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M.Tech in INDUSTRIAL AI

Assignment - 2

pls find the notebook at : https://github.com/aymuos/masters-practise-repo/tree/main/TERM4/Assignment2.ipynb

```
%pip install numpy scikit-learn pandas tabulate matplotlib statsmodels

```

```
Requirement already satisfied: numpy in /usr/local/lib/python3.12/dist-packages (2.0.2)
Requirement already satisfied: scikit-learn in /usr/local/lib/python3.12/dist-packages (1.6.1)
Requirement already satisfied: pandas in /usr/local/lib/python3.12/dist-packages (2.2.2)
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Requirement already satisfied: matplotlib in /usr/local/lib/python3.12/dist-packages (3.10.0)
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Requirement already satisfied: scipy>=1.6.0 in /usr/local/lib/python3.12/dist-packages (from scikit-learn) (1.16.
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Requirement already satisfied: kiwisolver>=1.3.1 in /usr/local/lib/python3.12/dist-packages (from matplotlib) (1.
Requirement already satisfied: packaging>=20.0 in /usr/local/lib/python3.12/dist-packages (from matplotlib) (25.0
Requirement already satisfied: pillow>=8 in /usr/local/lib/python3.12/dist-packages (from matplotlib) (11.3.0)
Requirement already satisfied: pyparsing>=2.3.1 in /usr/local/lib/python3.12/dist-packages (from matplotlib) (3.2
Requirement already satisfied: patsy>=0.5.6 in /usr/local/lib/python3.12/dist-packages (from statsmodels) (1.0.2)
Requirement already satisfied: six>=1.5 in /usr/local/lib/python3.12/dist-packages (from python-dateutil>=2.8.2->
```

Assignment 1

(Q1)

Approach used:

Alternating Location Allocation heuristics.

Breaks the problem into 2 alternating steps till the problem converges.

Phase - A : Allocation.

Warehouse location are fixed. We assign each dealership to the nearest warehouse based on Euclidian distance to minimise cost.

Assign dealership into warehouse j

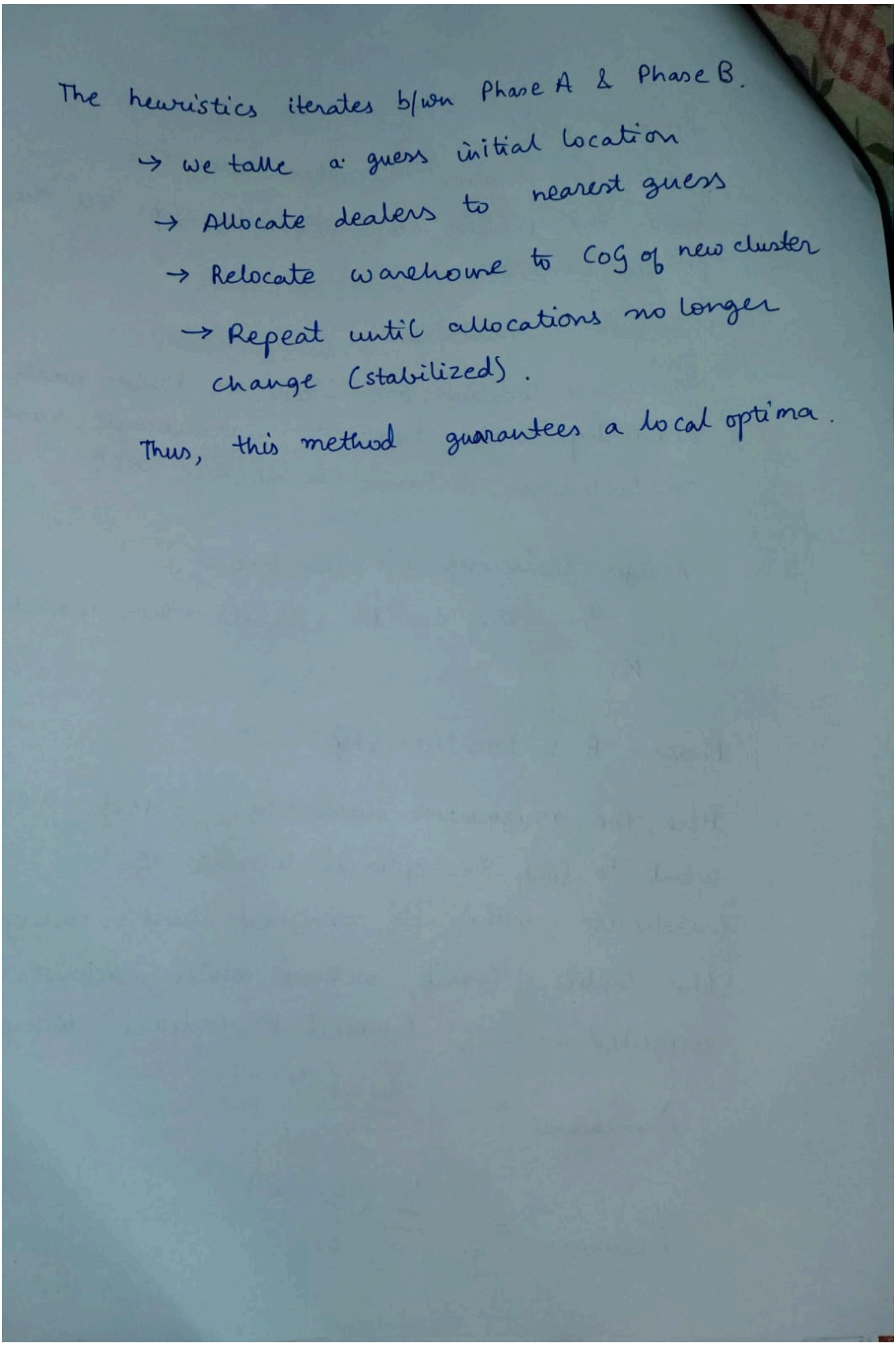
if $d_{ij} < d_{iK}$ for all other warehouses K.

Phase - B : Location Step.

Here, the assignment clusters are fixed. We want to find the optimal location of the warehouse within its assigned cluster using the Centre of Gravity method which minimises the weighted sum of Squared Euclidian distances

$$X_{warehouse} = \frac{\sum (x_i \cdot w_i)}{\sum w_i}$$

$$Y_{warehouse} = \frac{\sum (y_i \cdot w_i)}{\sum w_i}$$



- Approach used :

- ALA (Alternating Location-Allocation) Heuristic, which breaks the problem into two alternating steps until the solution stabilizes (converges).

Phase A: The Allocation Step

In this step, the locations of the warehouses are fixed. We assign each dealership to the nearest warehouse based on Euclidean distance to minimize cost.

Rule: Assign Dealership i to Warehouse j if $d_{ij} < d_{ik}$ for all other warehouses k. Theory: This creates a "Voronoi Partition" of the space, ensuring every customer is served by the closest facility.

