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ScienceDirect
International Journal of Forecasting 24 (2008) 630–644
www.elsevier.com/locate/ijforecast

Modeling and forecasting short-term electricity load: A comparison of methods with an application to Brazilian data

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Abstract

The goal of this paper is to describe a forecasting model for the hourly electricity load in the area covered by an electric utility located in the southeast of Brazil. A different model is constructed for each hour of the day. Each model is based on a decomposition of the daily series of each hour in two components. The first component is purely deterministic and is related to trends, seasonality, and the special days effect. The second is stochastic, and follows a linear autoregressive model. Nonlinear alternatives are also considered in the second step. The multi-step forecasting performance of the proposed methodology is compared with that of a benchmark model, and the results indicate that our proposal is useful for electricity load forecasting in tropical environments.

Keywords: Short-term load forecasting; Time series; Seasonality; Linear models; SARIMA; Time-series decomposition

1. Introduction

One kind of time series which is of major interest, both academic and practical, is the hourly electricity load. From an academic point of view, the series are remarkable because they have a number of interesting features, such as trends, annual and daily seasonal patterns, an influence of external variables, and possible nonlinearities. In addition, load series have been used over the years as a benchmark data set for different forecasting models.

From the applied point of view, short-term load forecasting is a very important task for electric utilities in order to manage the production, transmission, and distribution of electricity in a more efficient and secure way. As an example of the importance of accurate forecasts, it was estimated that an increase of only 1% in the forecast error (in 1984) caused an increase of 10 million pounds in operating costs per year for one electric utility in the United Kingdom (Bunn & Farmer, 1985b).

Over the years, different forecasting techniques have been developed to model the electricity load, both in the classical time series literature (Al-Hamadi & Soliman, 2004; Amjady, 2001; Bunn & Farmer, 1985a;

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doi:10.1016/j.ijforecast.2008.08.003

Cancelo, Espasa, & Grafe, 2008-this issue; Dordonnat, Koopman, Ooms, Dessertaine, & Collet, 2008-this issue; Huang, 2003; Huang, Huang, & Wang, 2005; Nowicka-Zagrajek & Weron, 2002; Taylor, 2008-this issue; Taylor, de Menezes, & McSharry, 2006; and Weron, 2006), and in the machine intelligence framework (da Silva, Ferreira, & Velasquez, 2008-this issue; Hippert, Bunn, & Souza, 2005; Hippert, Pedreira, & Souza, 2001; Khotanzad, Afkhami-Rohani, & Maratukulam, 1998; and Metaxiotis, Kagiannas, Askounis, & Psarras, 2003). Feinberg and Genethliou (2005) provide an updated review of different methods.

In this paper, we consider a methodology based solely on rigorous statistical arguments for modelling and forecasting the hourly electricity load of part of the southeast of Brazil. The area covered by the electric utility represents 25% of the state of Rio de Janeiro, totalling 11,132 km^2, and with a population of more than ten million people. The energy consumption corresponds to 75% of the total consumption in the Rio de Janeiro state. It is worth mentioning that this is one of the most important regions for tourism in Latin America. We adopt the same strategy as Fiebig, Bartels, and Aigner (1991), Peirson and Henley (1994), Ramanathan, Engle, Granger, Vahid-Arahi, and Brace (1997), Cottet and Smith (2003), and Soares and Souza (2006), treating each hour as a separate time series, such that 24 different models are estimated, one for each hour of the day. The approach considered in this paper is based on a two-step decomposition of the load series. In the first step, a component based on Fourier series, dummy variables, and a linear trend, is estimated to describe the long-run trend, the annual seasonality, the effects of the days of the week, and any other special days effects such as public holidays. In the second step, different linear autoregressive (AR) models are estimated. Neural network models are also considered in the second step. The type of decomposition considered here is not new. Similar proposals have been discussed in the literature during the last two decades; see, for example, Harvey and Koopman (1993), Temraz, Salama, and Quintana (1996), and Nowicka-Zagrajek and Weron (2002). However, we contribute to the literature in several different ways. First, to the best of our knowledge, the way in which we specify the models in each component is not common in the load forecasting literature, and relies only on rigorous classical statistical arguments. Recently, Cottet and Smith (2003) proposed a similar approach, but their methodology is fully based on Bayesian statistics and is computationally very demanding. Our methodology is simpler and can be used efficiently for real-time online load forecasting. Second, although very simple, this methodology is very flexible, allowing for different specifications in the second step. For example, neural networks and other nonlinear models may be estimated instead of a simple AR model. However, we show that the nonlinear effects are mainly related to the time-varying conditional variance and are not present in the conditional mean. Thus, the linear model is adequate to describe the dataset considered here. Furthermore, based on the bootstrap resampling technique, confidence intervals may easily be constructed under mild assumptions on the errors of the model. Finally, exogenous variables, when available, may easily be incorporated into the model.

The plan of the paper is as follows. Section 2 describes the dataset used in the paper. Section 3 presents the model and the modeling strategy. The benchmark model is discussed in Section 4. Section 5.1 shows the modeling results, and Section 5.2 presents the forecasting results. Final remarks are found in Section 6.

2. The Data

We consider a dataset containing hourly loads from January 1, 1990 to December 31, 2000. The period from January 1, 1990 to December 31, 1998 is used for estimation purposes (in-sample), and the data from the years 1999 and 2000 are left for forecast evaluation (out-of-sample). The data were obtained from an utility company from Rio de Janeiro, Brazil, and are shown in Fig. 1. This is the same dataset considered by Soares and Souza (2006). Fig. 1 shows the daily loads for each hour of the day during the in-sample period.

3. The Model

3.1. Mathematical Definition

The hourly load is modeled as the sum of two components. The first component is deterministic, representing the trend, the annual cycle, and the effects

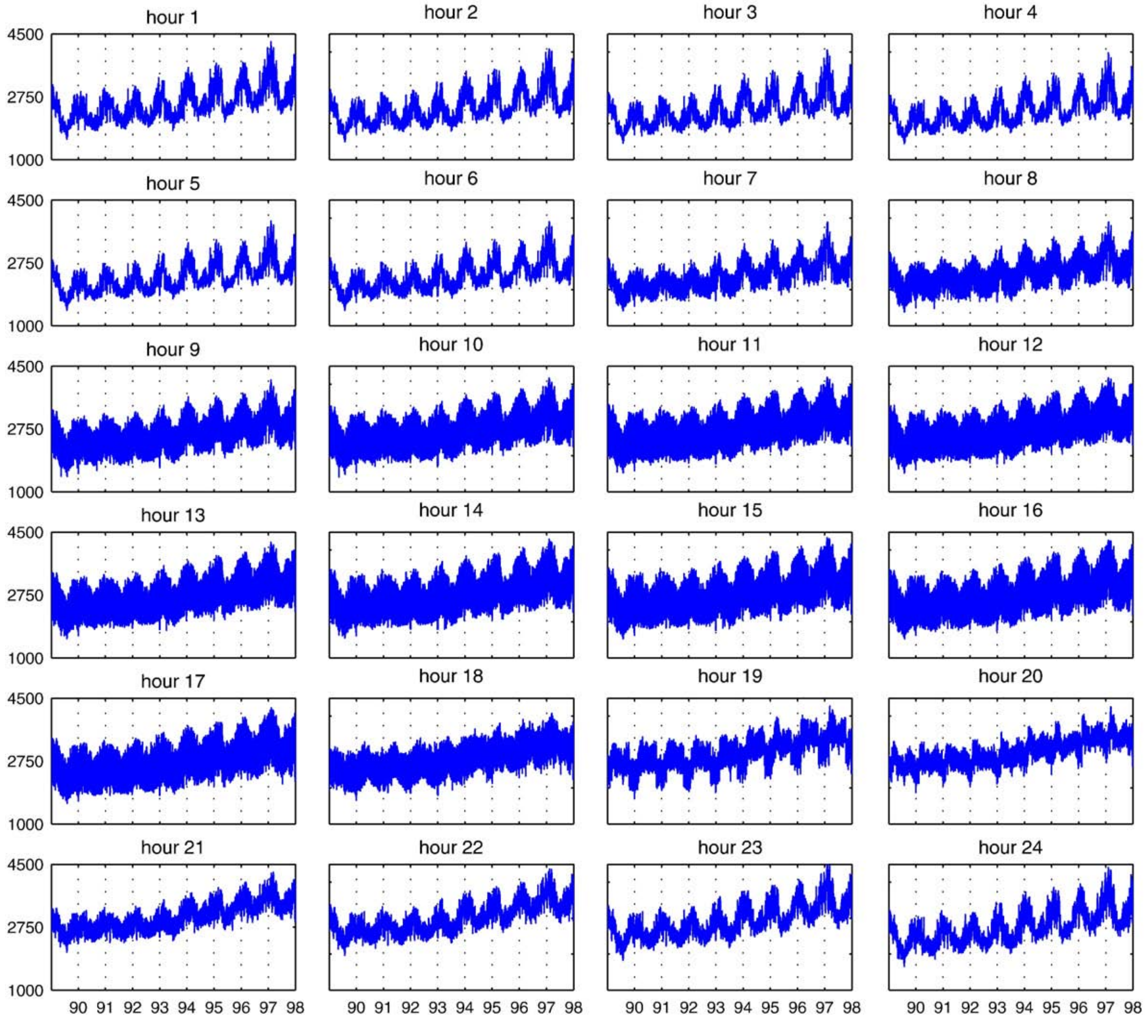

Fig. 1. Load for each hour from January 1, 1990 to December 31, 1998 (in-sample period).

of different types of days. The second component is stochastic.

