Aynur Süne – 21041

EE417 Post-Lab #8

```
%% Lab#8 Assignment starts here.
%% Transform pixel coordinates and construct X matrix using Equations 1
and 2
Kinv= inv(K);
x1 = Kinv*u1(:,:);
x2 = Kinv*u2(:,:);

a1 = [x1(1,1)*x2(1,1) x1(1,1)*x2(2,1) x1(1,1)*x2(3,1) x1(2,1)*x2(1,1)
x1(2,1)*x2(2,1) x1(2,1)*x2(3,1)...
    x1(3,1)*x2(1,1) x1(3,1)*x2(2,1) x1(3,1)*x2(3,1)]';

a2 = [x1(1,2)*x2(1,2) x1(1,2)*x2(2,2) x1(1,2)*x2(3,2) x1(2,2)*x2(1,2)
x1(2,2)*x2(2,2) x1(2,2)*x2(3,2)...
    x1(3,2)*x2(1,2) x1(3,2)*x2(2,2) x1(3,2)*x2(3,2)]';

a3 = [x1(1,3)*x2(1,3) x1(1,3)*x2(2,3) x1(1,3)*x2(3,3) x1(2,3)*x2(1,3)
x1(2,3)*x2(2,3) x1(2,3)*x2(3,3)...
    x1(3,3)*x2(1,3) x1(3,3)*x2(2,3) x1(3,3)*x2(3,3)]';

a4 = [x1(1,4)*x2(1,4) x1(1,4)*x2(2,4) x1(1,4)*x2(3,4) x1(2,4)*x2(1,4)
x1(2,4)*x2(2,4) x1(2,4)*x2(3,4)...
    x1(3,4)*x2(1,4) x1(3,4)*x2(2,4) x1(3,4)*x2(3,4)]';

a5 = [x1(1,5)*x2(1,5) x1(1,5)*x2(2,5) x1(1,5)*x2(3,5) x1(2,5)*x2(1,5)
x1(2,5)*x2(2,5) x1(2,5)*x2(3,5)...
    x1(3,5)*x2(1,5) x1(3,5)*x2(2,5) x1(3,5)*x2(3,5)]';

a6 = [x1(1,6)*x2(1,6) x1(1,6)*x2(2,6) x1(1,6)*x2(3,6) x1(2,6)*x2(1,6)
x1(2,6)*x2(2,6) x1(2,6)*x2(3,6)...
    x1(3,6)*x2(1,6) x1(3,6)*x2(2,6) x1(3,6)*x2(3,6)]';

a7 = [x1(1,7)*x2(1,7) x1(1,7)*x2(2,7) x1(1,7)*x2(3,7) x1(2,7)*x2(1,7)
x1(2,7)*x2(2,7) x1(2,7)*x2(3,7)...
    x1(3,7)*x2(1,7) x1(3,7)*x2(2,7) x1(3,7)*x2(3,7)]';

a8 = [x1(1,8)*x2(1,8) x1(1,8)*x2(2,8) x1(1,8)*x2(3,8) x1(2,8)*x2(1,8)
x1(2,8)*x2(2,8) x1(2,8)*x2(3,8)...
    x1(3,8)*x2(1,8) x1(3,8)*x2(2,8) x1(3,8)*x2(3,8)]';

X = [a1';a2';a3';a4';a5';a6';a7';a8';];

%% Estimate E, cure it and check for Essential Matrix Characterization

[U,S,V] = svd(X);
Es = V(:,end);
E = [Es(1) Es(4) Es(7); Es(2) Es(5) Es(8); Es(3) Es(6) Es(9)];

[U1,S1,V1] = svd(E);
S1(1,1) = 1;
S1(2,2) = 1;
S1(3,3) = 0;
detU1 = det(U1);
detV1 = det(V1);

if detU1 ~= 1
    disp('det not U1 1');
end
```

```
if detV1 ~= 1
    disp('det not V1 1');
end
E = U1*S1*V1;
%% Find epipoles and epipolar lines
l1 = E'*x2(:,1);
l2 = E*x1(:,1);

e1 = null(E);
e2 = null(E');

%% Verify epipoles and epipolar lines
c1 = l1'*e1;
c2 = l2'*e2;
if c1 == 0
    disp('l1 e1 verified');
end

if c2 == 0
    disp('l2 e2 verified');
end


%% Recover the rotation and the translation
Rz1 = [0 -1 0;
       1  0 0;
       0  0 1]; % 90 degree

  Rz2 = [0 1 0;
        -1 0 0;
         0 0 1]; % -90 degree

T1 = U1*Rz1*S1*U1';
R1 = U1*Rz1'*V1';
T2 = U1*Rz2*S1*U1';
R2 = U1*Rz2'*V1';
T1 = [T1(3,2) ; T1(1,3); T1(2,1)];
T2 = [T2(3,2) ; T2(1,3); T2(2,1)];

%% Compare your results with ground truth
disp('True E =')
disp(Etrue)
disp('Estimated E = ')
% disp(your estimated E variable)
disp(E)

disp('True R =')
disp(TrueR)
disp('Estimated R = ')
% disp(your estimated R variable)
disp(R1)
disp(R2)

disp('True T =')
disp(TrueT)
disp('Estimated T = ')
% disp(your estimated T variable)
disp(T1)
disp(T2)
```