Phase B: The Location Step (Center of Gravity)

In this step, the assignments (clusters) are fixed. We want to find the optimal location for the warehouse within its assigned cluster using the Center of Gravity (COG) method which minimizes the weighted sum of squared Euclidean distances. The coordinates (X, Y) for a warehouse are calculated as the weighted average of the coordinates of all dealerships assigned to it:

$$X_{warehouse} = \frac{\sum(x_i \cdot w_i)}{\sum w_i} \quad , \quad Y_{warehouse} = \frac{\sum(y_i \cdot w_i)}{\sum w_i}$$

Convergence of the algorithm :

The heuristic iterates between Phase A and Phase B.

- Guess initial locations.
- Allocate dealers to nearest guess.
- Re-locate the warehouse to the Center of Gravity of the new cluster.
- Repeat until the allocations no longer change (Stabilization).

This method guarantees a local optimum

```
import pandas as pd
import numpy as np
from scipy.spatial.distance import euclidean
from scipy.optimize import linprog
```

```
data = {
    'Dealership': ['D1', 'D2', 'D3', 'D4', 'D5', 'D6', 'D7', 'D8'],
    'x': [10, 20, 90, 85, 60, 65, 15, 20],
    'y': [50, 45, 10, 15, 70, 75, 25, 30],
    'Demand': [200, 150, 180, 170, 160, 140, 130, 120]
}
df = pd.DataFrame(data)
```

```
# 2. Function to Calculate Total Weighted Cost and Allocate

def calculate_total_cost(df, W):

    """Calculates the total weighted cost and performs the allocation step."""


    W1, W2 = W

    # Calculate Euclidean distances to current warehouse locations
    df['dist_W1'] = df.apply(lambda row: euclidean((row['x'], row['y']), W1), axis=1)
    df['dist_W2'] = df.apply(lambda row: euclidean((row['x'], row['y']), W2), axis=1)

    # Allocation: Assign to the closest warehouse
    df['Assigned_W'] = np.where(df['dist_W1'] <= df['dist_W2'], 1, 2)
    df['Min_Dist'] = np.minimum(df['dist_W1'], df['dist_W2'])
    df['Weighted_Cost'] = df['Demand'] * df['Min_Dist']

    total_cost = df['Weighted_Cost'].sum()
    return total_cost, df


# 3. Center of Gravity (COG) Method
def center_of_gravity(cluster_df):

    """Finds the Center of Gravity (COG) for a fixed cluster."""

    if cluster_df.empty:
        return np.array([0.0, 0.0])

    total_demand = cluster_df['Demand'].sum()

    x_cog = (cluster_df['x'] * cluster_df['Demand']).sum() / total_demand
    y_cog = (cluster_df['y'] * cluster_df['Demand']).sum() / total_demand
```

