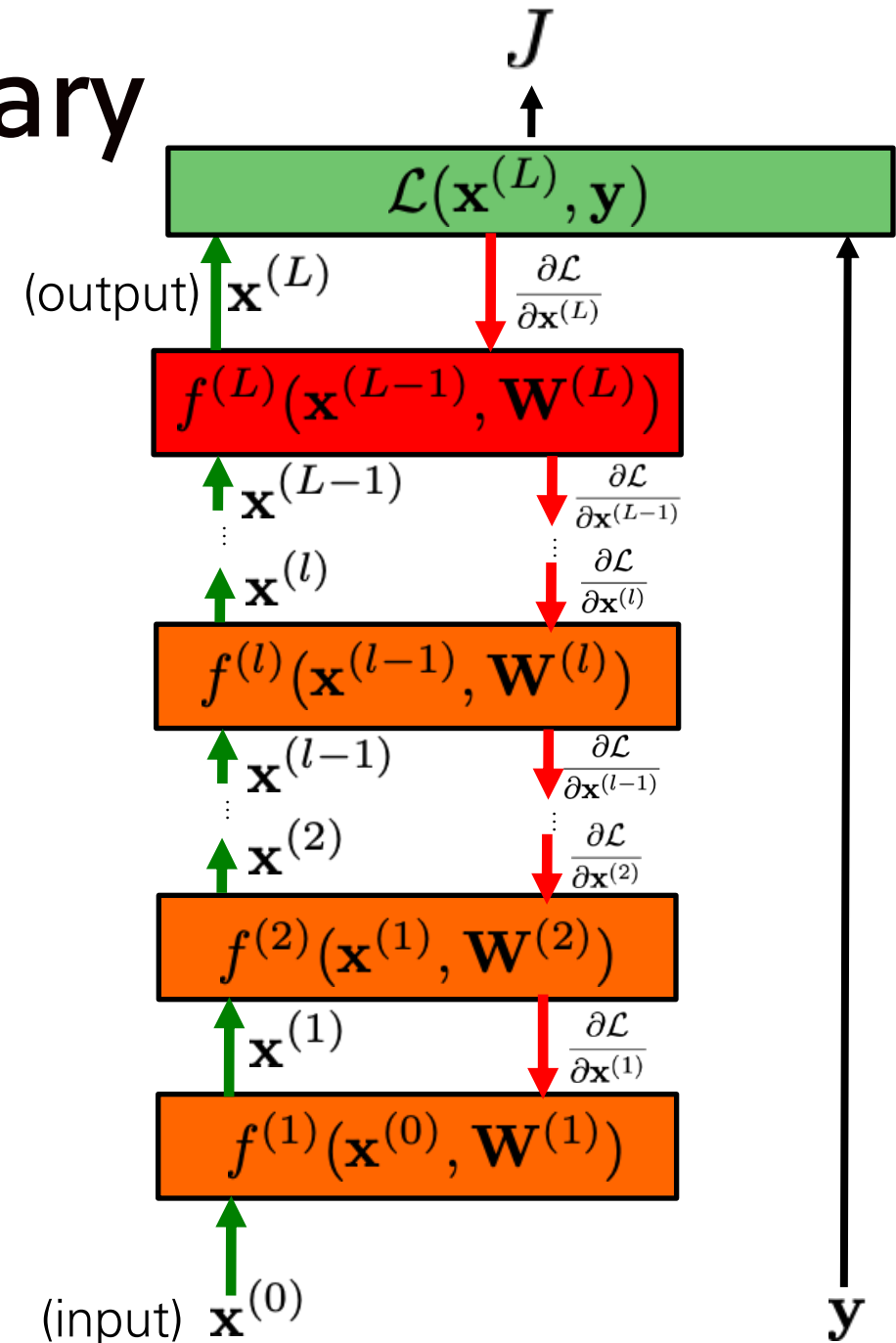
$$\mathbf{x}^{(l)} = f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})$$

- Backwards pass: compute loss derivatives iteratively from top to bottom:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l-1)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}} \cdot \frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{x}^{(l-1)}}$$

- Compute gradients w.r.t. weights, and update weights:

$$\frac{\partial \mathcal{L}}{\partial \mathbf{W}^{(l)}} = \frac{\partial \mathcal{L}}{\partial \mathbf{x}^{(l)}} \cdot \frac{\partial f^{(l)}(\mathbf{x}^{(l-1)}, \mathbf{W}^{(l)})}{\partial \mathbf{W}^{(l)}}$$



# Differentiable programming

Deep nets are popular for a few reasons:

1. High capacity
2. Easy to optimize (differentiable)
3. Compositional “block based programming”

An emerging term for general models with these properties is **differentiable programming**.



Yann LeCun

January 5 · 🌐

OK, Deep Learning has outlived its usefulness as a buzz-phrase. Deep Learning est mort. Vive Differentiable Programming!



Thomas G. Dietterich

@tdietterich

Following

DL is essentially a new style of programming--"differentiable programming"--and the field is trying to work out the reusable constructs in this style. We have some: convolution, pooling, LSTM, GAN, VAE, memory units, routing units, etc. 8/

8:02 AM - 4 Jan 2018

65 Retweets 194 Likes



6

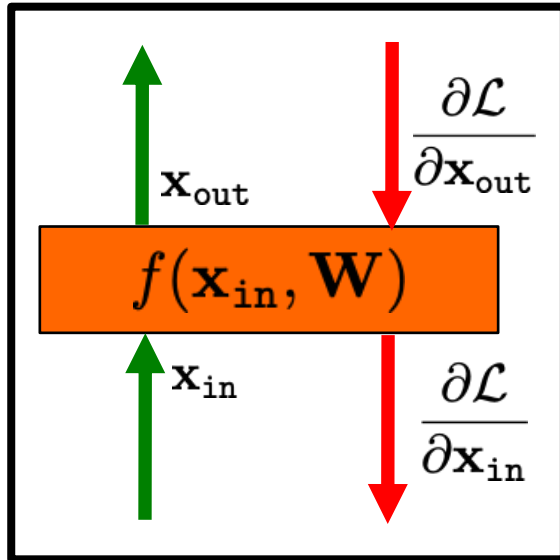
65

194

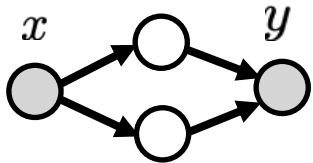
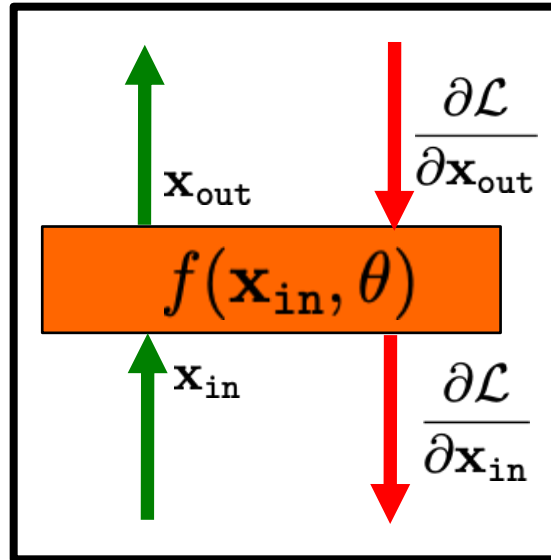


# Differentiable programming

Deep learning



Differentiable programming



```
1 for i, data in enumerate(dataset):
2     iter_start_time = time.time()
3     if total_steps % opt.print_freq == 0:
4         t_data = iter_start_time - iter_data_time
5         visualizer.reset()
6         total_steps += opt.batch_size
7         epoch_iter += opt.batch_size
8         model.set_input(data)
9         model.optimize_parameters()
```



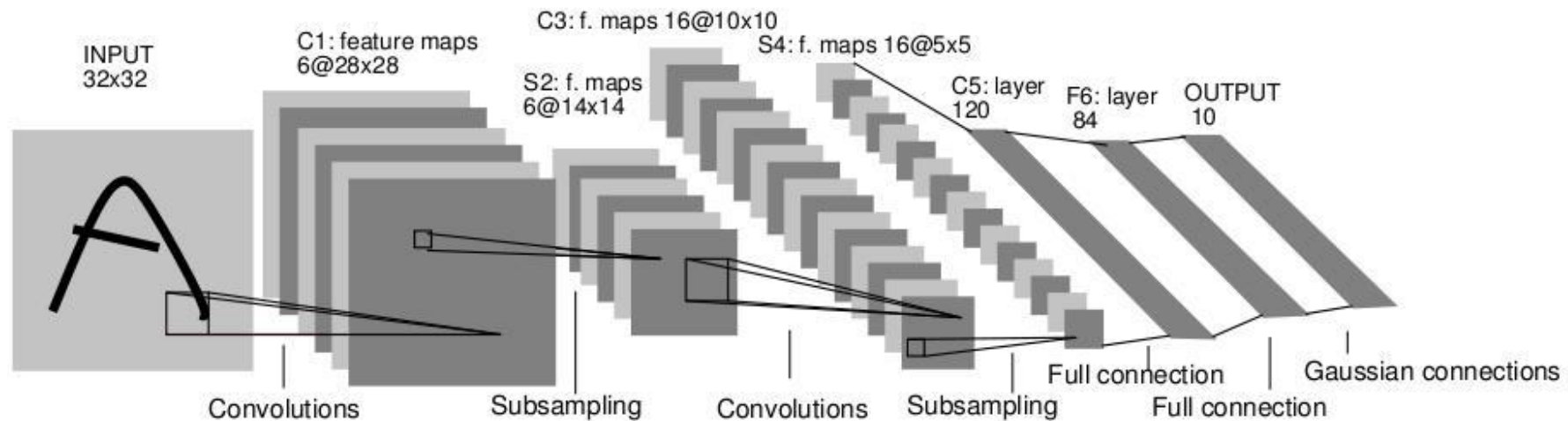
# Convolutional Neural Networks

# Convolutional Neural Networks

LeCun et al. 1989

Neural network with specialized connectivity

Tailored to processing natural signals with a grid topology (e.g., images).





# Image classification

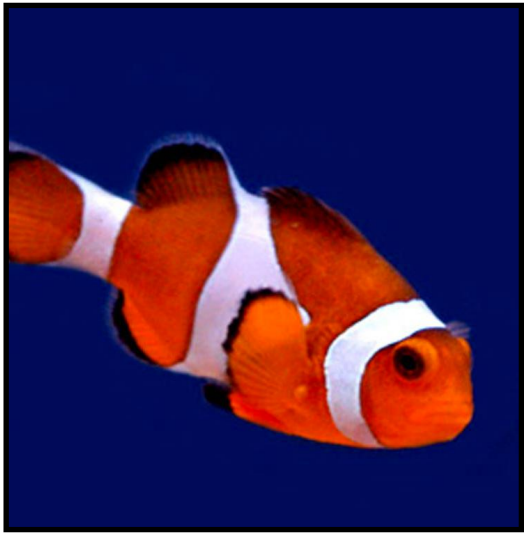


image  $x$

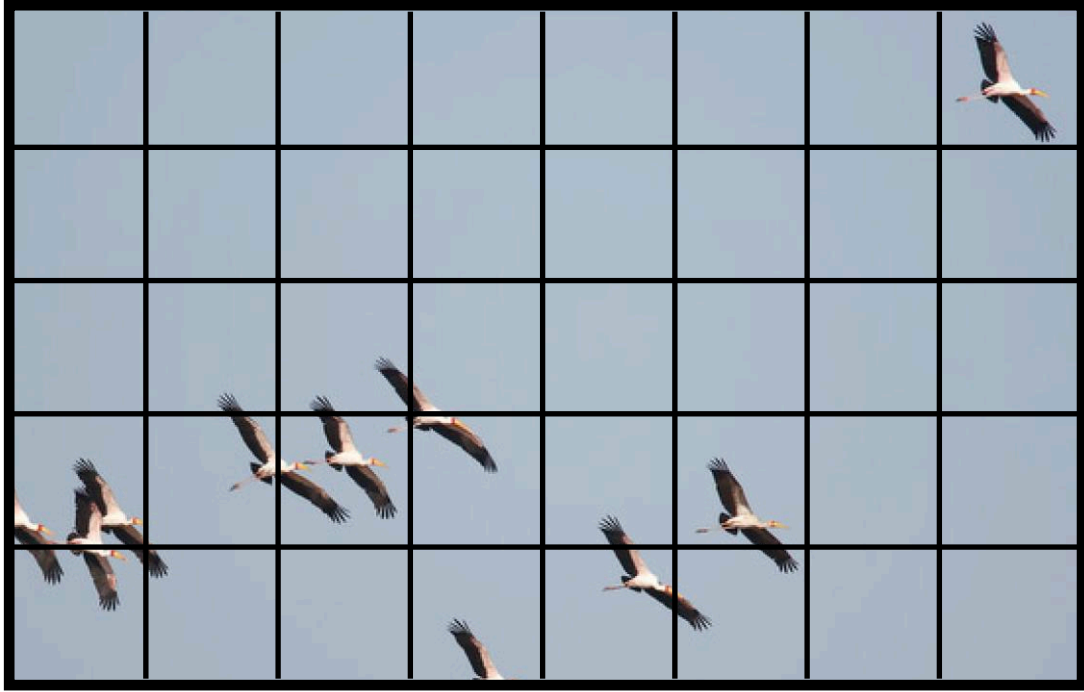


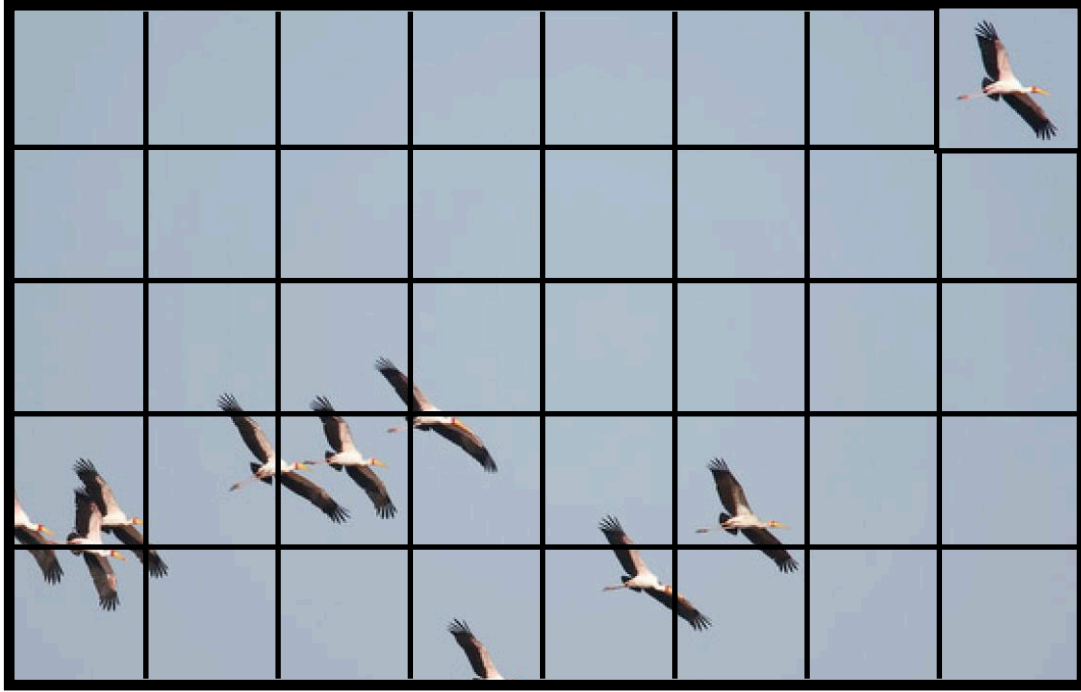
"Fish"

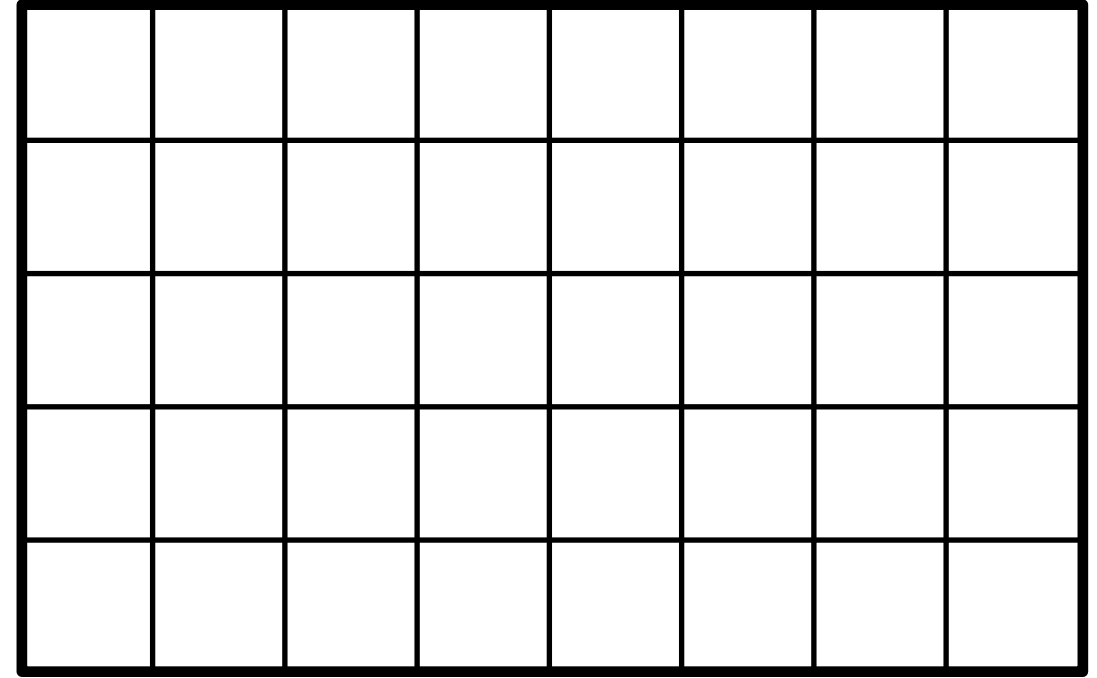
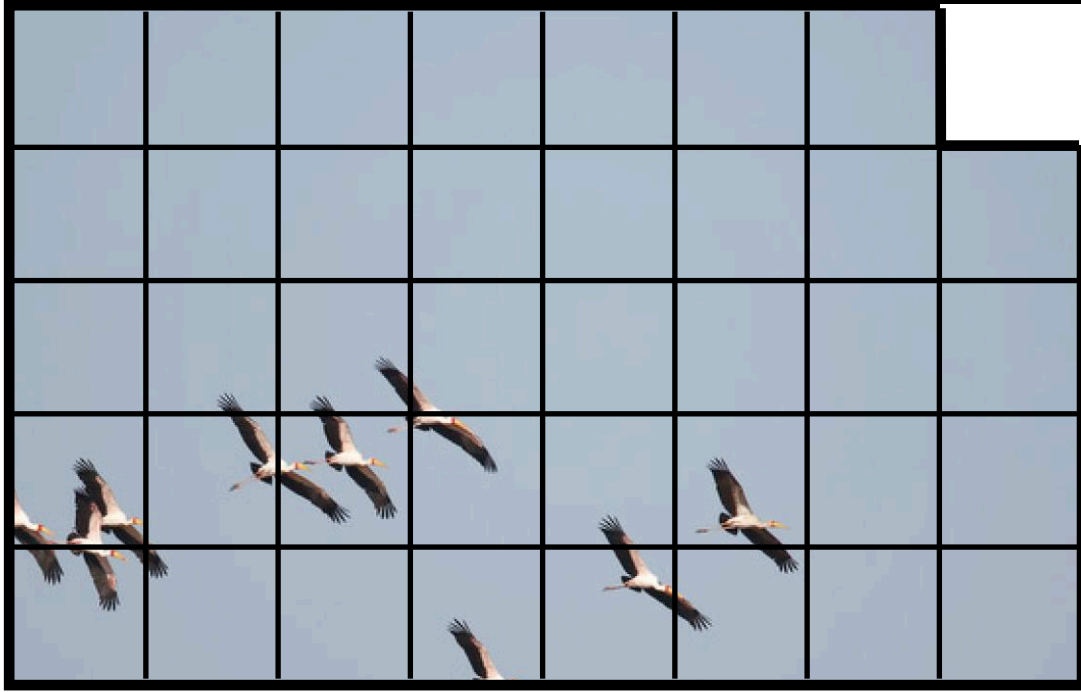
label  $y$

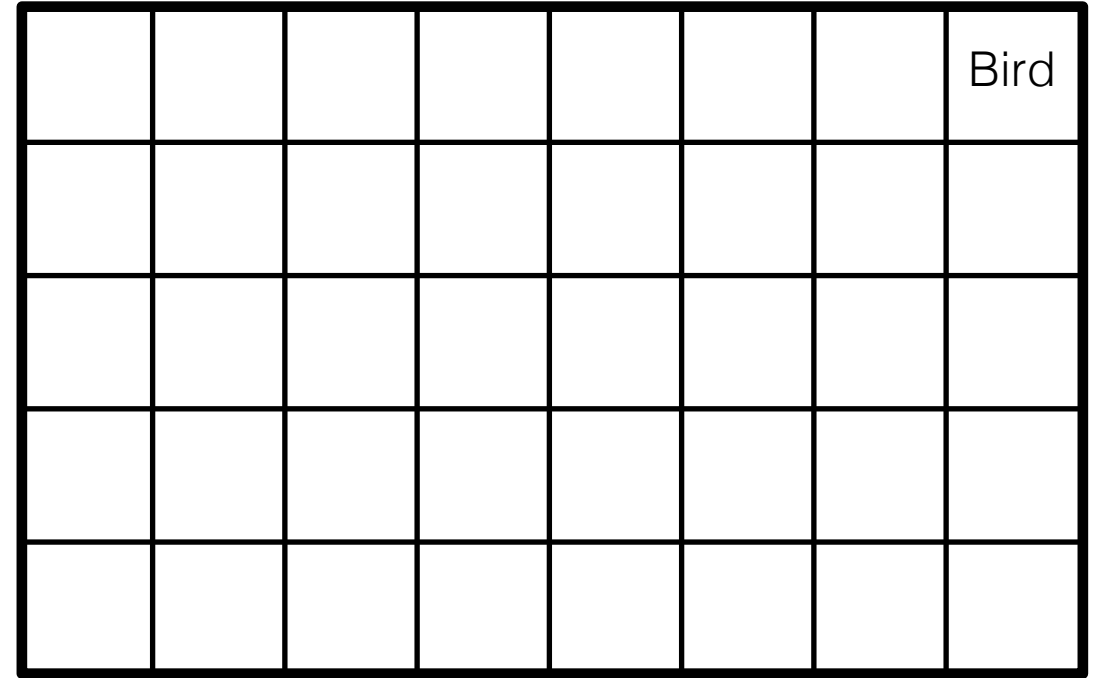
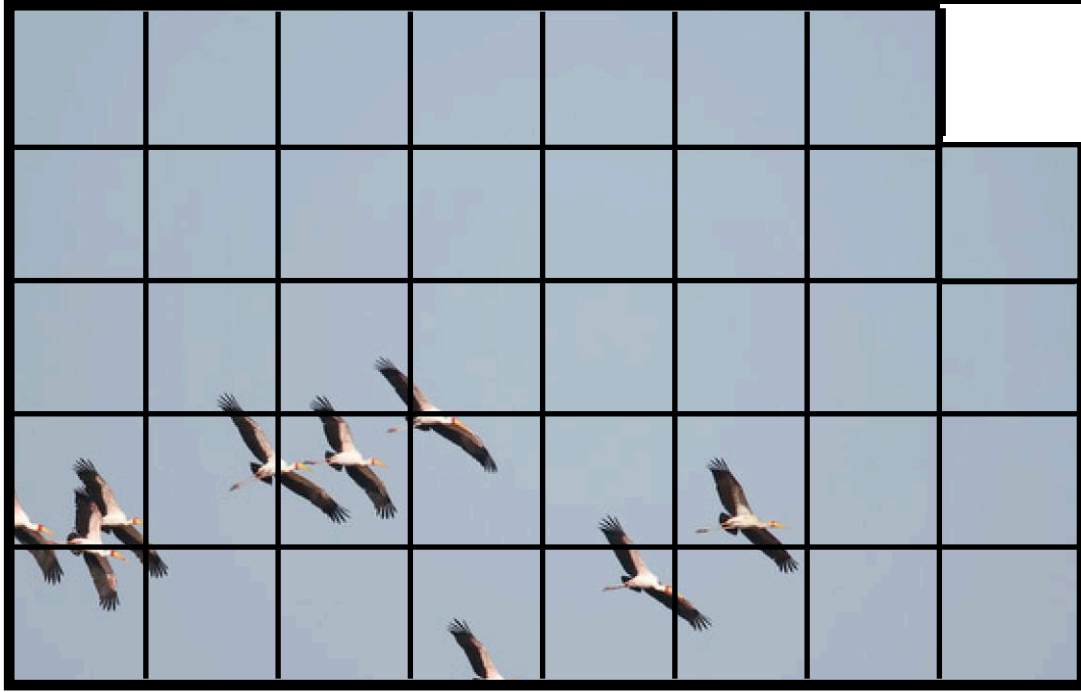


