Acing UTME Maths

A Comprehensive Guide with Past Questions and Solutions

By

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Dedication and Acknowledgements

This work is dedicated to our families, whose unwavering support has been our greatest strength throughout this journey. Their constant encouragement and belief in our abilities have fueled our passion and perseverance in creating this resource.

We would also like to express our sincere gratitude to the following individuals and institutions for their invaluable contributions:

- Our mentors and teachers, who instilled in us a love for mathematics and equipped us with the knowledge and skills needed to succeed.
- The examiners and administrators of the UTME, whose dedication to educational standards ensures a fair and effective assessment process.
- Our colleagues and friends, who provided feedback and support throughout the development of this book.
- The wider academic community, whose research and publications have laid the foundation for our understanding of mathematics.

We are truly grateful for the collective effort that has made this book possible. We hope that it will be a valuable resource for students preparing for the UTME and beyond.

A Note on Preparation

This book was meticulously prepared using the LaTeX document processing system, a powerful tool for typesetting high-quality scientific and mathematical texts. The diagrams herein were crafted using the TikZ package. We extend our thanks to the developers of the LaTeX system and the numerous packages that made this work possible.

About the Authors

Ayodeji Adesegun: He is a teenager. In his spare time, he enjoys solving mathematical problems from various contests not excluding the Olympiads. You can find more about him here.

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Contents

Preface

Welcome to “Acing UTME Maths”, a comprehensive guide designed to help you conquer the upcoming UTME Maths exam. This book provides you with a wealth of past questions, detailed solutions, and insightful strategies to enhance your understanding and preparation.

This book is organized into chapters that follow the official UTME Maths syllabus, covering all key topics and subtopics. Each chapter includes a variety of past questions carefully selected to reflect the types and difficulty levels encountered in the actual exam.

In addition to past questions, we have provided detailed solutions that explain the reasoning behind each step and highlight common mistakes to avoid. We encourage you to work through these solutions carefully and utilize them as learning tools to improve your problem-solving skills.

Furthermore, we have incorporated valuable strategies throughout the book, offering tips and techniques to maximize your efficiency and performance on the exam. These strategies will help you manage your time effectively, approach different question types confidently, and overcome any challenges you may encounter.

We are confident that “Acing UTME Maths” will be your ultimate companion on your journey to success. By diligently working through the material and utilizing the resources provided, you will gain the knowledge, skills, and confidence needed to achieve your desired score on the UTME Mathematics exam.

Best of luck!

Ayodeji Adesegun and Chimobi Nwafor

Chapter 1

Number and Numeration

1.1 Number Bases

1.1.1 Questions

1. The number 25 when converted from the tens and units base to the binary base (base 2) is one of the following

   A. 10011

   B. 1111011

   C. 111000

   D. 11001

   E. 110011

2. The currency used in a country bought 4 bags of rice at $N56$ per bag and 3 tins of milk at $N4$ per tin. What is the total cost of the items she bought?

   A. $N245$

   B. $N242$

   C. $N236$

   D. $N341$

   E. $N338$

3. Evaluate $212_3 - 121_3 + 222_3$.

   A. 313_3

   B. 1000_3

   C. 1020_3

   D. 1222_3

   E. 1213_3

4. A trader in a country where their currency 'MONT' (M) is in base five bought 103_5 oranges at $M14_5$ each. If he sold the oranges at $M24_5$ each, what would be his gain?

   A. $M103_5$

   B. $M1030_5$

   C. $M102_5$

   D. $M2002_5$

   E. $M3024_5$

5. Find x if $(x_4)^2 = (100100)_2$

   A. 6

   B. 12

   C. 100

   D. 210

   E. 10042

6. Convert 241_5 to base 8.

   A. 71_8

   B. 107_8

   C. 176_8

   D. 241_8

7. In the equation $\frac{11_2}{x_2} = \frac{1000_2}{x_2 + 101_2}$, solve for x.

   A. 101

   B. 11

   C. 110

   D. 111

   E. 10

8. $4243_5 - 12x4_5 = y344_5$. What is the difference between x and y?

   A. 4

   B. 2

   C. 1

   D. 3

E. 5

9. In base ten, the number 101101_2 equals?

A. 15
B. 45
C. 23
D. 12

10. Convert the number 39 to base 2.

A. 100111
B. 111001
C. 110111
D. 111111
E. 100101

11. Find n if $34_n = 10110_2$.

A. 5
B. 6
C. 7
D. 8
E. 9

12. If $2_9 \times (Y3)_9 = 3_5 \times (Y3)_5$. Find the value of Y.

A. 4
B. 3
C. 2
D. 1
E. 5

13. Simplify $213_4 \times 23_4$.

A. 10321_4
B. 12231_4
C. 13221_4
D. 10311_4
E. 13021_4

14. $55_x + 52_x = 77_{10}$, find x.

A. 5
B. 6
C. 7
D. 8
E. 10

15. If $x_{10} = 23_5$, find x.

A. 15
B. 12
C. 14
D. 13
E. 16

16. Find the sum of 25_6, 52_6, 43_6 in base 8.

A. 411
B. 141
C. 114
D. 417

17. $2A3_3 = 77_8$, find A.

A. 1
B. 2
C. 0
D. 4

18. Evaluate $(202_3)^2$ - $(112_3)^2$

A. 21112
B. 21121
C. 21011
D. 21120

19. If 321_4 is divided by 23_4 and leaves a remainder r, what is the value of r ?

A. 4
B. 2
C. 3
D. 0
E. 1

20. Convert 521_{10} to a number in base 5

A. 1404_5
B. 4041_5
C. 4140_5
D. 4014_5
E. 4104_5

21. If $6R7_8 = 511_9$, find R.

A. 6
B. 5
C. 3
D. 2
E. 8

22. Find the value of x if $121_x + 112_x = 30_{10}$.

A. 5

B. 7

C. $-\frac{9}{2}$

D. 3

E. 4

23. Evaluate $(1011_2)^2$ - 1012_2.

A. 110000_2

B. 110000_2

C. 101011_2

D. 110110_2

24. Add 1101_2, 11011_2 and 111_2.

A. 110110_2

B. 101011_2

C. 111011_2

D. 101010_2

E. 110011_2

25. Find the value of m if $13_m + 24_m = 41_m$

A. 8

B. 5

C. 4

D. 6

E. 3

26. If $125_x = 20_10$, find x.

A. 2

B. 3

C. 4

D. 6

E. 5

27. If $(K2)_6 \times 3_6 = 3_5(K4)_5$, what is the value of k?

A. 2

B. 1

C. 3

D. 4

E. 5

28. Find P, if 451_6 - $P_7 = 305_6$

A. 116_7

B. 62_7

C. 611_7

D. 142_7

29. The sum of four numbers is 1214_5. What is the average expressed in base 5?

A. 141

B. 411

C. 417

D. 114

E. 471

30. $(1P03)_4 = 115_{10}$, find P.

A. 2

B. 0

C. 1

D. 4

E. 3

31. $(P344)_6$ - $(23P2)_6 = (2PP2)_6$, find the digit P.

A. 1

B. 2

C. 3

D. 4

E. 5

32. 4243_5 - $(12X4)_5 = Y344$. What is the difference between X and Y?

A. 1

B. 2

C. 3

D. 4

E. 5

33. A.

B.

C.

D.

34. A.

B.

C.

D.

35. A.

B.

C.

D.

36. A.

B.
C.
D.

37. A.
B.
C.
D.

38. A.
B.
C.
D.

39. A.
B.
C.
D.

40. A.
B.
C.
D.

41. A.
B.
C.
D.

42. A.
B.
C.
D.

43. A.
B.
C.
D.

44. A.
B.
C.
D.

45. A.
B.
C.
D.

46. A.
B.
C.
D.

47. A.
B.
C.
D.

48. A.
B.
C.
D.

49. A.
B.
C.
D.

50. A.
B.
C.
D.

51. A.
B.
C.
D.

52. A.
B.
C.
D.

53. A.
B.
C.
D.

54. A.
B.
C.
D.

55. A.
B.
C.
D.

56. A.
B.

C.
D.

57. A.
B.
C.
D.

58. A.
B.
C.
D.

59. A.
B.
C.
D.

60. A.
B.
C.
D.

61. A.
B.
C.
D.

62. A.
B.
C.
D.

63. A.
B.
C.
D.

64. A.
B.
C.
D.

65. A.
B.
C.
D.

66. A.
B.
C.
D.

67. A.
B.
C.
D.

68. A.
B.
C.
D.

69. A.
B.
C.
D.

70. A.
B.
C.
D.

71. A.
B.
C.
D.

72. A.
B.
C.
D.

73. A.
B.
C.
D.

74. A.
B.
C.
D.

75. A.
B.
C.
D.

76. A.
B.
C.

D.