First, we remove the daily cycle, following the ideas of Ramanathan et al. (1997).[1] We consider a separate model for each hour of the day, which avoids the need to model complicated intra-day patterns in the hourly load, commonly called the load profile. Each hour has a distinct weekly pattern; see also Fiebig et al. (1991), Connor, Atlas, and Martin (1992), Peirson and Henley (1994), Infield and Hill (1998), Cottet and Smith (2003), and Soares and Souza (2006). Hippert et al. (2001) report that difficulties in modeling the load profile are common to several load forecasting models.

The data seem to have a linear positive trend; see Fig. 1. This is corroborated by the traditional Phillips-Perron unit-root test (Phillips & Perron, 1988), where the null hypothesis of a stochastic trend (unit root) is strongly rejected for each of the 24 individual series.[2]

[1] The model described by Ramanathan et al (1997) was the winner of a load forecast competition organized by Puget Sound Power and Light Company, USA.

[2] We run the test including a constant and a linear trend in the test equation. For the nonparametric correction, we use the quadratic spectral kernel.

Furthermore, the positive trend in the load is correlated with economic and demographic factors. Hence, it is expected that the trend will be closely related to the potential Gross Domestic Product (GDP), which in the case of Brazil is known to be almost linear; see Carneiro (2000) for a discussion. All that said, we model the trend as a deterministic linear function of time. Most papers in the load forecasting literature take first-order differences of the load series without previously testing for unit roots; see Reis and Silva (2005) for an example. This has a major drawback. When the trend is deterministic, taking first differences introduces a non-invertible moving average component into the data generating process, causing serious estimation problems. Furthermore, there is no linear autoregressive model that is able to correctly describe the dynamics of the data; see the discussion in Chapter 4 of Enders (2004). Finally, it seems that there is a break in the trend after 1999. As this break occurs in the out-of-sample period, we ignore it during the specification and estimation of the proposed model. This is important in order to test the robustness of the proposed model.

As is shown in Fig. 1, the time series displays a clear daily, weekly, and annual seasonality. We can see that the annual seasonality is more apparent during the night. This is mainly due to the fact that during the night the effects of the days of the week are less significant. The weekly seasonality – effects of the days of the week and special days, such as holidays – is modeled with different dummy variables. Several authors claim that Tuesdays, Wednesdays, Thursdays and Fridays can be modeled as a single type of day. Since we have a large amount of data, we prefer to model each day as a dummy variable. We also consider dummies for holidays, part-time holidays, and special days. Table 1 gives a summary of the variables used. Cottet and Smith (2003) adopted a similar approach. The classification of days are based on the particularities of Brazil. There are many special holidays when people have to work only during the morning or only in the afternoon. For example, the Wednesday immediately after the Carnival is a part-time working day and must be treated differently.

The annual cycle is modeled as a sum of sines and cosines, as in a Fourier decomposition. The motivation for this can easily be seen by a graphical inspection of Fig. 1. The number of trigonometric functions is determined by the Bayesian Information Criterion (SBIC) proposed by Schwarz (1978). Schneider, Takenawa, and Schiffman (1985) and El-Keib, Ma, and Ma (1995) have considered the same strategy. However, they applied the Fourier decomposition to a single hourly series instead of 24 different daily series. Furthermore, they determined the number of terms in the decomposition using a different method to that considered here. More recently, Cottet and Smith (2003) used trigonometric functions to model the seasonality in the load series. The authors also considered a distinct model for each hour of the day, but held the number of sines and cosines fixed. In addition, their approach was based on Bayesian statistics.

Table 1
Types of days used in the TLSAR model

Code	Description
1	Sunday
2	Monday
3	Tuesday
4	Wednesday
5	Thursday
6	Friday
7	Saturday
8	Holiday (official or religious)
9	Working day after a holiday
10	Working day before a holiday
11	Working day between a holiday and a weekend
12	Saturday after a holiday
13	Work only during the mornings
14	Work only during the afternoons
15	Special holidays

We do not include external variables, such as those related to temperature. This is an important point, as some temperature measures (maxima, averages, and others) could substantially improve the prediction if used, particularly in the summer, when the air conditioning appliances constitute a great part of the load. The reasons for not using weather variables are twofold. First, as mentioned in the Introduction, the area covered by the electric utility considered in this paper represents 25% of the state of Rio de Janeiro, totalling 11,132 km^2, which includes some sub-regions with temperatures that range from 10 degrees Celsius during the winter to 24 degrees during the summer; and other sub-regions with temperatures that vary between 23 (winter) and 42 (summer) degrees Celsius. For example, on any given day, it is common to observe two or more subregions with temperature differences of around 10 degrees Celsius at the one time. However, the hourly temperature measures available are collected at few points in the city of Rio de Janeiro (not the state), and do not give a complete picture of the temperature profile of the area covered. Second, the data available have a large number of deficient observations, including outliers and missing values, which distort the results and do not make any relevant contribution to the forecasting performance of the model. All that being said, we decide not to include the temperature in our model. Nevertheless, it should be mentioned that it is straightforward to include weather variables in this model whenever they are available. Many other recent papers in the literature have adopted the same strategy and do not include temperatures; see, for example, Carpinteiro, Reis, and Silva (2004), Taylor et al. (2006) and Soares and Souza (2006).

The proposed model is called the Two-Level Seasonal Autoregressive (TLSAR) model.

Definition 1. *The time series $L_{h,d}$, representing the load of the hour h, $h=1,\ldots,24$, and day d, $d=1,\ldots,D$, where D is the total number of days, follows a Two-Level Seasonal Autoregressive (TLSAR) model if*

$$L_{h,d} = L^P_{h,d} + L^I_{h,d}, \tag{1}$$

where

$$L^P_{h,d} = \alpha_0 + \rho d + \sum_{r=1}^{H} [\alpha_r cos(\omega r d) + \beta_r sin(\omega r d)] + \sum_{i=1}^{K} \mu_i \delta_i \tag{2}$$

is the "potential load";

$$L^I_{h,d} = \phi' \mathbf{z}_{h,d} + \varepsilon_{h,d} \tag{3}$$

is the "irregular load"; $\alpha_r\cos(\omega r d)+\beta_r\sin(\omega r d)$ is known as the r^{th} harmonic; $\omega=2\pi/365$; δ_i, $i=1,\ldots,K$ are dummy variables identifying the days of the week, public holidays, special days, etc; α_0, ρ, α_r, β_r, $r=1,\ldots,H$, and μ_i, $i=1,\ldots,K$ are unknown parameters. The vector $\mathbf{z}_{h,d}$ is formed by a constant and a subset of p lags of $L^I_{h,d}$; $\phi\in\mathbb{R}^{p+1}$ is a vector of unknown parameters; and $\varepsilon_{h,d}$ is an error term.

We make the following assumption about the error term.

Assumption 1. *The sequence of random variables $\{\varepsilon_{h,d}\}$, $h=1,\ldots,24$, is drawn from a continuous (with respect to the Lebesgue measure on the real line), positive everywhere density, and bounded in a neighborhood of 0. Furthermore, $\mathbf{E}(\varepsilon_{h,d}|\mathcal{F}_{d-1})=0$ and $\mathbf{E}(\varepsilon^2_{h,d}|\mathcal{F}_{d-1})=\sigma^2_{h,d}<\infty$, $\forall d$, where $\mathcal{F}_{d-1}$ is the full information set at day $d-1$.*

Assumption 1 is weak. Like Huang and Shih (2003) and Al-Hamadi and Soliman (2004), we do not assume that the errors are Gaussian. The distribution of the errors may be skewed and leptokurtic. However, unlike those authors, we allow for possible conditional heteroskedasticity, as $\mathbf{E}(\varepsilon^2_{h,d}|\mathcal{F}_{d-1})$ is not assumed to be constant; see Bollerslev (1986) for a discussion. Although we are not modeling the second moments explicitly, possible heteroskedasticity is taken into account in the statistical inference. Finally, the specification of the "irregular load" does not need to be linear. In this paper, we focus on a simple linear autoregressive model because more complicated neural network models do not improve the forecasting performance significantly (see Section 5.2). Moving average terms are not considered either, as their inclusion does not bring any benefits in terms of forecasting performance.