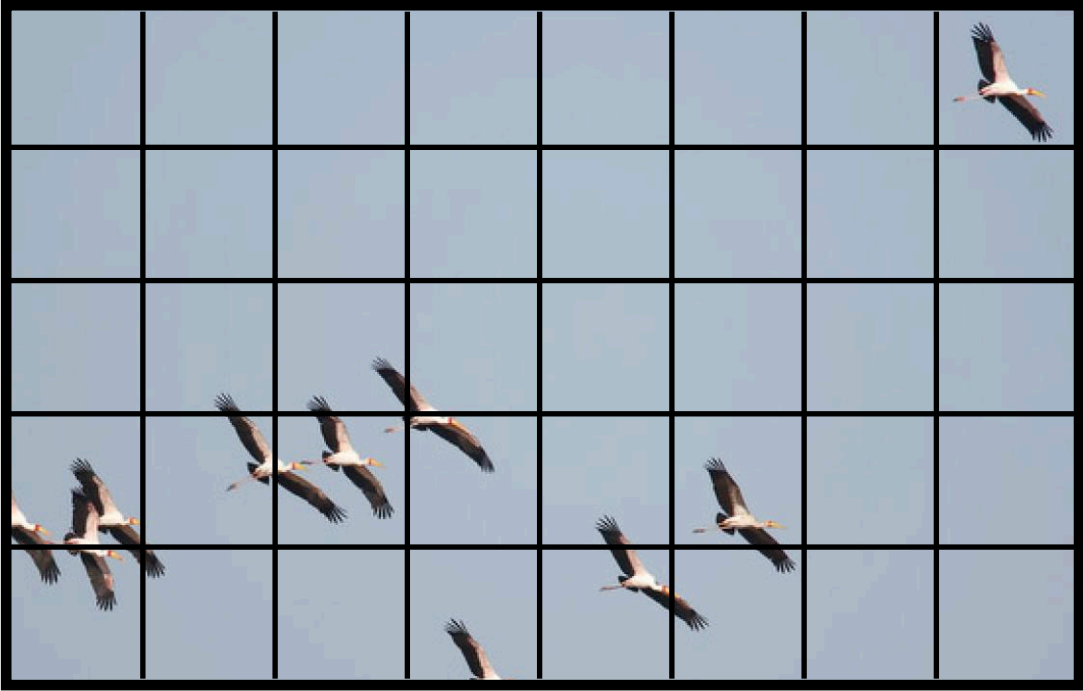
Photo credit: Fredo Durand



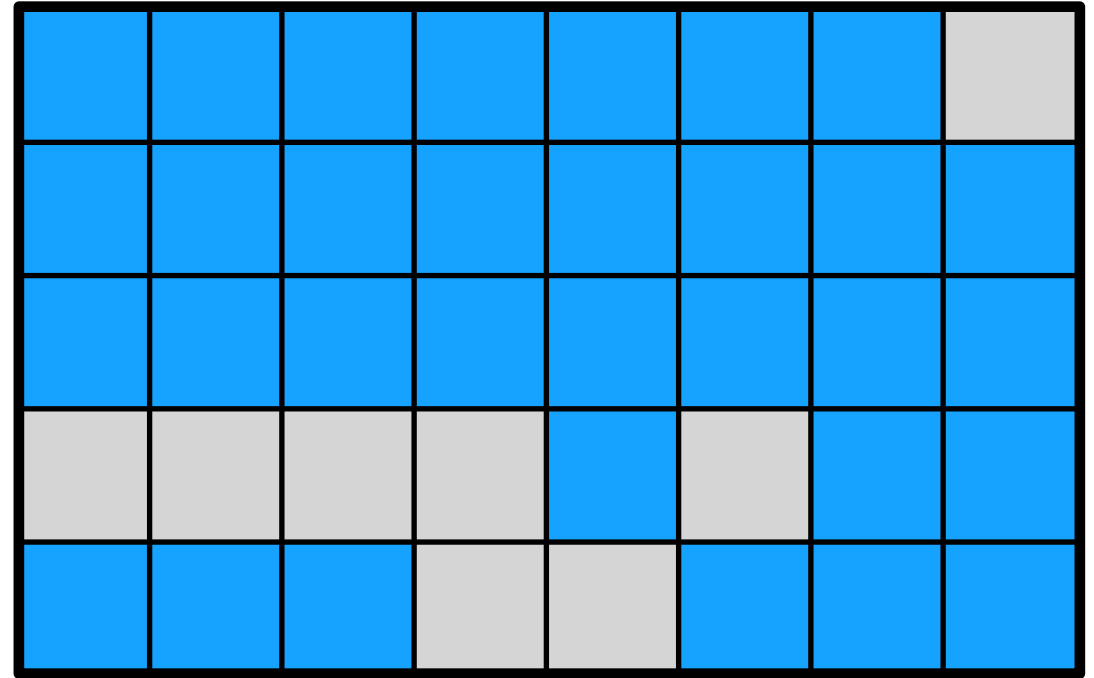
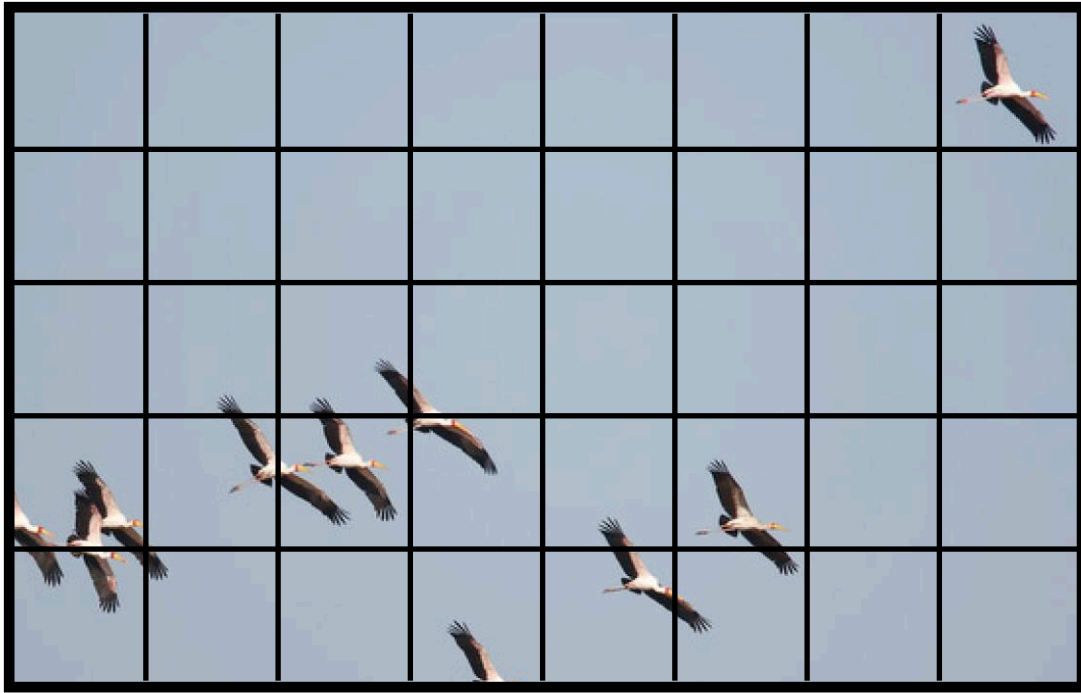




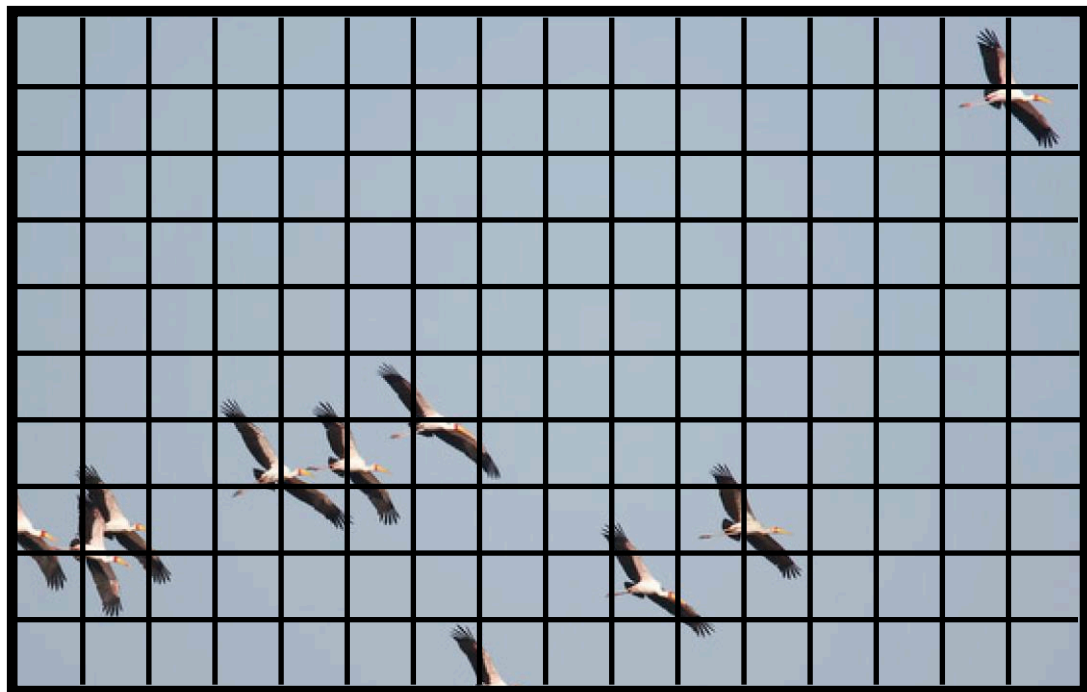


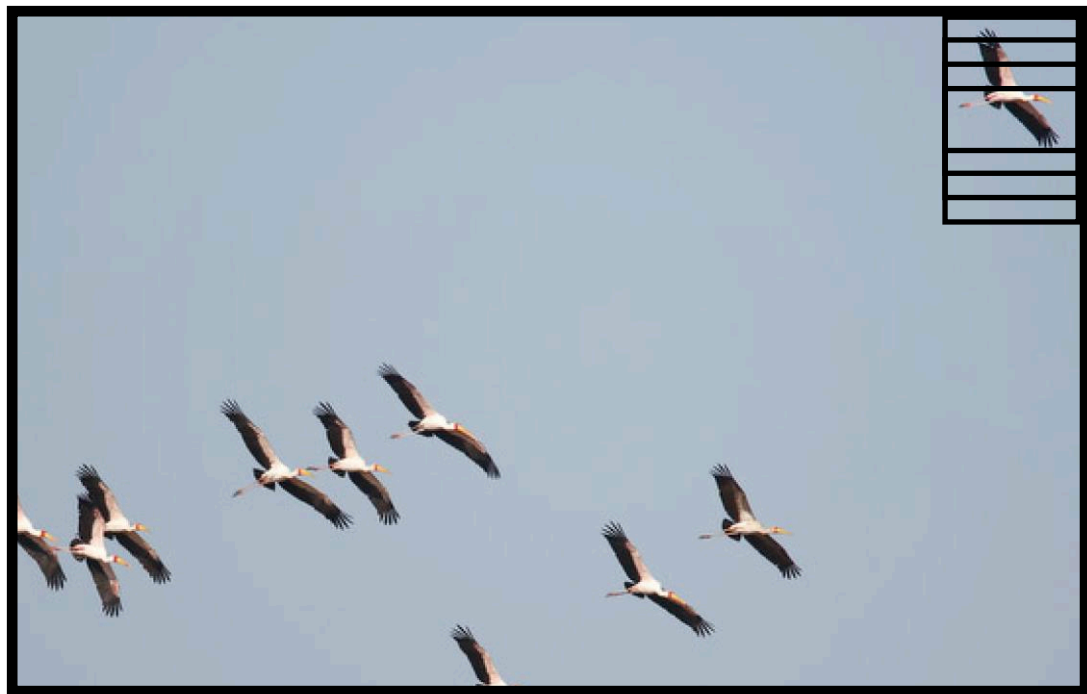


Sky	Sky	Sky	Sky	Sky	Sky	Sky	Bird
Sky	Sky	Sky	Sky	Sky	Sky	Sky	Sky
Sky	Sky	Sky	Sky	Sky	Sky	Sky	Sky
Bird	Bird	Bird	Sky	Bird	Sky	Sky	Sky
Sky	Sky	Sky	Bird	Sky	Sky	Sky	Sky

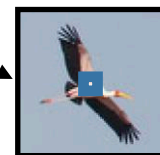




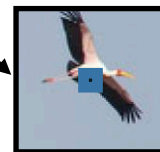




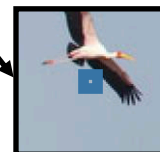
What's the object class of the center pixel?



"Bird"



"Bird"



"Sky"



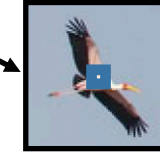
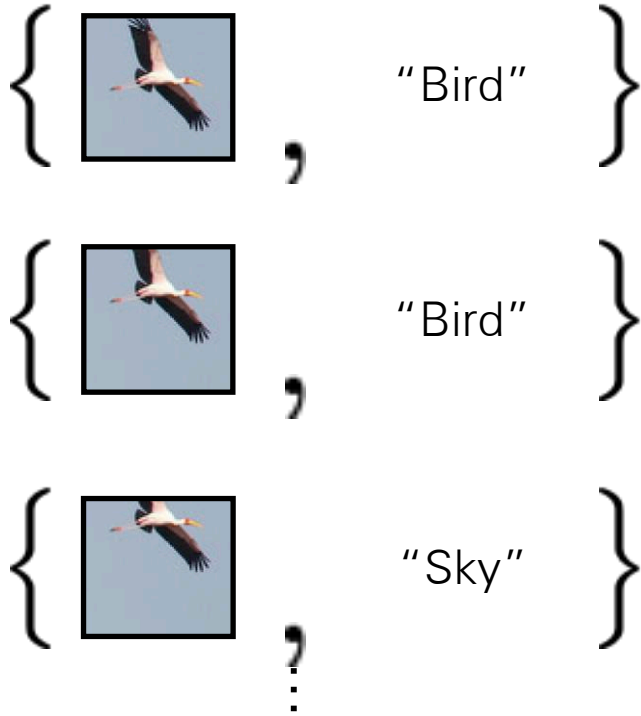
"Sky"

What's the object class of the center pixel?

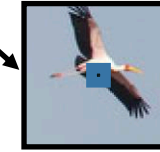
Training data

$\mathbf{x}$

$y$



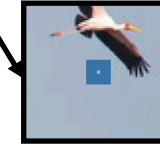
"Bird"



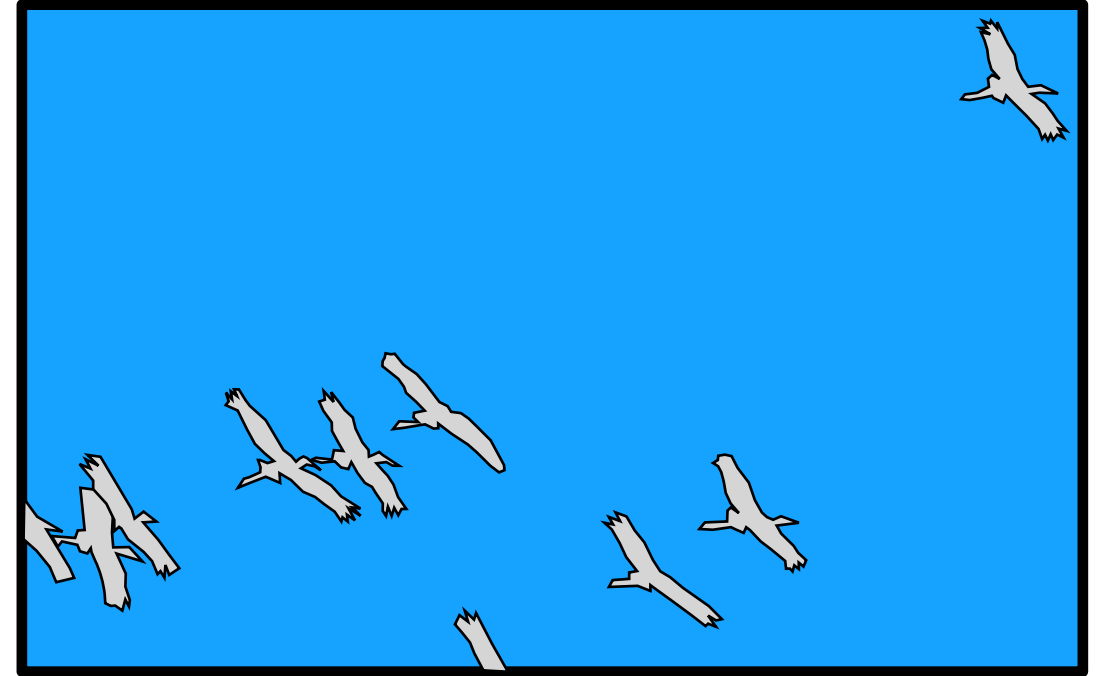
"Bird"



"Sky"

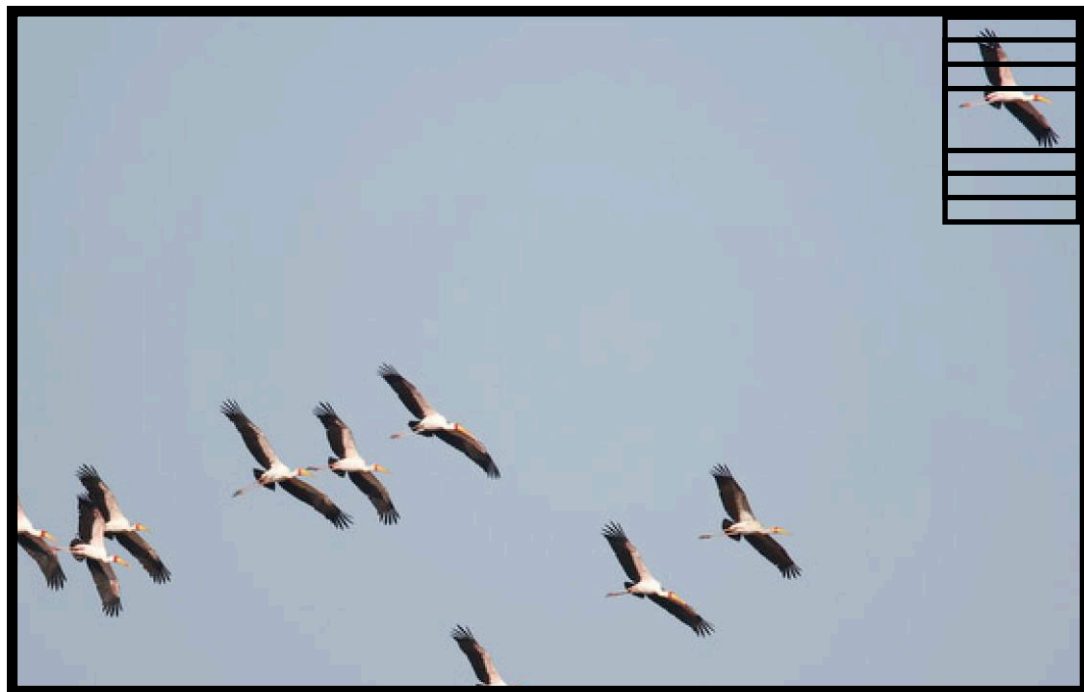


"Sky"

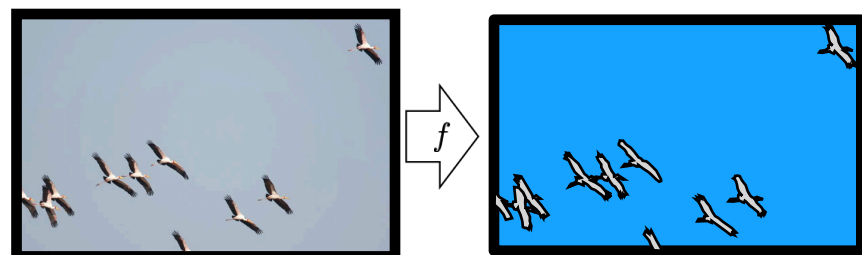
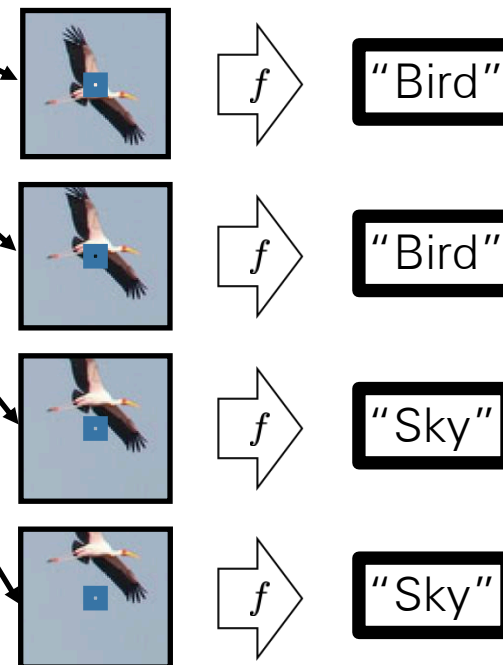


(Colors represent one-hot codes)

This problem is called **semantic segmentation**



What's the object class of the center pixel?

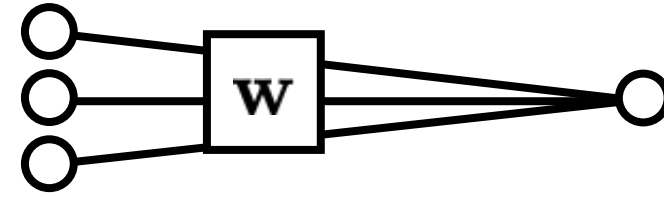


Translation invariance: process each patch in the same way.

An equivariant mapping:

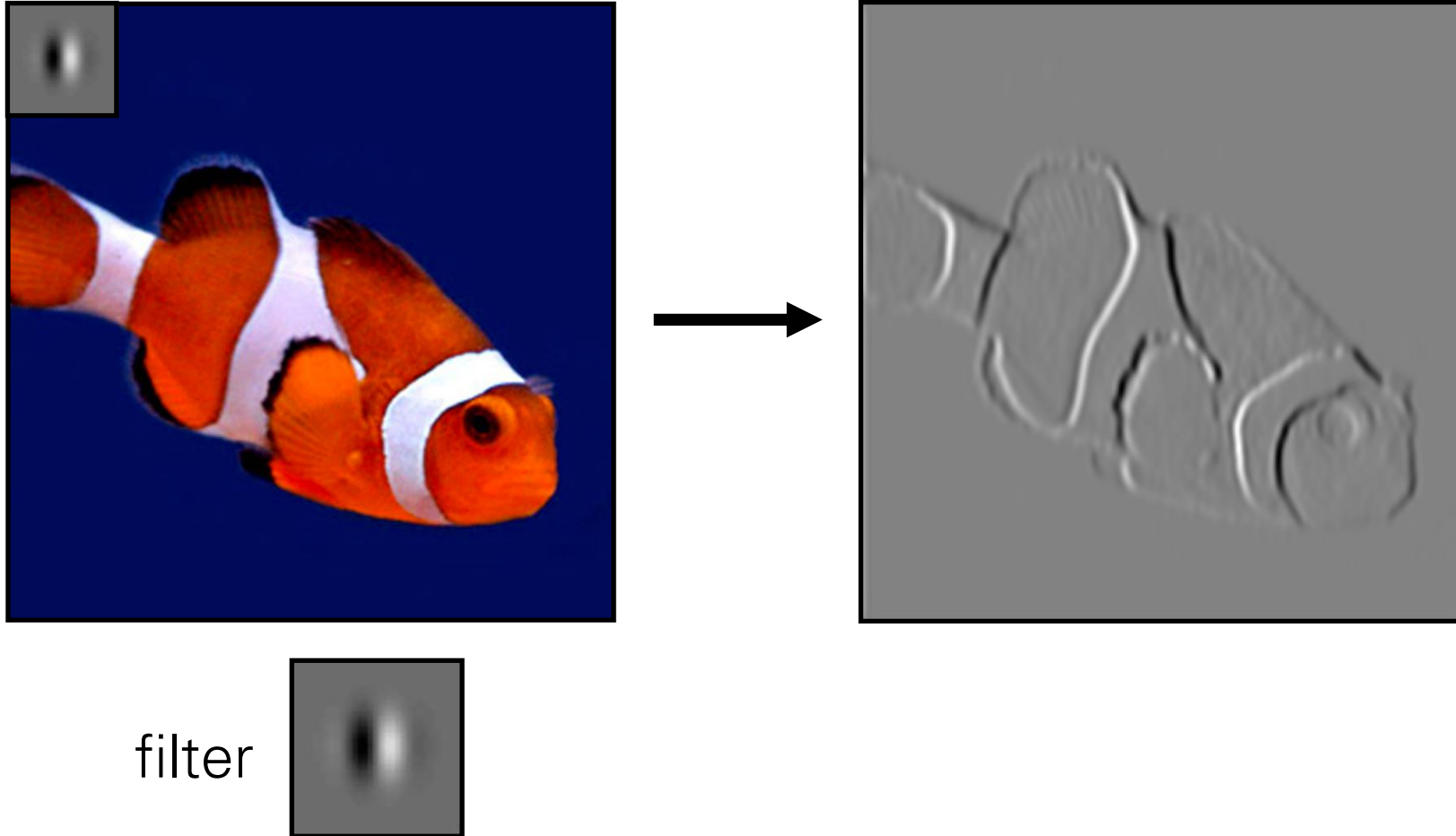
$$f(\text{translate}(x)) = \text{translate}(f(x))$$

$W$  computes a weighted sum of all pixels in the patch



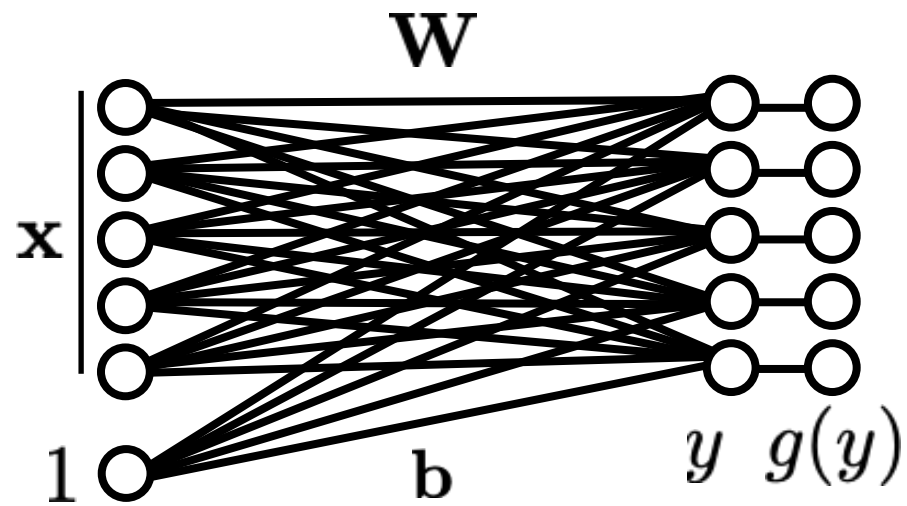
$W$  is a convolutional kernel applied to the full image!

