Steenrod Operations and the Steenrod Algebra

by

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ABSTRACT OF THE THESIS

This document is a summary of some relevant commands needed to create a Master's thesis for the Department of Mathematics and Statistics using LaTeX. Included are examples of equations, figures, tables, and theorems. The formats listed in this document have been approved by the Department of Mathematical Sciences and the Graduate Division and Research. If you have any difficulties with any of the driver or style files, please see your graduate adviser.

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ACKNOWLEDGMENTS

TO DO

CHAPTER 1
INTRODUCTION

The first three sections offer an overview of homology, cohomology, and homotopy theories, and may be skipped by a reader familiar with these topics. Section introduces whatever that is a bit more...Then we go into category theory...

1.1 Homology

1.2 Cohomology

1.2.1 Introduction

1.2.2 Cup product and cohomology ring

1.2.3 Hurewicz theorem

1.2.4 Bockstein Homomorphism

mathematics[1].

1.3 Homotopy

Bibliographical citations are relatively easy. Here is one [1] and another citation [12] and we cannot forget Milnor [14].

You type new paragraphs by just leaving an empty line between them.

1.4 Interplay between homotopy and cohomology: Eilenberg MacLane spaces and the Brown representability theorem

1.5 Category Theory and the Yoneda Lemma

Cohomology operations are operations between cohomology groups that commute with homomorphisms induced by continuous maps. Thus, they provide us with another means of distinguishing spaces.

[1] `http://en.wikibooks.org/wiki/LaTeX` provides a wealth of information regarding LaTeX, check it out.

Definition 1.1 (*Cohomology Operation*)**.** *A cohomology operation is a map* $\Theta = \Theta_X : H^m(X;G) \to H^n(X;H)$ *between cohomology groups, for any space* X*, and fixed* m*,* n*,* G*,* H*, satisfying the naturality property that for any map* $f : X \to Y$ *between spaces, the following diagram commutes:*

$$\begin{array}{ccc} H^m(Y;G) & \xrightarrow{\theta_Y} & H^n(Y;H) \\ \downarrow{\scriptstyle f^*} & & \downarrow{\scriptstyle f^*} \\ H^m(X;G) & \xrightarrow[\Theta_X]{} & H^n(X;H) \end{array}$$

Theorem 1.2. *[?, Proposition 4L.1] Fix* m*,* $n \in \mathrm{Z}$*,* G*,* H *groups. For a space* Z*, there is a bijection between the set of cohomology operations* $\Theta : H^m(Z;G) \to H^n(Z;H)$ *and* $H^n(K(G,m);H)$ *given by* $\Theta \mapsto \Theta(\iota)$*, where* ι *is a fundamental class in* $H^m(K(G,m);G)$*.*

We will state and prove a more general result in category theory, the *Yoneda Lemma*, and apply this result to prove Theorem 1.2. We first introduce the more general concept of a natural transformation between contravariant functors.

Definition 1.3 (*Natural Transformation*)**.** *If* F *and* G *are contravariant functors between categories* C *and* D*, a natural transformation* $\eta : F \to G$ *is a transformation such that for all maps* $g : X \to Y$ *in* C*, the following diagram commutes:*

$$\begin{array}{ccc} X & & FY & \xrightarrow{\eta_Y} & GY \\ \downarrow{\scriptstyle g} & & \downarrow{\scriptstyle Fg} & & \downarrow{\scriptstyle Gg} \\ Y & & FX & \xrightarrow{\eta_X} & GX \end{array}$$

Proposition 1.4. *Let* C *be the category of CW complexes and morphisms homotopy classes of continuous maps. Then cohomology operations are natural transformations from* C *to* C*.*

Proof. Immediate from definitions. □

We give a contravariant argument of the Yoneda lemma, so as to apply it to cohomology operations directly. The covariant argument is analogous.

Theorem 1.5 (*Yoneda Lemma*)**.** *Let* C *be a category, and* X *an object of* C*. Let* $h^X : C^{\mathrm{op}} \to \mathbf{Set}$ *be the contravariant functor* $h^X = \mathrm{Hom}(-, X)$*. Then for any contravariant set-valued functor* $F : C^{\mathrm{op}} \to \mathbf{Set}$*, we have a bijection between the natural transformations from* h^X *to* F *and* $FX \in \mathbf{Set}$*, that is,*

$$FX \simeq \mathrm{Nat}(h^X, F)$$

Proof. Consider a natural transformation $\eta : h^X \to F$. Then for any object Y in C and a map $g : Y \to X$, the following square commutes, where $(h^X g)(\beta) = \beta \circ g$ for $\beta \in h^X X$, and η_X, η_Y are the components of η at X, Y respectively.

$$\begin{array}{ccc} Y & & h^X X \xrightarrow{\eta_X} FX \\ \downarrow{\scriptstyle g} & & \downarrow{\scriptstyle h^X g} \quad\quad \downarrow{\scriptstyle Fg} \\ X & & h^X Y \xrightarrow{\eta_Y} FY \end{array}$$

Let $1_X \in h^X X$ be the identity map. Then $\eta_X(1_X) \in FX$ and

$$\eta_Y(h^X g)(1_X) = \eta_Y(g) = Fg(\eta_X(1_X)).$$

Thus for every object Y, η_Y is determined by $\eta_X(1_X)$. We define $\tau : \mathrm{Nat}(h^X, F) \to FX$ by $\tau(\eta) = \eta_X(1_X)$.