3.2. Modeling Strategy

In summary, the estimation procedure is carried out as follows:

- For each hour, estimate α_0, ρ, α_r, β_r, and μ_i, $r=1,\ldots,H$, and $i=1,\ldots K$, in Eq. (2) by ordinary least squares (OLS). The number of harmonics (H) is determined by minimizing the Schwartz Bayesian Information Criterion (SBIC). The number of dummies (K), representing the different types of days, is kept fixed and equal to 15, as described in Table 1.[3]

[3] Allowing the number of dummies to vary does not provide any benefit in terms of fitting and/or forecasting.

- After estimating the "potential load", we compute the residuals $\widehat{L}^{I}_{h,d}=L_{h,d}-\widehat{L}^{P}_{h,d}$, where

$$\widehat{L}^{P}_{h,d} = \widehat{\alpha}_0 + \widehat{\rho}d + \sum_{r=1}^{H}\left[\widehat{\alpha}_r cos(\omega rd) + \widehat{\beta}_r sin(\omega rd)\right] + \sum_{i=1}^{K}\widehat{\mu}_i\delta_i.$$

- Again, using the SBIC, we select the best combination of lags in $\mathbf{z}_{h,d}$ in Eq. (3) from among the first seven lags in the series.[4] The autoregressive model is estimated by OLS, and standard errors that are robust to heteroskedasticity are computed using White's estimator (White, 1980). Apart from the intercept, statistically insignificant lags are excluded from the model.[5]

The specification and estimation of Eqs. (2) and (3) can be done jointly. We decided to estimate them separately in order to reduce the computational burden of determining the number of harmonics and lags simultaneously. However, the final results do not change if joint specification and estimation is used instead.

After a model has been estimated, it is submitted to a number of misspecification tests. First, we test for no remaining serial correlation in the residuals using the Ljung-Box test (Godfrey, 1988). We also test for possible nonlinearities in the conditional mean, using the neural network test proposed by Teräsvirta, Lin, and Granger (1993), with the heteroskedasticity correction discussed by Medeiros, Teräsvirta, and Rech (2006). This test is important to justify the linear specification of the "irregular component". The Lagrange Multiplier test for conditional heteroskedasticity proposed by Engle (1982) is also considered. Conditional heteroskedasticity, if present, must be taken into account when computing confidence intervals. Finally, in order to verify whether the trend is modeled correctly, we apply the Phillips-Perron unit root test to the estimated residuals. Any evidence of nonstationarity is an indication that the trend has been modeled incorrectly. Although misspecification testing is a standard procedure in time series econometrics, it has been neglected in most short-term load forecasting applications.

In order to check the forecasting performance of the TLSAR model in forecasting, we consider a benchmark model, as described in Section 4.

4. The Benchmark Model

The benchmark model considered in this paper is a modified version of the Seasonal Integrated Autoregressive Moving Average (SARIMA) model, with dummy variables to correct for the effects of holidays and special days. Several other authors have used similar models as benchmarks; see, for example, Darbellay and Slama (2000) and Nowicka-Zagrajek and Weron (2002), among others. The main difference between this and previous studies is that we model each hour separately.[6]

Definition 2. *The time series $L_{h,d}$ representing the load of hour h, $h=1,\ldots,24$, and day d, $d=1,\ldots,D$, where D is the total number of days, follows a Dummy-Adjusted SARIMA (DASARIMA) model if*

$$\left(1-\phi_1 B-\phi_2 B^2-\cdots-\phi_p B^p\right)\overline{\Delta_7\Delta_1 L_{h,d}} = \left(1+\theta_1 B+\cdots+\theta_q B^q\right)\left(1+\beta B^7\right)\varepsilon_{h,d}, \quad (4)$$

where

$$\overline{\Delta_7\Delta_1 L_{h,d}} = \Delta_7\Delta_1 L_{h,d} - \sum_{i=1}^{M}\alpha_i\delta_{i,d} - \sum_{i=1}^{M}\lambda_i\delta_{i,d-1} - \sum_{i=1}^{M}\gamma_i\delta_{i,d-7}; \quad (5)$$

$\Delta_j=(1-B^j)$, $j=1,7$; B is the lag operator;[7] ϕ_l, $l=1,\ldots,p$, θ_k, $k=1,\ldots,q$, β, α_i, λ_i, and γ_i, $i=1,\ldots,M$, are parameters; δ_i, $i=1,\ldots,M$ are dummy variables identifying public holidays, special days, etc; and $\varepsilon_{h,d}$ is a zero mean error term with finite second-moments.

The first difference (Δ_1) is important to remove the long-term trend in the data. It seems that the effects of the seasons of the year are also removed by taking the first differences of the hourly series. It is important to

[4] Seven lags are enough to model the remaining autocorrelation in the load series considered in this paper. For other datasets it is possible that higher orders may be necessary.

[5] We consider the standard 5% significance level.

[6] This type of SARIMA model was also considered by Soares and Souza (2006).

[7] The lag operator is defined as $B^j L_{h,d}=L_{h,d-j}$.

mention that the DASARIMA model considers that the trend is stochastic instead of deterministic. This is a major difference between the DASARIMA and TLSAR models, and may be of extreme importance, as there is an apparent break in the trend in the out-of-sample period. The inclusion of the seventh-order difference and the seasonal moving average term are important to remove the weekly seasonality. The inclusion of higher order terms is not necessary, as simple residual analysis shows. In Table 2 we illustrate the types of days included in the DASARIMA model. The classification of days is different from the one considered in the TLSAR methodology, because the seventh-order difference in Eq. (5) removes the days-of-the-week effect. Consequently, we consider only the anomalous days, such as special holidays.[8]

The selection of lags is based on the analysis of the autocorrelation function (ACF), the partial autocorrelation (PACF) function and the SBIC. The ACF and PACF for each $\overline{\Delta_7 \Delta_1 L_{h,d}}$, $h=1,\ldots, 24$, $d=1,\ldots, D$, are used to roughly identify the orders of the ARMA component, which are further refined using the SBIC.

Table 2
Types of days used in the DASARIMA model

Code	Description
1	Weekdays (Sun, Mon, Tue, Wed, Thu, Fri, and Sat)
2	Holiday (official or religious)
3	Working day after a holiday
4	Working day before a holiday
5	Working day between a holiday and a weekend
6	Saturday after a holiday
7	Work only during the mornings
8	Work only during the afternoons
9	Special holidays

5. The Experiment

The experiment considered in this paper consists of computing multi-step forecasts of the hourly load from 1 to 7 days ahead using both the TLSAR and DASARIMA models. Section 5.1 shows the specification and estimation results. The forecasting results and comparisons are described in Section 5.2. All models are estimated on a computer with a Pentium V 2.2 GHz processor with 1 Gb of Ram memory and running Matlab 7.[9] The computational time for specifying and estimating all 24 models is negligible, not being over 60 seconds.

5.1. Specification and Estimation

Table 3 shows, for each hour of the day, the estimated number of harmonics and the estimated parameters of the autoregressive model with standard errors robust to heteroskedasticity. All autoregressive coefficients are significant at the 5% level. The table also shows the p value of the Ljung-Box test for no error serial autocorrelation of order 7. It is clear that the errors are not serially correlated, which indicates that the lags are correctly specified. Although this is not shown in the table, the Phillips-Perron test strongly rejects the null of nonstationarity (unit-roots) for all of the series, indicating that the linear detrending has successfully removed the trend from all 24 series.[10] The p values of the neural network linearity test proposed by Teräsvirta et al. (1993), with the heteroskedasticity correction discussed by Medeiros et al. (2006), are also reported in Table 3. At the 5% level, the null of linearity is rejected only for hours 10, 13, and 14, although not strongly. When the 1% level is considered, there is no evidence of nonlinearity for any series, apart from hour 14. In order to compare the results for the linear specification with a nonlinear alternative, a neural network (NN) model is estimated with Bayesian regularization in conjunction with the Levenberg-Marquadt algorithm for all series; see MacKay (1992a,b). However, in general, the forecasting results are inferior to the ones from the linear model. A similar result has been reported in the literature by Darbellay and Slama (2000). They found that the short-term evolution of the Czech electricity load is primarily a linear problem. On the other hand, when conditional heteroskedasticity is tested using Engle's ARCH LM test, the null hypothesis of homoskedasticity is strongly rejected for all series, indicating the presence of time-varying conditional

[8] The effects of the special days are removed from the differenced data because the raw series are nonstationary.

[9] For the estimation of the TLSAR and DASARIMA models, no additional Matlab toolbox is necessary. For the neural network estimation, the neural network toolbox is used.

