

# Towards High-Assurance Cryptographic Software: the F\* Proof Assistant

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# Outline

- Previously: Proving the security of cryptographic protocols
- Today:
  - Verifying **implementations** of cryptographic protocols
  - The F\* proof assistant
    - The functional core of F\*
    - Exercises
    - Try it online at <https://fstar-lang.org/tutorial/>
    - Or install it locally: <https://github.com/FStarLang/FStar>

# What can go wrong?

Protocol model:

*secret s, key k*

`r <- sample()`

`m <- encrypt(k, concat(r, s))`

`send m`

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`send m`

Protocol implementation:

`let r = random() in`

`let m = encrypt(k, r . s) in`

`send m`

# What can go wrong?

Protocol model:

*secret s, key k*

`r <- sample()`

`m <- encrypt(k, concat(r, s))`

`send m`

Protocol implementation:

`let random () = 0`

`let r = random() in`

`let m = encrypt(k, r . s) in`

`send m`

# What can go wrong?

Protocol model:

*secret s, key k*

`r <- sample()`

`m <- encrypt(k, concat(r, s))`

`send m`

Protocol implementation:

`print(k)`

`let r = random() in`

`let m = encrypt(k, r . s) in`

`send m`

# What can go wrong?

Protocol model:

*secret s, key k*

`r <- sample()`

`m <- encrypt(k, concat(r, s))`

`send m`

Protocol implementation:

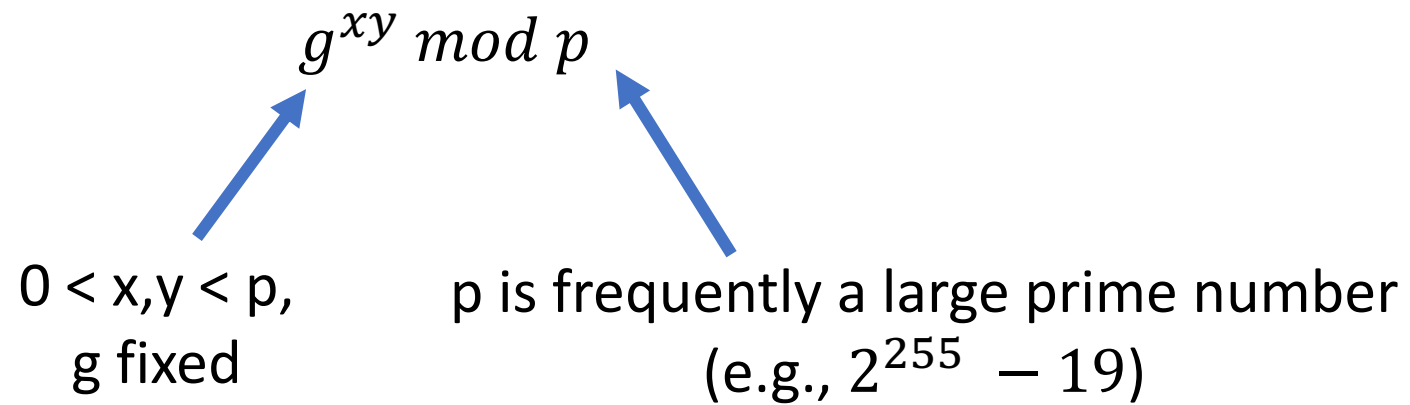
`let r = random() in`

`let m = encrypt(k, r . s) in`

`send (r . s)`

# A Concrete Example: Modular Arithmetic

- Modular arithmetic is frequently used in cryptographic primitives

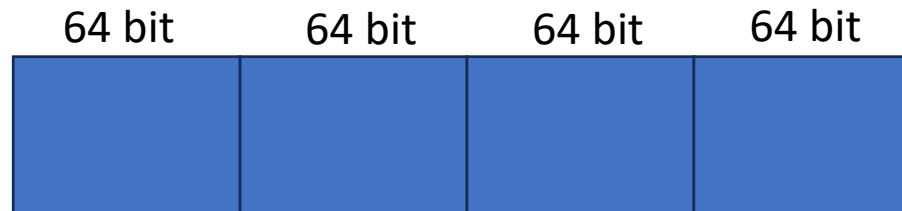




# Implementing Modular Exponentiation

$$a^b \bmod n = a * a * \dots * a \bmod n$$

- $a$  is a big integer (e.g.,  $2^{255} - 19$ )
- Exponentiation is even bigger
- Machine integers are (at most) 64 bits
- How to implement this? Need a bignum library