First, we find camera frame coordinate correspondences x_1 and x_2 of the pixel coordinates by multiplying them with the inverse of the K matrix. Then we select 8 points from world coordinates given. We calculate *a* vector with multiplication of x_1 and x_2 as following structure:

$$a = [x_1x_2 \quad x_1y_2 \quad x_1z_2 \quad y_1x_2 \quad y_1y_2 \quad y_1z_2 \quad z_1x_2 \quad z_1y_2 \quad z_1z_2]^T$$

We calculate vectors $a_1, a_2, a_3 \ldots a_8$ for all points and stack them to create matrix X as following structure:

$$X = [a_1^T;\ a_2^T;\ \ldots\ a_8^T]$$

We can obtain the stacked version of the Essential Matrix from matrix X since we know that $XE^s = 0$ where E^s is the stacked version of E. In order to find E^s, we can apply SDV decomposition on matrix X. We know that SVD decomposition creates [U,S,V] matrices. Last column of V matrix gives E^s which is written in form

$$E^s = [e_1, e_4, e_7, e_2, e_5, e_8, e_3, e_6, e_9]^T$$

In order to cure the Essential Matrix, we can simply apply SVD decomposition on estimated E^s and replace the singular values by [1,1,0] and check determinant of resulting U and V matrices. We replace the first and second element of the diagonal of the S matrix by 1 and last element by 0. We compute determinant of U and V and both of them should be equal to 1. After these steps, we re-compute E matrix by multiplying new S matrix that diagonals were changed with U and V matrices which have determinant equals to 1.

After creating the Essential Matrix, we find epipoles (e_1,e_2) and epipolar lines. In order to find epipoles, we find nullspace of the Essential Matrix from left and right. To find nullspace from the left, we should look for the nullspace of the transpose of the E. Equation for finding both epipolar lines are as follows:

$$l_1 \sim E^T \mathbf{x}_2 \qquad\qquad l_2 \sim E\mathbf{x}_1$$

After finding epipolar lines, we can verify that epipolar point e_i we found is on the epipolar line l_i by multiplying the transpose of the line with the epipolar point. This operation is like putting the point into the line equation. If the result is 0, or a value really close to 0, we can say that the epipolar point is on the epipolar line.

Last step is to recover the rotation and the translation of the camera. Two example poses are shown below.

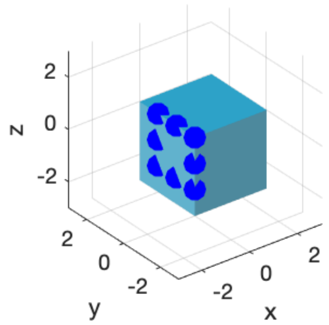
$$\begin{aligned}(\widehat{T}_1, R_1) &= \ (UR_Z(+\tfrac{\pi}{2})\Sigma U^T, UR_Z^T(+\tfrac{\pi}{2})V^T) \\ (\widehat{T}_2, R_2) &= \ (UR_Z(-\tfrac{\pi}{2})\Sigma U^T, UR_Z^T(-\tfrac{\pi}{2})V^T)\end{aligned}$$

We already have U and V matrices of Essential matrix. We can compute Rz values by calculating necessary cos and sin values. Later, we take necessary element from skew-symmetric matrix T to find rotation. After doing these operation, we have estimations of rotation and translation matrices and also estimation of Essential Matrix from previous operations.

