

Towards High-Assurance Cryptographic Software: the F* Proof Assistant

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Outline

- Previously: Proving the security of cryptographic protocols
- Today:
 - Verifying **implementations** of cryptographic protocols
 - The F* proof assistant
 - The functional core of F*
 - Exercises
 - Try it online at <https://fstar-lang.org/tutorial/>
 - Or install it locally: <https://github.com/FStarLang/FStar>

What can go wrong?

Protocol model:

secret s, key k

$r \leftarrow \text{sample}()$

$m \leftarrow \text{encrypt}(k, \text{concat}(r, s))$

send m

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Protocol implementation:

```
let r = random() in  
let m = encrypt(k, r . s) in  
send m
```

What can go wrong?

Protocol model:

secret s, key k

$r \leftarrow \text{sample}()$

$m \leftarrow \text{encrypt}(k, \text{concat}(r, s))$

send m

Protocol implementation:

let random () = 0

let r = random() in

let m = encrypt(k, r . s) in

send m

What can go wrong?

Protocol model:

secret s, key k

$r \leftarrow \text{sample}()$

$m \leftarrow \text{encrypt}(k, \text{concat}(r, s))$

send m

Protocol implementation:

print(k)

let $r = \text{random}()$ in

let $m = \text{encrypt}(k, r . s)$ in

send m

What can go wrong?

Protocol model:

secret s, key k

$r \leftarrow \text{sample}()$

$m \leftarrow \text{encrypt}(k, \text{concat}(r, s))$

send m

Protocol implementation:

```
let r = random() in  
let m = encrypt(k, r . s) in  
send (r . s)
```

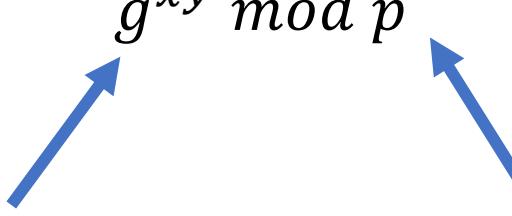
A Concrete Example: Modular Arithmetic

- Modular arithmetic is frequently used in cryptographic primitives

$$g^{xy} \bmod p$$

$0 < x, y < p,$
 g fixed

p is frequently a large prime number
(e.g., $2^{255} - 19$)



Implementing Modular Multiplication

$$a * b \bmod n$$

- a is a big integer (e.g., $2^{255} - 19$)
- Multiplication is even bigger
- Machine integers are (at most) 64 bits
- How to implement this? Need a bignum library