77. A.
B.
C.
D.

78. A.
B.
C.
D.

79. A.
B.
C.
D.

80. A.
B.
C.
D.

81. A.
B.
C.
D.

82. A.
B.
C.
D.

83. A.
B.
C.
D.

84. A.
B.
C.
D.

85. A.
B.
C.
D.

86. A.
B.
C.
D.

87. A.
B.
C.
D.

88. A.
B.
C.
D.

89. A.
B.
C.
D.

90. A.
B.
C.
D.

91. A.
B.
C.
D.

92. A.
B.
C.
D.

93. A.
B.
C.
D.

94. A.
B.
C.
D.

95. A.
B.
C.
D.

96. A.
B.
C.
D.

97. A.
    B.
    C.
    D.
98. A.
    B.
    C.
    D.
99. A.
    B.
    C.
    D.
100. A.
    B.
    C.
    D.

1.2 Fraction and Decimals

1.2.1 Questions

1. The sum of $3\frac{7}{8}$ and $1\frac{1}{3}$ is greater than the difference between $\frac{3}{8}$ and $1\frac{2}{3}$ by.

   A. $3\frac{2}{3}$ C. $8\frac{1}{8}$ E. $5\frac{1}{4}$
   B. $1\frac{1}{2}$ D. $3\frac{11}{12}$

2. After getting a rise of 15%, a man's new monthly salary is N 345. How much per month did he earn before the increase?

   A. N360 C. N293.25 E. N396.75
   B. N300 D. N330

3. Find correct to 3 significant figures, the value of $\sqrt{41830}$

   A. 647 C. 205 E. 6470
   B. 2050 D. 647

4. 12 men complete a job in 9 days. How many men working at the same rate, would be required to complete the job in 6 days?.

   A. 24 C. 8 E. 18
   B. 9 D. 12

5. Simplify $2\frac{5}{12} - 1\frac{7}{8} \times \frac{6}{5}$.

   A. $\frac{11}{30}$ C. $\frac{1}{6}$ E. $\frac{13}{20}$
   B. $\frac{9}{4}$ D. $\frac{5}{3}$

6. By selling an article for N45.00 a man makes a profit of 8%. For how much should he have sold it in order to make a profit of 32%?

   A. N59.00 C. N180.00 E. N42.00
   B. N55.00 D. N63.00

7. Which of the following fractions is less than one-third?

   A. $\frac{4}{11}$ C. $\frac{15}{44}$ E. $\frac{6}{14}$
   B. $\frac{122}{383}$ D. $\frac{22}{63}$

8. The ratio of the price of loaf of bread to the price of a packet of sugar in 1975 was $a : x$. In 1980, the price of a loaf of bread went up by 25% and that of a packet of sugar by 10%. Their new ratio is now ?

   A. $50a : 44x$ C. $40a : 44x$ E. $44a : 55x$
   B. $44a : 50x$ D. $55a : 44x$

9. Simplify: $1 + \cfrac{2}{3 + \cfrac{4}{5 + \cfrac{6}{7}}}$

   A. $\frac{7}{95}$
   B. $\frac{177}{95}$
   C. $\frac{233}{151}$
   D. $\frac{17}{10}$
   E. $\frac{3}{10}$

10. Evaluate and correct to 4 decimal places 827.51×0.015 .

    A. 124.1265
    B. 8.8415
    C. 12.4127
    D. 12.4120
    E. 124.1265

11. A micrometer is defined as one millionth of a millimeter. A length of of 12,000 micrometer may be represented as

    A. $0.000012m$
    B. $0.12m$
    C. $0.00000012m$
    D. $0.00000000012m$
    E. $0.0000012m$

12. The difference between $4\frac{5}{7}$ and $2\frac{1}{4}$ is greater than the sum of $\frac{1}{14}$ and $1\frac{1}{2}$ by.

    A. $\frac{27}{28}$

B. $\frac{23}{28}$

C. $\frac{50}{56}$

D. $\frac{48}{56}$

E. $\frac{24}{48}$

13. When a dealer sells a bicycle for N81, he makes a profit of 8%. What did he pay for the bicycle.

A. N75

B. N75.52

C. N74.52

D. N87.48

E. N73

14. A man and wife went to by an article costing N400. The woman had 10% of the cost and the man 40% of the remainder. How much did they have altogether?

A. N186

B. N184

C. N200

D. N144

E. N100

15. A sum of money invested at 5% per annum simple interest amount to N285.20 after 3 years. How long will it take the same sum to amount to N434.00 at $7\frac{1}{2}$% per annum simple interest?

A. 10 years

B. 12 years

C. $7\frac{1}{2}$ years

D. 14 years

E. 5 years

16. A construction company is owned by two partners A and B and it is agreed that their profit will be divided in ratio 4 : 5, at the end of the year, B recieved N5, 000 more than A. What is the total profit of the company for the year?

A. N45, 000

B. N30, 000

C. N150, 000

D. N25, 000

E. N30, 000

17. The diameter of metal rod is meased as 23.40cm to 4 significant figures. What isthe maximum error in the measurement?

A. 0.0004cm

B. 0.05cm

C. 0.005cm

D. 0.5cm

E. 0.45cm

18. Simplify: $3 - \dfrac{2}{\frac{4}{5} + \frac{1}{2}}$

A. $1\frac{9}{10}$

B. $1\frac{3}{10}$

C. $1\frac{3}{4}$

D. -1

E. 1

19. Given that $x : y = \frac{1}{3} : \frac{1}{2}$ and $\psi : \theta = \frac{2}{5} : \frac{4}{7}$, find $x : \theta$.

A. 20 : 21

B. 7 : 15

C. 3 : 20

D. 2 : 35

E. 4 : 105

20. If N560 is shared in the ratio 7 : 2 : 1, what is the smallest share?

A. N392

B. N113.40

C. N56.00

D. N87.48

E. N126.41

21. Simplify: $\dfrac{1}{2} + \dfrac{1}{2 + \dfrac{1}{2 - \dfrac{1}{4 + \frac{1}{5}}}}$

A. $\frac{169}{190}$

B. $-\frac{1}{3}$

C. $\frac{13}{15}$

D. $-\frac{3}{4}$

E. $-\frac{14}{27}$

22. $22\frac{1}{2}\%$ of the Nigerian Naira equals $17\frac{1}{10}\%$ of a foreign currency M. What is the conversion rate of M to Naira?

A. $2\frac{11}{57}$ Naria

B. $1\frac{18}{57}$ Naria

C. $\frac{15}{59}$ Naria

D. $\frac{15}{57}$ Naria

E. $38\frac{1}{4}$ Naria

23. Divide the LCM of 48, 64, and 80 by their HCF.

A. 30

B. 48

C. 52

D. 20

E. 60

24. A sum of money was invested at 8% per annum simple interest. If after 4 years the money amounts to N330.00, find the amount originally invested.

A. N150

B. N200

C. N165

D. N180

E. N250

25. P sold his bicycle to Q at a profit of 10%. Q sold to R for N209 at a loss of 5%. How much did the bicycle cost P?

A. N150

B. N205

C. N180

D. N196

E. N200

26. Find the smallest number by which 252 can be multiplied to obtain a perfect square.

A. 2

B. 3

C. 5

D. 7

E. 9

27. Find the reciprocal of: $\dfrac{\frac{2}{3}}{\frac{1}{2}+\frac{1}{3}}$

A. $\frac{4}{5}$

B. $\frac{2}{5}$

C. $\frac{6}{9}$

D. $\frac{5}{4}$

E. $\frac{3}{4}$

28. Three boys shared some oranges, the first recieved $\frac{1}{3}$ of the oranges, the second received $\frac{2}{3}$ of the remainder, if the third boy recieved the remaining 12 oranges. How many oranges did they share?

A. 48

B. 72

C. 54

D. 42

E. 60

29. Udoh deposited N150.00 in the bank. At the end of 5 years, the simple interest on the principal was N55.00. At what rate per annum was the interest paid

A. $7\frac{1}{3}\%$

B. 5%

C. 11%

D. $3\frac{1}{2}\%$

E. $4\frac{2}{5}\%$

30. A number of pencil were shared among Desmond, Florence, and Kevin in ratio 2 : 3 : 5 respectively. If Desmond got 5, how many were shared out?