[10] Detailed results can be obtained from the authors.

Table 3
Estimated parameters for the TLSAR model

Hour	Number of Harmonics	$\widehat{\phi}_0$	$\widehat{\phi}_1$	$\widehat{\phi}_2$	$\widehat{\phi}_3$	$\widehat{\phi}_4$	$\widehat{\phi}_5$	$\widehat{\phi}_6$	$\widehat{\phi}_7$	Ljung-Box	NN
1	2	−0.052	0.855	−0.137	–	0.046	–	–	0.057	0.982	0.318
		(2.443)	(0.023)	(0.026)		(0.018)			(0.015)		
2	2	−0.082	0.875	−0.148	–	0.044	–	–	0.058	0.936	0.226
		(2.321)	(0.023)	(0.027)		(0.018)			(0.015)		
3	2	−0.067	0.912	−0.181	–	0.051	–	–	0.052	0.943	0.456
		(2.174)	(0.023)	(0.026)		(0.018)			(0.015)		
4	2	−0.081	0.924	−0.191	–	0.056	–	–	0.049	0.922	0.322
		(2.080)	(0.022)	(0.026)		(0.018)			(0.015)		
5	2	−0.083	0.933	−0.199	–	0.060	–	–	0.047	0.702	0.306
		(1.991)	(0.022)	(0.025)		(0.017)			(0.014)		
6	2	−0.103	0.916	−0.175	–	0.054	–	–	0.053	0.791	0.145
		(1.886)	(0.022)	(0.025)		(0.018)			(0.014)		
7	5	−0.164	0.814	−0.090	–	0.047	–	–	0.068	0.995	0.068
		(1.773)	(0.022)	(0.023)		(0.018)			(0.015)		
8	3	−0.136	0.721	–	–	–	–	–	0.086	0.335	0.659
		(1.770)	(0.018)						(0.014)		
9	2	−0.140	0.660	–	–	–	–	–	0.115	0.972	0.424
		(1.972)	(0.022)						(0.015)		
10	2	−0.164	0.610	–	–	–	–	–	0.143	0.645	0.038
		(2.092)	(0.024)						(0.016)		
11	2	−0.211	0.577	–	–	–	–	–	0.156	0.988	0.094
		(2.169)	(0.026)						(0.016)		
12	2	−0.219	0.583	–	–	–	–	–	0.159	0.958	0.113
		(2.135)	(0.026)						(0.016)		
13	2	−0.224	0.592	–	–	–	–	–	0.154	0.911	0.027
		(2.165)	(0.025)						(0.016)		
14	3	−0.201	0.582	−0.004	0.042	–	–	–	0.138	0.940	0.001
		(2.302)	(0.034)	(0.026)	(0.019)				(0.015)		
15	3	−0.219	0.588	−0.007	0.045	–	–	–	0.130	0.884	0.133
		(2.402)	(0.033)	(0.026)	(0.019)				(0.014)		
16	2	−0.229	0.611	–	–	–	–	–	0.135	0.683	0.090
		(2.363)	(0.023)						(0.015)		
17	2	−0.267	0.555	–	–	0.047	–	–	0.136	0.972	0.146
		(2.367)	(0.026)			(0.022)			(0.017)		
18	1	−0.283	0.519	–	–	0.084	–	–	0.127	0.886	0.282
		(2.167)	(0.023)			(0.024)			(0.017)		
19	5	−0.318	0.597	–	–	0.072	–	0.043	0.092	0.410	0.393
		(1.887)	(0.021)			(0.021)		(0.020)	(0.018)		
20	6	−0.291	0.641	–	–	0.063	–	–	0.110	0.661	0.053
		(1.672)	(0.017)			(0.016)			(0.014)		
21	3	−0.215	0.706	−0.043	0.049	–	–	0.056	0.077	0.862	0.457
		(1.654)	(0.027)	(0.026)	(0.018)			(0.018)	(0.018)		
22	2	−0.210	0.784	−0.080	–	0.045	–	0.031	0.058	0.787	0.101
		(1.891)	(0.023)	(0.023)		(0.017)		(0.020)	(0.019)		
23	2	−0.185	0.825	−0.121	–	0.054	–	–	0.058	0.938	0.382
		(2.201)	(0.022)	(0.023)		(0.017)			(0.015)		
24	2	−0.163	0.858	−0.144	–	0.046	–	–	0.059	0.815	0.388
		(2.374)	(0.021)	(0.024)		(0.017)			(0.015)		

variances. In terms of estimation and point forecasts, this is not a drawback. However, in order to compute confidence intervals for the future loads it is important to take the conditional heteroskedasticity into account. Finally, a point that deserves attention is the fact that the final model specification differs across the hours, corroborating our view that the different hours need to be modelled separately because they have different structures and dynamics.

Fig. 2 shows the sum of the estimated harmonics for each hour of the day. As can be seen, for most of the hours two harmonics are sufficient to model the annual pattern. Apart from hours 7, 8, 14, 15, and 18–21, the annual pattern is fairly clear. First, there is a "summer regime" that begins more or less in November and goes approximately until March. The "winter regime" starts in April and ends in July, as the temperatures usually start to rise in August. However, the extremely high temperatures (over 30 Degrees Celsius) are most common from November to March. For that reason, there is a "spring regime" starting approximately in August and ending in October. This pattern is clearer during the night, due to the use of air-conditioning. Hours 18–20 are quite different because of several factors: public lighting, daylight saving period, holidays, etc. For example, public illumination starts earlier in the winter than in the summer.

Concerning the DASARIMA model, the selected order of the autoregressive term is one for all of the series. The selected order of the moving average term

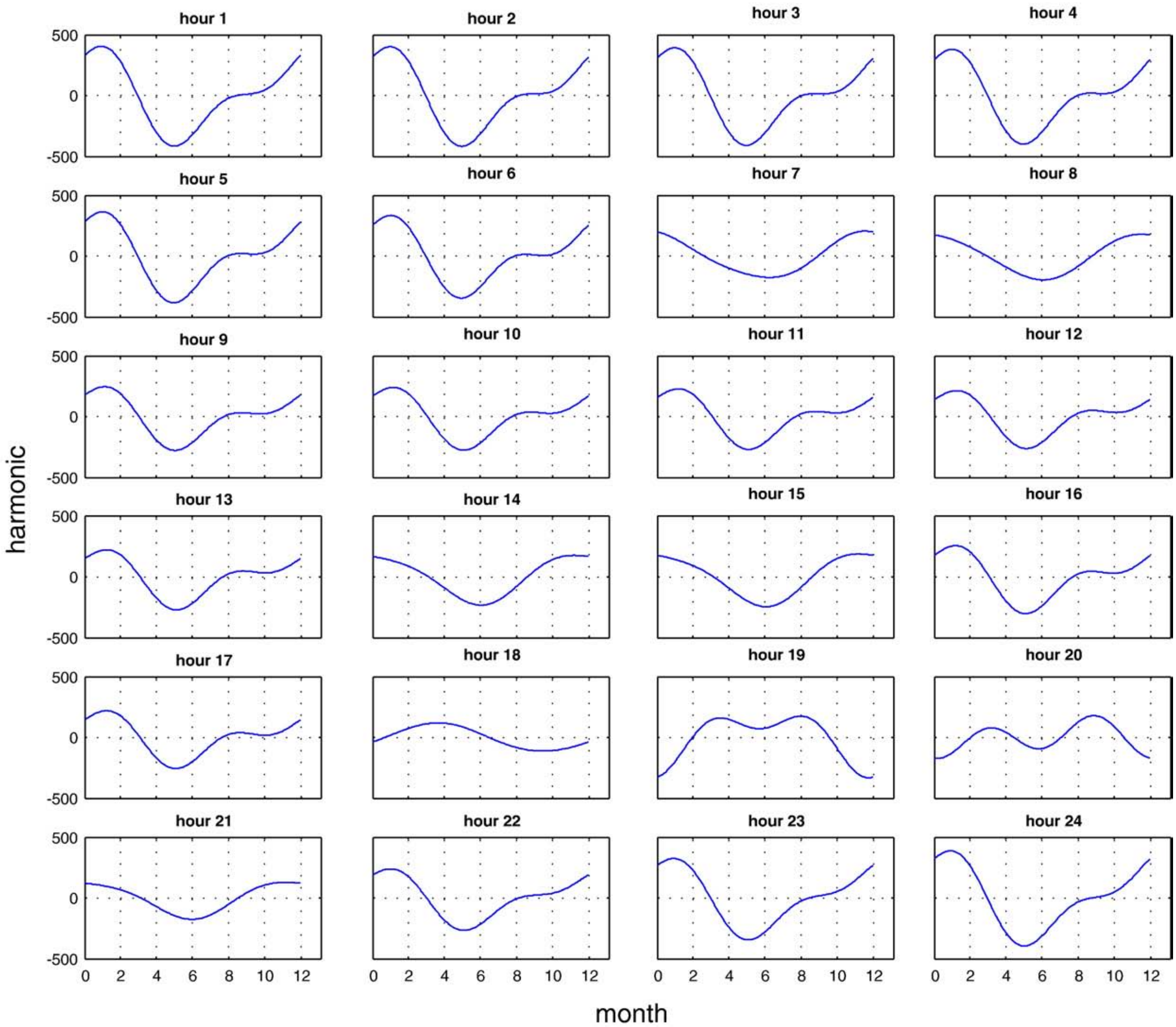

Fig. 2. Estimated harmonic shape for each hour.

is three for 19 series, two for two series, and one for three series. Table 4 shows the estimated parameters of the DASARIMA model, as well as the p values of the Ljung-Box test for autocorrelation up to order seven. The values in parentheses are the respective standard errors robust to heteroskedasticity. There is remaining autocorrelation for several hours. Increasing the lag order of both the autoregressive and moving average terms does not attenuate the problem. The inclusion of higher-order seasonal terms does not solve the problem either. These facts point to the possible misspecification of the DASARIMA model.