# Convolution



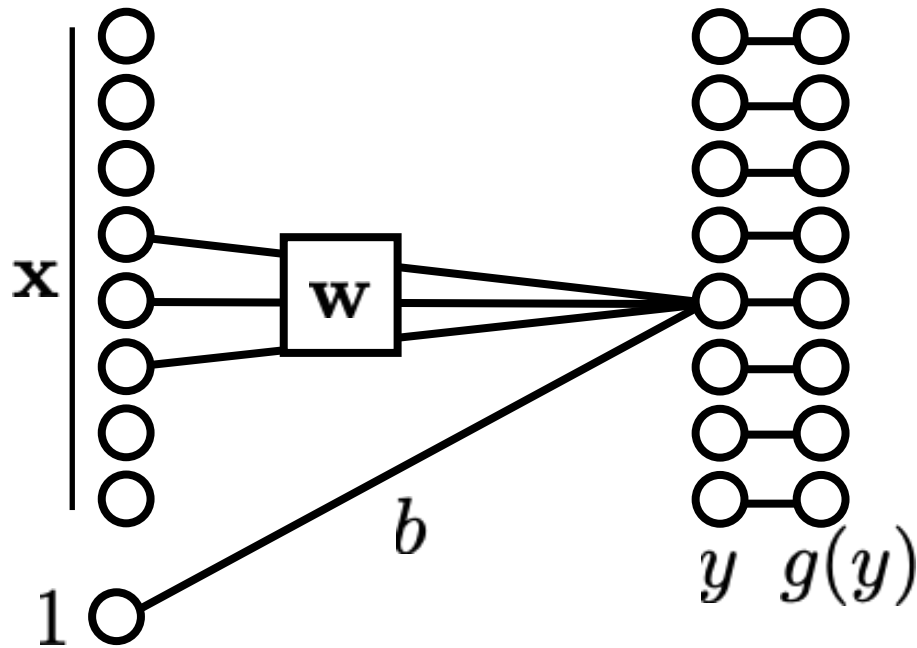
# Fully-connected network

Fully-connected (fc) layer





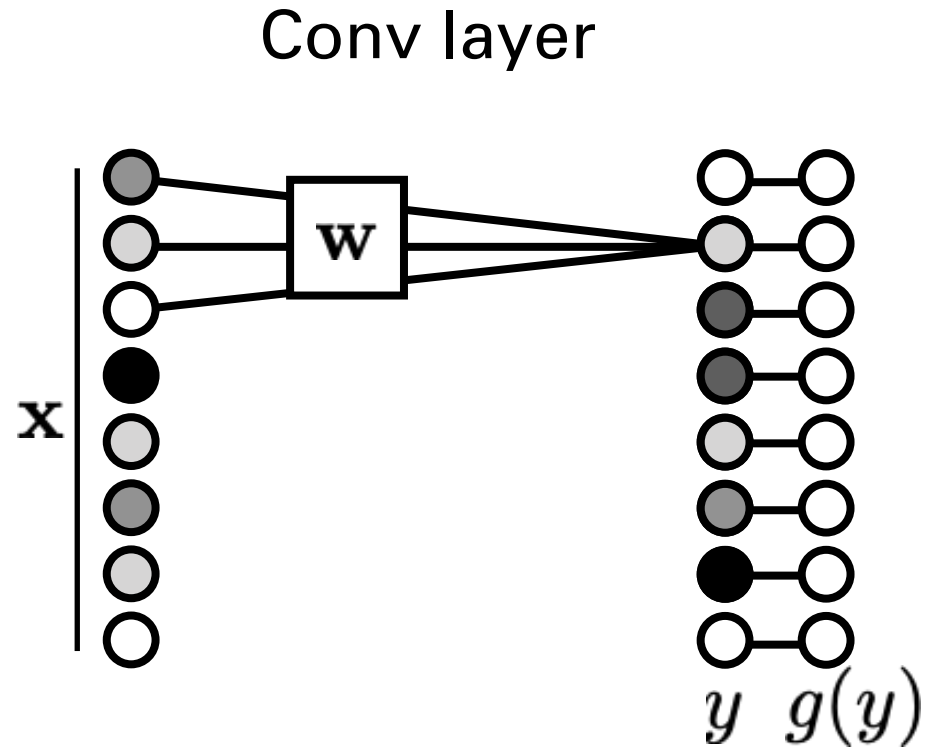
# Locally connected network



Often, we assume output is a **local** function of input.

If we use the same weights (**weight sharing**) to compute each local function, we get a convolutional neural network.

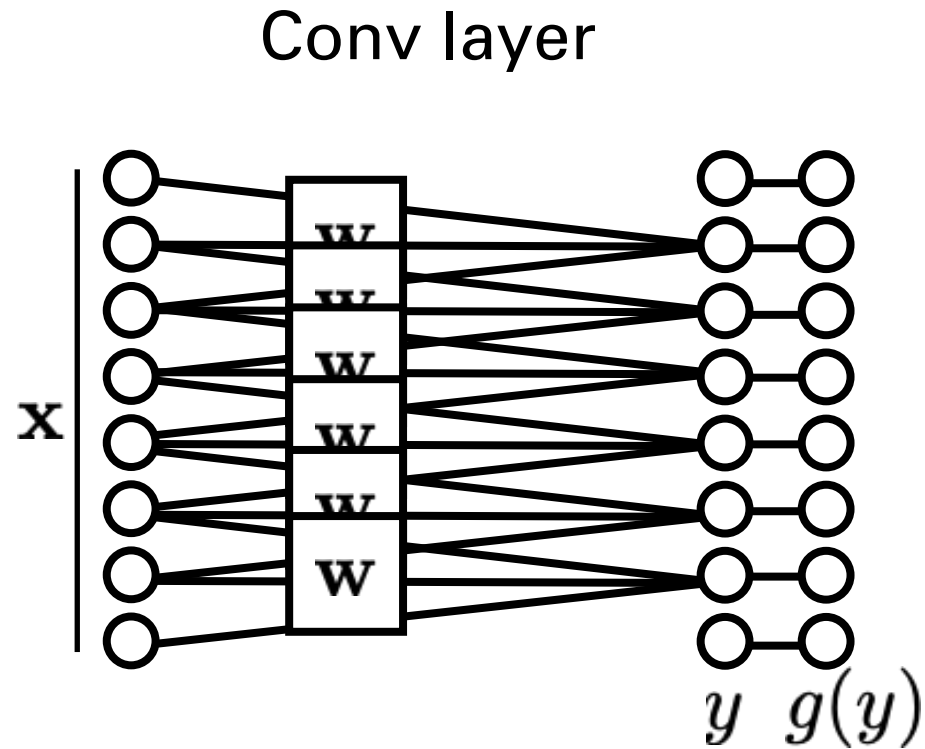
# Convolutional neural network



Often, we assume output is a **local** function of input.

If we use the same weights (**weight sharing**) to compute each local function, we get a convolutional neural network.

# Weight sharing



Often, we assume output is a **local** function of input.

If we use the same weights (**weight sharing**) to compute each local function, we get a convolutional neural network.

Toeplitz matrix

$$\begin{pmatrix} a & b & c & d & e \\ f & a & b & c & d \\ g & f & a & b & c \\ h & g & f & a & b \\ i & h & g & f & a \end{pmatrix}$$

$$\mathbf{x}^{(l+1)} = \begin{matrix} \text{[Toeplitz Matrix]} \end{matrix} * \mathbf{x}^{(l)}$$

e.g., pixel image

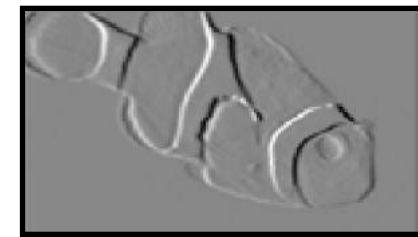
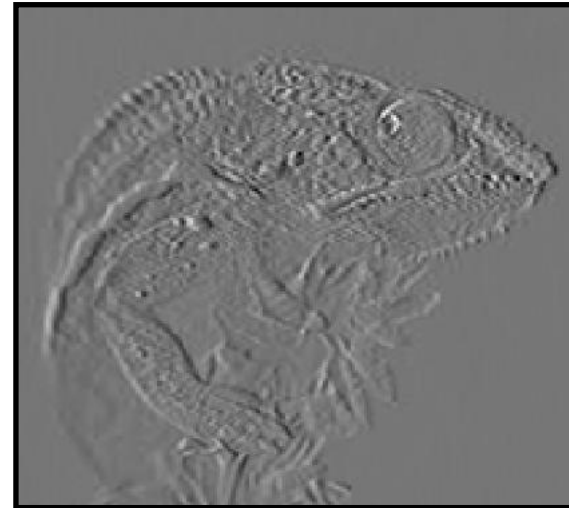
- Constrained linear layer (infinitely strong regularization)
- Fewer parameters —> easier to learn, less overfitting

$$\mathbf{x}^{(l+1)} = \begin{bmatrix} \text{matrix} \end{bmatrix} * \mathbf{x}^{(l)}$$

The diagram illustrates a vector-matrix multiplication. On the left, a vertical white rectangle with a black border represents the vector  $\mathbf{x}^{(l+1)}$ . In the center, a square matrix with a black and white checkerboard pattern along its main diagonal and dark gray elsewhere represents the transformation matrix. To the right of the matrix is an equals sign. To the left of the matrix is another vertical white rectangle with a black border, representing the vector  $\mathbf{x}^{(l)}$ . To the right of the matrix is an asterisk symbol, indicating multiplication.

$$\mathbf{x}^{(l+1)} = \begin{bmatrix} \text{diagonal matrix} \end{bmatrix} * \mathbf{x}^{(l)}$$

The diagram illustrates a vector-matrix multiplication. On the left, a vertical vector is labeled  $\mathbf{x}^{(l+1)}$ . In the center, a square matrix is shown with a dark gray background and a light gray diagonal line of pixels, representing a diagonal matrix. To the right of the matrix is an equals sign (=). Further right is a multiplication symbol (\*). On the far right, another vertical vector is labeled  $\mathbf{x}^{(l)}$ .

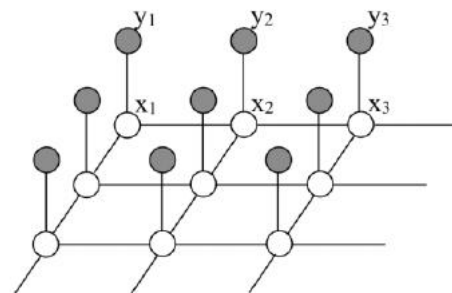


Conv layers can be applied to arbitrarily-sized inputs

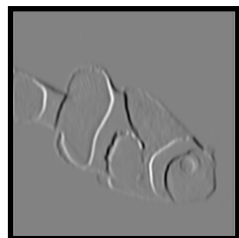
# Five views on convolutional layers

1. Equivariant with translation (stationarity)  $f(\text{translate}(x)) = \text{translate}(f(x))$

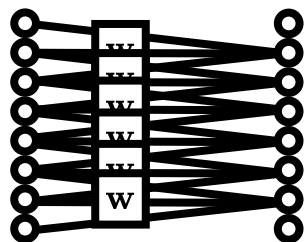
2. Patch processing (Markov assumption)



3. Image filter



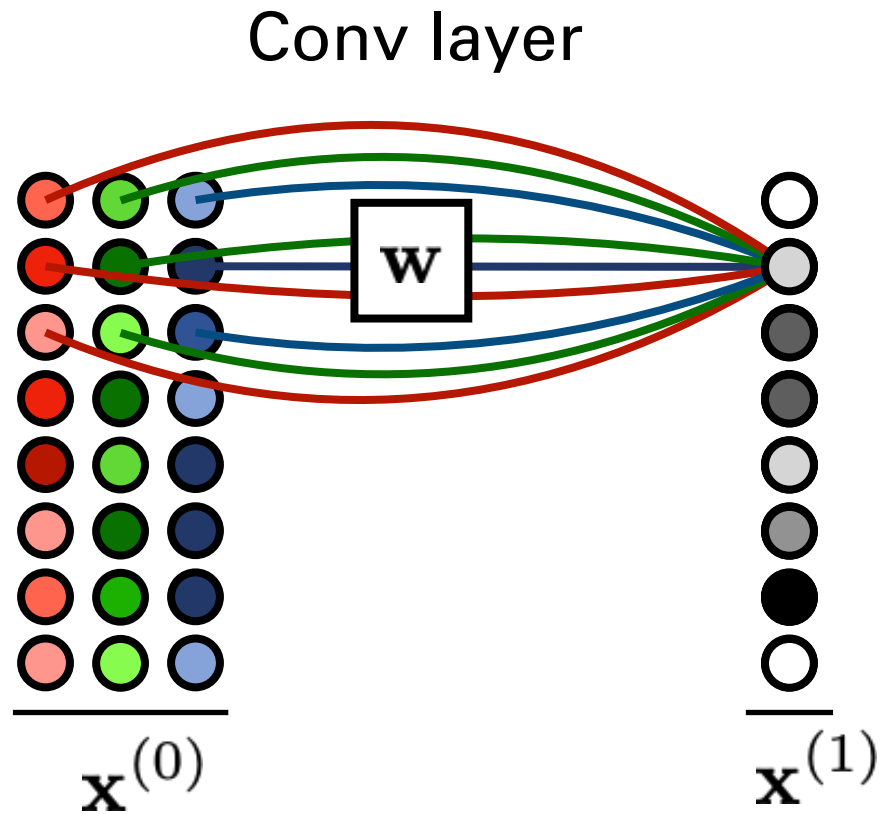
4. Parameter sharing



5. A way to process variable-sized tensors

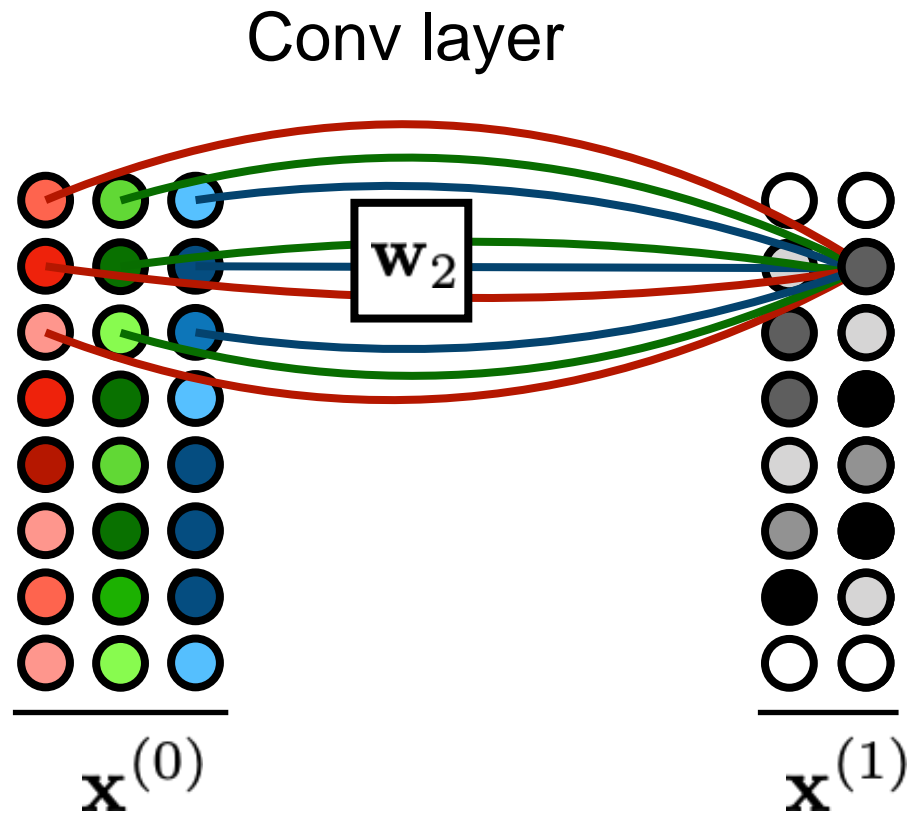


# Multiple channels



$$\mathbb{R}^{N \times C} \rightarrow \mathbb{R}^{N \times 1}$$

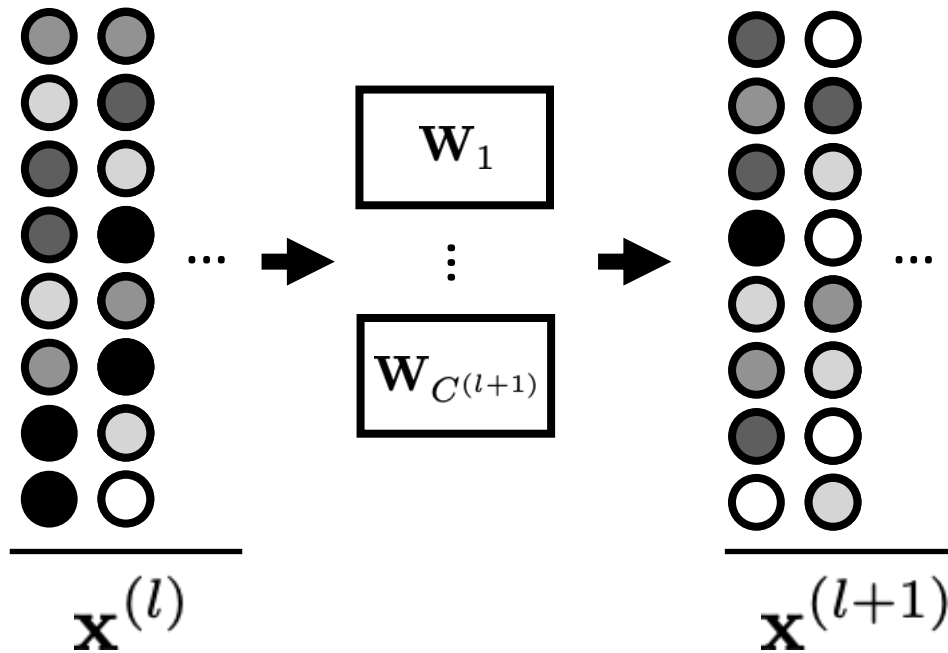
# Multiple channels



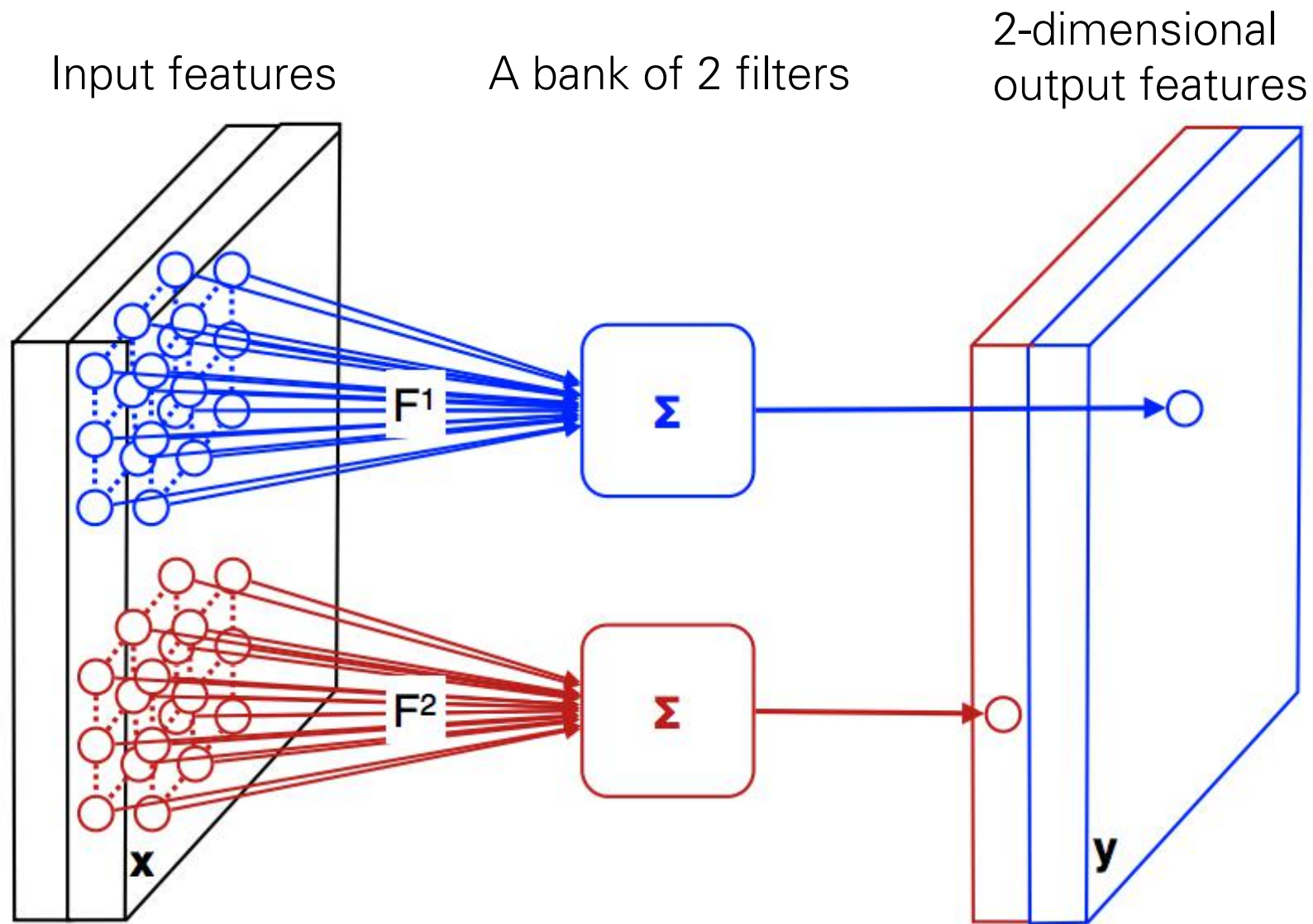
$$\mathbb{R}^{N \times C^{(0)}} \rightarrow \mathbb{R}^{N \times C^{(1)}}$$

# Multiple channels

Conv layer



$$\mathbb{R}^{N \times C^{(l)}} \rightarrow \mathbb{R}^{N \times C^{(l+1)}}$$

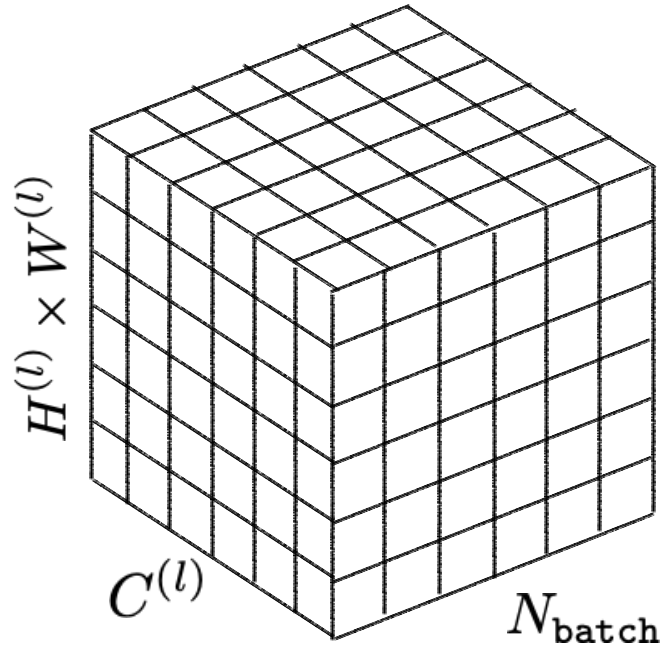


$$\mathbb{R}^{H \times W \times C^{(l)}} \rightarrow \mathbb{R}^{H \times W \times C^{(l+1)}}$$