A. 30

B. 15

C. 25

D. 20

E. 35

31. Find the least length of a rod which can be cut into exactly equal strips, each of 40 cm or 48 cm in length.

A. 240 cm

B. 480 cm

C. 360 cm

D. 120 cm

E. 480 cm

32. A rectangular lawn has an area of 1815 square yards. If its length is 50 metres, find its width in meters. Given that 1 metre equals 1.1 yard.

A. 30.00 m

B. 33.00 m

C. 32.00 m

D. 39.93 m

E. 36.45 m

33. Reduce each number to two significant figures and then evaluate $\dfrac{0.021741 \times 1.2047}{0.023789}$

A. 0.8

B. 1.2

C. 1.1

D. 0.9

E. 0.6

34. A cinema hall contains a certain number of people. If $27\frac{1}{2}\%$ are children, $47\frac{1}{2}\%$ are men and 84 are women, find the number of men in the hall

A. 133

B. 84

C. 63

D. 113

35. A woman buys 270 oranges for N1, 800 and sells at 5 for N40. What is her profit?

A. N 1, 620

B. N 630

C. N 360

D. N 2, 160

36. If a car travels 120km on 45 litres of petrol, how much petrol is needed for a journey of 600km?

A. 720 litres

B. 225 litres

C. 960 litres

D. 160 litres

37. Simplify $1 - \left(\frac{1}{7} \times 3\frac{1}{2}\right) \div \frac{3}{4}$

A. 2

B. 1

C. $\frac{1}{3}$

D. $\frac{2}{3}$

38. Evaluate: $\dfrac{12.02 \times 20.06}{26.04 \times 60.06}$, correct to 3 significant figures

A. 0.154

B. 0.155

C. 0.158

D. 0.157

39. Evaluate: $\dfrac{0.8 \times 0.43 \times 0.031}{0.05 \times 0.72 \times 0.021}$, correct to 3 significant figures

A. 14.1

B. 14.09

C. 14.12

D. 14.11

40. A man bought a car for N500, 000 and was able to sell it for N350, 000, what was his percentage loss?

A. 50%

B. 30%

C. 70%

D. 60%

41. Simplify: $1\frac{2}{3} + 4\frac{1}{4} + 1\frac{5}{12}$

A. $4\frac{1}{3}$

B. $4\frac{2}{3}$

C. $4\frac{12}{17}$

D. $4\frac{3}{17}$

42. A man donates 16% of his monthly net earning to the church. If it amounts to N4, 500, what is his monthly income?

A. N40, 500

B. N52, 000

C. N52, 500

D. N45, 000

43. If a student measured the length of a table to be 2.30 m insted of 2.50 m. What was his percentage error in measuring the length?

    A. 7%

    B. 10%

    C. 9%

    D. 8%

44. A man bought a second-hand photocopy machine for 34, 000. He serviced it at a cost of N2, 000 and then sold it at a profit of 15%. What was the selling price?

    A. 37, 550

    B. 40, 000

    C. 41, 400

    D. 42, 400

45. A student spent $\frac{1}{5}$ of his allowance on books, $\frac{1}{3}$ of the remainder on food and kept the rest for contingencies. What fraction was kept?

    A. $\frac{8}{15}$

    B. $\frac{4}{5}$

    C. $\frac{2}{3}$

    D. $\frac{7}{15}$

46. If $p : q = \frac{2}{3} : \frac{5}{6}$ and $\frac{3}{4} : \frac{1}{2}$, find $p : q : r$

    A. 12 : 15 : 10

    B. 10 : 15 : 24

    C. 9 : 10 : 15

    D. 12 : 15 : 16

47. Simplify: $\dfrac{3\frac{2}{3} \times \frac{5}{6} \times \frac{2}{3}}{\frac{11}{25} \times \frac{3}{4} \times \frac{2}{27}}$

    A. $4\frac{1}{3}$

    B. 30

    C. $5\frac{2}{3}$

    D. 50

48. A man earns N3, 500 per month out of which he spend 15% on his children's education. If he spends additional N1, 950 on food, how much does he have left?

    A. N2, 975

    B. N1, 950

    C. N525

    D. N1025

49. Evaluate $\frac{21}{9}$ to 3 significant figures

    A. 2.30

    B. 2.31

    C. 2.32

    D. 2.33

50. A girl shares a number of apples in the ratio 5 : 3 : 2. If the highest share is 40, find the smallest share.

    A. 74

    B. 38

    C. 36

    D. 16

51. Calculate the time taken for N3, 000 to earn N600 at 8% simple interest.

    A. 3 years

    B. $2\frac{1}{2}$ years

    C. $1\frac{1}{2}$ years

    D. $3\frac{1}{2}$ years

52. Find the tax on an income of N20, 000 if no tax is paid on the first N10, 000 and tax is paid at N50 and in N1, 000 on the next N5, 000 and at N55 and N1000 on the remainder.

    A. N225

    B. N525

    C. N552

    D. N500

53. The time taken to do a piece of work is inversely proportional to the number of men employed. If it takes 30 men to do a piece of work in 6days, how many men are required to do the work in 4 days?

    A. 35

    B. 45

    C. 25

    D. 60

54. Three boys shared oranges. The first received $\frac{1}{3}$ of the oranges and the second received $\frac{2}{3}$ of the remainder. If the third boy received the remaining 12 oranges, how much oranges did they share?

A. 42

B. 60

C. 54

D. 48

55. A farmer planted $5,000$ grains of maize and harvested $5,000$ cobs, each bearing 500 grains. What is the ratio of the number of grains sowed to the number harvested?

A. $1:5,000$

B. $1:25,000$

C. $1:500$

D. $1:250,000$

56. Evaluate: $\dfrac{0.21\times 0.072\times 0.00054}{0.006\times 1.68\times 0.063}$

A. 0.1286

B. 0.01285

C. 0.01286

D. 0.1285

57. A man's initial salary is N540 a month and increases after a period of six months by N36 a month. Find his salary in the eight month of the third year.

A. 828

B. 756

C. 720

D. 684

58. Find correct to 3 decimal places:

$$\left(\frac{1}{0.05}\right)\div\left(\frac{1}{5.005}\right)-(0.05\times 2.05)$$

A. 99.998

B. 9.998

C. 98.999

D. 89.899

59. Express $62\div 3$ as a decimal correct to 3 significant figures.

A. 20.667

B. 20.6

C. 20.7

D. 20.67

60. Factory P produces 20,000 bags of cement per day while factory Q produces 15,000 bags of per day. If P reduces production by 5% and Q increases production by 5% determine the effective loss in the number of bags produced per day by the two factories

A. 750

B. 250

C. 1000

D. 1250

61. If 3 gallons of spirit containing 20% water are added to 5 gallons of another spirit containing 15% water, what percentage of the mixture is water?

A. $2\dfrac{4}{5}$

B. $18\dfrac{7}{8}$

C. $18\dfrac{1}{8}$

D. $16\dfrac{7}{8}$

62. The radius of a circle is given as 5cm subject to an error of 0.1cm. What is the percentage erro in the area of the circle.

A. $\dfrac{1}{4}$

B. $\dfrac{1}{25}$

C. 25

D. 4

63. A man invested a sum of N280.00 partly at 5% and partly at 4%. If the total interest is N12.8 per annum, find the amount invested at 5%.

A. N120

B. N160

C. N200

D. N140

64. Evaulate $\dfrac{3524}{0.05}$ correct to 3 significant figures

A. 70000

B. 70480

C. 705

D. 70500

65. If N225 yields N27 in x years simple interest at the rate of 4% per annum, find x

A. 12

B. 4

C. 27

D. 3

66. A man's initial salary is N540 a month and increases after each period of six month by N36 a month. Find his salary in teh eighth month of the third year.

A. N756

B. N648

C. N720

D. N828

67. $\frac{1}{3} \div \left[\frac{5}{7}\left(\frac{9}{10} - 1 + \frac{3}{4}\right)\right]$

A. $\frac{13}{24}$

B. $\frac{39}{28}$

C. $\frac{28}{39}$

D. $\frac{84}{13}$

68. Two sisters, Taiwo and Kehinde, own a store. The ratio of Taiwo's share to Kehinde's is 11 : 9. Later Kehinde sells $\frac{2}{3}$ of her share to Taiwo of N720. Find the value of the store.

A. 1080

B. 3000

C. 3600

D. 2400

69. A tax payer has allowed $\frac{1}{8}$th of his income tax free, and pays 20% on the remainder. If he pays N490 tax, what is his income?