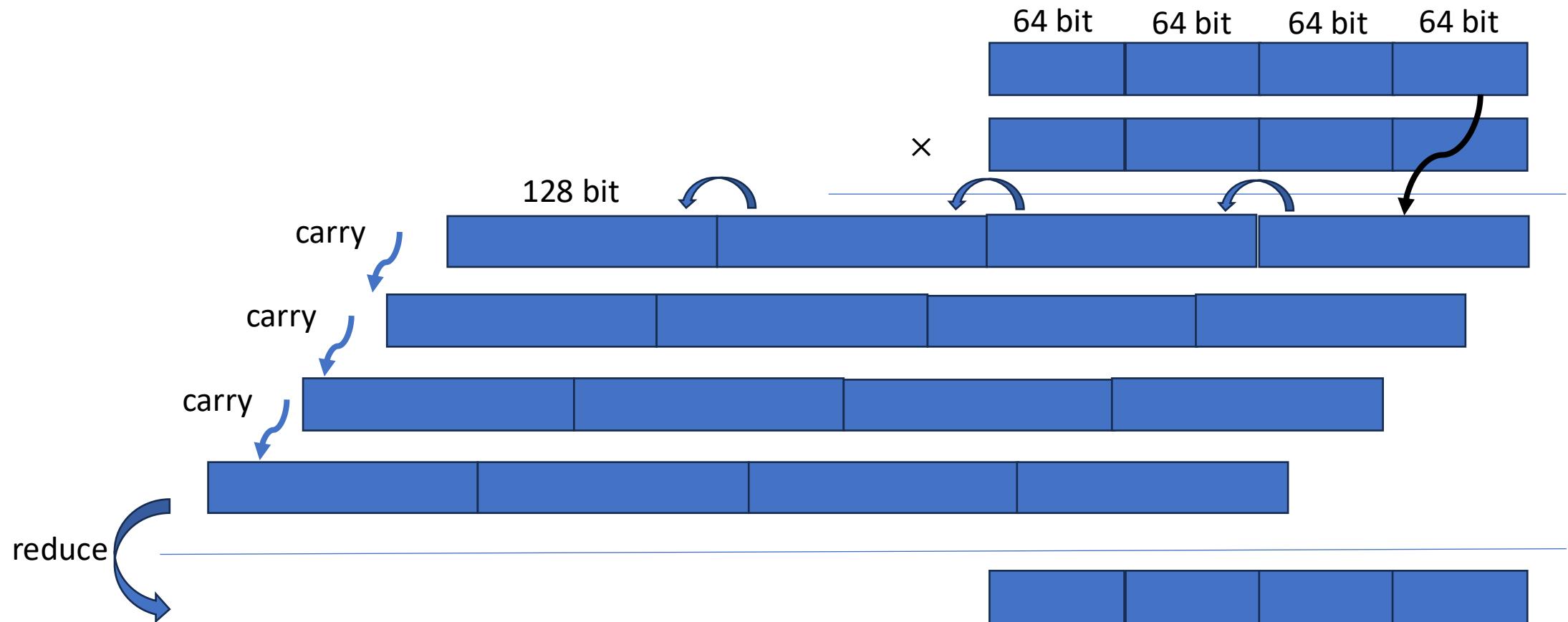
Textbook Multiplication

$$\begin{array}{r} 1101 & 13 \\ * 1010 & *10 \\ \hline - - - & = 130 \end{array}$$

$$\begin{array}{r} \text{carry} & 0000 \\ + 1101 \\ \text{carry} & + 1101 \\ + 0000 \\ \hline + 1101 \\ \hline - - - - - \end{array}$$

10000010

256-bit Modular Multiplication



256-bit Modular Multiplication

What can go wrong?

- Integer overflow (undefined output)
- Buffer overflow/underflow (memory error)
- Missing carry steps (wrong answer)
- Side-Channel attacks (leaks secrets)

Modular Arithmetic Optimizations

- For many primitives, modular arithmetic dominates the crypto overhead
 - n^2 64-bit multiplications
 - Long intermediate arrays
 - Many carry steps
- Many specific optimizations
 - Use only 51 out of 64 bits to reduce carries
 - Precompute reusable intermediate values
 - Use alternative modular reductions (Montgomery, Barrett)
 - Parallelize (vectorize) multiplication and squaring
- **Complex optimizations imply more chances of bugs!**

Many Bugs in Optimized Bignum Code

[2013] Bug in amd-64-64-24k Curve25519

“Partial audits have revealed **a bug in this software** ($r1 += 0 + \text{carry}$ should be $r2 += 0 + \text{carry}$ in amd-64-64-24k) **that would not be caught by random tests**”

– D.J. Bernstein, W.Janssen, T.Lange, and P.Schwabe

[2014] Arithmetic bug in TweetNaCl’s Curve25519

[2014] Carry bug in Langley’s Donna-32 Curve25519

[2016] Arithmetic bug in OpenSSL Poly1305

[2017] Arithmetic bug in Mozilla NSS GF128

...

TweetNaCL Arithmetic Bug

```
sv pack25519(u8 *o, const gf n)
{
    int i,j,b;
    gf m,t;
    FOR(i,16) t[i]=n[i];
    car25519(t);
    car25519(t);
    car25519(t);
    FOR(j,2) {
        m[0]=t[0]-0xffed;
        for(i=1;i<15;i++) {
            m[i]=t[i]-0xffff-((m[i-1]>>16)&1);
            m[i-1]&=0xffff;
        }
        m[15]=t[15]-0x7fff-((m[14]>>16)&1);
        b=(m[15]>>16)&1;
        m[15]&=0xffff;
        sel25519(t,m,1-b);
    }
    FOR(i,16) {
        o[2*i]=t[i]&0xff;
        o[2*i+1]=t[i]>>8;
    }
}
```

seb.dbzteam.org

This bug is triggered when the last limb **n[15]** of the input argument **n** of this function is greater or equal than **0xffff**. In these cases the result of the scalar multiplication is not reduced as expected resulting in a wrong packed value. This code can be fixed simply by replacing **m[15]&=0xffff;** by **m[14]&=0xffff;**.

