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Evolutionary Algorithms for Multi-Objective Optimization: Performance Assessments and Comparisons

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Abstract. Evolutionary techniques for multi-objective (MO) optimization are currently gaining significant attention from researchers in various fields due to their effectiveness and robustness in searching for a set of trade-off solutions. Unlike conventional methods that aggregate multiple attributes to form a composite scalar objective function, evolutionary algorithms with modified reproduction schemes for MO optimization are capable of treating each objective component separately and lead the search in discovering the global Pareto-optimal front. The rapid advances of multi-objective evolutionary algorithms, however, poses the difficulty of keeping track of the developments in this field as well as selecting an existing approach that best suits the optimization problem in-hand. This paper thus provides a survey on various evolutionary methods for MO optimization. Many well-known multi-objective evolutionary algorithms have been experimented with and compared extensively on four benchmark problems with different MO optimization difficulties. Besides considering the usual performance measures in MO optimization, e.g., the spread across the Pareto-optimal front and the ability to attain the global trade-offs, the paper also presents a few metrics to examine the strength and weakness of each evolutionary approach both quantitatively and qualitatively. Simulation results for the comparisons are analyzed, summarized and commented.

Keywords: evolutionary algorithms, multi-objective optimization, Pareto optimality, survey

1. Introduction

Many real-world design tasks involve complex optimization problems of various competing design specifications and constraints (Beale 1988; Ben-Tal 1980; Deb 1995; Greenwood et al. 1996; Horn et al. 1994; Lis and Eiben 1997; Murata and Ishibuchi 1995; Reklaitis et al. 1983). For such multi-objective optimization problems, it is highly improbable that all the conflicting criteria would be extremized by a single design, and hence trade-off among the conflicting design objectives is often inevitable. In mathematics, multi-objective (MO) optimization seeks to optimize a vector of non-commensurable and often competing objectives or cost functions within a feasible decision variable space, viz, considering a minimization problem, it tends to find a parameter set $\boldsymbol{P}$ for (Eshenauer et al. 1990; Osyczka 1985),

$$\mathrm{Min}_{P \in \Phi} F(P),\ P \in R^n \tag{1}$$

where $\boldsymbol{P} = \{p_1, p_2, \ldots, p_n\}$ is a n-dimensional individual vector having n decision variables or parameters while Φ defines a feasible set of $\boldsymbol{P}$. $\boldsymbol{F} = \{f_1, f_2, \ldots, f_m\}$ is an objective vector with m objectives to be minimized. The feasible set of candidate vectors in the decision variable space can be confined to equality, inequality and/or discrete constraints, defined as:

$$q \text{ equality constraints:} \quad h_i(P) = 0 \ \ i = 1, 2, \ldots, q \tag{2a}$$

$$r \text{ inequality constraints:} \quad g_i(P) \geq 0 \ \ i = 1, 2, \ldots, r \tag{2b}$$

$$s \text{ discrete constraints:} \quad p_i \in \{\chi_1, \chi_2, \ldots, \chi_z\} \ \ i = 1, 2, \ldots, s \tag{2c}$$

It is clear from the simple formulation that just as the choice of $\boldsymbol{P}$ determines $\boldsymbol{F}(\boldsymbol{P})$, the choice of $\boldsymbol{F}(\boldsymbol{P})$ determines the preferred value of $\boldsymbol{P}$. If the objective functions change, the decision variables may be concentrating on a different set of performance aspects and this obviously has significant bearing on what is perceived to be the efficient solutions. Keeney and Raiffa (1976) provided some desirable properties for objective functions that may be used to compare solutions. In short, the objective functions must be:

1. Complete so that all pertinent aspects of the decision problem are presented;
2. Operational in that they can be used in a meaningful manner;
3. Decomposable if disaggregation of objective functions is required or it is desirable;
4. Non-redundant so that no aspect of the decision problem is considered twice;
5. Minimal such that there is no other set of objective functions capable of representing the problem with the smaller number of elements.

Solution to the above MO optimization problem is a family of points known as the Pareto-optimal set, where each objective component of any point along the Pareto-front can only be improved by degrading at least one of its other objective components (Fonseca and Fleming 1993; Goldberg and Richardson 1987). Before discussing on various methods of evolutionary MO optimization, some useful terms in MO optimization are introduced below:

1.1. *Pareto dominance*

In the totally absence of information for preferences of the objectives, Pareto dominance is regarded as an appropriate approach to compare the strength between any two solutions in MO optimization (Fonseca and Fleming 1993;

Steuer 1986). For a minimization problem, an objective vector $\boldsymbol{F}_a$ is said to dominate another objective vector $\boldsymbol{F}_b$, denoted by $\boldsymbol{F}_a \prec \boldsymbol{F}_b$, *iff*

$$f_{a,i} \leq f_{b,i} \forall\, i \in \{1, 2, \ldots, m\} \text{ and } f_{a,j} < f_{b,j} \text{ for some } j \in \{1, 2, \ldots, m\} \quad (3)$$

1.2. *Local Pareto-optimal set*

If there exists no solution $\boldsymbol{P}_i$ in set ψ, where $\psi \subseteq \Phi$, dominating any member $\boldsymbol{P}_j$ in a set Ω, where $\Omega \subseteq \psi$, then Ω denotes the local Pareto-optimal set. It usually refers to a Pareto-optimal set found in each iteration or at the end of the optimization in a single run. "Pareto-optimal" solutions are also termed "non-inferior", "admissible", or "efficient" solutions (Van Veldhuizen and Lamont 1999). Their corresponding vectors are termed "non-dominated" (Horn 1997).

1.3. *Global Pareto-optimal set*

If there exist no solution $\boldsymbol{P}_i$ in the set Φ, dominating any member $\boldsymbol{P}_k$ in a set Γ, where $\Gamma \subseteq \psi$, then Γ denotes the global Pareto-optimal set. Since $\psi \subseteq \Phi$, it is always true that there is no solution $\boldsymbol{P}_j$ in a local Pareto-optimal set Ω dominating any solution $\boldsymbol{P}_k$ in the global Pareto-optimal set Γ. Γ usually refers to the actual Pareto-optimal set in a MO optimization problem. It can be either obtained via solving the objective functions concerning the space of Φ or approximated through many-repeated optimization runs.

1.4. *Pareto front and Pareto front's structure*

According to (Van Veldhuizen and Lamont 2000), for a given MO optimization function $\boldsymbol{F}(\boldsymbol{P})$ and Pareto-optimal set Ω, the Pareto-front (PF^*) is defined as:

$$PF^* : \ = \{\vec{u} = F(P) = (f_1(P), \ldots, f_m(P)) \,|\, P \in \Omega\} \quad (4)$$

Concerning the Pareto-front's structure, Horn and Nafpliotis (1993) stated that the Pareto-front is a m–1 dimensional surface in a m-objective optimization problem. This statement was later revised by Van Veldhuizen and Lamont (1999) in which they stated that the Pareto-front of an optimization with m = 2 objectives is at most a (restricted) curve, and is at most a (restricted) m–1 dimensional surface if $m \geq 3$.

1.5. *Totally conflicting, none conflicting and partially conflicting objective functions*

For any given MO optimization problem, the objective functions can be categorized as totally conflicting, none conflicting or partially conflicting. In a given solution set Φ, the objective functions $\boldsymbol{F} = \{f_1, f_2, \ldots, f_m\}$ are said to be totally-conflicting if there exists no two solutions $\boldsymbol{P}_a$ and $\boldsymbol{P}_b$ in set Φ such that $(\boldsymbol{F}_a \prec \boldsymbol{F}_b) \vee (\boldsymbol{F}_b \prec \boldsymbol{F}_a)$. In this class of problem, an optimization process is not needed since the solution set in Φ is the global Pareto-optimal set, i.e., $\Gamma = \Phi$

The objective functions are said to be none conflicting if any two selected solutions $\boldsymbol{P}_a$ and $\boldsymbol{P}_b$ in set Φ always satisfy $(\boldsymbol{F}_a \prec \boldsymbol{F}_b) \vee (\boldsymbol{F}_b \prec \boldsymbol{F}_a)$. This class of problem can be easily converted into single objective optimization either by arbitrarily considering one of the objective components throughout the optimization process or linearly combining the objective vector into a scalar function via non-negative weight values. This is because the improvement in one objective component will always lead to the improvement of the rest of the objective components, i.e., the size of the global or local Pareto-optimal set is always equal to one.

A MO optimization problem is belonged to the third class if it does not belong to the first or the second class. In this case, the objective functions $\boldsymbol{F} = \{f_1, f_2, \ldots, f_m\}$ are said to be partially-conflicting if there exists a non-empty set of $\boldsymbol{P}_a$ and $\boldsymbol{P}_b$ such that $(\boldsymbol{F}_a \prec \boldsymbol{F}_b) \vee (\boldsymbol{F}_b \prec \boldsymbol{F}_a)$. This is a class that most of the MO optimization problems belong to, which requires the application of MO optimization techniques for finding the set of Pareto-optimal solutions.

