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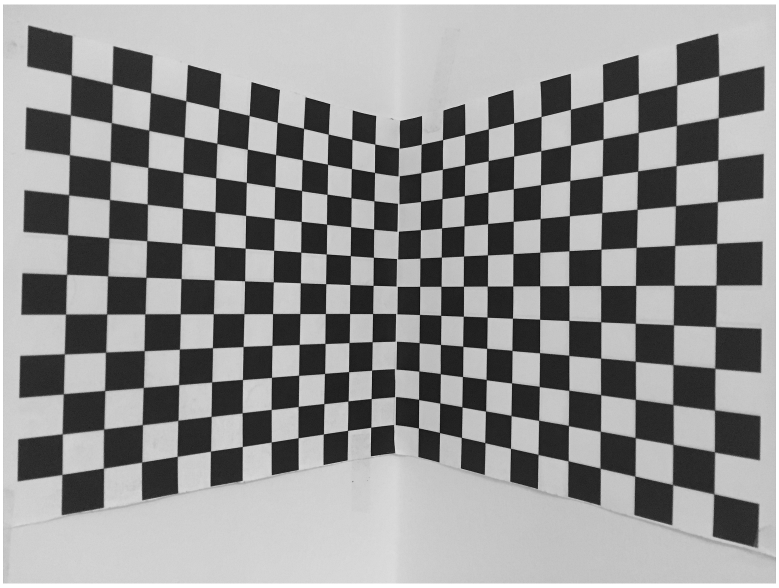
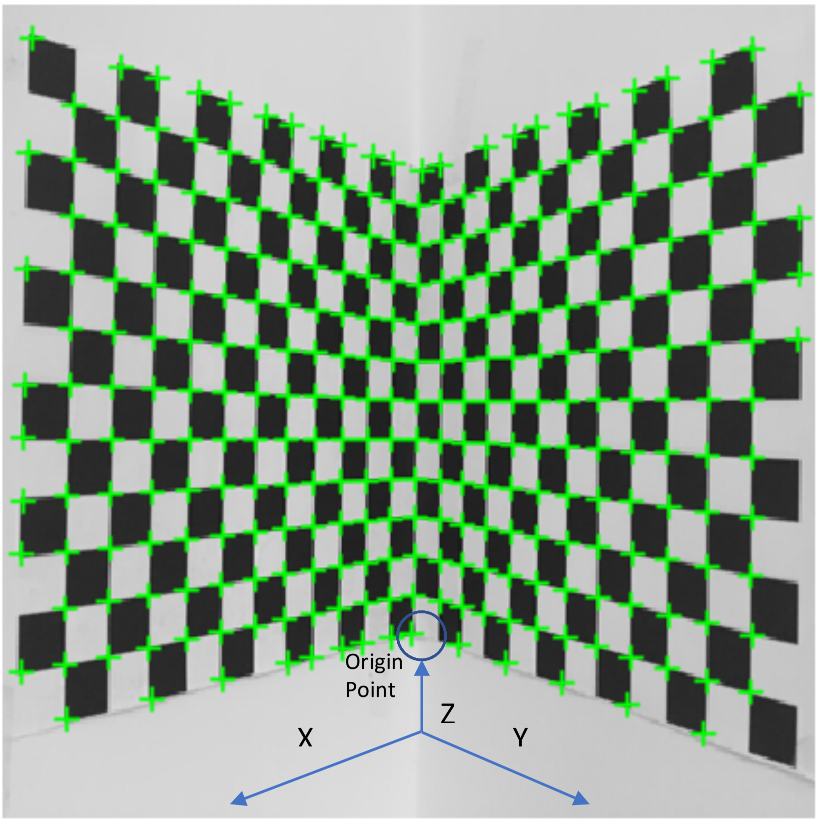
EE 417 Assignment #2

Camera Calibration Project

<u>Calibration Rig: Each Square Length = 15mm</u>



On calibration rig, corners were detected using Harris Corner Detection method. Total 300 corners were detected on the rig and origin point is set as follows:



In order to find the corresponding pixel coordinates of the world points, we use matrix equations we saw in lectures. When we multiply rotation and translation matrices with world coordinates P_w of the point, we get corresponding camera point P. When we multiply internal parameters of the camera with the camera point, we get corresponding pixel coordinate P' of the world point P_w.

$$P = \begin{bmatrix} R & T \\ 0 & 1 \end{bmatrix}_{4\times4} P_w \qquad P' = M\,P_w = \underbrace{K}_{\text{Internal parameters}}\underbrace{\begin{bmatrix} R & T \end{bmatrix}}_{\text{External parameters}} P_w$$

Pixel coordinate can be represented as below equation as well, where (u,v) are pixel coordinates of the point found by Harris Corner detection with a scale factor lambda, and 1 is given to convert both world point coordinates and pixel coordinates into homogeneous coordinates for linear mapping. Lambda is set to be 1, as usual.

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \overbrace{\boxed{K[R|T]}}^{\text{Projection Matrix}} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

$$\lambda \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} = \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \end{bmatrix} \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix} = K[R\,|\,T] \begin{bmatrix} X_w \\ Y_w \\ Z_w \\ 1 \end{bmatrix}$$

We have 11 values in M matrix to estimate. In order to do such estimation, for each point we get two equations as follows:

$$u_i = \frac{\lambda u_i}{\lambda} = \frac{m_{11}X_i + m_{12}Y_i + m_{13}Z_i + m_{14}}{m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}}$$

$$v_i = \frac{\lambda v_i}{\lambda} = \frac{m_{21}X_i + m_{22}Y_i + m_{23}Z_i + m_{24}}{m_{31}X_i + m_{32}Y_i + m_{33}Z_i + m_{34}}$$

To solve these equations, we make cross-multiplication and get followings:

$$\mathrm{u_i}(\mathbf{m}_3\ \mathrm{P_i}) = \mathbf{m}_1\ \mathrm{P_i} \ \rightarrow\ u_i(\mathbf{m}_3\ P_i) - \mathbf{m}_1\ P_i = 0$$

$$\mathrm{v_i}(\mathbf{m}_3\ \mathrm{P_i}) = \mathbf{m}_2\ \mathrm{P_i} \ \rightarrow\ v_i(\mathbf{m}_3\ P_i) - \mathbf{m}_2\ P_i = 0$$

For all points we have, we get a set of equations as below, and we need to find m values. We can turn these equations into a homogeneous linear system to solve it.

$$\begin{cases} -u_1(\mathbf{m}_3\ P_1) + \mathbf{m}_1\ P_1 = 0 \\ -v_1(\mathbf{m}_3\ P_1) + \mathbf{m}_2\ P_1 = 0 \\ \qquad \vdots \\ -u_n(\mathbf{m}_3\ P_n) + \mathbf{m}_1\ P_n = 0 \\ -v_n(\mathbf{m}_3\ P_n) + \mathbf{m}_2\ P_n = 0 \end{cases} \longrightarrow \boxed{\underbrace{\mathcal{P}}_{\text{known}}\underbrace{\boldsymbol{m}}_{\text{unknown}} = 0}$$

Homogenous linear system

$$\mathcal{P} \stackrel{\text{def}}{=} \begin{pmatrix} \boldsymbol{P}_1^T & \mathbf{0}^T & -u_1\boldsymbol{P}_1^T \\ \mathbf{0}^T & \boldsymbol{P}_1^T & -v_1\boldsymbol{P}_1^T \\ \dots & \dots & \dots \\ \boldsymbol{P}_n^T & \mathbf{0}^T & -u_n\boldsymbol{P}_n^T \\ \mathbf{0}^T & \boldsymbol{P}_n^T & -v_n\boldsymbol{P}_n^T \end{pmatrix}_{2n \times 12} \quad (\boldsymbol{P}_1^T:\ 1\text{x}4) \qquad \boldsymbol{m} \stackrel{\text{def}}{=} \begin{pmatrix} \mathbf{m}_1^{\mathrm{T}} \\ \mathbf{m}_2^{\mathrm{T}} \\ \mathbf{m}_3^{\mathrm{T}} \end{pmatrix}_{12\text{x}1} \quad (\mathbf{m}_1^{\mathrm{T}}:\ 4\text{x}1)$$

In order to solve this homogenous system, we can compute SVD decomposition of $\mathcal{P}$. Decomposition operation gives three matrices $\mathrm{U_{2nx12} D_{12x12} V_{12x12}^T}$ and last column of matrix V gives us $\mathrm{m_{12x1}}$ which is also matrix M.

After finding $\mathrm{M_{3x4}}$ matrix, we drop last column, take the 3x3 submatrix and use QR factorization to decompose it into the product of an upper triangular matrix K and a rotation matrix R. T can be found by multiplication of K inverse and dropped column of M.

$$T = K^{-1}\begin{bmatrix} m_{14} \\ m_{24} \\ m_{34} \end{bmatrix}$$

In my implementation, there is a hand-written $\mathcal{P}$ matrix for total of 60 points, half picked from left checkerboard and half from right checkerboard of the calibration rig. $\mathcal{P}$ matrix is written in following format where X Y and Z are point coordinates according to origin I picked at the beginning, multiplied by 15 to represent the world coordinates; u an v are coordinates computed by Harris Corner Detection method.

$$X_i\ Y_i\ Z_i\ 1\ 0\ 0\ 0\ 0\ -u*X_i\ \ -u*Y_i\ -u*Z_i\ -u*1;$$
$$0\ 0\ 0\ 0\ X_i\ Y_i\ Z_i\ 1\ -v*X_i\ \ -v*Y_i\ -v*Z_i\ -v*1;$$

