

Side-Channel Attacks and Non-Interference

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Outline

- Last week:
 - Safety and correctness bugs in cryptographic implementations
 - Introduction to the F* proof assistant
- Today:
 - Side-channel attacks
 - Establishing non-interference in implementations
- Exam will be on Feb 27

Leaking Secrets

secret s, key k

`m <- encrypt(k, s)`

`send m`

Assumption: **k is secret**

Implementation:

`print(k)`

`let m = encrypt(k, s) in`

`send(m)`

Indirectly Leaking Secrets

```
if k = 0xDEADBEEF then
```

```
    print(foo)
```

```
else
```

```
    print(bar)
```

```
let m = encrypt(k, s) in
```

```
send(m)
```

Leaking Information through Observations

```
let verify_pwd(string msg, string pwd) =  
  if msg.length <> pwd.length then return false  
  for (k = 0; k < msg.length; k++) {  
    if msg[k] <> pwd[k] then return false  
  }  
  return true
```

Possible attack:

- Measure execution time
- *Observe* longer execution time when msg has the same length as pwd
- *Observe* longer execution time when msg and pwd match on the first k characters

Side-Channel Attacks

- A ***side-channel attack*** exploits *physical observations* due to running a program to *infer information* about secrets
 - Execution time
 - Power consumption
 - Cache patterns
 - Keyboard sounds
 - ...
- Can leak cryptographic keys, plaintexts, state information, ...

Timing Attacks [Kocher, CRYPTO' 96]

- First published side-channel attack on cryptography
- Focuses on modular exponentiation
- Able to find fixed Diffie-Hellman exponents, factor RSA keys, ...
- Let's look at this on RSA

Background on RSA [Rivest, Shamir, Adleman, 78]

- Public-key encryption algorithm (can also be used for signing)
- Relies on a public key (N, e) , and a private key d
- N is the product of two large prime numbers p and q
- e and d are related through $ed = 1 \bmod (p - 1)(q - 1)$
- Security relies on p and q being unknown to the attacker (i.e., factoring large numbers is hard)

RSA Encryption

- Public key (N, e) , private key d , plaintext M
- Encryption: Ciphertext is $M^e \bmod N$
- Decryption: We receive a ciphertext C . We return $C^d \bmod N$
- Correctness: For any plaintext M , $\text{decrypt}(\text{encrypt}(M)) == M$
Mathematically: $(M^e)^d \bmod N = M \bmod N$
Proof relies on Fermat's little theorem
- Can also be used for signing:
 - Send $(M, M^d \bmod N)$
 - Anybody can check that $(M^d)^e \bmod N = M \bmod N$

Timing Attack on RSA

- Attacker goal: Guess private key d
- Attacker capabilities: Can query decryption for any ciphertext C

$C^d \bmod N$ implementation (assume d contains w bits):

$x = 1$

for $k = 0$ to $w - 1$ do

 if $d[k] = 1$ then $x = xC \bmod N$

$x = x^2 \bmod N$

return x

Timing Attack on RSA

$x = 1$

for $k = 0$ to $w - 1$ do

 if $d[k] = 1$ then $x = xC \bmod N$

$x = x^2 \bmod N$

return x

Example: Take $d = 10$ (binary: 1010)

(Iteration 0): $d[0] = 0$

$$x = x^2 \bmod N // = 1 \bmod N$$

(Iteration 1): $d[1] = 1$

$$x = xC \bmod N // = C \bmod N$$

$$x = x^2 \bmod N // = C^2 \bmod N$$

(Iteration 2): $d[2] = 0$

$$x = x^2 \bmod N // = C^4 \bmod N$$

(Iteration 3): $d[3] = 1$

$$x = xC \bmod N // = C^5 \bmod N$$

$$x = x^2 \bmod N // = C^{10} \bmod N$$



Timing Attack on RSA

$x = 1$

for $k = 0$ to $w - 1$ do

 if $d[k] = 1$ then $x = xC \bmod N$

$x = x^2 \bmod N$

return x

- Attacker goal: Guess $d[0]$
- Assumption: $y \bmod N$ is slower for some values of y
 - Ex: When $y \geq N$ depending on mod impl

Attack:

- Call decrypt with two ciphertexts C_1, C_2 , such that $C_1^2 < N \leq C_2^2$
- If execution times differ, then $d[0] = 1$, else $d[0] = 0$
- In practice, statistical analysis with a family of C_1, C_2 to account for noise, network delay, ...