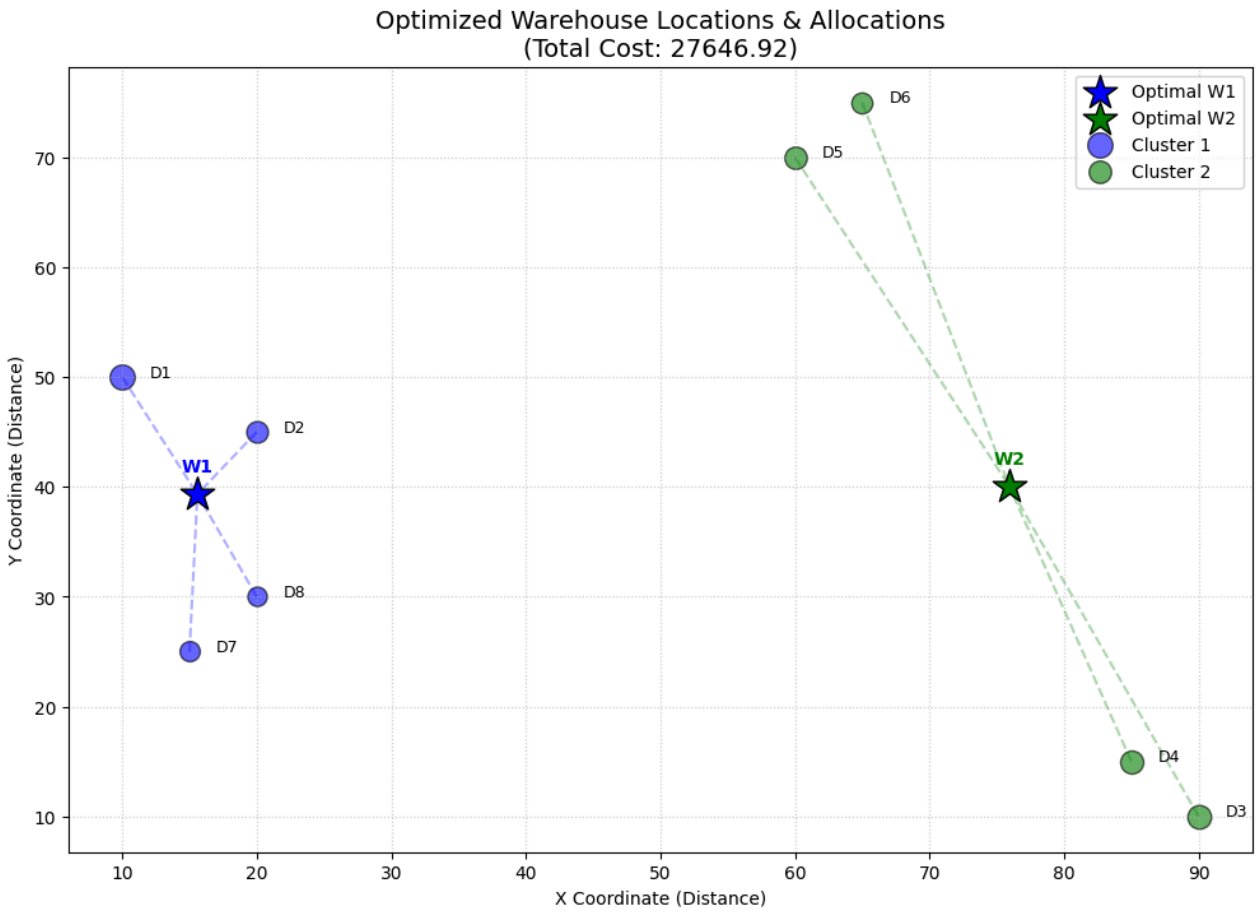
```
    return np.array([x_cog, y_cog])

#MAIN lOOP

# 4. Alternating Location-Allocation (ALA) Heuristic Main Loop
def ala_heuristic_cog(df, initial_W):
    W = initial_W
    prev_allocation = None


    for iteration in range(1, 10):

        # --- Allocation
        cost, current_df = calculate_total_cost(df.copy(), W)
        current_allocation = current_df['Assigned_W'].tolist()

        # Check for convergence (Allocation stabilization)
        if current_allocation == prev_allocation:
            # Re-run the COG calculation one last time with the final allocation for precise coordinates
            cluster1_df = current_df[current_df['Assigned_W'] == 1]
            cluster2_df = current_df[current_df['Assigned_W'] == 2]
            W1_new = center_of_gravity(cluster1_df)
            W2_new = center_of_gravity(cluster2_df)
            W = (W1_new, W2_new)
            final_cost, final_df = calculate_total_cost(df.copy(), W)
            return W, final_cost, final_df

        prev_allocation = current_allocation

        # --- Location Step (COG)

        cluster1_df = current_df[current_df['Assigned_W'] == 1]
        W1_new = center_of_gravity(cluster1_df)

        cluster2_df = current_df[current_df['Assigned_W'] == 2]
        W2_new = center_of_gravity(cluster2_df)

        W = (W1_new, W2_new)

    return W, cost, current_df
```

```


W_initial = (   np.array([16.75, 39.50]),
                np.array([75.31, 40.08]))

optimal_W, min_cost, final_df = ala_heuristic_cog(df.copy(), W_initial)


W1_final = optimal_W[0]
W2_final = optimal_W[1]
output_df = final_df[['Dealership', 'x', 'y', 'Demand', 'Assigned_W', 'Min_Dist', 'Weighted_Cost']].copy()
output_df.rename(columns={'Assigned_W': 'Warehouse', 'Min_Dist': 'Distance (km)', 'Weighted_Cost': 'Weighted

output_df['Warehouse'] = 'W' + output_df['Warehouse'].astype(str)
output_df['Distance (km)'] = output_df['Distance (km)'].round(2)
output_df['Weighted Cost (units*km)'] = output_df['Weighted Cost (units*km)'].round(2)

print(f"\nOptimal Warehouse 1 Location (W1): ({W1_final[0]:.2f}, {W1_final[1]:.2f}) km")

print(f"Optimal Warehouse 2 Location (W2): ({W2_final[0]:.2f}, {W2_final[1]:.2f}) km")

print(f"Minimum Total Transportation Cost: {min_cost:.2f} units * km")

print("--------------------------------------------------------------------------------")
print("\nFinal Dealership Assignments and Costs (Table View):")
print(output_df.to_markdown(index=False, floatfmt=".2f"))
```

```
Optimal Warehouse 1 Location (W1): (15.58, 39.33) km
Optimal Warehouse 2 Location (W2): (75.92, 40.08) km
Minimum Total Transportation Cost: 27646.92 units * km
--------------------------------------------------------------------------------

Final Dealership Assignments and Costs (Table View):
| Dealership   |   x |   y |   Demand | Warehouse   |   Distance (km) |   Weighted Cost (units*km) |
|:-------------|----:|----:|---------:|:------------|----------------:|---------------------------:|
| D1           |  10 |  50 |      200 | W1          |           12.04 |                    2407.92 |
| D2           |  20 |  45 |      150 | W1          |            7.18 |                    1077.69 |
| D3           |  90 |  10 |      180 | W2          |           33.21 |                    5977.47 |
| D4           |  85 |  15 |      170 | W2          |           26.67 |                    4533.75 |
| D5           |  60 |  70 |      160 | W2          |           33.90 |                    5423.35 |
```

```
| D6          |  65 |  75 |      140 | W2          |           36.59 |                  5122.80 |
| D7          |  15 |  25 |      130 | W1          |           14.35 |                  1864.88 |
| D8          |  20 |  30 |      120 | W1          |           10.33 |                  1239.07 |
```

```python
import matplotlib.pyplot as plt

def plot_warehouse_allocation(df, warehouses):

    W1, W2 = warehouses

    plt.figure(figsize=(12, 8))

    colors = {1: 'blue', 2: 'green'}

    # 1. Plot the Warehouses

    # Warehouse 1
    plt.scatter(W1[0], W1[1], c='blue', marker='*', s=400,
                label='Optimal W1', edgecolors='black', zorder=10)
    plt.text(W1[0], W1[1]+2, 'W1', fontweight='bold', ha='center', color='blue')

    # Warehouse 2
    plt.scatter(W2[0], W2[1], c='green', marker='*', s=400,
                label='Optimal W2', edgecolors='black', zorder=10)
    plt.text(W2[0], W2[1]+2, 'W2', fontweight='bold', ha='center', color='green')

    # 2. Plot Dealerships and Connections
    for index, row in df.iterrows():
        w_id = int(row['Assigned_W'])
        color = colors[w_id]
        warehouse_coords = W1 if w_id == 1 else W2

        # A. Draw Connection Line (Spider line)
        plt.plot([row['x'], warehouse_coords[0]], [row['y'], warehouse_coords[1]],
                 c=color, linestyle='--', alpha=0.3)

        # B. Plot Dealership
        plt.scatter(row['x'], row['y'], c=color, s=row['Demand'],
                    alpha=0.6, edgecolors='black', label=f'Cluster {w_id}' if index in [0, 4] else "")

        # C. Label Dealership
        plt.text(row['x']+2, row['y'], row['Dealership'], fontsize=9)

    # 3. Formatting the Graph
    plt.title(f'Optimized Warehouse Locations & Allocations\n(Total Cost: {min_cost:.2f})', fontsize=14)
    plt.xlabel('X Coordinate (Distance)')
    plt.ylabel('Y Coordinate (Distance)')
    plt.legend(loc='upper right')
    plt.grid(True, linestyle=':', alpha=0.6)

    plt.show()


plot_warehouse_allocation(final_df, optimal_W)
```



Answer 1

Optimal Warehouse 1 Location (W1): (15.58, 39.33) km

Optimal Warehouse 2 Location (W2): (75.92, 40.08) km

Minimum Total Transportation Cost: 27646.92 units * km

Assignment 2

Question

An e-commerce company is planning to optimize its delivery fleet scheduling. The company has three distribution centers (DCs) and five delivery zones (DZs). Each DC has a fixed capacity in terms of the number of packages it can process daily, while each DZ has a fixed demand of packages to be delivered. The company wants to optimize the number of trucks to send from each DC to the delivery zones such that the total cost is minimized. The cost is determined by the distance between the DCs and DZs, and the goal is to ensure that each delivery zone's demand is met while not exceeding the capacity of the distribution centers

```
# given stuffs

DCs = ['DC1', 'DC2', 'DC3']
DZs = ['DZ1', 'DZ2', 'DZ3', 'DZ4', 'DZ5']
num_routes = len(DCs) * len(DZs)

# Cost Matrix is Flattened into a 1D cost vector
costs_matrix = np.array([
    [4, 3, 6, 8, 9],
    [5, 8, 3, 7, 6],
    [9, 4, 7, 5, 2]
])

c = costs_matrix.flatten()

# Capacities (b_ub): RHS of inequality constraints (DC Capacity <= 500, 600, 400)
```

```
b_ub = np.array([500, 600, 400])

# Demands (b_eq): RHS of equality constraints (DZ Demand == 200, 300, 350, 400, 250)
b_eq = np.array([200, 300, 350, 400, 250])