Conventional MO optimization techniques include linear programming, gradient methods, methods of inequalities, goal attainment or weighted sum approach. To obtain a good solution, however, these methods require a set of precise settings of weights or goals that are usually not well manageable or understood (Coello 1996; Fonseca and Fleming 1993). If the solution produced is not satisfactory, the weights or goals must be changed and the optimization process has to be repeated. The shortcomings of these approaches were explicitly pointed out by Deb (1999a):

> "In dealing with multi-criterion optimization problems, classical search and optimization methods are not efficient, simply because (i) most of them cannot find multiple solutions in a single run, thereby requiring them to be applied as many times as the number of desired Pareto-optimal solutions, (ii) multiple application of these methods do not guarantee finding widely different Pareto-optimal solutions, and (iii) most of them cannot

> efficiently handle problems with discrete variables and problems having multiple optimal solutions"

Emulating the biological evolution mechanism and Darwin's principal on "survival-of-the-fittest", evolutionary algorithms (EAs) have been recognized to be well suited for MO optimization problems where conventional tools fail to work well (Anderson et al. 1998; Bäck 1996; Davidor 1991; Deb and Goldberg 1989; Deb 2001; Fonseca and Fleming 1998; Forrest et al. 1993; Fujita et al. 1998; Mahfoud 1995; Miller and Shaw 1996; Pétrowski 1996). The growing interest of evolutionary methods for MO optimization can be reflected by the high volume of publications in this topic as well as the numerous special/tutorial sessions in reputable conferences like IEEE Congress on Evolutionary Computation (CEC) and Genetic and Evolutionary Computation Conference (GECCO) over the last few years. The importance of this research area is also shown by the recent success of the first international conference on Evolutionary Multi-criteria Optimization (EMO'2001) held in March 2001 at Zurich, Switzerland.

Unlike conventional gradient-guided search methods, EAs require no gradient information, which makes it a unique and robust tool for solving multi-objective optimization problems. In addition, EAs are capable of producing the set of Pareto-optimal solutions in a single optimization run due to their nature of parallelism through the process of recombination and reproduction (Fonseca and Fleming 1997; Goldberg and Richardson 1987; Srinivas and Deb 1994; Tan et al. 1999). Owing to these features, evolutionary algorithms have been successfully applied to numerous optimization problems in scientific and engineering disciplines including treatment of cancer in medical fields (Haas et al. 1997), control engineering design in power systems (Reformat et al. 1998), physiological processes of biological plants (Morimoto et al. 1995), recognition of Chinese characters (Lin and Leou 1997) and so forth.

There are a number of surveys on evolutionary techniques for MO optimization (Coello 1996; 1999; Deb 1999b; Fonseca and Fleming 1995a; Tan et al. 2001a; Van Veldhuizen and Lamont 1999, 2000; Zitzler and Thiele 1999; Zitzler et al. 2000). Among these works, the publication from Coello (1996) is a comprehensive survey, which aims to summarize and organize the information by classifying various techniques into three main groups, based on different implemented strategies in cost assignments and selection methods. The three groups include naïve approaches, non-aggregation approaches, and Pareto-based approaches. In each group, a fairly detailed implementation of the methods with relevant comments was given. Besides Coello (1996), there are other studies that attempt to classify existing approaches in different ways. For example, Fonseca and Fleming (1995a) classified existing techniques from a broad algorithmic perspective, Bentley and Wakerfield (1997) from the perspective of range dependency, while others (e.g., Horn 1997; Van Veldhuizen and Lamont 2000) from the decision maker's perspective.

This paper provides an up-do-date survey on various evolutionary methods for MO optimization. The performances of many well-known multi-objective evolutionary algorithms are compared extensively on four benchmark problems with different MO optimization difficulties. A few important performance metrics are also presented to examine the strength and weakness of each approach, both quantitatively and qualitatively. Section 2 gives a general overview of evolutionary MO optimization methods and extracts the feature elements of each approach for discussion. Section 3 describes the various MO performance measures, and Section 4 presents the four benchmark MO optimization problems. The simulation results of each approach are compared and summarized in Section 5. Conclusions are drawn in Section 6.

2. Developments in Evolutionary Multi-objective Optimization

In this section, existing evolutionary approaches for MO optimization are surveyed through different perspective. Instead of classify existing methods into groups, the feature elements of each approach are extracted and discussed. Table 1 shows a list of 51 publications on different evolutionary approaches for MO optimization. In the table, the MO handling techniques and important operators (apart from standard genetic operators like selection, crossover and mutation) for each algorithm are summarized in the 2nd and 3rd column, respectively. The approaches are sorted chronologically according to the year of publication.

It can be observed from Table 1 that the MO handling techniques for each algorithm can be decomposed into one or more basic element(s), as shown in Tables 2 and 3. The dots in Table 3 represent ownership of each element under various techniques surveyed in the paper. These elements can be divided into two major groups according to their roles in MO optimization. The first group contains the elements that have a direct relationship with MO handling techniques in finding the Pareto-optimal set. These include *Weights*, *Min-Max*, *Pareto*, *Ranking*, *Goals*, *Pref.*, *Gene*, *Sub-pop.*, *Fuzzy*, *Agents* and *Others*. On the other hand, the second group of elements plays an indirect role which aims to support the algorithms for better MO optimization. The description of these two groups of elements is provided in Table 2. It can be observed that some of the elements are common for more than one technique. At the same time, there exist several algorithms applying more than one element for MO handling techniques and/or supporting operators. This observation suggests that evolutionary MO optimization methods should not be classified into

Table 1. Summary of existing evolutionary methods for MO optimization

Evolutionary Methods for MO	**MO Handling Techniques**	**Other Operators Applied**
Charnes and Cooper (1961)	• Goal programming.	
Ijiri (1965)	• Goal programming.	
Jutler (1967)	• Weighted min-max approach.	
Solich (1969)	• Weighted min-max approach.	
Fourman (1985)	• Lexicographic ordering, starting with the most important one and proceeding to the order of importance of objectives.	
Schaffer (1985): VEGA	• Main population was divided into sub-populations and selection was performed according to each objective function in each sub-population.	• Proportional selection.
Goldberg and Richardson (1987)	• Simple Pareto domination scheme.	• Sharing on whole population. • Proportional Selection.
Allenson (1992)	• Sex was used to distinguish between two objectives and was assigned at birth.	
Chen et al. (1992)	• Transformation of qualitative relationships between objectives into quantitative attributes for an appropriate weight of each objective in a way similar to linguistic ranking methods. The weight generated can be used with a aggregating approach or Pareto-ranking.	
Hajela and Lin (1992): HLGA	• Weighted-sum method was used for fitness assignment. To search for multiple solutions in parallel, the weights were not fixed but encoded in the genotype instead.	• The diversity of the weight combinations was promoted by phenotype fitness sharing. • Mating restriction was employed for faster convergence and better stability.
Jakob et al. (1992)	• Linear combination of objective functions.	
Fonseca and Fleming (1993): MOGA	• Pareto domination scheme. It was extendable for single set of goals and priorities.	• Sharing on whole population. • Mating restriction was employed for faster convergence and better stability.
Wilson and Macleod (1993)	• Goal attainment.	
Adeli and Cheng (1994)	• Use of Penalty function.	
Horn et al. (1994): NPGA	• Randomly-selected comparison set was used to determine the winner of the competitors.	• Phenotype sharing was applied if the competitors end in a tie in domination. • Tournament selection.
Ritzel et al. (1994)	• Minimizing one objective function, while considering other objective functions as constraints. • Reduction to a single objective.	
Srinivas and Deb (1994): NSGA	• Several layers of classification of the individuals according to domination were applied.	• Sharing on dummy fitness value in each layer. • Proportional Selection.
Sandgren (1994)	• Goal programming.	