Timing attack on RSA

for $k = 0$ to $w - 1$ do

 if $d[k] = 1$ then $x = xC \bmod N$

$x = x^2 \bmod N$

return x

- Assume $d[0], \dots, d[k-1]$ are known
- Attacker goal: Guess $d[k]$
- Assumption: $y \bmod N$ is slower when $N \leq y$

Attack:

- The attacker can compute the first k iterations for any ciphertext C
- Call decrypt with two ciphertexts C_1, C_2 , such that $x_1^2 < N \leq x_2^2$ where x_1, x_2 are intermediate results after k iterations for C_1, C_2
- If execution times differ, then $d[k] = 1$, else $d[k] = 0$

Timing Attack on RSA

- Recursively applying this methodology, we can guess all bits of d
- Original results:
 - 128-bit key could be broken with about 10,000 samples (4 bits/sec)
 - 512-bit key could be broken in a few minutes with ~350,000 measurements
- Further attacks on optimized RSA implementations intended to circumvent timing attacks also shown effective

Remote Timing Attacks are Practical, Brumley and Boneh, USENIX' 03

Cache-based Side Channel Attacks

- Exploit timing differences due to accesses to memory caches
- Especially demonstrated on the AES block cipher

Bernstein, D. J. (2005). *Cache-timing attacks on AES*.

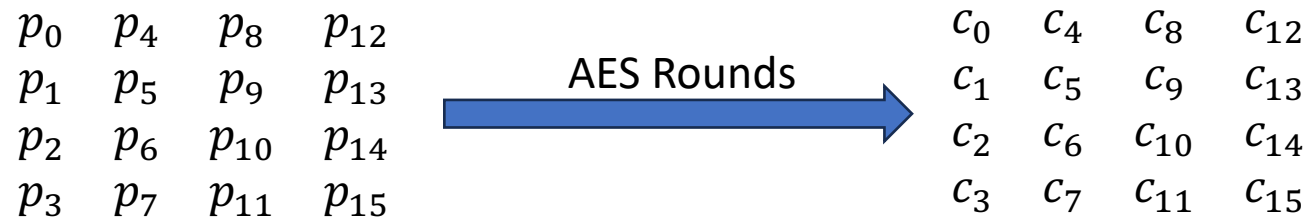
Osvik, D. A., Shamir, A., & Tromer, E. (2006). *Cache attacks and countermeasures: the case of AES*.

Bonneau, J., & Mironov, I. (2006). *Cache-collision timing attacks against AES*.

Tromer, E., Osvik, D. A., & Shamir, A. (2010). *Efficient cache attacks on AES, and countermeasures*

Background on AES

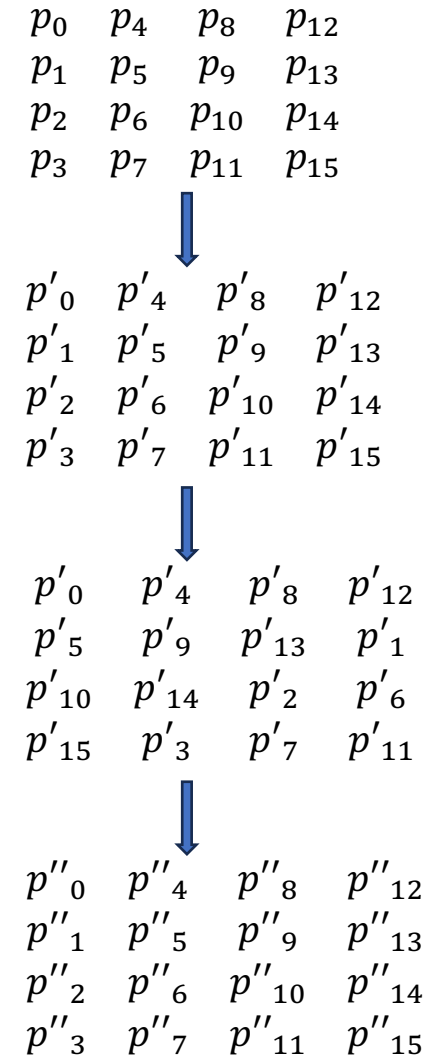
- Block cipher: transforms a fixed-size plaintext (128 bits) into a ciphertext using a secret key k
 - Many encryption modes to support arbitrary-sized plaintexts (AES-GCM, AES-CTR, ...)
- Initially, xor plaintext with key
- Followed by several rounds of encryption operating on a state of 16 bytes



AES Round

Several Successive Transformations:


- Substitute bytes through affine transformation (SubBytes)
- Different shifts in each row (ShiftRows)
- Apply linear transformation to each column (MixColumns):
- Xor with (a derived sub)key (AddRoundKey): $c_i = p_i'' \oplus k_i$



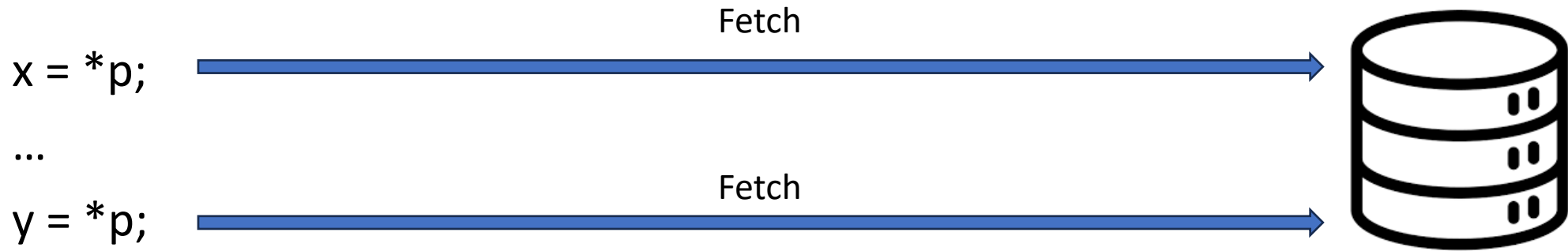
Optimized AES Round

- The first three transformations (SubBytes, ShiftRows, MixColumns) only depend on the input state
- The result can be precomputed for all p_i , and stored in tables T_k .

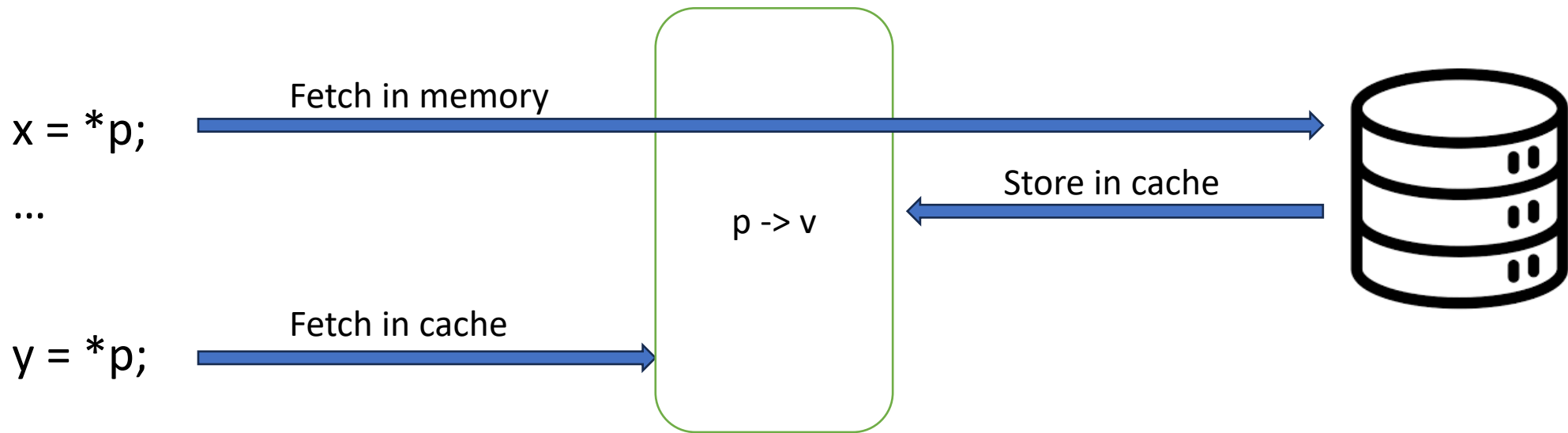
Optimized AES round:

x_0	x_4	x_8	x_{12}		$T_0[x_0] \oplus T_1[x_5] \oplus T_2[x_{10}] \oplus T_3[x_{15}] \oplus \{k_0, k_1, k_2, k_3\}$
x_1	x_5	x_9	x_{13}		$T_0[x_4] \oplus T_1[x_9] \oplus T_2[x_{14}] \oplus T_3[x_3] \oplus \{k_4, k_5, k_6, k_7\}$
x_2	x_6	x_{10}	x_{14}		$T_0[x_8] \oplus T_1[x_{13}] \oplus T_2[x_2] \oplus T_3[x_7] \oplus \{k_8, k_9, k_{10}, k_{11}\}$
x_3	x_7	x_{11}	x_{15}		$T_0[x_{12}] \oplus T_1[x_1] \oplus T_2[x_6] \oplus T_3[x_{11}] \oplus \{k_{12}, k_{13}, k_{14}, k_{15}\}$