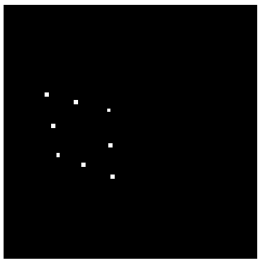
First Plane

World Points Used:

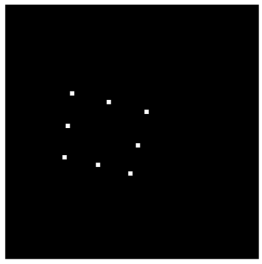
```
P_W=[0   2    0    1;
     0    1    0    1;
     0    0    0    1;
     0    2   -1    1;
     0    0   -1    1;
     0    2   -2    1;
     0    1   -2    1;
     0    0   -2    1;];
```



View 1



View 2



True E =

$$\begin{matrix} 0 & -1.0000 & 0 \\ -0.3615 & 0 & -3.1415 \\ 0 & 3.0000 & 0 \end{matrix}$$

Estimated E =

$$\begin{matrix} -0.8688 & 0.4734 & 0.0182 \\ -0.4743 & -0.8789 & -0.0393 \\ 0.1423 & -0.0400 & -0.0013 \end{matrix}$$

True R =

$$\begin{matrix} 0.9063 & 0 & -0.4226 \\ 0 & 1.0000 & 0 \\ 0.4226 & 0 & 0.9063 \end{matrix}$$

Estimated R =

$$R_1 \quad \begin{matrix} -0.9764 & 0.1389 & 0.1657 \\ -0.1288 & -0.9892 & 0.0699 \\ 0.1736 & 0.0469 & 0.9837 \end{matrix}$$

$$R_2 \quad \begin{matrix} 0.9841 & -0.1290 & 0.1223 \\ 0.1306 & 0.9914 & -0.0046 \\ -0.1207 & 0.0205 & 0.9925 \end{matrix}$$

True T =

$$\begin{matrix} 3 \\ 0 \\ 1 \end{matrix}$$

Estimated T =

$$T_1 \quad \begin{matrix} 0.1441 \\ 0.0327 \\ 0.9890 \end{matrix}$$

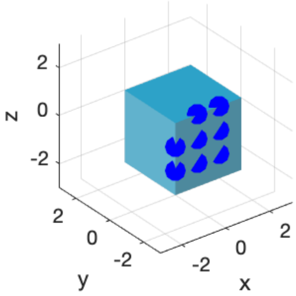
$$T_2 \quad \begin{matrix} -0.1441 \\ -0.0327 \\ -0.9890 \end{matrix}$$

We can see that R_2 value estimation is little bit close to true R value, but estimation of Essential matrix is not very similar to real R matrix. Also T matrix similarity is not well.

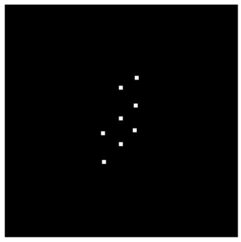
Aynur Süne – 21041

Second Plane

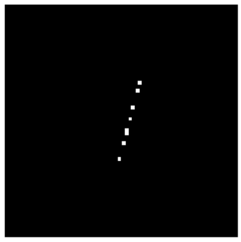
```
P_W=[0    0    -1   1;
     0    0    -2   1;
     1    0    0    1;
     2    0    0    1;
     1    0    -1   1;
     2    0    -1   1;
     1    0    -2   1;
     2    0    -2   1;];
```



View 1



View 2



```
True E =
          0   -1.0000         0
    -0.3615         0   -3.1415
          0    3.0000         0
```

```
Estimated E =
     0.5757   -0.1199   -0.8088
    -0.8156   -0.1113   -0.5641
     0.0532    0.0072    0.0362
```

```
True R =
     0.9063         0   -0.4226
          0    1.0000         0
     0.4226         0    0.9063
```

