Assignment 8

Consider the two decision problems E and D:

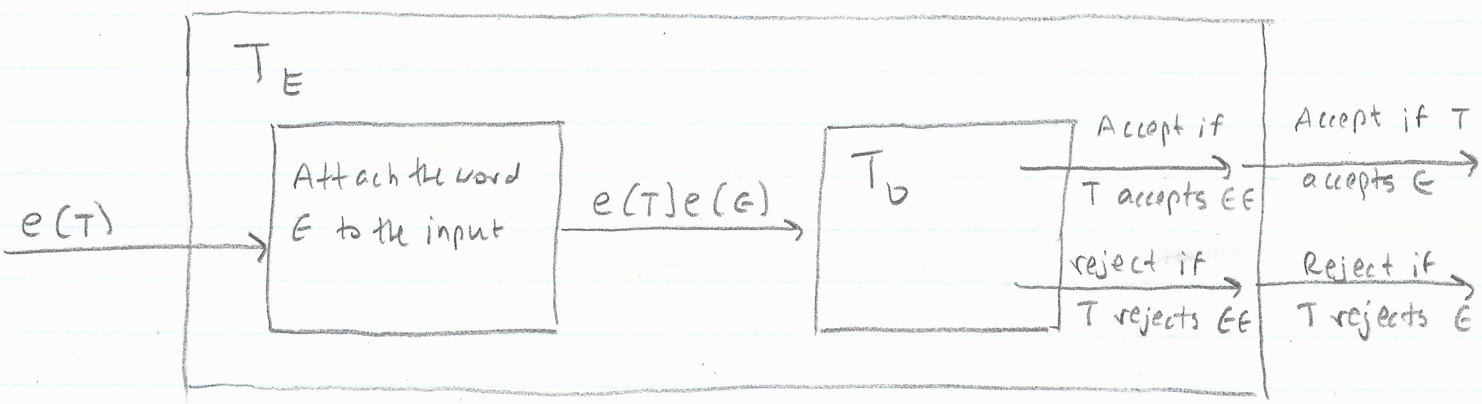
E: Given a Turing Machine T, does T accept ϵ?

D: Given a Turing Machine T and input w, does T accept ww?

We will prove that D is undecidable using a reduction to a known undecidable problem E.

Now, suppose that D is solveable. Then there is a turing machine T_D that takes a turing machine T and word w as input, and always halts and decides if T accepts ww.

We will use T_D to create T_E, which is a turing machine that takes any turing machine as input and halts and decides if the machine accepts ϵ



So if a turing machine T accepts $\epsilon\epsilon$, it must mean that it accepts ϵ because $\epsilon\epsilon = \epsilon$

And if a turing machine T does not accept $\epsilon\epsilon$, it must mean that it doesnt accept ϵ.

So T_E decides the problem E. However, we know from class that E is undecidable, so we get a contradiction.

The only assumption we made was that D is decidable.

Thus, the problem D is undecidable, and so T_D cannot exist.

2) The decidable problem: given a CFG G, does $G \Rightarrow^* \epsilon$?

How to solve it:

1) Convert G into chomsky normal form.
Let the Chomsky normal form version of G be called G'

2) Determine the nullable variable set of G' (the set of variables of G' that can derive epsilon).

Here is the recursive algorithm for finding the nullable variable set:

Base case: Let T be the set of all variables A such that $A \rightarrow \epsilon$ is a production of G'

Recursive case: For all right hand sides of a production $B \rightarrow X_1 X_2 \dots X_{r_i}$
if all X_i are nullable then B is nullable.
Add B to our set T of nullable variables

Continue until no new variables can be added to our nullable variable set.

3) If the start variable S of G' is in the nullable variable set, then $S \Rightarrow^* \epsilon$, so we accept.

Otherwise, if S is not in the nullable variable set, it means that S cannot derive ϵ, so we reject