Cache Model (Simplified)



Cache Model (Simplified)



- Accesses to the cache are faster than to main memory
- Storage in the cache is smaller than memory
- When the cache is full, storing a new value removes older mappings

AES First Round Cache Attack

- For the first round, the inputs x_i are equal to $p_i \oplus k_i$
- We are accessing memory at address $T_k[x_i]$
- The attacker controls input p
- We access $T_0[x_0], T_0[x_4], T_0[x_8], T_0[x_{12}]$
- If (e.g.) $x_0 = x_4$, execution time is lower as $T_0[x_4]$ is stored in cache when accessing $T_0[x_0]$
- Trying different samples, we can find values of p_0, p_4 , such that $x_0 = p_0 \oplus k_0 = x_4 = p_4 \oplus k_4$
- We can determine the value of $k_0 \oplus k_4$

AES Cache-Based Attacks

- Similar attacks allow to infer more information about the key, leading to key retrieval
- Omitted details
 - Attacker needs to control the initial state of the cache
 - Cache does not allow to reason about lower bits of accessed addresses
 - Other computations can lead to timing differences
- There exists technical solutions for all of this

Speculative Side-Channel Attacks: Spectre

```
if (0 <= x < a.length) {  
    i = a[x];  
    r = b[i];  
}
```

- Assume that all values in a are in $[0; b.length[$
- Can this code lead to a buffer overflow?
- In theory, no, all accesses are in bound, but...

CPU Branch Prediction

- CPU instruction pipeline: Fetch, Decode, Execute, Access Memory, Write results in registers
- Modern CPUs anticipate and start executing next instructions early
- When branching occur, CPUs “guess” which branch is most likely to start the instruction pipeline
- When wrong, rollback to earlier CPU state
- **Problem: Rollback does not include the entire microarchitectural state, e.g., cache state**

Speculative Side-Channel Attacks: Spectre

```
if (0 <= x < a.length) {  
    i = a[x];  
    r = b[i];  
}
```

- Run program with $x = a.length + n$
- CPU predicts that the if branch will be taken
- Pre-executes the two memory accesses
- When rolling back, the cache contains a mapping for i

- Attack:
 - Train branch predictor for if branch
 - Pick n such that $a[a.length + n]$ contains a secret
 - Launch a cache side channel attack to infer i

Physical Side-Channel Attacks

- Similar attacks exploit the power consumption or electromagnetic leakage.
- Ex: Power consumption of a given instruction is correlated to the number of bits set in its operands (Hamming weight model)
- Infer information about secrets manipulated by the program
- Require some access to the device

Recent Physical Side-Channel Attacks

Video-Based Cryptanalysis: Extracting Cryptographic Keys from Video Footage of a Device's Power LED, Nassi et al., 2023

- **Core idea:**
 - Direct access to device is not needed, a video of its use might be enough
 - The power consumption of a device affects the brightness of its power LED
 - In some cases, this is sufficient to launch a remote power-based side-channel attack
- Today: Focus on *digital* side-channel attacks

Non-Interference [Goguen-Meseguer, 82]

- Goal: We want to ensure that *secret data* does not impact *public observations* available to an attacker
- Information-flow property based on *secrecy labels*:
 - High (H) == Secret data
 - Low (L) == Public data
- High-level idea: There is no flow from high data to low data

Non-Interference, Formally

For a given program p ,

$\forall (s_1 \ s_2 \colon state),$

$$s_{1|L} = s_{2|L} \Rightarrow$$

// States agree on low values

$$s_1 \rightarrow_p^* s'_1 \Rightarrow$$

// Executing p in s_1 yields s'_1

$$s_2 \rightarrow_p^* s'_2 \Rightarrow$$

// Executing p in s_2 yields s'_2

$$s'_{1|L} = s'_{2|L}$$

// Results agree on low values

Non-Interference Example

if $x = 1$ then $y := 1$ else $y := 0$

- If $x : H, y : H$: No low values, non-interference
 - If $x : L, y : L$: Initial agreement on x , non-interference
 - If $x : L, y : H$: Initial agreement on x , non-interference
 - If $x : H, y : L$: Observing the result of y leaks information about x
-
- Goal: Statically ensure noninterference

Non-Interference by Typing [Volpano et al., 96]