Conversely, any $g : Y \to X$, gives rise to $Fg : FX \to FY$. Let $x \in FX$. We wish to define a natural transformation $\lambda(x) : h^X \to F$. We define components map $(\lambda(x))_Y : h^X Y \to FY$ given by

$$(\lambda(x))_Y(g) = Fg(x).$$

Then $\lambda(x)$ is a natural transformation, that is given any $f : Z \to Y$, we claim we have the following commuting:

$$\begin{array}{ccc} Z & & h^X Y \xrightarrow{(\lambda(x))_Y} FY \\ \downarrow{\scriptstyle f} & & \downarrow{\scriptstyle h^X f} \quad\quad \downarrow{\scriptstyle Ff} \\ Y & & h^X Z \xrightarrow{(\lambda(x))_Z} FZ \end{array}$$

Indeed, $g : Y \to X$, $g \in h^X Y$ has $Ff(Fg(x)) = F(g \circ f)(x) = F(h^X f)(g)(x)$. Therefore we can define $\lambda : FX \to \mathrm{Nat}(h^X, F)$, $x \mapsto \lambda(x)$.

Finally, we need to show that τ and λ are inverses. For $x \in FX$, we have $\tau(\lambda(x)) = (\lambda(x))_X(1_X) = F(1_X)(x) = 1_{FX}(x)$, so that $\tau \circ \lambda = 1_{FX}$. For $\eta \in \mathrm{Nat}(h^X, F)$, we have $\lambda(\tau(\eta)) = \lambda(\eta_X(1_X))$. Then for any object Y and any $g : Y \to X$, $\lambda(\eta_X(1_X))_Y(g) = Fg(\eta_X(1_X)) = \eta_Y(g)$ by above. Thus $\lambda(\eta_X(1_X)) = \eta$, so that $\lambda \circ \tau = 1_{\mathrm{Nat}(h^X, F)}$.

□

Corollary 1.6. *Let C be a category, X, Y objects in C. Then,*

$$\mathrm{Hom}(X, Y) \simeq \mathrm{Nat}\left(\mathrm{Hom}(-, X), \mathrm{Hom}(-, Y)\right)$$

Proof. Let $F = h^Y$, and apply Yoneda lemma. □

We are now in a position to prove Theorem 1.2.

Proof of Theorem 1.2. By CW-approximation, it suffices to prove the statememt for the case of Z a CW-complex. Then we can identify $H^m(Z;G)$ with $[Z, K(G,m)]$ and likewise $H^n(Z;H)$ with $[Z, K(H;n)]$.

By Corollary 1.6,

$$\mathrm{Hom}(K(G,m), K(H,n)) \simeq \mathrm{Nat}\left(\mathrm{Hom}(Z, K(G,m)), \mathrm{Hom}(Z, K(H,n)\right) \simeq \mathrm{Nat}(H^m(Z;G), H^n(Z;H)),$$

but the natural transformations between the cohomology groups are cohomology operations by Proposition 1.4, and $\mathrm{Hom}(K(G,m), K(H,n))$ is $H^n(K(G,m);H)$.

Let $K = K(G,m))$. The map τ from the proof of the Yoneda lemma sends a cohomology operation Θ to $\Theta_K(1_K)$, where 1_K is the identity map on K. Then $\Theta_K(1_K) = \Theta(\iota)$ for $\iota \in H^m(K,G) = H^m(K(G,m);G)$ with ι a fundamental class since $1_K = 1^*\iota = \iota$. □

CHAPTER 2
MISCELLANEOUS COMMANDS: AN INTRODUCTION TO EQUATIONS, THEOREMS, FIGURES AND TABLES

In this chapter we see how equations, theorems, figures and tables are created, enumerated and referenced. We also play around with lengths of chapter and section headings. For example, this chapter begins with a long chapter heading that must conform to the thesis manual. Later on there is a very long section heading. These examples show how the SDSU thesis class file automatically handles formatting.

2.1 Basic Math

You can have fun formulas, such as $x = 7y^x$. If you want the equations displayed you can use two dollar signs, $$ to enclose the mathematics, or you can use

```
\begin{equation*}
  math stuff
\end{equation*}
```

as in

```
\begin{equation*}
  \int_{\partial \Omega} \omega = \int_{\Omega} d\omega.
\end{equation*}
```

which produces

$$\int_{\partial\Omega} \omega = \int_{\Omega} d\omega.$$

There are several other ways to display equations. The code for this one (which you can see in `body.tex`) aligns all the equal signs.

$$\begin{aligned} (x+2)^3 &= (x+2)(x+2)^2 && (2.1)\\ &= (x+2)(x^2+4x+4) && (2.2)\\ &= x^3+6x^2+12x+8 && (2.3) \end{aligned}$$

Notice that this last set of equations is numbered, but the previous one is not. The * in the LaTeX code eliminates the numbering.

2.2 Equations

Enumeration of equations, theorems, definitions, tables, is handled automatically by LaTeX. Each of these items may be given a label using `\label{<labelname>}`. The item can then be refereed to by `\ref{<labelname>}`. Below we demonstrate how to create and label an equation. Our first is a general differential equation,

$$\dot{x} = f(t, x), \qquad x(0) = x_0. \tag{2.4}$$

To see that the numbering is going fine we insert a matrix system as follows:

$$\dot{y} = \begin{bmatrix} a_1 & 0 & \cdots & 0 \\ 0 & a_2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & a_n \end{bmatrix} y. \tag{2.5}$$

The numbering is valuable when one wants to refer to the Equations (2.4) and (2.5). Note that when referring to Equation (2.4) you must capitalize the word equation. Also, when you enter a specific equation, figure, or table, *e.g.,* Eqn. (2.4), then you should type a ~ between the word Eqn., Fig., or Table and its labeling number to prevent inappropriate division of the label at the end of a line.