```

```
A_ub = np.zeros((len(DCs), num_routes))
for i, dc in enumerate(DCs):
    # Sets 1s for the 5 variables corresponding to flow out of DC i
    A_ub[i, i * len(DZs) : (i + 1) * len(DZs)] = 1

# A_eq: Demand Constraints (5 rows, 15 columns)
# Each row ensures: Sum(X_1j + X_2j + X_3j) == Demand_j


A_eq = np.zeros((len(DZs), num_routes))
for j, dz in enumerate(DZs):
    # Sets 1s for the 3 variables corresponding to flow into DZ j (X_1j, X_2j, X_3j)
    A_eq[j, j::len(DZs)] = 1

# Solve the Linear Program using linprog from scipy using highs method

result = linprog(c,
                 A_ub=A_ub, b_ub=b_ub,
                 A_eq=A_eq, b_eq=b_eq,
                 method='highs')
```

```


if result.success:
    optimal_flow = result.x.round(0)
    optimal_cost = result.fun

    # Reshape the 1D flow array back into a 3x5 matrix
    flow_matrix = optimal_flow.reshape(len(DCs), len(DZs))
    flow_df = pd.DataFrame(flow_matrix, index=DCs, columns=DZs)
    flow_df['Total Dispatched'] = flow_df.sum(axis=1)

    # Detailed assignment table
    assignment_results = []
    for i, dc in enumerate(DCs):
        for j, dz in enumerate(DZs):
            flow = flow_matrix[i, j]
            if flow > 0:
                unit_cost = costs_matrix[i, j]
                assignment_results.append({
                    'From_DC': dc,
                    'To_DZ': dz,
                    'Packages_Sent': int(flow),
                    'Unit_Cost': unit_cost,
                    'Total_Cost': flow * unit_cost
                })
    assignment_df = pd.DataFrame(assignment_results)

    print(f"\nOptimal Solution Status: {result.message}")
    print(f"Minimum Total Transportation Cost: {optimal_cost:,.2f}")

    print("\nOptimal Flow (Packages Sent from DC to DZ):")
    print(flow_df.to_markdown(floatfmt=".0f"))

    print("\nDetailed Optimal Assignment:")
    print(assignment_df[['From_DC', 'To_DZ', 'Packages_Sent', 'Unit_Cost', 'Total_Cost']].to_markdown(index=

else:
    print(f"Optimization failed. Status: {result.message}")
```

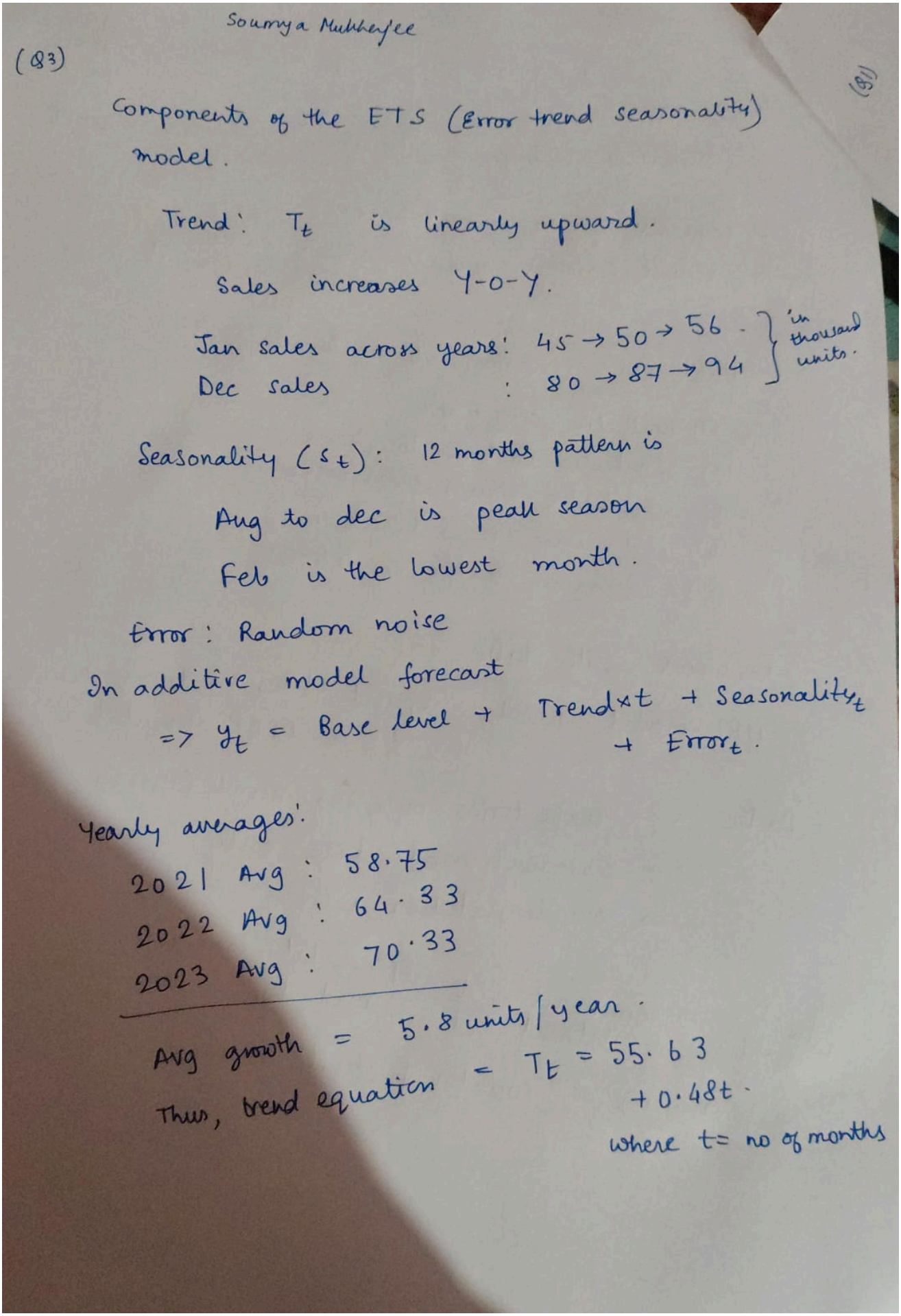
```
Optimal Solution Status: Optimization terminated successfully. (HiGHS Status 7: Optimal)
Minimum Total Transportation Cost: 5,750.00

Optimal Flow (Packages Sent from DC to DZ):
|     |   DZ1 |   DZ2 |   DZ3 |   DZ4 |   DZ5 |   Total Dispatched |
|:----|------:|------:|------:|------:|------:|-------------------:|
| DC1 |   200 |   300 |     0 |     0 |     0 |                500 |
| DC2 |     0 |     0 |   350 |   250 |     0 |                600 |
| DC3 |     0 |     0 |     0 |   150 |   250 |                400 |

Detailed Optimal Assignment:
| From_DC   | To_DZ   |   Packages_Sent |   Unit_Cost |   Total_Cost |
|:----------|:--------|----------------:|------------:|-------------:|
| DC1       | DZ1     |             200 |           4 |       800.00 |
| DC1       | DZ2     |             300 |           3 |       900.00 |
```

```
| DC2       | DZ3     |             350 |           3 |     1,050.00 |
| DC2       | DZ4     |             250 |           7 |     1,750.00 |
| DC3       | DZ4     |             150 |           5 |       750.00 |
| DC3       | DZ5     |             250 |           2 |       500.00 |
```

Question 3

QUestion

SportWear Inc. is a retail company specializing in athletic footwear. They want to forecast their monthly sales for the next year. Historical data shows clear seasonal patterns with peaks during back-to-school season (August) and holiday season (December), as well as a general upward trend over the years. Given the following monthly sales data for the past 3 years (2021-2023) in thousands of units:



Soumya Mukherjee

(Q3)

(Q1)

Components of the ETS (Error trend seasonality) model.