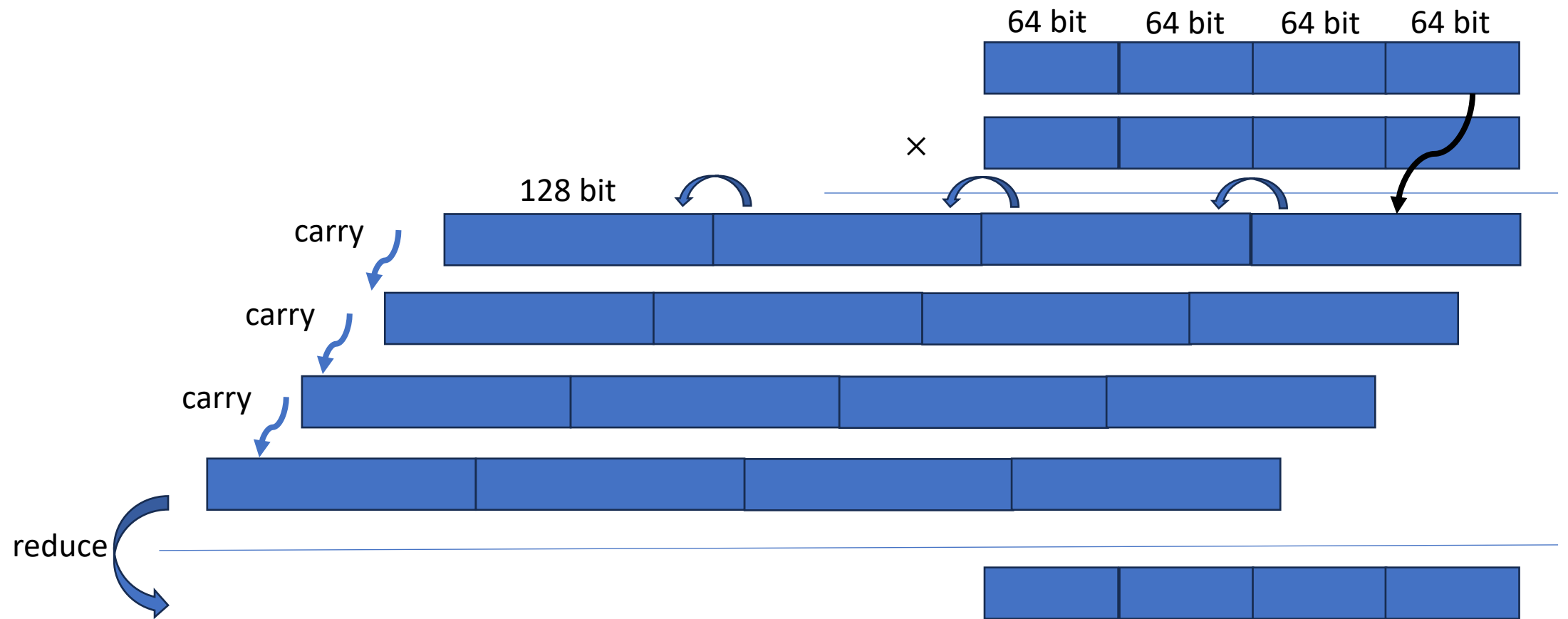
# Textbook Multiplication

$$\begin{array}{r} 1101 \\ * 1010 \\ \hline \end{array}$$

$$\begin{array}{r} 13 \\ * 10 \\ \hline = 130 \end{array}$$

$$\begin{array}{r} \text{carry} \quad 0000 \\ \text{carry} + 1101 \\ \text{carry} + 0000 \\ + 1101 \\ \hline 10000010 \end{array}$$

# 256-bit Modular Multiplication



# 256-bit Modular Multiplication

What can go wrong?

- Integer overflow (undefined output)
- Buffer overflow/underflow (memory error)
- Missing carry steps (wrong answer)
- Side-Channel attacks (leaks secrets)

# Modular Arithmetic Optimizations

- For many primitives, modular arithmetic dominates the crypto overhead
  - $n^2$  64-bit multiplications
  - Long intermediate arrays
  - Many carry steps
- Many specific optimizations
  - Use only 51 out of 64 bits to reduce carries
  - Precompute reusable intermediate values
  - Use alternative modular reductions (Montgomery, Barrett)
  - Parallelize (vectorize) multiplication and squaring
- Complex optimizations imply more chances of bugs!

# Many Bugs in Optimized Bignum Code

[2013] Bug in amd-64-64-24k Curve25519

“Partial audits have revealed **a bug in this software** ( $r1 += 0 + \text{carry}$  should be  $r2 += 0 + \text{carry}$  in amd-64-64-24k) **that would not be caught by random tests**”

– D.J. Bernstein, W.Janssen, T.Lange, and P.Schwabe

[2014] Arithmetic bug in TweetNaCl’s Curve25519

[2014] Carry bug in Langley’s Donna-32 Curve25519

[2016] Arithmetic bug in OpenSSL Poly1305

[2017] Arithmetic bug in Mozilla NSS GF128

...

