Assignment 2

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1) We will prove that $L = \{a^i b^j c^k : i = j \wedge i = k\}$ is not regular using the pumping lemma and contradiction

Proof. Suppose L is regular. Then it has a pumping length $n \geq 0$
Let $w = a^n b^n c^n$. Then $w \in L$ because $n = n$, and note that $|w| = |a^n b^n c^n| = 3n \geq n$
Suppose $w = xyz$ such that $|xy| \leq n$ and $|y| \geq 1$ (so $y \neq \epsilon$)
Then since $w = a^n b^n c^n$ and $|xy| \leq n$, we must have that $xy = a^p a^q$ for some $0 < p + q \leq n$ (that is, xy can only consist of a's).
Let $x = a^p, y = a^q$ (where $p \geq 0$ and $q \geq 1$ since $y \neq \epsilon$)
Then $z = a^{n-p-q} b^n c^n$
Now consider the word $w^{(0)} = xy^0 z$
$w^{(0)} = xy^0 z = a^p b^{n-p-q} b^n c^n = a^{n-q} b^n c^n$
Since $n - q \neq n$ (because $q \geq 1$), it means that $w^{(0)}$ is not an element of L.
So the pumping lemma does not hold for this language, which contradicts the assumption that this language is regular.
Thus, L is not regular □

2) We will prove that $L = \{0^i 1^j : i \neq j\}$ is not regular using pumping lemma, contradiction and closure properties.

Proof. Suppose L is regular. Then it's complement $\overline{L} = \Sigma^\star - L$ should also be regular.
Note that $\overline{L} = \{0^i 1^j : i = j\}$
Since $\overline{L}$ is regular, it has a pumping length $n \geq 0$
Let $w = 0^n 1^n$. Then $w \in \overline{L}$ because $n = n$, and note that $|w| = |0^n 1^n| = 2n \geq n$
Now suppose $w = xyz$ where $|xy| \leq n$ and $|y| \geq 1$ (so $y \neq \epsilon$).

Since $w = 0^n 1^n$ and $|xy| \leq n$, it must be that $xy = 0^a 0^b$ for some $0 < a + b \leq n$ (That is, xy can only consist of zeros)
Let $x = 0^a, y = 0^b$ (where $a \geq 0$ and $b \geq 1$ because $y \neq \epsilon$)
Then $z = 0^{n-a-b} 1^n$
Now, consider the word $w^{(0)} = xy^0 z$
$w^{(0)} = xy^0 z = 0^a 0^{n-a-b} 1^n = 0^{n-b} 1^n$
However, since $n - b \neq n$ (where $b \geq 1$), it means that $w^{(0)}$ is not an element of $\overline{L}$. So the pumping lemma does not hold for $\overline{L}$, which means that $\overline{L}$ is not regular.
And since $\overline{L}$ is not regular, it means that L must have not been regular to begin with, so we get a contradiction.
Thus, L is not regular. $\square$

3) Let $\Sigma = \{\emptyset, \epsilon, a, b, (,), +, \star\}$
Then the context free grammar that generates all strings in $\Sigma^\star$ that are valid regular expressions would be:

$S \to SS | S + S | S\star$
$S \to (S)$
$S \to \emptyset | \epsilon | a | b$