Chapter 1: Matrix & Matrix Operations

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Matrices

A matrix is a rectangular array of numbers (or other mathematical objects), called the "entries" of the matrix. Matrices are subject to standard operations such as addition and multiplication. Most commonly, a matrix over a field

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{bmatrix}$$

$\Rightarrow$ this is a 3x3 matrix.
$\Rightarrow$ now this is a 3x2 & 3x1 Matrix:

$$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$$

$$C = \begin{bmatrix} 1 & 2 & 3 \end{bmatrix}$$

Transpose

$$A^T = \begin{bmatrix} 1 & 4 & 9 \\ 2 & 5 & 9 \\ 3 & 6 & 9 \end{bmatrix}$$

Addition / Subtraction

$$A + B = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} = \begin{bmatrix} a_{11} + b_{11} & a_{12} + b_{12} \\ a_{21} + b_{21} & a_{22} + b_{22} \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 9 & 9 & 9 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 3 \\ 4 & 6 & 6 \\ 9 & 9 & 10 \end{bmatrix}$$

Multiplication

$$
\begin{aligned}
AB &= \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 9 & 9 & 9 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} 1\cdot 1+2\cdot 0+3\cdot 1 & 1\cdot 0+2\cdot 1+3\cdot 0 & 1\cdot 2+2\cdot 0+3\cdot 1 \\ 4\cdot 1+5\cdot 0+6\cdot 1 & 4\cdot 0+5\cdot 1+6\cdot 0 & 4\cdot 2+5\cdot 0+6\cdot 1 \\ 9\cdot 1+9\cdot 0+9\cdot 1 & 9\cdot 0+9\cdot 1+9\cdot 0 & 9\cdot 2+9\cdot 0+9\cdot 1 \end{bmatrix} \\
&= \begin{bmatrix} 4 & 2 & 5 \\ 10 & 5 & 14 \\ 18 & 9 & 27 \end{bmatrix}
\end{aligned}
$$

Inverse Matrix

$$A^{-1} \cdot A = A \cdot A^{-1} = I$$

$$A^{-1} = \frac{1}{\det(A)} \cdot {}^{t}\mathrm{Com}(A)$$