To display an equation without numbering, one uses the math displaystyle mode which works as follows:

$$\dot{y} = g(y),$$

which is an autonomous equation in y. The y at the end of the last sentence is in standard math mode. Further information on equations is provided in Appendix A.

2.3 Theorems, etc.

The student needs to highlight important results such as theorems, hypotheses, or definitions. In this section we investigate how LaTeX handles definitions, theorems, corollaries, etc.

Definition 2.1. *A linear differential equation is asymptotically stable if and only if all eigenvalues, λ, of the operator matrix have negative real part.*

We follow this with a couple of theorems and a corollary.

Theorem 2.2. *If the matrix A in the linear differential equation,*

$$\dot{y} = Ay, \qquad y(0) = y_0, \tag{2.6}$$

is symmetric, then the solution of (2.6) *is non-oscillatory.*

Corollary 2.3. *If the matrix A in* (2.6) *is symmetric and has negative eigenvalues, then the solution is non-oscillatory and asymptotically stable.*

In order to check how the numbering proceeds we insert here another theorem.

Theorem 2.4. *If the matrix H in the linear differential equation,*

$$\dot{y} = Hy, \qquad y(0) = y_0, \tag{2.7}$$

is antisymmetric, then the solution of (2.7) *is oscillatory.*

The `thesis.tex` also defines environments for `lemma` and `proposition` though you can add more if you wish. For example sometimes it is useful to add an `example` style environment. See the preamble of the document for more information.

2.4 Numbering of Theorems, etc...

Everyone has their own favorite way to number things; by default all environments of types { THEOREM, COROLLARY, DEFINITION, EXAMPLE, LEMMA, PROPOSITION, REMARK } share the same counter, which is reset at the start of a new chapter. If you want to change this, search for ``Independent Counters'' in `thesis.tex`.

2.5 Figures or How to Get into Real Trouble if You Take Advantage of What LaTeX Can Do

This section shows how to display figures and refer to them in the text. LaTeX does have the ability to insert postscript files using the `graphicx` package. Make sure to include `\usepackage{graphicx}` in your preamble, that is between the LaTeX commands `\documentclass` and `\begin{document}`. *See* `http://en.wikibooks.org/wiki/LaTeX/Importing_Graphics` *for information about importing graphics into your document.*

To insert a figure that is formatted in encapsulated postscript, which must include a Bounding Box line which is named `fname.ps` you do the following:

```
\begin{figure}[ht]
   \includegraphics[width=\linewidth]{fname.eps}
   \caption{Insert a caption here.  \label{figlabel} }
\end{figure}
```

to produce the figure. The [**ht**] argument to the figure command is a *suggestion* to LaTeX to put the figure [**h**]ere, or at the [**t**]op of the page; [**p**] for a separate page is also possible. Avoid putting tables and figures at the [**b**]ottom of the page as this is frowned upon by the thesis manual; the preference is to put tables and figures right after they

Style note **NEVER put anything in the margin like this!!!**

are first referenced, *i.e.* [**h**]ere, but at the [**t**]op of the following page is acceptable in cases where it does not fit [**h**]ere. You can make the suggestion stronger by saying [**h!**] for "[**h**]ere**!**," but the internal rules may still override your suggestion. "`\linewidth`" above can be replaced by some number of inches (or other size LaTeX size measure such as `pt`, `em`, or `ex`). This will left justify the figure. Centering is a little more complicated. We place everything in a `minipage` environment:

```
\begin{figure}[ht]
   \centering
   \begin{minipage}{xin}
      \includegraphics[width=\linewidth]{fname.ps}
      \caption{Insert a caption here.  \label{figlabel} }
   \end{minipage}
\end{figure}
```

To demonstrate how the department would like to see figures in the thesis the following is provided. If you are examining these files with `xdvi`, you will only see a blank spot. However, both printed and ghostview methods described in the previous chapter will allow viewing. Suppose that we create a figure to graph the curve

$$y = \sin(\omega t), \tag{2.8}$$

where ω is the circular frequency. Figure 2.1 is a graph of Equation (2.8), and figure 2.2 is an illustration of a mapping in the complex plane. The interval of time viewed is $t \in [-5, 5]$. The figure reference should be denoted by either Fig. 2.1 or by Figure 2.1 with specific figures capitalized as noted here.

When you have a collection of figures and large figures, you may want to delay insertion of them until the end of the chapter. At the end of this chapter we are including a full page figure (Fig. 2.3) to demonstrate this LaTeX command. Note that if you cannot obtain postscript figures or are having too much trouble using the technique described above, then you can use the `\vspace` command to provide an empty space in the manuscript, then use the old-fashioned technique of taping in your figure and photocopying it.

2.6 Tables

The Department of Mathematical Sciences does not have specific requirements on the exact layout of a table. However, the tables should be easily readable and properly labeled according to the regulations in the SDSU Thesis Manual. In this section we want to demonstrate how LaTeX handles tables. More complicated examples can be found in Lamport's book [10, 11]. We begin with a small table, given by Table

