Introduction to Real-World Graphs

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Outline

1. Graphs and networks
2. Time and memory complexity
3. Algorithms

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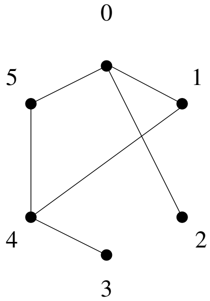
Graph

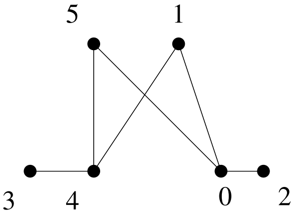
A graph $G = (V, E)$ is a couple of sets.

- V is a set of *vertices* (or *nodes*) ***fr:*** *sommet ou nœud*
- $E \subseteq (V \times V)$ is a set of *edges* (or *links*) ***fr:*** *lien, arête*

Example

- $V = \{0, 1, 2, 3, 4, 5\}$
- $E = \{(0, 1), (0, 2), (3, 4), (4, 5), (5, 0), (1, 4)\}$





Graph theory

Graph theory is a very important and well studied field:
`https://en.wikipedia.org/wiki/Graph_theory`

Question: Why is this useful to us?

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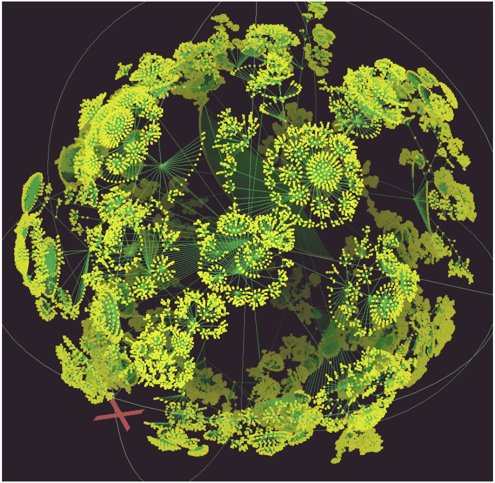
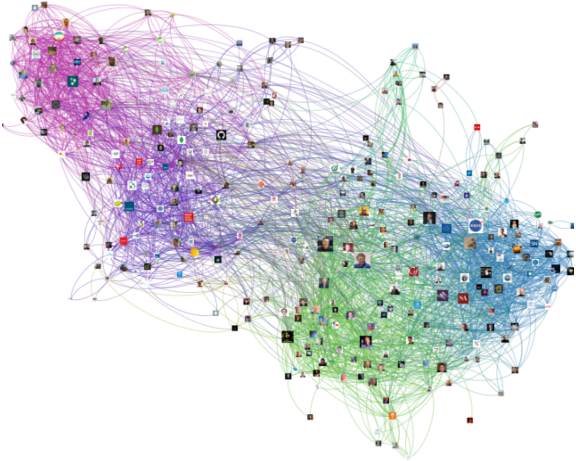
Question: Why is this useful to us?
Many systems can be modeled using graphs!

Internet: computers connected by internet connections



Twitter: profiles connected by follow-links



Many more examples

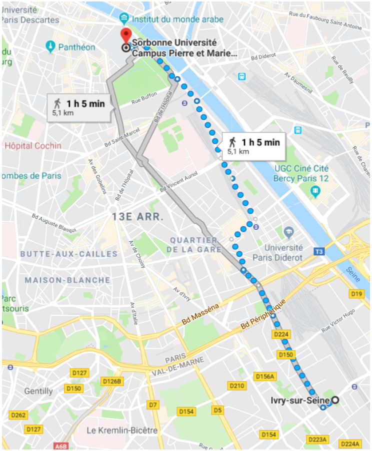
Graphs extracted from the “real-world” are called networks, complex networks or real-world graphs.

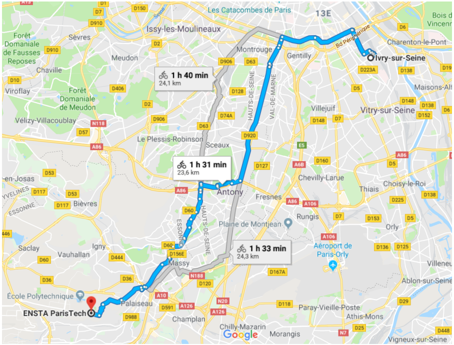
Question: Give some more examples of real-world graphs?

Common properties of real-world graphs

- **Very large:**
  Several G edges
- **Sparse:**
  Not many edges compared to what could be
- **Short distances and diameter:**
  $\sim \log(n)$; Six degrees of separation; Kevin Bacon game
- **Heterogeneous degree distribution:**
  $\sim$ power law; contains hubs (very high degree nodes)
- **Dense locally (many triangles):**
  High clustering coefficient & transitivity ratio
- ...

