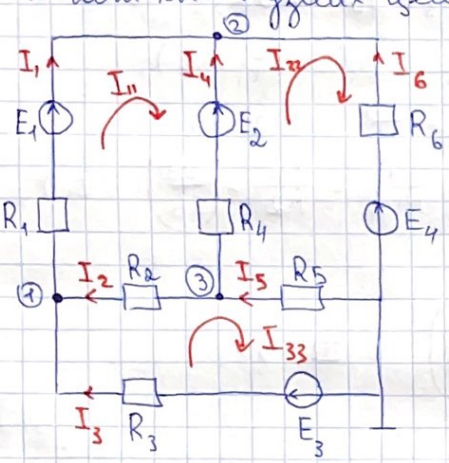
Задача 1.

Задание: составить систему ур-ий по методу контурных токов и методу узл. пот-лов, определить токи в ветвях и пот-ли в узлах цепи.



$E_1 = 10$ В $\quad R_1 = R_5 = 10$ Ом

$E_2 = 20$ В $\quad R_2 = R_3 = 5$ Ом

$E_3 = 15$ В $\quad R_4 = R_6 = 15$ Ом

$E_4 = 25$ В

Решение:

1. **Метод контурных токов.**

$$\begin{cases} I_{11}(R_1 + R_4 + R_2) - I_{22} \cdot R_4 - I_{33} \cdot R_2 = E_1 - E_2 \\ I_{22}(R_4 + R_6 + R_5) - I_{11} \cdot R_4 - I_{33} \cdot R_5 = E_2 - E_4 \\ I_{33}(R_2 + R_5 + R_3) - I_{11} \cdot R_2 - I_{22} \cdot R_5 = E_3 \end{cases}$$

$$\begin{cases} I_{11} \cdot 30 - I_{22} \cdot 15 - I_{33} \cdot 5 = -10 \\ I_{22} \cdot 40 - I_{11} \cdot 15 - I_{33} \cdot 10 = -5 \\ I_{33} \cdot 20 - I_{11} \cdot 5 - I_{22} \cdot 10 = 15 \end{cases}$$

$$\begin{cases} 6I_{11} - 3I_{22} - I_{33} = -2 & (I) \\ -3I_{11} + 8I_{22} - 2I_{33} = -1 & (II) \\ -I_{11} - 2I_{22} + 4I_{33} = 3 & (III) \end{cases}$$

$I_{11} = -2I_{22} + 4I_{33} - 3$ (III)

$-3I_{11} = 6I_{22} - 12I_{33} + 9$ (3 · III)

$6I_{22} - 12I_{33} + 9 + 8I_{22} - 2I_{33} = -1$ (II)

$14I_{22} - 14I_{33} + 10 = 0$ (II)

$I_{22} - I_{33} + \frac{10}{14} = 0$ (II)

$I_{22} = I_{33} - \frac{10}{14}$ (II)

$-15I_{22} = -15I_{33} + \frac{150}{14}$ (II)

$I_{22} = \frac{37}{56} - \frac{10}{14} = \frac{37-40}{56} = -\frac{3}{56}$ (II)

$I_{11} = \frac{6}{56} + \frac{4 \cdot 37}{56} - 3 = \frac{148+6-168}{56} = -\frac{14}{56} = -\frac{1}{4}$ (III)

$6I_{11} = -12I_{22} + 24I_{33} - 18$ (III)

$-12I_{22} + 24I_{33} - 18 - 3I_{22} - I_{33} = -2$ (I)

$-15I_{22} + 23I_{33} = 16$ (I)

$-15I_{33} + \frac{150}{14} + 23I_{33} = 16$ (I)

$8I_{33} = \frac{224}{14} - \frac{150}{14}$ (I)

$8I_{33} = \frac{74}{14}$ (I)

$I_{33} = \frac{74}{14 \cdot 8} = \frac{74}{112} = \frac{37}{56}$ (I)

Torga: $I_{11} = -\frac{1}{4}$, $I_{22} = -\frac{3}{56}$, $I_{33} = \frac{34}{56}$.