Table 1. Continue

Evolutionary Methods for MO	MO Handling Techniques	Other Operators Applied
Murata and Ishibuchi (1995): MIMOGA	• The weights that were attached to the MO functions were not constant but randomly specified for each selection.	• A tentative set of Pareto-optima was stored and updated at every generation. A number of individuals were randomly selected from the set and used as elite individuals.
Vemuri and Cedeño (1995)	• Individuals were ranked for each objective. Total ranking for each individual was then determined by the sum of all the ranking numbers.	• Similarity among individuals was used during selection and replacement.
Coello (1996): Monte Carlo method 1	• Space was explored twice, first searching for ideal vector and then searching for min-max optimum.	• Mating Restriction was applied. • Constraints were handled by death penalty method.
Coello (1996): Monte Carlo method 2	• Space was explored only once, and Pareto set was generated while searching for ideal vector. Then this set was analyzed to check for min-max optimum.	• Mating Restriction was applied. • Sharing was used to overcome high selection pressure. • Min-max tournament selection was applied.
Greenwood et al. (1996)	• Compromise between no preference information (in the case of pure Pareto rankings) and aggregation methods like the weighted-sum to perform imprecise ranking of attributes.	• Sharing was employed to distribute the Pareto-front. • Tournament selection.
Kita et al. (1996)	• Use of concepts of entropy and temperature, combined with Pareto-based ranking technique, in selection.	
Sakawa et al. (1996)	• Fuzzy goals of objective functions were quantified by eliciting linear membership functions.	
Viennet et al. (1996)	• Each function was separately optimized. The populations from each run were processed and set Pareto-optima was obtained via elimination of Pareto inefficient points.	• Elitist selection was employed when optimizing each function separately.
Bently and Wakefield (1997): SWR	• Linear combination of objective functions which have been converted into ratios by using the best and worst solution in the current population.	
Bently and Wakefield (1997): SWGR	• Sum of weighted global ratios.	
Bently and Wakefield (1997): WAR	• Weighted average ranking.	• Non-generational selection. Fitness of an individual was counted increasingly. • Crossover and mutation to produce new individual which substituted the worst individual.
Lis and Eiben (1997): MSGA	• Generalized version from (Allenson 1992) where the sex was not restricted to male and female.	• Multi-parent crossover was applied for recombination, requiring one parent from each sex. • A solution was represented by a string, like in classical GA, and the sex marker.
Marcu (1997)	• Adaptation of goal and use of goal values in Pareto-ranking to direct the search towards the middle region of the trade-off.	

Table 1. Continue

Evolutionary Methods for MO	MO Handling Techniques	Other Operators Applied
Fujita et al. (1998)	• Multiple functions were unified into scalar function so that 1 for every Pareto optima while other non-Pareto optima has value equal to how far it was from a set of Pareto optima.	• Sharing was employed to the scalar function. • The algorithm limited the crossover between a pair of similar solutions.
Jaszkiewicz (1998)	• Weights are randomly selected. Then a temporary population is created composed of best known solution on the current weights.	• Each individual in the population is optimized locally according to the randomly selected weight function.
Laumanns et al. (1998)	• Predators were applied to chase the prey (candidate solution) according to one of the objectives. As there are several predators with different selection criteria, those prey individuals, which are best with respect to all objectives, are able to produce more.	
Voget and Kolonko (1998)	• The method is similar to goal attainment, except that membership functions are used to express goals in vague terms.	
Cvetković and Parmee (1999)	• Transformation of qualitative relationships between objectives into quantitative attributes for an appropriate weight of each objective in a way similar to linguistic ranking methods. The weight generated can be used with a aggregating approach or Pareto-ranking.	
Hiroyasu et al. (1999)	• Simple Pareto domination scheme.	• Population was divided into several islands where simple genetic operations were performed in each island. After certain generations, migration was performed. • When the size of the frontier solution exceeds a criterion, sharing was performed.
Knowles and Corne (1999): PAES	• Simple Pareto domination scheme.	• Based on (1 + 1) evolution strategy. • Local search was used form a population of one but using a reference archive of previously found solutions to store Pareto-optima and to identify dominance ranking of candidate solutions. • Tracking the degree of crowding in different regions of solution space to spread reference archive.
Romero and Manzanares (1999): MOAQ	• A family of agents for each objective and each family tried to optimize an objective considering the solutions found for other objectives.	
Sait et al. (1999)	• Fuzzy goal-based cost computation measure combined with fuzzy allocation scheme was applied. It used fuzzy rules and membership functions to combine multiple objectives and added controlled randomness in placing a cell on an empty location within a fuzzy window.	
Tagami and Kawabe (1999)	• Based on Pareto neighborhood search method on the basis of distribution in objective space divided into pre-specified regions.	

Table 1. Continue

Evolutionary Methods for MO	**MO Handling Techniques**	**Other Operators Applied**
Tan et al. (1999): MOEA	• Pareto domination scheme, extendable for soft/hard goals and priorities and even set of goals and priorities.	• Dynamic sharing on whole population. • Switching Preserved Strategy (SPS) for elitism. • Tournament selection.
Zitzler and Thiele (1999): SPEA	• Simple Pareto domination scheme. Fitness of solutions was determined only from solutions stored in the external non-dominated set.	• Use of clustering to reduce the number of non-dominated solutions stored. • Non-dominated solutions found so far were store externally. • Binary tournament selection.
Andrzej and Stanislaw (2000)	• Simple Pareto domination scheme.	• Constraint tournament selection where functions are evaluated only for feasible solutions.
Knowles and Corne (2000): M-PAES	• Simple Pareto domination scheme.	• Based on PAES (Knowles and Corne 1999), but uses a population of solutions and employs crossover. • Besides the main population, two archives were required for elitism.
Mariano and Morales (2000): MDQL	• Each agent proposes a solution for its corresponding objective function. Solutions were then evaluated using non-dominated criterion and solutions in the final Pareto set were rewarded.	
Rekiek et al. (2000)	• Use of preference ranking organization method for enrichment evaluation (Brans et al. 1986).	
Sefrioui and Periaux (2000)	• Based on non-cooperative game theory to find Nash equilibria.	
Khor et al. (2000): IMOEA	• Pareto domination scheme, extendable for soft/hard goals and priorities and even set of goals and priorities.	• Dynamic population size based upon on-line discovered Pareto-front for desired population distribution density and Dynamic local fine-tuning to achieve broader neighborhood.
Khor et al. (2001): EMOEA	• Pareto domination scheme, extendable for soft/hard goals and priorities and even set of goals and priorities.	• Tabu list and Tabu constraint were used for individual examination and preservation. • Lateral interference for uniform distribution.

groups, which may lead to imprecise classification where intersection among groups are neglected. Instead, it could be broken into basic elements, since these elements represent part of the approach's behavior such that analysis can be made based upon the basic elements that each approach belongs to.

Figure 1 shows the development trends of MO handling elements in evolutionary methods for MO optimization. Figure 1a illustrates the number of methods incorporating MO handling elements in each year, where the fraction of each MO handling element within the year could be observed. It

Table 2. Description of elements in evolutionary MO optimization approaches

Label		**Description**
MO Handling Elements	*Weights*	Multiple objectives are combined into scalar objective via weight vector. Weights may be assigned through: direct assignment, eigenvector method, entropy method, minimal information method, randomly determined or adaptively determined. It is difficult to precisely pre-determine the weights. If the objective functions are simply weighted and added to produce a single fitness, the function with the largest range would dominate evolution. A poor input value for the objective with the larger range makes the overall value much worse than a poor value for the objective with the smaller range (Bently and Wakefield 1997). It suffers the disadvantage of missing concave portions of the trade-off curve (Coello 1996).
	Min-Max	It uses the distance between an efficient design and a pre-defined ideal design. It attempts to find from the feasible domain an efficient design which is nearest to the ideal design in the min-max sense. It is able of discovering all efficient solutions of a multi-objective problem whether the problem is convex or non-convex.
	Pareto	It uses Pareto dominance scheme defined in eqn. 3 for individual comparison. The comparison results among the individuals will influence the selection and reproduction process in the evolution.
	Ranking	Individuals are sorted from the most preferable to the least preferable, or vice versa. Individuals are then assigned rank according to their preferences. If two or more individuals are equally preferable, they will be assigned the same rank.
	Goals	It requires a designer to set goals for the objectives that he wishes to achieve and adopts the decision rule that the best compromise design should be the one which minimizes the deviation from the set goals.
	Pref.	It requires a designer to set preferences/priorities of the objectives to optimize and adopts the decision rule that the objectives with higher priorities are given higher privilege to optimize than the objectives with lower priorities.
	Gene	Chromosome genes do not only store the information of decision variables or parameter values for each individual but they also influence the way where fitness/cost assignment process for each individual is performed. The genes for the latter purpose can be either altered stochastically through normal/special evolution process or assigned through deterministic rule.