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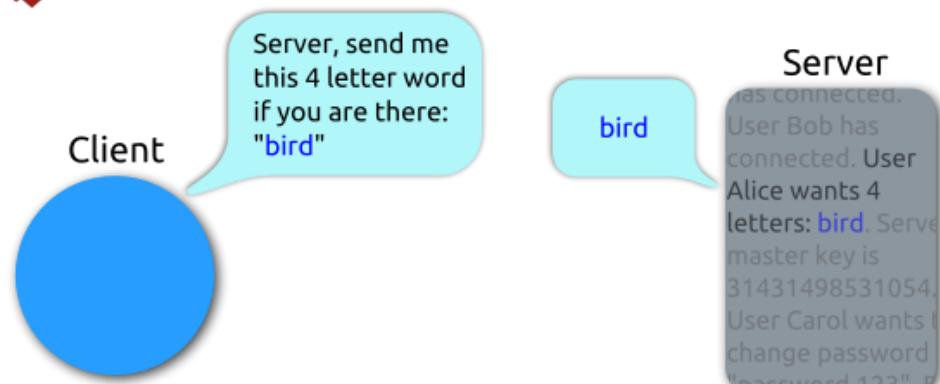
Heartbleed (CVE-2014-0160)



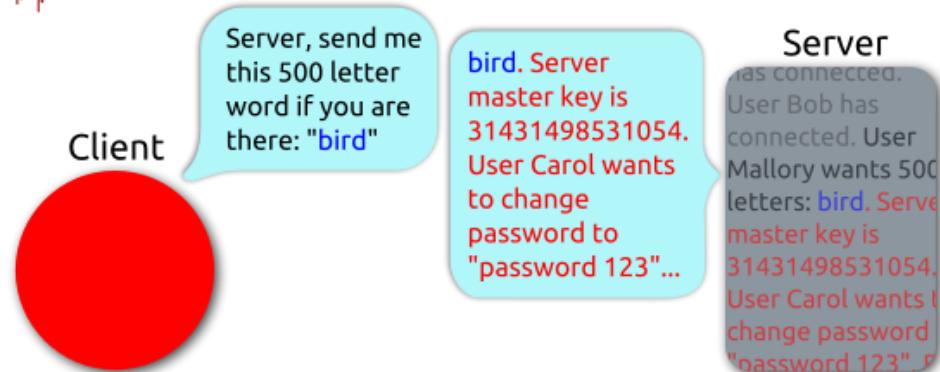
- Major vulnerability in OpenSSL TLS implementation
- Affected 17% of all SSL servers
- “Compromises the secret keys used to identify the service providers and to encrypt the traffic, the names and passwords of the users, and the actual content”
- “Allows attackers to eavesdrop on communications, steal data [...] and impersonate services and users.”
- Attacks do not leave a trace

Heartbleed (CVE-2014-0160)

Heartbeat – Normal usage



Heartbeat – Malicious usage



- Missing bound check during a memcpy

response = malloc(length);
memcpy(response, recv.heartbeat, length);



response = malloc(length);
if length > ssl_state.heartbeat {return 0;}
memcpy(response, recv.heartbeat, length);



GotoFail (CVE-2014-1266)

```
status SSLVerifyExchange (...) { ...
    if ((err = update(&hashCtx, &signedParams)) != 0)
        goto fail;
    goto fail;
    if ((err = final(&hashCtx, &hashOut)) != 0)
        goto fail;

...
fail:
    SSLFreeBuffer(&signedHashes);
    SSLFreeBuffer(&hashCtx);
    return err;
}
```

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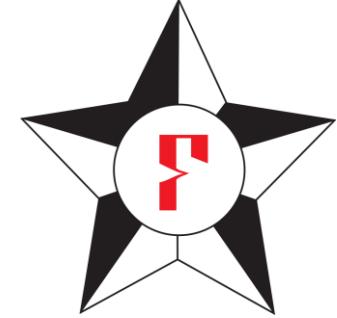
GotoFail (CVE-2014-1266)

- Duplicated goto statement in Apple's TLS implementation
- Bad copy/paste? Faulty merge?
- Impact:
 - Many invalid certificates were accepted
 - Allows using an arbitrary private key for signing or skipping the signing step
 - Enables Man-in-the-Middle attacks
- Many other vulnerabilities: SKIP, FREAK, many memory bugs, correctness issues, infinite loops, ...

