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① overthinking $\to O$ procastination $\to pro$ perfectionism $\to per$ negativity $\to N$

الف) $O \perp\!\!\!\perp pro \mid per, N$ $per \perp\!\!\!\perp N \mid O$

ب) $$p(N, per=true) = \sum_O \sum_{pro} p(per=true)\,p(O|per)\,p(N|O)\,p(pro|N,per)$$

$$= p(per=true)\left(\sum_O p(O|per)p(N|O)\right)\left(\sum_{pro} p(pro|N,per)\right)$$

$$f_1 = \sum_{pro} p(pro|N,per) = 1 \qquad f_2 = \sum_O p(O|per)p(N|O)$$

$$f_3 = p(per=true) f_1 f_2 \qquad p(N|per=true) = \alpha f_3$$

② الف) $$p(+c|+a,+b,+d) = \frac{p(+a,+b,+c,+d)}{p(+a,+b,+d)}$$

$$p(+a,+b,+c,+d) = p(+a)\,p(+b|+a)\,p(+c|+b)\,p(+d|+a,+c) = 0.5\times0.8\times0.1\times0.6 = 0.024$$

$$p(+a,+b,+d) = \sum_c p(+a,+b,c,+d) = 0.024 + \underbrace{0.5\times0.8\times0.9\times0.1}_{p(+a,+b,-c,+d)} = 0.06$$

$$p(+c|+a,+b,+d) = \frac{0.024}{0.06} = 0.4$$

ب) $-a,-b$, $+a,-b$

ج)

$-a$	$+b$	$-c$	$+d$	w_1
$+a$	$+b$	$-c$	$+d$	w_2
$+a$	$+b$	$-c$	$+d$	w_3
$-a$	$+b$	$+c$	$+d$	w_4
$+a$	$+b$	$+c$	$+d$	w_5

$w_1 = 1 \times p(+b|-a) \times p(+d|-a,-c) = 0.4 \times 0.5 = 0.2$

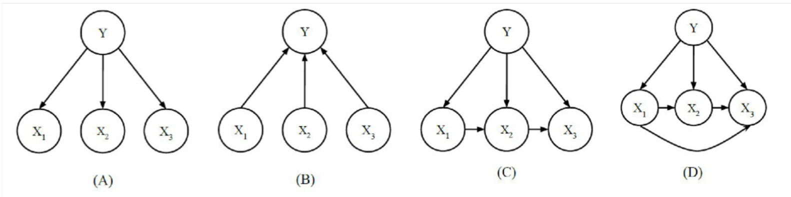
$w_2 = w_3 = 1 \times p(+b|+a)\, p(+d|+a,-c) = 0.8 \times 0.1 = 0.08$

$w_4 = 1 \times p(+b|-a)\, p(+d|-a,+c) = 0.4 \times 0.2 = 0.08$

$w_5 = 1 \times p(+b|+a)\, p(+d|+a,+c) = 0.8 \times 0.6 = 0.48$

$$p(+a|+b,+d) = \frac{w_2+w_3+w_5}{w_1+w_2+w_3+w_4+w_5} = \frac{0.64}{0.72} \simeq 0.7$$

3) با فرض اینکه هر کدام از متغیرهای تصادفی n مقدار متفاوت می‌توانند بپذیرند.



جدول هایی که با توجه به استقلال متغیرها باید تشکیل دهیم

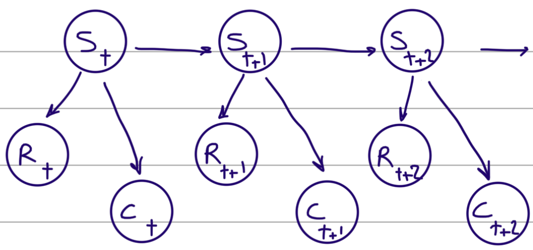
$A \longrightarrow P(Y),\ P(X_1|Y),\ P(X_2|Y),\ P(X_3|Y) \longrightarrow n+3n^2$

$B \longrightarrow P(X_1),\ P(X_2),\ P(X_3),\ P(Y|X_1,X_2,X_3) \longrightarrow 3n+n^4$

$C \longrightarrow P(Y),\ P(X_1|Y),\ P(X_2|X_1,Y),\ P(X_3|X_2,Y) \longrightarrow n+n^2+2n^3$

$D \longrightarrow P(Y),\ P(X_1|Y),\ P(X_2|X_1,Y),\ P(X_3|X_2,X_1,Y) \longrightarrow n+n^2+n^3+n^4$

4) الف)