Estimated R =

R_1

```
    -0.2280   -0.1659    0.9594
     0.2378    0.9460    0.2201
    -0.9442    0.2784   -0.1763
```

R_2

```
     0.2290    0.1656   -0.9592
    -0.3578   -0.9021   -0.2411
    -0.9053    0.3984   -0.1474
```

```
True T =
     3
     0
     1
```

Estimated T =

T_1

```
     0.0005
    -0.0647
    -0.9979
```

T_2

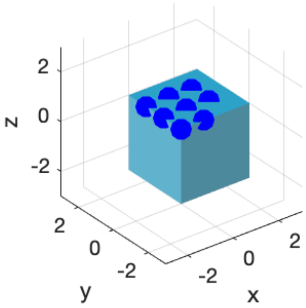
```
    -0.0005
     0.0647
     0.9979
```

For this plane, estimation of E, R and T matrices are all very different from their real values. Estimations are not good. As in first plane, second and third elements of T_2 may be counted as similar to real values, but first element is not close.

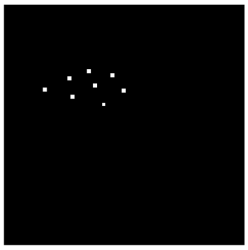
Aynur Süne – 21041

Third Plane

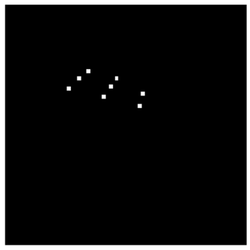
```
P_W=[0    2    0    1;
     0    1    0    1;
     0    0    0    1;
     1    0    0    1;
     1    1    0    1;
     2    1    0    1;
     1    2    0    1;
     2    2    0    1];
```



View 1



View 2



True E =

$$\begin{bmatrix} 0 & -1.0000 & 0 \\ -0.3615 & 0 & -3.1415 \\ 0 & 3.0000 & 0 \end{bmatrix}$$

Estimated E =

$$\begin{bmatrix} -0.9611 & 0.0810 & 0.2374 \\ -0.0596 & -0.9786 & -0.0038 \\ -0.1265 & -0.1885 & 0.0275 \end{bmatrix}$$

True R =

$$\begin{bmatrix} 0.9063 & 0 & -0.4226 \\ 0 & 1.0000 & 0 \\ 0.4226 & 0 & 0.9063 \end{bmatrix}$$

Estimated R =

$$R_1 \quad \begin{bmatrix} 0.8802 & 0.4650 & 0.0949 \\ -0.4586 & 0.8848 & -0.0822 \\ -0.1221 & 0.0288 & 0.9921 \end{bmatrix}$$

$$R_2 \quad \begin{bmatrix} -0.8500 & -0.4186 & -0.3200 \\ 0.5100 & -0.8059 & -0.3006 \\ -0.1320 & -0.4187 & 0.8985 \end{bmatrix}$$

True T =

$$\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$$

Estimated T =

$$T_1 \quad \begin{bmatrix} -0.1159 \\ -0.1971 \\ 0.9735 \end{bmatrix}$$

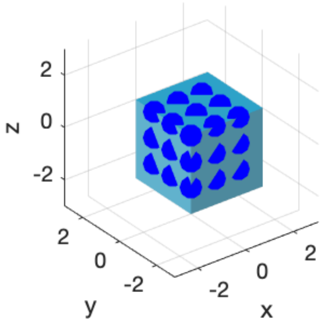
$$T_2 \quad \begin{bmatrix} 0.1159 \\ 0.1971 \\ -0.9735 \end{bmatrix}$$

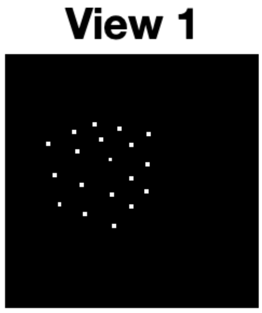
We can consider some of elements of R_1 similar to real values, but all other vaşues are very different. Essential matrix estimation and translation matrices are not close to real values.

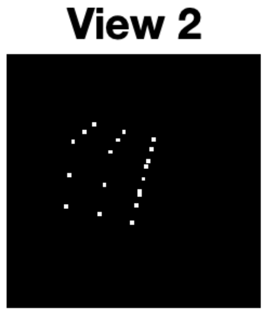
When we choose all of the 8 points we need to estimate the Essential Matrix from the same plane, it is considered as degenerate case which happens when projection matrix (intrinsic+extrinsic parameters) is not unique. Points chosen from same plane cause these problems.