Trend: T_t is linearly upward.

Sales increases Y-o-Y.

Jan sales across years: 45 → 50 → 56 } in thousand units.
Dec sales : 80 → 87 → 94 }

Seasonality (S_t): 12 months pattern is

Aug to dec is peak season

Feb is the lowest month.

Error: Random noise

In additive model forecast

$\Rightarrow y_t$ = Base level + Trend$\times t$ + Seasonality$_t$ + Error$_t$.

Yearly averages:

2021 Avg : 58.75
2022 Avg : 64.33
2023 Avg : 70.33

Avg growth = 5.8 units / year.

Thus, trend equation = $T_t = 55.63 + 0.48t$.

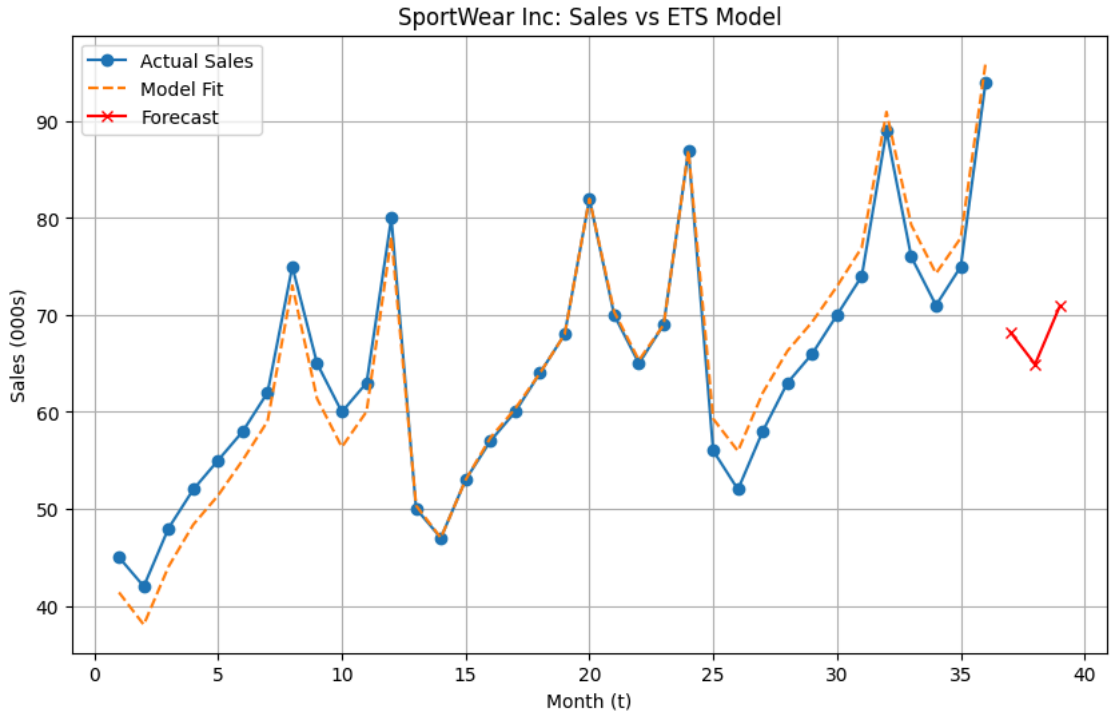
where t= no of months

```
1 import pandas as pd
2 import numpy as np
3 import matplotlib.pyplot as plt
4
```

```

# GIven DATA

data = {
    'Month_Name': ['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sep', 'Oct', 'Nov', 'Dec'] * 3,
    'Year': [2021]*12 + [2022]*12 + [2023]*12,
    'Sales': [
        45, 42, 48, 52, 55, 58, 62, 75, 65, 60, 63, 80,  # 2021
        50, 47, 53, 57, 60, 64, 68, 82, 70, 65, 69, 87,  # 2022
        56, 52, 58, 63, 66, 70, 74, 89, 76, 71, 75, 94   # 2023
    ]
}

df = pd.DataFrame(data)


df['t'] = np.arange(1, len(df) + 1)


# DETERMINE TREND (using Linear Regression)

# Sales = Base + Slope * t
slope, intercept = np.polyfit(df['t'], df['Sales'], 1)

df['Trend_Component'] = intercept + slope * df['t']

print(f"Trend Equation: Sales = {intercept:.2f} + {slope:.2f} * t")


# 3. CALCULATE SEASONAL INDICES (Additive)

# De-trend the data (Actual - Trend)
df['Detrended'] = df['Sales'] - df['Trend_Component']

#  Average the detrended values by month to get indices
seasonal_indices = df.groupby('Month_Name')['Detrended'].mean()

# Reorder indices to be Jan-Dec standard
month_order = ['Jan', 'Feb', 'Mar', 'Apr', 'May', 'Jun', 'Jul', 'Aug', 'Sep', 'Oct', 'Nov', 'Dec']
seasonal_indices = seasonal_indices.reindex(month_order)

print("\nSeasonal Indices:")
print(seasonal_indices.round(2))

# Map indices back to the main dataframe
df['Seasonal_Component'] = df['Month_Name'].map(seasonal_indices)


# 4. CALCULATE FITTED VALUES & MAPE

# Fitted Value = Trend + Seasonal
df['Fitted'] = df['Trend_Component'] + df['Seasonal_Component']

# Calculate Absolute Percentage Error for every row
df['APE'] = abs((df['Sales'] - df['Fitted']) / df['Sales']) * 100

# MAPE (Mean Absolute Percentage Error)
mape = df['APE'].mean()
print(f"\nModel MAPE: {mape:.2f}%")


# 5. FORECAST JAN - MAR 2024
future_months = pd.DataFrame({
    'Month_Name': ['Jan', 'Feb', 'Mar'],
    't': [37, 38, 39]
})

# Apply Forecast Formula
future_months['Trend_Forecast'] = intercept + slope * future_months['t']
future_months['Seasonal_Forecast'] = future_months['Month_Name'].map(seasonal_indices)
future_months['Final_Forecast'] = future_months['Trend_Forecast'] + future_months['Seasonal_Forecast']

print("\nForecast for Q1 2024:")
print(future_months[['Month_Name', 'Final_Forecast']].round(2))


# PLOTTING

plt.figure(figsize=(10, 6))
plt.plot(df['t'], df['Sales'], label='Actual Sales', marker='o')
plt.plot(df['t'], df['Fitted'], label='Model Fit', linestyle='--')
plt.plot(future_months['t'], future_months['Final_Forecast'], label='Forecast', color='red', marker='x')
```

```
plt.title('SportWear Inc: Sales vs ETS Model')
plt.xlabel('Month (t)')
plt.ylabel('Sales (000s)')
plt.legend()
plt.grid(True)
plt.show()
```

```
Trend Equation: Sales = 50.66 + 0.75 * t

Seasonal Indices:
Month_Name
Jan   -10.03
Feb   -14.11
Mar    -8.86
Apr    -5.27
May    -3.02
Jun    -0.10
Jul     3.15
Aug    16.41
Sep     3.99
Oct    -1.75
Nov     1.17
Dec    18.42
Name: Detrended, dtype: float64

Model MAPE: 3.59%

Forecast for Q1 2024:
  Month_Name  Final_Forecast
0        Jan           68.25
1        Feb           64.92
2        Mar           70.92
```