Formally Verifying Implementations

- Cryptographic implementations must be correct and secure, but also **fast**
- Cryptographic implementations are notoriously complex
 - Many tricky optimizations
 - Written in low-level, unsafe languages (C, Assembly)
 - Multiplicity of parameters and variants
- We need strong, formal guarantees about the **safety, correctness, and security** of cryptographic implementations

The F* Proof Assistant



- A functional programming language
(like OCaml, Haskell, F#, ...)
 - With support for dependent types (like Coq, Agda), refinement types, ...
 - Semi-automated verification by relying on SMT solving
(like Dafny, Why3, LiquidHaskell, ...)
 - Also offers a metaprogramming and tactic framework (Meta-F*)
 - Extraction to OCaml, F#, C (under certain conditions)
-
- Try it online at <https://fstar-lang.org/tutorial/>
 - Or install it locally: <https://github.com/FStarLang/FStar>



F* Applications

- Wide range of applications, mostly security-critical
 - **HACL***: High-Assurance cryptographic library
 - **miTLS**: Verified reference implementation of TLS (1.2 and 1.3)
 - **Noise***: End-to-end verified Implementations of 59 protocols in the Noise family
 - **EverParse**: Verified binary parsers and serializers
 - **StarMalloc**: Verified, concurrent, security-oriented memory allocator

The Functional Core of F*

- Recursive Functions

```
val factorial : nat -> nat
```

```
let rec factorial n =  
    if n = 0 then 1 else n * (factorial (n-1))
```

The Functional Core of F*

- Inductive types and pattern-matching

```
type list (a:Type) =  
| Nil : list a  
| Cons : hd: a -> tl: list a -> list a
```

```
let rec map (f: a -> b) (l:list a) : list a = match l with  
| [] -> []  
| hd :: tl -> f hd :: map f tl
```

```
map (fun x -> x + 3) [1; 2; 3]
```

Dependent Types in F*

- Types can be indexed by values, or other types

```
val vec (a:Type) : nat -> Type
```

```
type vec (a:Type) =
| Nil : vec a 0
| Cons : #n: nat -> hd: a -> tl: vec a n -> vec a (n+1)
```

```
let rec append #a #n #m (v1: vec a n) (v2: vec a m) : vec a (n + m) =
  match v1 with
  | Nil -> v2
  | Cons hd tl -> Cons hd (append tl v2)
```

Dependent Typechecking

```
let rec append #a #n #m (v1: vec a n) (v2: vec a m) : vec a (n + m) =  
  match v1 with  
  | Nil -> v2  
  | Cons hd tl -> Cons hd (append tl v2)
```

- Two typechecking goals:
 - $v1 = \text{Nil} \vdash v2 : \text{vec } a (n + m)$
 - $v1 = \text{Cons } \text{hd } \text{tl} \vdash \text{Cons } \text{hd } (\text{append } \text{tl } v2) : \text{vec } a (n + m)$
- Case 1: Goal is $\text{vec } a m = \text{vec } a (n + m)$
 - $v1 = \text{Nil} \Rightarrow n = 0$. Goal is $0 + m = m$.
Ok by SMT, using F* extensional type theory

Refinement Types

- A *refinement type* is a base type qualified with a logical formula; the formula can express invariants, preconditions, postconditions
- Refinement types are types of the form $x : T \{ \varphi \}$ where
 - T is the base type
 - x refers to the result of the expression, and
 - φ is a logical formula
- The values of this type are the values M of type T such that $\varphi\{M/x\}$ holds

Refinement Types in F*

```
type nat = n : int { n >= 0 }
```

```
type pos = n : int { n > 0 }
```

```
type neg = n : int { n < 0 }
```

```
type empty = n : int { False }
```

```
type empty_list (a:Type) = l : list a { l == [] }
```

```
type nonempty_list (a:Type) = l : list a { l != [] }
```

```
let nonempty_hd (l : nonempty_list a) = match l with  
| hd :: _ -> hd
```

```
nonempty_hd [1; 2; 3]      // Returns 1
```

```
nonempty_hd []            // Typing error returned by F*
```

Refinement Subtyping

```
type nat = n : int { n >= 0 }
type pos = n : int { n > 0 }
```

- How to ensure that a given integer can be typed as a nat?
 - Ex: $0:\text{int} <: \text{nat}$
- When given an $n : \text{pos}$, how to use it as a $n : \text{nat}$?
 - Ex: $2 : \text{pos} <: \text{nat}$
- We need rules for *Refinement Subtyping*

Refinement Subtyping: Elimination

- The type $x : t \{ \varphi \}$ is a subtype of t
For any expression $e : (x : t \{ \varphi \})$, it is always safe to eliminate the refinement φ
- Examples:
 - $x : \text{nat} (= \text{int} \{ x \geq 0 \}) <: x : \text{int}$
 - $f : \text{list } a \rightarrow \text{list } a, l : \text{nonempty_list } a,$
 $=> f l : \text{list } a$