# TweetNaCL Arithmetic Bug

```
sv pack25519(u8 *o, const gf n)
{
    int i,j,b;
    gf m,t;
    FOR(i,16) t[i]=n[i];
    car25519(t);
    car25519(t);
    car25519(t);
    FOR(j,2) {
        m[0]=t[0]-0xffed;
        for(i=1;i<15;i++) {
            m[i]=t[i]-0xffff-((m[i-1]>>16)&1);
            m[i-1]&=0xffff;
        }
        m[15]=t[15]-0x7fff-((m[14]>>16)&1);
        b=(m[15]>>16)&1;
        m[15]&=0xffff;
        sel25519(t,m,1-b);
    }
    FOR(i,16) {
        o[2*i]=t[i]&0xff;
        o[2*i+1]=t[i]>>8;
    }
}
```

seb.dbzteam.org

This bug is triggered when the last limb `n[15]` of the input argument `n` of this function is greater or equal than `0xffff`. In these cases the result of the scalar multiplication is not reduced as expected resulting in a wrong packed value. This code can be fixed simply by replacing `m[15]&=0xffff;` by `m[14]&=0xffff;`.

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# Heartbleed (CVE-2014-0160)

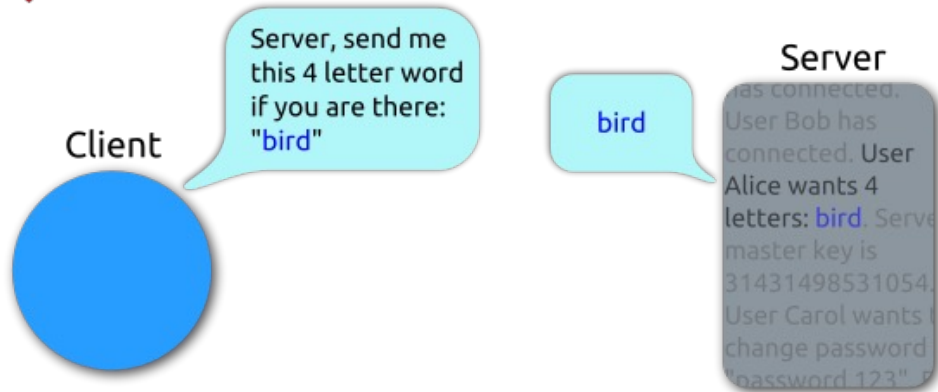


- Major vulnerability in OpenSSL TLS implementation
- Affected 17% of all SSL servers
- “Compromises the secret keys used to identify the service providers and to encrypt the traffic, the names and passwords of the users, and the actual content”
- “Allows attackers to eavesdrop on communications, steal data [...] and impersonate services and users.”
- Attacks do not leave a trace



# Heartbleed (CVE-2014-0160)

## Heartbeat – Normal usage

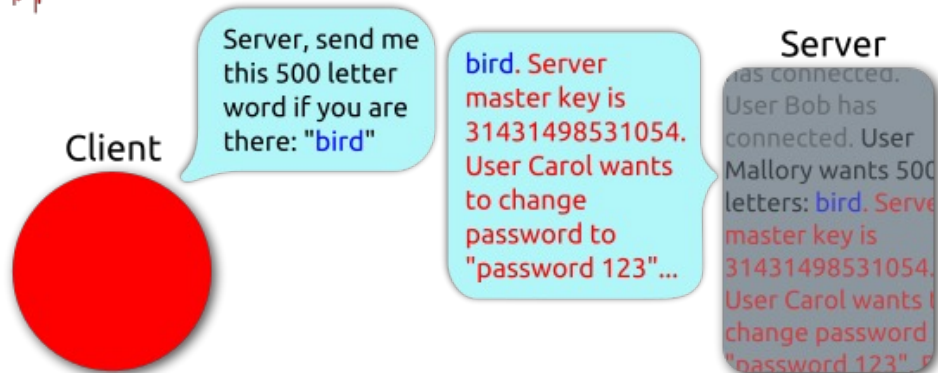


- Missing bound check during a memcpy

```
response = malloc(length);  
memcpy(response, recv.heartbeat, length);
```



## Heartbeat – Malicious usage



```
response = malloc(length);  
if length > ssl_state.heartbeat {return 0;}  
memcpy(response, recv.heartbeat, length);
```



# GotoFail (CVE-2014-1266)

```
status SSLVerifyExchange (...) { ...  
    if ((err = update(&hashCtx, &signedParams)) != 0)  
        goto fail;  
    goto fail;  
    if ((err = final(&hashCtx, &hashOut)) != 0)  
        goto fail;  
    ...  
fail:  
    SSLFreeBuffer(&signedHashes);  
    SSLFreeBuffer(&hashCtx);  
    return err;  
}
```