5.2. Forecasting

This section reports the forecasting results for both the TLSAR and DASARIMA models. The results for the generalized long memory (GLM) model discussed by Soares and Souza (2006) are also included. Although the linearity test does not show evidence of nonlinearity, NN models are estimated as an alternative to the linear specification in the second step of the TLSAR methodology. The NN models are estimated by Bayesian regularization with 10 hidden units, and the first seven lags of the estimated "irregular component" in the first step.[11] We use the Mean Absolute Percentage Error (MAPE) to compare the models. An important point deserves attention. Several authors achieve MAPEs as low as 2% when predicting the total daily load,[12] but the results of different models cannot be compared on different datasets because of the differences among load curves in different countries. For example, the load profile of a country with tropical weather, such as Brazil, is distinct from one like the USA or the United Kingdom. Hence, if different datasets are used, the same model(s) must be used, and the comparison should be made among data sets and not models. If the researcher wants to compare the performance of different models, the same data with the same forecasting period must be used.

As far as the present dataset is concerned, Tables 5 and 6 show the MAPEs for one- to seven-days-ahead for the years of 1999 and 2000. The bold figures indicate which model attains the lowest MAPE. Considering 1999, the TLSAR model outperforms all of

Table 4
Parameter estimates and diagnostic tests for the DASARIMA model

Hour	$\widehat{\phi}_1$	$\widehat{\theta}_1$	$\widehat{\theta}_2$	$\widehat{\theta}_3$	$\widehat{\beta}$	Ljung-Box
1	0.281	−0.300	−0.164	−0.080	−0.817	0.461
	(0.143)	(0.143)	(0.022)	(0.038)	(0.012)	
2	0.307	−0.324	−0.165	−0.087	−0.823	0.201
	(0.132)	(0.132)	(0.023)	(0.037)	(0.013)	
3	0.305	−0.284	−0.173	−0.098	−0.825	0.211
	(0.124)	(0.123)	(0.022)	(0.034)	(0.012)	
4	0.282	−0.252	−0.174	−0.112	−0.819	0.195
	(0.119)	(0.118)	(0.020)	(0.033)	(0.012)	
5	0.262	−0.219	−0.177	−0.114	−0.813	0.084
	(0.122)	(0.121)	(0.020)	(0.032)	(0.012)	
6	0.261	−0.230	−0.149	−0.111	−0.787	0.141
	(0.128)	(0.128)	(0.021)	(0.031)	(0.013)	
7	–	−0.043	−0.094	−0.105	−0.726	0.231
		(0.021)	(0.020)	(0.020)	(0.014)	
8	–	−0.076	−0.041	−0.049	−0.623	0.023
		(0.024)	(0.021)	(0.020)	(0.018)	
9	–	−0.114	–	−0.033	−0.576	0.023
		(0.017)		(0.017)	(0.014)	
10	–	−0.143	–	–	−0.530	0.001
		(0.028)			(0.022)	
11	–	−0.157	–	–	−0.542	0.002
		(0.029)			(0.021)	
12	–	−0.133	–	–	−0.544	0.000
		(0.029)			(0.021)	
13	–	−0.135	−0.042	–	−0.567	0.003
		(0.017)	(0.017)		(0.014)	
14	–	−0.129	−0.064	–	−0.563	0.005
		(0.029)	(0.019)		(0.020)	
15	–	−0.122	−0.063	−0.035	−0.570	0.012
		(0.028)	(0.020)	(0.020)	(0.020)	
16	–	−0.125	−0.056	−0.053	−0.576	0.030
		(0.028)	(0.020)	(0.021)	(0.020)	
17	–	−0.179	−0.041	−0.055	−0.607	0.035
		(0.028)	(0.020)	(0.020)	(0.018)	
18	−0.538	0.292	−0.162	−0.080	−0.629	0.002
	(0.229)	(0.228)	(0.062)	(0.023)	(0.017)	
19	−0.476	0.264	−0.158	−0.070	−0.633	0.006
	(0.437)	(0.435)	(0.095)	(0.033)	(0.016)	
20	–	−0.167	−0.052	−0.065	−0.669	0.006
		(0.022)	(0.019)	(0.018)	(0.015)	
21	–	−0.131	−0.108	−0.062	−0.693	0.120
		(0.025)	(0.019)	(0.020)	(0.015)	
22	0.191	−0.269	−0.119	−0.082	−0.734	0.271
	(0.170)	(0.171)	(0.026)	(0.034)	(0.014)	
23	0.233	−0.289	−0.158	−0.092	−0.776	0.407
	(0.137)	(0.138)	(0.024)	(0.034)	(0.014)	
24	0.292	−0.330	−0.168	−0.076	−0.778	0.336
	(0.136)	(0.137)	(0.023)	(0.037)	(0.014)	

[11] Bayesian regularization is a pruning technique that avoids overfitting.
[12] See, for example, Park, El-Sharkawi, Marks II, Atlas, and Damborg (1991).

Table 5
Forecasting comparison for each hour of the day for 1999

Hour	TLSAR							DASARIMA						
	Forecasting horizon (in days)							Forecasting horizon (in days)						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	**3.76**	5.12	5.67	**5.82**	5.95	6.07	6.18	3.93	**4.93**	**5.63**	5.84	**5.83**	**5.63**	**4.72**
2	**3.68**	5.17	5.74	5.93	6.06	6.19	6.33	3.85	**4.79**	**5.37**	**5.50**	**5.45**	**5.39**	**4.66**
3	3.60	5.06	5.67	5.89	5.96	6.10	6.25	3.77	**4.65**	**5.30**	**5.43**	**5.28**	**5.21**	**4.49**
4	3.63	4.93	5.54	5.76	5.87	6.01	6.15	3.72	**4.65**	**5.31**	**5.46**	**5.38**	**5.23**	**4.43**
5	3.38	4.55	5.14	5.37	5.47	5.62	5.73	3.47	**4.24**	**5.02**	**5.21**	**5.18**	**4.89**	**4.08**
6	3.08	4.15	**4.63**	**4.84**	**4.93**	**5.07**	5.17	3.34	4.38	5.34	5.93	5.97	5.42	**4.36**
7	**2.83**	3.60	**3.94**	**4.06**	**4.13**	**4.20**	**4.27**	3.69	6.26	8.26	9.13	9.20	8.42	6.32
8	**2.69**	3.28	**3.53**	**3.61**	**3.67**	**3.74**	**3.80**	4.09	8.43	11.04	11.8	11.99	11.38	8.38
9	2.74	3.29	**3.47**	**3.51**	**3.56**	**3.59**	**3.60**	4.63	10.33	13.18	13.76	13.98	13.62	10.05
10	**2.76**	3.28	**3.46**	**3.46**	**3.49**	**3.50**	**3.51**	4.91	11.38	14.42	14.89	15.07	14.84	11.05
11	**2.76**	3.28	**3.45**	**3.44**	**3.44**	**3.46**	**3.47**	4.95	11.81	15.19	15.48	15.67	15.57	11.40
12	**2.71**	3.21	**3.32**	**3.33**	**3.33**	**3.33**	**3.34**	5.02	12.06	15.61	15.81	15.93	15.96	11.58
13	**2.86**	3.38	**3.52**	**3.55**	**3.55**	**3.56**	**3.56**	5.04	11.69	15.32	15.64	15.72	15.50	11.19
14	**3.11**	3.71	**3.88**	**3.92**	**3.90**	**3.90**	**3.89**	5.10	12.01	15.91	16.14	16.20	16.04	11.45
15	3.23	3.84	**4.02**	**4.07**	**4.08**	**4.07**	**4.07**	5.16	12.45	16.78	17.08	17.16	16.89	11.79
16	**3.18**	3.70	**3.97**	**3.99**	**4.01**	**4.01**	**4.02**	5.11	12.40	16.92	17.33	17.34	16.93	11.70
17	**3.04**	3.50	**3.72**	**3.76**	**3.78**	**3.79**	**3.80**	4.82	11.57	16.16	16.57	16.64	16.11	10.97
18	**2.81**	3.25	**3.43**	**3.51**	**3.53**	**3.54**	**3.55**	4.27	8.97	11.95	12.46	12.56	12.19	8.65
19	**2.73**	**3.27**	**3.41**	**3.46**	**3.51**	**3.54**	**3.56**	3.54	6.04	7.55	8.02	8.01	7.89	6.12
20	**2.33**	**2.76**	**2.80**	**2.82**	**2.87**	**2.89**	**2.93**	3.07	4.90	5.63	5.98	5.94	5.77	4.88
21	2.31	**2.78**	**2.89**	**2.94**	**2.92**	**2.95**	**3.02**	2.85	4.47	5.44	5.84	5.82	5.44	4.42
22	**2.56**	**3.18**	**3.45**	**3.46**	**3.44**	**3.47**	**3.53**	3.08	4.85	6.21	6.73	6.70	6.09	4.76
23	3.50	4.35	4.68	4.69	**4.67**	**4.73**	**4.82**	4.19	5.66	6.96	7.43	7.33	6.85	5.47
24	4.71	5.85	6.28	6.36	6.35	6.43	6.56	5.42	6.60	7.57	7.93	7.95	7.54	6.43