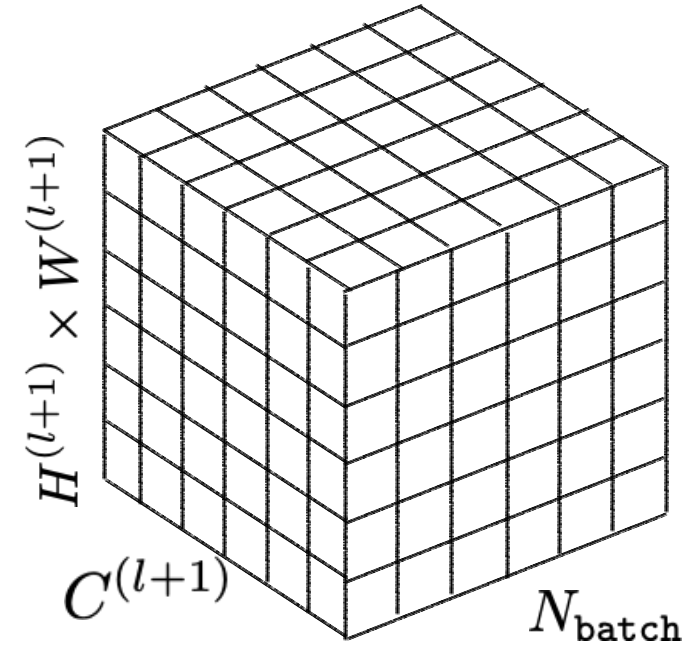
[Figure from Andrea Vedaldi]

# "Tensor flow"

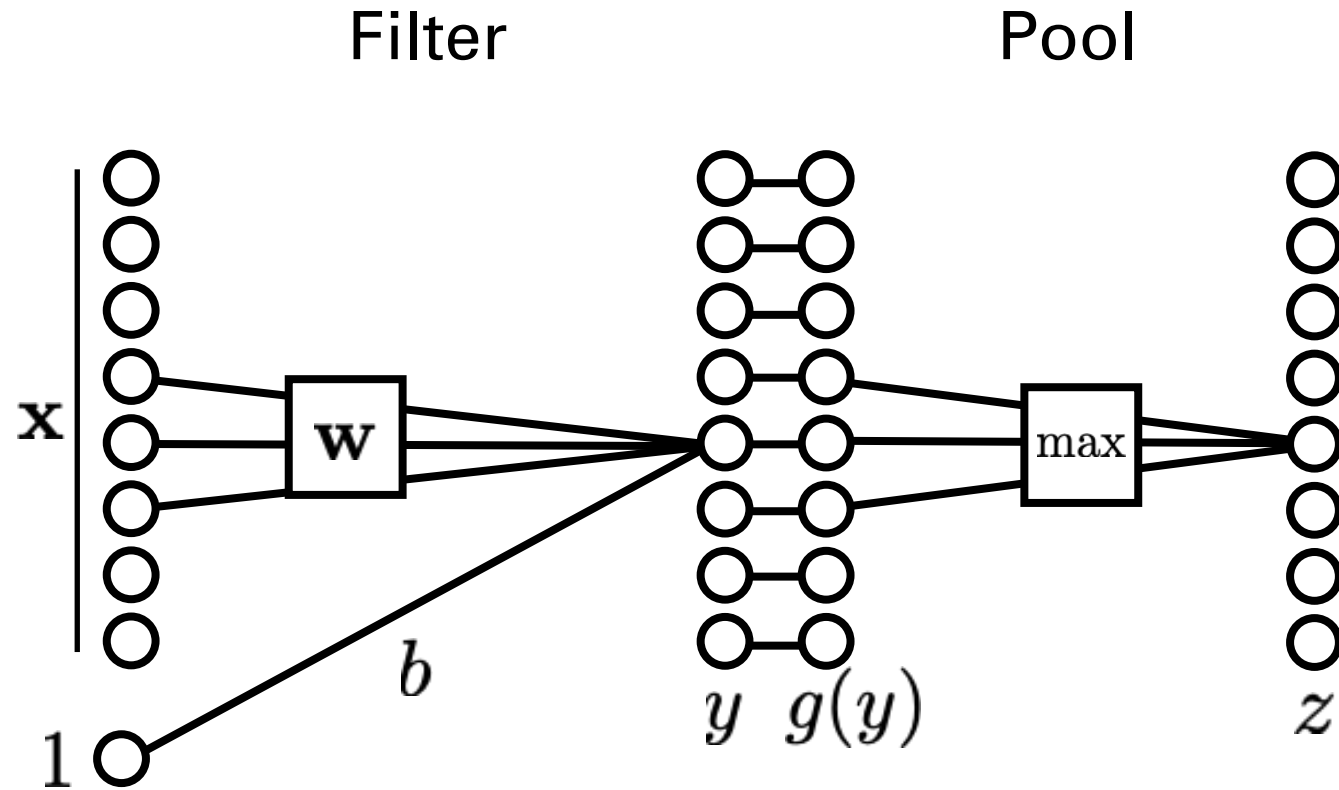
$$\mathbf{x}^{(l)} \in \mathbb{R}^{N_{\text{batch}} \times H^{(l)} \times W^{(l)} \times C^{(l)}}$$



$$\mathbf{x}^{(l+1)} \in \mathbb{R}^{N_{\text{batch}} \times H^{(l+1)} \times W^{(l+1)} \times C^{(l+1)}}$$



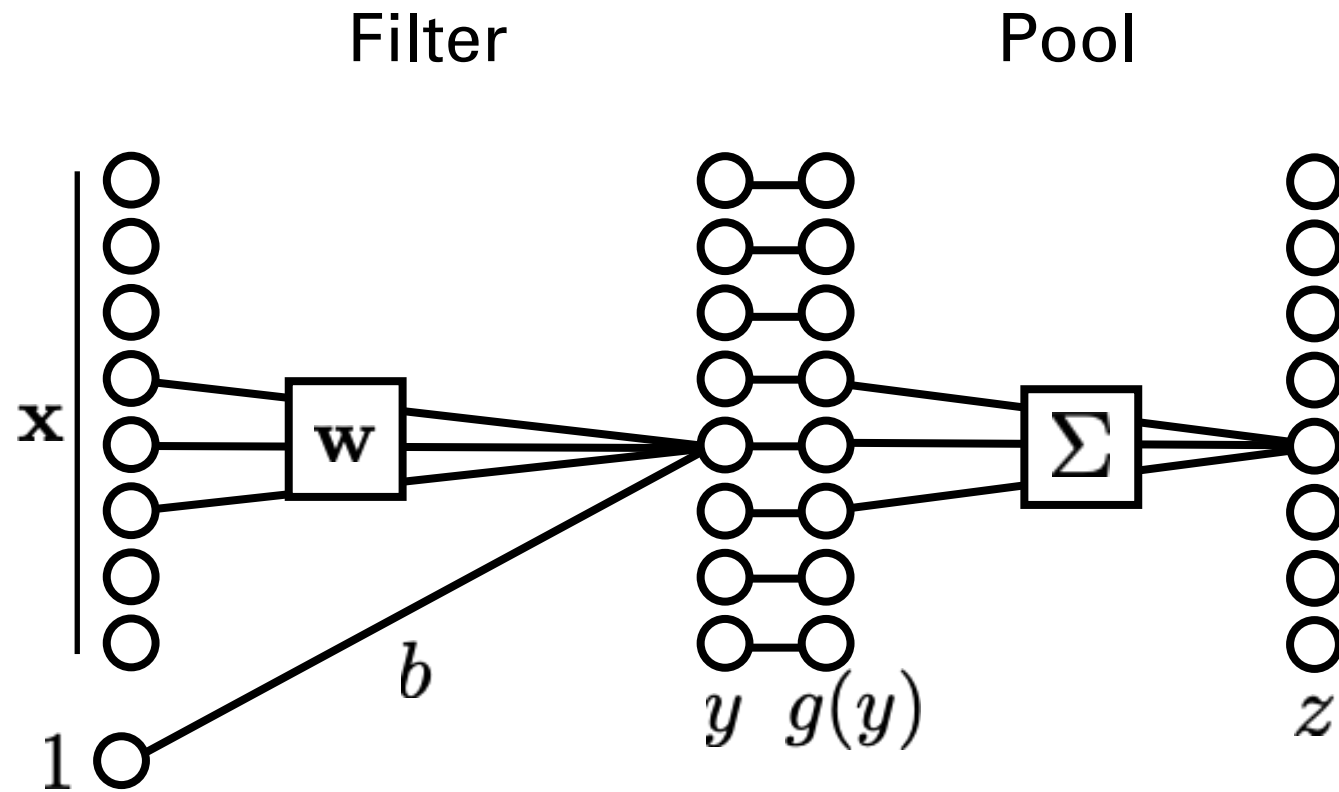
# Pooling



Max pooling

$$z_k = \max_{j \in \mathcal{N}(j)} g(y_j)$$

# Pooling



Max pooling

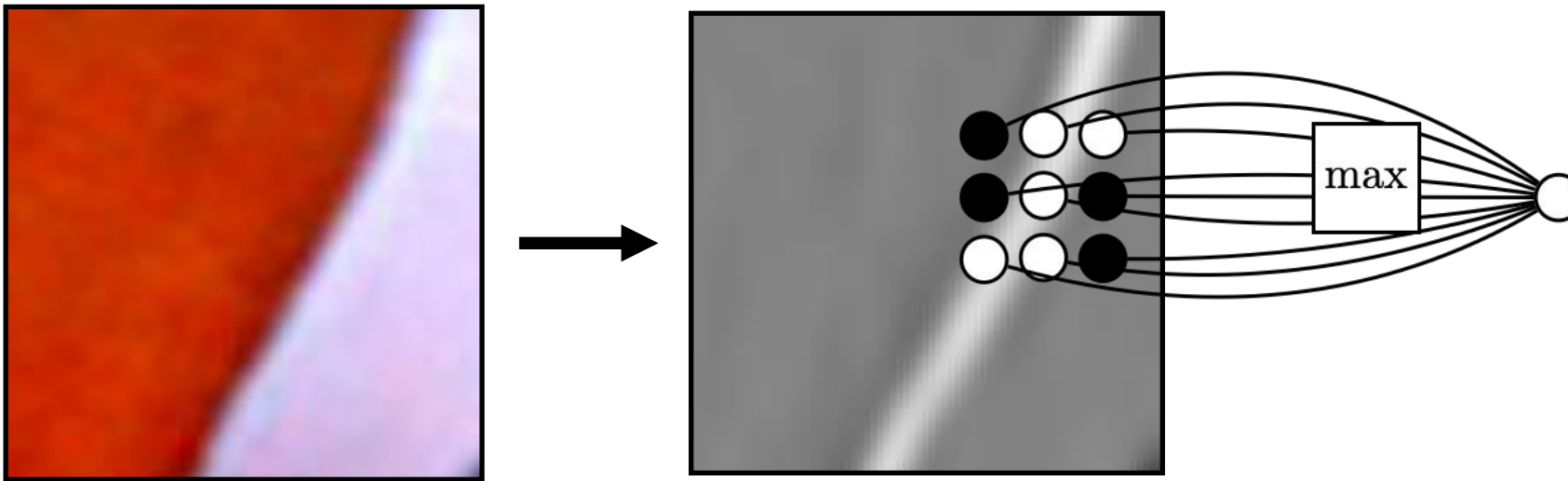
$$z_k = \max_{j \in \mathcal{N}(j)} g(y_j)$$

Mean pooling

$$z_k = \frac{1}{|\mathcal{N}|} \sum_{j \in \mathcal{N}(j)} g(y_j)$$

# Pooling – Why?

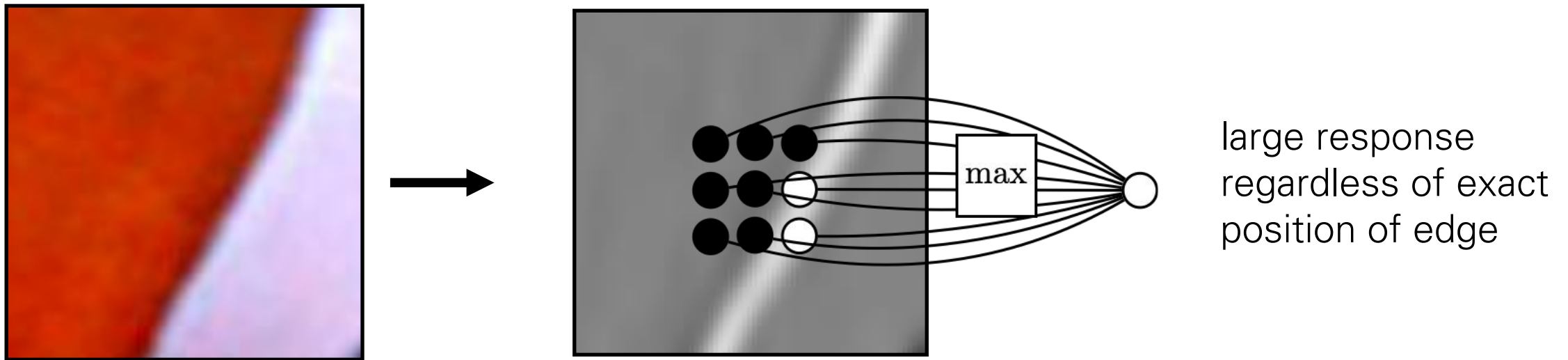
Pooling across spatial locations achieves stability w.r.t. small translations:





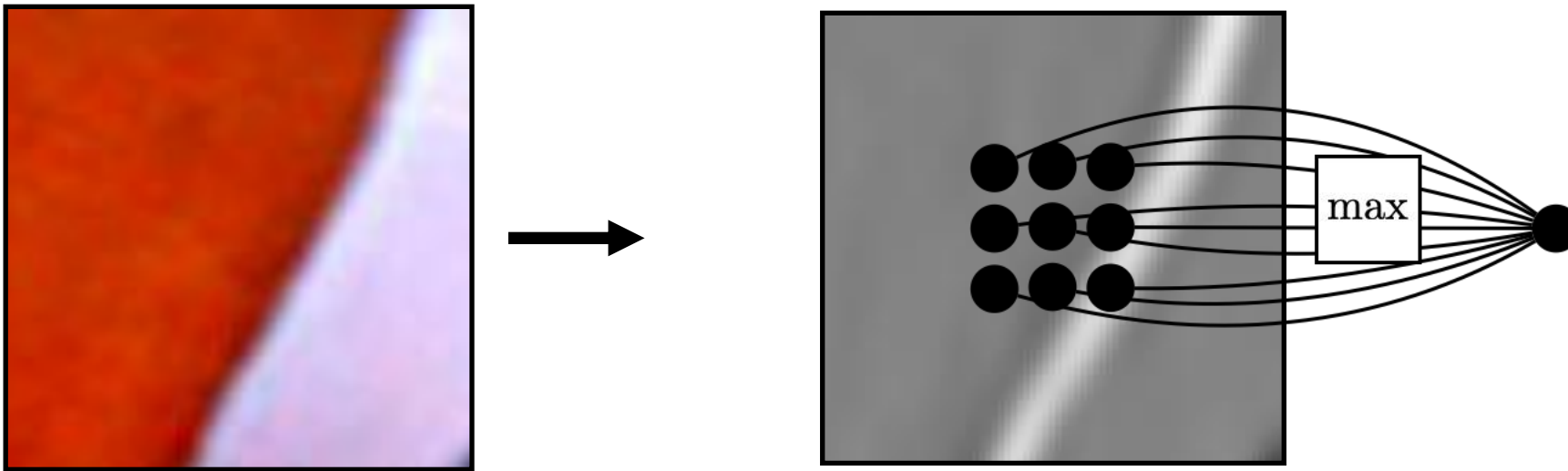
# Pooling – Why?

Pooling across spatial locations achieves stability w.r.t. small translations:

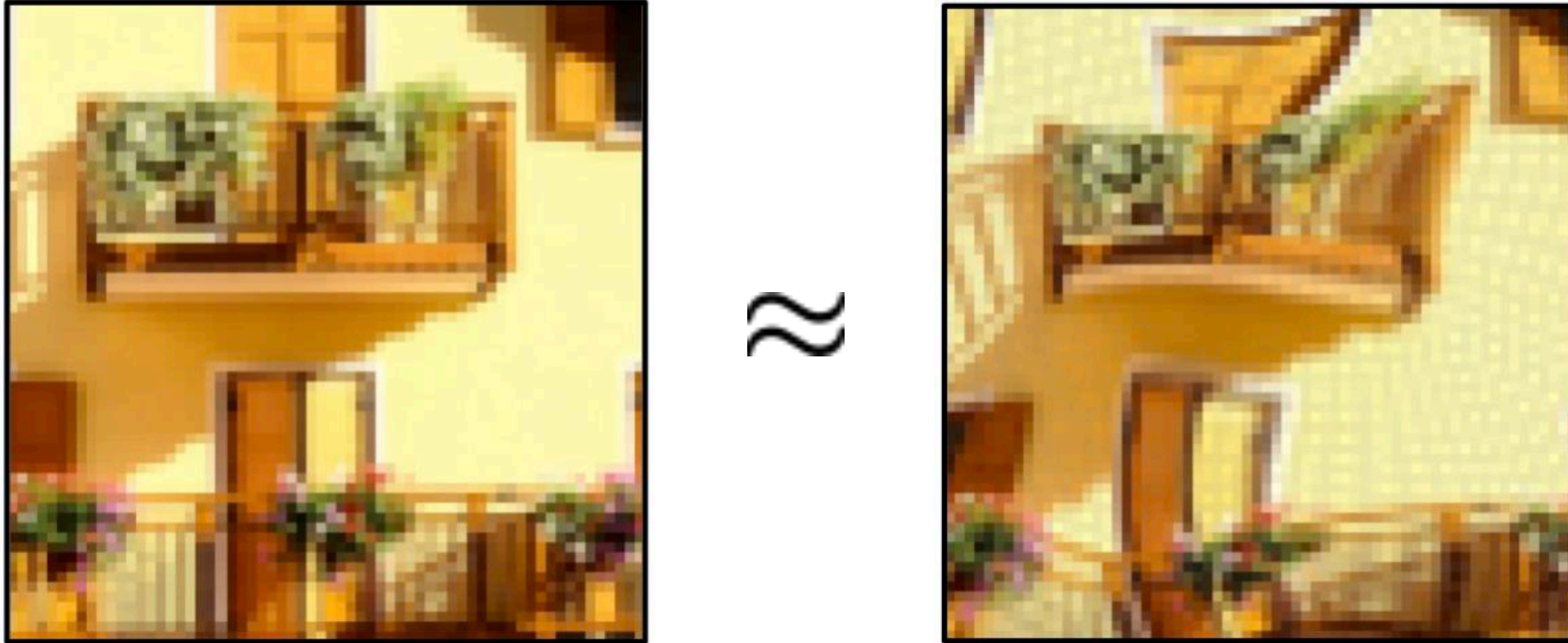


# Pooling – Why?

Pooling across spatial locations achieves stability w.r.t. small translations:



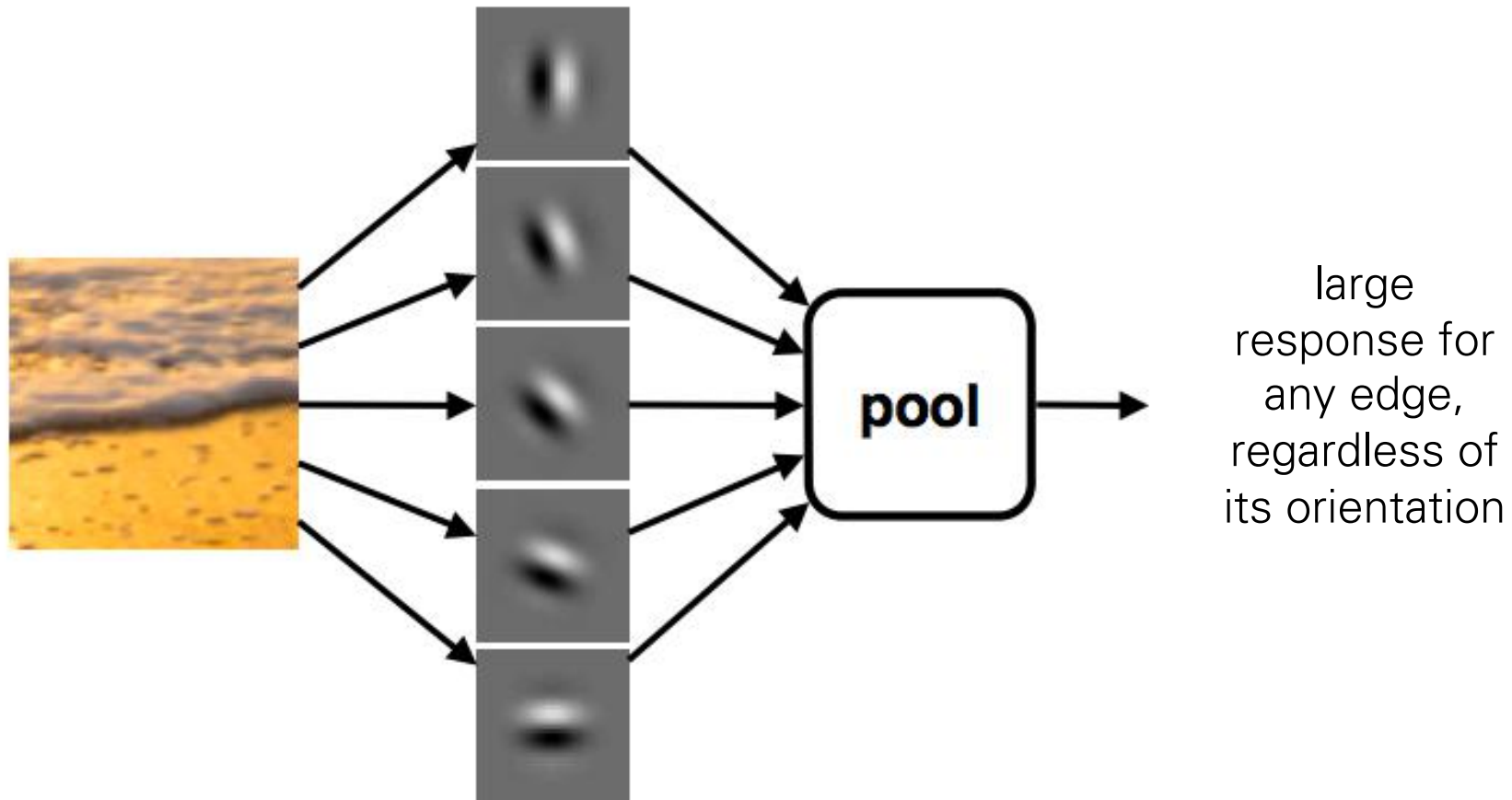
# CNNs are stable w.r.t. diffeomorphisms



[“Unreasonable effectiveness of Deep Features as a Perceptual Metric”, Zhang et al. 2016]

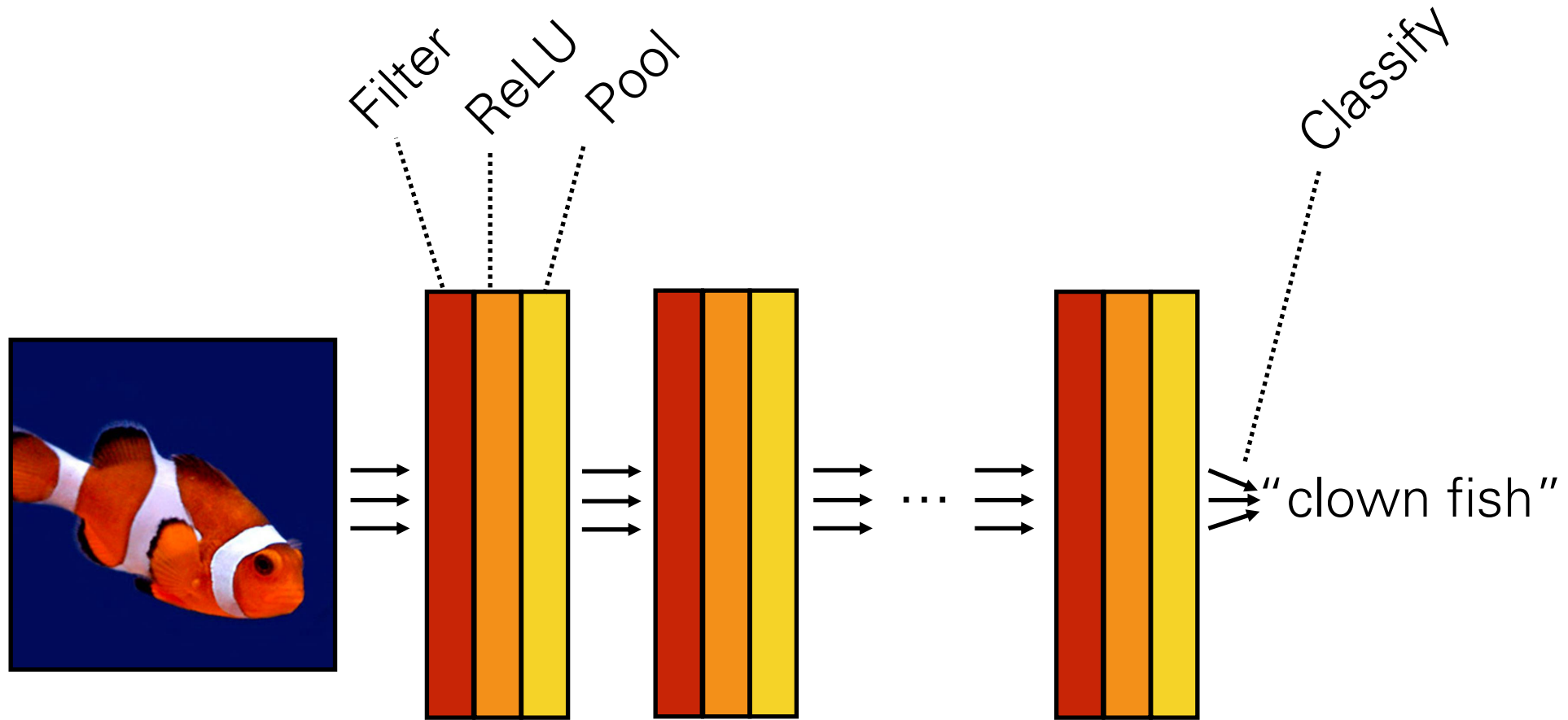
# Pooling – Why?