# GotoFail (CVE-2014-1266)

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status SSLVerifyExchange (...) { ...  
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    ...  
fail:  
    SSLFreeBuffer(&signedHashes);  
    SSLFreeBuffer(&hashCtx);  
    return err;  
}
```

# GotoFail (CVE-2014-1266)

- Duplicated goto statement in Apple's TLS implementation
- Bad copy/paste? Faulty merge?
- Impact:
  - Many invalid certificates were accepted
  - Allows using an arbitrary private key for signing or skipping the signing step
  - Enables Man-in-the-Middle attacks
- Many other vulnerabilities: SKIP, FREAK, many memory bugs, correctness issues, infinite loops, ...

# Formally Verifying Implementations

- Cryptographic implementations must be correct and secure, but also **fast**
- Cryptographic implementations are notoriously complex
  - Many tricky optimizations
  - Written in low-level, unsafe languages (C, Assembly)
  - Multiplicity of parameters and variants
- We need strong, formal guarantees about the **safety, correctness,** and **security** of cryptographic implementations

# The F\* Proof Assistant



- A functional programming language  
(like OCaml, Haskell, F#, ...)
- With support for dependent types (like Coq, Agda), refinement types, ...
- Semi-automated verification by relying on SMT solving  
(like Dafny, Why3, LiquidHaskell, ...)
- Also offers a metaprogramming and tactic framework (Meta-F\*)
- Extraction to OCaml, F#, C (under certain conditions)
- Try it online at <https://fstar-lang.org/tutorial/>
- Or install it locally: <https://github.com/FStarLang/FStar>



# F\* Applications

- Wide range of applications, mostly security-critical
  - **HACL\***: High-Assurance cryptographic library
  - **miTLS**: Verified reference implementation of TLS (1.2 and 1.3)
  - **Noise\***: End-to-end verified Implementations of 59 protocols in the Noise family
  - **EverParse**: Verified binary parsers and serializers
  - **StarMalloc**: Verified, concurrent, security-oriented memory allocator

# The Functional Core of F\*

- Recursive Functions

```
val factorial : nat -> nat
```

```
let rec factorial n =  
    if n = 0 then 1 else n * (factorial (n-1))
```



# The Functional Core of F\*

- Inductive types and pattern-matching

```
type list (a:Type) =
```

```
  | Nil : list a
```

```
  | Cons : hd: a -> tl: list a -> list a
```

```
let rec map (f: a -> b) (l:list a) : list b = match l with
```

```
  | [] -> []
```

```
  | hd :: tl -> f hd :: map f tl
```

```
map (fun x -> x + 3) [1; 2; 3]
```

# Dependent Types in F\*

- Types can be indexed by values, or other types

```
val vec (a:Type) : nat -> Type
```

```
type vec (a:Type) =
```

```
| Nil : vec a 0
```

```
| Cons : #n: nat -> hd: a -> tl: vec a n -> vec a (n+1)
```

```
let rec append #a #n #m (v1: vec a n) (v2: vec a m) : vec a (n + m) =
```

```
match v1 with
```

```
| Nil -> v2
```

```
| Cons hd tl -> Cons hd (append tl v2)
```

# Dependent Typechecking

```
let rec append #a #n #m (v1: vec a n) (v2: vec a m) : vec a (n + m) =  
  match v1 with  
  | Nil -> v2  
  | Cons hd tl -> Cons hd (append tl v2)
```

- Two typechecking goals:
  - $v1 = \text{Nil} \mid - v2 : \text{vec } a (n + m)$
  - $v1 = \text{Cons } hd \ tl \mid - \text{Cons } hd \ (\text{append } tl \ v2) : \text{vec } a (n + m)$
- Case 1: Goal is  $\text{vec } a \ m = \text{vec } a (n + m)$ 
  - $v1 = \text{Nil} \Rightarrow n = 0$ . Goal is  $0 + m = m$ .  
Ok by SMT, using  $F^*$  extensional type theory

# Refinement Types

- A *refinement type* is a base type qualified with a logical formula; the formula can express invariants, preconditions, postconditions
- Refinement types are types of the form  $\mathbf{x} : \mathbf{T} \{ \varphi \}$  where
  - $\mathbf{T}$  is the base type
  - $\mathbf{x}$  refers to the result of the expression, and
  - $\varphi$  is a logical formula
- The values of this type are the values  $M$  of type  $T$  such that  $\varphi\{M/x\}$  holds

# Refinement Types in F\*

```
type nat = n : int { n >= 0 }
```

```
type pos = n : int { n > 0 }
```

```
type neg = n : int { n < 0 }
```

```
type empty = n : int { False }
```

```
type empty_list (a:Type) = l : list a { l == [] }
```

```
type nonempty_list (a:Type) = l : list a { l != [] }
```

```
let nonempty_hd (l : nonempty_list a) = match l with  
  | hd :: _ -> hd
```

```
nonempty_hd [1; 2; 3]      // Returns 1
```

```
nonempty_hd []             // Typing error returned by F*
```

# Refinement Subtyping