Hour	GLM							NN						
	Forecasting horizon (in days)							Forecasting horizon (in days)						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	**3.76**	5.14	5.82	6.08	6.30	6.49	6.71	5.01	5.02	5.78	6.26	6.29	6.25	6.11
2	**3.68**	5.09	5.84	6.12	6.33	6.59	6.81	5.17	5.18	5.97	6.47	6.57	6.51	6.38
3	**3.54**	4.97	5.75	6.00	6.18	6.47	6.70	5.08	5.10	5.88	6.48	6.62	6.51	6.35
4	**3.35**	4.67	5.54	5.73	5.92	6.18	6.40	5.05	5.06	5.75	6.40	6.55	6.47	6.34
5	**3.21**	4.39	5.20	5.39	5.56	5.83	6.04	4.59	4.60	5.33	5.95	6.12	6.02	5.96
6	**2.91**	**4.09**	4.70	4.93	5.07	5.35	5.54	4.13	4.13	4.84	5.40	5.57	5.48	5.48
7	2.85	3.69	4.16	4.30	4.43	4.63	4.72	3.51	**3.51**	4.08	4.59	4.72	4.63	4.66
8	2.74	3.37	3.75	3.88	4.04	4.22	4.31	2.87	**2.87**	3.57	4.06	4.13	4.04	4.06
9	2.75	3.45	3.76	3.86	3.98	4.12	4.21	**2.70**	**2.72**	3.52	3.96	3.99	3.91	3.78
10	2.82	3.46	3.78	3.86	3.93	4.00	4.07	2.77	**2.79**	3.52	3.97	4.02	3.91	3.77
11	2.92	3.59	3.86	3.97	4.01	4.04	4.05	2.80	**2.82**	3.55	3.97	4.06	3.84	3.68
12	2.89	3.61	3.88	3.96	3.97	3.98	4.00	2.78	**2.78**	3.49	3.87	3.98	3.80	3.61
13	2.99	3.79	4.07	4.17	4.19	4.19	4.24	2.97	**3.00**	3.67	4.05	4.22	4.00	3.84
14	3.15	4.06	4.40	4.51	4.49	4.49	4.52	3.40	**3.42**	4.02	4.43	4.52	4.35	4.14
15	**3.22**	4.16	4.52	4.63	4.67	4.67	4.73	3.58	**3.60**	4.18	4.51	4.59	4.40	4.20
16	3.21	4.11	4.48	4.60	4.70	4.74	4.77	3.57	**3.59**	4.13	4.45	4.46	4.28	4.12
17	3.26	3.84	4.17	4.27	4.35	4.40	4.43	3.33	**3.36**	3.97	4.31	4.23	4.13	4.00
18	2.95	3.49	3.71	3.79	3.81	3.85	3.90	3.22	**3.22**	3.61	3.80	3.77	3.76	3.69
19	2.82	3.38	3.57	3.73	3.87	3.94	3.99	3.85	3.87	4.13	4.30	4.15	4.15	4.12

Table 5 (*continued*)

Hour	GLM							NN						
	Forecasting horizon (in days)							Forecasting horizon (in days)						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
20	2.35	2.87	2.93	3.04	3.05	3.07	3.12	3.19	3.19	3.58	3.68	3.58	3.57	3.45
21	**2.23**	2.83	3.10	3.21	3.27	3.35	3.47	3.05	3.04	3.43	3.51	3.41	3.45	3.44
22	2.56	3.39	3.75	3.88	3.93	4.02	4.13	3.55	3.54	3.88	3.89	3.80	3.78	3.79
23	**3.04**	**3.99**	**4.45**	**4.66**	4.72	4.87	5.01	4.62	4.61	4.98	5.04	4.92	4.81	**4.78**
24	**3.54**	**4.75**	**5.33**	**5.60**	**5.72**	**5.92**	**6.10**	6.24	6.22	6.66	6.71	6.61	6.46	6.45

Mean absolute percentage errors from the TLSAR, DASARIMA, GLM and NN models.

Table 6
Forecasting comparison for each hour of the day for 2000

Hour	TLSAR							SARIMA						
	Forecasting horizon (in days)							Forecasting horizon (in days)						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	**4.34**	6.58	7.51	7.95	8.22	8.30	8.33	4.53	**5.77**	**6.55**	**6.71**	**6.73**	**6.65**	**5.70**
2	**4.45**	6.70	7.70	8.20	8.50	8.60	8.62	4.53	**5.73**	**6.42**	**6.55**	**6.54**	**6.50**	**5.65**
3	**4.30**	6.56	7.67	8.20	8.50	8.60	8.63	4.44	**5.58**	**6.27**	**6.36**	**6.35**	**6.33**	**5.48**
4	**4.18**	6.34	7.49	7.98	8.30	8.40	8.45	4.35	**5.30**	**6.05**	**6.15**	**6.15**	**6.06**	**5.24**
5	**3.99**	6.18	7.24	7.72	8.03	8.14	8.18	4.24	**5.07**	**5.82**	**5.92**	**5.97**	**5.87**	**5.11**
6	**3.74**	5.66	6.61	7.06	7.31	7.41	7.48	4.14	5.10	6.21	**6.48**	**6.46**	**6.16**	**5.14**
7	**3.55**	5.06	5.74	6.13	**6.35**	**6.45**	**6.51**	4.47	6.93	8.68	9.12	9.16	8.72	6.57
8	3.29	4.55	5.15	5.42	**5.58**	**5.64**	**5.69**	4.70	9.08	11.41	12.08	12.13	11.58	8.61
9	3.23	4.36	4.82	4.99	**5.11**	**5.15**	**5.17**	4.92	10.69	13.48	14.28	14.33	13.86	10.25
10	3.21	4.20	4.56	**4.65**	**4.70**	**4.73**	**4.75**	5.24	12.01	15.02	15.80	15.85	15.52	11.48
11	3.20	4.12	4.44	**4.52**	**4.57**	**4.59**	**4.60**	5.39	12.69	16.11	16.81	16.79	16.53	11.99
12	3.27	4.06	4.41	**4.47**	**4.51**	**4.52**	**4.53**	5.60	12.78	16.47	17.14	17.12	16.84	12.10
13	3.28	4.17	4.48	**4.54**	**4.57**	**4.59**	**4.59**	5.40	12.56	16.27	16.94	16.93	16.49	11.71
14	3.52	4.50	4.83	**4.90**	**4.93**	**4.93**	**4.94**	5.55	12.85	16.80	17.65	17.60	17.09	12.11
15	3.61	4.68	5.08	**5.15**	**5.17**	**5.18**	**5.18**	5.74	13.29	17.70	18.40	18.45	17.89	12.41
16	3.61	4.72	5.11	**5.22**	**5.26**	**5.26**	**5.26**	5.77	13.17	17.77	18.47	18.57	17.90	12.29
17	**3.50**	4.42	**4.71**	**4.75**	**4.79**	**4.78**	**4.79**	5.39	12.19	16.65	17.26	17.39	16.63	11.25
18	**3.29**	4.04	**4.32**	**4.42**	**4.47**	**4.48**	**4.48**	4.79	9.24	12.18	12.66	12.68	12.09	8.67
19	**3.15**	**3.77**	**4.12**	**4.26**	**4.31**	**4.32**	**4.33**	4.03	6.43	7.81	8.13	8.14	7.80	6.18
20	2.85	3.48	3.75	3.82	3.91	**3.93**	3.95	3.47	5.15	5.74	5.95	6.05	5.80	4.99
21	2.72	3.58	3.93	**4.09**	4.18	**4.22**	4.23	3.17	4.89	5.67	5.84	5.95	5.69	4.56
22	3.04	4.22	4.76	5.00	5.13	5.18	5.19	3.36	5.30	6.61	6.92	6.90	6.56	4.92
23	3.68	5.26	5.88	**6.21**	**6.39**	**6.43**	6.43	3.99	5.64	6.95	7.28	7.24	6.79	**5.30**
24	4.26	6.08	6.96	7.38	7.62	7.67	7.70	4.50	**5.79**	**6.78**	**7.09**	**6.98**	**6.70**	**5.59**