Table 2. Continue

Label		**Description**
	Sub-pop.	The main population is divided into several sub-populations where each sub-population is optimized based on similar/different selection criteria. If different selection criteria are applied, it may refer to either different objective component or utility function of the objectives. The shuffling and merging of all sub-populations are in fact corresponding to fitness averaging for each of the objective component (Richardson et al. 1989). If gender is applied to classify the population, a relative large population size with lots of computational effort is required to maintain a reasonably diverse spread of genders across the entire population (Coello 1999).
	Fuzzy	Fuzzy rules and fuzzy membership functions are applied to combine the multiple objectives to handle the vague term of user's specifications. The resulted fuzzy reasoning process is then used in selection process.
	Agents	It involves the use of a family of agent where each agent participated in improving the individuals for its corresponding objective.
	Others	The objective handling techniques that are apart from the above techniques.
Supporting Elements	*Dist.*	Involving any explicit operator to distribute the individuals either in phenotype or genotype space. It includes fitness sharing (Deb and Goldberg 1989), niching (Beasley et al. 1993), crowding (De Jong 1975), clearing (Pétrowski 1996), clustering (Zitzler and Thiele 1999) and others.
	Mat.	Mating restriction (Fonseca and Fleming 1995b) to only allow mating between "similar" solutions (over some metrics) in the hope to increase algorithm effectiveness and efficiently.
	Sub-reg.	Phenotype or genotype space is divided into pre-defined regions to keep track of the degree of crowding of the space.
	Ext.	Besides the evolving population, external population is applied to store the non-dominated solutions found do far.
	Elitism	The Pareto optimum solutions are preserved and updated at each generation. In some methods, the diversity and uniform distribution are also taken into account in updating the non-dominated individuals.

can be seen that there is an increasing effort of incorporating MO handling elements into evolutionary MO optimization methods. The recent popular ones are *Pareto* and *Ranking* schemes, which occupy a large fraction among the methods incorporating MO handling elements in the years of 1999 and 2000. Figure 1b shows the cumulative representation for the number of methods incorporating MO handling elements that “have been” proposed over the years, where each line represents the trend of a particular MO handling element. It can be seen that the increased numbers of schemes such as *Weights*, *Min-Max*, *Gene*, *Sub-pop*, *Fuzzy* and *Others* are getting saturated, while others like *Pareto*, *Ranking*, *Goals*, *Pref.*, and *Agents* continue to gain interests from researchers over the years. In particular, the *Pareto* scheme is increasing consistently since the early nineties, and is expected to grow in the coming years.

The development trends of supporting elements are depicted in Figure 2, which are represented in both types of bar and cumulative graphs. It can be seen from the graphs that these supporting elements only started to gain attention from the nineties, particularly in the year of 1999 (See Figure 2a). Among the supporting elements, the distribution operator (*Dist.*) for distributing population along the trade-offs has been applied in most of the methods as compared to other supporting elements (see Figure 2b). Although mating restriction (*Mat.*) was second popular till the year of 1998, it has been overtaken by *Elitism* since then (see Figure 2b). Concerning the rest of the supporting elements, *Ext.* and *Sub-reg.* are less popular as compared to others.

3. Performance Measures in Multi-objective Optimization

The quality of multi-objective optimization is often difficult to be defined precisely (Zitzler et al. 2000). Until 1998, there was a lack of well-established quantitative performance measures for evolutionary MO optimization. The arguments for/against the methods are mainly based on qualitative aspects of the evolution process and the final population distribution. Since then, Deb (1999c); Shaw et al. (1999); Van Veldhuizen and Lamont (1998, 1999); Zitzler and Thiele (1999) began to explore new comparison methods and formalized the quantitative metrics as a common basis for performance comparison in evolutionary MO optimization. However, as stated by Shaw et al. (1999), these quantitative metrics mainly examine the optimization performance in two aspects, e.g., the spread across the Pareto-optimal front and the ability to attain the global trade-off surface.

In this section, a total of six performance metrics are recommended and applied to examine the strengths and weaknesses of various multi-objective evolutionary algorithms. These performance metrics examine the fraction of

Table 3. Basic elements in evolutionary MO optimization approaches

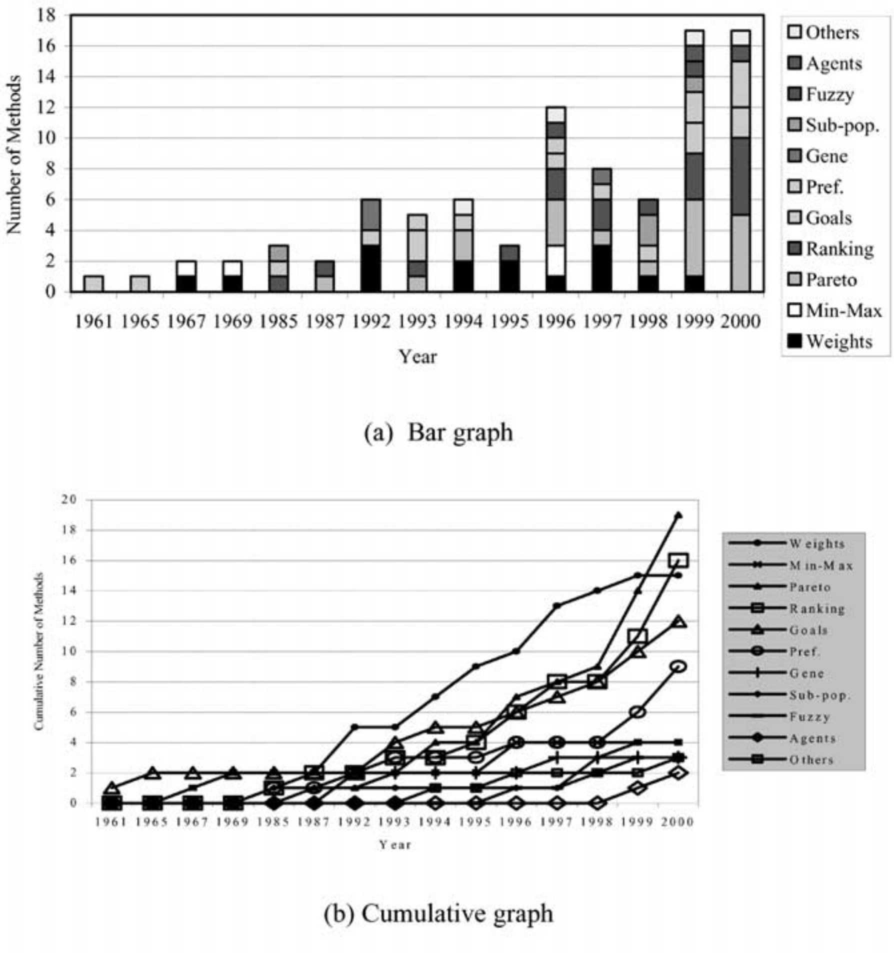
Evolutionary MO	***MO Handling Elements***											***Supporting Elements***				
Optimization Methods	*Weights*	*Min-Max*	*Pareto*	*Ranking*	*Goals*	*Pref.*	*Gene*	*Sub-pop.*	*Fuzzy*	*Agents*	*Others*	*Dist.*	*Mat.*	*Sub-reg.*	*Ext.*	*Elitism*
Charnes and Cooper (1961)					•											
Ijiri (1965)					•											
Jutler (1967)	•	•														
Solich (1969)	•	•														
Fourman (1985)				•		•										
Schaffer (1985): VEGA								•								
Goldberg and Richardson (1987)			•	•												
Allenson (1992)							•									
Chen et al. (1992)	•					•										
Hajela and Lin (1992): HLGA	•						•					•	•			
Jakob et al. (1992)	•															
Fonseca and Fleming (1993): MOGA			•	•	•	•						•	•			
Wilson and Macleod (1993)					•											
Adeli and Cheng (1994)	•															
Horn et al. (1994): NPGA			•									•				
Ritzel et al. (1994)											•					
Ritzel et al. (1994)	•															

Table 3. Continue

Evolutionary MO	***MO Handling Elements***											***Supporting Elements***				
Optimization Methods	*Weights*	*Min-Max*	*Pareto*	*Ranking*	*Goals*	*Pref.*	*Gene*	*Sub-pop.*	*Fuzzy*	*Agents*	*Others*	*Dist.*	*Mat.*	*Sub-reg.*	*Ext.*	*Elitism*
Srinivas and Deb (1994): NSGA			•									•				
Sandgren (1994)					•											
Murata and Ishibuchi (1995): MIMOGA	•														•	•
Vemuri and Cedeño (1995)	•			•												
Coello (1996): Monte Carlo I	•	•											•			
Coello (1996): Monte Carlo II		•	•									•				
Greenwood et al. (1996)			•	•		•						•				
Kita et al. (1996)			•	•												
Sakawa et al. (1996)					•				•							
Viennet et al. (1996)											•					•
Bently and Wakefield (1997): SWR	•															
Bently and Wakefield (1997): SWGR	•															
Bently and Wakefield (1997): WAR	•			•												
Lis and Eiben (1997): MSGA							•									
Marcu (1997)			•	•	•											
Fujita et al. (1998)			•									•	•			
Jaszkiewicz (1998)	•							•								•
Laumanns et al. (1998)								•								