(*expressions*) $e ::= x \mid l \mid n \mid e + e' \mid e - e' \mid e = e' \mid e < e'$

(*commands*) $c ::= e := e' \mid c; c' \mid \mathbf{if\ } e \mathbf{\ then\ } c \mathbf{\ else\ } c' \mid$
 $\mathbf{while\ } e \mathbf{\ do\ } c \mid \mathbf{letvar\ } x := e \mathbf{\ in\ } c$

(*data types*) $\tau ::= s$

(*phrase types*) $\rho ::= \tau \mid \tau \textit{ var} \mid \tau \textit{ cmd}$

- Data types s are security labels (in our case, H and L)
- Each expression and command is annotated with a security label

Typing Judgement

$$\lambda; \gamma \vdash p : \rho$$

- λ is a memory store: It associates to each *location* its security label
- γ is a variable environment: It maps variables to their type
- Under this context, this judgement gives program p the type ρ

Typing Rules

(INT) $\lambda; \gamma \vdash n : \tau$

(VAR) $\lambda; \gamma \vdash x : \tau \text{ var}$ if $\gamma(x) = \tau \text{ var}$

(VARLOC) $\lambda; \gamma \vdash l : \tau \text{ var}$ if $\lambda(l) = \tau$

(ARITH)
$$\frac{\lambda; \gamma \vdash e : \tau, \quad \lambda; \gamma \vdash e' : \tau}{\lambda; \gamma \vdash e + e' : \tau}$$

(ASSIGN)
$$\frac{\lambda; \gamma \vdash e : \tau \text{ var}, \quad \lambda; \gamma \vdash e' : \tau}{\lambda; \gamma \vdash e := e' : \tau \text{ cmd}}$$

Typing Rules

$$\text{(COMPOSE)} \quad \frac{\lambda; \gamma \vdash c : \tau \text{ cmd}, \quad \lambda; \gamma \vdash c' : \tau \text{ cmd}}{\lambda; \gamma \vdash c; c' : \tau \text{ cmd}}$$

$$\text{(IF)} \quad \frac{\lambda; \gamma \vdash e : \tau, \quad \lambda; \gamma \vdash c : \tau \text{ cmd}, \quad \lambda; \gamma \vdash c' : \tau \text{ cmd}}{\lambda; \gamma \vdash \mathbf{if} \ e \ \mathbf{then} \ c \ \mathbf{else} \ c' : \tau \text{ cmd}}$$

$$\text{(WHILE)} \quad \frac{\lambda; \gamma \vdash e : \tau, \quad \lambda; \gamma \vdash c : \tau \text{ cmd}}{\lambda; \gamma \vdash \mathbf{while} \ e \ \mathbf{do} \ c : \tau \text{ cmd}}$$

Typing Example

if $x = 1$ then $y := 1$ else $y := 0$

Assume that $x : H \text{ var}$, $y : H \text{ var}$

Goal : Give this program the type $H \text{ cmd}$

Typing Example

Goal: $x: H \text{ var}, y: H \text{ var} \vdash \text{if } x = 1 \text{ then } y := 1 \text{ else } y := 0 : H \text{ cmd}$

$$\begin{array}{c} \text{(IF)} \quad \frac{\begin{array}{l} \lambda; \gamma \vdash e : \tau, \\ \lambda; \gamma \vdash c : \tau \text{ cmd}, \\ \lambda; \gamma \vdash c' : \tau \text{ cmd} \end{array}}{\lambda; \gamma \vdash \mathbf{if } e \mathbf{ then } c \mathbf{ else } c' : \tau \text{ cmd}} \end{array}$$

Need to prove

- $x: H \text{ var}, y : H \text{ var} \vdash x = 1 : H$
- $x: H \text{ var}, y : H \text{ var} \vdash y := 1 : H \text{ cmd}$
- $x: H \text{ var}, y : H \text{ var} \vdash y := 0 : H \text{ cmd}$

Typing Example

Goal: $x: H \text{ var}, y: H \text{ var} \vdash x = 1 : H$

$$\text{(ARITH)} \quad \frac{\lambda; \gamma \vdash e : \tau, \quad \lambda; \gamma \vdash e' : \tau}{\lambda; \gamma \vdash e + e' : \tau}$$