```
type nat = n : int { n >= 0 }
```

```
type pos = n : int { n > 0 }
```

- How to ensure that a given integer can be typed as a nat?
  - Ex: `0:int <: nat`
- When given an `n : pos`, how to use it as a `n : nat` ?
  - Ex: `2 : pos <: nat`
- We need rules for *Refinement Subtyping*

# Refinement Subtyping: Elimination

- The type  $\mathbf{x : t \{ \varphi \}}$  is a subtype of  $\mathbf{t}$

For any expression  $e : (\mathbf{x : t \{ \varphi \}})$ , it is always safe to eliminate the refinement  $\varphi$

- Examples:

- $\mathbf{x : nat (= int \{ x \geq 0 \})} <: \mathbf{x : int}$
- $\mathbf{f : list\ a \rightarrow list\ a, l : nonempty\_list\ a,}$   
 $\mathbf{=> f\ l : list\ a}$

# Refinement Subtyping: Introduction

- For a term  $\mathbf{e} : \mathbf{t}$ ,  $\mathbf{t}$  is a subtype of the refinement type  $\mathbf{x} : \mathbf{t} \{ \varphi \}$  if  $\varphi[e/x]$
- Examples:
  - $[x] : \text{nonempty\_list } a$
  - If  $x : \text{even}$ , then  $x + 1 : \text{odd}$



# Refinement Subtyping

**let** incr\_even (x : even) : odd = x + 1

**let** incr\_odd (x : odd) : even = x + 1

**let** f (x: int) : int =

**if** x % 2 = 0 **then** incr\_even x

**else** incr\_odd x

If branch, two goals:

- $x \% 2 = 0 \mid = x : \text{int} <: x : \text{even}$
- $x \% 2 = 0 \mid = \text{incr\_even } x <: \text{int}$

Else branch, two goals:

- $\text{not } (x \% 2 = 0) \mid = x : \text{int} <: x : \text{odd}$
- $\text{not } (x \% 2 = 0) \mid = \text{incr\_odd } x <: \text{int}$

# Combining Refinement and Dependent Types

```
val incr (x:int) : (y:int{y = x + 1})
```

```
let incr x = x + 1      // Correctly typechecks
```

```
let incr x = x + 2      // Subtyping check failed, expected type y:int{y = x + 1}
```

```
val append (#a:Type) (l1 l2:list a) : (l:list a{length l == length l1 + length l2})
```