S_0	$P(S_0)$
$+S$	0.6
$-S$	0.4

R_t	S_t	$P(R_t \mid S_t)$
$+r$	$+S$	0.1
$-r$	$+S$	0.9
$+r$	$-S$	0.7
$-r$	$-S$	0.3

S_{t+1}	S_t	$P(S_{t+1} \mid S_t)$
$+S_{t+1}$	$+S_t$	0.9
$-S_{t+1}$	$+S_t$	0.1
$+S_{t+1}$	$-S_t$	0.2
$-S_{t+1}$	$-S_t$	0.8

C_t	S_t	$P(C_t \mid S_t)$
$+C$	$+S$	0.2
$-C$	$+S$	0.8
$+C$	$-S$	0.4
$-C$	$-S$	0.6

• $p(S_1 | -r_1, -c_1)$ (ب

$p(+S_1) = \sum_{S_0} p(+s_1 | s_0) p(S_0) = 0.6 \times 0.9 + 0.4 \times 0.2 = 0.62$ $\quad p(-S_1) = 0.38$

$p(+S_1, -r_1, -c_1) = p(S_1) p(-r_1 | S_1) p(-c_1 | S_1) = 0.62 \times 0.9 \times 0.8 = 0.4464$

$p(-S_1, -r_1, -c_1) = p(-S_1) p(-r_1 | -S_1) p(-c_1 | -S_1) = 0.38 \times 0.3 \times 0.6 = 0.0684$

$(0.0684 + 0.4464)\alpha = 1$ $\quad \alpha = \frac{1}{0.5148}$

$p(+S_1 | -r_1, -c_1) = \alpha p(+S_1, -r_1, -c_1) = 0.867$ $\quad p(-S_1 | -r_1, -S_1) = 0.133$

• $p(S_2 | c_{1:2}, r_{1:2})$

$p(+S_2 | -r_1, -c_1) = \sum_{S_1} p(S_1 | -r_1, -c_1) \times p(+S_2 | S_1) = 0.9 \times 0.867 + 0.2 \times 0.133 = 0.807$

$p(-S_2 | -r_1, -c_1) = 0.193$

$p(+S_2, +r_2, -c_2 | -r_1, -c_1) = p(+S_2 | -r_1, -c_1) p(+r_2, -c_2 | +S_2) = 0.807 \times 0.1 \times 0.8 = 0.06456$

$p(-S_2, +r_2, -c_2 | -r_1, -c_1) = p(-S_2 | -r_1, -c_1) p(+r_2, -c_2 | -S_2) = 0.193 \times 0.6 \times 0.7 = 0.08156$

$\alpha(0.06456 + 0.08156) = 1$ $\quad \alpha = \frac{1}{0.14612}$

$p(+S_2 | c_{1:2}, r_{1:2}) = \alpha p(+S_2, +r_2, -c_2 | -r_1, -c_1) = 0.442$

$p(-S_2 | c_{1:2}, r_{1:2}) = \alpha p(-S_2, +r_2, -c_2 | -r_1, -c_1) = 0.558$

• $p(S_3 | c_{1:3}, r_{1:3})$

$p(+S_3 | c_{1:2}, r_{1:2}) = \sum_{S_2} p(S_2 | c_{1:2}, r_{1:2}) p(+S_3 | S_2) = 0.442 \times 0.9 + 0.558 \times 0.2 = 0.5094$

$p(-S_3 | c_{1:2}, r_{1:2}) = 1 - 0.5094 = 0.4906$

$p(+S_3 | c_{1:3}, r_{1:3}) = \alpha p(+S_3 | c_{1:2}, r_{1:2}) p(+r_3, +c_3 | +S_3) = \alpha \times 0.5094 \times 0.1 \times 0.2 = 0.0102\alpha$

$p(-s_3|c_{1:3}, r_{1:3}) = \alpha p(-s_3|c_{1:2}, r_{1:2})\, p(+r_3,+c_3|-s_3) = \alpha \times 0.4906 \times 0.7 \times 0.4 = 0.1374\alpha$

$0.0102\alpha + 0.1374\alpha = 1$ $\qquad \alpha = \dfrac{1}{0.0102+0.1374}$ $\qquad p(+s_3|c_{1:3}, r_{1:3}) = 0.069$

$p(-s_3|c_{1:3}, r_{1:3}) = 0.931$

- $p(S_2|c_{1:3}, r_{1:3}) = \alpha p(S_2|c_{1:2}, r_{1:2})\, p(+c_3, +r_3|S_2)$

$p(+r_3, +c_3|+s_2) = \sum_{S_3} p(+r_3,+c_3|S_3)\, p(S_3|+s_2) = 0.02\times 0.9 + 0.28\times 0.1 = 0.046$