Refinement Subtyping: Introduction

- For a term $e : t$, t is a subtype of the refinement type $x : t \{ \varphi \}$ if $\varphi[e/x]$
- Examples:
 - $[x] : \text{nonempty_list } a$
 - If $x : \text{even}$, then $x + 1 : \text{odd}$

Refinement Subtyping

```
let incr_even (x : even) : odd = x + 1
```

```
let incr_odd (x : odd) : even = x + 1
```

```
let f (x: int) : int =
  if x % 2 = 0 then incr_even x
  else incr_odd x
```

If branch, two goals:

- $x \% 2 = 0 |= x : \text{int} <: x : \text{even}$
- $x \% 2 = 0 |= \text{incr_even } x <: \text{int}$

Else branch, two goals:

- $\text{not}(x \% 2 = 0) |= x : \text{int} <: x : \text{odd}$
- $\text{not}(x \% 2 = 0) |= \text{incr_odd } x <: \text{int}$

Combining Refinement and Dependent Types

```
val incr (x:int) : (y:int{y = x + 1})
```

```
let incr x = x + 1      // Correctly typechecks
```

```
let incr x = x + 2      // Subtyping check failed, expected type y:int{y = x + 1}
```

```
val append (#a:Type) (l1 l2:list a) : (l:list a{length l == length l1 + length l2})
```

```
val seq_map (#a:Type) (f: a -> a) (s:seq a) : (s': seq a{
```

```
length s' == length s ∧
```

```
∀ (i:nat). i < length s ⇒ s'.[i] == f s.[i]})
```

Combining Refinement and Dependent Types

```
// Sample cryptographic library interface in F*
module AES

type key // Abstract type for secrets
type block = b: bytes{length b == 16}

val encrypt: k: key -> p:block -> c:block {c == AES(k, p)}
val decrypt: k: key -> c:block -> p:block {c == AES(k, p)}
```

Type Safety

- Safety means that all logical refinements hold at runtime
- **Theorem (safety):**
For a program A and a type T, if $\emptyset \vdash A : T$, then A is safe

Interfaces and Modular Typing

```
val seq (a: Type) : Type
```

Seq.fsti

```
val index (#a:Type) (s: seq a)  
  (i:nat{i < length s}) : a
```

```
val upd (#a:Type) (s: seq a)  
  (i:nat{i < length s}) (v: a) : seq a
```



```
let seq (a: Type) = list a
```

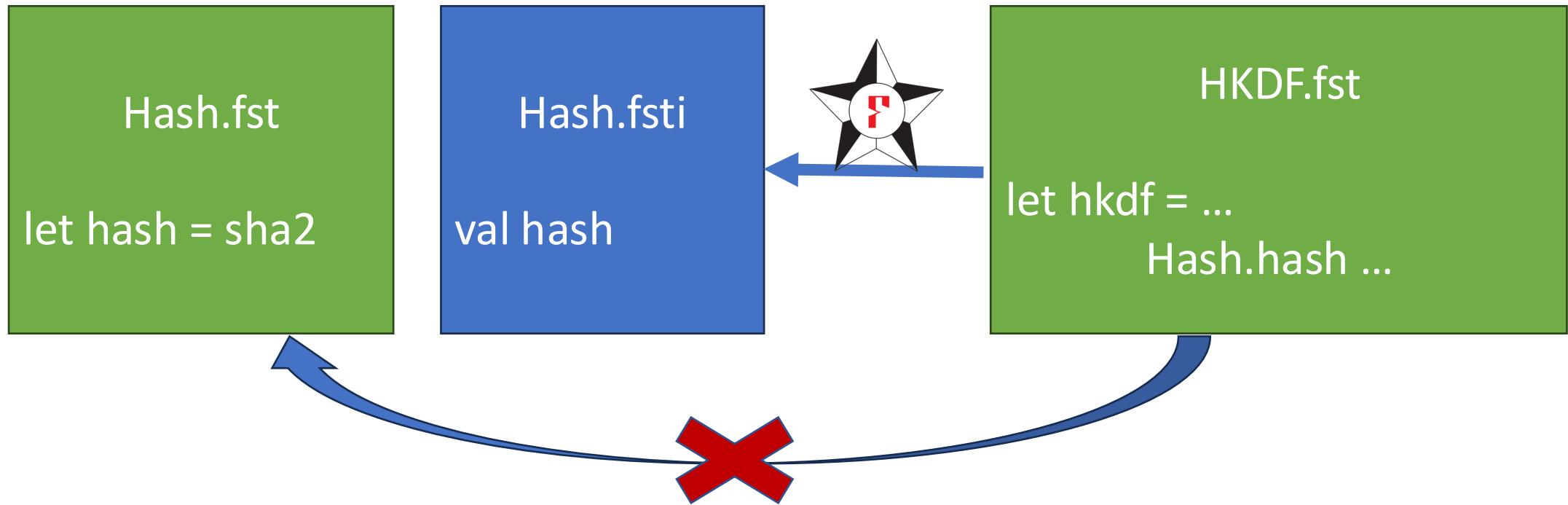
Seq.fst

```
let rec index #a s i =  
  if i = 0 then List.hd s else index (List.tl s) (i - 1)
```

```
let rec upd #a s i v =  
  if i = 0 then v :: List.tl s  
  else (List.hd s) :: upd (List.tl s) (i-1) v
```