Need to prove

- $x: H \text{ var}, y : H \text{ var} \vdash 1 : H$

$$\text{(INT)} \quad \lambda; \gamma \vdash n : \tau$$

- $x: H \text{ var}, y : H \text{ var} \vdash x : H$

$$\text{(VAR)} \quad \lambda; \gamma \vdash x : \tau \text{ var} \quad \text{if } \gamma(x) = \tau \text{ var}$$

$$\text{(R-VAL)} \quad \frac{\lambda; \gamma \vdash e : \tau \text{ var}}{\lambda; \gamma \vdash e : \tau}$$

Typing Example

Goal: $x: H \text{ var}, y: H \text{ var} \vdash y := 1 : H \text{ cmd}$

$$\text{(ASSIGN)} \quad \frac{\begin{array}{l} \lambda; \gamma \vdash e : \tau \text{ var}, \\ \lambda; \gamma \vdash e' : \tau \end{array}}{\lambda; \gamma \vdash e := e' : \tau \text{ cmd}}$$

Need to prove

- $x: H \text{ var}, y: H \text{ var} \vdash y : H \text{ var}$ (VAR) $\lambda; \gamma \vdash x : \tau \text{ var}$ if $\gamma(x) = \tau \text{ var}$
- $x: H \text{ var}, y: H \text{ var} \vdash 1 : H$ (INT) $\lambda; \gamma \vdash n : \tau$



Label Subtyping

- The type system is sufficient when x and y have the same label
- What about $x : L \text{ var}$, $y : H \text{ var}$?

$$\text{(IF)} \quad \frac{\begin{array}{l} \lambda; \gamma \vdash e : \tau, \\ \lambda; \gamma \vdash c : \tau \text{ cmd}, \\ \lambda; \gamma \vdash c' : \tau \text{ cmd} \end{array}}{\lambda; \gamma \vdash \mathbf{if } e \mathbf{ then } c \mathbf{ else } c' : \tau \text{ cmd}}$$

- The If rule requires the condition and the commands to have the same label!

Label Subtyping

$$\text{(BASE)} \quad \frac{\tau \leq \tau'}{\vdash \tau \subseteq \tau'}$$

$$\text{(SUBTYPE)} \quad \frac{\lambda; \gamma \vdash p : \rho, \quad \vdash \rho \subseteq \rho'}{\lambda; \gamma \vdash p : \rho'}$$

- We consider that label L is “lower” than label “H”
- Models that a public value can always be hidden as secret
- Given $x = 0 : L$, this allows us to derive $x = 0 : H$

Label Subtyping

$$(\text{CMD}^-) \quad \frac{\vdash \tau \subseteq \tau'}{\vdash \tau' \text{ cmd} \subseteq \tau \text{ cmd}}$$

- Different variance compared to expression rule
- Intuitively: If a program is “secure” when operating on/accessing secret variables, then it is also when accessing less privileged data
- Alternative proof: $y := 1 : H \text{ cmd} \Rightarrow y := 1 : L \text{ cmd}$

Exercises

- For the following programs, either give a typing derivation showing non-interference, or explain why the program does not typecheck
- $x: L \text{ var}, y: H \text{ var} \vdash \text{while } (x < 10) \text{ do } (x := x + 1; y := y + 1)$
- $x: H \text{ var}, y: L \text{ var} \vdash \text{while } (x < 10) \text{ do}$
 if $y = 2$ then $x := x + 1$ else $x := x + 2$

Back to Digital Side-Channels

- The typing approach so far avoids indirect leaks, e.g., by observing public values
- However, it allows typechecking if $\text{key} = \dots$ then $x = \dots$, which leaks the key by observing the timing of the attack
- Need to extend formalism beyond leaking values!

Instrumenting Semantics

- Previously: $s_1 \rightarrow_p^* s'_1$
- We record traces containing all branching and memory accesses

(Trace) $l ::= \varepsilon \mid \text{Branch}(b) . l \mid \text{Access}(n) . l$

$$s_1 \rightarrow_p^* s'_1, l_1$$

When executing *if b then p else p'*, we record Branch(b)

When executing *a[n]*, we record Access(n)

Non-Interference with Observations

For a given program p ,

$\forall (s_1 \ s_2 \vdash \text{state}),$

$$s_{1|L} = s_{2|L} \Rightarrow s_1 \xrightarrow{*}_p s'_1, l_1 \Rightarrow s_2 \xrightarrow{*}_p s'_2, l_2 \Rightarrow \\ s'_{1|L} = s'_{2|L} \wedge l_1 = l_2$$

Captures that the program executes the same program paths, and performs identical memory (and hence cache) accesses for the same attacker-controlled inputs

The “Constant-Time” Programming Discipline

Cryptographic implementations must follow a “constant-time” programming discipline, which forbids

- Branching involving secrets
- Using instructions which execute in variable time with secrets (e.g., division)
- Accessing memory based on secret indices

The “Constant-Time” Programming Discipline

- Is this enough?