A real-world problem: GPS





Many more examples

Question: Give some more real-world problems related to graphs?

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Time and memory complexity

Let G be a graph of n nodes and m edges.

Question: What do you think about the following complexities?

$$\mathcal{O}(m) \text{ and } \mathcal{O}(n^2)$$

Storing a graph in main memory

Three main formats:

1. List of edges
2. Adjacency matrix
3. Adjacency array

Question: Implementation? Pros and cons? Complexities?

`https://github.com/maxdan94/LoadGraph`

Adjacency Array in C

```c
//compact adjacency array datastructure
typedef struct {
  unsigned n; //number of nodes
  unsigned m; //number of edges
  unsigned *cd; //cumulative degree cd[0]=0 length=n+1
  unsigned *adj; //concatenated lists of neighbors
} adjarray;
```

Question: Remarks? How to list the neighbors of node *u*?

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Breadth First Search algorithm (BFS)

Algorithm 1 BFS from node *s*

```
function BFS($G$,$s$)
    FIFO $\leftarrow$ CreateFIFO()
    FIFO.add($s$)
    Mark($s$)
    while FIFO not empty do
        $u \leftarrow$ FIFO.pop()
        output $u$
        for each $v$ neighbor of $u$ in $G$ do
            if $v$ is not marked then
                FIFO.add($v$)
                Mark($v$)
```

Question: Implementation? Complexity?

Breadth First Search algorithm (BFS)

- **Question:** How to compute the diameter of a graph? Complexity?
- **Question:** How to compute a good lower bound to the diameter using only a few BFS? Complexity?
- **Question:** How to generalize BFS if edges have weights?

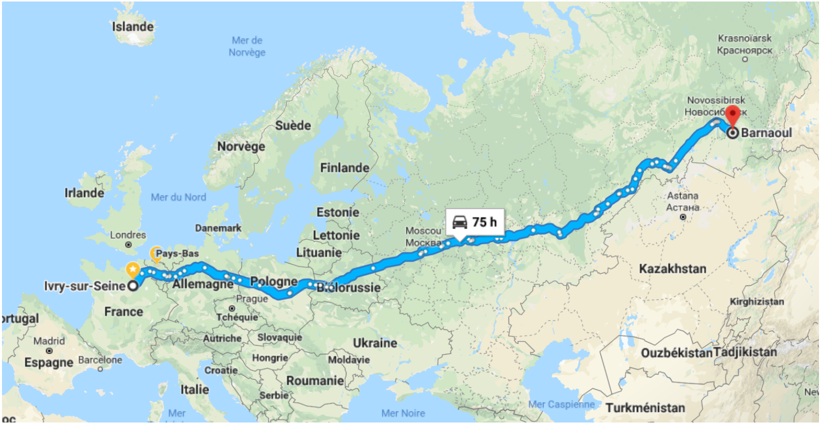
Dijkstra algorithm

Algorithm 2 Dijkstra on a weighted graph G_w from node s

```
function DIJKSTRA($G_w$,$s$)
    for each node $u$ in $G_w$ do
        $D[u] \leftarrow$ INFINITY
    $D[s] = 0$
    while $G_w$ not empty do
        $u \leftarrow$ node in $G_w$ with minimum $D[u]$
        output $u$
        for each $v$ neighbor of $u$ in $G$ do
            $tmp \leftarrow D[v] + w_{uv}$
            if $tmp < D[v]$ then
                $D[v] = tmp$
        remove $u$ from $G_w$
```

Question: Remarks? Complexity?

Dijkstra algorithm



Question: Is Dijkstra fast enough to answer that query in 1sec?

Listing triangles

Question: Suggest an algorithm to list all triangles?

(Fast) computation of the number of triangles

Algorithm: for each edge (u, v) in the graph, compute the intersection of the neighborhoods of u and v.

Remarks:

- Algorithm slow if u or v have high degree
- Triangle (u, v, w) is found 3 times

Idea: reduce as much as possible the size of the lists involved in the computation of the intersection:

- sort the nodes in non-increasing order of degree and re-index the graph (such that $u < v \Rightarrow d(u) \geq d(v)$)
- given an edge (u, v), we consider only the neighbors w of u (resp. v) such that $u < w$ (resp. $v < w$)

(Fast) computation of the number of triangles

Algorithm 3 List all triangles in G

```
function TRLIST(G)
    for each node u in G do
        tsl[u] ← truncated and sorted list of neighbors of u
    for each edge (u, v) in G do
        W ← Intersect(tsl[u], tsl[v])
        for each node w in W do
            output triangle {u, v, w}
```

Question: Remarks? Complexity?

Listing k-cliques

Question: Given an integer k, suggest an algorithm to list all k-cliques?

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`https://papers-gamma.link/paper/32`