- Interfaces abstract the underlying implementation and definitions
- Interfaces are optional

Modular Typing, Taming Proof Complexity



- Implementation details are not available for verification
- Replacing, e.g., SHA2 by another algorithm does not impact other modules
- Interfaces can be used as abstractions

Modular Typing, Formally

- We write $I_0 \vdash A \rightsquigarrow I$ when, in the typing environment I_0 , the module A is well-typed and exports the interface I
- **Theorem (Modular Typing):**

For programs A_0, A , interface I_0 and type T ,
If $\emptyset \vdash A_0 \rightsquigarrow I_0$ and $I_0 \vdash A : T$, then $\emptyset \vdash A_0 . A : T$
- This gives us safety of the program $A_0 . A$ based on the previous theorem

Assertions and Assumptions

Like many other languages, F* supports assertions and assumptions.

- **assert (P)** : Introduce a proof obligation for predicate P
- **assume (P)** : Adds predicate P to the current context.

Examples:

```
let f (x: int) : unit =  
  assume (x % 2 == 0);  
  assert ((x + 1) % 2 == 1)
```

```
let f (x: int) : unit =  
  assume (False);  
  assert (x == x + 1)
```

One can also use **admit ()** to introduce False in the context and admit the remaining of a proof

Intrinsic vs Extrinsic Verification

- Intrinsic Proof: The type of a term includes properties of interest

`val` list (a:Type) : Type

`val` length (#a:Type) (l: list a) : nat

`val` append (#a:Type) (l1 l2: list a) : (l: list a{length l == length l1 + length l2})

- Pros:
 - The proof easily follows the program
 - The property is directly available when calling the function
- Cons:
 - Proving while programming can be tedious
 - The type signature becomes harder to read
 - What about many different properties?

Extrinsic Verification: Lemmas

- F* supports built-in syntax for stating theorems.

```
val list (a:Type) : Type
```

```
val length (#a:Type) (l: list a) : nat
```

```
val append (#a:Type) (l1 l2: list a) : list a
```

```
val append_length (#a:Type) (l1 l2: list a) :
```

```
Lemma (length l1 + length l2 == length (append l1 l2))
```

Exercises

- Write the length and append functions, and prove the append_length theorem
- Write a list reverse function, and prove that reverse is involutive
- Write a recursive sum function that sums integers from 1 to n, and prove that it is equal to $\frac{n * (n+1)}{2}$
(You will need the command *open FStar.Mul* to use the * operator)

F*'s Effect System

- By default, F* functions are **total**

```
let rec factorial (n:nat) : nat =  
  if n = 0 then 1 else n * (factorial (n-1))
```

F*'s Effect System

- By default, F* functions are **total**

```
let rec factorial (n:nat) : Tot nat =  
  if n = 0 then 1 else n * (factorial (n-1))
```

- **Tot** is an **effect**, capturing that functions always terminate, and that they have no side-effects.
- What happens if we try to give this weaker type to factorial?

```
let rec factorial (n:int) : Tot int =  
  if n = 0 then 1 else n * (factorial (n-1))
```

F* Termination Checker

```
let rec factorial (n:int) : Tot int =  
  if n = 0 then 1 else n * (factorial (n-1))
```



Subtyping check failed, expected type (x:int{x << n}), got type int

factorial (-1) loops!