A. N2450

B. N2800

C. N3920

D. N560

70. A basket contains green, black and blue balls in the ratio 5 : 2 : 1. If there are 10 blue balls, find the correspondin new ratio when 10 green and 10 black balls are removed from the basket

A. 1 : 1 : 1

B. 4 : 2 : 1

C. 5 : 1 : 1

D. 4 : 1 : 1

71. The prime factors of $2,520$ are.

A. $2, 3, 5, 7$

B. $2, 9, 5$

C. $2, 3, 7, 9$

D. $2, 9, 7$

72. A.

B.

C.

D.

73. A.

B.

C.

D.

74. A.

B.

C.

D.

75. A.

B.

C.

D.

76. A.

B.

C.

D.

77. A.

B.

C.

D.

78. A.

B.

C.

D.

79. A.

B.

C.

D.

80. A.

B.

C.

D.

81. A.

B.

C.

D.

82. A.

B.
C.
D.

83. A.
B.
C.
D.

84. A.
B.
C.
D.

85. A.
B.
C.
D.

86. A.
B.
C.
D.

87. A.
B.
C.
D.

88. A.
B.
C.
D.

89. A.
B.
C.
D.

90. A.
B.
C.
D.

91. A.
B.
C.
D.

92. A.
B.
C.
D.

93. A.
B.
C.
D.

94. A.
B.
C.
D.

95. A.
B.
C.
D.

96. A.
B.
C.
D.

Chapter 2

Algebra

2.1 Factorization and Remainder Theorem

2.1.1 Questions

1. If the function f is defined by $f(x+2) = 2x^2 + 7x - 5$, find $f(-1)$

   A. -8

   B. 4

   C. 10

   D. -10

2. Factorize $a^2x - b^2y - b^2x + a^2y$

   A. $(y-x)(a-b)(a+b)$

   B. $(a-b)(x+y)$

   C. $(x-y)(a-b)$

   D. $(x+y)(a-b)(a+b)$

3. If $x-1$ and $x+1$ are both factors of the equation: $x^3 + px^2 + qx + 6 = 0$, evaluate p and q

   A. $-6, -1$

   B. $1, -1$

   C. $6, 1$

   D. $6, -6$

4. If $f(x) = \dfrac{1}{x-1} + \dfrac{x-1}{x^2-1}$, find $f(1-x)$

   A. $\dfrac{1}{x} + \dfrac{1}{x-2}$

   B. $-\dfrac{1}{x} - \dfrac{1}{x-2}$

   C. $x + \dfrac{1}{2x-1}$

   D. $\dfrac{1}{x} + \dfrac{1}{x^2-1}$

   E. $-\dfrac{1}{x} - \dfrac{1}{2x-1}$

5. Multiply $(x+3y+5)$ by $(2x^2+5y+2)$

   A. $2x^3 + 3x^2y + 10xy^2 + 13y + 10x^2 + 2x + 10$

   B. $2x^3 + 2x^2y + 10xy + 10y^2 + 31y + 5x^2 + 2x + 10$

   C. $2x^3 + 6x^2y + 5xy + 10y^2 + 13y + 5x^2 + 2x + 10$

   D. $2x^3 + 6x^2y + 5xy + 15y^2 + 31y + 5x^2 + 2x + 10$

   E. $2x^3 + 3x^2y + 5xy + 10y^2 + 13y + 5x^2 + 2x + 10$

6. If $kx^3 + 10x^2 + lx - 3$ is divisible by $(x-1)$ and if when it is divided by $(x+2)$ the remainder is 27, find the constant k and l.