Table 3. Continue

Evolutionary MO Optimization Methods	***MO Handling Elements***											***Supporting Elements***				
	Weights	*Min-Max*	*Pareto*	*Ranking*	*Goals*	*Pref.*	*Gene*	*Sub-pop.*	*Fuzzy*	*Agents*	*Others*	*Dist.*	*Mat.*	*Sub-reg.*	*Ext.*	*Elitism*
Voget and Kolonko (1998)					•				•							
Cvetković and Parmee (1999)	•					•										
Hiroyasu et al. (1999)			•		•			•				•				
Knowles and Corne (1999): PAES			•	•								•		•		•
Romero and Manzanares (1999): MOAQ										•						
Sait et al. (1999)					•				•							
Tagami and Kawabe (1999)			•									•		•		
Tan et al. (1999): MOEA			•	•		•						•				•
Zitzler and Thiele (1999): SPEA			•	•								•			•	•
Andrzej and Stanislaw (2000)			•	•												
Knowles and Corne (2000): M-PAES			•	•										•	•	•
Mariano and Morales (2000): MDQL			•							•						
Rekiek et al. (2000)				•		•										
Sefrioui and Periaux (2000)											•					
Khor et al. (2000): IMOEA			•	•	•	•						•				•
Khor et al. (2001): EMOEA			•	•	•	•						•			•	•
Total	15	4	19	16	12	9	3	4	3	2	3	14	4	3	4	9
Percentage (%)	12.1	3.2	15.3	12.9	9.7	7.3	2.4	3.2	2.4	1.6	2.4	11.3	3.2	2.4	3.2	7.3



(a) Bar graph

(b) Cumulative graph

Figure 1. Development trends of MO handling elements.

useful individuals in a population, the uniform distribution of individuals along the Pareto-front, the computational effort of an optimization algorithm, the robustness to disturbances as well as the average best performance of tracking optimal regions in changing environments. Some of these metrics are referred from other articles (Branke 1999; Deb 1999c; Shaw et al. 1999; Van Veldhuizen and Lamont 1998, 1999; Zitzler and Thiele 1999), while others are designed in this section for a more comprehensive comparison. These metrics are chosen since they have been widely used for performance comparisons in MO optimization. Moreover, these metrics do not based upon the actual trade-off surface which often needs to be obtained through the

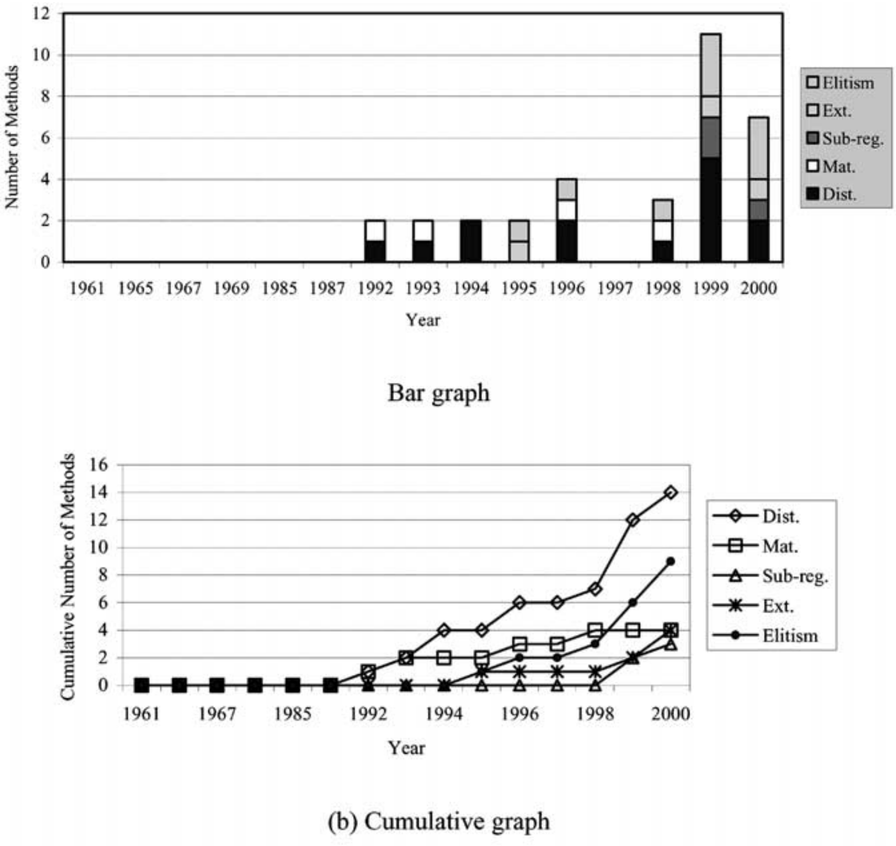


Figure 2. Development trends of MO supporting elements.

method of deterministic enumeration or is not obtainable in certain practical problems (Veldhuizen and Lamont 1999).

3.1. *Algorithm effort (AE)*

The performance in MO optimization is often evaluated not only in terms of how good the final Pareto-front is, but also in terms of the computational effort required in obtaining the optimal solutions. For this purpose, the algorithm effort (*AE*) is defined as the ratio of the total number of function evaluations N_{eval} over a fixed period of simulation time T_{run} (Tan et al. 2001a),

$$AE = \frac{T_{run}}{N_{eval}}, \quad \text{provided that: } (T_{run} > T_{1stgen}) \cap (T_{eval} \propto N_{eval}) \qquad (5)$$

As shown in equation 5, for a fixed period of T_{run}, more number of function evaluations being performed indirectly indicates that less computational effort is required by the optimization algorithm and hence resulting in a smaller *AE*. The condition of $T_{run} > T_{1stgen}$, where T_{1stgen} is the computation time for the 1st generation, should be hold so that T_{run} and N_{eval} are > 0. This results *AE* to be bounded in the range of $(0, \infty)$.

3.2. *Ratio of non-dominated individuals (RNI)*

This performance metric is defined as the ratio of non-dominated individuals (*RNI*) for a given population *X*,

$$RNI(X) = \frac{nondom\_indiv}{P} \tag{6}$$

where *nondom_indiv* is the number of non-dominated individuals in population *X* and *P* is the size of population *X*. Therefore the value *RNI* = 1 means all the individuals in the population are non-dominated, and *RNI* = 0 represents the situation where none of the individuals in the population are non-dominated. Since a population size of more than zero is often desired, there is always at least one non-dominated individual in the population within the range of $0 < RNI \leq 1$.

3.3. *Size of space covered (SSC)*

Zitzler and Thiele (1999) proposed a quantitative measure to evaluate the overall size of phenotype space covered by all the optimized solutions for maximization of objective function. This measure was later generalized for both maximization and minimization problems (Tan et al. 2001a). Let $x_i \in X$ be an $i^{-\text{th}}$ individual member in a population *X*. The function *SSC*(*X*) gives the volume enclosed by the union of area in the objective domain where any point within the area is always dominated by at least one individual x_i in the population *X*. The higher the value of *SSC*(*X*), the larger the dominated volume in the objective domain and hence the better the MO optimization performance is.

3.4. *Uniform distribution (UD) of non-dominated population*

One of the important metric in MO optimization is uniform distribution (*UD*) that measures the distribution of non-dominated individuals (Srinivas and Deb 1994; Tan et al. 2001b). The distribution should be as uniform as possible to achieve consistent gaps among neighboring individuals in the population. Mathematically, $UD(X')$ for a given set of non-dominated individuals X' in a population X, where $X' \subseteq X$, is defined as,

$$UD(X') = \frac{1}{1 + S_{nc}} \tag{7}$$

where S_{nc} is the standard deviation of niche count of the overall set of non-dominated individuals X', and is formulated as,

$$S_{nc} = \sqrt{\frac{\sum_i^{N_{x'}} \left(nc(x_i') - \overline{nc}(X')\right)^2}{N_{x'} - 1}} \tag{8}$$

where $N_{x'}$ is the size of the set X'; $nc(x'_i)$ is the niche count of i^{th} individual x'_i where $x'_i \in X'$; and $\overline{nc}(X')$ is the mean value of $nc(x'_i)$, $\forall\, i = 1, 2, \ldots, N_{x'}$, as shown in the following equations (Tan et al. 2001c),

$$nc(x_i') = \sum_{j, j \neq i}^{N_{x'}} f(i, j), \quad where\ f(i, j) = \begin{cases} 1, & dis(i, j) < \sigma_{share} \\ 0, & else \end{cases} \tag{9}$$

$$\overline{nc}(X') = \frac{\sum_i^{N_{x'}} nc\left(X_i'\right)}{N_{x'}} \tag{10}$$

where $dis(i, j)$ is the distance between individual i and j in the objective domain.

3.5. *Noise sensitivity (NS)*

In addition to the above metrics, noise sensitivity is also introduced in this paper to evaluate the robustness of evolutionary optimization in a noisy environment (Tan et al. 2001a). Generally, the error of evolutionary optimization E_{op} in the parameter domain can be defined as,

$$E_{op} = |X_{actual} - X_{best}| \tag{11}$$

where X_{actual} is the actual solution, and X_{best} is the best solution found at each generation that may be corrupted with noises. Intuitively, the optimization error E_{op} at the initial and the final stage of evolution is caused by the initial unfit solutions and the corruption of noises in the environment, respectively. The noise sensitivity function, denoted as S_n, can be defined as,

$$S_n = \frac{\left|E_{op}(t \to \infty)\right|_\infty}{|N(t \to \infty)|_\infty} \tag{12}$$

where $|\,.\,|_\infty$ represents the H_∞ norm and N denotes the noise in the environment. From equation 12, a smaller S_n means that the optimization system is less sensitive to the noise, and thus is more robust in a noisy environment.