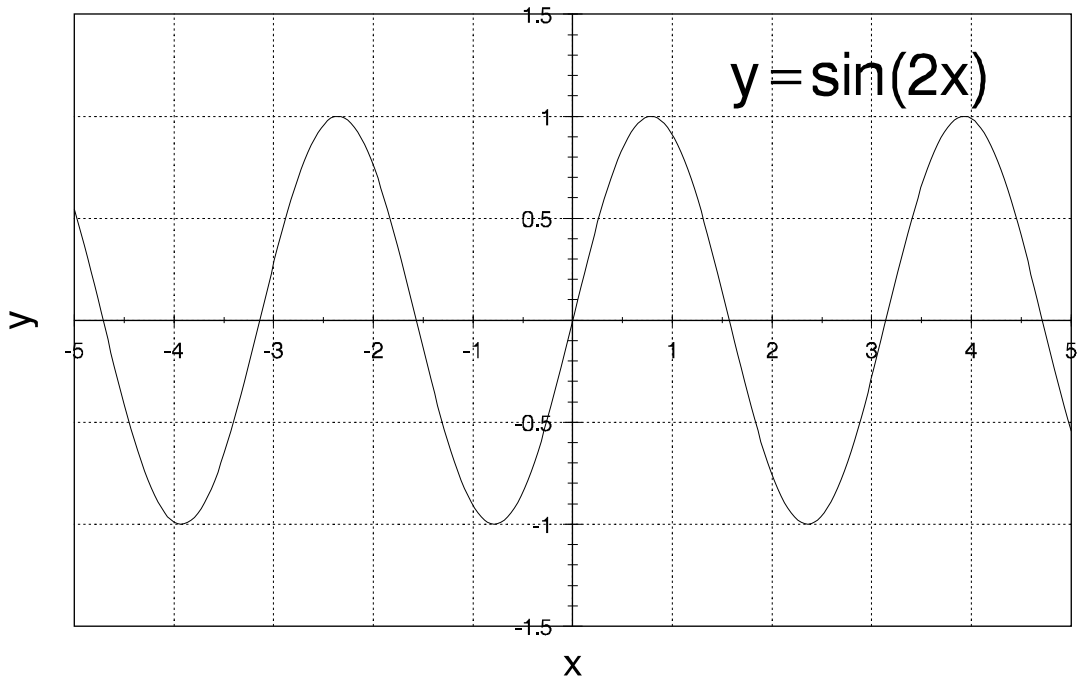

Figure 2.1. This is a graph of the above equation, where the circular frequency is taken as $\omega = 2$. Note: *if you need to cite a source (of e.g. a figure) in the caption, include the FULL CITATION, e.g.* [§4.10.4 Figures, MONTEZUMA PUBLISHING, *San Diego State University Dissertation and Thesis Manual: Policies, Procedures and Format*, Spring 2010.] — The Easiest way to achieve this is to first use the `\cite{...}` command, build the document, then replace the cite command with the appropriate text copied from the generated `thesis.bib` file. If the figure caption is the *only* place the source is cited, add a `\nocite{...}` command to ensure that it shows up in the bibliography.

2.1 which inserts nicely into the text. Note that the same centering trick as was employed for figures is done here and we set the width of the `minipage` environment to 1.9 inches.

Style note

The manual however allows for the caption to be a little wider if the table is really small and so we can use a wider `minipage` and then center the table inside there. See for example Table 2.2 where we used width of 3.5 inches.

Note that you can use the `center` environment instead of `\centering` but that might add a little bit of unwanted whitespace. With `\centering` on the other hand, you might have to put braces around the text you wish to center and sometimes need to add a `\par`. If you use it inside a `minipage`, `table` or `figure` environment, you don't have to really worry about that. Note however that without the use of `minipage` you

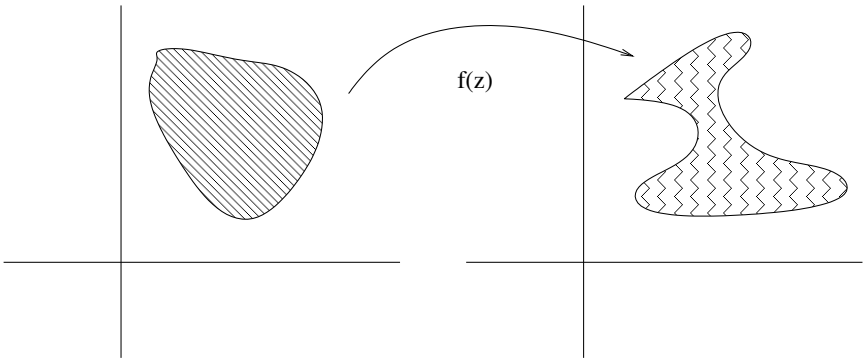

Figure 2.2. Mapping $f(x)$ from the complex plane to itself.

cannot center the caption as it automatically left aligns itself to conform with the thesis manual.

Tables can also be left aligned see for example Table 2.3. Here we don't use the `minipage` environment, but we must then add linebreaks so that the table caption does not go wider then the table itself. We need to add then two titles, one for the list of tables and one for the caption here. The former will not have line breaks and the latter will.

Sometimes a table might not fit onto a single page, in this case you must not use the `table` environment, but instead the `longtable` environment. Do note that `longtable` automatically centers so you need not worry about that. See Table 2.4 for some absolutely random numbers. To use `longtable` environment you must include the `longtable` package in your preamble. **see the note in `thesis.tex` on how to fix the longtable entries in the "List of Tables" if they are incorrect.**

Table 2.1. A Small Table for Listing Some Parameters Used in Some Numerical Procedure. LONG CAPTION— The Department of Mathematical Sciences does not have specific requirements on the exact layout of a table. However, the tables should be easily readable and properly labeled according to the regulations in the SDSU Thesis Manual.

Trial	a	b	c	ω
1	5	10	15	π
2	10	20	15	2π

Table 2.2. Another Small Table for Listing Some Parameters Used in a Numerical Procedure.

