Q7:

$f_s: \{0,1\}^3 \longrightarrow \{0,1\}^3$

$f_s(000) = f_s(111) = 000$
$f_s(001) = f_s(110) = 001$
$f_s(010) = f_s(101) = 010$
$f_s(011) = f_s(100) = 011$

1. $f(x) = f(y)$ if $x \oplus y \in \{0^n, s\}$

$x \oplus y = s$ for $x \neq y$ and $s \neq 0^n$ implies $y = s \oplus x$ and

$$\underline{f(x) = f(y) = f(x \oplus s)}$$

$000 \oplus 111 = \underline{111 = s}$ f_{111} so $f_{111}(000) = f_{111}(111) = f_{111}(000 \oplus 111)$
$\Leftrightarrow 001 \oplus 110 = 010 \oplus 101 = 011 \oplus 100$

satisfie simon's problem:
→ $f_{111}(001) = f_{111}(110) = f_{111}(001 \oplus 111) = f_{111}(110)$
→ $f_{111}(010) = f_{111}(101) = f_{111}(010 \oplus 111) = f_{111}(101)$
→ $f_{111}(011) = f_{111}(100) = f_{111}(011 \oplus 111) = f_{111}(100)$

2. Starting State: $|000\rangle|1\rangle$

3. Hadamard $H^{\otimes 4}$: $\frac{1}{4}(|0000\rangle + |0001\rangle + |0010\rangle + |0011\rangle + |1100\rangle + |1101\rangle + |1110\rangle + |1111\rangle)(|0\rangle - |1\rangle)$

4. Oracle: phase shift $(-1)^{f(x)}$: $(-1)^{f(000)=f(111)} = 1$, $(-1)^{f(001)=f(110)} = -1$,
$(-1)^{f(010)=f(101)} = -1$, $(-1)^{f(011)f(100)} = 1$

$$\frac{1}{4}(|0000\rangle - |0001\rangle - |0010\rangle + |0011\rangle + |1100\rangle - |1101\rangle - |1110\rangle + |1111\rangle)(|0\rangle - |1\rangle)$$

5. 011

6. Hadamard: $|011\rangle$

7. the measurement outcome does not conclude to s, becaus with j we get

$j \oplus f_s(x) = 000$, which is not unique.

8. $s = j_1 \oplus j_2 = 011 \oplus 101 = 110$

9. If $s = 000$ is allowed the function f is one-to-one, so you can not use Simon's Algorithem.

The algorithem depends on $s \neq 0 \ldots 0$ because if not given for $f(x) = f(y)$ y is not necessarily unique. Every input with $x \oplus s$ for $s = 0 \ldots 0$ will result in itself and the definition $f(x) = f(y)$ with unique y is not given. Without two-to-one and uniqueness of y the patterns might not exist and no information over s can be gained from the entanglement and following superposition.