This 120x12 matrix and complete implementation is as follows:

```
img1 = imread('IMG_2807.JPG');
img = imresize(img1,[360,360]);
[r,c,color] = size(img);
if(color==3)
    img = rgb2gray(img);
end
 %-------- harris corners
harriscorners = detectHarrisFeatures(img);
imshow(img);
hold on;
plot(harriscorners.selectStrongest(300));

p = [1*15 0*15 1*15 1 0 0 0 0 -172.1*1*15 -172.1*0*15 -172.1*1*15 -172.1*1;...
     0 0 0 0 1*15 0*15 1*15 1 -263.6*1*15 -263.6*0*15 -263.6*1*15 -263.6*1;... %1
     2*15 0*15 1*15 1 0 0 0 0 -161.3*2*15 -161.3*0*15 -161.3*1*15 -161.3*1;...
     0 0 0 0 2*15 0*15 1*15 1 -265.5*2*15 -265.5*0*15 -265.5*1*15 -265.5*1;... %2
     3*15 0*15 2*15 1 0 0 0 0 -150.2*3*15 -150.2*0*15 -150.2*2*15 -150.2*1;...
     0 0 0 0 3*15 0*15 2*15 1 -248.9*3*15 -248.9*0*15 -248.9*2*15 -248.9*1;... %3
     4*15 0*15 2*15 1 0 0 0 0 -138.4*4*15 -138.4*0*15 -138.4*2*15 -138.4*1;...
     0 0 0 0 4*15 0*15 2*15 1 -250.6*4*15 -250.6*0*15 -250.6*2*15 -250.6*1;... %4
     6*15 0*15 2*15 1 0 0 0 0 -112.3*6*15 -112.3*0*15 -112.3*2*15 -112.3*1;...
     0 0 0 0 6*15 0*15 2*15 1 -254.6*6*15 -240.6*0*15 -254.6*2*15 -254.6*1;... %5
     6*15 0*15 3*15 1 0 0 0 0 -112.4*6*15 -112.4*0*15 -112.4*3*15 -112.4*1;...
     0 0 0 0 6*15 0*15 3*15 1 -234*6*15 -234*0*15 -234*3*15 -234*1;...         %6
     9*15 0*15 3*15 1 0 0 0 0 -66.28*9*15 -66.28*0*15 -66.28*3*15 -66.28*1;...
     0 0 0 0 9*15 0*15 3*15 1 -238.8*9*15 -238.8*0*15 -238.8*3*15 -238.8*1;... %7
     11*15 0*15 3*15 1 0 0 0 0 -28.89*11*15 -28.89*0*15 -28.89*3*15 -28.89*1;...
     0 0 0 0 11*15 0*15 3*15 1 -242.9*11*15 -242.9*0*15 -242.9*3*15 -242.9*1;... %8
     4*15 0*15 0*15 1 0 0 0 0 -137.3*4*15 -137.3*0*15 -137.3*0*15 -137.3*1;...
     0 0 0 0 4*15 0*15 0*15 1 -289.2*4*15 -289.2*0*15 -289.2*0*15 -289.2*1;... %9
     1*15 0*15 3*15 1 0 0 0 0 -172.5*1*15 -172.5*0*15 -172.5*3*15 -172.5*1;...
     0 0 0 0 1*15 0*15 3*15 1 -228.2*1*15 -228.2*0*15 -228.2*3*15 -228.2*1;... %10
     3*15 0*15 4*15 1 0 0 0 0 -150.6*3*15 -150.6*0*15 -150.6*4*15 -150.6*1;...
     0 0 0 0 3*15 0*15 4*15 1 -211.5*3*15 -211.5*0*15 -211.5*4*15 -211.5*1;... %11
     5*15 0*15 4*15 1 0 0 0 0 -126.1*5*15 -126.1*0*15 -126.1*4*15 -126.1*1;...
     0 0 0 0 5*15 0*15 4*15 1 -212.7*5*15 -212.7*0*15 -212.7*4*15 -212.7*1;... %12
     9*15 0*15 4*15 1 0 0 0 0 -66.5*9*15 -66.5*0*15 -66.5*4*15 -66.5*1;...
     0 0 0 0 9*15 0*15 4*15 1 -216*9*15 -216*0*15 -216*4*15 -216*1;...         %13
     11*15 0*15 5*15 1 0 0 0 0 -29.73*11*15 -29.73*0*15 -29.73*5*15 -29.73*1;...
     0 0 0 0 11*15 0*15 5*15 1 -193.5*11*15 -193.5*0*15 -193.5*5*15 -193.5*1;... %14
     10*15 0*15 6*15 1 0 0 0 0 -49.4*10*15 -49.4*0*15 -49.4*6*15 -49.4*1;...
     0 0 0 0 10*15 0*15 6*15 1 -169.7*10*15 -169.7*0*15 -169.7*6*15 -169.7*1;... %15
     11*15 0*15 7*15 1 0 0 0 0 -30.8*11*15 -30.8*0*15 -30.8*7*15 -30.8*1;...
     0 0 0 0 11*15 0*15 7*15 1 -143.8*11*15 -143.8*0*15 -143.8*7*15 -143.8*1;... %16
     6*15 0*15 7*15 1 0 0 0 0 -113.5*6*15 -113.5*0*15 -113.5*7*15 -113.5*1;...
     0 0 0 0 6*15 0*15 7*15 1 -152.4*6*15 -152.4*0*15 -152.4*7*15 -152.4*1;... %17
     8*15 0*15 7*15 1 0 0 0 0 -83.88*8*15 -83.88*0*15 -83.88*7*15 -83.88*1;...
     0 0 0 0 8*15 0*15 7*15 1 -149.2*8*15 -149.2*0*15 -149.2*7*15 -149.2*1;... %18
     11*15 0*15 9*15 1 0 0 0 0 -31.72*11*15 -31.72*0*15 -31.72*9*15 -31.72*1;...
     0 0 0 0 11*15 0*15 9*15 1 -94.4*11*15 -94.4*0*15 -94.4*9*15 -94.4*1;... %19
     4*15 0*15 9*15 1 0 0 0 0 -139.5*4*15 -139.5*0*15 -139.5*9*15 -139.5*1;...
     0 0 0 0 4*15 0*15 9*15 1 -116.3*4*15 -116.3*0*15 -116.3*9*15 -116.3*1;... %20
     6*15 0*15 9*15 1 0 0 0 0 -114*6*15 -114*0*15 -114*9*15 -114*1;...
     0 0 0 0 6*15 0*15 9*15 1 -111.4*6*15 -111.4*0*15 -111.4*9*15 -111.4*1;... %21
     9*15 0*15 9*15 1 0 0 0 0 -68.34*9*15 -68.34*0*15 -68.34*9*15 -68.34*1;...
     0 0 0 0 9*15 0*15 9*15 1 -102.1*9*15 -102.1*0*15 -102.1*9*15 -102.1*1;... %22
```

```
12*15 0*15 10*15 1 0 0 0 0 -12.54*12*15 -12.54*0*15 -12.54*10*15 -12.54*1;...
0 0 0 0 12*15 0*15 10*15 1 -65.76*12*15 -65.76*0*15 -65.76*10*15 -65.76*1;... %23
10*15 0*15 10*15 1 0 0 0 0 -51.33*10*15 -51.33*0*15 -51.33*10*15 -51.33*1;...
0 0 0 0 10*15 0*15 10*15 1 -74.41*10*15 -74.41*0*15 -74.41*10*15 -74.41*1;... %24
7*15 0*15 10*15 1 0 0 0 0 -100.2*7*15 -100.2*0*15 -100.2*10*15 -100.2*1;...
0 0 0 0 7*15 0*15 10*15 1 -86.91*7*15 -86.91*0*15 -86.91*10*15 -86.91*1;... %25
7*15 0*15 11*15 1 0 0 0 0 -101.3*7*15 -101.3*0*15 -101.3*11*15 -101.3*1;...
0 0 0 0 7*15 0*15 11*15 1 -65.63*7*15 -65.63*0*15 -65.63*11*15 -65.63*1;... %26
4*15 0*15 11*15 1 0 0 0 0 -140.6*4*15 -140.6*0*15 -140.6*11*15 -140.6*1;...
0 0 0 0 4*15 0*15 11*15 1 -77.8*4*15 -77.8*0*15 -77.8*11*15 -77.8*1;... %27
5*15 0*15 12*15 1 0 0 0 0 -128.6*5*15 -128.6*0*15 -128.6*12*15 -128.6*1;...
0 0 0 0 5*15 0*15 12*15 1 -54.29*5*15 -54.29*0*15 -54.29*12*15 -54.29*1;... %28
2*15 0*15 12*15 1 0 0 0 0 -164.5*2*15 -164.5*0*15 -164.5*12*15 -164.5*1;...
0 0 0 0 2*15 0*15 12*15 1 -67.79*2*15 -67.79*0*15 -67.79*12*15 -67.79*1;... %29
9*15 0*15 12*15 1 0 0 0 0 -69.33*9*15 -69.33*0*15 -69.33*12*15 -69.33*1;...
0 0 0 0 9*15 0*15 12*15 1 -33.02*9*15 -33.02*0*15 -33.02*12*15 -33.02*1;... %30
%
0*15 1*15 12*15 1 0 0 0 0 -194.5*0*15 -194.5*1*15 -194.5*12*15 -194.5*1;...
0 0 0 0 0*15 1*15 12*15 1 -71.27*0*15 -71.27*1*15 -71.27*12*15 -71.27*1;... %31
0*15 6*15 12*15 1 0 0 0 0 -251.6*0*15 -251.6*6*15 -251.6*12*15 -251.6*1;...
0 0 0 0 0*15 6*15 12*15 1 -49.46*0*15 -49.46*6*15 -49.46*12*15 -49.46*1;... %32
0*15 11*15 12*15 1 0 0 0 0 -332.4*0*15 -332.4*11*15 -332.4*12*15 -332.4*1;...
0 0 0 0 0*15 11*15 12*15 1 -20.01*0*15 -20.01*11*15 -20.01*12*15 -20.01*1;... %33
0*15 11*15 11*15 1 0 0 0 0 -333.6*0*15 -333.6*11*15 -333.6*11*15 -333.6*1;...
0 0 0 0 0*15 11*15 11*15 1 -45.69*0*15 -45.69*11*15 -45.69*11*15 -45.69*1;... %34
0*15 8*15 11*15 1 0 0 0 0 -280.2*0*15 -280.2*8*15 -280.2*11*15 -280.2*1;...
0 0 0 0 0*15 8*15 11*15 1 -61.39*0*15 -61.39*8*15 -61.39*11*15 -61.39*1;... %35
0*15 3*15 11*15 1 0 0 0 0 -215.7*0*15 -215.7*3*15 -215.7*11*15 -215.7*1;...
0 0 0 0 0*15 3*15 11*15 1 -81.68*0*15 -81.68*3*15 -81.68*11*15 -81.68*1;... %36
0*15 6*15 11*15 1 0 0 0 0 -251.4*0*15 -251.4*6*15 -251.4*11*15 -251.4*1;...
0 0 0 0 0*15 6*15 11*15 1 -70.38*0*15 -70.38*6*15 -70.38*11*15 -70.38*1;... %37
0*15 12*15 11*15 1 0 0 0 0 -353.9*0*15 -353.9*12*15 -353.9*11*15 -353.9*1;...
0 0 0 0 0*15 12*15 11*15 1 -40.27*0*15 -40.27*12*15 -40.27*11*15 -40.27*1;... %38
0*15 11*15 10*15 1 0 0 0 0 -333.7*0*15 -333.7*11*15 -333.7*10*15 -333.7*1;...
0 0 0 0 0*15 11*15 10*15 1 -71.44*0*15 -71.44*11*15 -71.44*10*15 -71.44*1;... %39
0*15 2*15 9*15 1 0 0 0 0 -205.3*0*15 -205.3*2*15 -205.3*9*15 -205.3*1;...
0 0 0 0 0*15 2*15 9*15 1 -121.5*0*15 -121.5*2*15 -121.5*9*15 -121.5*1;... %40
0*15 6*15 9*15 1 0 0 0 0 -251.1*0*15 -251.1*6*15 -251.1*9*15 -251.1*1;...
0 0 0 0 0*15 6*15 9*15 1 -112.9*0*15 -112.9*6*15 -112.9*9*15 -112.9*1;... %41
0*15 8*15 8*15 1 0 0 0 0 -279.5*0*15 -279.5*8*15 -279.5*8*15 -279.5*1;...
0 0 0 0 0*15 8*15 8*15 1 -130.2*0*15 -130.2*8*15 -130.2*8*15 -130.2*1;... %42
0*15 12*15 8*15 1 0 0 0 0 -353.5*0*15 -353.5*12*15 -353.5*8*15 -353.5*1;...
0 0 0 0 0*15 12*15 8*15 1 -120*0*15 -120*12*15 -120*8*15 -120*1;... %43
0*15 10*15 7*15 1 0 0 0 0 -313.5*0*15 -313.5*10*15 -313.5*7*15 -313.5*1;...
0 0 0 0 0*15 10*15 7*15 1 -150.5*0*15 -150.5*10*15 -150.5*7*15 -150.5*1;... %44
0*15 7*15 7*15 1 0 0 0 0 -264.5*0*15 -264.5*7*15 -264.5*7*15 -264.5*1;...
0 0 0 0 0*15 7*15 7*15 1 -153.7*0*15 -153.7*7*15 -153.7*7*15 -153.7*1;... %45
0*15 5*15 7*15 1 0 0 0 0 -238.4*0*15 -238.4*5*15 -238.4*7*15 -238.4*1;...
0 0 0 0 0*15 5*15 7*15 1 -155.6*0*15 -155.6*5*15 -155.6*7*15 -155.6*1;... %46
0*15 1*15 7*15 1 0 0 0 0 -195*0*15 -195*1*15 -195*7*15 -195*1;...
0 0 0 0 0*15 1*15 7*15 1 -158.4*0*15 -158.4*1*15 -158.4*7*15 -158.4*1;... %47
0*15 2*15 5*15 1 0 0 0 0 -204.7*0*15 -204.7*2*15 -204.7*5*15 -204.7*1;...
0 0 0 0 0*15 2*15 5*15 1 -194.2*0*15 -194.2*2*15 -194.2*5*15 -194.2*1;... %48
0*15 6*15 5*15 1 0 0 0 0 -250.6*0*15 -250.6*6*15 -250.6*5*15 -250.6*1;...
0 0 0 0 0*15 6*15 5*15 1 -196.5*0*15 -196.5*6*15 -196.5*5*15 -196.5*1;... %49
0*15 9*15 5*15 1 0 0 0 0 -295.2*0*15 -295.2*9*15 -295.2*5*15 -295.2*1;...
0 0 0 0 0*15 9*15 5*15 1 -199*0*15 -199*9*15 -199*5*15 -199*1;...       %50
0*15 10*15 4*15 1 0 0 0 0 -312.6*0*15 -312.6*10*15 -312.6*4*15 -312.6*1;...
0 0 0 0 0*15 10*15 4*15 1 -224.7*0*15 -224.7*10*15 -224.7*4*15 -224.7*1;... %51
0*15 9*15 2*15 1 0 0 0 0 -295.5*0*15 -295.5*9*15 -295.5*2*15 -295.5*1;...
0 0 0 0 0*15 9*15 2*15 1 -270.5*0*15 -270.5*9*15 -270.5*2*15 -270.5*1;... %52
0*15 6*15 2*15 1 0 0 0 0 -250.2*0*15 -250.2*6*15 -250.2*2*15 -250.2*1;...
0 0 0 0 0*15 6*15 2*15 1 -259.4*0*15 -259.4*6*15 -259.4*2*15 -259.4*1;... %53
0*15 3*15 2*15 1 0 0 0 0 -214.6*0*15 -214.6*3*15 -214.6*2*15 -214.6*1;...
0 0 0 0 0*15 3*15 2*15 1 -251.1*0*15 -251.1*3*15 -251.1*2*15 -251.1*1;... %54
0*15 1*15 1*15 1 0 0 0 0 -194.2*0*15 -194.2*1*15 -194.2*1*15 -194.2*1;...
0 0 0 0 0*15 1*15 1*15 1 -264.2*0*15 -264.2*1*15 -264.2*1*15 -264.2*1;... %55
0*15 5*15 1*15 1 0 0 0 0 -237.1*0*15 -237.1*5*15 -237.1*1*15 -237.1*1;...
0 0 0 0 0*15 5*15 1*15 1 -276.6*0*15 -276.6*5*15 -276.6*1*15 -276.6*1;... %56
0*15 8*15 1*15 1 0 0 0 0 -278.7*0*15 -278.7*8*15 -278.7*1*15 -278.7*1;...
0 0 0 0 0*15 8*15 1*15 1 -289*0*15 -289*8*15 -289*1*15 -289*1;...       %57
0*15 11*15 1*15 1 0 0 0 0 -331.7*0*15 -331.7*11*15 -331.7*1*15 -331.7*1;...
0 0 0 0 0*15 11*15 1*15 1 -305*0*15 -305*11*15 -305*1*15 -305*1;...     %58
0*15 8*15 0*15 1 0 0 0 0 -277.6*0*15 -277.6*8*15 -277.6*0*15 -277.6*1;...
```

```
     0 0 0 0 0*15 8*15 0*15 1 -310.5*0*15 -310.5*8*15 -310.5*0*15 -310.5*1;...   %59
     0*15 4*15 0*15 1 0 0 0 0 -223.7*0*15 -223.7*4*15 -223.7*0*15 -223.7*1;...
     0 0 0 0 0*15 4*15 0*15 1 -291.6*0*15 -291.6*4*15 -291.6*0*15 -291.6*1  %60
];

[u,s,v] = svd(p, 'econ');
m = v(:,12);
M = [m(1:4)'; m(5:8)'; m(9:12)'];

[R,K] = qr(M(:,1:3));
T = inv(K)*M(:,4);

disp(m);
disp(M);
disp(R);
disp(K);
disp(T);
```