   A. $k = -7, l = -15$

   B. $k = \dfrac{-21}{5}, l = \dfrac{-61}{5}$

   C. $k = -15, l = -7$

   D. $k = -\dfrac{5}{3}, l = \dfrac{19}{5}$

   E. $k = -\dfrac{5}{3}, l = -\dfrac{19}{3}$

7. Factorize $3x^3 + 4x^2 - 13x + 6$ completely, given that $x-1$ is a factor

   A. $(x-1)(x-3)(x+2)$

   B. $(x-1)(x-3)(3x+2)$

   C. $(x-1)(x-2)(x+3)$

   D. $(x-1)(x+3)(3x+2)$

   E. $(x-1)(x+3)(3x-2)$

8. Multiply $x^2 + x + 1$ by $x^2 - x + 1$

   A. $x^4 + 3x^2 + x + 1$

   B. $x^4 + 4x^2 - 6x + 1$

C. $x^4 + 4x^2 + 1$

D. $x^4 - x^3 - x^2 + x + 1$

E. $x^4 - 6x^2 - 4x + 1$

9. If $x = 1$ is a root of the equation: $x^3 - 2x^2 - 5x + 6$, find the other roots.

A. -3 and 2

B. 1 and 3

C. -2 and 2

D. 3 and -2

E. -3 and 1

10. If $x + 2$ and $x - 1$ are factors of the expression: $lx^3 + 2kx^2 + 24$, find the values of l and k

A. $l = -6, k = -9$

B. $l = -2, k = -1$

C. $l = -2, k = 1$

D. $l = 0, k = 1$

E. $l = 6, k = 0$

11. Factorize completely: $81a^4 - 16b^4$

A. $(3a + 2b)(2a - 3b)(9a^2 + 4b^2)$

B. $(3a - 2b)(2a - 3b)(9a^2 - 4b^2)$

C. $(3a - 2b)(3a - 2b)(9a^2 + 4b^2)$

D. $(3a - 2b)(2a - 3b)(4a^2 - 9b^2)$

E. $(3a - 2b)(3a + 2b)(9a^2 + 4b^2)$

12. The factor which is common to all three bionomial expressions: $4a^2 - 9b^2$, $8a^2 + 27b^3$, $(4a + 6b)^2$

A. $4a - 6b$

B. $4a + 6b$

C. $2a - 3b$

D. $2a + 3b$

E. $3a - 2b$

13. If $x - 2$ and $x + 1$ are factors of the expression: $x^3 + px^2 + qx + 1$

A. -3

B. 0

C. $-\frac{17}{3}$

D. $-\frac{2}{3}$

E. 3

14. The factors of $9 - \left(x^2 - 3x - 1\right)^2$ are

A. $(x - 4)(x - 1)(x - 1)(x + 2)$

B. $(x - 4)(x + 1)(x - 2)(x - 1)$

C. $(x - 2)(x + 2)(x + 1)(x + 4)$

D. $(x - 2)(x + 2)(x + 1)(x - 1)$

E. $(x - 4)(x - 3)(x - 2)(x + 1)$

15. If $f(x - 2) = 4x^2 + x + 7$, find $f(1)$

A. 27

B. 7

C. 17

D. 46

E. 12

16. If $g(y) = \frac{y - 3}{11} + \frac{11}{y^2 - 9}$ what is $g(y + 3)$

A. $\frac{y}{11} + \frac{11}{y(y + 5)}$

B. $\frac{y + 30}{11} + \frac{11}{y(y + 3)}$

C. $\frac{y}{11} + \frac{11}{y(y + 3)}$

D. $\frac{y + 3}{11} + \frac{11}{y(y - 6)}$

17. Factorize completely $3a + 125a^3$

A. $(2a + 5x)(4 + 10ax + 25ax^2)$

B. $(2a + 5x^2)(4 + 25ax)$

C. $a(2 + 5x)(4 - 10x + 25ax^2)$

D. $a(2 + 5x)(4 + 10ax + 25ax^2)$

18. Factorize $x^2 + 2a + ax + 2x$

A. $(x^2 - 1)(x + a)$

B. $(x + 2)(x + a)$

C. $(x + 2a)(x + 3)$

D. $(x + 2a)(x - 1)$

19. The graphical method of solving the equation: $x^3 + 3x^2 + 4x - 28 = 0$ is by drawing the graphs of the curves

A. $y = x^3$ and $y = 3x^2 + x - 28$

B. $y = x^3 + 3x^2 + 4x$ and y

C. $y = x^3 + 3x^2 + 4x - 28$ and the line $y = \frac{28}{x}$

D. $y = x^2 + 3x + 4$ and $y = \frac{28}{x}$

E. $y = x^2 + 3x + 4$ and line $y = 28x$

20. Factorize $(4a + 3)^2 - (3a - 2)^2$

A. $(x + 2a)(x - 1)$

B. $(x+1)(x+2a)$

C. $(x+2)(x+a)$

D. $(x^2-1)(x+a)$

21. If $x^3-12x-16=0$ has $x-2$ as a solution, then the equation has

A. 3 roots all different

B. $x-4$ as a solution also

C. 3 roots all equal

D. 3 roots with two equal and the third different

E. only one root

22. The expression: x^3-4x^2+cx+d is such that $x+1$ is a factor and its values is 1 when x is -2. Find c and d

A. $c=-4$ and $d=9$

B. $c=20$ and $d=9$

C. $c=-20$ and $d=15$

D. $c=-20$ and $d=-15$

E. $c=20$ and $d=-15$

23. What factor is common to all the expressions: $x+1$, $2x^2+x+1$, and x^2-1

A. $x+1$

B. 1

C. No common factor

D. $2x-1$

E. x

24. Factorize completely: $(x^2+x)^2-(2x+2)^2$

A. $(x+y)(x+2)(x-2)$

B. $(x+1)^2(x+2)(x-2)$

C. $(x+y)^2(y-2)^2$

D. $(x+1)^2(x+2)^2$

25. If $f(x)=2x^2+5x+3$, find $f(x+1)$

A. $2x^2-x+10$

B. $2x^2-x$

C. $4x^2+3x+12$

D. $4x^2+3x+2$

26. If one factor of x^3-8^{-1} is $x-2^{-1}$, the other factor is

A. $x^2+2^{-1}x-4^{-1}$

B. $x^2-2^{-1}x-4^{-1}$

C. $x^2-2^{-1}x-4^{-1}$

D. $x^2+4^{-1}x-2^{-1}$

27. Factorize $9(x+y)^2-4(x-y)^2$

A. $(x+y)(5x+y)$

B. $(x+y)^2$

C. $5(x+y)^2$

D. $(x+5y)(5x-y)$

28. Factorize $4a^2-12ab-c^2+9b^2$

A. $(2a+3b-c)(2a+3b+c)$

B. $4a(a-3b)+(3b-c)^2$

C. $(2a-3b-c)(2a-3b+c)$

D. $4a(a-3b)+(3b+c)^2$

29. What are the values of k and l respectively if: $\frac{1}{2}(3y-4x)^2=(8x^2+kxy+ly^2)$

A. $12, \frac{9}{2}$

B. $-12, \frac{9}{2}$

C. $6, 9$

D. $-6, 9$

30. If $f(x-4)=x^2+2x+3$, find $f(2)$

A. 11

B. 6

C. 51

D. 27

31. Factorize completely: $y^3-4xy+xy^3-4y$

A. $y(1-x)(y+2)(y-2)$

B. $y(1+x)(y-2)(y-2)$

C. $(y+xy)(y+2)(y-2)$

D. $(y+xy)(y+2)(y-2)$

32. If $g(x)=x^2+3x+4$, find $g(x+1)-g(x)$

A. $2(x+2)$

B. $(x+2)$

C. $(2x+1)$

D. x^2+4

33. Factorize: m^3-2m^2-m+2

A. $(m+1)(m+1)(m+2)$

B. $(m^2+1)(m-2)$

C. $(m^2+2)(m-1)$

D. $(m-2)(m+1)(m-1)$

34. Which of the following is a factor of $rs+tr-pt-ps$?

A. $(p - s)$

B. $(r - p)$

C. $(s - p)$

D. $(r + p)$

35. If $x + 1$ is a factor of: $x^3 + 3x^2 + kx + 4$, find the value of k

A. -6

B. 6

C. 8

D. -8

36. Factorize: $9p^2 - q^2 + 6qr - 9r^2$

A. $(3p - 3q + r)(3p - q)$

B. $(6p - 3q - 3r)(3p - q - 4r)$

C. $(3p - 3q + r)(3p - q - 3r)$

D. $(3p - 3q + r)(3p - q + 4r)$

37. If a function is defined by $f(x + 1) = 3x^2 - x + 4$, find $f(0)$

A. 6

B. 8

C. 0

D. 2

E. 4

38. If $f(x + 2) = 3x^2 + 4x + 1$, find the value of $f(1)$

A. 32

B. 40

C. 8

D. 32

E. 21

39. Factorize: $6x^2 - 14x - 12$

A. $2(x + 3)(3x - 2)$

B. $2(x - 3)(3x + 2)$

C. $6(x - 2)(x + 1)$

D. $6(x + 2)(x - 1)$

E. $(3x + 4)(2x + 3)$

40. Factorize: $abx^2 + 8y - 4bx - 2axy$

A. $(ax - 4)(bx - 2y)$

B. $(bx - 4)(ax - 2y)$

C. $(ax + b)(x - 8y)$

D. $(x - 2y)(abx - 4)$

E. $(ax - 2y)(by - 4)$

41. Factorize: $1 - (a - b)^2$

A. $(1 + a + b)(1 - a - b)$

B. $(1 - a + b)(1 + a - b)$

C. $(1 + a - b)(1 - a + b)$

D. $(1 - a - b)(1 - a + b)$

E. $(1 - a + b)(1 + a + b)$

42. Which of the following is a factor of: $15 + 7x - 2x^2$

A. $x + 3$

B. $x - 5$

C. $x + 5$

D. $x - 3$

43. Divide the expression: $x^3 + 7x^2 - x - 7$ by $-1 + x^2$

A. $x + 7$

B. $-x^3 + 7x^2 - x - 7$

C. $x - 7$

D. $-x^2 + 7x + 7$

44. Find a positve value of p if the expression: $2x^2 - px + p$ leaves a remainder 6 when divided by $x - p$

A. 1

B. 2

C. 3

D. 4

45. When the expression: $pm^2 + qm + 1$ is divided by $(m - 1)$, it has a remainder of 2 and when divided by $(m + 1)$ the remainder is 4, find p and q respectively