Hour	GLM							NN						
	Forecasting horizon (in days)							Forecasting horizon (in days)						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
1	4.37	6.42	7.38	7.92	8.18	8.20	8.16	5.93	5.94	6.99	7.73	7.99	8.37	8.65
2	**4.43**	6.54	7.62	8.15	8.43	8.45	8.42	6.06	6.08	7.19	7.93	8.29	8.67	8.96
3	4.37	6.45	7.61	8.15	8.46	8.44	8.45	5.98	5.98	7.14	7.95	8.33	8.73	9.05
4	4.23	6.24	7.40	7.93	8.26	8.28	8.26	5.71	5.71	6.92	7.78	8.16	8.58	8.97
5	4.03	5.99	7.12	7.69	8.01	8.06	8.09	5.48	5.48	6.74	7.52	7.87	8.28	8.61
6	3.81	5.60	6.58	7.05	7.32	7.40	7.46	4.92	**4.91**	**6.12**	6.92	7.25	7.60	7.91

(continued on next page)

Table 6 (*continued*)

Hour	GLM							NN						
	Forecasting horizon (in days)							Forecasting horizon (in days)						
	1	2	3	4	5	6	7	1	2	3	4	5	6	7
7	3.60	5.08	5.88	6.21	6.43	6.53	6.59	4.28	**4.27**	**5.35**	**6.05**	6.42	6.65	6.99
8	**3.22**	4.61	5.26	5.54	5.73	5.81	5.88	3.57	**3.56**	**4.66**	**5.32**	5.58	5.85	6.10
9	**3.03**	4.28	4.75	5.03	5.22	5.30	5.35	3.29	**3.28**	**4.36**	**4.94**	5.26	5.42	5.49
10	**2.97**	4.06	4.49	4.67	4.80	4.86	4.92	3.25	**3.24**	**4.25**	4.72	4.91	5.04	5.11
11	**3.00**	3.97	4.39	4.56	4.69	4.76	4.81	3.33	**3.32**	**4.27**	4.69	4.83	4.96	4.98
12	**3.05**	3.99	4.41	4.56	4.68	4.73	4.75	3.40	**3.39**	**4.32**	4.72	4.81	4.96	4.97
13	**3.00**	4.00	**4.44**	4.59	4.71	4.76	4.78	3.58	**3.56**	4.49	4.83	4.92	5.06	5.10
14	**3.17**	4.29	**4.72**	4.93	5.05	5.10	5.10	3.92	**3.90**	4.80	5.15	5.28	5.41	5.36
15	**3.38**	4.51	**4.98**	5.19	5.31	5.35	5.36	4.14	**4.13**	4.99	5.39	5.53	5.66	5.58
16	**3.43**	4.55	**4.98**	5.23	5.33	5.41	5.43	4.16	**4.14**	5.04	5.42	5.55	5.72	5.66
17	3.60	4.43	4.77	4.89	4.98	5.04	5.02	3.91	**3.91**	4.83	5.17	5.33	5.53	5.44
18	3.44	4.19	4.43	4.52	4.58	4.58	4.59	3.96	**3.95**	4.53	4.78	4.93	5.19	5.11
19	3.21	3.77	**4.07**	4.30	4.44	4.50	4.53	4.17	4.16	4.58	4.80	5.02	5.23	5.14
20	**2.71**	**3.30**	**3.60**	**3.74**	**3.87**	3.93	**3.93**	3.65	3.64	4.14	4.34	4.50	4.67	4.58
21	**2.63**	**3.45**	**3.85**	4.11	**4.16**	4.23	**4.22**	3.57	3.58	4.22	4.49	4.68	4.83	4.83
22	**2.98**	**4.12**	**4.62**	**4.94**	**5.11**	**5.17**	5.16	4.33	4.32	4.95	5.24	5.50	5.64	5.59
23	**3.58**	**5.15**	**5.82**	6.22	6.41	**6.43**	6.41	5.51	5.50	6.17	6.44	6.72	6.90	6.84
24	**4.14**	5.96	6.81	7.34	7.56	7.60	7.56	6.61	6.63	7.33	7.61	7.90	8.14	8.15

Mean absolute percentage errors from the TLSAR, DASARIMA, GLM and NN models.

the concurrent specifications for most of the hours and for almost all forecasting horizons. There are only few exceptions: the DASARIMA is the best model during the night (hours 1–5) for forecasting horizons greater than one; the GLM model is the best alternative for hours 1–6 and when one-step-ahead forecasts are considered; and, on average, the NN specification attains the lowest MAPEs for hours 7–18 for two-step-ahead forecasts. The superiority of the proposed model over the DASARIMA specification is huge when the middle hours are analyzed. For example, consider hour 13 for 1999 (Table 5). The MAPEs of the TLSAR model range from 2.86% to 3.56%, while the MAPEs of the DASARIMA model go from 5.04% to 15.72%. One interesting point is that the TSLAR model attains its lowest MAPEs during the peak hours (19–21). Furthermore, the superior performance of the DASRIMA model during the night hours may be due to the less challenging dynamics of these hours, and hence the simpler benchmark method is adequate.

Considering the year 2000, the results are slightly different. However, it is still clear that the DASARIMA is the worst model, as it attains extremely high MAPEs during the day. The TLSAR has an evident advantage over the GLM and NN alternatives, especially for longer forecasting horizons. It is important to note that the TLSAR model is clearly the winner for hours 17–19.

Comparing the results between years, we do see some differences in the comparative performance between the models. The results in 2000 are slightly worse than the ones obtained from the same models in 1999, mainly because the linear trend is not re-estimated but seems to suffer a break in the former year, as explained before. Even so, the results are good and are qualitatively equal to the year 1999. This shows that the TSLAR model is quite robust. It is important to note that when we speak of h-steps-ahead, we consider the sectional data, and hence refer to days. As the primary data are hourly, one must interpret it as 24h-steps-ahead, so that 1,2,…, 7 daily steps ahead actually correspond to 24, 48,…, 168 hourly steps ahead. In practice, it would be interesting to use the model proposed here and the benchmark for the hours and time horizons at which each one fares best, or even in a combined way. However, forecast combination is beyond the scope of this paper. Confidence intervals may be computed by taking the conditional heteroskedasticity into account. One way of proceeding is by estimating a GARCH (Generalized Autoregressive Conditional Heteroskedastic) model (Bollerslev, 1986). Another option is to use a block bootstrap (Efron & Tibshirani, 1993) or the stationary bootstrap (Politis & Romano, 1994) to resample the residuals.

Although not reported in this paper, the forecast performances of the TSLAR model do not differ among different standard weekdays (from Sunday to Saturday). However, as expected, during anomalous days, such as Christmas, New Year's Eve, Carnival, etc, the MAPEs are higher. On the other hand, there are huge differences in performance depending on the time of the year. The forecasts are worse during the warmer months, like January and February. Furthermore, the best performance is attained during the colder months (May–July). These additional results can be obtained from the authors.

6. Conclusions

In this paper we have considered a two-level model for the hourly electricity load from the area covered by a specific utility in the southeast of Brazil. The proposed model is applied to sectional data; that is, the load for each hour of the day is treated separately as a series. A forecasting exercise against a specific class of seasonal ARIMA models (the benchmark) and the Generalized Long Memory (GLM) model discussed by Soares and Souza (2006) is highly favorable to our proposal. Furthermore, considering more complex nonlinear models, such as neural networks, does not bring any clear benefit in terms of forecasting performance, in particular when the year 1999 is considered. Possible extensions of the methodology proposed here are the combination of forecasts, interval forecasts and forecast density evaluation.