Pooling across feature channels (filter outputs) can achieve other kinds of invariances:



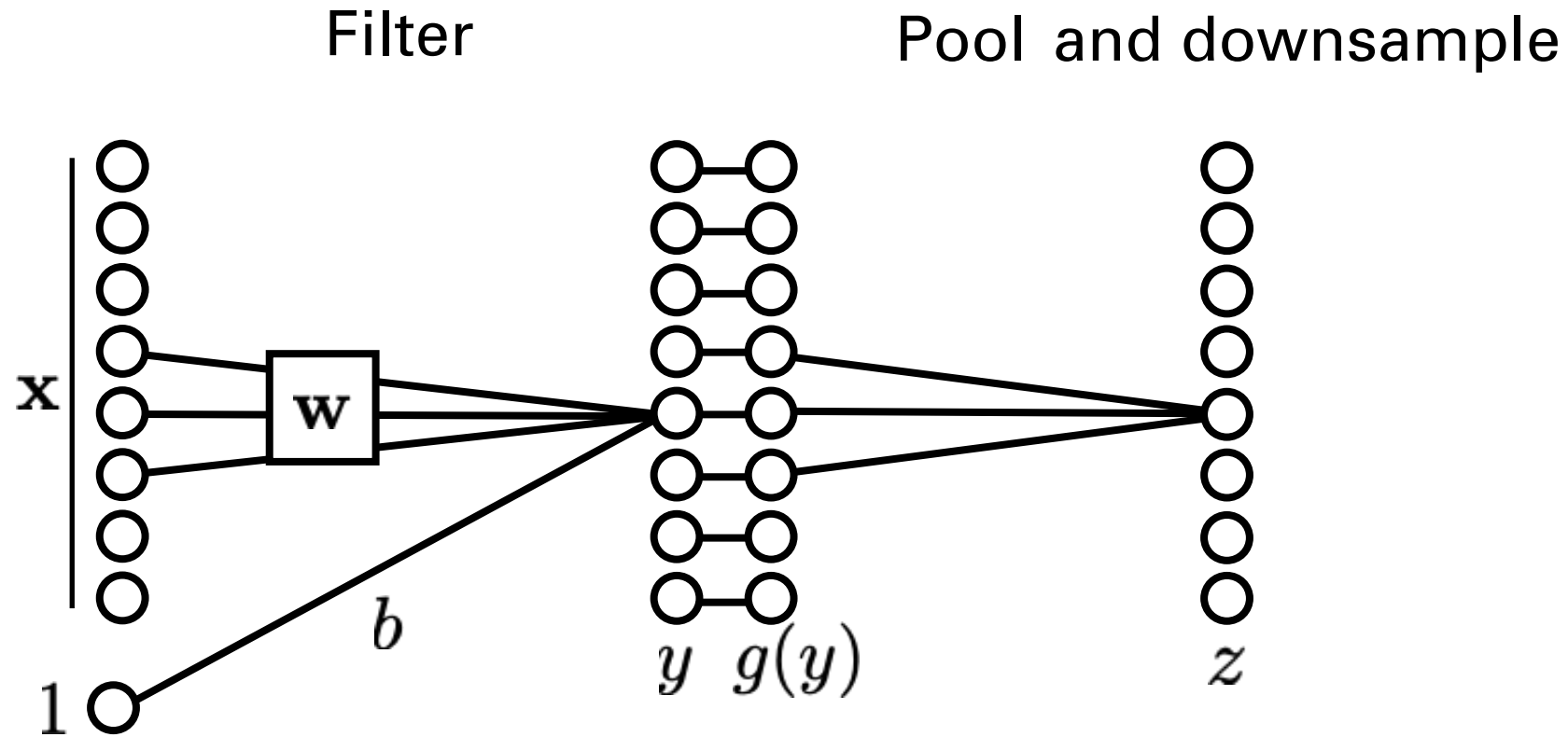
[Derived from slide by Andrea Vedaldi]

# Computation in a neural net

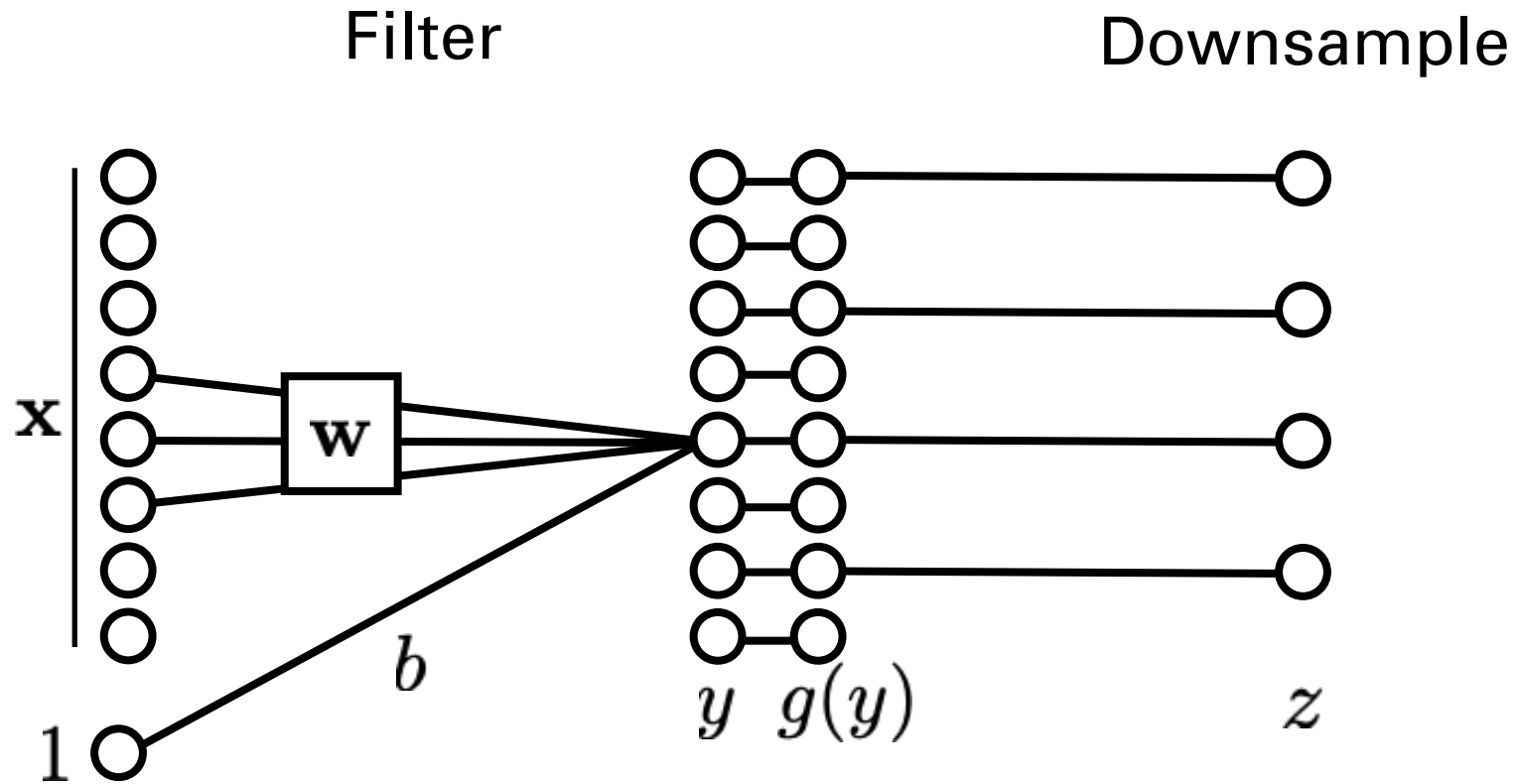


$$f(\mathbf{x}) = f_L(\dots f_2(f_1(\mathbf{x})))$$

# Downsampling

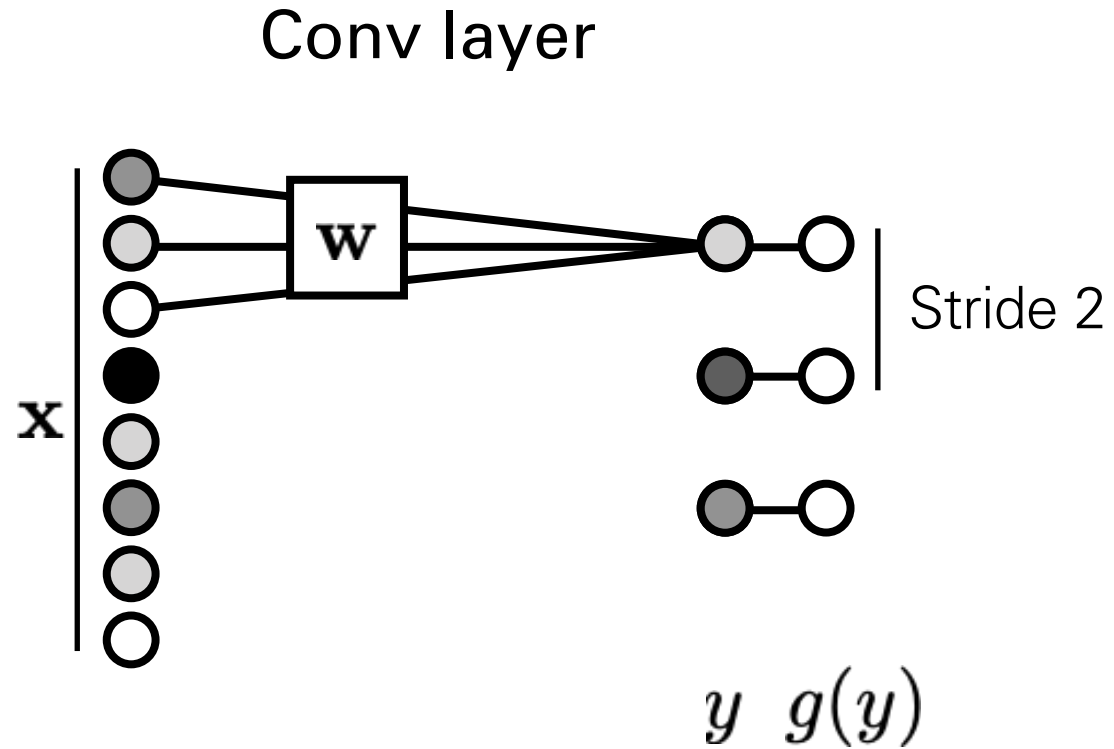


# Downsampling



$$\mathbb{R}^{H^{(l)} \times W^{(l)} \times C^{(l)}} \rightarrow \mathbb{R}^{H^{(l+1)} \times W^{(l+1)} \times C^{(l+1)}}$$

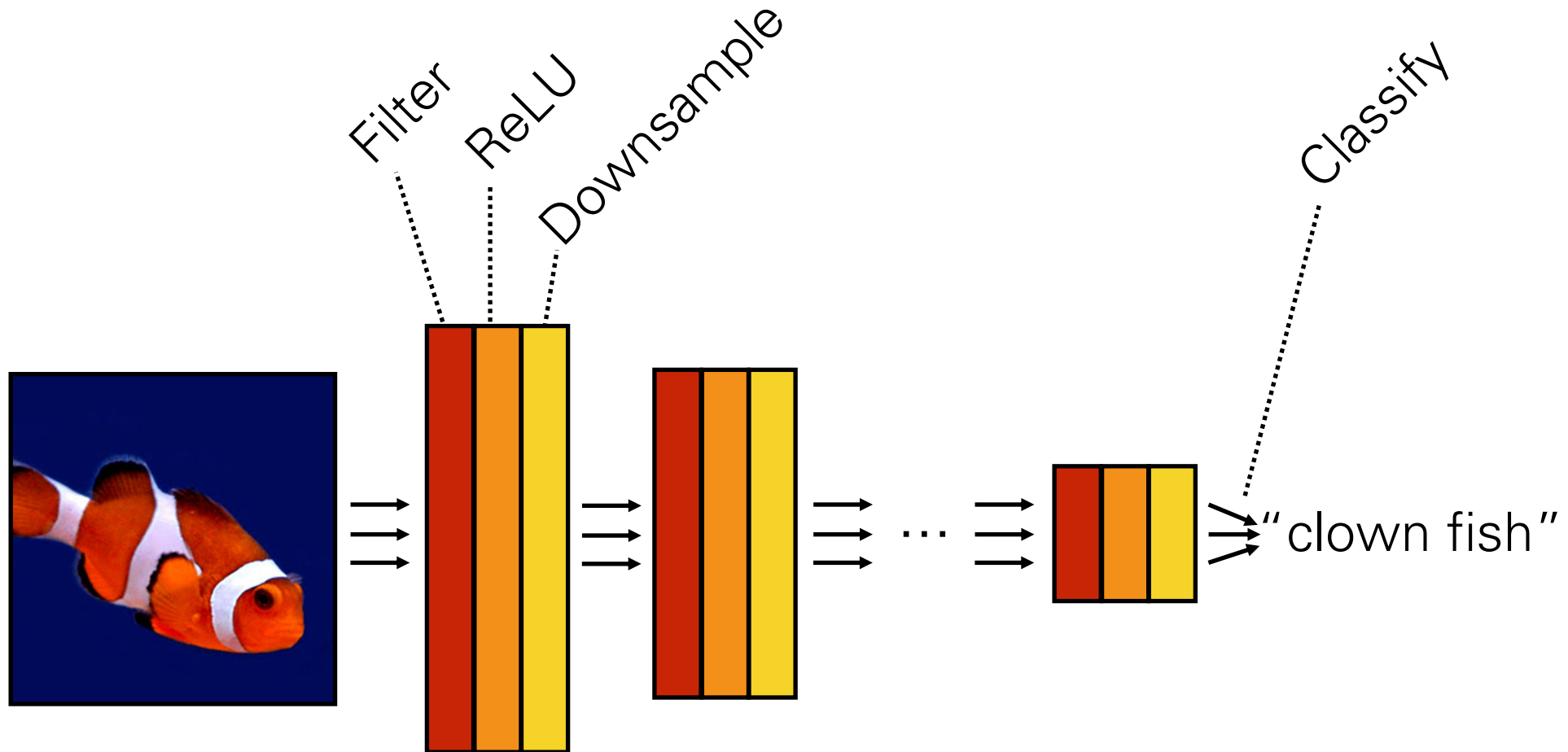
# Strided convolution



**Strided convolutions** combine convolution and downsampling into a single operation.

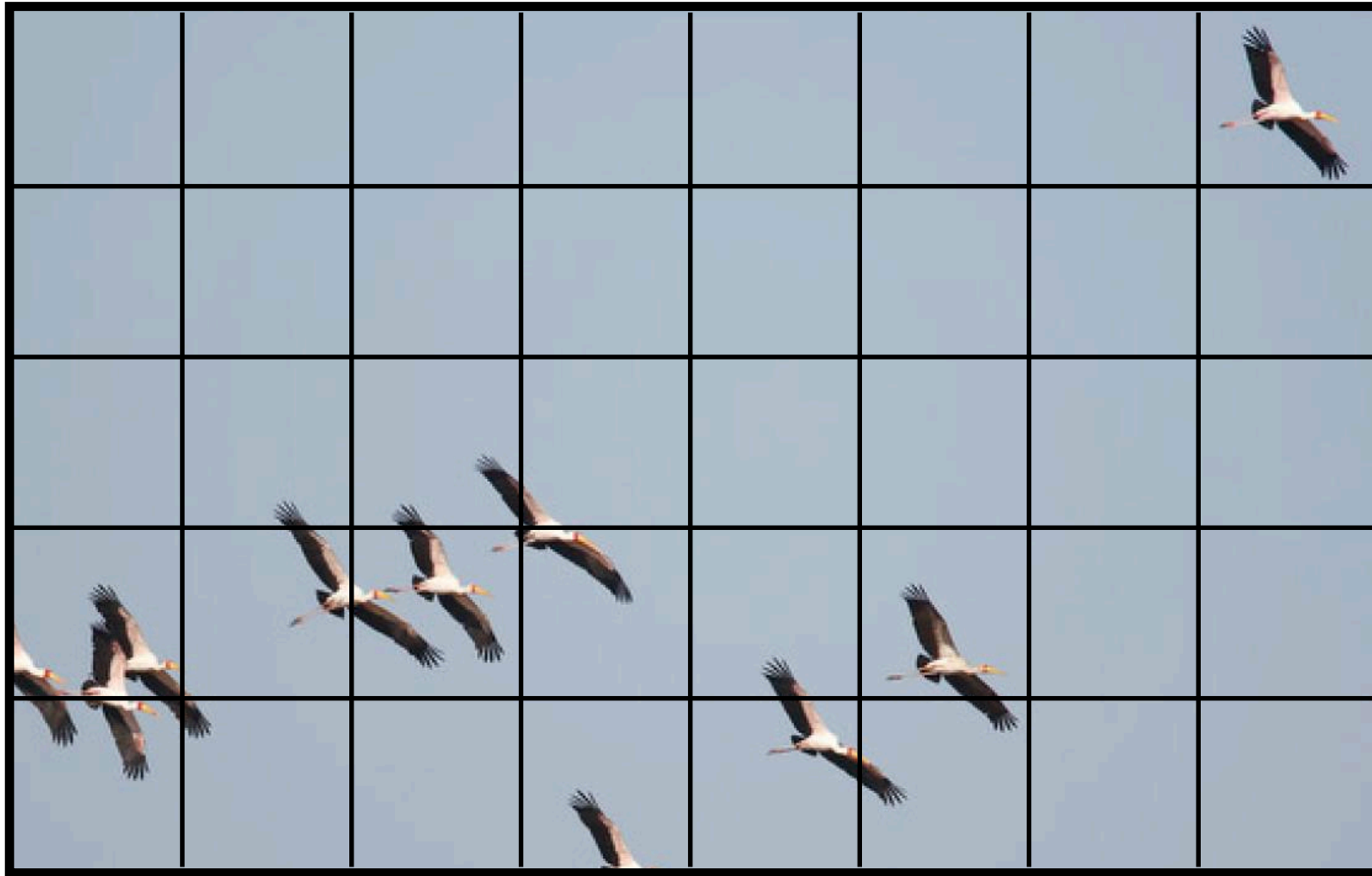


# Computation in a neural net

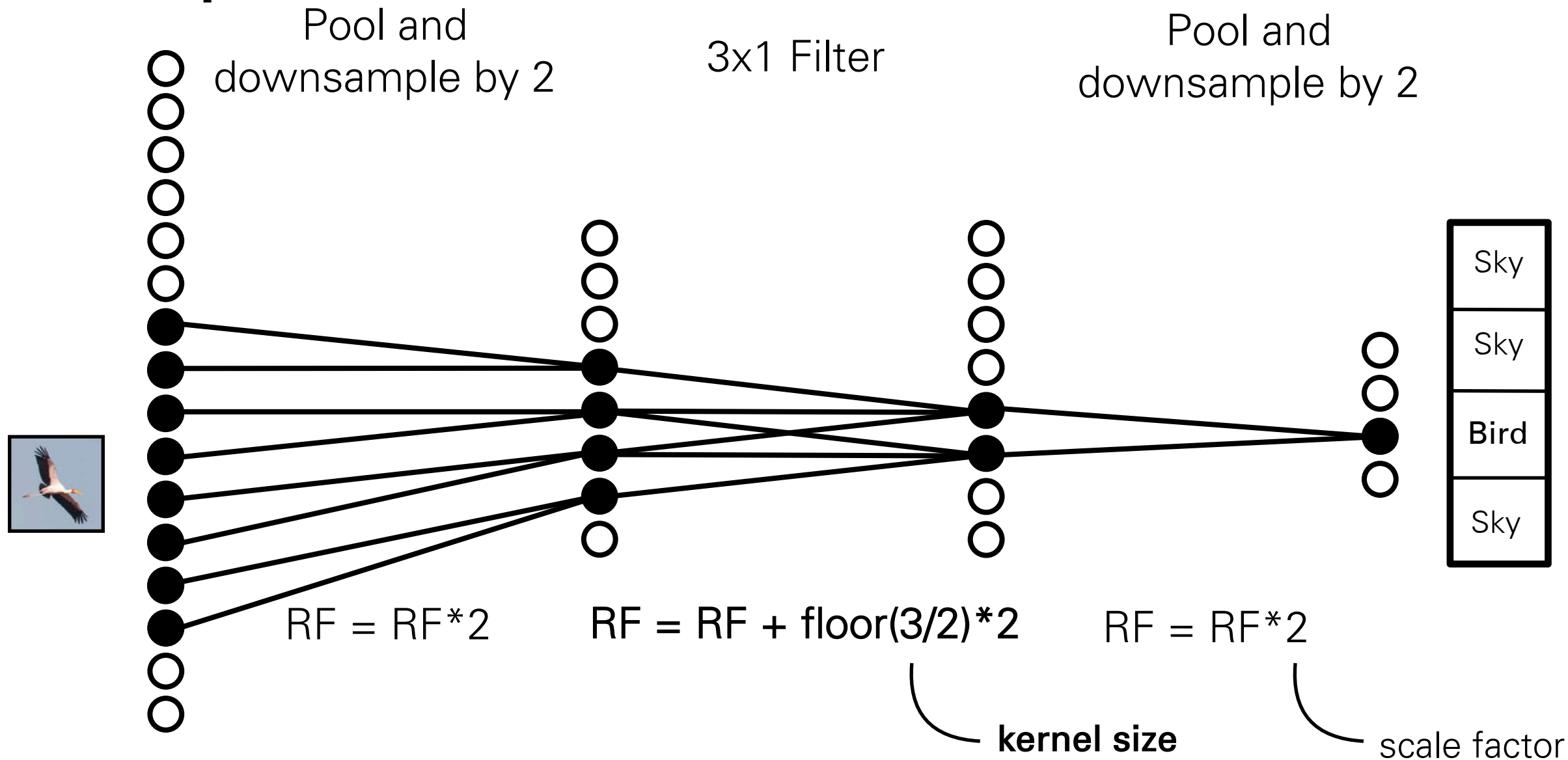


$$f(\mathbf{x}) = f_L(\dots f_2(f_1(\mathbf{x})))$$

# Receptive fields



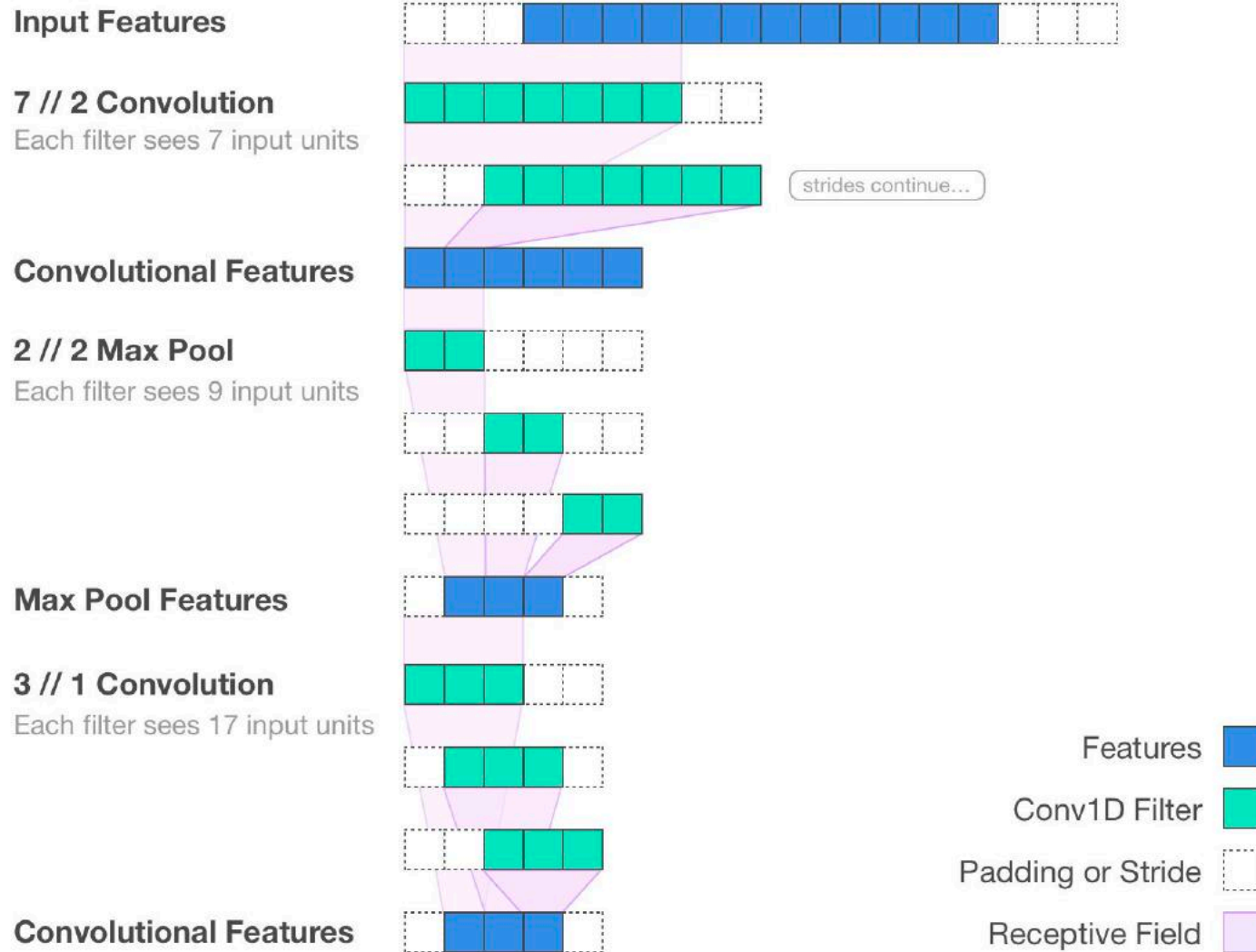
# Receptive fields



# Effective Receptive Field

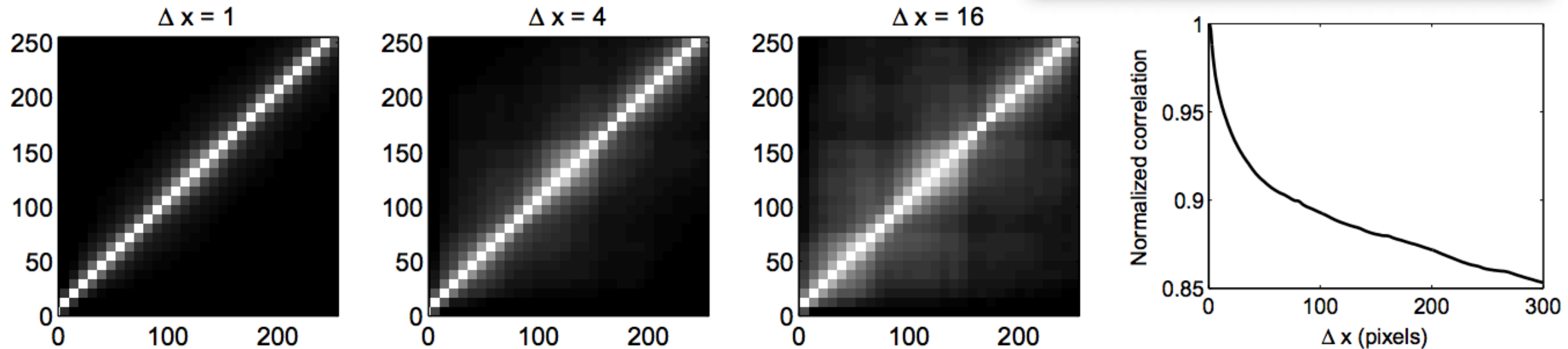
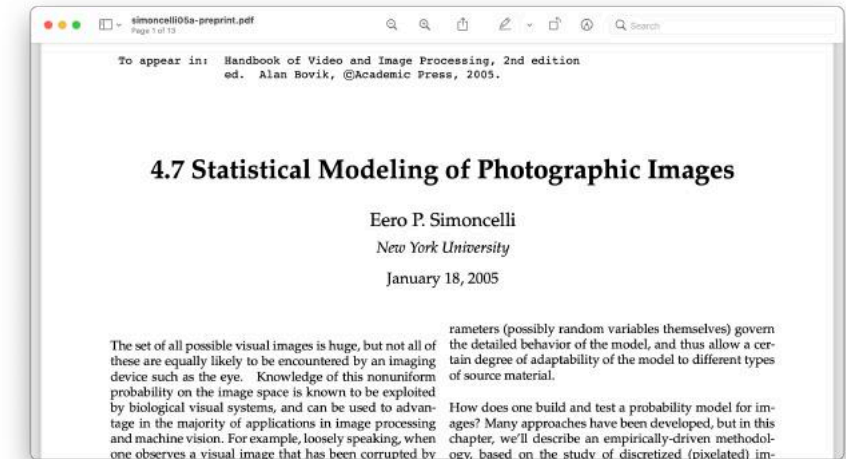
Contributing input units to a convolutional filter.