A. $-2, 3$

B. $2, -1$

C. $-1, 2$

D. $3, -2$

46. Factorize: $r^2 - r(2p + q) + 2pq$

A. $(r - q)(r - 2p)$

B. $(r - 2q)(2r - p)$

C. $(2r - p)(r + p)$

D. $(r - p)(r + q)$

47. Divide $2x^3 + 11x^2 + 17x + 6$ by $2x + 1$

A. $x^2 + 5x + 6$

B. $x^2 - 5x + 6$

C. $2x^2 + 5x + 6$

D. $x^2 + 5x + 6$

48. Factorize completely: $x^2 + 2xy + y^2 + 3x + 3y - 18$

A. $(x + y + 6)(x + y - 3)$

B. $(x - y - 6)(x - y + 3)$

C. $(x - y + 6)(x - y - 3)$

D. $(x + y - 6)(x + y + 3)$

49. Divide $4x^3 - 3x + 1$ by $2x - 1$

A. $2x^2 - x + 1$

B. $2x^2 + x - 1$

C. $2x^2 - x - 1$

D. $2x^2 + x + 1$

50. If $(x-1)$, $(x+1)$ and $(x-2)$ are factors of the polynomial: $ax^3 + bx^2 + cx - 1$, find a, b,c respectively.

A. $\frac{-1}{2}, 1, \frac{1}{2}$

B. $\frac{1}{2}, 1, \frac{1}{2}$

C. $\frac{1}{2}, -1, \frac{1}{2}$

D. $\frac{1}{2}, 1, 1\frac{1}{2}$

51. Factorize: $4x^2 - 9y^2 + 20x + 25$

A. $(2x - 3y + 5)(2x - 3y - 5)$

B. $(2x - 3y + 5)(2x + 3y + 5)$

C. $(2x + 3y - 5)(2x + 3y + 5)$

D. $(2x + 3y - 5)(2x - 3y - 5)$

E. $(2x - 3y + 5)(2x - 3y - 5)$

52. Divide $a^{3x} - 26a^{2x} + 156a^x - 216$ by $a^{2x} - 24a^x + 108$

A. $a^x + 2$

B. $a^x - 2$

C. $a^x - 18$

D. $a^x - 6$

53. Find the values of x where the curve: $y = x^3 + 2x^2 - 5x - 6$ crosses the x axis

A. $2, -1$ and -3

B. $2, 1$ and 3

C. $2, 1$ and -3

D. $2, -1$ and 3

54. The polynomial whose roots are $\frac{4}{3}$ and $-\frac{3}{5}$ is

A. $15x^2 - 11x - 12$

B. $12x^2 + 11x - 15$

C. $12x^2 - x - 12$

D. $15x^2 + 11x - 12$

55. If $9x^2 + 6xy + 4y^2$ is a factor of $27x^3 - 8y^3$, find the other factor

A. $2y + 3x$

B. $2y - 3x$

C. $3x + 2y$

D. $3x - 2y$

56. Factorize completely: $\frac{x^3 + 3x^2 - 10x}{2x^2 - 8}$

A. $\frac{x(x - 5)}{2(x - 2)}$

B. $\frac{x(x - 5)}{2(x + 2)}$

C. $\frac{x(x + 5)}{2(x + 2)}$

D. $\frac{x^2 + 5}{2x + 4}$

57. Find the remainder when $3x^3 + 5x^2 + 11x - 4$ is divided by $x + 3$

A. 1

B. 4

C. -4

D. -1

58. Factorize completely: $ac - 2bc + a^2 + 4b^2$

A. $(a - 2b)(c + a - 2b)$

B. $(a - 2b)(c - a + 2b)$

C. $(a - 2b)(c + a + 2b)$

D. $(a - 2b)(c - a - 2b)$

59. Factorize $2y^2 - 15xy + 18x^2$

A. $(2y - 3x)(y + 6x)$

B. $(2y - 3x)(y - 6x)$

C. $(3y + 2x)(y - 6x)$

D. $(2y + 3x)(y - 6x)$

60. Find the value of k if $y-1$ is a factor of: $y^3 + 4y^2 + ky - 6$

A. -6

B. -4

C. 0

D. 1

61. Divide: $6x^2 + 13x + 5$ by $2x + 1$

A. $3x - 5$

B. $3x + 5$

C. $5x - 3$

D. $5x + 3$

62. The polynomial whose roots are -2, -1 and 3 is

A. $x^3 - 7x + 6$

B. $x^3 + 7x - 6$

C. $x^3 - 7x - 6$

D. $x^3 + 7x + 6$

63. Find the value of k if the expression: $kx^3 + x^2 - 5x - 2$ if it leaves a remainder of 2 when divided by $2x + 1$

A. 10

B. -8

C. -10

D. 8

64. Find the roots of: $x^3 - 2x^2 - 5x + 6 = 0$

A. $-1, -2, 3$

B. $-1, 2, -3$

C. $1, 2, -3$

D. $1, -2, 3$

65. Factorize: $2t^2 + t - 15$

A. $(t + 3)(2t - 5)$

B. $(2t + 3)(t - 5)$

C. $(t + 3)(t - 5)$

D. $(2t - 3)(t + 5)$

66. If $2x^2 - kx - 12$ is divisible by $x - 4$, find the value of k

A. 4

B. 5

C. 6

D. 7

67. Solve for x in the equation: $x^3 - x + 5 = 0$

A. $-1, 1, -5$

B. $1, 1, 5$

C. $1, 1, -5$

D. $1, -1, 5$

68. Find the remainder when $x^3 - 2x^2 + 3x - 3$ is divided by $x^2 + 1$

A. $2x - 1$

B. $x + 3$

C. $2x + 1$

D. $3x - 3$

69. Find the remainder when: $2x^3 - 11x^2 + 8x - 1$ is divided by $x + 3$

A. -871

B. -781

C. -187

D. -178

70. Factorize: $(2x+3y)^2 + 2(2x+3y)(2x-3y) + (2x-3y)^2$

A. $16x^2$

B. $18x^2$

C. $12x^2$

D. none of the above

71. Factorize: $45a^3b + 5ab^3 - 30a^2b^2$

A. $5ab(3a - b)^2$

B. $7ab(5a - b)^2$

C. $5ab(5a - b)^2$

D. none of the above

72. Find the factors of $(a - b)^3 + (b - c)^3 + (c - a)^3$

A. $3(a + b)(b + c)(c + a)$

B. $3(a - b)(b - c)(c - a)$

C. $5(a - b)(b - c)(c - a)$

D. $5(a + b)(b - c)(c - a)$

73. Factorize: $a^2 + \dfrac{1}{a^2} + 3 - 2a - \dfrac{2}{a}$

A. $\left(a + \dfrac{1}{a} - 1\right)\left(a - \dfrac{1}{a} + 1\right)$

B. $\left(a + \dfrac{1}{a} + 1\right)\left(a + \dfrac{1}{a} + 1\right)$

C. $\left(a + \dfrac{1}{a} + 1\right)\left(a + \dfrac{1}{a} - 1\right)$

D. $\left(a + \dfrac{1}{a} - 1\right)\left(a + \dfrac{1}{a} - 1\right)$

74. Resolve into factors: $9(3x + 5y)^2 - 12(3x + 5y)(2x + 3y) + 4(2x + 3y)^2$

A. $(5x + 9y)$

B. $(5x-9y)^2$

C. $(7x+9y)^2$

D. None of the above

75. Factorize: $(a-b+c)^2+(b-c+a)^2+2(a-b+c)(b-c+a)$

A. $6a^2$

B. $4a^2$

C. $8a^2$

D. $10a^2$

76. Resolve into factors: $81x^2y^2-108xyz+36z^2$

A. $(6xy+9z)^2$

B. $(9xy-7z)^2$

C. $(9xy+6z)^2$

D. $(6xy-7z)^2$

77. What value should a possess so that $x+1$ may be a factor of the polynomial

$$f(x)=2x^3-ax^2-(2a-3)x+2$$

A. 3

B. 2

C. -2

D. none of the above

78. If $(x-2)$ is a factor of the polynomial x^3-2ax^2+ax-1, find the value of a.

A. $\frac{5}{6}$

B. $\frac{7}{6}$

C. $\frac{11}{6}$

D. none of the above

79. The remainder when: $6p^3-p^2-47p+30$ is divided by $p-3$ is

A. 21

B. 63

C. 18

D. 42

80. Factorize: $k^2-2kp+p^2$

A. $(k+p)^2$

B. $(k-p)^2$

C. k^2-p^2

D. k^2+p^2

81. If $x+1$ is a factor of x^3+3x^2+mx+4, find the value of m.

A. 8

B. -6

C. 6

D. -8

82. Divide the expression: x^3+7x^2-x-7 by x^2-1

A. $x-1$

B. $x+1$

C. $x-7$

D. $x+7$

83. If $x+a$ is a factor of the polynomial $x^3+ax^2-2x+a+4$, then find the value of a

A. $-\frac{4}{3}$

B. $\frac{4}{3}$

C. $\frac{2}{3}$

D. $-\frac{2}{3}$

84. Resolve into factors: $16x^2-72xy+81y^2-12x+27y$

A. $(6x-7y)(6x-7y-5)$

B. $(6x-7y)(6x-7y+5)$

C. $(4x+9y)(4x+9y+3)$

D. $(4x-9y)(4x-9y-3)$

85. Resolve into factors: $16(x-y)^2-9(x+y)^2$

A. $(x-5y)(5x-y)$

B. $(x+7y)(7x+y)$

C. $(x-7y)(7x-y)$

D. $(x+5y)(5x+y)$

86. Factorize the expression: $a^2+\frac{1}{4}+a$

A. $\left(a+\frac{1}{2}\right)\left(a-\frac{1}{2}\right)$

B. $\left(a+\frac{1}{2}\right)^2$

C. $\left(a+\frac{1}{2}\right)^3$

D. $\left(a+\frac{1}{2}\right)\cdot a$

87. Resolve into factors: $(a+b)^2-2(a^2-b^2)+(a-b)^2$

A. $6b^2$

B. $4b^2$

C. $2b^2$

D. $8b^2$

88. Factorize the Expression: $(a+b)^2 - 14c(a+b) + 49c^2$

A. $(a-b-9c)^3$

B. $(a+b+9c)^2$

C. $(a+b-7c)^2$

D. none of the above

89. When a polynomial $f(x)$ is divided by $x-3$ and $x+6$, the respective remainders are 7 and 22. What is the remainder when $f(x)$ is divided by $(x-3)(x+6)$?

A. $\frac{-5}{3}x + 12$

B. $\frac{-7}{3}x + 14$

C. $\frac{-5}{3}x + 16$

D. $\frac{-7}{3}x + 12$

90. When $4x^3 - ax^2 + bx + 4$ is divided by $x-2$ and $x+1$, the respective remainders are 20 and -13. Find the values of a and b.