```
val seq_map (#a:Type) (f: a -> a) (s:seq a) : (s': seq a{  
    length s' == length s ∧  
    ∀ (i: nat). i < length s ⇒ s'[i] == f s[i]})
```

# Combining Refinement and Dependent Types

// Sample cryptographic library interface in F\*

module AES

type key // Abstract type for secrets

type block = b: bytes{length b == 16}

val encrypt: k: key -> p:block -> c:block {c == AES(k, p)}

val decrypt: k: key -> c:block -> p:block {c == AES(k, p)}

# Type Safety

- Safety means that all logical refinements hold at runtime
- **Theorem (safety):**  
For a program  $A$  and a type  $T$ , if  $\emptyset \vdash A : T$ , then  $A$  is safe

# Interfaces and Modular Typing

**val** seq (a: Type) : Type Seq.fsti

**val** index (#a:Type) (s: seq a)  
    (i:nat{i < length s}) : a

**val** upd (#a:Type) (s: seq a)  
    (i:nat{i < length s}) (v: a) : seq a

**let** seq (a: Type) = list a Seq.fst

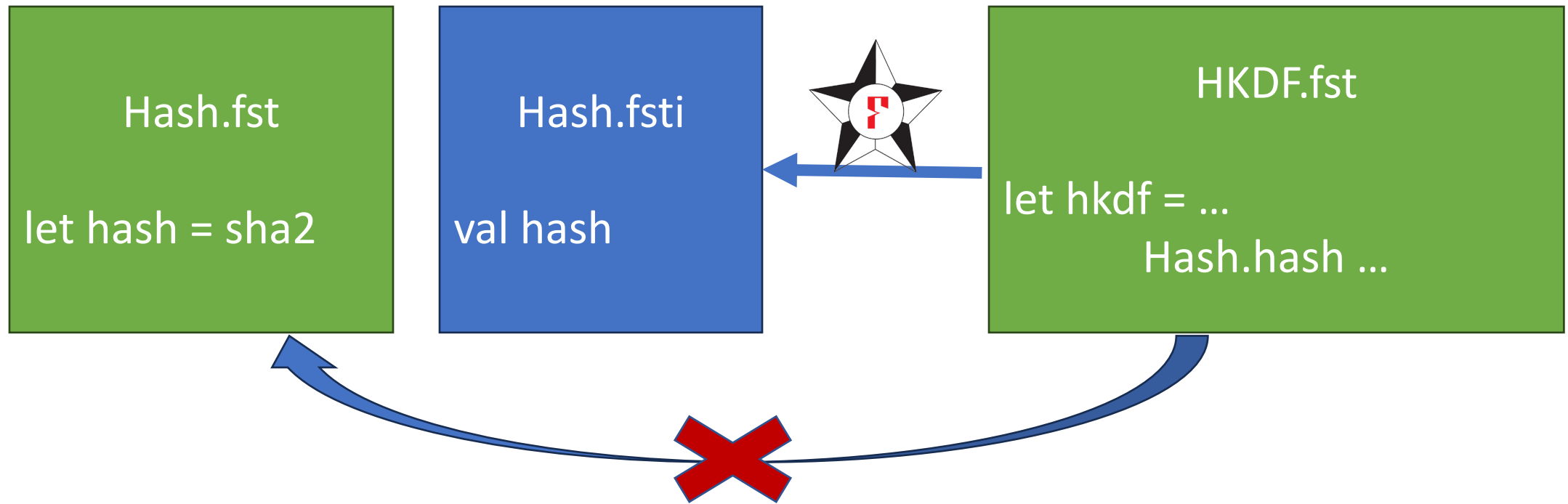
**let rec** index #a s i =  
    if i = 0 then List.hd s else index (List.tl s) (i - 1)

**let rec** upd #a s i v =  
    if i = 0 then v :: List.tl s  
    else (List.hd s) :: upd (List.tl s) (i-1) v



- Interfaces abstract the underlying implementation and definitions
- Interfaces are optional

# Modular Typing, Taming Proof Complexity



- Implementation details are not available for verification
- Replacing, e.g., SHA2 by another algorithm does not impact other modules
- Interfaces can be used as abstractions

# Modular Typing, Formally

- We write  $I_0 \vdash A \rightsquigarrow I$  when, in the typing environment  $I_0$ , the module  $A$  is well-typed and exports the interface  $I$
- **Theorem (Modular Typing):**  
For programs  $A_0, A$ , interface  $I_0$  and type  $T$ ,  
If  $\emptyset \vdash A_0 \rightsquigarrow I_0$  and  $I_0 \vdash A : T$ , then  $\emptyset \vdash A_0 . A : T$
- This gives us safety of the program  $A_0 . A$  based on the previous theorem

# Assertions and Assumptions

Like many other languages, F\* supports assertions and assumptions.

- `assert (P)` : Introduce a proof obligation for predicate P
- `assume (P)` : Adds predicate P to the current context.

Examples:

```
let f (x: int) : unit =  
    assume (x % 2 == 0);  
    assert ((x + 1) % 2 == 1)
```

```
let f (x: int) : unit =  
    assume (False);  
    assert (x == x + 1)
```

One can also use `admit ()` to introduce False in the context and admit the remaining of a proof



# Intrinsic vs Extrinsic Verification

- Intrinsic Proof: The type of a term includes properties of interest

```
val list (a:Type) : Type
```

```
val length (#a:Type) (l: list a) : nat
```

```
val append (#a:Type) (l1 l2: list a) : (l: list a {length l == length l1 + length l2})
```

- Pros:
  - The proof easily follows the program
  - The property is directly available when calling the function
- Cons:
  - Proving while programming can be tedious
  - The type signature becomes harder to read
  - What about many different properties?

# Extrinsic Verification: Lemmas

- F\* supports built-in syntax for stating theorems.

```
val list (a:Type) : Type
```

```
val length (#a:Type) (l: list a) : nat
```

```
val append (#a:Type) (l1 l2: list a) : list a
```

```
val append_length (#a:Type) (l1 l2: list a) :
```

```
    Lemma (length l1 + length l2 == length (append l1 l2))
```

# Exercises

- Write the length and append functions, and prove the append\_length theorem
- Write a list reverse function, and prove that reverse is involutive
- Write a recursive sum function that sums integers from 1 to n, and prove that it is equal to  $\frac{n * (n+1)}{2}$   
(You will need the command *open FStar.Mul* to use the \* operator)

# F\*'s Effect System

- By default, F\* functions are **total**

```
let rec factorial (n:nat) : nat =  
  if n = 0 then 1 else n * (factorial (n-1))
```

# F\*'s Effect System

- By default, F\* functions are **total**

```
let rec factorial (n:nat) : Tot nat =  
  if n = 0 then 1 else n * (factorial (n-1))
```

- **Tot** is an **effect**, capturing that functions always terminate, and that they have no side-effects.

- What happens if we try to give this weaker type to factorial?