@jimmfleming // fomoro.com



[<http://fomoro.com/tools/receptive-fields/index.html>]

# Why CNNs?



**Fig. 1.** (a) Scatterplots of pairs of pixels at three different spatial displacements, averaged over five example images. (b) Autocorrelation function. Photographs are of New York City street scenes, taken with a Canon 10D digital camera, and processed in RAW linear sensor mode (producing pixel intensities are in roughly proportional to light intensity). Correlations were computed on the logs of these sensor intensity values [41].

# Why CNNs?

Statistical dependences between pixels decay as a power law of distance between the pixels.

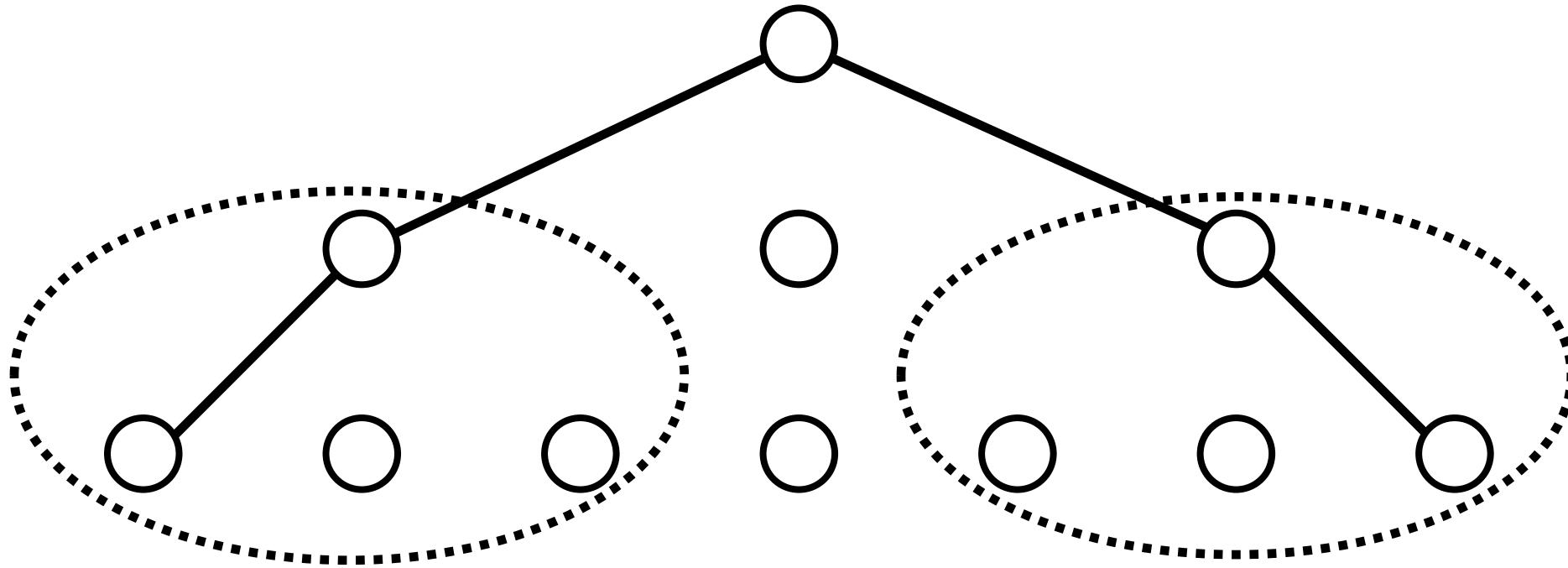
It is therefore often sufficient to model local dependences only. —> **Convolution**

More generally, we should allocate parameters that model dependences in proportion to the strength of those dependences. —> **Multiscale, hierarchical representations**

[For more discussion, see “Why does Deep and Cheap Learning Work So Well?”, Lin et al. 2017]

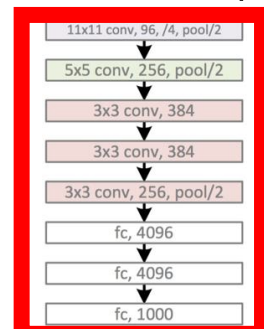
# Why CNNs?

Capturing long-range dependences:



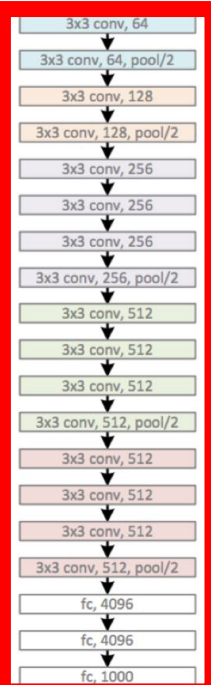
# Deep Neural Networks for Visual Recognition

2012: AlexNet  
5 conv. layers



Error: 15.3%

2014: VGG  
16 conv. layers



Error: 8.5%

2015: GoogLeNet  
22 conv. layers



Error: 7.8%

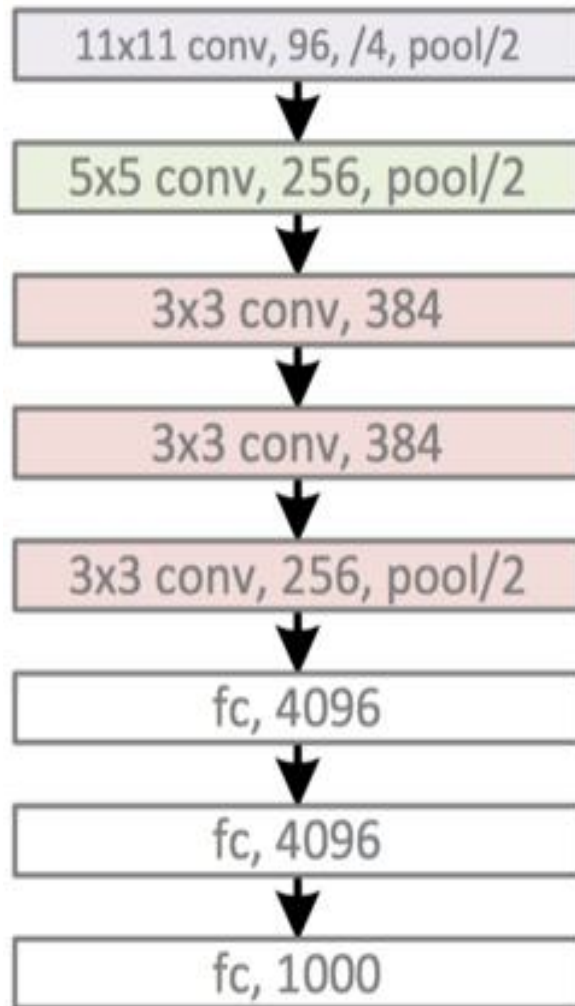
2016: ResNet  
>100 conv. layers



Error: 4.4%

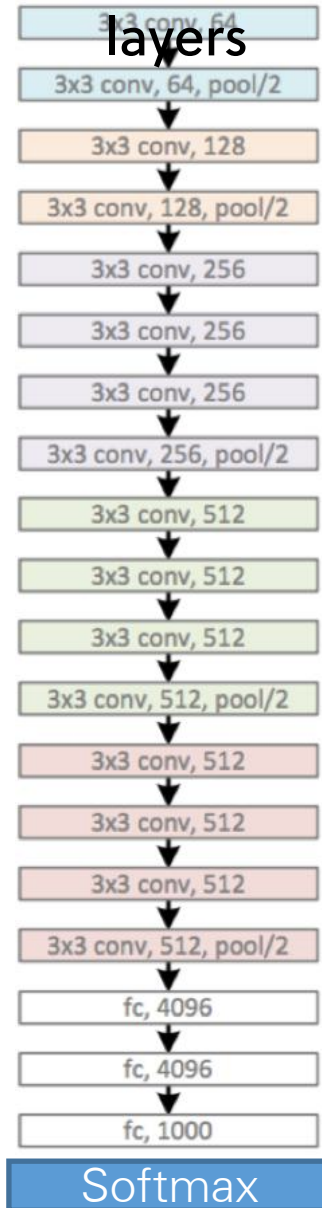


## 2012: AlexNet 5 conv. layers



Error: 15.3%

## 2014: VGG 16 conv. layers

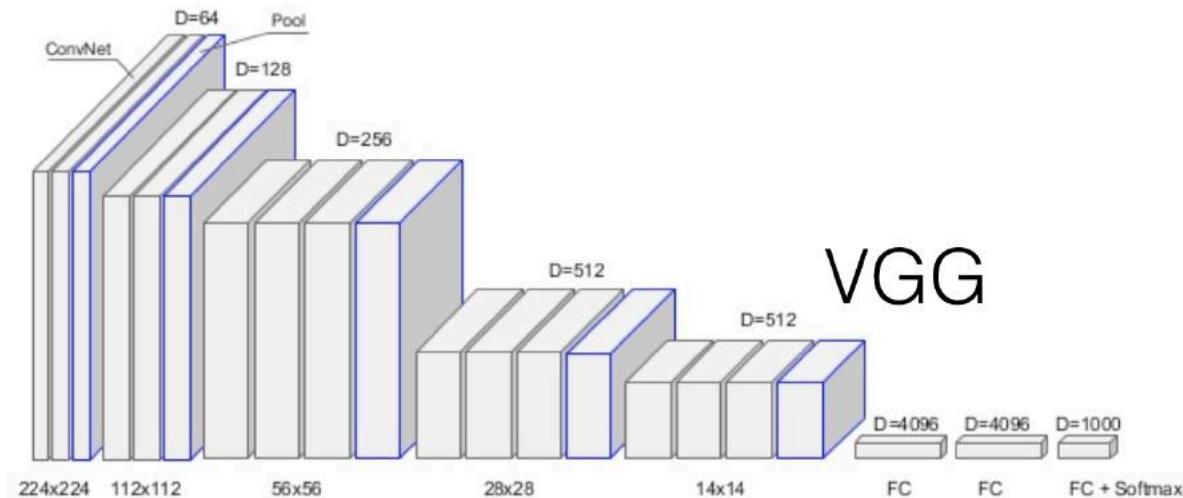
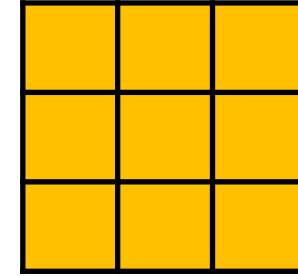


Error: 8.5%

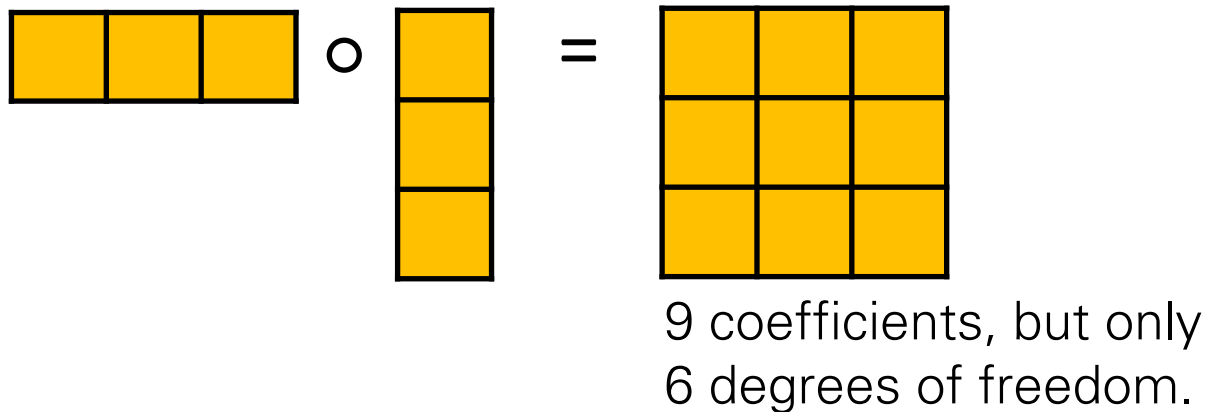
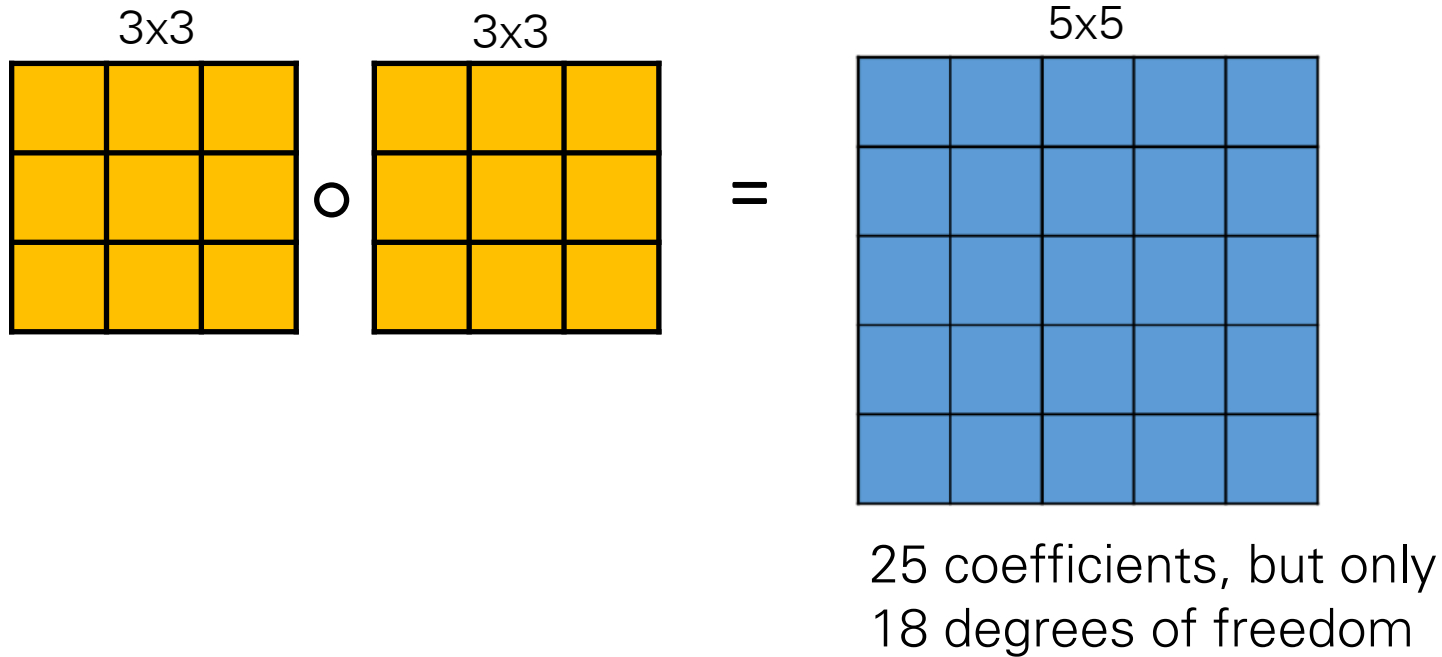
# VERY DEEP CONVOLUTIONAL NETWORKS FOR LARGE-SCALE IMAGE RECOGNITION

<https://arxiv.org/pdf/1409.1556.pdf>

Small convolutional kernels: 3x3  
ReLu non-linearities  
>100 million parameters.

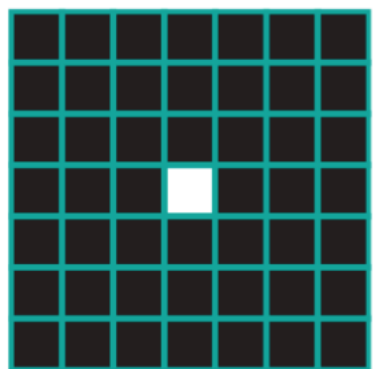
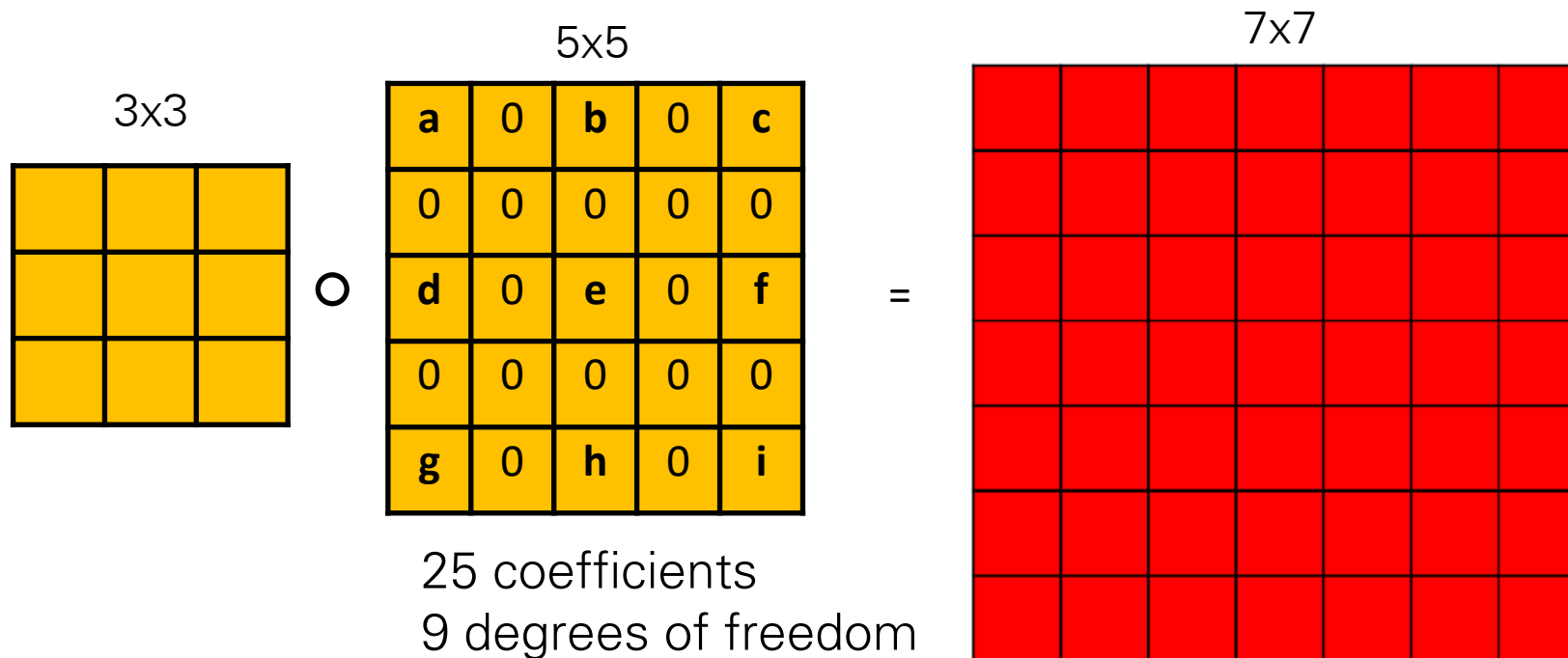


# Chaining convolutions

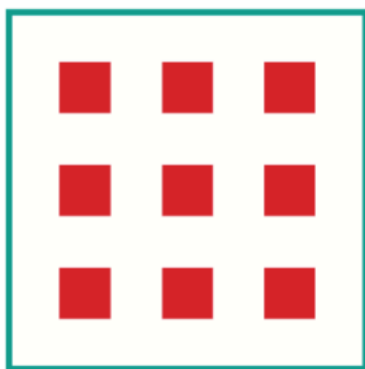


Only separable filters... would this be enough?

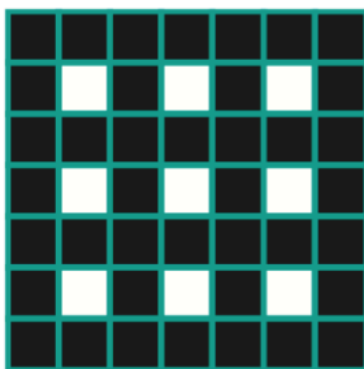
# Dilated convolutions



(a) Input



(b) Dilation 2



(c) Output

What is lost?