After filling $\mathcal{P}$ matrix, I used SVD decomposition and find m_{12x1}. Then translate m into matrix M_{3x4}. Later, I take the submatrix M_{3x3} and use QR factorization to find K and R matrices. From K matrix, I am also able to find T matrix.

m:

```
   0.0030
  -0.0007
  -0.0001
  -0.5481
   0.0011
   0.0011
   0.0034
  -0.8364
   0.0000
   0.0000
  -0.0000
  -0.0030
```

M:

```
   0.0030  -0.0007  -0.0001  -0.5481
   0.0011   0.0011   0.0034  -0.8364
   0.0000   0.0000  -0.0000  -0.0030
```

R:

```
  -0.9426   0.3339   0.0001
  -0.3339  -0.9426  -0.0055
  -0.0017  -0.0052   1.0000
```

K:

```
  -0.0032   0.0003  -0.0011
        0  -0.0013  -0.0032
        0        0  -0.0000
```

T:

```
  142.7538
  258.7777
  158.0048
```

Aynur Süne - 21041

Camera Calibration Toolbox

I used 20 different images of the checkerboard to use with the toolbox, window size is 11x11 (k=5).

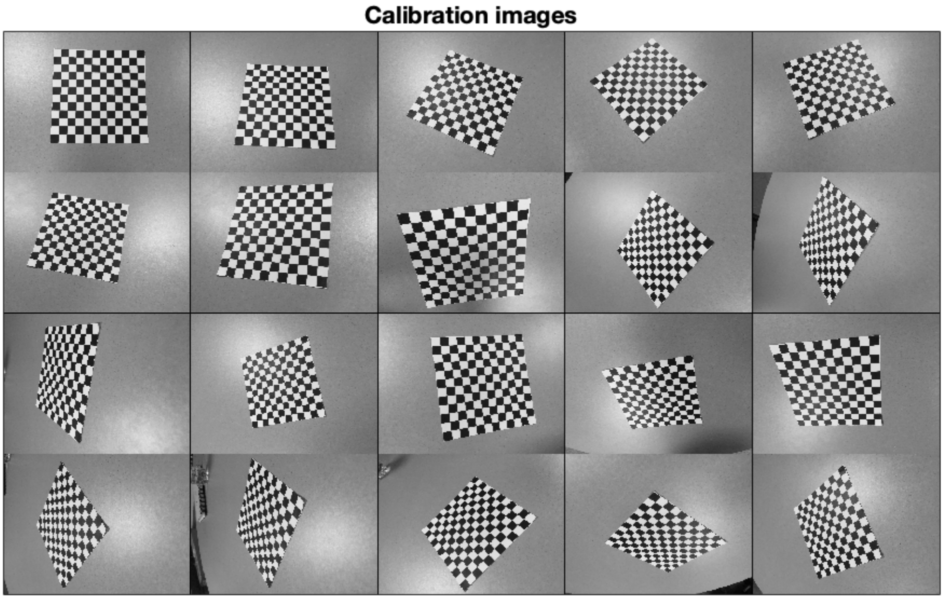


image1

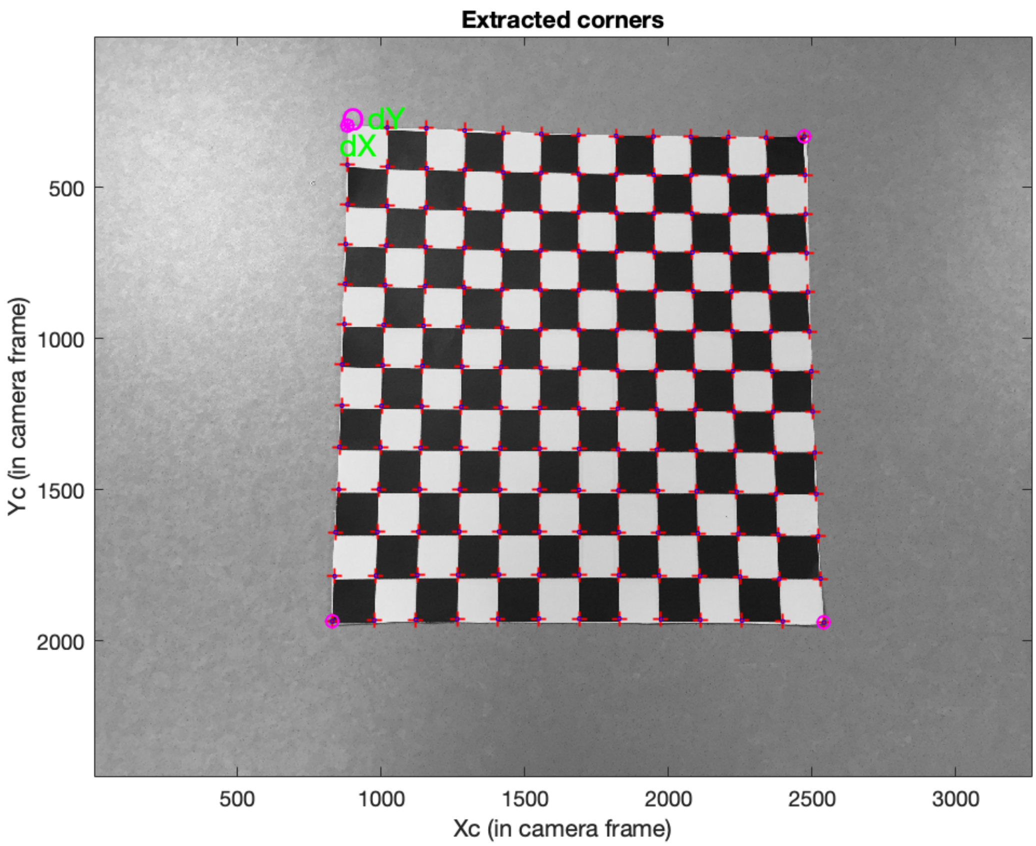


image10

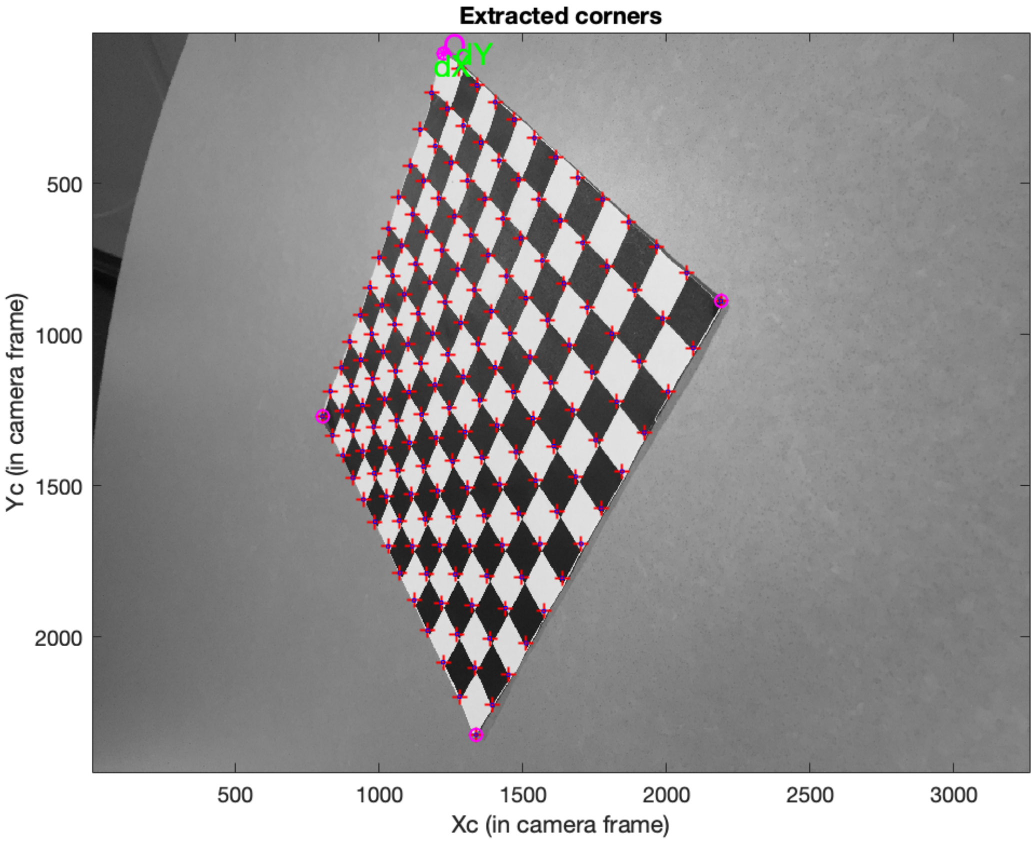
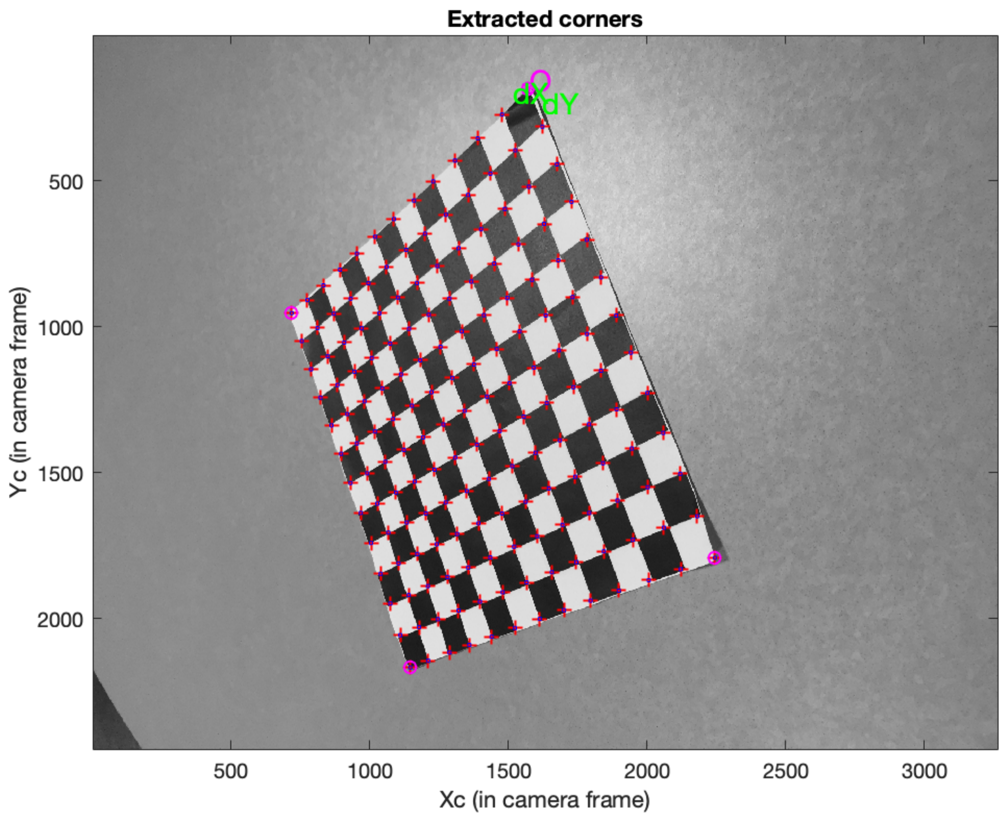

Extracted corners
dX
dY
500
1000
1500
2000
Yc (in camera frame)
500
1000
1500
2000
2500
3000