Acknowledgments

L. J. Soares is with UENF, Campos, Brazil. M.C. Medeiros is with the Department of Economics, Pontifical Catholic University of Rio de Janeiro, Brazil. The first author's research has been partially supported by CEPEL - Electric Power Research Center. The authors also acknowledge the partial financial support of CNPq. The responsibility for any errors or shortcomings in the paper remains solely ours. The comments of Leonardo Souza, James Taylor and two anonymous referees are gratefully acknowledged. This paper is partly based on the first author's Ph.D. thesis at the Department of Electrical Engineering, Pontifical Catholic University of Rio de Janeiro (PUC-Rio).

References

Al-Hamadi, H., & Soliman, S. (2004). Short-term electric load forecasting based on Kalman filtering algorithm with moving window weather and load model. *Electric Power Systems Research*, *68*, 47–59.

Amjady, N. (2001). Short-term hourly load forecasting using time-series modeling with peak load estimation capability. *IEEE Transactions on Power Systems*, *16*, 498−504.

Bollerslev, T. (1986). Generalized autoregressive conditional heteroskedasticity. *Journal of Econometrics*, *21*, 307−328.

Bunn, D., & Farmer, E. (1985a). *Comparative Models for Electrical Load Forecasting.* New York: Wiley.

Bunn, D., & Farmer, E. (1985b). Economic and operational context of electric load prediction. In D. Bunn & E. Farmer (Eds.), *Comparative Models for Electrical Load Forecasting* (pp. 3−11).Wiley.

Cancelo, J., Espasa, A., & Grafe, R. (2008). Forecasting from one day to one week ahead for the Spanish system operator. *International Journal of Forecasting*, *24*, 588−602 (this issue).

Carneiro, D. (2000). *Inflation targeting in Brazil: What difference does a year make?Textos para Discussão*, *Vol. 429*. Pontifícia Universidade Católica do Rio de Janeiro.

Carpinteiro, O., Reis, A., & Silva, A. (2004). A hierarchical neural model in short-term load forecasting. *Applied Soft Computing*, *4*, 405−412.

Connor, J., Atlas, L., & Martin, D. (1992). Recurrent networks and NARMA modelling. *Advances in Neural Information Processing Systems*, *4*, 301−308.

Cottet, R., & Smith, M. (2003). Bayesian modeling and forecasting of intraday electricity load. *Journal of the American Statistical Association*, *98*, 839−849.

da Silva, A., Ferreira, V., & Velasquez, R. (2008). Input space to neural network based load forecasting. *International Journal of Forecasting*, *24*, 616−629 (this issue).

Darbellay, G., & Slama, M. (2000). Forecasting the short-term demand for electricity: Do neural networks stand a better chance? *International Journal of Forecasting*, *16*, 71−83.

Dordonnat, V., Koopman, S., Ooms, M., Dessertaine, A., & Collet, J. (2008). An hourly periodic state space model for modelling French national electricity load. *International Journal of Forecasting*, *24*, 566−587 (this issue).

Efron, B., & Tibshirani, R. J. (1993). *An Introduction to Bootstrap.* New York: Chapman & Hall.

El-Keib, A., Ma, X., & Ma, H. (1995). Advancement of statistical based modeling techniques for short-term load forecasting. *Electric Power Systems Research*, *35*, 51−58.

Enders, W. (2004). Applied Econometric Time Series. *Wiley Series in Probability and Statistics*, 2nd Ed. New Jersey: Wiley.

Engle, R. F. (1982). Autoregressive conditional heteroskedasticity with estimates of the variance of UK inflation. *Econometrica*, *50*, 987−1007.

Feinberg, E., & Genethliou, D. (2005). Load forecasting. In J. Chow, F. Wu, & J. Momoh (Eds.), *Applied Mathematics for Restructured Electric Power Systems: Optimization, Control, and Computational Intelligence* (pp. 269−285). Springer.

Fiebig, D., Bartels, R., & Aigner, D. (1991). A random coefficient approach to the estimation of residual end-use load profiles. *Journal of Econometrics*, *50*, 297−327.

Godfrey, L. G. (1988). *Misspecification Tests in Econometrics, 2nd Ed. Econometric Society Monographs*, *Vol. 16*. New York: Cambridge University Press.

Harvey, A., & Koopman, S. (1993). Forecasting hourly electricity demand using time-varying splines. *Journal of the American Statistical Association*, *88*, 1228−1237.

Hippert, H., Bunn, D., & Souza, R. (2005). Large neural networks of electricity load forecasting: Are they overfitted? *International Journal of Forecasting*, *21*, 425−434.

Hippert, H., Pedreira, C., & Souza, R. (2001). Neural networks for short-term load forecasting: A review and evaluation. *IEEE Transactions on Power Systems*, *16*, 44−55.

Huang, S. -J. (2003). Short-term load forecasting via ARMA model identification including non-Gaussian process considerations. *IEEE Transactions on Power Systems*, *18*, 673−679.

Huang, C. -M., Huang, C. -J., & Wang, M. -L. (2005). A particle swarm optimization to identify the ARMAX model for short-term load forecasting. *IEEE Transactions on Power Systems*, *20*, 1126−1133.

Huang, S. -J., & Shih, K. -R. (2003). Short-term load forecasting via ARMA model identification including non-Gaussian process considerations. *IEEE Transactions on Power Systems*, *18*, 673−679.

Infield, D., & Hill, D. (1998). Optimal smoothing and trend removal in short-term electricity demand forecasting. *IEEE Transactions on Power Systems*, *13*, 1115−1120.

Khotanzad, A., Afkhami-Rohani, R., & Maratukulam, D. (1998). ANNSTLF - Artificial neural network short-term load forecaster - Generation three. *IEEE Transactions on Power Systems*, *13*, 1413−1422.

MacKay, D. J. C. (1992a). Bayesian interpolation. *Neural Computation*, *4*, 415−447.

MacKay, D. J. C. (1992b). A practical Bayesian framework for backpropagation networks. *Neural Computation*, *4*, 448−472.

Medeiros, M. C., Teräsvirta, T., & Rech, G. (2006). Building neural network models for time series: A statistical approach. *Journal of Forecasting*, *25*, 49−75.

Metaxiotis, K., Kagiannas, A., Askounis, D., & Psarras, J. (2003). Artificial intelligence in short term electric load forecasting: A state-of-art survey for the researcher. *Energy Conversion and Management*, *44*, 1525−1534.

Nowicka-Zagrajek, J., & Weron, R. (2002). Modeling electricity loads in California: ARMA models with hyperbolic noise. *Signal Processing*, *82*, 1903−1915.

Park, D., El-Sharkawi, M., Marks, R., II, Atlas, L., & Damborg, M. (1991). Electric load forecasting using an artificial neural network. *IEEE Transactions on Neural Networks*, *6*, 442−449.

Peirson, J., & Henley, A. (1994). Electricity load and temperature. *Energy Economics*, *16*, 235−243.

Phillips, P., & Perron, P. (1988). Testing for a unit root in time series regression. *Biometrika*, *75*, 335−346.

Politis, D., & Romano, J. (1994). The stationary bootstrap. *Journal of the American Statistical Association*, *89*, 1303−1313.

Ramanathan, R., Engle, R. F., Granger, C., Vahid-Arahi, F., & Brace, C. (1997). Short-run forecasts of electricity loads and peaks. *International Journal of Forecasting*, *13*, 161−174.

Reis, A., & Silva, A. (2005). Feature extraction via multiresolution analysis for short-term load forecasting. *IEEE Transactions on Power Systems*, *20*, 189−198.

Schneider, A., Takenawa, T., & Schiffiman, D. (1985). 24-hour electric utility load forecasting. In D. Bunn & E. Farmer (Eds.), *Comparative Models for Electrical Load Forecasting*. Wiley.

Schwarz, G. (1978). Estimating the dimension of a model. *Annals of Statistics*, *6*, 461−464.

Soares, L., & Souza, L. (2006). Forecasting electricity demand using generalized long memory. *International Journal of Forecasting*, *22*, 17−28.

Taylor, J. (2008). An evaluation of methods for very short-term load forecasting using minute-by-minute British data. *International Journal of Forecasting*, *24*, 645−658 (this issue).

Taylor, J., de Menezes, L., & McSharry, P. (2006). A comparison of univariate methods for forecasting electricity demand up to a day ahead. *International Journal of Forecasting*, *22*, 1−16.

Temraz, H., Salama, M., & Quintana, V. (1996). Application of the decomposition technique for forecasting the load of a large electric power network. *IEEE Proceedings — Generation, Transmission, and Distribution*, *143*, 13−18.

Teräsvirta, T., Lin, C. F., & Granger, C. W. J. (1993). Power of the neural network linearity test. *Journal of Time Series Analysis*, *14*, 309−323.

Weron, R. (2006). *Modeling and Forecasting Electricity Loads and Prices: A Statistical Approach.* Chichester: Wiley.

White, H. (1980). A heteroskedasticity-consistent covariance matrix estimator and a direct test for heteroskedasticity. *Econometrica*, *48*, 827−838.