[<https://arxiv.org/pdf/1511.07122.pdf>]

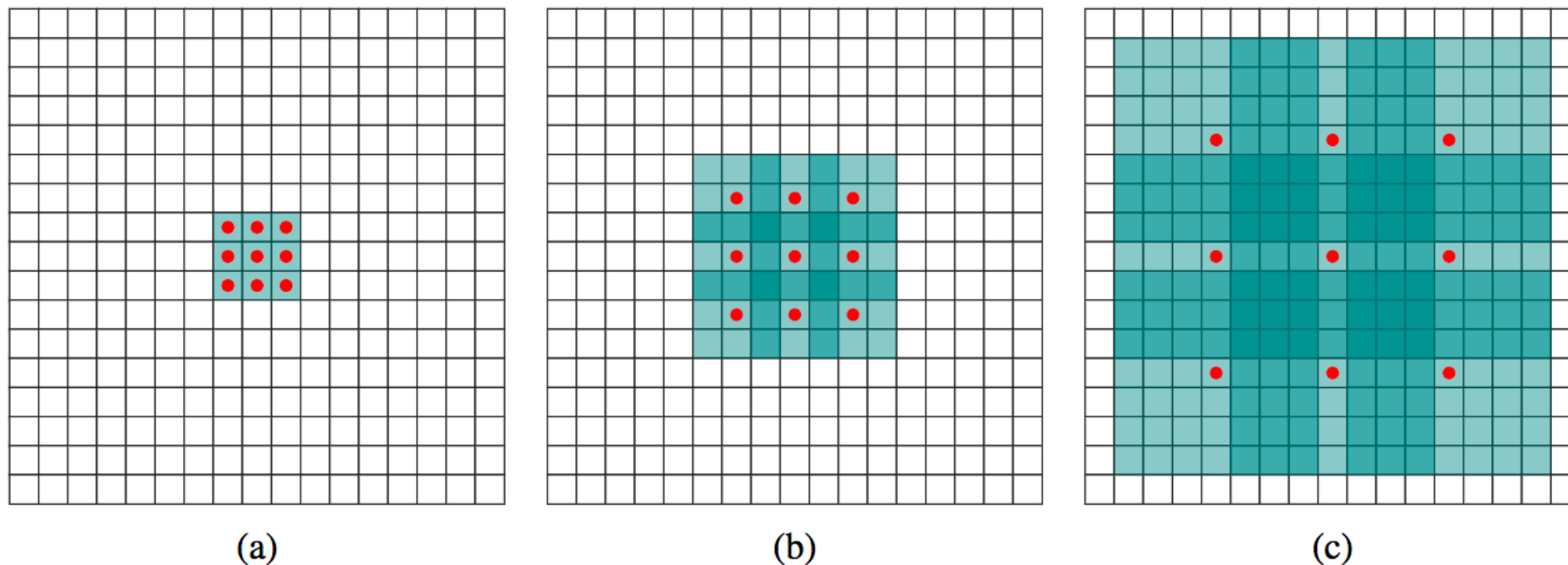
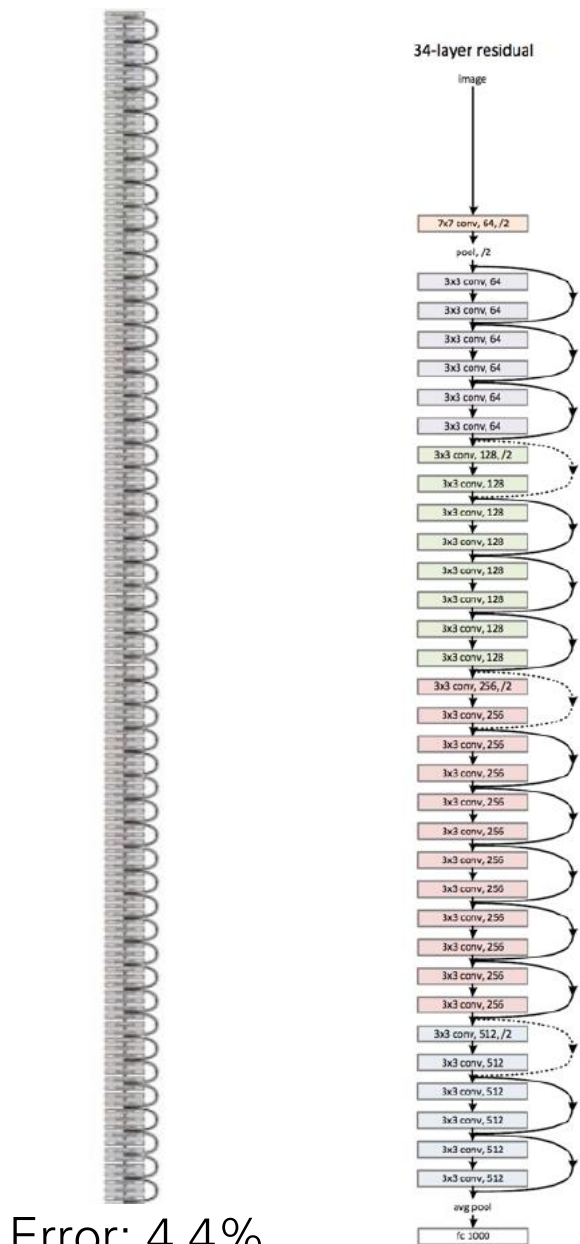


Figure 1: Systematic dilation supports exponential expansion of the receptive field without loss of resolution or coverage. (a)  $F_1$  is produced from  $F_0$  by a 1-dilated convolution; each element in  $F_1$  has a receptive field of  $3 \times 3$ . (b)  $F_2$  is produced from  $F_1$  by a 2-dilated convolution; each element in  $F_2$  has a receptive field of  $7 \times 7$ . (c)  $F_3$  is produced from  $F_2$  by a 4-dilated convolution; each element in  $F_3$  has a receptive field of  $15 \times 15$ . The number of parameters associated with each layer is identical. The receptive field grows exponentially while the number of parameters grows linearly.

2016: ResNet  
>100 conv. layers

# Deep Residual Learning for Image Recognition

<https://arxiv.org/pdf/1512.03385.pdf>



Error: 4.4%

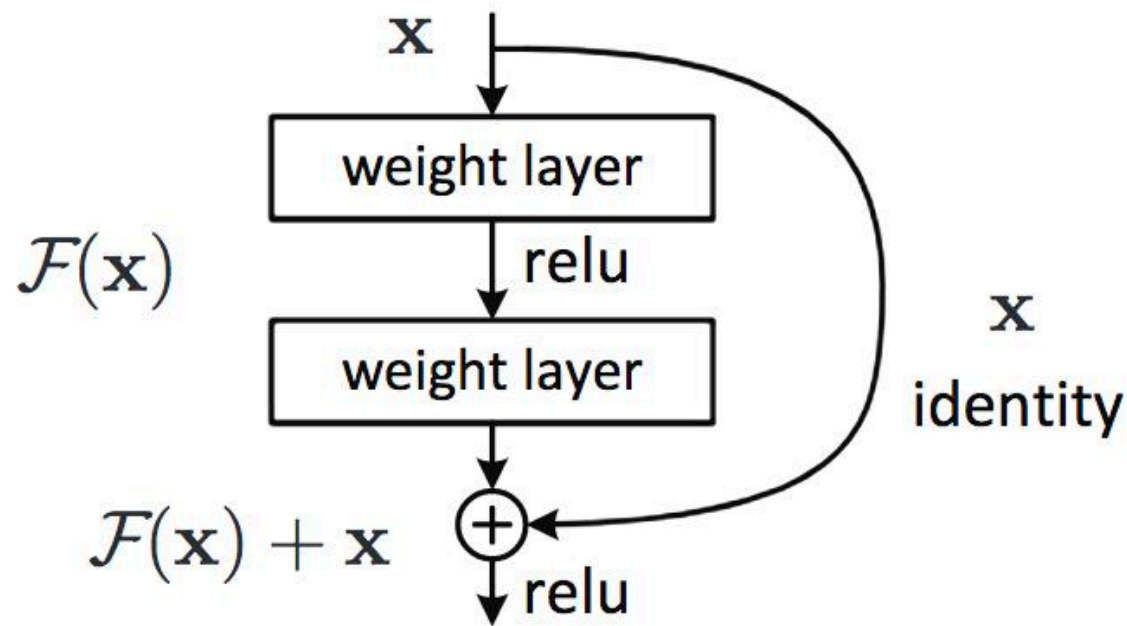
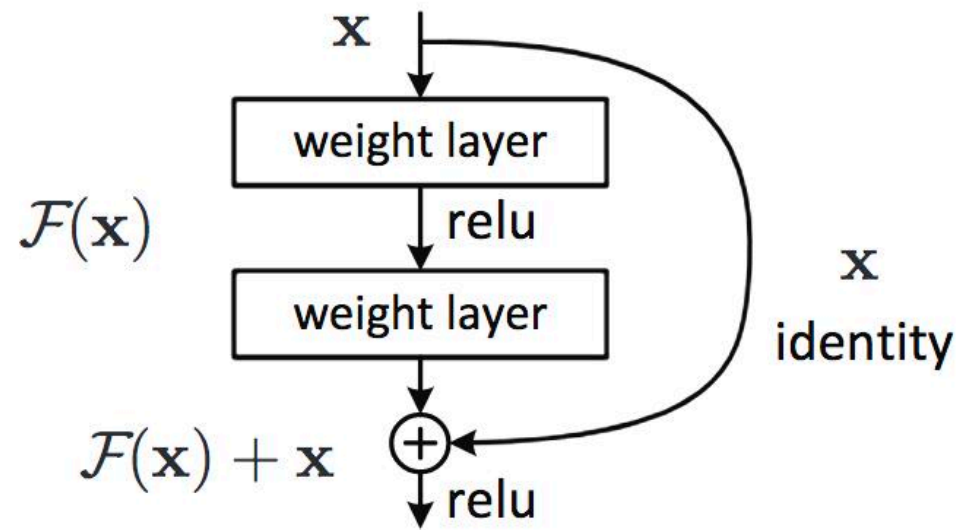
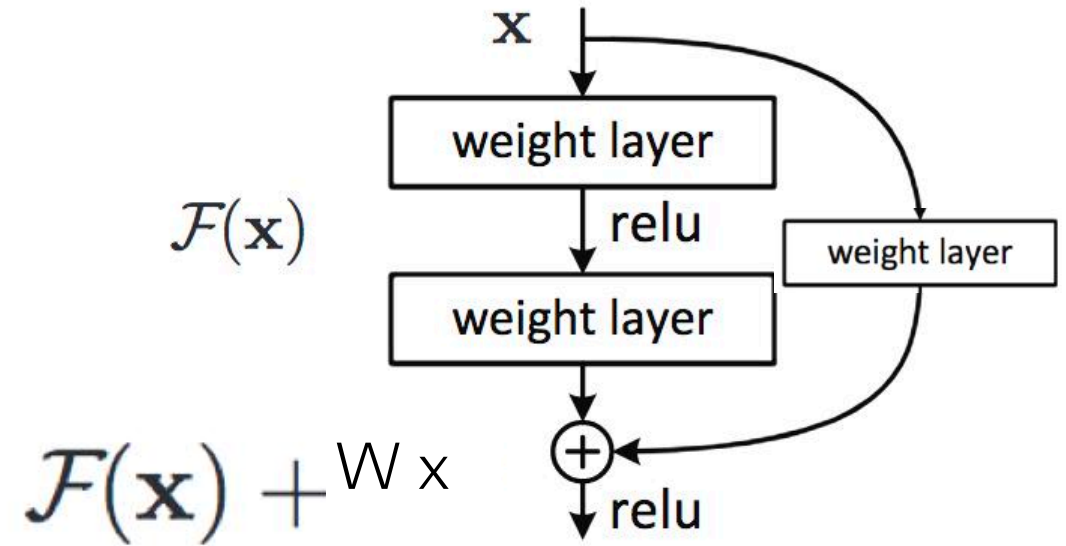


Figure 2. Residual learning: a building block.

If output has same size as input:

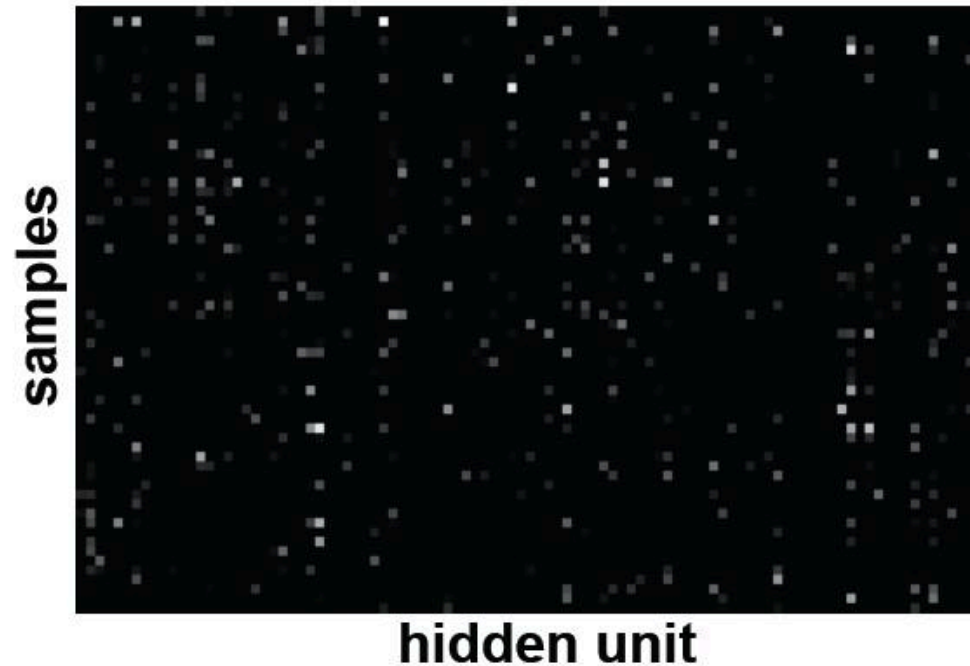


If output has a different size:



# Other good things to know

- Check gradients numerically by finite differences
- Visualize hidden activations — should be uncorrelated and high variance



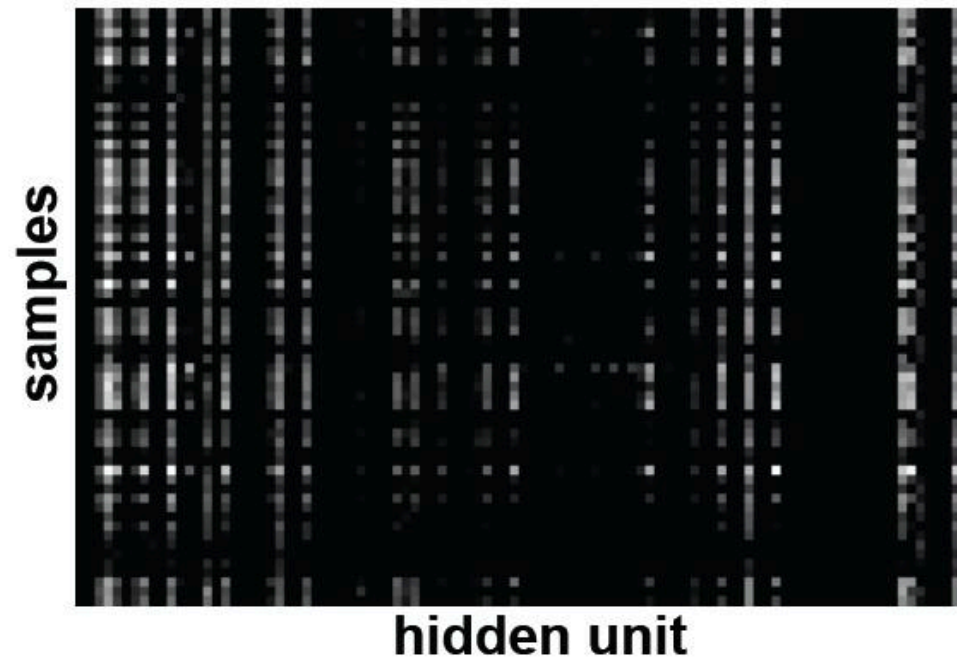
**Good training:** hidden units are sparse across samples and across features.

[Derived from slide by Marc'Aurelio Ranzato]



# Other good things to know

- Check gradients numerically by finite differences
- Visualize hidden activations — should be uncorrelated and high variance

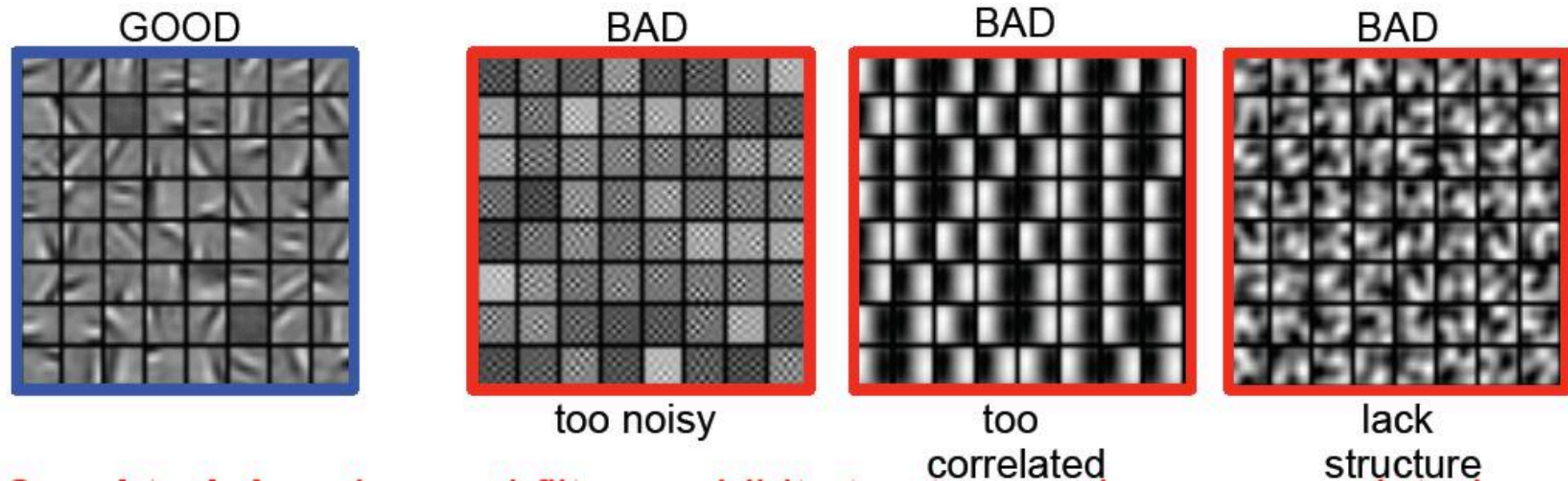


**Bad training:** many hidden units ignore the input and/or exhibit strong correlations.

[Derived from slide by Marc'Aurelio Ranzato]

# Other good things to know

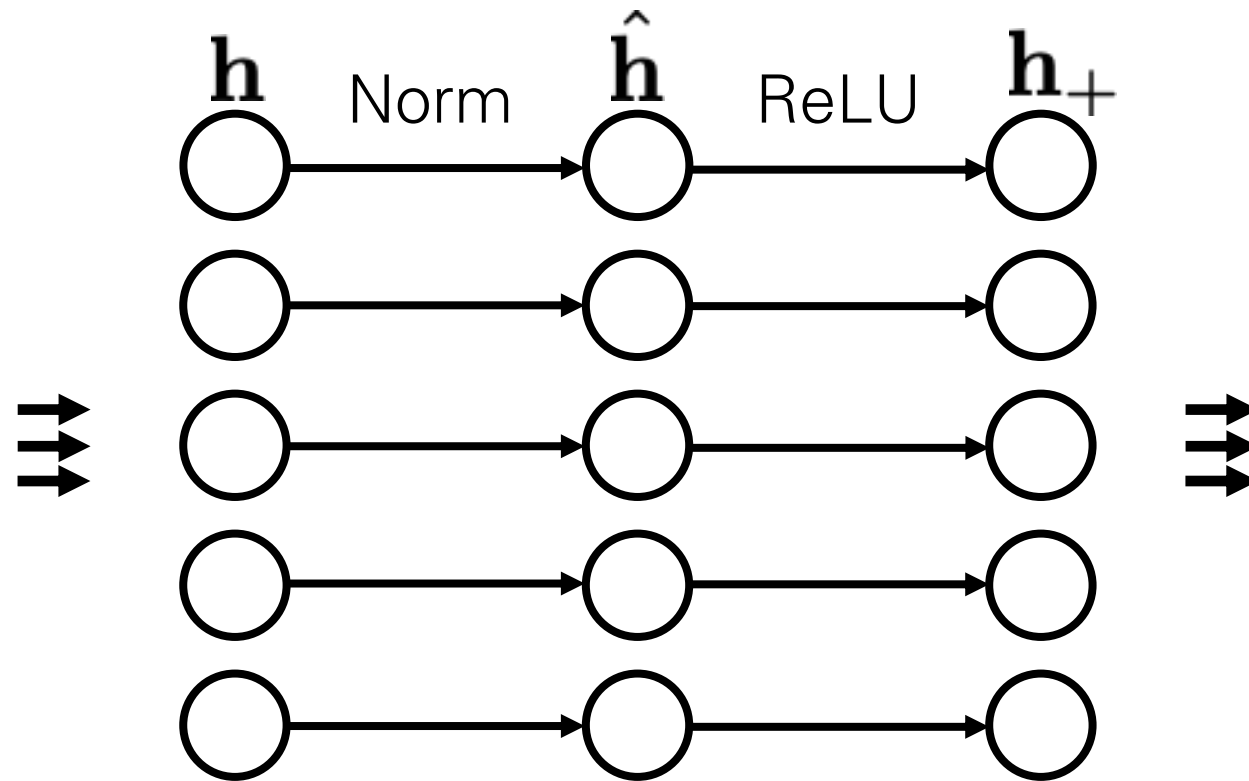
- Check gradients numerically by finite differences
- Visualize hidden activations — should be uncorrelated and high variance
- Visualize filters



**Good training:** learned filters exhibit structure and are uncorrelated.

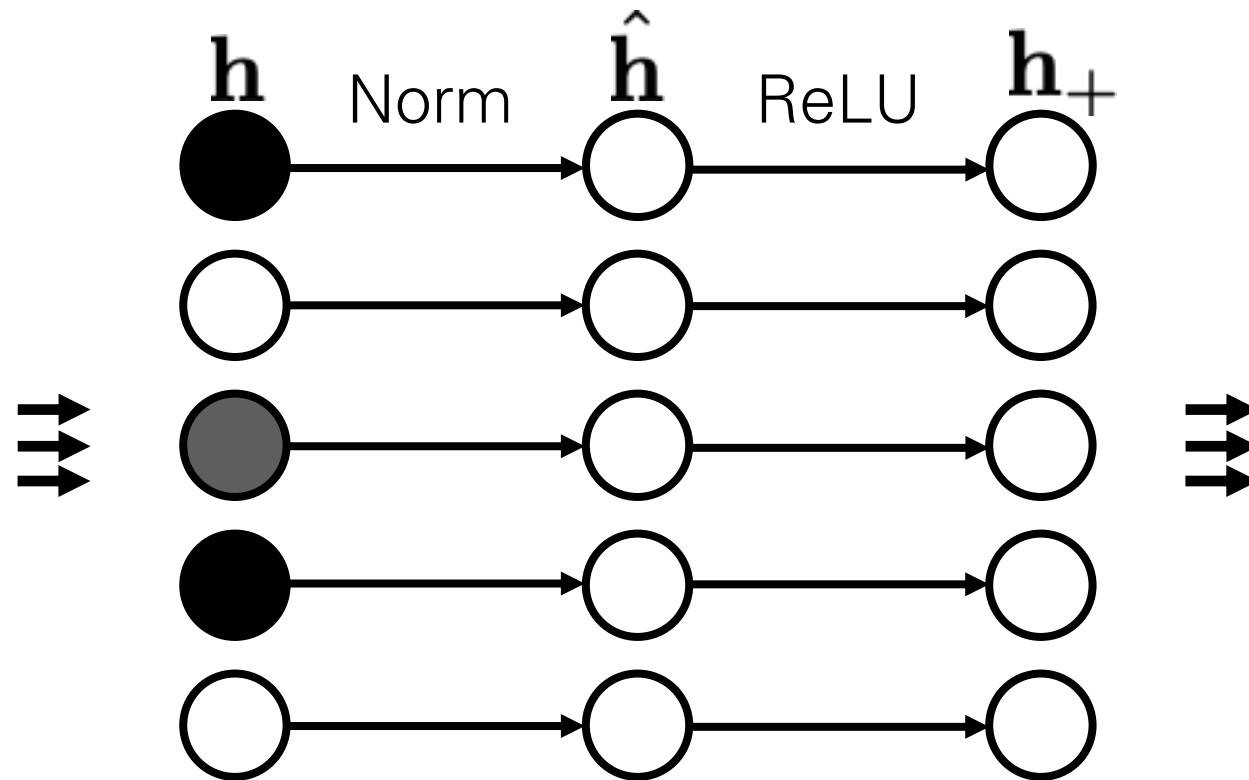
[Derived from slide by Marc'Aurelio Ranzato]

# Normalization layers



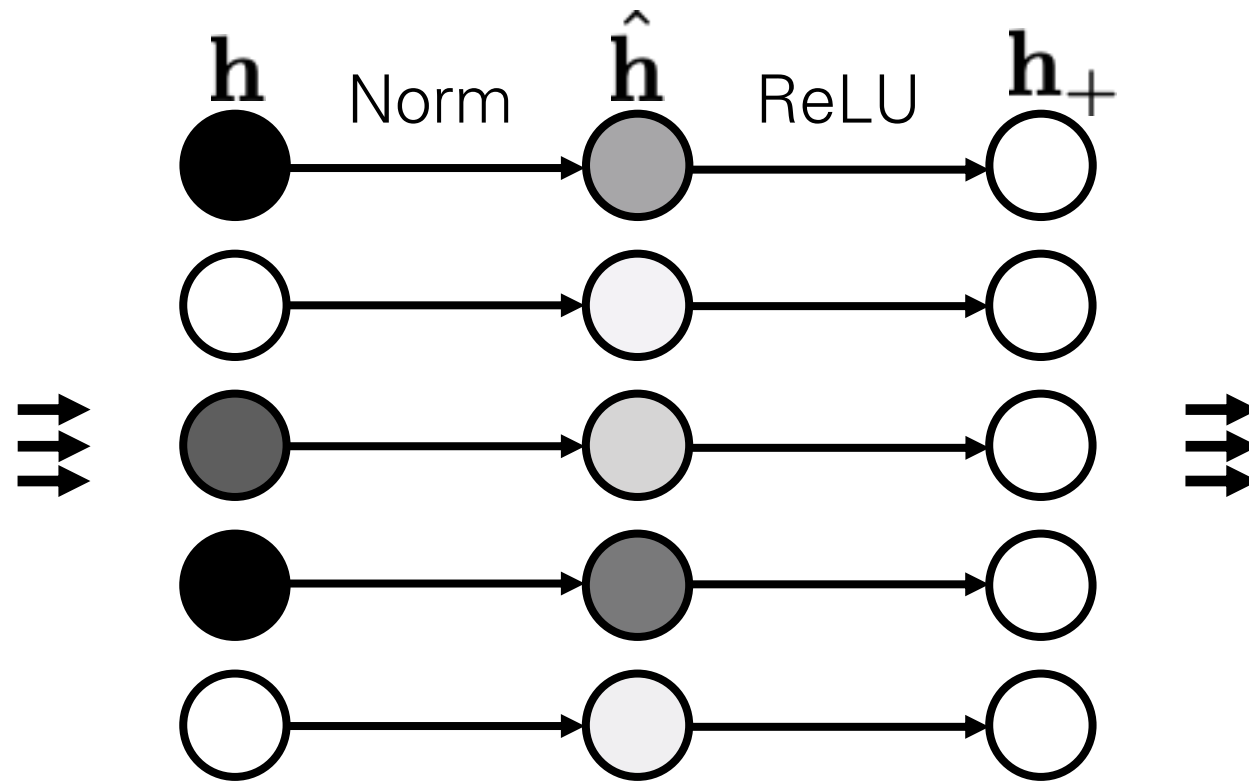
$$\hat{h}_k = \frac{h_k - \mathbb{E}[h_k]}{\sqrt{\text{Var}[h_k]}}$$