Xc (in camera frame)

image20


Extracted corners
dX
dY
500
1000
1500
2000
Yc (in camera frame)
500
1000
1500
2000
2500
3000
Xc (in camera frame)

Results of Calibration:

Main calibration optimization procedure - Number of images: 20
Gradient descent iterations:
1...2...3...4...5...6...7...8...9...10...11...12...13...14...15...16...17...18...19...20...done
Estimation of uncertainties...done
Calibration results after optimization (with uncertainties):

Focal Length: fc = [2860.43156 2859.14403] +/- [15.53703 16.24683]
Principal point: cc = [1631.50000 1223.50000] +/- [0.00000 0.00000]
Skew: alpha_c = [0.00000] +/- [0.00000] => angle of pixel axes = 90.00000 +/- 0.00000 degrees
Distortion: kc = [0.15524 -0.07821 0.00053 -0.00098 0.00000] +/- [0.02721 0.13186 0.00111 0.00120 0.00000]
Pixel error: err = [3.97052 4.00792]

Note: The numerical errors are approximately three times the standard deviations (for reference).

Number(s) of image(s) to show ([] = all images) = []
Pixel error: err = [3.97052 4.00792] (all active images)

Reprojection:

Image1

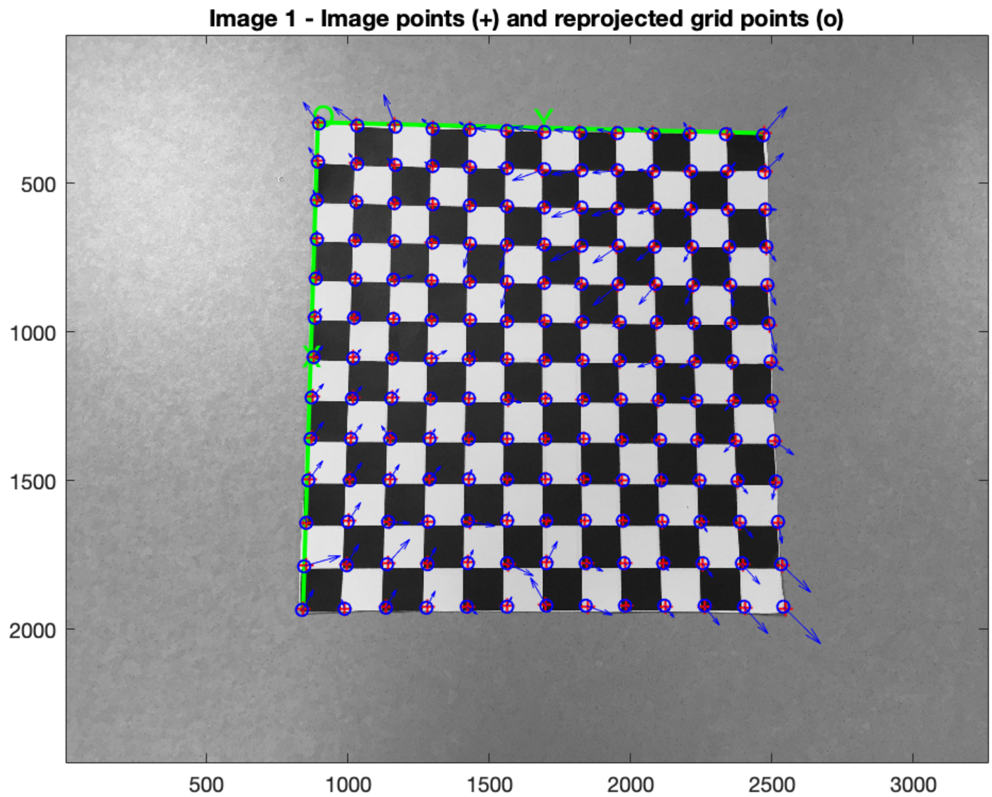


Image10

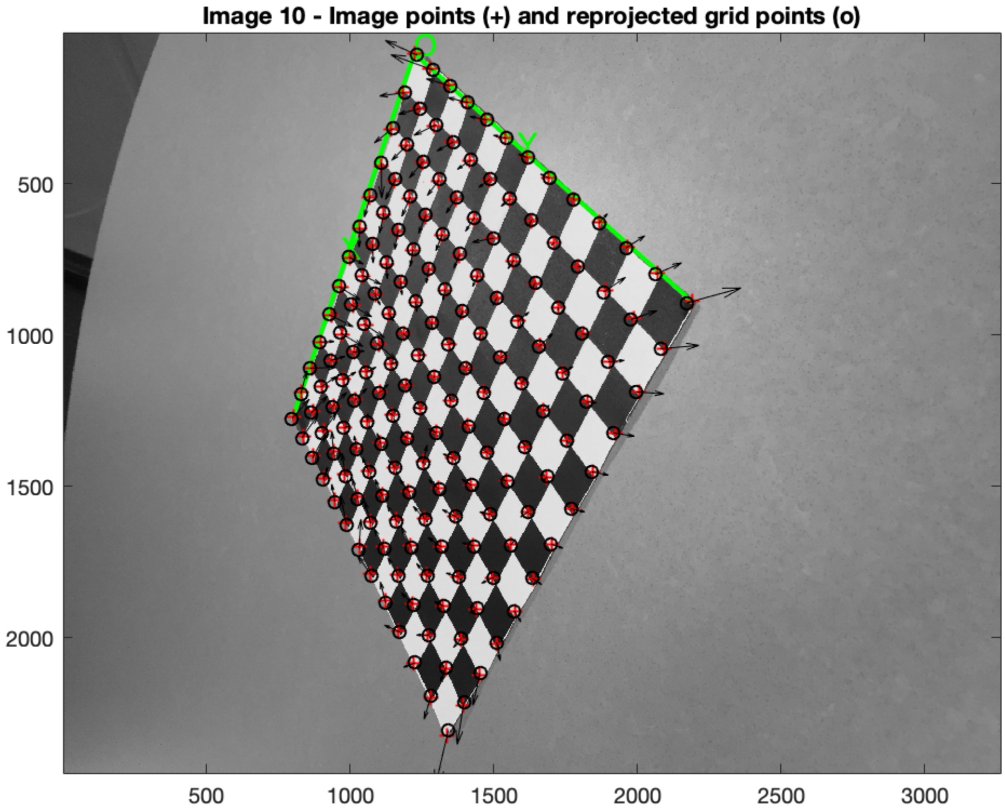
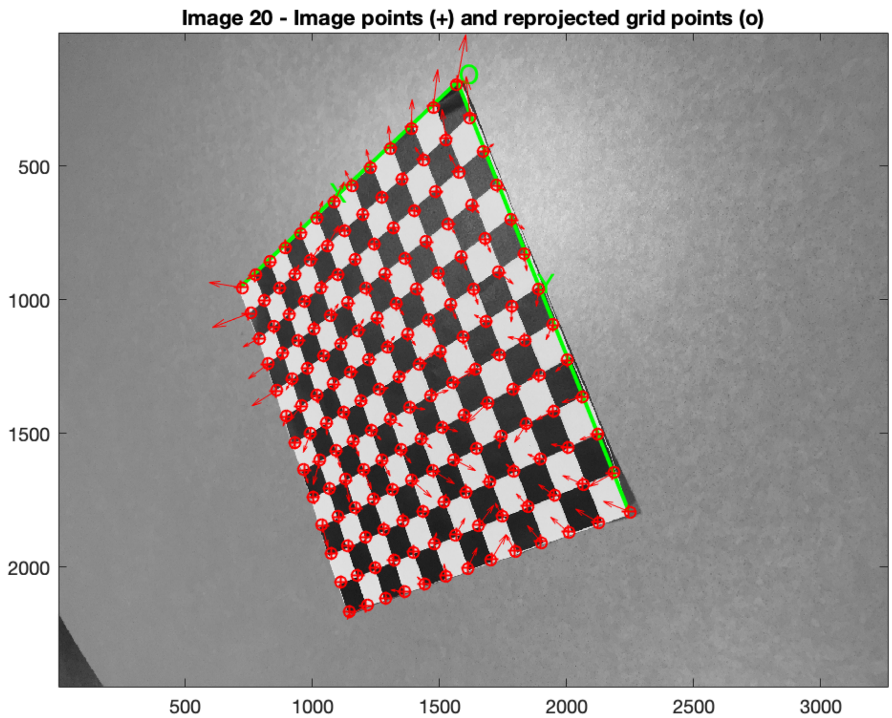