```
let rec factorial (n:int) : Tot int =  
  if n = 0 then 1 else n * (factorial (n-1))
```

# F\* Termination Checker

```
let rec factorial (n:int) : Tot int =  
  if n = 0 then 1 else n * (factorial (n-1))
```



Subtyping check failed, expected type  $(x:\text{int}\{x \ll n\})$ , got type `int`

`factorial (-1)` loops!

Arguments in recursive calls must decrease according to a well-founded ordering  $\ll$

**Definition:** An ordering is well-founded if it does not admit any infinite descending chain

# Semantic Termination Checking

- Natural numbers related by  $<$  (e.g.,  $1 \ll 2$  since  $1 < 2$ )
- Inductives related by subterm ordering (e.g.,  $tl \ll \text{Cons } hd \ tl$ )
- By default, a recursive function with several arguments uses a lexicographical order on the arguments

# Termination Checking, Examples

```
let rec factorial (n:nat) : Tot nat =  
  if n = 0 then 1 else n * (factorial (n-1))
```

- Goal:  $n - 1 << n$ .
  - Ordering on naturals is  $<$ , SMT can prove automatically  $n - 1 < n$

```
let rec append #a (l1 l2: list a) : list a =  
  match v1 with  
  | Nil -> v2  
  | Cons hd tl -> Cons hd (append tl v2)
```

- Goal:  $\%[tl; l2] << \%[l1; l2]$ .
  - $tl << l1$  or  $(tl == l1 \wedge l2 << l2)$
  - Subterm ordering on  $l1$  gives  $tl << l1$ .



# Termination Checking, Examples

```
let rec ackermann (n m:nat) : Tot nat =  
  if m=0 then n + 1  
  else if n = 0 then ackermann 1 (m - 1)  
  else ackermann (ackermann (n - 1) m) (m - 1)
```

Does this function pass termination checking?

# Termination Checking, Examples

```
let rec ackermann (n m:nat) : Tot nat =  
  if m=0 then n + 1  
  else if n = 0 then ackermann 1 (m - 1)  
  else ackermann (ackermann (n - 1) m) (m - 1)
```

Does this function pass termination checking?

```
let rec ackermann (n m:nat) : Tot nat (decreases %[m; n]) =  
  if m=0 then n + 1  
  else if n = 0 then ackermann 1 (m - 1)  
  else ackermann (ackermann (n - 1) m) (m - 1)
```

# F\* Effect System: Divergence

- We might want to write non-terminating code:
  - Web servers, operating systems, TLS protocol implementation, ...

- F\* provides a built-in *effect* for divergence

```
let rec factorial (n:int) : Dv int =  
  if n = 0 then 1 else n * (factorial (n-1))
```

- Code must still typecheck, but termination checker is disabled

# Divergence: Avoiding inconsistencies

- Termination is required for consistency in proof assistants

```
let rec loop () : Dv False = loop ()           // This typechecks!
```

```
let f (x : int) : Tot (y:int{y == x + 1}) = let _ = loop () in x    // What prevents this?
```

- F\* effect system encapsulates effectful code: By default, different effects cannot interact

```
let f (x : int) : Tot (y:int{y == x + 1}) = let _ = loop () in x
```



Computed type "int" and effect "DIV" is not compatible with the annotated type "int" effect "Tot"

# Subeffecting

- Pure code cannot call potentially divergent code, and only pure code can appear in specifications and proofs.
- But including pure code in divergent code can be useful

```
let rec factorial (n:int) : Dv int = if n = 0 then 1 else n * (factorial (n-1))
```



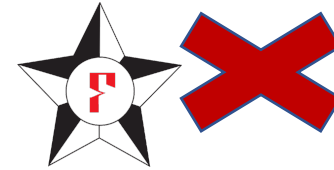
We do not want to redefine each basic operator

- F\* supports sub-effecting:  $\text{Tot } t \leq \text{Dv } t$

# Intrinsic Divergence Verification

```
let rec factorial (n:int) : Dv int = if n = 0 then 1 else n * (factorial (n-1))
```

```
val factorial_lemma (n:int) : Lemma (n ≥ 0 => factorial n ≥ 0)
```



- Only pure code can appear in specifications

```
let rec factorial (n:int) : Dv (y:int{n ≥ 0 => y ≥ 0}) =  
  if n = 0 then 1 else n * (factorial (n-1))
```



# The GTot effect

- F\* also allows writing Ghost code for specifications, proofs, ... which will be erased during extraction.

// Specification of factorial, using natural numbers

val factorial\_spec: nat -> GTot nat

// Implementation, using machine integers

val factorial: n:uint64 -> Tot (y:uint64{to\_nat y == factorial (to\_nat n)})

# GTot Subeffecting

- Total code can be used in Ghost functions:  $\text{Tot } t <: \text{GTot } t$
- Ghost code **cannot** be used in total functions

```
val f: nat -> GTot nat
```

f is ghost, hence erased at runtime.