All Points

```
P_W=[0   2    0    1;
     0   1    0    1;
     0   0    0    1;
     0   2   -1    1;
     0   1   -1    1;
     0   0   -1    1;
     0   2   -2    1;
     0   1   -2    1;
     0   0   -2    1;
     1   0    0    1;
     2   0    0    1;
     1   0   -1    1;
     2   0   -1    1;
     1   0   -2    1;
     2   0   -2    1;
     1   1    0    1;
     2   1    0    1;
     1   2    0    1;
     2   2    0    1];
```







Note that when all points are taken into account, determinant of matrix V is calculated as 0, violates Essential Matrix rules.

```
True E =
         0   -1.0000         0
   -0.3615         0   -3.1415
         0    3.0000         0
```

```
Estimated E =
   -0.0773    0.3023    0.0702
    0.0731    0.2310   -0.9702
    0.2281   -0.8986   -0.1954
```

```
True R =
    0.9063         0   -0.4226
         0    1.0000         0
    0.4226         0    0.9063
```

Estimated R =

R_1
```
   -0.9800   -0.0093   -0.1990
   -0.0104    0.9999    0.0048
   -0.1989   -0.0068    0.9800
```

R_2
```
   -0.9001   -0.0037    0.4356
    0.0023   -1.0000   -0.0038
   -0.4356    0.0024   -0.9001
```

```
True T =
     3
     0
     1
```

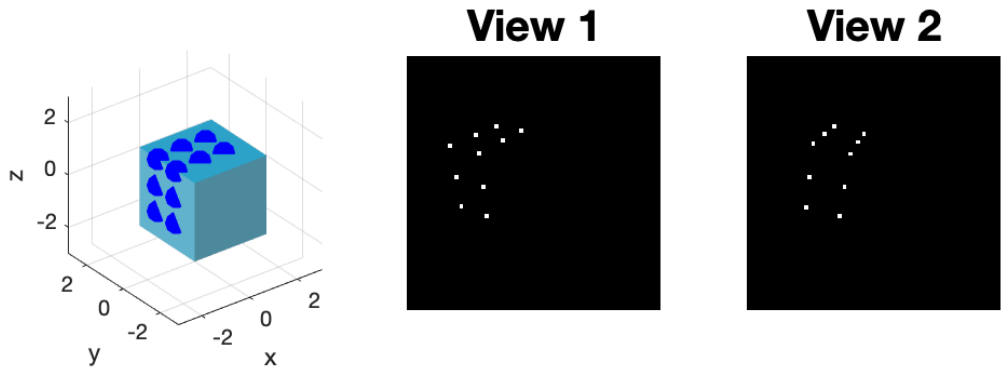
Estimated T =

T_1
```
   -0.9475
   -0.0041
   -0.3198
```

T_2
```
    0.9475
    0.0041
    0.3198
```

When we consider all points, Essential Matrix estimation is again not good (since it violates the rules), some of the R_1 values are close but not significantly well, T matrix is not similar either.

In addition, I considered points only from left and upper plane.




```
True E =
         0   -1.0000         0
   -0.3615         0   -3.1415
         0    3.0000         0

Estimated E =
   -0.0345    0.0135   -0.0618
    0.3453    0.2879    0.8910
    0.3644   -0.9180    0.1530
```

```
True R =
    0.9063         0   -0.4226
         0    1.0000         0
    0.4226         0    0.9063

Estimated R =
```

R_1

```
    0.8899    0.0140   -0.4560
   -0.0070    0.9998    0.0172
    0.4562   -0.0121    0.8898
```

R_2

```
    0.9112    0.1393   -0.3878
    0.1213   -0.9901   -0.0707
   -0.3938    0.0174   -0.9190
```

```
True T =
     3
     0
     1
```

```
Estimated T =
   -0.9974
   -0.0633
   -0.0345

    0.9974
    0.0633
    0.0345
```

Most of the R_1 values seems similar with the real rotation matrix however, translation and essential matrices are again not similar.