Image20



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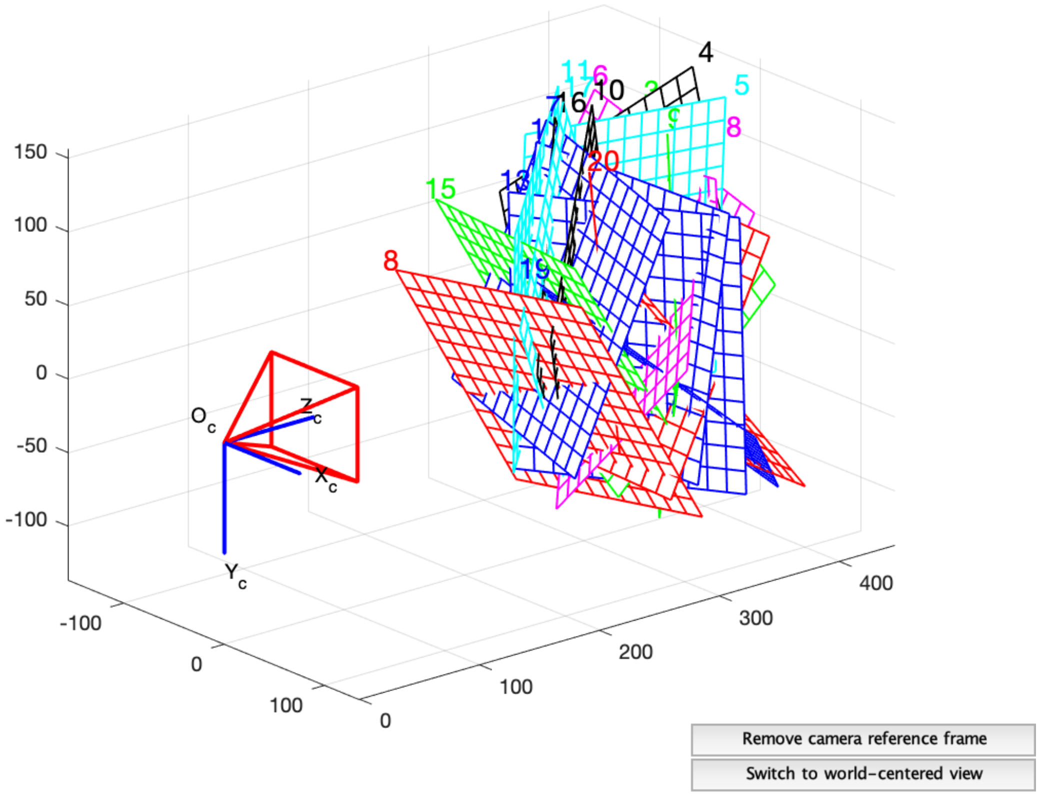
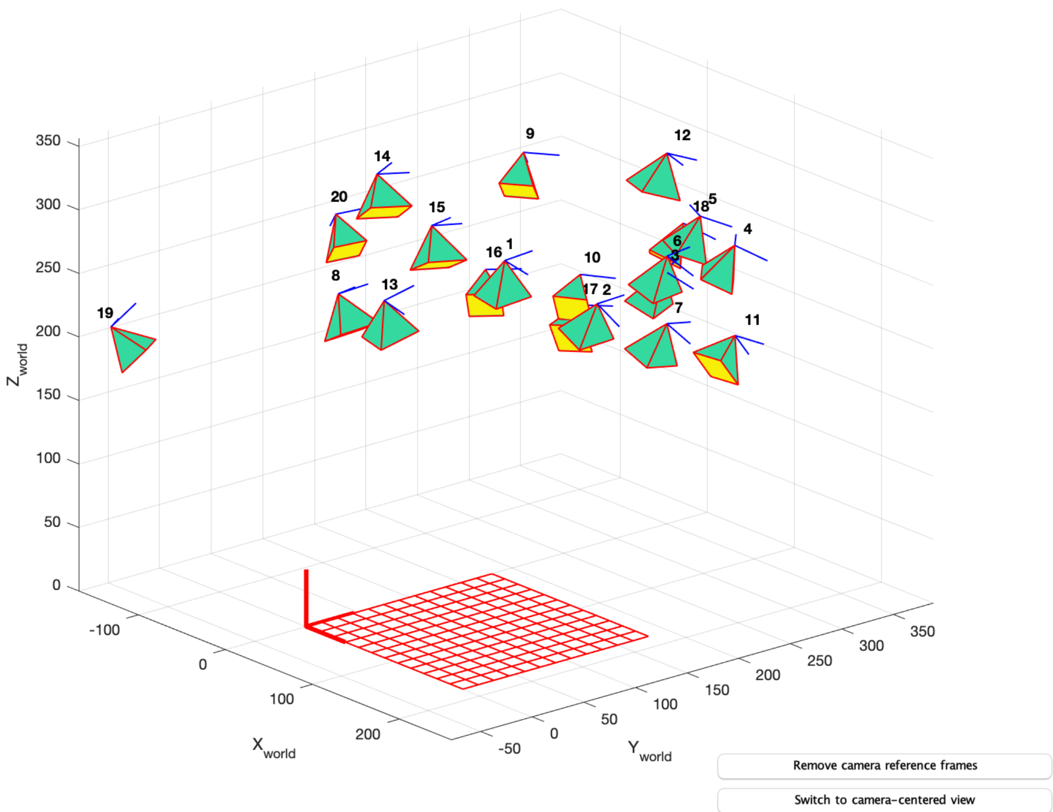
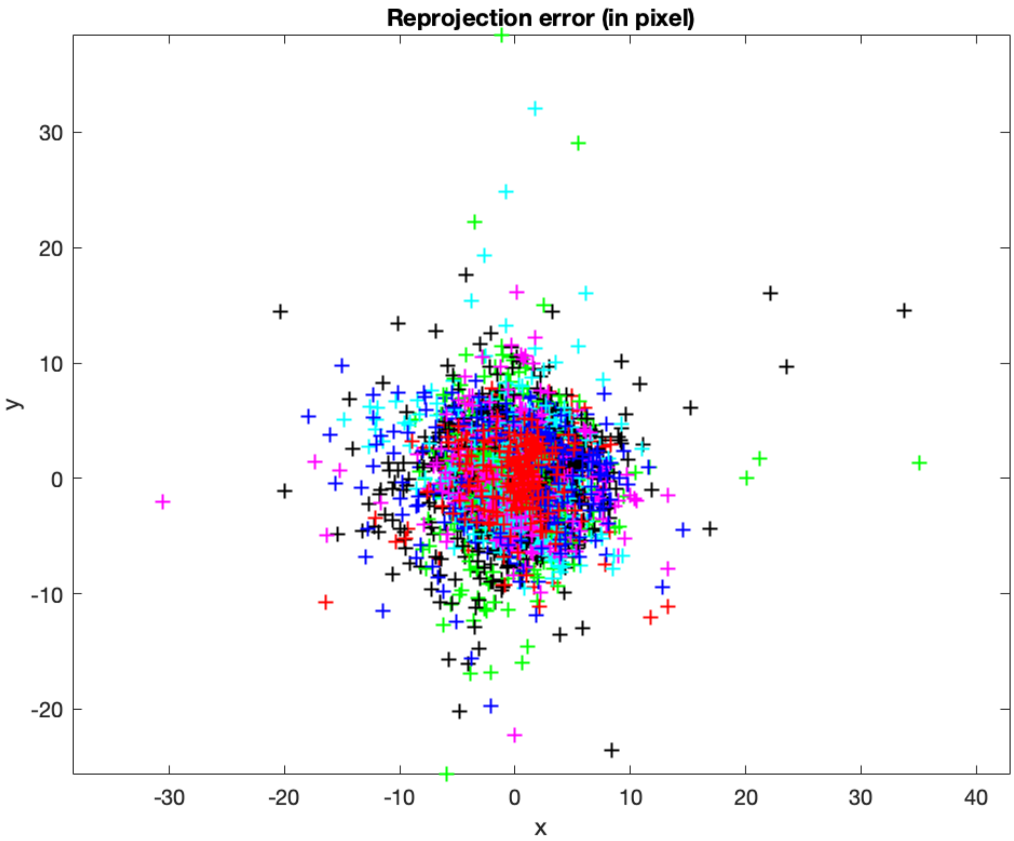
Extrinsic parameters (camera-centered)



Extrinsic parameters (world-centered)





Re-extraction of the corners and Re-Calibration

Main calibration optimization procedure - Number of images: 20
Gradient descent iterations:
1...2...3...4...5...6...7...8...9...10...11...12...13...14...15...16...17...18...done
Estimation of uncertainties...done

Calibration results after optimization (with uncertainties):

Focal Length: fc = [2870.23930 2869.40617] +/- [6.39767 6.69219]
Principal point: cc = [1631.50000 1223.50000] +/- [0.00000 0.00000]
Skew: alpha_c = [0.00000] +/- [0.00000] => angle of pixel axes = 90.00000 +/- 0.00000 degrees
Distortion: kc = [0.15569 -0.07342 0.00096 -0.00021 0.00000] +/- [0.01121 0.05465 0.00045 0.00049 0.00000]
Pixel error: err = [1.73418 1.53098]

Note: The numerical errors are approximately three times the standard deviations (for reference).

Note that we needed less gradient descent iterations this time. Pixel error is almost 2.5 times less than first calibration. Point accuracy is improved.