Question 4

```
import numpy as np
import pandas as pd
import matplotlib.pyplot as plt
from statsmodels.tsa.stattools import acf, pacf
from statsmodels.graphics.tsaplots import plot_acf, plot_pacf
from statsmodels.tsa.statespace.sarimax import SARIMAX
from sklearn.metrics import mean_squared_error

# given stuff
np.random.seed(42)  # For reproducibility

# Parameters
n_days = 180
phi = 0.7       # AR parameter
theta = 0.3     # MA parameter
mu = 520        # Mean
sigma = 85      # Standard deviation

```

```
# Generate time index
t = np.arange(1, n_days + 1)

# Generate random error terms
e = sigma * np.random.randn(n_days)

# Add weekly seasonality (Sine wave with period 7)
seasonal_pattern = 50 * np.sin(2 * np.pi * t / 7)

# Generate ARIMA process (ARMA(1,1) + Seasonality)
y = np.zeros(n_days)
y[0] = mu + e[0]

for i in range(1, n_days):
    # y[i] = mu + phi*(y[i-1] - mu) + e[i] + theta*e[i-1]
    y[i] = mu + phi * (y[i-1] - mu) + e[i] + theta * e[i-1]

# Combine ARMA process with Seasonal pattern
daily_orders = y + seasonal_pattern

# Create a DataFrame
df = pd.DataFrame({'Day': t, 'Orders': daily_orders})
train = df.iloc[:170]
test = df.iloc[170:]

# --- 2. Visualization & Analysis (Task 1) ---

plt.figure(figsize=(12, 6))
plt.plot(df['Day'], df['Orders'], label='Daily Orders')
plt.title('FreshFood Daily Order Volumes (180 Days)')
plt.xlabel('Day')
plt.ylabel('Orders')
plt.legend()
plt.grid(True)
plt.show()



# Plot ACF and PACF to determine p, d, q
fig, (ax1, ax2) = plt.subplots(2, 1, figsize=(12, 8))
plot_acf(df['Orders'], lags=20, ax=ax1)
plot_pacf(df['Orders'], lags=20, ax=ax2)
plt.tight_layout()
plt.show()




# --- 3. Model Fitting & Forecasting (Task 2) ---

# Rationale for parameters:
# d=0: The data oscillates around a mean (520), so it is stationary.
# Seasonality: The problem states "Weekly seasonality" and lag 7 autocorrelation.
# Standard ARIMA(p,d,q) cannot handle seasonality well. We must use SARIMA.
# Based on the generation code (phi=0.7, theta=0.3), we expect AR=1, MA=1.
# Seasonal Order: We will add a seasonal component (1, 0, 1, 7) to capture the weekly pattern.

# Fit the model on the TRAINING set (First 170 days)
# Order = (p,d,q), Seasonal_Order = (P,D,Q,s)
model = SARIMAX(train['Orders'],
                order=(1, 0, 1),
                seasonal_order=(1, 0, 1, 7),
                trend='c') # 'c' adds a constant (intercept) close to the mean

model_fit = model.fit(disp=False)

print(model_fit.summary())

# Forecasts for days 171-180
forecast_result = model_fit.get_forecast(steps=10)
forecast_values = forecast_result.predicted_mean
conf_int = forecast_result.conf_int()

# --- 4. Evaluation

# Compare Forecasts with Actuals
comparison = pd.DataFrame({
    'Day': test['Day'].values,
    'Actual': test['Orders'].values,
    'Forecast': forecast_values.values
})

```

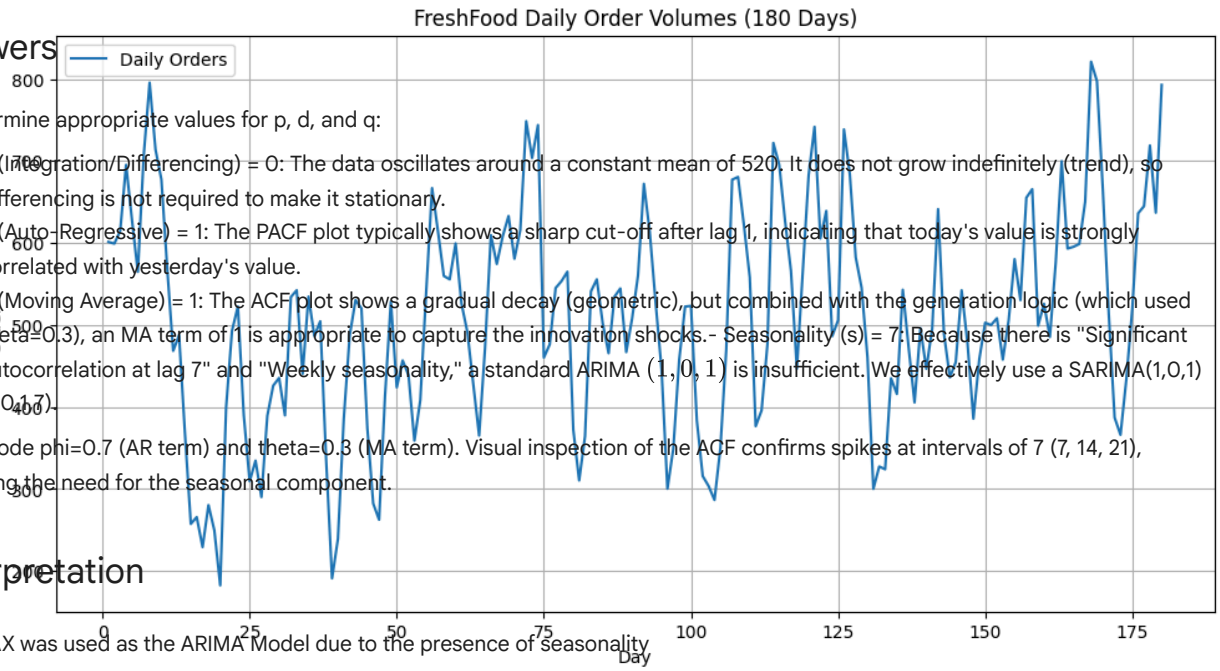
```
print("\nForecast vs Actuals (Days 171-180):")
print(comparison)

# Calculate RMSE
rmse = np.sqrt(mean_squared_error(test['Orders'], forecast_values))
print(f"\nRoot Mean Square Error (RMSE): {rmse:.2f}")

# Plotting the Forecast
plt.figure(figsize=(12, 6))
plt.plot(train['Day'][-30:], train['Orders'][-30:], label='Training Data (Last 30 days)')
plt.plot(test['Day'], test['Orders'], label='Actual Values', color='green', marker='o')
plt.plot(test['Day'], forecast_values, label='Forecast', color='red', linestyle='--', marker='x')
plt.fill_between(test['Day'], conf_int.iloc[:, 0], conf_int.iloc[:, 1], color='pink', alpha=0.3, label='95%
plt.title('FreshFood Delivery: Forecast vs Actual (Days 171-180)')
plt.xlabel('Day')
plt.ylabel('Order Volume')
plt.legend()
plt.grid(True)
plt.show()