Arguments in recursive calls must decrease according to a well-founded ordering <<

Definition: An ordering is well-founded if it does not admit any infinite descending chain

Semantic Termination Checking

- Natural numbers related by $<$ (e.g., $1 << 2$ since $1 < 2$)
- Inductives related by subterm ordering (e.g., $\text{tl} << \text{Cons hd tl}$)
- By default, a recursive function with several arguments uses a lexicographical order on the arguments

Termination Checking, Examples

```
let rec factorial (n:nat) : Tot nat =  
    if n = 0 then 1 else n * (factorial (n-1))
```

- Goal: $n - 1 \ll n$.
 - Ordering on naturals is $<$, SMT can prove automatically $n - 1 < n$

```
let rec append #a (l1 l2: list a) : list a =  
    match v1 with  
    | Nil -> v2  
    | Cons hd tl -> Cons hd (append tl v2)
```

- Goal: $\%[tl; l2] \ll \%[l1; l2]$.
 - $tl \ll l1$ or $(tl == l1 \wedge l2 \ll l2)$
 - Subterm ordering on $l1$ gives $tl \ll l1$.

Termination Checking, Examples

```
let rec ackermann (n m:nat) : Tot nat =  
  if m=0 then n + 1  
  else if n = 0 then ackermann 1 (m - 1)  
  else ackermann (ackermann (n - 1) m) (m - 1)
```

Does this function pass termination checking?

Termination Checking, Examples

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```

Does this function pass termination checking?

```
let rec ackermann (n m:nat) : Tot nat (decreases %[m; n]) =  
  if m=0 then n + 1  
  else if n = 0 then ackermann 1 (m - 1)  
  else ackermann (ackermann (n - 1) m) (m - 1)
```

F* Effect System: Divergence

- We might want to write non-terminating code:
 - Web servers, operating systems, TLS protocol implementation, ...

- F* provides a built-in *effect* for divergence

```
let rec factorial (n:int) : Dv int =
  if n = 0 then 1 else n * (factorial (n-1))
```

- Code must still typecheck, but termination checker is disabled

Divergence: Avoiding inconsistencies

- Termination is required for consistency in proof assistants

```
let rec loop () : Dv False = loop () // This typechecks!
```

```
let f (x : int) : Tot (y:int{y == x + 1}) = let _ = loop () in x // What prevents this?
```

- F* effect system encapsulates effectful code: By default, different effects cannot interact

```
let f (x : int) : Tot (y:int{y == x + 1}) = let _ = loop () in x
```



Computed type "int" and effect "DIV" is not compatible with the annotated type
"int" effect "Tot"

Subeffecting

- Pure code cannot call potentially divergent code, and only pure code can appear in specifications and proofs.
- But including pure code in divergent code can be useful

```
let rec factorial (n:int) : Dv int = if n = 0 then 1 else n * (factorial (n-1))
```



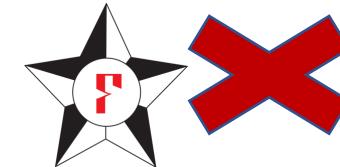
We do not want to redefine each basic operator

- F* supports sub-effecting: $\text{Tot } t <: \text{Dv } t$

Intrinsic Divergence Verification

```
let rec factorial (n:int) : Dv int = if n = 0 then 1 else n * (factorial (n-1))
```

```
val factorial_lemma (n:int) : Lemma (n ≥ 0 => factorial n ≥ 0)
```



- Only pure code can appear in specifications

```
let rec factorial (n:int) : Dv (y:int{n ≥ 0 => y ≥ 0}) =  
  if n = 0 then 1 else n * (factorial (n-1))
```



The GTot effect

- F* also allows writing Ghost code for specifications, proofs, ... which will be erased during extraction.

```
// Specification of factorial, using natural numbers
```

```
val factorial_spec: nat -> GTot nat
```

```
// Implementation, using machine integers
```

```
val factorial: n:uint64 -> Tot (y:uint64{to_nat y == factorial_spec (to_nat n)})
```

GTot Subeffecting

- Total code can be used in Ghost functions: $\text{Tot } t <: \text{GTot } t$
- Ghost code **cannot** be used in total functions

```
val f: nat -> GTot nat
```

f is ghost, hence erased at runtime.

```
let g (n: nat) : Tot nat =
```

```
  let x = f n in ← How to compile this statement?  
  x + 1
```

- Small subtlety: Ghost code for non-informative types (e.g., ghost values) is allowed (useful for proof purposes)