Reprojection:

Image1

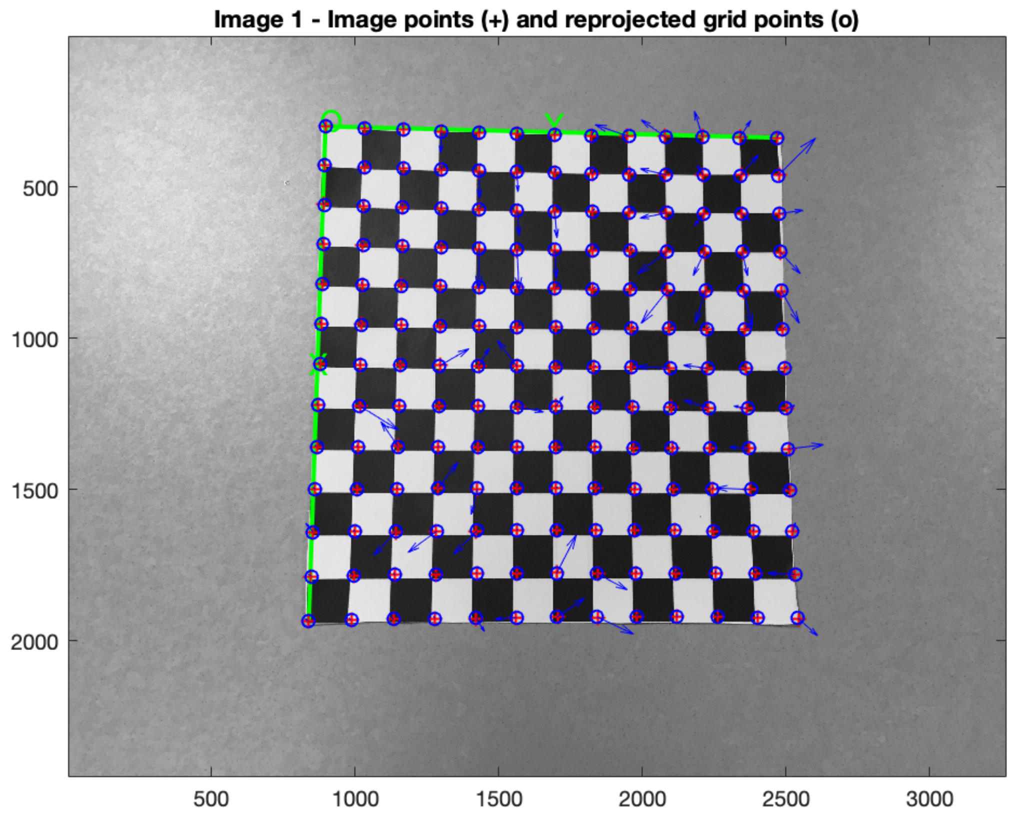


Image10

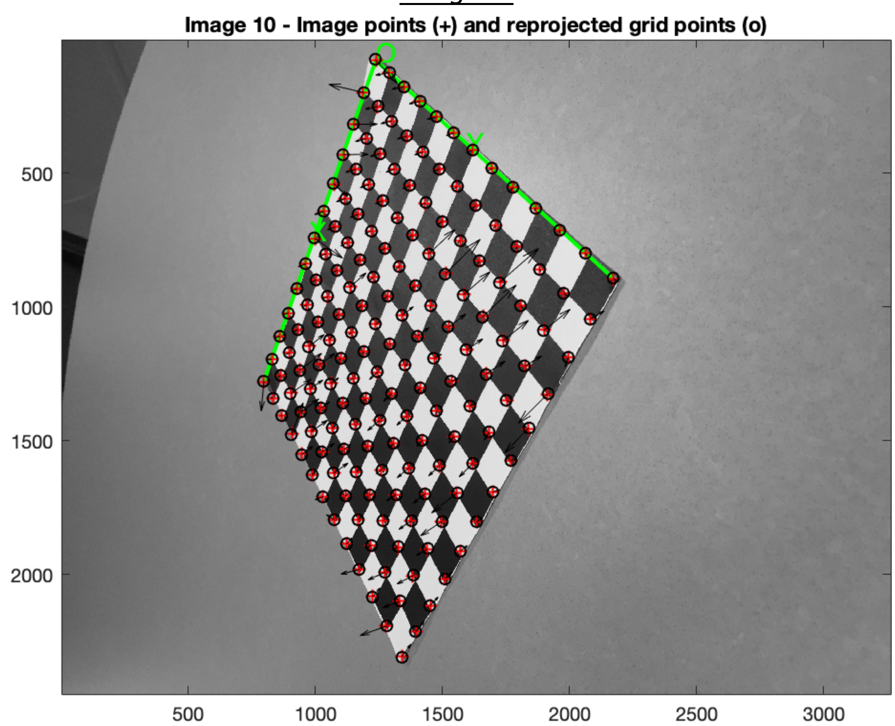


Image20

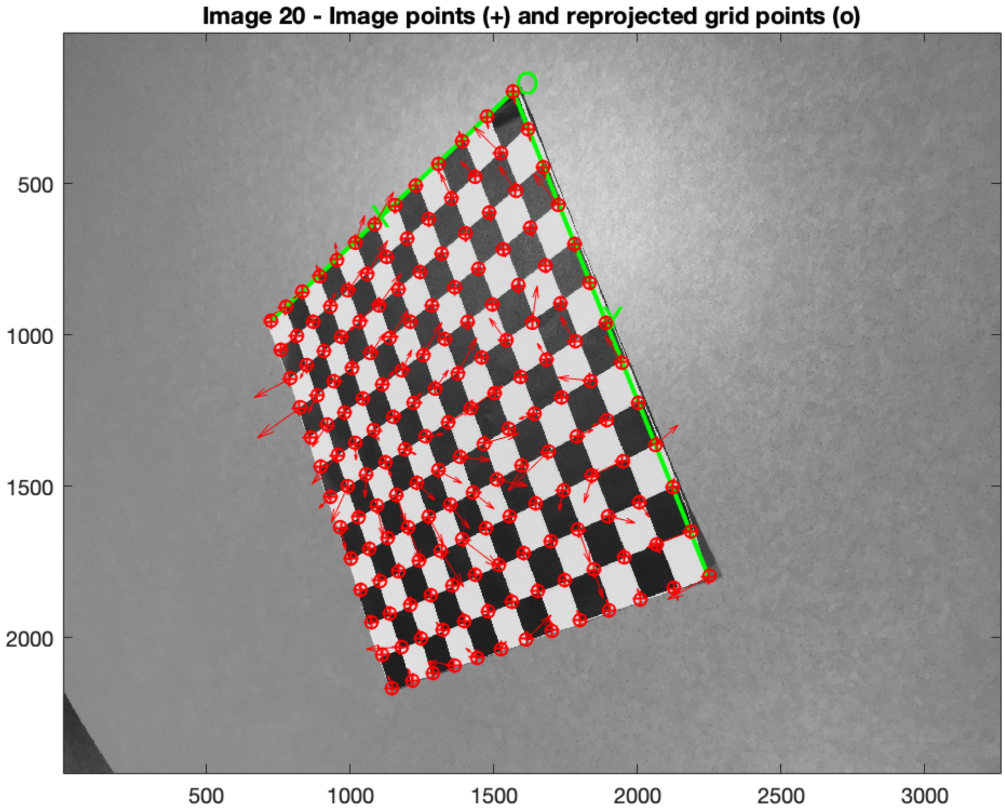
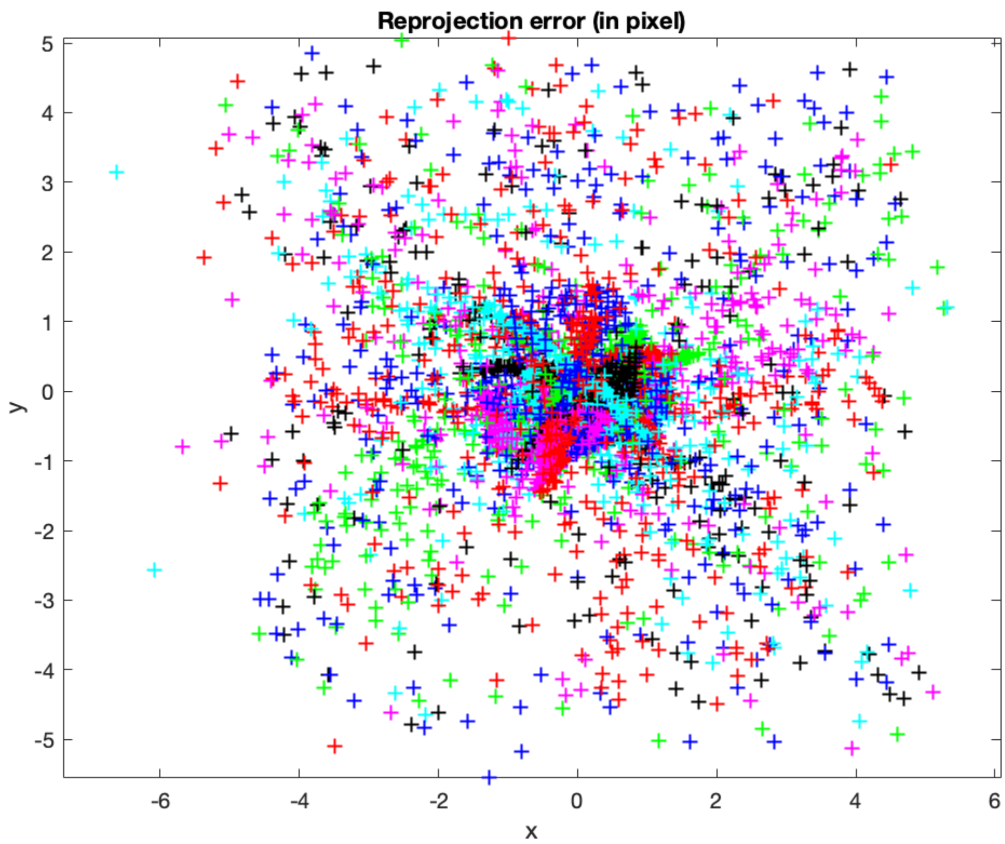
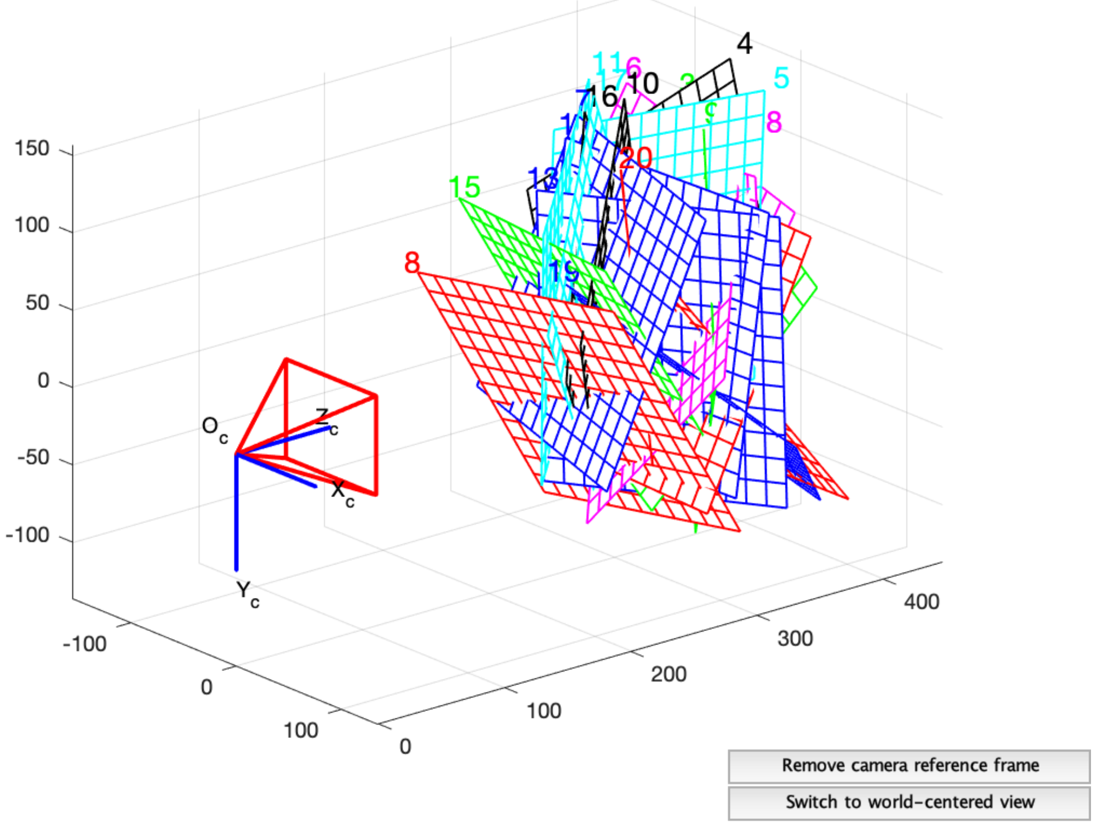