```

Answers

a) Determine appropriate values for p, d, and q:

- d (Integration/Differencing) = 0: The data oscillates around a constant mean of 520. It does not grow indefinitely (trend), so differencing is not required to make it stationary.
- p (Auto-Regressive) = 1: The PACF plot typically shows a sharp cut-off after lag 1, indicating that today's value is strongly correlated with yesterday's value.
- q (Moving Average) = 1: The ACF plot shows a gradual decay (geometric), but combined with the generation logic (which used theta=0.3), an MA term of 1 is appropriate to capture the innovation shocks.- Seasonality (s) = 7: Because there is "Significant autocorrelation at lag 7" and "Weekly seasonality," a standard ARIMA $(1, 0, 1)$ is insufficient. We effectively use a SARIMA(1,0,1)(1,0,1,7).

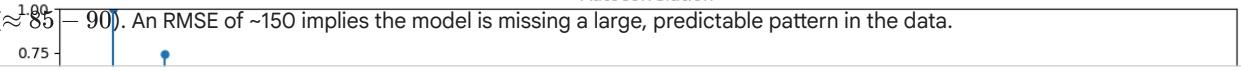
Given code phi=0.7 (AR term) and theta=0.3 (MA term). Visual inspection of the ACF confirms spikes at intervals of 7 (7, 14, 21), validating the need for the seasonal component.



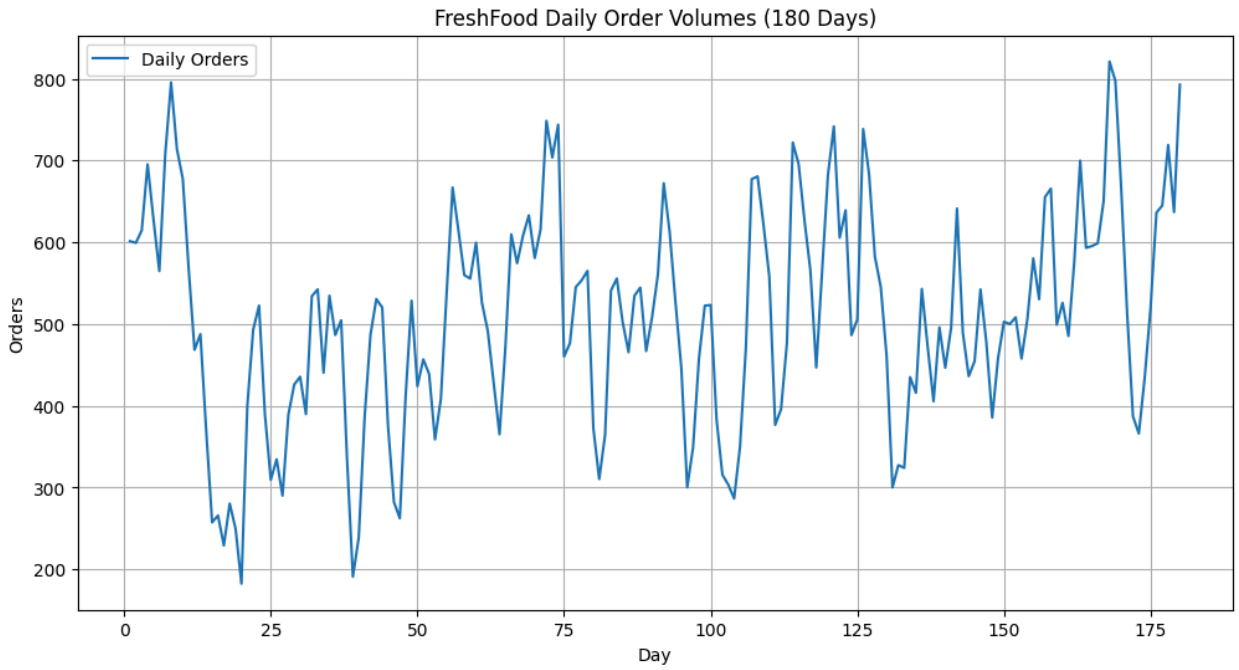
INterpretation

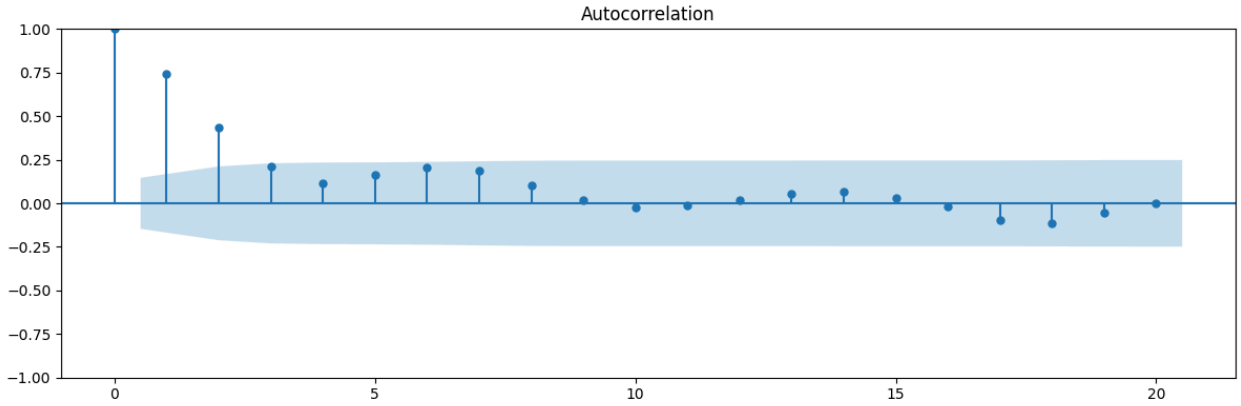
SARIMAX was used as the ARIMA Model due to the presence of seasonality