Trial	a	b	c	ω
1	5	10	15	π
2	10	20	15	2π

Table 2.3. Another Such Table but Left Aligned

Trial	a	b	c	ω
1	5	10	15	π
2	10	20	15	2π

Table 2.4. A Table of Some Totally Random Numbers. Often (when there are other `table`s defined before the `longtable`) it is necessary to issue the command `\clearpage` prior to the longtable environment; otherwise table-numbering and page-ordering of table/longtable object may get VERY strange.

First	Second	Third
$16883.20050 \times 64.19591$	23174^{2905}	$(5112, 5468, 27117)$
$7216.3398 \times 12239.16770$	19961^{9127}	$(16136, 21997, 26051)$
$15977.29588 \times 5732.19698$	14995^{26728}	$(28634, 14278, 17183)$
$24699.2338 \times 8803.18474$	19221^{28853}	$(18539, 6044, 19259)$
$21444.11156 \times 24727.15793$	18372^{28126}	$(28032, 2375, 15319)$
$4391.18511 \times 4548.30442$	1720^{1369}	$(3406, 21419, 16364)$
$30135.17285 \times 30643.14550$	9216^{213}	$(23353, 27690, 19435)$
$19438.13461 \times 25479.5929$	2137^{3868}	$(30657, 17930, 22240)$
$26015.13194 \times 24615.8566$	17585^{10358}	$(13114, 15259, 12079)$
$14483.18666 \times 730.30848$	16033^{18015}	$(28723, 30583, 27231)$
$28936.21168 \times 22153.15603$	7838^{2847}	$(8315, 13767, 4984)$
$12183.11656 \times 22915.1655$	4903^{3341}	$(26271, 13469, 20927)$
$3861.26584 \times 3418.15940$	8299^{22084}	$(16670, 6379, 5349)$
$1917.2334 \times 3164.29148$	31271^{24332}	$(18534, 14106, 32170)$
$21381.22421 \times 13170.26365$	1836^{24826}	$(16512, 3492, 29730)$
$19854.29763 \times 10431.8013$	856^{4247}	$(11431, 16797, 12547)$
$748.699 \times 18926.6097$	2617^{21261}	$(9262, 31765, 19764)$
$826.17531 \times 1102.229$	6144^{23524}	$(13399, 32510, 25360)$
$5457.16254 \times 28852.2419$	3340^{25847}	$(12851, 11353, 26704)$
$17098.22785 \times 10733.29645$	23533^{11432}	$(15804, 29630, 14049)$
$4297.6124 \times 13047.24061$	6951^{30578}	$(25163, 7180, 3955)$
$15919.20579 \times 3697.8512$	26036^{19951}	$(4596, 28456, 23292)$
$30444.8539 \times 1877.24380$	25637^{24662}	$(2345, 22515, 15427)$
$13777.5551 \times 12290.27827$	9848^{18414}	$(8106, 1141, 25365)$
$5916.26304 \times 32545.9871$	9456^{20356}	$(13568, 17968, 13625)$
$752.22564 \times 9313.24044$	20240^{17852}	$(25921, 11852, 10721)$
$17816.14197 \times 468.475$	27975^{6019}	$(12765, 23034, 15867)$
$31180.31140 \times 17008.23777$	4288^{10545}	$(23555, 14160, 20001)$
$11143.27728 \times 5201.24768$	28480^{27765}	$(1313, 19756, 15238)$

(table continues)

Table 2.4 (Continued)

First	Second	Third
$19165.12910 \times 27090.29887$	30726^{8520}	$(30355, 31201, 3727)$
$3607.11199 \times 26761.19474$	9611^{25133}	$(3715, 620, 29421)$
$14260.24175 \times 10813.1493$	2551^{5774}	$(6694, 27319, 1486)$
$1691.28633 \times 21243.16929$	15030^{1385}	$(11252, 12149, 32111)$
$19772.9737 \times 30544.23499$	13344^{8975}	$(17492, 50, 18586)$
$9857.3765 \times 19207.6510$	18025^{10614}	$(17324, 19518, 13165)$

A larger table, given by Table 2.5 and reproduced from another document, then you may need to allow an entire page for the table. This is done by typing the command `\begin{table}[p]`. This test example is included in the minipage environment to show how a footnote[1] can be added to a table. Several problems have been noted before on how LaTeX handles the location of the table in the text.

2.7 Potential Pitfalls

(Oh yeah, there must be text between sectioning commands...)

2.7.1 Tables and Figures

There is a conflict between the `\usepackage{subfig}`, `\usepackage{caption}` and the `sdsu-thesis.cls` class specification. The long table captions show up correctly (bold and left aligned with table). Use `\usepackage{subfigure}` instead and all captions, as well as the list of tables page show up ok.

If you insist on `\usepackage{subfig}`, make sure to **first** issue the command `\usepackage[bf,labelsep=period,textfont=bf]{caption}` where the first "`bf`" makes the labels "Figure n" bold; `labelsep=period` says "use '.' instead of ':'; and `textfont=bf` makes the caption text bold. This may solve your subfig problems.

Style note

Table captions ("table titles" [15]) go ABOVE the table, must be in *headline style* where "all major words are capitalized," and there is no period at the end of the caption; in figure captions only the first word is capitalized, and there is a period at the end. — **THE STYLE DOES NOT CURRENTLY ENFORCE THIS, *YOU* HAVE TO DO IT MANUALLY.**

Charts, graphs, diagrams, maps, photographs, and other graphic illustrations should all be labeled as *Figures* [15, §4.6.9, and §4.10.4]. Figure captions are capitalized

[1] We also need to see how a regular footnote appears in the text, so one was inserted here. Multiple lines are easily handled by LaTeX.