A. $a = 3, b = 2$

B. $a = 9, b = 8$

C. $a = 7, b = 6$

D. $a = 5, b = 4$

91. When $(x^3 - 2x^2 + px - q)$ is divided by $x^2 - 2x - 3$ the remainder is $(x-6)$. The values of p and q are:

A. $p = -2, q = -6$

B. $p = 2, q = -6$

C. $p = 2, q = 6$

D. $p = -2, q = 6$

92. If $(x-1)$ is a factor of $ax^3 + bx^2 - 36x + 22$ and $2^b = 64^a$. Find a and b.

A. $a = 4, b = 16$

B. $a = 6, b = 24$

C. $a = 2, b = 12$

D. $a = 8, b = 16$

93. If $2apq = (p+q)^2 - (p-q)^2$, then the value of a is

A. 1

B. 2

C. 4

D. 8

94. What is the value of $\frac{(a^2+b^2)(a-b) - (a-b)^3}{a^2b - ab^2}$

A. -1

B. -2

C. 1

D. 2

95. Find the value of m and n in the polynomials $2x^3 + mx^2 + nx - 14$, such that $(x-1)$ and $(x+2)$ are its factors

A. $m = 4, n = 5$

B. $m = 9, n = 3$

C. $m = 6, n = 7$

D. none of the above

96. Divide the polynomial $4y^3 - 3y^2 + 2y - 4$ by $y+2$ and find the quotient and remainder

A. $4y^2 - 11y + 24, -52$

B. $6y^2 - 13y + 36, -64$

C. $4y^2 + 13y - 24, 52,$

D. none of the above

97. Suppose that b and c are constants and

$$(x+2)(x+b) = x^2 + cx + 6$$

what is c?

A. -5

B. -3

C. -1

D. 3

E. 5

98. Simplify the expression: $x^3 + 3x + \frac{3}{x} + \frac{1}{x^3}$

A. $\left(x - \frac{1}{3}\right)^3$

B. $\left(x - \frac{1}{x}\right)^3$

C. $\left(x - \frac{3}{x}\right)^3$

D. $\left(x + \frac{1}{x}\right)^3$

99. If $(x^{3/2} - xy^{1/2} + x^{1/2}y - y^{3/2})$ is divided by $(x^{1/2} - y^{1/2})$, the quotient is

A. $x^{1/2} + y^{1/2}$

B. $x^2 - y^2$

C. $x - y$

D. $x + y$

100. Let $f(x) = a_0x^n + a_1x^{n-1} + a_2x^{n-2} + \cdots + a_{n-1}x + a_n$, where, $a_0, a_1, a_2, \ldots, a_n$ are constants. If $f(x)$ is divided by $ax - b$, then the remainder is

A. $f\left(\frac{b}{a}\right)$

B. $f\left(\frac{-b}{a}\right)$

C. $f\left(\frac{a}{b}\right)$

D. $f\left(\frac{-a}{b}\right)$

2.2 Indices and Standard Form

2.2.1 Questions

1. If $(25)^{x-1} = 64\left(\frac{5}{2}\right)^6$, then x has the value

   A. 7
   B. 4
   C. 32
   D. 5
   E. 64

2. Simplify $\frac{5^x \times 25^{x-1}}{125^{x+1}}$

   A. 5^{x+2}
   B. 5^{2x-1}
   C. 5^{x+1}
   D. 5^3
   E. 5^{-5}

3. Express 37.05×0.0042 in standard form

   A. 15.561×10^2
   B. 1.556×10^1
   C. 1.5561×10^{-4}
   D. 1.5561×10^{-1}
   E. 1.5561×10^2

4. Simplify: $\sqrt[3]{(64r^{-6})^{\frac{1}{2}}}$

   A. $\frac{1}{2r}$
   B. $\frac{2}{r}$
   C. 2
   D. $\frac{1}{2}$

5. What are the values of y that satisfy this equation:

   $$9^y - 4(3^y) + 3 = 0$$

   A. -1 and 0
   B. 1 and 3
   C. 0 and 1
   D. -1 and 1

6. Simplify: $\frac{9^{\frac{1}{3}} \times 27^{-\frac{1}{2}}}{3^{-\frac{1}{6}} \times 3^{-\frac{2}{3}}}$

   A. $\frac{1}{3}$
   B. $\frac{1}{9}$
   C. 3
   D. 1
   E. 9

7. If $\sqrt{3^x} = \sqrt[3]{9}$

   A. $\frac{3}{4}$
   B. $\frac{4}{3}$
   C. $\frac{1}{3}$
   D. $\frac{2}{3}$
   E. $\frac{1}{2}$

8. Find the value of $\left(4^{\frac{1}{2}}\right)^6$

   A. 6
   B. 2
   C. 1
   D. 4
   E. 8

9. Simplify: $\frac{3(2^{n-1}) - 4(2^{n-1})}{2^{n+1} - 2^n}$

   A. -2^{n-1}
   B. 2^{n+1}
   C. -2^1
   D. -2^{-1}

10. Evaluate: $\frac{27^{\frac{1}{3}} - 8^{\frac{2}{3}}}{16^{\frac{2}{4}} \times 2}$

    A. $-\frac{1}{8}$
    B. $\frac{21}{7}$
    C. $\frac{23}{5}$
    D. $-\frac{23}{5}$

E. $-\frac{23}{6}$

11. If $\frac{4^{x+3}}{16^{2x-3}} = 1$, find x

A. 1

B. -1

C. -3

D. 3

E. -3

12. Evaluate without using tables: $(0.008)^{-\frac{1}{3}} \times (0.16)^{-\frac{3}{2}}$

A. $\frac{8}{625}$

B. 8

C. $\frac{625}{8}$

D. $\frac{1}{8}$

13. Simplify: $\frac{3^n - 3^{n-1}}{3^3 \times 3^n - 27 \times 3^{n-1}}$

A. 0

B. $\frac{1}{27}$

C. $3^n - 3^{n-1}$

D. 1

E. $\frac{2}{27}$

14. Evaluate and leave your answer in standard form:

$$\sqrt{\frac{0.0048 \times 0.81 \times 10^{-7}}{0.027 \times 0.04 \times 10^{6}}}$$

A. 6×10^{-14}

B. 6×10^{-7}

C. 6×10^{7}

D. 6×10^{14}

15. If $3^{2y} - 6(3^y) = 27$, find y

A. 3

B. -1

C. 2

D. -3

E. 1

16. If it is given that $5^{x+1} + 5^x = 150$, then the value of x is equal to

A. 2

B. 3

C. $\frac{1}{2}$

D. 1

E. 4

17. Given that $10^{2n+1} = 0.0000001$, find n

A. -7

B. -6

C. $-\frac{3}{4}$

D. 4

E. -4

18. The result of dividing $\left(\frac{x^a}{x^b}\right)^{a-b}$ by $\left(\frac{x^{a+b}}{x^{a-b}}\right)^{\frac{a^2}{b}}$

A.

B.

C.

D.

19. If $\sqrt[3]{81} = 3^x$, find the value of x

A. $\frac{4}{3}$

B. $-\frac{4}{3}$

C. $\frac{3}{4}$

D. $-\frac{3}{4}$

20. Simplify: $\frac{x(x+1)^{-\frac{1}{2}} - (x+1)^{\frac{1}{2}}}{(x+1)^{\frac{1}{2}}}$

A. $\frac{1}{x+1}$

B. $-\frac{1}{x+1}$

C. $\frac{1}{x}$

D. $-\frac{1}{\sqrt{x+1}}$

21. Express in standard form

$$\frac{0.8 \times 0.8 \times 0.8 - 0.5 \times 0.5 \times 0.5}{0.8 \times 0.8 + 0.8 \times 0.5 + 0.5 \times 0.5}$$