Refined Computation Types

- So far, refinement in value types:

```
val incr (n:int) : Tot (y:int{even n => odd y})
```

- F* also allows refined computation types:

```
val factorial (n:int) : Pure int (requires n ≥ 0) (ensures fun y -> y ≥ 0)
```

- Three elements:

- Effect (here, Pure), result type (here, int), specification (e.g., pre and post)
- **Tot** t is defined as an *abbreviation* of
Pure t (requires True) (ensures fun _ -> True)

Refined Computation Types

- Other effects are defined in a similar fashion

```
let rec loop (_:unit) : Div unit (requires True) (ensures fun _ -> False) = loop ()
```

```
Dv t == Div t (requires True) (ensures fun _ -> True)
```

```
val append_length (#a:Type) (l1 l2: list a) : Ghost unit  
  (requires True)  
  (ensures fun _ -> length l1 + length l2 == length (append l1 l2))
```

```
GTot t == Ghost t (requires True) (ensures fun _ -> True)
```

```
Lemma (requires P) (ensures Q) = Ghost unit (requires P) (ensures fun _ -> Q)
```

Working around the SMT solver

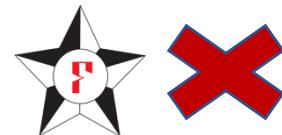
- So far, all F* proofs were discharged by SMT.
- Convenient, automated, but:
 - Cannot reason about induction (manual inductive proofs)
 - Struggles with some theories (e.g., complex modular arithmetic)
 - Performance degrades as the context grows (requires clever abstractions/interfaces for large programs)
- F* provides other reasoning facilities: normalization, the calc statement, and tactics

Proof by Normalization

- Dependently typed proof assistants include a *normalizer* which reduces computations during typechecking.
- F* provides access to the normalizer for proof purposes.

```
let rec length #a (l: list a) = match l with  
| [] -> 0 | hd :: tl -> 1 + length tl
```

```
assert (length [1; 2; 3; 4; 5; 6; 7; 8; 9; 10] == 10)
```



```
assert_norm (length [1; 2; 3; 4; 5; 6; 7; 8; 9; 10] == 10)
```



Proof by Normalization, Example

```
let rec length #a (l: list a) = match l with  
| [] -> 0 | hd :: tl -> 1 + length tl
```

```
assert_norm (length [1; 2; 3; 4; 5; 6; 7; 8; 9; 10] == 10)
```

```
match [1; 2; 3; 4; 5; 6; 7; 8; 9; 10] with | [] -> 0 | hd :: tl -> 1 + length tl == 10 ~
```

```
1 + match [2; 3; 4; 5; 6; 7; 8; 9; 10] with | [] -> 0 | hd :: tl -> 1 + length tl == 10 ~
```

...

```
10 == 10 ~
```

```
True
```

- Extremely useful for proofs involving recursive functions and **concrete terms**

Proof by Normalization

- The normalizer only performs reductions, it does not use logical facts in the context

```
assert_norm (length [1; 2; 3; 4; 5; 6; 7; 8; 9; 10] == 10)
```



```
let f (l:list a { length l == 10}) = assert_norm (length l == 10)
```



- The normalizer cannot reduce symbolic terms
- The normalizer can be fine-tuned (only include certain reduction steps, only unfold some definitions, definitions with a given attribute, ...)

Calc Statement

- Many (mathematical) proofs consist of a succession of equalities/comparisons:

$$(a + b * 2^c) * 2^d == a * 2^d + b * 2^c * 2^d == a * 2^d + b * 2^{c+d}$$

- F* provides a construct to emulate this:

```
calc (==) {  
    e1;  
    (==) { // proof of e1 == e2 }  
    e2;  
    (==) { // proof of e2 == e3 }  
    e3;  
}
```

```
calc (≥) {  
    e1;  
    (==) { // proof of e1 == e2 }  
    e2;  
    (≥) { // proof of e2 ≥ e3 }  
    e3;  
}
```

F* Tactics

- F* provides a metaprogramming and tactics framework, called Meta-F*
`assert (pow2 19 == 524288) by (compute (); dump "after compute")`
- Works well for:
 - Small rewritings/goal manipulation
 - Specific types of goals (separation logic, ring normalization)
 - F* goal inspection
- Not recommended as the main proof technique, better to use as a help to SMT

Exercises

- Stack, StackClient
- QuickSort: https://fstar-lang.org/tutorial/book/part1/part1_quicksort.html#exercises
- Arithmetic proofs using calc