$I_1 = I_{11} = -\frac{1}{4}$ A

$I_2 = I_{11} - I_{33} = \frac{-14-37}{56} = -\frac{51}{56}$ A

$I_3 = I_{33} = \frac{37}{56}$ A

$I_4 = -I_{11} + I_{22} = \frac{11}{56}$ A

$I_5 = I_{22} - I_{33} = -\frac{5}{7}$ A

$I_6 = -I_{22} = \frac{3}{56}$ A

2. Метод узловых пот-лов.

$$U_k \sum_{i=1}^{n} G_{ki} - \sum_{i=1}^{n} U_i G_{ki} = \sum_{i=1}^{n} E_{ki} G_{ki}$$

- U_k - пот. узла, отн. которого сост-о ур-ие
- G_{ki} - проводимости ветвей, соед. этот узел с сосед. узл-и
- U_i - пот. сосед. узлов
- E_{ki} - э.д.с. в соотв. их ветвях (знак "+", если э.д.с. напр. к узлу)

$$\begin{cases} U_1\left(\frac{1}{R_1}+\frac{1}{R_2}+\frac{1}{R_3}\right) - U_2\cdot\frac{1}{R_1} - U_3\cdot\frac{1}{R_2} = -E_1\cdot\frac{1}{R_1} + E_3\cdot\frac{1}{R_3} \\ U_2\left(\frac{1}{R_1}+\frac{1}{R_4}+\frac{1}{R_6}\right) - U_1\cdot\frac{1}{R_1} - U_3\cdot\frac{1}{R_4} = E_1\cdot\frac{1}{R_1} + E_2\cdot\frac{1}{R_4} + E_4\cdot\frac{1}{R_6} \\ U_3\left(\frac{1}{R_2}+\frac{1}{R_4}+\frac{1}{R_5}\right) - U_1\cdot\frac{1}{R_2} - U_2\cdot\frac{1}{R_4} = -E_2\cdot\frac{1}{R_4} \end{cases}$$

$$\begin{cases} \frac{1}{2}U_1 - \frac{1}{10}U_2 - \frac{1}{5}U_3 = 2 & (I) \\ -\frac{1}{10}U_1 + \frac{7}{30}U_2 - \frac{1}{15}U_3 = 4 & (II) \\ -\frac{1}{5}U_1 - \frac{1}{15}U_2 + \frac{11}{30}U_3 = -\frac{4}{3} & (III) \end{cases}$$

$$\begin{cases} 5U_1 - U_2 - 2U_3 = 20 & (I) \\ -3U_1 + 7U_2 - 2U_3 = 120 & (II) \\ -6U_1 - 2U_2 + 11U_3 = -40 & (III) \end{cases}$$

$\boxed{U_2 = 5U_1 - 2U_3 - 20}$ (I)

$-2U_2 = -10U_1 + 4U_3 + 40$ (I)

$-6U_1 - 10U_1 + 4U_3 + 40 + 11U_3 = -40$ (III)

$-16U_1 + 15U_3 = -80$ (III)

$-16U_1 = -80 - 15U_3$ (III)

$\boxed{U_1 = 5 + \frac{15}{16}U_3}$ (III)

$-3U_1 = -15 - \frac{45}{16}U_3$ (III)

$-15 - \frac{45}{16}U_3 + 35 + \frac{301}{16}U_3 - 2U_3 = 120$ (II)

$\frac{-45 + 301 - 32}{16}U_3 = 100$ (II)

$\frac{224}{16}U_3 = 100$ (II)

$14U_3 = 100$ (II)

$U_3 = \frac{100}{14}$ (II)

$7U_2 = 35U_1 - 14U_3 - 140$ (I)

$7U_2 = 175 + \frac{525}{16}U_3 - 14U_3 - 140$ (I)

$7U_2 = 35 + \frac{301}{16}U_3$ (I)

$35U_1 = 175 + \frac{525}{16}U_3$ (III)

$U_1 = 5 + \frac{15}{16} \cdot \frac{100}{14}$ (III)

$U_1 = \frac{655}{56}$ (III)

$U_2 = \frac{5 \cdot 655}{56} - \frac{200}{14} - 20 = \frac{1355}{56}$

Ответ:

1) $I_1 = -\frac{1}{4}$ A, $I_2 = -\frac{51}{56}$ A, $I_3 = \frac{37}{56}$ A, $I_4 = \frac{11}{56}$ A, $I_5 = -\frac{5}{7}$ A, $I_6 = \frac{3}{56}$ A

2) $U_1 = \frac{655}{56}$ В ≈ 11.7 В, $U_2 = \frac{1355}{56}$ В ≈ 24.2 В, $U_3 = \frac{100}{14}$ В ≈ 7.1 В