Table 2.5. Computations for Products of the *RRN* Genes at Different Growth Rates

τ(min)	100	60	40	30	24
C period	67	50	45	43	42
D period	30	27	25	24	23
V_0	0.437	0.577	0.815	1.15	1.63
$\bar{c}^a$	11.1	16.8	22.1	28.1	31.4
${\bar{c}_{85}}^b$	1.73	2.68	3.65	4.81	5.57
${\bar{c}_{57}}^c$	1.36	1.98	2.43	2.87	2.96
$\bar{c}_{85}(\times 100)/\bar{c}^d$	15.6	15.9	16.5	17.1	17.7
$\bar{c}_{57}(\times 100)/\bar{c}^e$	12.3	11.8	11.0	10.2	9.44
$\bar{c}_{85}/\bar{c}_{57}$	1.27	1.35	1.50	1.68	1.88
r^f	3.75	10.27	22.56	38.42	56.98
${c_{max}}^g$	11.28	17.04	22.33	28.36	31.77
${c_{max}/c_{min}}^h$	1.041	1.036	1.027	1.024	1.026

[a] $\times 1000$ ribosomes/μm^3.

[b] $\times 1000$ ribosomes/μm^3, representing the average concentration of the product of the *rrn* gene located at $85'$.

[c] $\times 1000$ ribosomes/μm^3, representing the average concentration of the product of the *rrn* gene located at $57'$.

[d] Percentage of $\bar{c}$ produced by the *rrn* gene located at $85'$.

[e] Percentage of $\bar{c}$ produced by the *rrn* gene located at $57'$.

[f] Initiations/min/gene.

[g] $\times 1000$ ribosomes/μm^3, representing the maximum concentration during the cell cycle.

[h] Ratio of maximum to minimum concentration during the cell cycle.

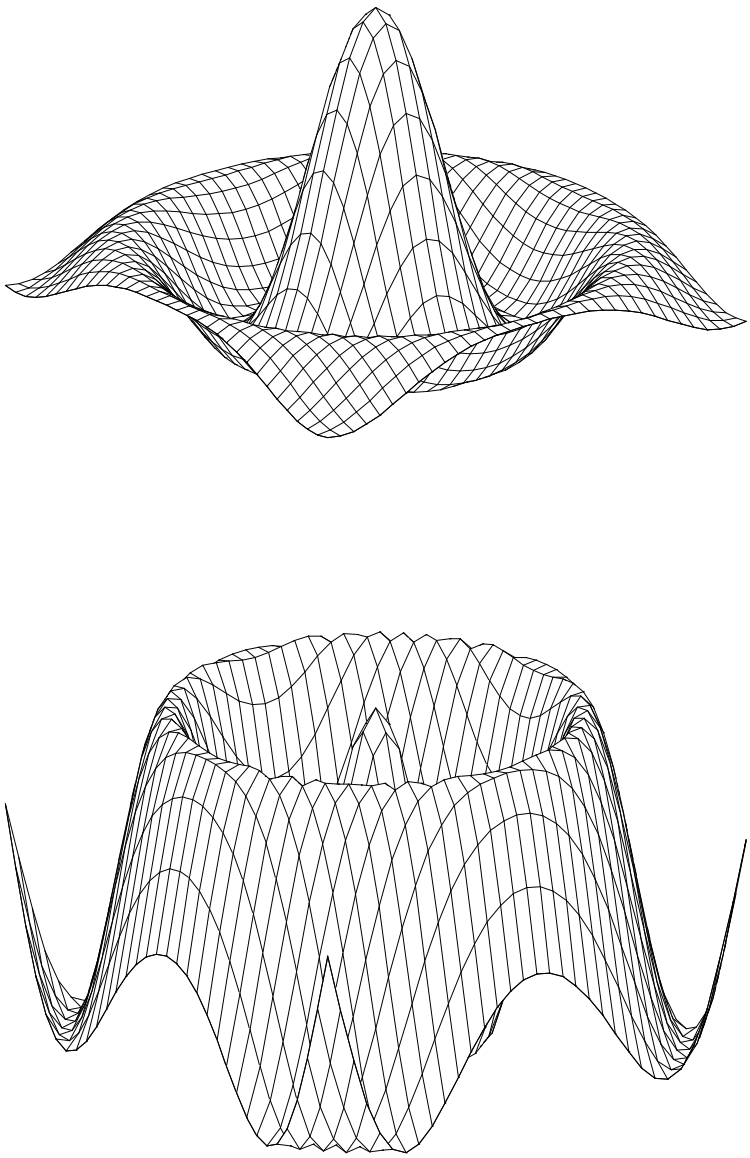

Figure 2.3. The top graph is the function $z = \sin(r)/r$, while the bottom surface is the function $z = \cos(r)$.

sentence style in the text; therefore, the List of Figures entries should be in sentence style.

All tables and figures must be referenced in text *prior* to their appearance. Those references should be by number.

2.7.1.1 Centered Tables Figures

It is not as simple as adding `\centering` into the figure or table environment as that will center the caption on the page rather then left align it with the left edge of the figure or table. So the way to solve this is to figure out the width of the figure or table and add it in a minipage and center that. For example if our table is 2 inches wide when typeset, then we could do

```
\begin{table}[ht]
  \centering
  \begin{minipage}{2in}
    \caption{Caption goes here}
    ... here is your table ...
  \end{minipage}
\end{table}
```

2.7.2 Margins

It is believed that the `sdsu-thesis.cls` template complies with the SDSU thesis manual: 1.25 inch left, 1 inch top, bottom and right. But your *printout* may not give the right measurement, if your printer/printer-driver scales the document. You may have to turn off scaling and/or tweak the settings in the `sdsu-thesis.cls` file.