System-level Non-interference for Constant-time Cryptography, Barthe et al., CCS' 14
studies this formally

- Easy programming discipline to follow?

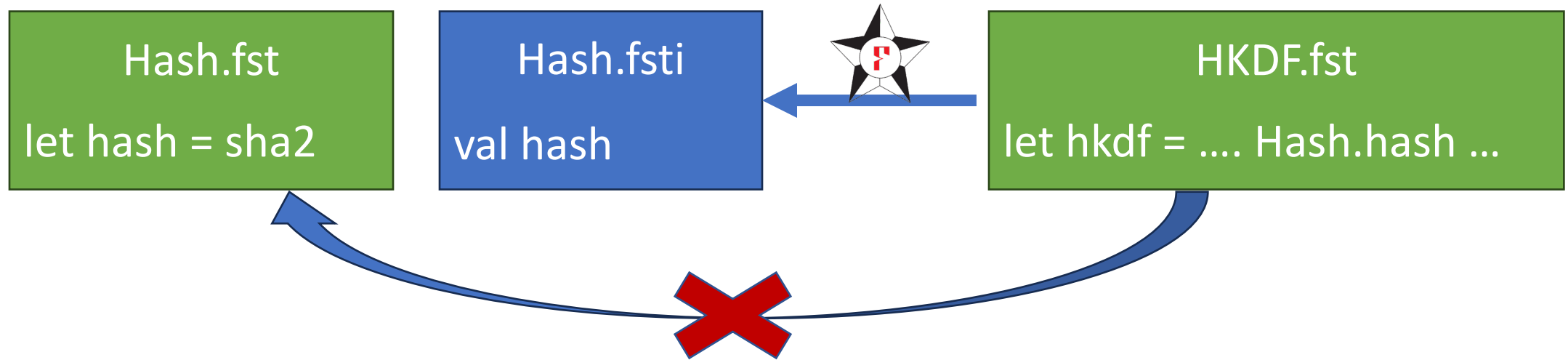
Jan 2024: **KyberSlash: division timings depending on secrets in Kyber software**

<https://kyberslash.cr.yp.to/> , <https://cryspen.com/post/ml-kem-implementation/>

- We need tools to enforce this

Non-Interference by Typing Abstraction

- Remember from last week:



- Client modules only have access to the interface
- Underlying implementation is hidden (true for other languages supporting abstraction)

Non-Interference by Typing Abstraction

SUInt32.fsti

```
val suint32: Type      // Abstract type for secret uint32 integers
```

```
val (+) : suint32 -> suint32 -> suint32
```

```
val (*) : suint32 -> suint32 -> suint32
```

```
// Non-constant time operations are not exposed
```

```
// val (/) : suint32 -> suint32 -> suint32
```

Implementing Abstract Secret Integers

SUInt32.fst

```
let uint32 = uint32    // Underlying definition is simply standard integers
```

```
let (+) n1 n2 = n1 + n2
```

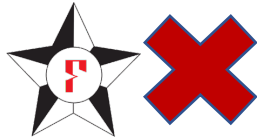
```
let (*) n1 n2 = n1 * n2
```

- Abstract type for opaque “secret integers”
- Exposes arithmetic and bitwise constant-time operations, but not comparison, division
- After extraction, compiled to standard integer, no runtime cost

Using Secret Integers

`n1, n2 : uint32` *// Secret integers*

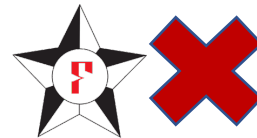
`if n1 > n2 then ...`



No comparison defined for secret integers

`val index (b: array uint8) (i: uint32) : ...`

`let x = b.[n1] in ...`



Expected type uint32, got type uint32

- Can be seen as an extension of previous typing discipline

Typing Limitations

- Only guarantees resistance against timing and cache-based side-channels (variants exist for speculative side-channels)
- Only provides guarantees within the semantics of the source language (C, OCaml, ...)
- Compilers can reintroduce side-channels

Compiler-Induced Side Channels

let login() =

x = read_passwd()

res = check_pwd(x)

x = 0

return res

Compile

let login() =

x = read_passwd()

res = check_pwd(x)

return res

Unused assignment

Password can leak after execution!