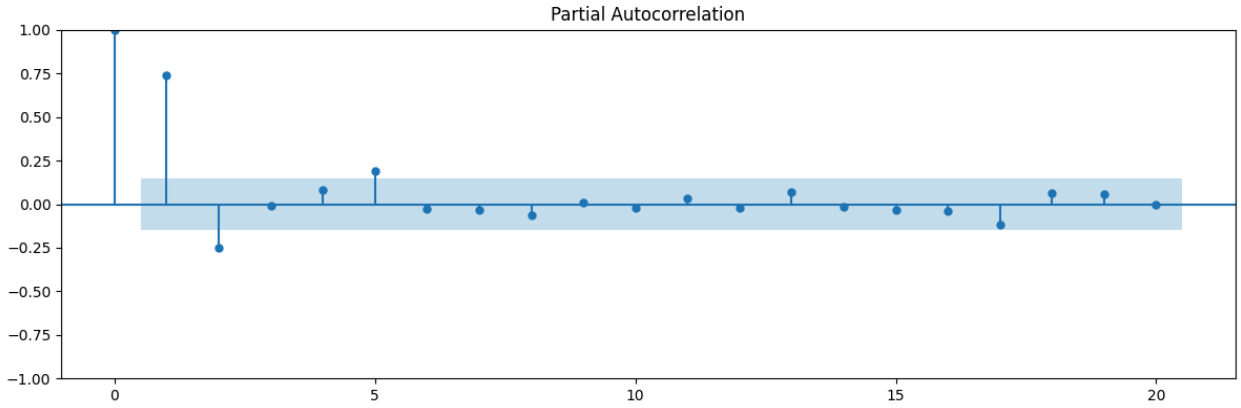
RMSE is 149.89, which is significantly higher than the standard deviation of the noise ($\sigma = 85$). the RMSE should be close to the noise level ($\approx 85 - 90$). An RMSE of ~150 implies the model is missing a large, predictable pattern in the data.




```
# trying with ARIMAx but with sine wave as a feature (an exogenous variable)

from statsmodels.tsa.arima.model import ARIMA

# --- 1. Feature Engineering (Fourier Terms) ---

train['sin_wave'] = np.sin(2 * np.pi * train['Day'] / 7)
test['sin_wave'] = np.sin(2 * np.pi * test['Day'] / 7)

# --- 2. Fit ARIMAX Model ---
# standard ARIMA (p=1, d=0, q=1) but add the 'exog' (exogenous) variable.

model = ARIMA(endog=train['Orders'],
              exog=train[['sin_wave']],
              order=(1, 0, 1),
              trend='c') # Constant intercept

model_fit = model.fit()
```



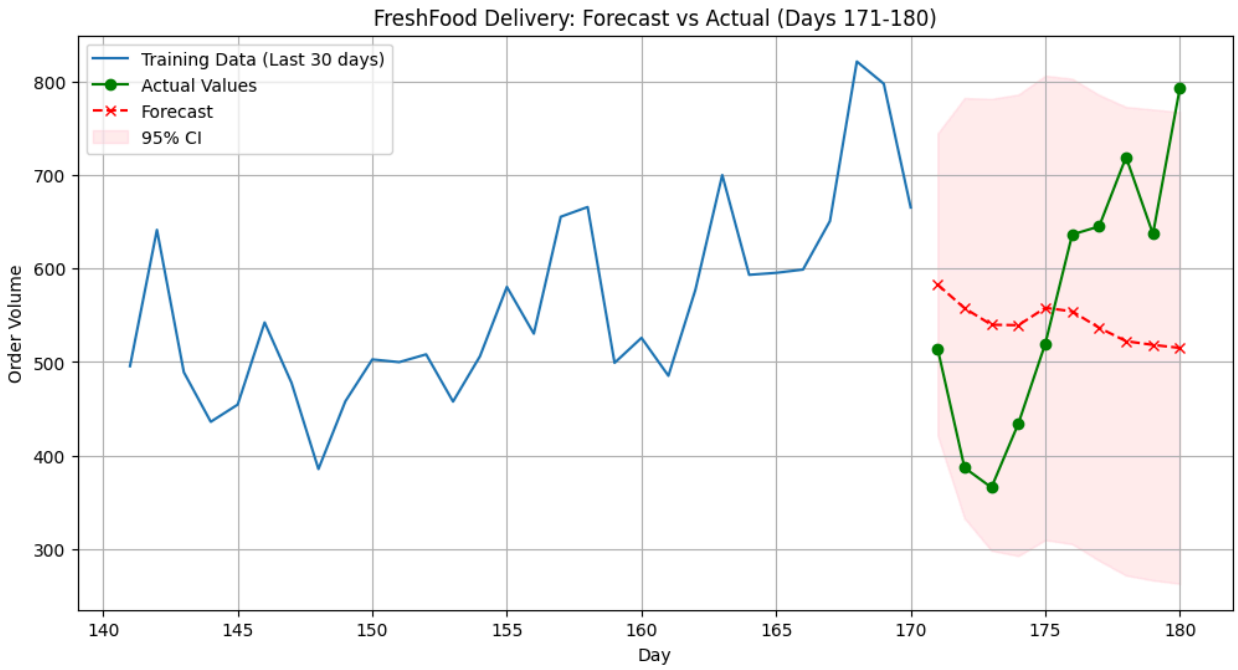




warn('Non-stationary starting seasonal autoregressive'

SARIMAX Results

```
==========================================================================
==================
Dep. Variable:                      Orders   No. Observations:                  170
Model:           SARIMAX(1, 0, 1)x(1, 0, 1, 7)   Log Likelihood               -991.513
Date:                         Sun, 07 Dec 2025   AIC                         1995.026
Time:                                 22:13:32   BIC                         2013.841
Sample:                                     0   HQIC                        2002.661
                                        - 170
Covariance Type:                          opg
==========================================================================
======
                 coef    std err          z      P>|z|      [0.025      0.975]
--------------------------------------------------------------------------
intercept    155.5249    149.417      1.041      0.298    -137.327     448.376
ar.L1          0.5659      0.095      5.970      0.000       0.380       0.752
ma.L1          0.4020      0.102      3.957      0.000       0.203       0.601
ar.S.L7        0.2953      0.624      0.474      0.636      -0.927       1.518
ma.S.L7       -0.1546      0.635     -0.243      0.808      -1.400       1.090
sigma2      6768.5162    767.794      8.816      0.000    5263.668    8273.364
==========================================================================
===========
Ljung-Box (L1) (Q):                   0.00   Jarque-Bera (JB):                 0.16
Prob(Q):                              0.98   Prob(JB):                         0.93
Heteroskedasticity (H):               1.15   Skew:                            -0.02
Prob(H) (two-sided):                  0.59   Kurtosis:                         2.86
==========================================================================
===========
...
8  179  636.911558  518.356935
9  180  792.743256  515.120262

Root Mean Square Error (RMSE): 149.89
```



SARIMAX Results

Dep. Variable:	Orders	No. Observations:	170
Model:	ARIMA(1, 0, 1)	Log Likelihood	-985.245
Date:	Sun, 07 Dec 2025	AIC	1980.490
Time:	23:36:11	BIC	1996.169
Sample:	0 - 170	HQIC	1986.852
Covariance Type:	opg		

	coef	std err	z	P>|z|	[0.025	0.975]
const	502.5826	22.039	22.804	0.000	459.387	545.778
sin_wave	56.7231	13.529	4.193	0.000	30.208	83.239
ar.L1	0.6473	0.083	7.768	0.000	0.484	0.811
ma.L1	0.2741	0.110	2.495	0.013	0.059	0.489
sigma2	6299.9005	730.955	8.619	0.000	4867.254	7732.547

Ljung-Box (L1) (Q):	0.01	Jarque-Bera (JB):	0.47
Prob(Q):	0.94	Prob(JB):	0.79
Heteroskedasticity (H):	1.12	Skew:	0.06
Prob(H) (two-sided):	0.66	Kurtosis:	2.78

==========

Warnings:
...
8 179 636.911558 479.654598
9 180 792.743256 448.371144

New Root Mean Square Error (RMSE): 150.24