```
let g (n: nat) : Tot nat =
```

```
  let x = f n in  
  x + 1
```



How to compile this statement?

- Small subtlety: Ghost code for non-informative types (e.g., ghost values) is allowed (useful for proof purposes)



# Refined Computation Types

- So far, refinement in value types:

`val incr (n:int) : Tot (y:int{even n => odd y})`

- F\* also allows refined computation types:

`val factorial (n:int) : Pure int (requires n ≥ 0) (ensures fun y -> y ≥ 0)`

- Three elements:

- Effect (here, Pure), result type (here, int), specification (e.g., pre and post)

- `Tot t` is defined as an *abbreviation* of

`Pure t (requires True) (ensures fun _ -> True)`

# Refined Computation Types

- Other effects are defined in a similar fashion

```
let rec loop (_:unit) : Div unit (requires True) (ensures fun _ -> False) = loop ()
```

```
Dv t == Div t (requires True) (ensures fun _ -> True)
```

```
val append_length (#a:Type) (l1 l2: list a) : Ghost unit  
  (requires True)  
  (ensures fun _ -> length l1 + length l2 == length (append l1 l2))
```

```
GTot t == Ghost t (requires True) (ensures fun _ -> True)
```

```
Lemma (requires P) (ensures Q) = Ghost unit (requires P) (ensures fun _ -> Q)
```

# Exercises

- Stack, StackClient
- QuickSort: [https://fstar-lang.org/tutorial/book/part1/part1\\_quicksort.html#exercises](https://fstar-lang.org/tutorial/book/part1/part1_quicksort.html#exercises)

# Working around the SMT solver

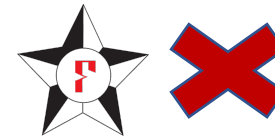
- So far, all F\* proofs were discharged by SMT.
- Convenient, automated, but:
  - Cannot reason about induction (manual inductive proofs)
  - Struggles with some theories (e.g., complex modular arithmetic)
  - Performance degrades as the context grows (requires clever abstractions/interfaces for large programs)
- F\* provides other reasoning facilities: normalization, the calc statement, and tactics

# Proof by Normalization

- Dependently typed proof assistants include a *normalizer* which reduces computations during typechecking.
- F\* provides access to the normalizer for proof purposes.

```
let rec length #a (l: list a) = match l with  
  | [] -> 0 | hd :: tl -> 1 + length tl
```

```
assert (length [1; 2; 3; 4; 5; 6; 7; 8; 9; 10] == 10)
```



```
assert_norm (length [1; 2; 3; 4; 5; 6; 7; 8; 9; 10] == 10)
```



# Proof by Normalization, Example

```
let rec length #a (l: list a) = match l with  
  | [] -> 0 | hd :: tl -> 1 + length tl
```

```
assert_norm (length [1; 2; 3; 4; 5; 6; 7; 8; 9; 10] == 10)
```

```
match [1; 2; 3; 4; 5; 6; 7; 8; 9; 10] with | [] -> 0 | hd :: tl -> 1 + length tl == 10 ~
```

```
1 + match [2; 3; 4; 5; 6; 7; 8; 9; 10] with | [] -> 0 | hd :: tl -> 1 + length tl == 10 ~
```

```
...
```

```
10 == 10 ~
```

```
True
```

- Extremely useful for proofs involving recursive functions and **concrete** terms

# Proof by Normalization

- The normalizer only performs reductions, it does not use logical facts in the context

`assert_norm` (length [1; 2; 3; 4; 5; 6; 7; 8; 9; 10] == 10)



`let f (l:list a { length l == 10}) = assert_norm (length l == 10)`



- The normalizer cannot reduce symbolic terms
- The normalizer can be fine-tuned (only include certain reduction steps, only unfold some definitions, definitions with a given attribute, ...)

# Calc Statement

- Many (mathematical) proofs consist of a succession of equalities/comparisons:

$$(a + b * 2^c) * 2^d == a * 2^d + b * 2^c * 2^d == a * 2^d + b * 2^{c+d}$$

- F\* provides a construct to emulate this:

```
calc (==) {  
  e1;  
  (==) { // proof of e1 == e2 }  
  e2;  
  (==) { // proof of e2 == e3 }  
  e3;  
}
```

```
calc (≥) {  
  e1;  
  (==) { // proof of e1 == e2 }  
  e2;  
  (≥) { // proof of e2 ≥ e3 }  
  e3;  
}
```



# F\* Tactics

- F\* provides a metaprogramming and tactics framework, called Meta-F\*  
`assert (pow2 19 == 524288) by (compute (); dump "after compute")`
- Works well for:
  - Small rewritings/goal manipulation
  - Specific types of goals (separation logic, ring normalization)
  - F\* goal inspection
- Not recommended as the main proof technique, better to use as a help to SMT

# Exercises

- Arithmetic proofs using calc