Someone said: *"Some laser printers don't do the margins correctly, for example my printer shifts the page a bit. You can correct this with the* `\hoffset` *and* `\voffset` *lengths as:*

```
\hoffset -0.0625in
\voffset  0.15625in
```

"

2.7.3 Bad Pagebreaks

Sometimes LaTeX does not do exactly what you want with respect to pagebreaks. To solve this you can manually add a `\pagebreak` command where it should break, or you could add `\enlargethispage{12pt}` to make a page slightly larger if needed; though I'm not sure how the thesis reviewer will look on such transgressions, so do that at own risk.

Bad pagebreaks in the table of contents (or list of tables/figures): If you get a bad pagebreak in a table of contents you can force a pagebreak by: `\addtocontents{toc}{\protect\pagebreak}` you add this at the point in your document that corresponds to that place in the table of contents. For list of tables and list of figures, replace 'toc' in the line above with 'lot' or 'lof.'

2.7.4 Bad Linebreaks

Bad linebreaks in chapter, section (subsection, etc...), or table/figure caption titles: This classfile tries to make all titles conform to the requirements of the thesis manual, but it is possible that it gets things wrong and you may want to add linebreaks (the `\\` command) yourself. However, the table of contents title should not have any linebreaks. The way you do it is to add an optional argument to `\chapter`, `\section`, `\caption` as in:

```
\chapter[Title for Table of Contents]{Title With\\Linebreaks}
```

Note that for `\caption`'s in figures and tables you might have to do this whenever you have a small figure or table as the table/figure environment cannot make the caption only as long as the figure since it doesn't know how large the figure is until it typesets everything. See example above and more examples in the long-example directory. You can also solve the `\caption` issue with minipage in the same way we do centering, see section 2.7.1.1.

2.7.5 Vertical Space

This classfile tries to make all the vertical space as required, but sometimes you may need to modify what it does, or you just need to insert some vertical space. You use the `\vspace` and `\vspace*` commands (see LaTeX manual). You can use positive or negative length there and `\vspace*` makes sure the space appears even if there is a pagebreak in between. For example to add 2 inches of space you can add `\vspace{2in}`.

2.7.6 Bold Math in the Thesis: $\mathbf{x} = \pi$

Math in section titles need to be **bold**, but cannot be bold in the Table of Contents.

CHAPTER 3

SECTIONING — THE MIDDLE

Middle chapter. Here we put the middle things, that is, things that are in the middle and not in the beginning or in the end. Here we also test all the section, subsection, and other headings.

Note that CHAPTER TITLES need to be in ALL CAPS — YOU have enter the chapter titles in ALL CAPS!!! ***Style note***

3.1 A Section

Some section text. Note that there should ALWAYS be some text in between two sectioning levels; a `\section` directly followed by a `\subsection` will not go through the review.

3.1.1 A Subsection With a Very Long Title To See How That Will Look When Printed

Some subsection text.

3.1.1.1 A Subsubsection

Some subsubsection text.

3.1.1.1.1 A Subsubsubsection

Some subsubsubsection text. If you are using this, you are ~~probably~~ over-organizing things.

3.1.1.1.1.1 A Paragraph. Some paragraph text. You never really get this deep — don't be ridiculous.

3.1.2 A Friend of §3.1.1

This subsection is here because §3.1.1 cannot stand alone; sub^n-sectioning commands are supposed to *divide* the text into logical chunks.

3.2 A 2nd Section

Yeah, no (sub)section can stand alone; so if there is a §3.1 there must be a §3.2. §3.1.1, needs a "friend"; and so does §3.1.1.1, and §3.1.1.1.1.

Bending the Rules :: A Single "Section" Without a Number

Occasionally, you may "need" a sub^n-single section; in that case you can circumvent the rules by not numbering it, currently the `\section*{...}` commands do not produce the desirable results; the best quick-fix is:

```
[blank line]
\vspace{\baselineskip}
\centerline{\textbf{\large A ‘‘Section’’ Without a Number}}
\vspace{0.25\baselineskip}
[blank line]
```

CHAPTER 4
REFERENCING

Below a list of references are provided in the acceptable format for Master's thesis submission. References are to be numbered and should appear either alphabetically or in the order of appearance in the text. (LaTeX does the former for the student.) For students using LaTeX these are obtained using the plain style with BibTeX. The Department of Mathematics and Statistics will accept either the plain style or the SIAM style. (For the SIAM style, it is recommended that you use the included `siammod.bst` version.) There are references for journal `article`s [1], `book`s and `booklet`s [5, 21], `inbook`s, `incollection`s, and `inproceedings` [4, 6, 19]. *Note that when you have more than one citation in a single bracket they must be in increasing numerical order!* Other sources may be `proceedings` [3], technical reports (`techreport`) [17], theses (`mastersthesis`, or `PhDthesis`) [7], or `unpublished` material [18]. This should provide a fairly comprehensive list for any material that the student may encounter. For additional assistance, see the graduate adviser in your area of concentration.

Style note

If you cite a website [16] and you cannot find the year on the website, you should put "n.d." (not dated) at the end. (this is true for other reference also.) It must also has the word "accessed" and the month and year you access the website. You can change how things with no author(s) are sorted in the bibliography by supplying a `key` entry (see `thbib.bib`), *e.g.* this news release [20] will be sorted under "U," the leading letter of the publishing agency (as preferred by the thesis publisher).

This [8] is an example of a patent. ***Notice:*** how the `month` and `year` fields in `thbib.bib` have been abused to force the "correct" format.