3.6. *Average best performance (ABP)*

The measure of best performance was applied to evaluate the ability of the optimization methods in tracking the optimal regions in changing environments for time-dependent objective functions (Branke 1999). For stochastic evolutionary optimization, it is more meaningful to assess the average best performance (*ABP*), which records the average of the best solutions found at each generation as given by,

$$ABP(T) = \frac{1}{T}\sum_{t=1}^{T} g^*(t) \tag{13}$$

where $g^*(t)$ is the performance of the best individual found at time t. Since the number of values that are used for averaging the performance grows with time, the curve of *ABP* tends to get smoother along the generations.

4. Test Problems for Multi-objective Optimization

According to Branke (1999), test problems should be simple, easy to describe, easy to analyze and also tunable in their parameters. On one hand they should be complex enough to allow conjectures to the real world, on the other hand they should be simple enough to gain new insights into working of the optimization algorithm. Many attempts have been made for finding the characteristics that make a problem hard for EAs to perform effectively, such as deception (Goldberg 1987; Whitely 1991), multimodality (Horn and Golberg 1995), epistasis (Davidor 1991), and noise (Kargupta 1995). Concerning the formation of test problems for MO optimization, Deb (1999b) suggested a few features that cause difficulties for evolutionary optimization to converge to the Pareto-optimal front as well as to maintain the population diversity. These features include multi-modality, deception, isolated optimum and collateral noise, which are accounted in various test functions in this section. Table 4 summarizes the features of the test problems adopted in this paper.

4.1. *Test problem 1*

As stated by Merz and Freisleben (1998), it is important to consider high dimensional landscapes for evaluating the performance of evolutionary algorithms. However, there is a lack of test functions for both high dimensional landscapes and MO optimization in the existing literature. Therefore test problem 1 is proposed in this section by extending the single

Table 4. Features of the test problems

Test Problem	**Features**
1	High-dimensional search space with many local optima
2	Multi-modal and deceptive problem with harmful local optima
3	Noisy landscapes
4	Non-stationary search environments

objective minimization problem on two-dimensional search space proposed by Schaffer et al. (1989). In this problem, the search algorithm is evaluated on the problem characteristics of two-objective minimization with high-dimensional search space and a lot of local optima as formulated below,

$$f_1 = x_1, \tag{14a}$$

$$f_2 = \frac{1}{x_1}\left\{1 + \left(\sum_{i=2}^{11} x_i^2\right)^{0.25}\left[sin^2\left(50\left(\sum_{i=2}^{11} x_i^2\right)^{0.1}\right) + 1.0\right]\right\}, \tag{14b}$$

It involves 11-dimensional search space with ranges of [0.1, 1] for x_1 and [–100, 100] for x_i $\forall\, i = 2, 3, \ldots, 11$. The 2-dimensional cross section of $f_2(x)x_1$ through the origin is shown in Figure 3. It can be seen that there exist a lot of local optima around the global optimum. Although f_1 involves only x_1 to be minimized, f_1 and f_2 are highly correlated and trade-offs to each other, i.e., the increase of f_1 will cause the decrease of f_2 and vice versa. This is illustrated in Figure 4 where the line represents the trade-off curve and the shaded region represents the unfeasible region in the objective domain. The aim is to evolve the global Pareto-optimal solutions of $x_i = 0\ \forall\, i = 2, 3, \ldots, 11$, and x_1 should be spread uniformly along its entire range.

4.2. *Test problem 2*

This problem was originated from Deb (1999b), which is a two-objective minimization problem with existing of local optima that search algorithms are easily to get trapped at. In this paper, the original test problem is modified and expanded such that the global optimum is farther away from the local optima and a higher dimensionality of the search space is considered. The purpose is to increase the optimization difficulties where unfit search algorithms are likely to converge pre-maturely to local optima instead of the global optimum.

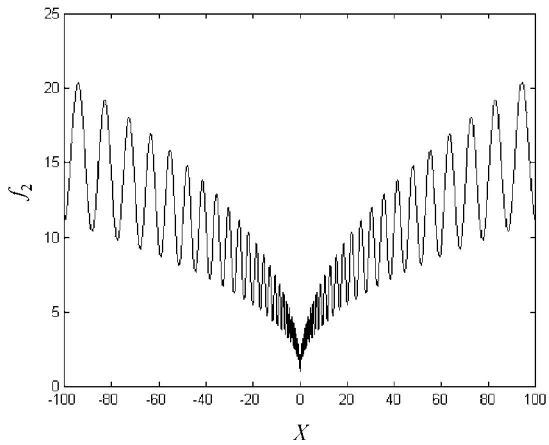


Figure 3. Central cross section of $f_{6,2}(x)x_1$.

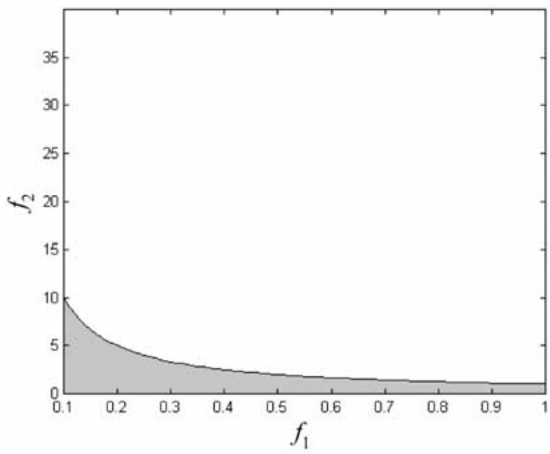


Figure 4. Pareto-optimal curve in objective domain.

The modified two-objective functions to be minimized are,

$$f_1 = x_1, \tag{15a}$$

$$f_2 = \frac{1}{x_1}\Pi_{i=1}^{3}\, g_i, \tag{15b}$$

where,

$$\begin{aligned} g_i &= 2.0 - \exp\left\{-\left(\frac{x_{i+1} - 0.1}{0.004}\right)^2\right\} - 0.8\exp\left\{-\left(\frac{x_{i+1} - 0.9}{0.4}\right)^2\right\}, \\ \forall i &= 1, 2, 3 \end{aligned} \tag{15c}$$

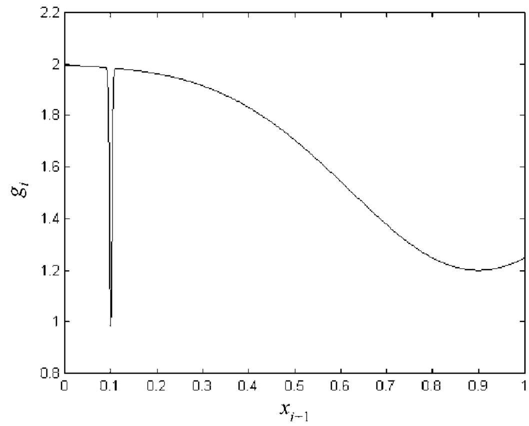


Figure 5. The function g_i has a global and local minimum.

where,

$$0.1 \leq x_1 \leq 1, \tag{15d}$$

and,

$$0 \leq x_j \leq 1, \quad \forall j = 2, 3, 4 \tag{15e}$$

Figure 5 depicts the function of g_i for $0 \leq x_{i+1} \leq 1$. It can be seen that g_i is a multi-modal function with x_{i+1} = 0.1 as the global minimum and x_{i+1} = 0.9 as the local minima, where their distance (0.8) has been increased by 0.4 or 100% from the original problem (distance = 0.4) of Deb (1999b). Figure 6 shows the plotting of f_1 against f_2, with the local and the global Pareto-optimal curve represented by dashed and solid line, respectively. The shaded region represents the unfeasible search area for this problem.