# Normalization layers



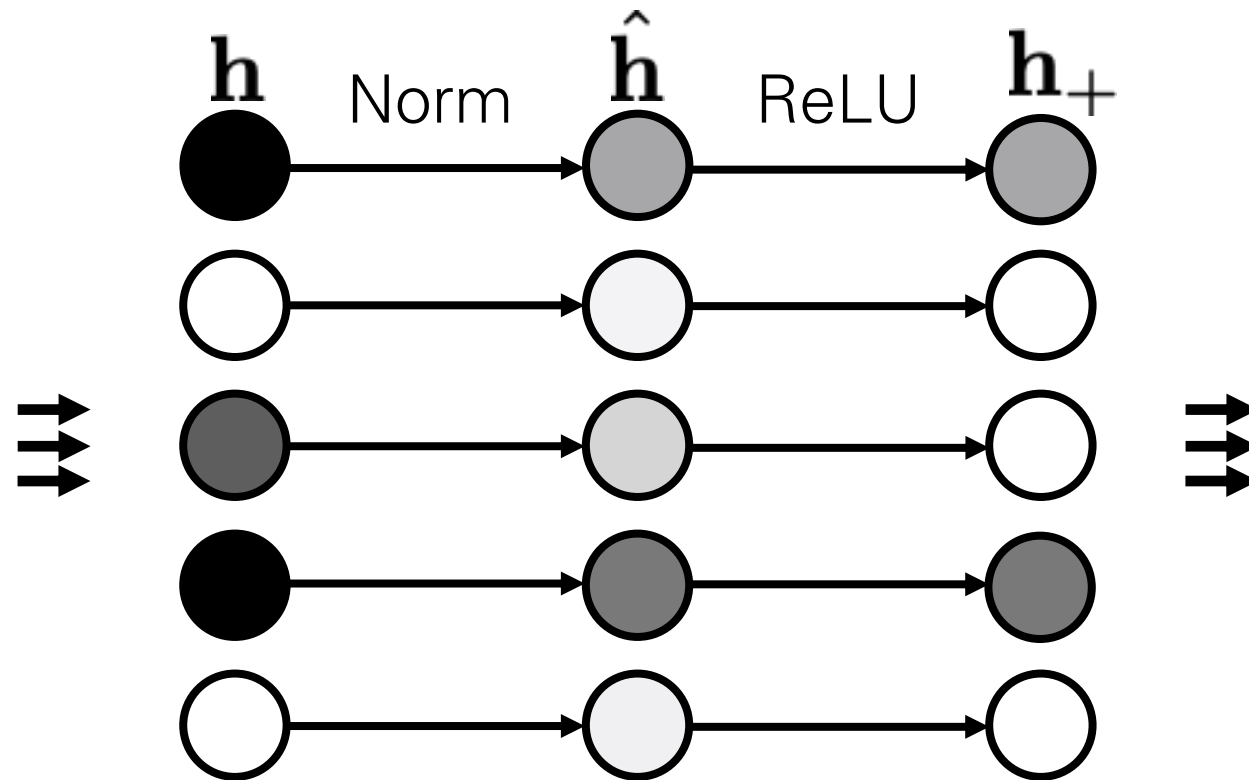
$$\hat{h}_k = \frac{h_k - \mathbb{E}[h_k]}{\sqrt{\text{Var}[h_k]}}$$

# Normalization layers



$$\hat{h}_k = \frac{h_k - \mathbb{E}[h_k]}{\sqrt{\text{Var}[h_k]}}$$

# Normalization layers



$$\hat{h}_k = \frac{h_k - \mathbb{E}[h_k]}{\sqrt{\text{Var}[h_k]}}$$

# Normalization layers

Keep track of mean and variance of a unit (or a population of units) over time.

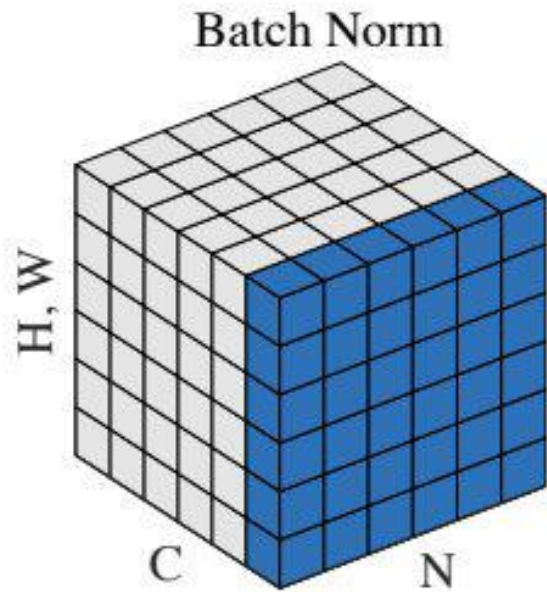
Standardize unit activations by subtracting mean and dividing by variance.

Squashes units into a **standard range**, avoiding overflow.

Also achieves **invariance** to mean and variance of the training signal.

Both these properties reduce the effective capacity of the model, i.e. regularize the model.

# Normalization layers

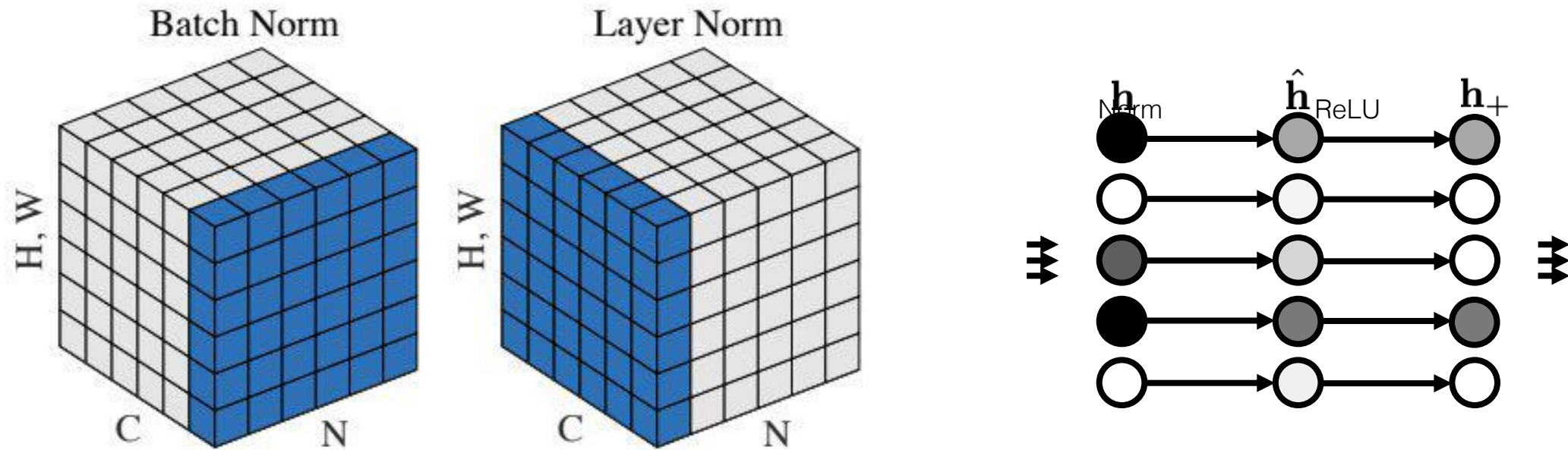


Normalize w.r.t. a single hidden unit's pattern of activation over training examples (a batch of examples).

[Figure from Wu & He, arXiv 2018]



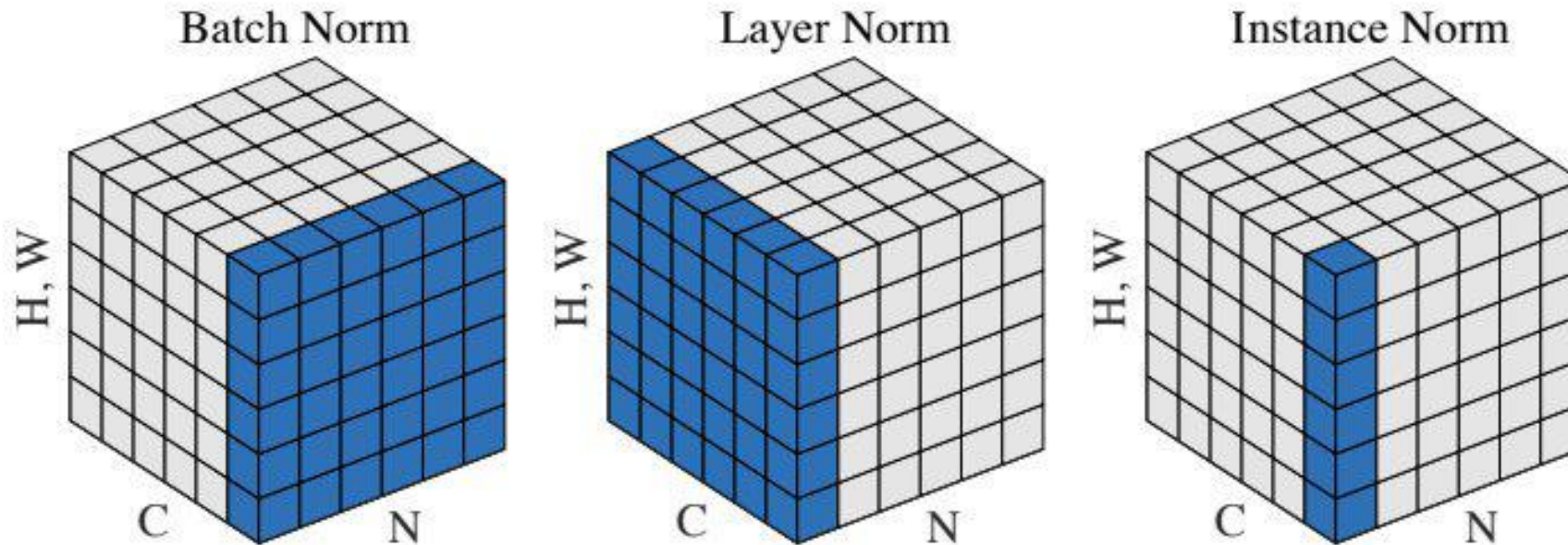
# Normalization layers



Normalize w.r.t. the mean and variance of the activations of all the hidden units (neurons) on this layer (c).

[Figure from Wu & He, arXiv 2018]

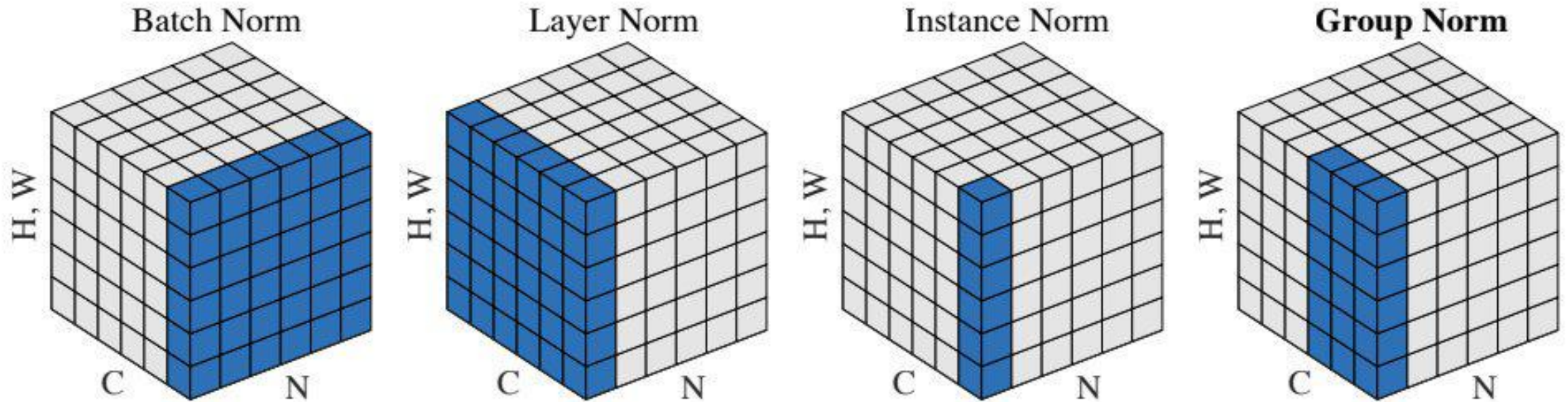
# Normalization layers



Normalize w.r.t. the mean and variance of the activations of all the hidden units (neurons) on this layer (c) that process this particular location (h,w) in the image.

[Figure from Wu & He, arXiv 2018]

# Normalization layers

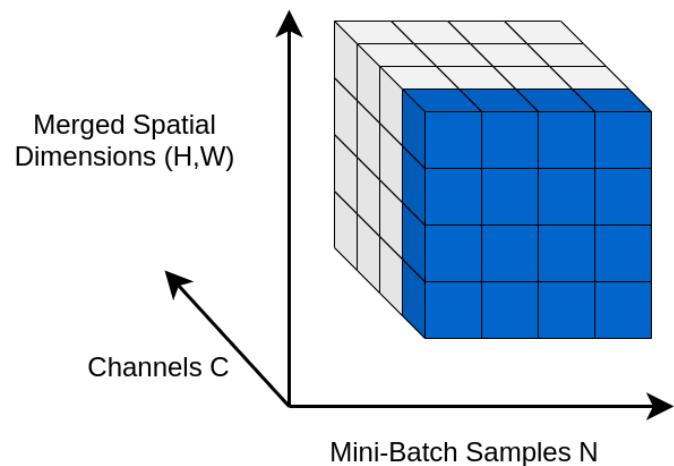


Might as well...

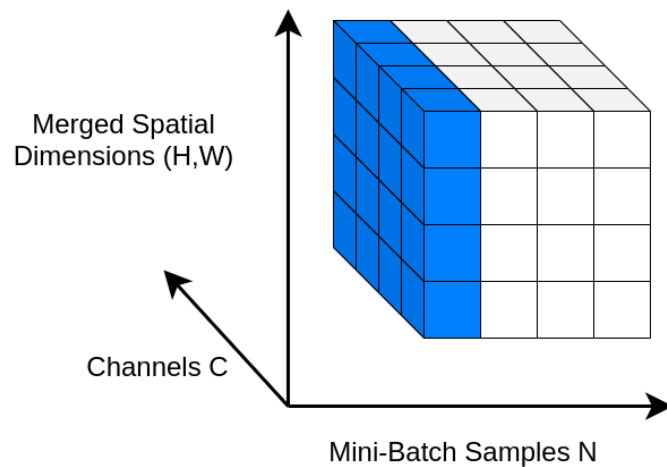
[Figure from Wu & He, arXiv 2018]

# Normalization layers

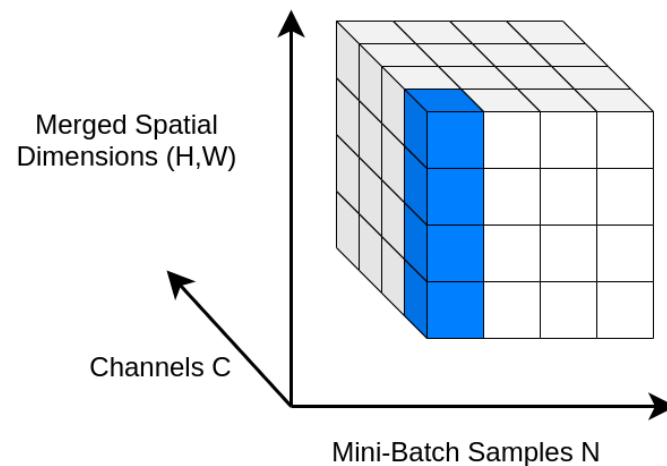
Batch Normalization (2015)



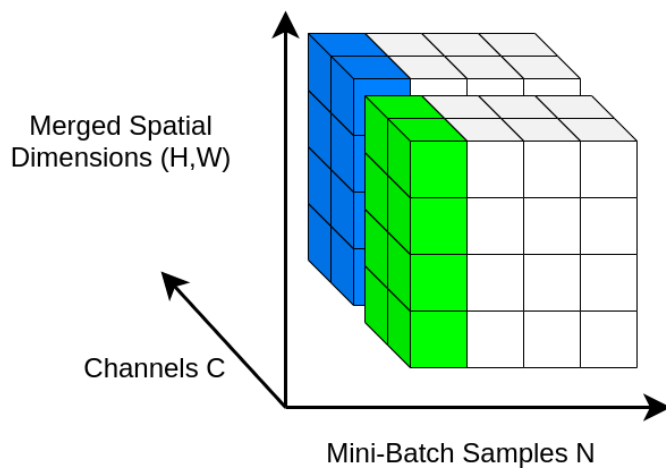
Layer Normalization (2016)



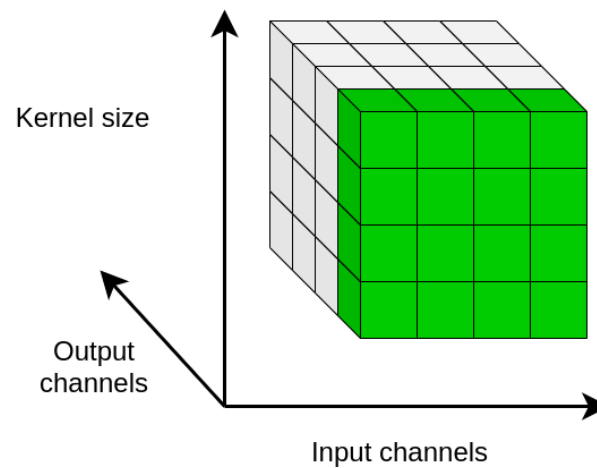
Instance Normalization (2016)



Group Normalization (2018)



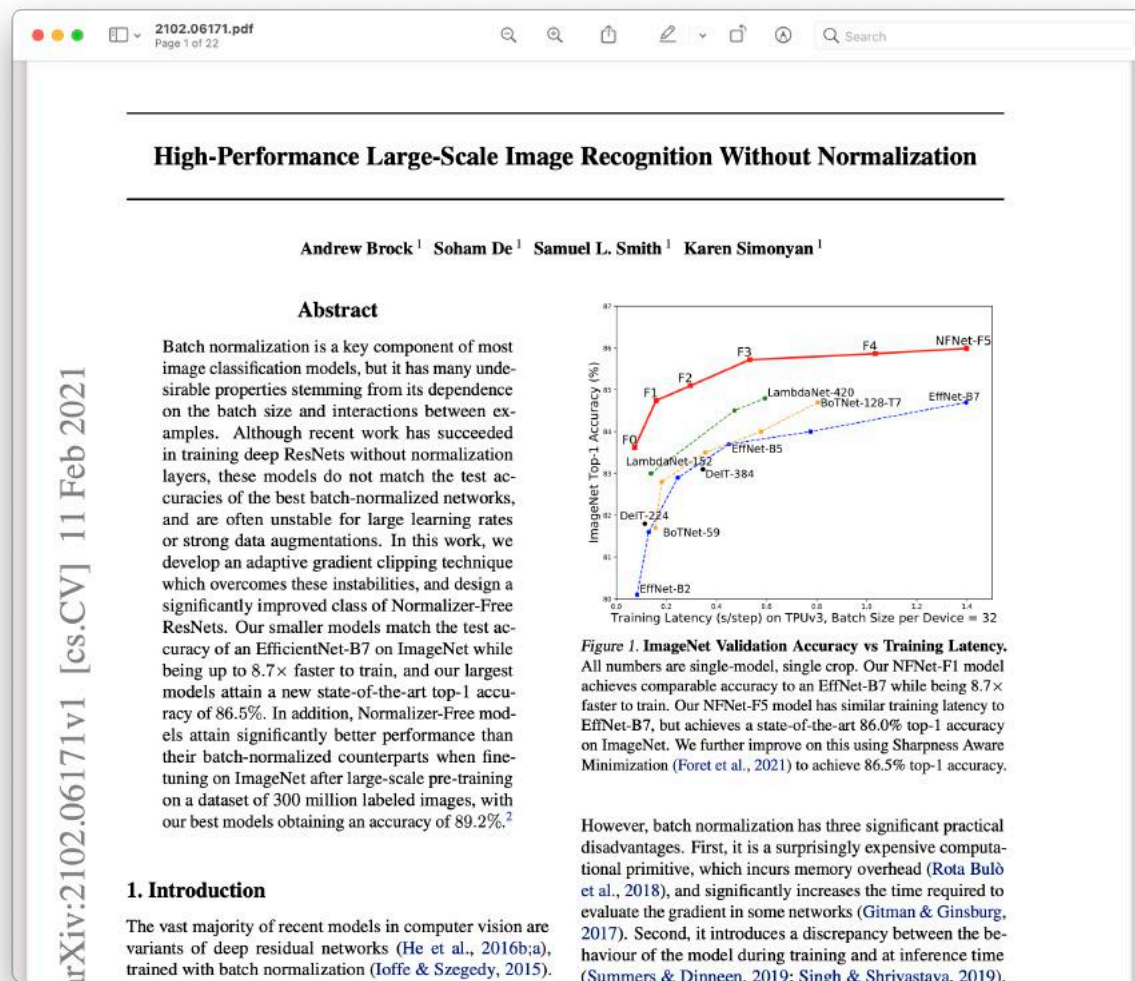
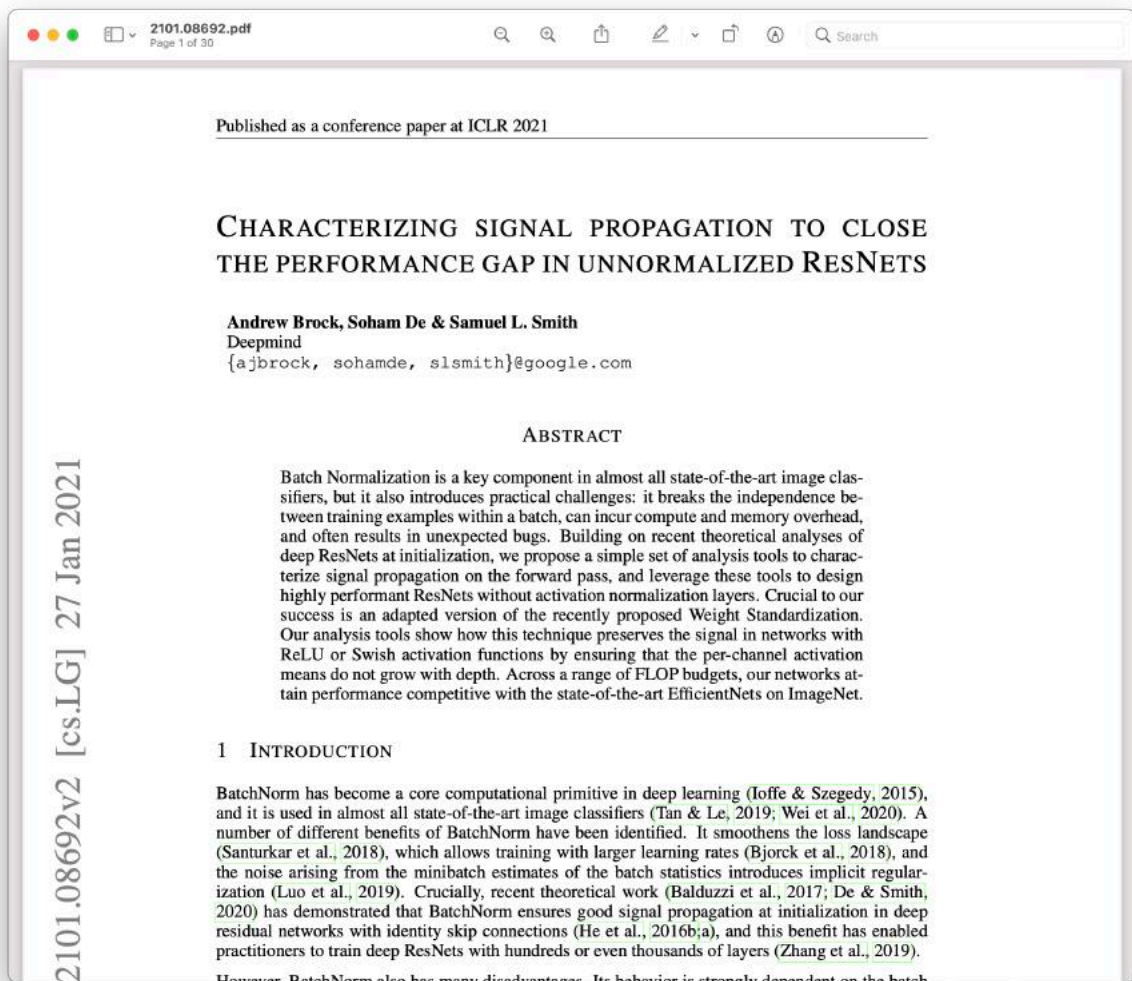
Weight Standardization (2019)



[\[https://theaisummer.com/normalization\]](https://theaisummer.com/normalization)



# No normalization layers



# **Next Lecture:**

# **Sequential Processing with RNNs**