Importance Sampling

$$\underline{\mathbb{E}_{x\sim p}(f(X))} = \int_{-\infty}^{+\infty} f(x)\, P_X(x)\, dx$$

$\underline{f(x)\uparrow}$ $\quad$ $\underline{P_X(x)\downarrow}$

$$X_1 \dots X_n \overset{iid}{\sim} P_X(x)$$

$$\mathbb{E}(f(X)) \simeq \frac{1}{n}\sum_{i=1}^{n} f(X_i)$$

$$\frac{f(X)\,P(X)}{q(X)}$$

$$X_1 \dots X_n \overset{iid}{\sim} q_X(x) \qquad \cdot P(x)$$

$$\mathbb{E}_{X\sim q}(f(X)) = \int_{-\infty}^{+\infty} f(x)\, \frac{\cancel{q(x)}\,dx}{\cancel{q(x)}} = \int_{-\infty}^{+\infty} f(x)\cdot P(x)\, dx$$

$$\mathbb{E}_{x\sim p}\left(f(x)\right) \simeq \underbrace{\frac{1}{n}\sum_{i=1}^{n} \frac{f(x_i)\,p(x_i)}{q(x_i)}}_{W}$$

$$x_1 \dots x_n \overset{iid}{\sim} q$$

$$\mathrm{Var}[W] = \frac{1}{n^2}\sum_{i=1}^{n} \mathrm{Var}\left[\frac{f(x_i)\,p(x_i)}{q(x_i)}\right]$$

$$= \frac{n \cdot}{n^2}\,\mathrm{Var}\left[\frac{f(x)\,p(x)}{q(x)}\right]$$

$$= 1/n\;\mathrm{Var}\left[\frac{f(x)\,p(x)}{q(x)}\right]$$

$$\mathrm{Var}[\underbrace{W}] = \mathbb{E}(W^2) - \mu^2$$

$$= \int \frac{f^2 p^2}{q^2} \cdot q \, dx - \mu^2$$

$$= \int \frac{f^2 p^2}{q} \, dx - \mu^2$$

$$\mu = \mathbb{E}_q(W)$$

$$= \int \frac{f \cdot p}{\cancel{q}} \cdot \cancel{q} \, dx = \mathbb{E}_p(f)$$

$$q = \frac{f \cdot p}{\mu}$$

Gibbs Samp.

$$q \propto f \cdot p$$

$$q \propto |f| \cdot p$$

$$= \mu \underbrace{\int f \cdot p \, dx}_{\mu} - \mu^2 = 0$$

$$I(X;Y) = H(X|Z) - H(X|Y,Z)$$

$$\begin{cases} H(X) = -\sum_{x \in D} p(x) \log p(x) \\ H(X|Y) = -\sum_{y \in D_y} \left[\sum_{x \in D_x} \overbrace{p(x|y)}^{p(x,y)} \log p(x|y) \right] \cdot p(y) \end{cases}$$

$$\rightarrow I(X;Y|Z) = \mathbb{E}_Z\, D\left(P_{X,Y|Z}(x,y|z) \,\|\, P_{X|Z}(x|z) \cdot P_{Y|Z}(y|z) \right)$$

$$I(X;Y|Z)$$

$$\{ \dots\, p(x|y=1),\ p(x|y=2) \dots \}$$

$$p(y=1)$$