4.3. *Test problem 3*

In this test problem, the search algorithms are evaluated in the noisy environments. The aim is to test the algorithm's robustness in the sense that disappearance of certain individuals from the population has little effect on its global optimization behavior (Collard and Escazut 1995). A few studies for optimization in the nosy environments that are restricted to single objective optimization have been presented (Aizawa and Wah 1993; Angeline 1996; Fitzpatrick and Grefenstette 1988; Miller and Goldberg 1995). To investigate the relative abilities of evolutionary algorithms for MO optimization in noisy environments, a noisy version of two-objective optimization function with three variables is constructed. The optimization functions contain elements of noise as given below,

$$f_1 = x_1', \tag{16a}$$

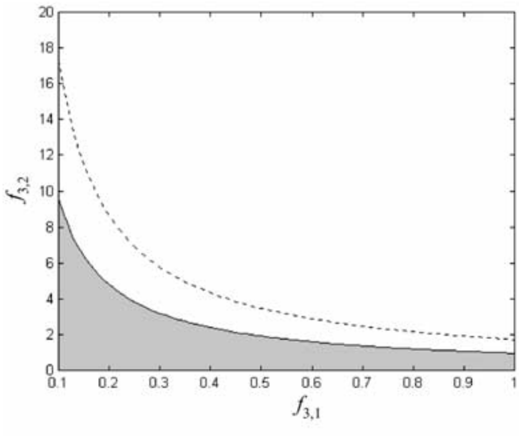


Figure 6. Global and local Pareto-optimal curve in objective domain.

$$f_2 = \frac{1}{x_1}\left\{1 + \left({x_2'}^2 + {X_3'}^2\right)^{0.25}\left[sin^2\left(50\left({x_2'}^2 + {x_3'}^2\right)^{0.1}\right) + 1.0\right]\right\}, \quad (16b)$$

Instead of performing the optimization on real-valued parameters, x_i, the optimization is performed on the "corrupted" parameters with additive noise elements,

$$x_i' = x_i + N(\sigma, \mu), \quad (16c)$$

where $0.1 \leq x_i \leq 1$ and $-100 \leq x_i \leq 100$ $\forall i = 2, 3$. $N(\sigma, \mu)$ is a white noise. $\mu = 0$ and $\sigma^2 = 0.1$ are the mean and variance of the probability distribution density, respectively. It should be noted that the noisy search environment is modeled with corrupted parameters instead of corrupted objective functions. Besides the noisy search environment, the optimization difficulty is further increased by multimodality. There are different patterns of well depths and heights of the barriers between wells as formulated in equation 16b (Schaffer et al. 1989). The 2-dimensional cross section of $f_2(x)x_1$ through the origin is shown in Figure 7. It can be seen that there exist a lot of local optima around the global optimum. The Pareto-optimal curve in the objective domain is shown by the line in Figure 8, where the shaded region represents the unfeasible search area.

4.4. *Test problem 4*

Unlike existing test problems for non-stationary environments that are limited to single objective optimization (Branke 1999; Collard and Escazut 1995;

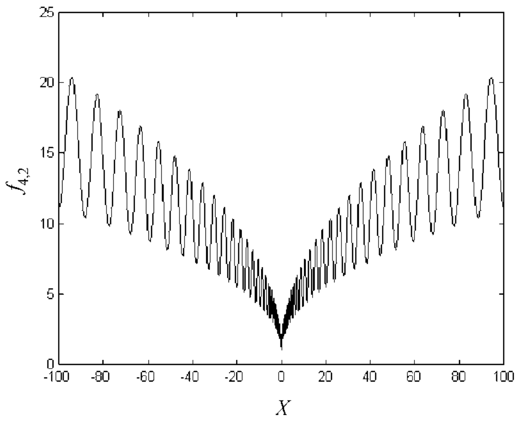


Figure 7. Central cross section of $f_{4,2}$.

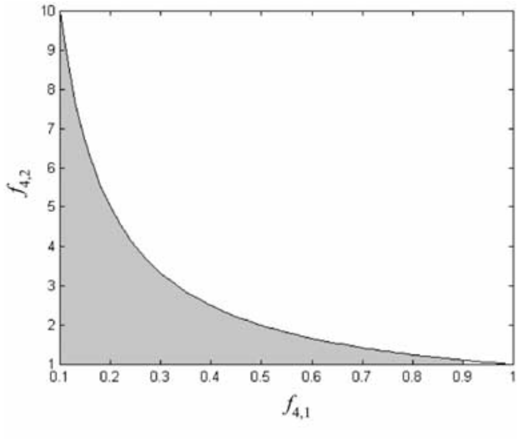


Figure 8. Pareto-optimal curve in objective domain.

Dasgupta and McGregor 1992; Gaspar and Collard 1997; Ghosh et al. 1996; Grefenstette 1992), the test problem proposed by Branke (1999) is extended in this section to evaluate the performance of evolutionary MO optimization in non-stationary environments. Branke's problem is considered since it provides an artificial multi-dimensional landscape consisting of several peaks. The height, the width and the position of each peak is altered every time a change is occurred in the environment. The extended objective functions are given as,

$$f_1 = x_1 \tag{17a}$$

$$f_2 = \frac{1}{x_1 \cdot g(t)} \tag{17b}$$

Table 5. Initial parameters for various peaks

	X_1	X_2	X_3	X_4	X_5	W	H
Peak 1	8.0	64.0	67.0	55.0	4.0	0.1	50.0
Peak 2	50.0	13.0	76.0	15.0	7.0	0.1	50.0
Peak 3	9.0	19.0	27.0	67.0	24.0	0.1	50.0
Peak 4	66.0	87.0	65.0	19.0	43.0	0.1	50.0
Peak 5	76.0	32.0	43.0	54.0	65.0	0.1	50.0

$$g(t) = \max_{i=1\ldots5} \frac{H_i(t)}{1 + W_i(t)\sum_{j=1}^{5}\left(x_{j+1} - X_j(t)\right)^2} \tag{17c}$$

where $0.1 \leq X_1 \leq 1$ and $0 \leq x_i \leq 100\ \forall i = 2, \ldots, 6$. The coordinates, the height H and the width W of each peak are initialized according to Table 5. At every Δe generation, the height and width of each peak are changed by adding a random Gaussian variable. The location of every peak is moved by a vector v of fixed length s in a random direction. Hence the parameter s allows the control of severity of the change and Δe determines the frequency of the change. A change can thus be described as,

$$\sigma \in N(0, 1) \tag{18a}$$

$$H_i(t) = H_i(t-1) + 7 \cdot \sigma \tag{18b}$$

$$W_i(t) = W_i(t-1) + 0.01 \cdot \sigma \tag{18c}$$

$$\vec{X}(t) = \vec{X}(t-1) + \vec{v} \tag{18d}$$

The function $g(t)$ is multimodal and changes slightly with respect to the time t. The values of $s = 1$ and $\Delta e = 50$ are used. At each generation, the highest peak is scaled to 50 and the rest of the peaks are multiplied by the same scale similar to the highest peak.

5. Performance Assessments and Comparisons

In this section, the six quantitative measures described in Section 3 are applied to examine the performance of various well-known evolutionary MO optimization techniques based upon the four benchmark problems shown in Section 4. The multi-objective evolutionary algorithms included in the study are: (1) VEGA from (Schaffer 1985); (2) MIMOGA from (Murata and Ishibuchi 1995); (3) HLGA from (Hajela and Lin 1992); (4) NPGA from (Horn et al. 1994); (5) MOGA from (Fonseca and Fleming 1995b); (6) NSGA from (Srinivas and Deb 1994); (7) SPEA from (Zitzler and Thiele 1999); (8) MOEA from (Tan et al. 1999); (9) IMOEA from (Khor et al. 2000); and (10) EMOEA from (Khor et al. 2001).

All methods under comparison were implemented with the same common sub-functions using the same programming language in Matlab (The MathWorks 1998) on an Intel Pentium II 450 MHz computer. Each of the simulation was terminated automatically when a fixed pre-specified simulation period for each test problem is reached, in the same platform that is free from other computation or being interrupted by other programs. The Pareto tournament selection scheme with t_{dom} = 10% of the population size is used in NPGA for a tight population distribution as recommended by (Horn et al. 1994). For consistency in the comparisons, all algorithms apply the same decimal coding scheme (Tan et al. 2001d), two-point crossover (probability = 0.7) and standard mutation (probability = 0.01). Phenotype sharing is applied to those algorithms that require fitness sharing operation. The sharing distance for HLGA, NPGA, MOGA and NSGA as well as the performance measure of *UD* for all methods is set to 0.01 in the normalized space. Since dynamic sharing (Tan et al. 1999) is used for both MOEA and IMOEA, the sharing distance is computed dynamically at each generation. No sharing distance parameter is needed for SPEA and EMOEA as proposed by (Zitzler and Thiele 1999) and (Khor et al. 2001). Except SPEA and EMOEA which include a secondary population, a population size of 100 is used in test problem 4 and a population size of 30 is adopted in test problems 1, 2 and 3. For SPEA and EMOEA, four combinations of {primary population size, secondary population size} are applied, e.g., {30, 10} on problems 1, 2, 3 and {90, 10} on problem 4. 30 independent simulation runs with randomly generated initial populations have been performed for each method on each test problem to minimize bias in the simulations.