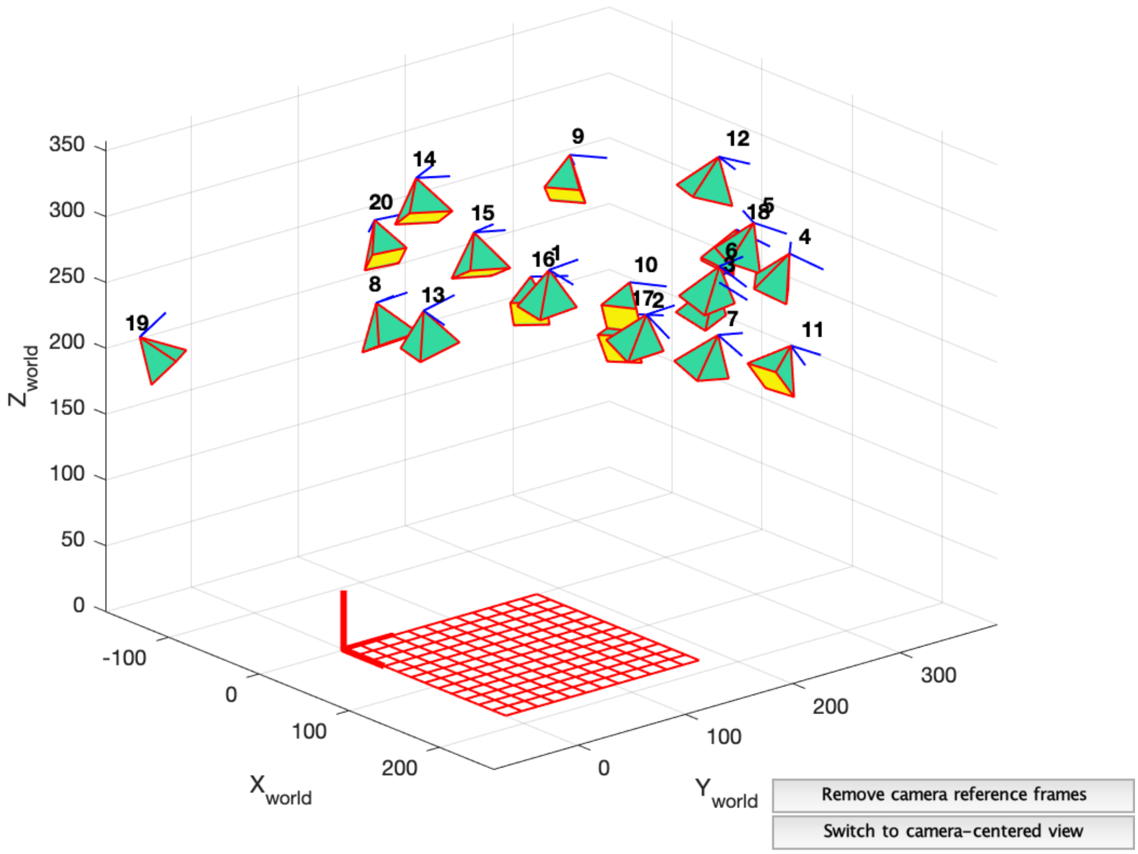
Image 20 - Image points (+) and reprojected grid points (o)
500
1000
1500
2000
500
1000
1500
2000
2500
3000


Reprojection error (in pixel)
5
4
3
2
1
0
-1
-2
-3
-4
-5
y
-6
-4
-2
0
2
4
6
x

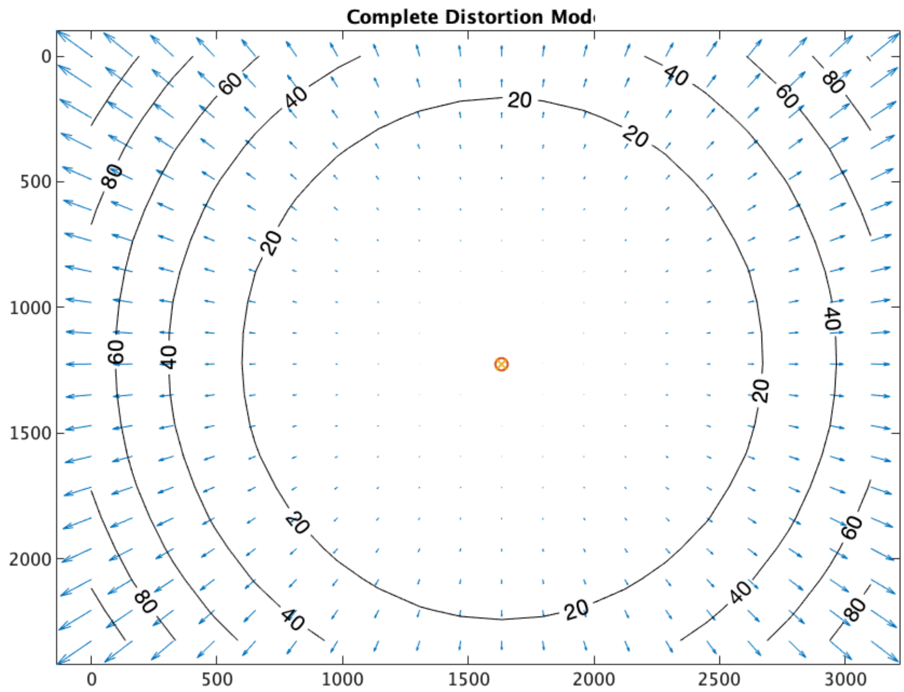
Extrinsic parameters (camera-centered)



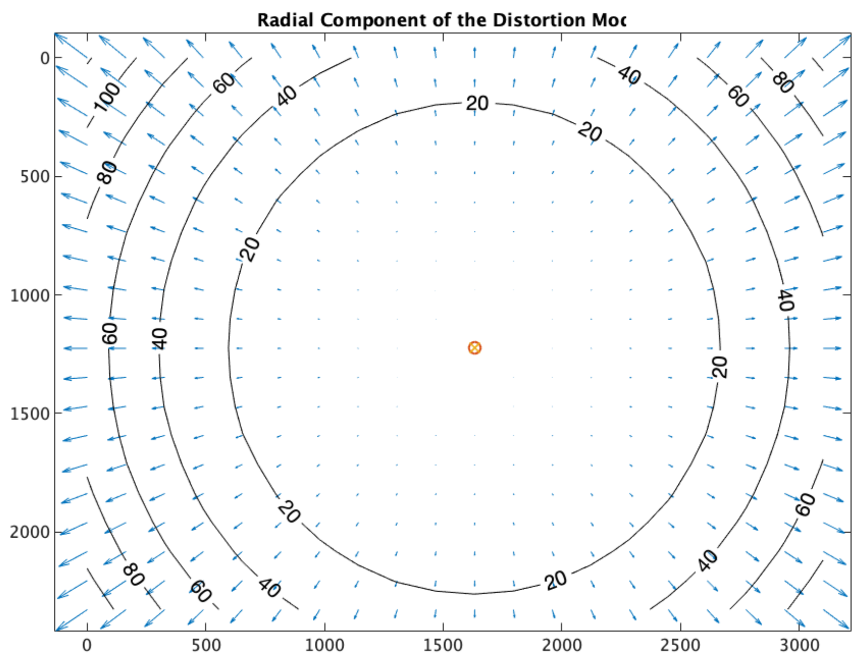
Extrinsic parameters (world-centered)



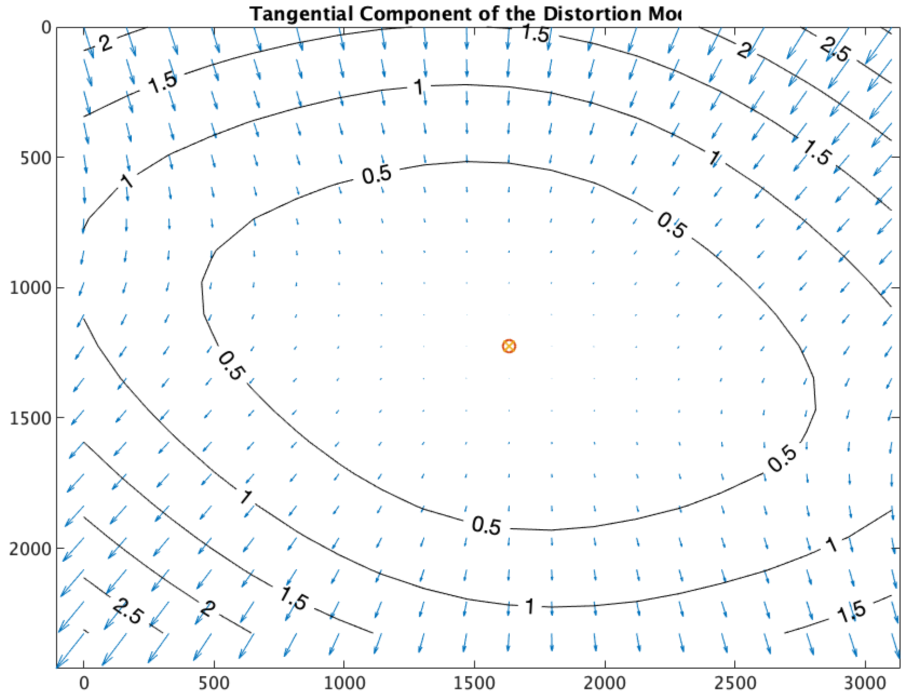
While extracting corners, I entered distortion coefficients for more accurate points. Visualization for distortions are shown below:



Pixel error	= [1.734, 1.531]	
Focal Length	= (2870.24, 2869.41)	+/- [6.398, 6.692]
Principal Point	= (1631.5, 1223.5)	+/- [0, 0]
Skew	= 0	+/- 0
Radial coefficients	= (0.1557, -0.07342, 0)	+/- [0.01121, 0.05465, 0]
Tangential coefficients	= (0.000964, -0.0002102)	+/- [0.0004541, 0.000495]



Pixel error	= [1.734, 1.531]	
Focal Length	= (2870.24, 2869.41)	+/- [6.398, 6.692]
Principal Point	= (1631.5, 1223.5)	+/- [0, 0]
Skew	= 0	+/- 0
Radial coefficients	= (0.1557, -0.07342, 0)	+/- [0.01121, 0.05465, 0]
Tangential coefficients	= (0.000964, -0.0002102)	+/- [0.0004541, 0.000495]



Pixel error	= [1.734, 1.531]	
Focal Length	= (2870.24, 2869.41)	+/− [6.398, 6.692]
Principal Point	= (1631.5, 1223.5)	+/− [0, 0]
Skew	= 0	+/− 0
Radial coefficients	= (0.1557, −0.07342, 0)	+/− [0.01121, 0.05465, 0]
Tangential coefficients	= (0.000964, −0.0002102)	+/− [0.0004541, 0.000495]

Using toolbox for calibration was easier and more time efficient. Being able to recalibrate quickly is very useful and increases accuracy.