Some articles [2] have many authors, pay attention to how (1) the entry looks in the `thbib.bib` file, and (2) in the final bibliography.

UNTOLD SECRETS

Some departments (*i.e.* the Department of Mathematics and Statistics) and programs (*i.e.* the Computational Sciences program) have their own LaTeX-thesis reviewer(s)[1]. This alternative review **bypasses the Montezuma Publishing review**, and tends to be FASTER and more understanding of LaTeXquirks. Please read `LaTeX_Thesis_Format_Details_[UNOFFICIAL].pdf` in the `Resources` folder.

This is not really a secret, but calling it a secret makes it more exciting than section **??**?

[1] Peter Blomgren ⟨`blomgren@sdsu.edu`⟩ is one of them!

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APPENDIX A

CW Complexes and CW approximation

CW Complexes and CW approximation

To demonstrate how an appendix should be inserted into the thesis we have provided two appendices. This first appendix illustrates some more advanced techniques to improve the appearance of your equations. Below is a system of partial differential equations from a model for cellular control by an external nutrient. The equations are complicated and LaTeX tends to allow them to run into each other. To prevent this we have used the `\vrule` command to separate them. Note this is an ordinary TeX command and is not in L. Lamport's book [10]. Furthermore, we have some complicated boundary conditions that we needed to align, so we used the array command, but to get the equations looking right we also needed the `\dfrac` command instead of the `\frac` command. The equations for our model are as follows:

$$\begin{aligned}
\dot{U}_1(t) &= \tilde{f}(W_1(t-T)) - U_1(t) + \gamma_1 U_2(R\sigma,t), \\
\dot{W}_1(t) &= -\hat{b}_3 W_1(t) + \gamma_3 W_2(R\sigma,t), \\
\frac{\partial U_2}{\partial t} &= D_1\nabla^2 U_2 - U_2 - \tilde{f}(W_1(t-T)) - \gamma_1 U_2(R\sigma,t), \\
\frac{\partial V_2}{\partial t} &= D_2\nabla^2 V_2 - b_2 V_2 + c_0\big(U_2 + U_1(t)\big), \\
\frac{\partial W_2}{\partial t} &= D_3\nabla^2 W_2 - b_3 W_2 + (\hat{b}_3 - b_3)W_1 - \gamma_3 W_2(R\sigma,t) \\
&\quad + k\left[\left[\left(\frac{D_3}{r^2}\right)\frac{d}{dr}\left(r^2\frac{dh}{dr}\right) - b_3 h\right] V_2(R,t) - h\dot{V}_2(R,t)\right],
\end{aligned} \tag{A.1}$$

for $t > 0$ and $R\sigma < r < R$ and with the boundary conditions:

$$\begin{aligned}
\frac{\partial U_2(R\sigma,t)}{\partial r} &= \beta_1 U_2(R\sigma,t), & \frac{\partial U_2(R,t)}{\partial r} &= 0, \\
\frac{\partial V_2(R\sigma,t)}{\partial r} &= 0, & \frac{\partial V_2(R,t)}{\partial r} &= 0, \\
\frac{\partial W_2(R\sigma,t)}{\partial r} &= \beta_3 W_2(R\sigma,t), & \frac{\partial W_2(R,t)}{\partial r} &= 0.
\end{aligned}$$

Notice that the system is numbered only once by (A.1) and that this is centered as best we can on one line. All other lines have the $\backslash nonumber$ command.

A.1 Theorems

The appendix can also include technical theorems and lemmas which are call in the same manner as before. For example,

Theorem A.1. *The system of equations (A.1) can exhibit periodic solutions for certain parameter values.*

Proof. The argument uses Hopf bifurcation techniques and is very complicated. See Mahaffy *et al* [13]. □

APPENDIX B

Graded modules and algebras, Hopf algebras, more algebraic stuff

Graded modules and algebras, Hopf algebras, more algebraic stuff

The thesis will rarely use list environments, but they are valuable for résumés. For more information on creating a résumé you may want to see the author of this document (you also need to learn quite a bit about `\parbox` commands). To create a list you will want to use one of `itemize, enumerate,` or `description`. For example:

continuous A function f is **continuous** at x if and only if for every $\varepsilon > 0$ there exists a $\delta(x) > 0$ such that whenever $|y - x| < \delta$, $|f(y) - f(x)| < \varepsilon$.

uniformily continuous A function f is **uniformly continuous** if and only if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that whenever $|y - x| < \delta$, $|f(y) - f(x)| < \varepsilon$ independent of x and y.

equicontinuous A family of functions f_n is **equicontinuous** at a point x if and only if for every $\varepsilon > 0$ there exists a $\delta > 0$ such that whenever $|y - x| < \delta$, $|f_n(y) - f_n(x)| < \varepsilon$ for all functions f_n.

LaTeX provides an environment for block quotations. To agree with the thesis manual follow the format below for a quotation exceeding four lines. From Lewis Carrol's *Hunting of the Snark* we hear the Bellman tell his crew:

> The Bellman himself they all praised to the skies–
> Such a carriage, such ease and such grace!
> Such solemnity, too! One could see he was wise,
> The moment one looked in his face!
>
> He had bought a large map representing the sea,
> Without the least vestige of land:
> And the crew were much pleased when they found it to be
> A map they could all understand.
>
> "What's the good of Mercator's, North Poles and Equators,
> Tropics, Zones, and Meridian Lines?"
> So the Bellman would cry: and the crew would reply,
> "They are merely conventional signs!"
>
> "Other maps are such shapes, with their islands and capes!
> But we've got our brave Captain to thank"
> (So the crew would protest) "that he's bought us the best–
> A perfect and absolute blank!"