The performance of each algorithm on the first three test problems, with respect to the algorithm effort (*AE*), ratio of non-dominated individuals (*RNI*), size of space covered (*SSC*) and uniform distribution (*UD*) is illustrated in Figure 9, where the distribution of the simulation data of 30 independent runs is compiled into box plot format (Chambers et al. 1983). The box plot format has been applied by Zitzler and Thiele (1999) to provide a clear visualization of the median, upper quartile, lower quartile and outside value for each distribution of the simulation data. In each graph, the sequence of box plots from left to right is based upon the list of algorithms indexed in the first paragraph of this section. It can be seen from Figure 9 that VEGA and

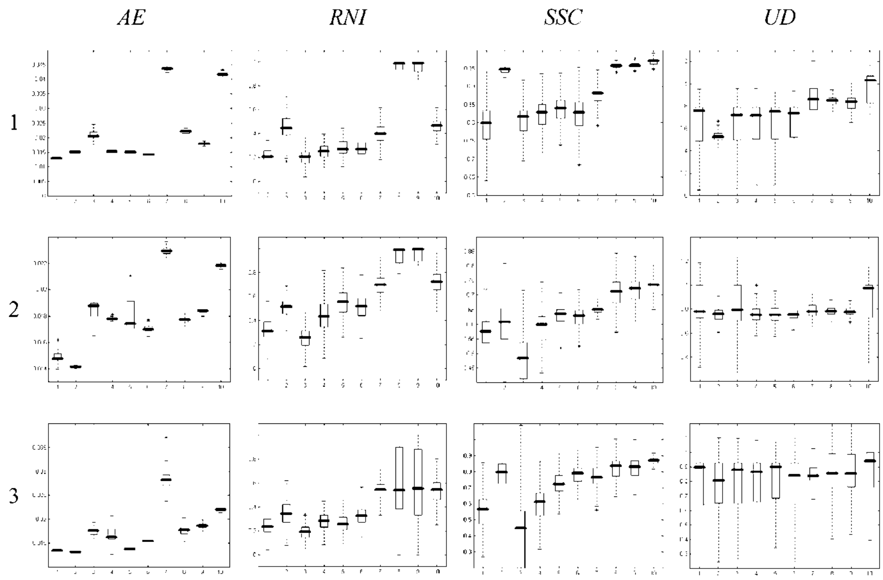


Figure 9. Box plots for the measure of algorithm effort (*AE*), ratio of non-dominated individuals (*RNI*), size of space covered (*SSC*) and uniform distribution (*UD*) of non-dominated population.

MIMOGA have a relatively low algorithm effort (*AE*) in all test problems. This indicates that these two algorithms are less computational expensive, which perform more function evaluations within a fixed period of CPU time as compared to others. As illustrated in Figure 9, MOEA and IMOEA have the highest ratio of non-dominated individuals (*RNI*) for all test problems, which shows their excellent abilities of preserving important non-dominated individuals at each generation via the method of switching preserved strategy (SPS) (Tan et al. 1999). It can also be observed that the performance of *RNI* for SPEA and EMOEA are varied according to the ratio of secondary (best-found) and primary population size. For the rest of algorithms without elitism strategy, there are no clear differences in the measure of *RNI*, although their values of *RNI* are generally low. As shown in Figure 9, the performances of MOEA, IMOEA and EMOEA are generally good in all test problems for the measure of size of space covered (*SSC*) in the objective domain. The results in Figure 9 also unveil that the performance of uniform distribution (*UD*) for all algorithms are about the same, i.e., none of the algorithms can be significantly distinguished from others in the measure of *UD*.

Figure 10 depicts the performance of noise sensitivity S_n on test problem 3. For the second and third parameters, S_n is computed with respect to each of the parameter by considering the best match solution to the actual solution

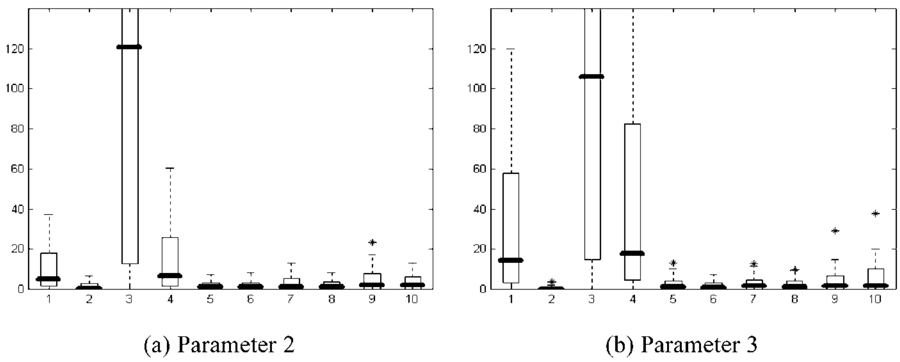


Figure 10. Noise sensitivity function versus optimization algorithms on test problem 3.

at the final generation in each simulation, assuming that the steady-state of evolution has been achieved. As can be seen, all methods except VEGA, HLGA and NPGA, provide a relatively low noise sensitivity function for both the parameters. This shows that many evolutionary MO optimization methods are robust and work well in the noisy environments.

Figure 11 illustrates the trace of average best performance (*ABP*) along the generation for all algorithms on test problem 4, where the non-stationary fitness environment as described in Section 3 is concerned. As expected, the graph for each algorithm tends to get smoother along the generation, as the number of values used in the computation increases with the generation. As shown in Figure 11, the values of *ABP* for VEGA and HLGA are relatively low along the generation, which indicates that they are less capable of tracking the optimal regions continuously in the changing environment. It can also be observed that the performances of *ABP* for MIMOGA, NPGA, MOGA and NSGA are moderate. As shown in Figure 11, the performances of *ABP* for SPEA, MOEA, IMOEA and EMOEA are generally good, which shows their abilities of tracking from one optimal region to another in a non-stationary fitness environment.

From these experiments, it can be concluded that although VEGA and MIMOGA require less computational effort, their performances in discovering the entire trade-off surface are less superior as compared to others. This is evidenced from their poor performances in the measures of *SSC* and *UD* in Figure 9, which are mainly due to the absence of elitism strategy and the lack of explicit operators to diversify the population in these algorithms. The results also show that feature elements such as elitism and sharing strategy as implemented in SPEA, MOEA, IMOEA and EMOEA are important for good convergence and population distribution along the discovered trade-offs in MO optimization. Generally, any feature elements

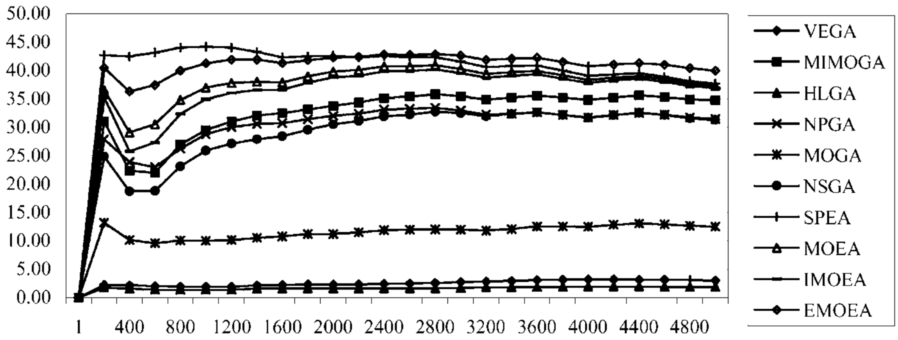


Figure 11. Average best performance *ABP*(*T*) versus generation *T*.

resulting in high algorithm effort are only suitable for problems with time-consuming function evaluations, e.g., their effects in algorithm effort become less significance in these problems. In the presence of noise, the algorithms of VEGA, HLGA and NPGA performed less superior as compared to others. For VEGA and HLGA, the poor performance in noisy environment could be due to the proportional selection used in these algorithms, which is often more sensitive with respect to fitness evaluation as compared to tournament or ranking schemes. For NPGA, it could be due to the size of comparison set used in the algorithm, as smaller comparison set often leads to higher noise sensitivity.

6. Conclusions

In this paper, existing multi-objective evolutionary algorithms have been surveyed and their performances have been assessed and compared based on four benchmark problems. Besides considering the usual performance measures in MO optimization, e.g., the spread across the Pareto-optimal front and the ability to attain the global trade-offs, a few performance metrics for MO optimization have been presented to examine the fraction of useful individuals in a population, the uniform distribution of individuals along the Pareto-front, the computational effort of an optimization algorithm, the robustness to disturbances as well as the average best performance of tracking optimal regions in changing environments. It can be concluded from the overall simulation results that there is no single algorithm excels in all performance measures. Feature elements such as elitism and sharing strategy as implemented in SPEA, MOEA, IMOEA and EMOEA are important for good convergence and population distribution along the discovered trade-offs in MO optimization. Generally, any feature elements resulting in high algorithm effort are only suitable for problems with time-consuming function evaluations, e.g., their effects in algorithm effort become less significance in these problems. It is believed that the findings in this paper could be useful for combining different feature elements from various methods into a new multi-objective evolutionary algorithm that performs well for the type of optimization problem in-hand.

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