1) Discrete Distributions:

Deals with variables that take distinct, countable values (like whole numbers, not fractions or continuous values).

Distribution	Definition	Parameters	When to Use?
Bernoulli	A single trial with two possible outcomes: success (1) or failure (0)	p (probability of success)	When you perform a single experiment (e.g., flipping a coin once).
Binomial	The number of successes in n independent Bernoulli trials	n, p (number of trials, probability of success)	When you have a fixed number of trials and want to count successes.
Geometric	The number of trials until the first success	p (probability of success)	When you want to know how long it takes for the first success (e.g., how many times you flip a coin before getting heads).
Poisson	The number of events occurring within a fixed interval, given a known average rate	λ (average number of occurrences)	When events happen randomly at a known average rate (e.g., number of emails received per hour).

2) Continuous Distributions

Deals with variables that can take any value within a certain range (like height, weight, time).

Distribution	Definition	Parameters	When to Use?
Uniform	All values within a given range are equally likely	a, b (minimum and maximum values)	When all outcomes have an equal probability (e.g., picking a random number between 1 and 10).
Normal (Gaussian)	A symmetric, bell-shaped distribution with most values centered around the mean	μ, σ^2 (mean, variance)	When data is naturally distributed around a central value (e.g., height, IQ scores).
Exponential	The time between occurrences of events in a Poisson process	λ (rate of occurrence)	When measuring time between random events (e.g., waiting time for the next customer in a queue).

3. Key Comparisons

<u>*</u> Binomial vs. Poisson:

- **Binomial**: Counts the number of successes in a fixed number of trials.
- **Poisson**: Counts the number of occurrences in a given time period, with no fixed number of trials.
- If n is large and p is small in Binomial, it can approximate Poisson.

<u>*</u> Geometric vs. Exponential:

- **Geometric**: Counts the number of trials until the first success (*Discrete*).
- **Exponential**: Measures the time until the next event (*Continuous*).
- They are related, but one deals with countable trials, and the other deals with time.

<u>*</u> Normal vs. Exponential:

- **Normal**: Data is symmetric and centered around a mean.
- **Exponential**: Data is skewed and represents waiting times or time until an event happens.

Notes

- If you have a fixed number of attempts, **Binomial** or **Geometric** is often the best choice.
- If you're measuring time or distances, **Exponential** or **Normal** might be better.
- If all values are equally likely, **Uniform** is the right model.

1. Unconditional Probability (Regular Probability)

Also called "Marginal Probability," this represents the likelihood of an event occurring on its own, without any prior knowledge of other events.

- **Notation: P(A)** (the probability of event A occurring).
- **Example:**
    - If you randomly pick a card from a deck, the probability of drawing an Ace is:

P(Ace) = 4/52

- **Key Characteristics:**
    - Doesn't consider any prior conditions.
    - Calculated based on total possible outcomes.

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2. Conditional Probability

The probability of an event occurring given that another event has already occurred.

- **Notation: P(A|B)** (the probability of event A happening given that event B has happened).
- **Formula:**

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

(if P(B) > 0)

- **Example:**
    - What is the probability of drawing an **Ace**, given that you already know the card is **a face card (J, Q, K, Ace)?**
    - There are **4 Aces** in the deck and **16 face cards** (4 Jacks, 4 Queens, 4 Kings, 4 Aces).
    - So, $$P(\text{Ace} \mid \text{Face Card}) = \frac{4}{16} = 0.25$$
- **Key Characteristics:**
    - Depends on a given condition.
    - Uses the probability of both events happening together P(A ∩ B).
    - Helps refine probability estimates when we have extra information.

4. Connection to Bayes' Theorem

Conditional probability is the foundation of **Bayes' Theorem**, which allows us to update probabilities based on new information:

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

This is widely used in:

- **Medical Diagnosis:** Given a positive test, what's the probability of actually having the disease?
- **Machine Learning:** Estimating the likelihood of a spam email given certain keywords.

Final Thoughts

- If you're calculating a probability **without any conditions**, use **Regular Probability** .
- If you already have **some prior knowledge** (another event occurred), use **Conditional Probability** .
- Understanding this helps in fields like **statistics, AI, finance, and decision-making**.

Parameter	**Conditional Probability**	**Joint Probability**	**Marginal Probability**
Definition	The probability of an event occurring given. that another event has already occurred.	The probability of two or more events occurring simultaneously.	The probability of an event occurring without considering any other events.
Calculation	P (A | B)	P (A ∩ B)	P(A)
Variables involved	Two or more events	Two or more events	Single event.

Conditional Probability and Bayes' Theorem

Bayes' Theorem is a fundamental concept in probability theory named after the Reverend Thomas Bayes. It provides a mathematical framework for updating beliefs or hypotheses in light of new evidence or information. This theorem is extensively used in various fields, including statistics, machine learning, and artificial intelligence.

At its core, Bayes' Theorem enables us to calculate the probability of a hypothesis being true given observed evidence. **The theorem is expressed mathematically as follows:**

P(A|B) = (P(B|A) × P(A)) / P(B)

Where:

- **P(A|B)** is the posterior probability of hypothesis **A** given evidence **B**.
- **P(B|A)** is the likelihood of observing evidence **B** given that hypothesis **A** is true.
- **P(A)** is the prior probability of hypothesis **A** before observing any evidence.
- **P(B)** is the probability of observing evidence **B** regardless of the truth of hypothesis **A**.

Random variable is a fundamental concept in statistics that bridges the gap between theoretical probability and real-world data. A R**andom variable** in statistics is a function that assigns a real value to an outcome in the sample space of a random experiment**. For example**: if you roll a die, you can assign a number to each possible outcome.

There are two basic types of random variables:

- Discrete Random Variables (which take on specific values).
- Continuous Random Variables (assume any value within a given range).

What is Probability Density Function(PDF)?

Probability Density Function is used for calculating the probabilities for continuous random variables. When the cumulative distribution function (CDF) is differentiated we get the probability density function (PDF). Both functions are used to represent the probability distribution of a continuous random variable.

What Does a Probability Density Function (PDF) Tell Us?

A Probability Density Function (PDF) is a function that describes the likelihood of a continuous random variable taking on a particular value. Unlike discrete random variables, where probabilities are assigned to specific outcomes, continuous random variables can take on any value within a range. Probability Density Function (PDF) tells us

- Relative Likelihood
- Distribution Shape
- Expected Value and Variance, etc.

How to find Probability Density Function (PDF):

Step 1: First check the PDF is valid or not using the necessary conditions.

Step 2: If the PDF is valid, use the formula and write the required probability and limits.

Step 3: Divide the integration according to the given PDF.

Step 4: Solve all integrations.

Step 5: The resultant value gives the required probability.