Crypto Compiler-Induced Side Channels

Assume b is secret

if b then $r := x$ else $r := y$

Rewrite into constant-time version

int mask = create_mask(b);
 $r := (x \& \text{mask}) \mid (y \& \sim \text{mask});$



: Did you mean

```
[@@ Comment "Returns 2^64 - 1 if a = b, otherwise returns 0."
static inline uint64_t FStar_UInt64_eq_mask(uint64_t a, uint64_t b)
{
    uint64_t x = a ^ b;
    uint64_t minus_x = ~x + (uint64_t)1U;
    uint64_t x_or_minus_x = x | minus_x;
    uint64_t xnx = x_or_minus_x >> (uint32_t)63U;
    return xnx - (uint64_t)1U;
}
```

if b then $r := x$ else $r := y$

Avoiding Compiler-Induced Side-Channels

- Several solutions:
 - Use a constant-time preserving compiler
 - e.g., *Formal verification of a constant-time preserving C compiler*, Barthe et al., POPL' 20
 - Analyze binary code after compilation
 - Verifying constant time implementations*, Almeida et al., USENIX' 16
 - BINSEC/REL: Efficient Relational Symbolic Execution for Constant-Time at Binary-Level*, Daniel et al., S&P' 20
 - ...

Non-Interference by Taint Analysis

- Taint analysis: a static analysis for non-interference
- **Core idea:** Mark some inputs as secret (“taint” them)
- Static analysis *propagates* the taint throughout the program
- If taint is propagated to attacker-observable components, raise error

Taint Analysis Example

```
let f (x : int) =
```

```
  y := x;
```

```
  z := 0;
```

```
  w := z + y;
```

```
let f (x : int) =
```

```
  y := x;
```

```
  z := 0;
```

```
  w := z + y;
```

- Mark input x as secret
- Propagate taint through program

Taint Analysis: Join Operator

```
let f (x : int, p: int) =
```

```
  z := p;
```

```
  if z > 0
```

```
    y := x;
```

```
  else
```

```
    y := 0;
```

```
  w := z + y;
```

```
let f (x : int, p: int) =
```

```
  z := p;
```

```
  if z > 0
```

```
    y := x;
```

```
  else
```

```
    y := 0;
```

```
  w := z + y;
```

- When joining two execution paths, we take the "highest" value for each variable

Taint Analysis: Raising Errors

```
let f (x : int) =  
  c := x + 2;  
  if c > 0  
    y := 1;  
  else  
    y := 2;
```

```
let g (x : int, a: int[]) =  
  y := a[x];
```

```
let f (x : int) =  
  c := x + 2;  
  if c > 0  
    y := 1;  
  else  
    y := 2;
```



```
let g (x : int, a: int[]) =  
  y := a[x];
```



Taint Analysis: Erasing Taint

```
let f (x : int) =  
  i := x + 2;  
  c := xor(x, x);  
  if c > 0  
    y := 1;  
  else  
    y := 2;
```

```
let f (x : int) =  
  i := x + 2;  
  c := xor(x, x);  
  if c > 0  
    y := 1;  
  else  
    y := 2;
```



- While tainted in theory, the output of some operations does not depend on its inputs
- We can soundly erase the taint in these cases

Taint Analysis: Memory Accesses

```
let f (x : int, y: int, a: int[]) =
```

```
  a[0] := x;
```

```
  c := a[y];
```

```
  if c > 0
```

```
    y := 1;
```

```
  else
```

```
    y := 2;
```

- Is this program constant-time?
- Depends on the values of y

Taint Analysis: Memory Accesses

```
let f (x : int, y: int, a: int[]) =
```

```
  a[0] := x;
```

```
  if y > 0
```

```
    c := a[y];
```

```
  else
```

```
    c := 2;
```

```
  if c > 0 ...
```

- Is this program constant-time?
- Yes, however tracking this requires tracking information about possible values of y
- We need a precise analysis to avoid false positives

Taint Analysis: Memory Accesses

```
let f (x : int, p1: *int, p2: *int) =  
    *p1 := x;  
    y := *p2;  
    if y > 0 ...
```

- Is this program constant-time?
- Depends on whether p1 and p2 alias
- We need aliasing information, either inferred (points-to analysis) or provided by programmer

Taint Analysis: Summary

- Mark secret inputs as “tainted”
- Propagate taint throughout the program
- If the taint reaches an attacker observation (return value, branching, memory access), possible secret leak
- Main difficulty: Reasoning about memory, which requires specific analyses
- Can be done on a variety of languages, including assembly
- Applicable beyond constant-time reasoning, e.g., to track possible leaks of private user information

Certification of Programs for Secure Information Flow, Denning and Denning, CACM' 77