All intrinsic extrinsic parameters of the images are included in Calib_Results.m file below:

% Intrinsic and Extrinsic Camera Parameters
%
% This script file can be directly executed under Matlab to recover the camera intrinsic and extrinsic parameters.
% IMPORTANT: This file contains neither the structure of the calibration objects nor the image coordinates of the calibration points.
%   All those complementary variables are saved in the complete matlab data file Calib_Results.mat.
% For more information regarding the calibration model visit http://www.vision.caltech.edu/bouguetj/calib_doc/

%-- Focal length:
fc = [2870.239301660451474 ; 2869.406168193617759];

%-- Principal point:
cc = [1631.500000000000000 ; 1223.500000000000000];

%-- Skew coefficient:
alpha_c = 0.000000000000000;

%-- Distortion coefficients:
kc = [0.155694057120704 ; -0.073417697334018 ; 0.000963979397752 ; -0.000210182023345 ; 0.000000000000000];

%-- Focal length uncertainty:
fc_error = [6.397671316439594 ; 6.692194340261080];

%-- Principal point uncertainty:
cc_error = [0.000000000000000 ; 0.000000000000000];

%-- Skew coefficient uncertainty:
alpha_c_error = 0.000000000000000;

%-- Distortion coefficients uncertainty:
kc_error = [0.011206802846470 ; 0.054652040606334 ; 0.000454078128833 ; 0.000494957784094 ; 0.000000000000000];

%-- Image size:
nx = 3264;
ny = 2448;

%-- Various other variables (may be ignored if you do not use the Matlab Calibration Toolbox):
%-- Those variables are used to control which intrinsic parameters should be optimized

n_ima = 20; % Number of calibration images
est_fc = [1 ; 1]; % Estimation indicator of the two focal variables
est_aspect_ratio = 1; % Estimation indicator of the aspect ratio fc(2)/fc(1)
center_optim = 0; % Estimation indicator of the principal point
est_alpha = 0; % Estimation indicator of the skew coefficient
est_dist = [1 ; 1 ; 1 ; 1 ; 0]; % Estimation indicator of the distortion coefficients

%-- Extrinsic parameters:
%-- The rotation (omc_kk) and the translation (Tc_kk) vectors for every calibration image and their uncertainties

%-- Image #1:

omc_1 = [2.114793e+00 ; 2.119362e+00 ; -9.448923e-02];
Tc_1 = [-8.276975e+01 ; -1.041320e+02 ; 3.304650e+02];
omc_error_1 = [2.141507e-03 ; 2.172498e-03 ; 4.364982e-03];
Tc_error_1 = [6.850689e-02 ; 6.755142e-02 ; 7.777905e-01];

%-- Image #2:
omc_2 = [1.936751e+00 ; 2.003509e+00 ; -4.353023e-01];
Tc_2 = [-8.212791e+01 ; -8.644788e+01 ; 3.728290e+02];
omc_error_2 = [1.672207e-03 ; 1.669881e-03 ; 3.104100e-03];
Tc_error_2 = [7.571927e-02 ; 1.095384e-01 ; 7.487355e-01];

%-- Image #3:
omc_3 = [1.541337e+00 ; 2.401814e+00 ; -5.002250e-01];
Tc_3 = [-5.070487e+01 ; -1.227683e+02 ; 3.929098e+02];
omc_error_3 = [1.383074e-03 ; 1.959367e-03 ; 3.342713e-03];
Tc_error_3 = [9.423329e-02 ; 1.247933e-01 ; 7.737733e-01];

%-- Image #4:
omc_4 = [1.066615e+00 ; 2.738755e+00 ; -6.461381e-01];
Tc_4 = [-1.649058e+01 ; -1.563061e+02 ; 4.051695e+02];
omc_error_4 = [1.030370e-03 ; 2.018823e-03 ; 2.934114e-03];
Tc_error_4 = [1.067330e-01 ; 1.434367e-01 ; 7.826659e-01];

%-- Image #5:
omc_5 = [6.042824e-01 ; 2.936213e+00 ; -6.269929e-01];
Tc_5 = [2.615166e+01 ; -1.495659e+02 ; 3.967040e+02];
omc_error_5 = [9.205777e-04 ; 2.156933e-03 ; 3.132147e-03];
Tc_error_5 = [9.985097e-02 ; 1.322159e-01 ; 7.904556e-01];

%-- Image #6:
omc_6 = [1.829158e+00 ; 2.215806e+00 ; -4.439982e-01];
Tc_6 = [-1.022860e+02 ; -1.190329e+02 ; 3.953572e+02];
omc_error_6 = [1.577998e-03 ; 2.044857e-03 ; 3.419399e-03];
Tc_error_6 = [9.303805e-02 ; 1.262570e-01 ; 7.863595e-01];

%-- Image #7:
omc_7 = [2.014046e+00 ; 2.219823e+00 ; -6.218709e-01];
Tc_7 = [-8.854361e+01 ; -1.145510e+02 ; 3.435520e+02];
omc_error_7 = [1.346202e-03 ; 1.523290e-03 ; 2.564866e-03];
Tc_error_7 = [8.790243e-02 ; 1.016316e-01 ; 6.483435e-01];

%-- Image #8:
omc_8 = [-2.081159e+00 ; -1.846387e+00 ; -5.598964e-01];
Tc_8 = [-9.859040e+01 ; -3.710783e+01 ; 2.262986e+02];
omc_error_8 = [1.176808e-03 ; 1.107401e-03 ; 2.062342e-03];
Tc_error_8 = [7.096999e-02 ; 8.226140e-02 ; 5.408615e-01];

```
%-- Image #9:
omc_9 = [ -1.061879e+00 ; -2.329885e+00 ; 3.748548e-01 ];
Tc_9  = [ -1.400482e+01 ; -1.168417e+02 ; 3.821869e+02 ];
omc_error_9 = [ 1.104279e-03 ; 1.838907e-03 ; 2.396014e-03 ];
Tc_error_9  = [ 1.172951e-01 ; 5.163479e-02 ; 8.678131e-01 ];

%-- Image #10:
omc_10 = [ -1.022856e+00 ; -2.104892e+00 ; 5.140075e-01 ];
Tc_10  = [ -4.687926e+01 ; -1.357300e+02 ; 3.468374e+02 ];
omc_error_10 = [ 8.810568e-04 ; 1.102048e-03 ; 1.397477e-03 ];
Tc_error_10  = [ 1.080661e-01 ; 4.896274e-02 ; 7.412588e-01 ];

%-- Image #11:
omc_11 = [ -1.492911e+00 ; -1.889848e+00 ; 7.183614e-01 ];
Tc_11  = [ -1.108058e+02 ; -1.246432e+02 ; 3.724841e+02 ];
omc_error_11 = [ 9.433726e-04 ; 9.354976e-04 ; 1.326595e-03 ];
Tc_error_11  = [ 1.276050e-01 ; 7.357625e-02 ; 6.928198e-01 ];

%-- Image #12:
omc_12 = [ -2.276893e+00 ; -1.809650e+00 ; 4.582234e-01 ];
Tc_12  = [ -1.127935e+02 ; -6.549142e+01 ; 4.298465e+02 ];
omc_error_12 = [ 2.297281e-03 ; 1.912787e-03 ; 4.172769e-03 ];
Tc_error_12  = [ 1.289778e-01 ; 1.001008e-01 ; 8.833986e-01 ];

%-- Image #13:
omc_13 = [ 2.158836e+00 ; 1.908567e+00 ; 1.719653e-01 ];
Tc_13  = [ -7.010607e+01 ; -8.156709e+01 ; 2.967938e+02 ];
omc_error_13 = [ 1.798741e-03 ; 1.526624e-03 ; 3.394837e-03 ];
Tc_error_13  = [ 8.366072e-02 ; 7.464430e-02 ; 7.415994e-01 ];

%-- Image #14:
omc_14 = [ -1.983163e+00 ; -1.600728e+00 ; -4.040460e-01 ];
Tc_14  = [ -1.146771e+02 ; -2.510567e+01 ; 3.411076e+02 ];
omc_error_14 = [ 1.389712e-03 ; 1.267317e-03 ; 2.221866e-03 ];
Tc_error_14  = [ 6.669033e-02 ; 1.079801e-01 ; 8.066634e-01 ];

%-- Image #15:
omc_15 = [ -1.963681e+00 ; -1.779995e+00 ; -3.278842e-01 ];
Tc_15  = [ -1.282168e+02 ; -6.353232e+01 ; 2.839062e+02 ];
omc_error_15 = [ 1.373406e-03 ; 1.204844e-03 ; 2.345998e-03 ];
Tc_error_15  = [ 6.014874e-02 ; 9.811837e-02 ; 6.466793e-01 ];

%-- Image #16:
omc_16 = [ -8.844112e-01 ; -2.156980e+00 ; 3.353435e-01 ];
Tc_16  = [ -7.083551e+01 ; -1.231667e+02 ; 3.354828e+02 ];
omc_error_16 = [ 8.574088e-04 ; 1.092176e-03 ; 1.397749e-03 ];
Tc_error_16  = [ 1.129127e-01 ; 4.723851e-02 ; 7.542195e-01 ];
```

```
%-- Image #17:
omc_17 = [ -1.071435e+00 ; -1.983585e+00 ; 4.651214e-01 ];
Tc_17  = [ -6.573868e+01 ; -1.343582e+02 ; 3.384025e+02 ];
omc_error_17 = [ 8.506477e-04 ; 9.724414e-04 ; 1.254100e-03 ];
Tc_error_17  = [ 1.092366e-01 ; 4.886664e-02 ; 7.207058e-01 ];

%-- Image #18:
omc_18 = [ -1.092346e+00 ; -2.583690e+00 ; 8.248690e-01 ];
Tc_18  = [ 8.193976e+00 ; -1.150042e+02 ; 3.938714e+02 ];
omc_error_18 = [ 1.039234e-03 ; 1.758485e-03 ; 2.500293e-03 ];
Tc_error_18  = [ 1.047379e-01 ; 1.024557e-01 ; 7.575688e-01 ];

%-- Image #19:
omc_19 = [ -1.074535e+00 ; -2.306330e+00 ; -9.229960e-01 ];
Tc_19  = [ -8.424699e+00 ; -4.973394e+01 ; 2.617826e+02 ];
omc_error_19 = [ 9.511878e-04 ; 1.252922e-03 ; 1.784256e-03 ];
Tc_error_19  = [ 1.050299e-01 ; 1.082801e-01 ; 7.661216e-01 ];

%-- Image #20:
omc_20 = [ -5.401658e-01 ; -2.397256e+00 ; 4.466232e-02 ];
Tc_20  = [ -6.828782e+00 ; -1.095052e+02 ; 3.102853e+02 ];
omc_error_20 = [ 8.362706e-04 ; 1.428432e-03 ; 1.930952e-03 ];
Tc_error_20  = [ 1.013381e-01 ; 6.898129e-02 ; 7.766475e-01 ];
```