$p(+r_3, +c_3|-s_2) = \sum_{S_3} p(+r_3,+c_3|S_3)\, p(S_3|-s_2) = 0.02\times 0.2 + 0.28\times 0.8 = 0.228$

$p(+s_2|c_{1:3}, r_{1:3}) = \alpha \times 0.442 \times 0.046 = 0.0203\alpha$ $\qquad p(-s_2|c_{1:3}, r_{1:3}) = \alpha \times 0.558 \times 0.228 = 0.127\alpha$

$p(+s_2|c_{1:3}, r_{1:3}) = 0.138$ $\qquad p(-s_2|c_{1:3}, r_{1:3}) = 1 - 0.138 = 0.862$

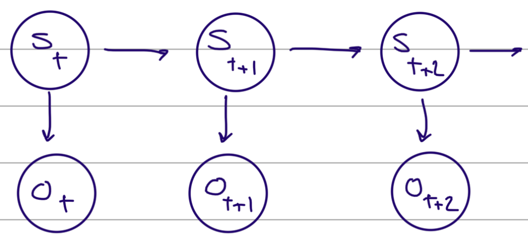
ج) $p(S_k|c_{1:n}, r_{1:n}) = p(S_k|c_{1:k}, r_{1:k}, c_{k+1:n}, r_{k+1:n}) = \alpha p(S_k|c_{1:k}, r_{1:k})\, p(c_{k+1:n}, r_{k+1:n}|S_k)$

$p(c_{k+1:n}, r_{k+1:n}|S_k) = \sum_{S_{k+1}} p(c_{k+1:n}, r_{k+1:n}|S_{k+1})\, p(S_{k+1}|S_k)$

$= \sum_{S_{k+1}} p(c_{k+1:n}, r_{k+1:n}|S_{k+1})\, \underbrace{p(c_{k+2:n}, r_{k+2:n}|S_{k+1})}_{\text{به صورت بازگشتی محاسبه می‌شود}}\, p(S_{k+1}|S_k)$

$p(S_k|c_{1:n}, r_{1:n}) = \alpha p(S_k|c_{1:k}, r_{1:k}) \sum_{S_{k+1}} p(c_{k+1:n}, r_{k+1:n}|S_{k+1})\, p(c_{k+2:n}, r_{k+2:n}|S_{k+1})\, p(S_{k+1}|S_k)$

د)



S_0	$p(S_0)$
$+S$	0.6
$-S$	0.4

S_{t+1}	S_t	$p(S_{t+1}\|S_t)$
$+S_{t+1}$	$+S_t$	0.9
$-S_{t+1}$	$+S_t$	0.1
$+S_{t+1}$	$-S_t$	0.2
$-S_{t+1}$	$-S_t$	0.8

O_t	S_t	$p(O_t \mid S_t)$
$+r, +c$	$+s$	0.02
$+r, +c$	$-s$	0.28
$+r, -c$	$+s$	0.08
$+r, -c$	$-s$	0.42
$-r, +c$	$+s$	0.18
$-r, +c$	$-s$	0.12
$-r, -c$	$+s$	0.72
$-r, -c$	$-s$	0.18

۵) نادرست - برابر می‌شوند.

۵) الف)

$$X_t = k \longrightarrow p(A=on \mid X_t=k)\, p(B=off \mid X_t=k)\, p(C=on \mid X_t=k)\, p(D=off \mid X_t=k)$$

$X_t = 2 \longrightarrow 1 \times 0.6 \times 0.4 \times 0.6 = 0.144$ $\qquad$ $X_t = 12 \longrightarrow 0.4 \times 0.6 \times 0.4 \times 0 = 0$

$X_t = 13 \longrightarrow 0.4 \times 0.6 \times 1 \times 0.6 = 0.144$

ب)

$p(C=on \mid X_t=k) = 1$

$X_t = 2 \longrightarrow 1 \times 0.6 \times 1 \times 0.6 = 0.36$ $\qquad$ $X_t = 12 \longrightarrow 0.4 \times 0.6 \times 1 \times 0 = 0$

$X_t = 13 \longrightarrow 0.4 \times 0.6 \times 1 \times 0.6 = 0.144$

ج)

$$p(X_t = 11) = \frac{0.16}{0.16 + 0.1 + 0.24} = 0.32 \qquad E[X_t = 11] = n\,p(X_t = 11) = 100 \times 0.32 = 32$$