A. 8×10^{-1}

B. 4×10^{-1}

C. 3×10^{-1}

D. 1.3×10^{-1}

22. Express in standard form

$$\frac{69842 \times 69842 - 30158 \times 30158}{69842 - 30158}$$

A. 3.0158×10^{-4}

B. 10^{-4}

C. 6.9842×10^{-5}

D. 10^{-5}

E. 10^{5}

23. The value of $\frac{9^2 \times 18^4}{3^{16}}$ is:

A. $\frac{2}{3}$

B. $\frac{4}{9}$

C. $\frac{32}{243}$

D. $\frac{16}{81}$

24. If m and n are whole numbers such that $m^n = 121$ then $(m-1)^{n+1} = ?$

A. 10

B. 10^2

C. 10^3

D. 10^4

25. Simplify: $\frac{a^{\frac{1}{2}} + a^{-\frac{1}{2}}}{1-a} + \frac{1 - a^{\frac{1}{2}}}{1 + \sqrt{a}}$

A. $\frac{a}{a-1}$

B. $\frac{a-1}{2}$

C. $\frac{2}{a-1}$

D. $\frac{2}{1-a}$

26. Simplify: $\left(\frac{1}{64}\right)^0 + (64)^{-\frac{1}{2}} + (-32)^{\frac{4}{5}}$

A. $17\frac{1}{8}$

B. $11\frac{7}{8}$

C. $17\frac{3}{8}$

D. $17\frac{7}{8}$

27. If $\left(\frac{x}{y}\right)^{5a-3} = \left(\frac{y}{x}\right)^{17-3a}$, what is the value of a

A. -7

B. -5

C. 0

D. 3

28. Evaluate: $\frac{(0.064 - 0.008)(0.16 - 0.04)}{(0.16 + 0.08 + 0.04)(0.4 + 0.2)^3}$

A. $\frac{1}{3}$

B. 3

C. $\frac{3}{2}$

D. $\frac{2}{3}$

29. The value of $\left[\left(\sqrt[n]{x^2}\right)^{n/2}\right]^2$

A. $\frac{1}{x^2}$

B. x

C. x^2

D. $x^{\frac{n}{2}}$

30. Solve for x if $3^x - 3^{x-1} = 486$

A. 5

B. 6

C. 7

D. 9

31. If $5\sqrt{5} \times 5^3 \div 5^{-\frac{3}{2}} = 5^{a+2}$, then the value of a is

A. 4

B. 5

C. 6

D. 8

32. If $\left(\sqrt{3}\right)^5 \times 9^2 = 3^n \times 3\sqrt{3}$, then find n

A. 2

B. 3

C. 4

D. 5

33. The value of $\frac{243^{\frac{n}{5}} \times 3^{2n+1}}{9^n \times 3^{n-1}}$

A. 3

B. 6

C. 9

D. 12

34. If $k^a k^b k^c = 1$, then the value of $a^3 + b^3 + c^3$ is:

A. 9

B. $a + b + c$

C. abc

D. $3abc$

35. The value of $\dfrac{81^{3.6} \times 9^{2.7}}{81^{4.2} \times 3}$ is

A. 3

B. 6

C. 9

D. 8.2

36. Simplify $\dfrac{6^{2n+1} \times 9^n \times 4^{2n}}{18^n \times 2^n \times 12^{2n}}$

A. 3^{2n}

B. $3 \times 2^{n+1}$

C. $2n$

D. 6

E. 1

37. Solve the systems of equations: $2^{x+y} = 32$ and $3^{3y-x} = 27$, find (x, y) respectively

A. $(-3, 2)$

B. $(-3, -2)$

C. $(3, 2)$

D. $(2, 2)$

E. $(3, -2)$

38. A.

B.

C.

D.

39. A.

B.

C.

D.

40. A.

B.

C.

D.

41. A.

B.

C.

D.

42. A.

B.

C.

D.

43. A.

B.

C.

D.

44. A.

B.

C.

D.

45. A.

B.

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46. A.

B.

C.

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47. A.

B.

C.

D.

48. A.

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49. A.

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50. A.

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93. A.
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95. A.
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96. A.
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97. A.
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98. A.
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99. A.
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100. A.
B.
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2.3 Surds

2.3.1 Questions

1. A.
   B.
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2. A.
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   C.
   D.
3. A.
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71. A.
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72. A.
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73. A.
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74. A.
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76. A.
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95. A.
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96. A.
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97. A.
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98. A.
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99. A.
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100. A.
B.
C.
D.

2.4 Logarithms

2.4.1 Questions

1. A.
   B.
   C.
   D.
2. A.
   B.
   C.
   D.
3. A.
   B.
   C.
   D.
4. A.
   B.
   C.
   D.
5. A.
   B.
   C.
   D.
6. A.
   B.
   C.
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Chapter 3

Geometry

3.1

3.1.1 Questions

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3.2 Co-ordinate Geometry

3.2.1 Questions

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Chapter 4

Calculus

4.1 Differentiation

4.1.1 Questions

1. The minimum point on the curve $y = x^2 - 6x + 5$ is at?

   A. (1,5)

   B. (2,3)

   C. (3,4)

   D. (-3,4)

   E. (3,-4)

2. At what value of x is the function $y = x^2 + x + 1$ minimum?

   A. -1

   B. $-\frac{1}{2}$

   C. $\frac{1}{2}$

   D. 1

3. At what value of x is the function $y = x^2 - 2x - 3$ minimum?

   A. 1

   B. -1

   C. -4

   D. 4

4. Find the maximum value of $y = -x^2 + 2x + 3$

   A. -4

   B. -1

   C. 1

   D. 4

5. Find the maximum value of $y = 3x^2 - x^3$

   A. 2

   B. 4

   C. 6

   D. 0

6. Find the minimum value of $y = x^3 - 3x + 1$

   A. -1

   B. 1

   C. 2

   D. -2

7. Find the value of x for which the function $f(x) = 2x^3 - x^2 - 4x + 4$ has a maximum value.

   A. $\frac{2}{3}$

   B. 1

   C. -1

   D. $-\frac{2}{3}$

8. Find the value of x for which the function $f(x) = 3x^3 - 9x^2$ is minimum

   A. 2

   B. 0

   C. 5

   D. 3

9. Find the maximum value of the function $f(x) = 2+x-x^2$

   A. $\frac{9}{4}$

   B. $\frac{7}{4}$

C. $\frac{3}{2}$

D. $\frac{1}{2}$

10. Find the maximum value of y in the equation: $y = 1 - 2x - 3x^2$

A. $\frac{4}{3}$

B. $\frac{5}{4}$

C. $\frac{3}{4}$

D. $\frac{5}{3}$

11. The minimum value of y in the equation: $y = x^2 - 6x + 8$ is

A. 8

B. 3

C. 0

D. -1

12. Obtain the maximum value of the function: $f(x) = x^3 - 12x + 11$

A. -5

B. -2

C. 2

D. 27

13. Find the value of h if the maximum value of $y = 1 + hx - 3x^2$ is 13.

A. 10

B. 11

C. 12

D. 13

14. A trader realizes $10x - x^2$ naira profit from the sale of x bags of corn. How many bags will give him the maximum profit?

A. 4

B. 5

C. 6

D. 7

15. Find the value of x for which the function $y = x^3 - x$ has a minimum value.

A. $\frac{\sqrt{3}}{3}$

B. $-\frac{\sqrt{3}}{3}$

C. $\sqrt{3}$

D. $-\sqrt{3}$

16. If $f(x) = x^2 - 2x - 3$, find the least value of $f(x)$ and the corresponding value of x.

A. $f(x) = -3, x = 1$

B. $f(x) = -3, x = 3$

C. $f(x) = -4, x = 1$

D. $f(x) = 1, x = -4$

17. If $y = 3\cos\left(\frac{x}{3}\right)$, find $\frac{dy}{dx}$ when $x = \frac{3\pi}{2}$.

A. -1

B. 1

C. 2

D. 3

18. What is the rate of change of the volume v of a hemisphere with respect to its radius r when $r = 2$? ($V = \frac{2}{3}\pi r^3$)

A. 2π

B. 4π

C. 8π

D. 16π

19. If $y = (1 - 2x)^3$, find the value of $\frac{dy}{dx}$ at $x = -1$.

A. 22

B. 57

C. -6

D. -54

20. Find the derivative of $y = \sin(2x^3 + 3x - 4)$.

A. $\cos(2x^3 + 3x - 4)$

B. $-\cos(2x^3 + 3x - 4)$

C. $-(6x^2 + 3)\cos(2x^3 + 3x - 4)$

D. $(6x^2 + 3)\cos(2x^3 + 3x - 4)$

21. The radius r of a circular disc is increasing at the rate of 0.5 cm/sec. At what rate is the area of the disc increasing when its radius is 6 cm?

A. 3π cm^2/sec

B. 18π cm^2/sec

C. 6π cm^2/sec

D. 36π cm^2/sec

22. Find $\frac{dy}{dx}$, if $y = \cos x$.

A. $\sin x$

B. $-\sin x$

C. $\tan x$

D. $-\tan x$

23. Differentiate: $(\cos\theta - \sin\theta)^2$ with respect to θ.

A. $1 - 2\cos 2\theta$

B. $-2\sin 2\theta$

C. $-2\cos 2\theta$

D. $1 - 2\sin 2\theta$

24. Differentiate: $\left(x^2 - \frac{1}{x}\right)^2$ with respect to x.

A. $4x^3 - 2 + \frac{2}{x^3}$

B. $4x^3 - 2 - \frac{2}{x^3}$

C. $4x^3 - 4x - \frac{2}{x}$

D. $4x^3 - 3x + \frac{2}{x}$

25. Find the point (x, y) on the Euclidean plane where the curve $y = 2x^2 - 2x + 3$ has 2 as the gradient.

A. $(1, 3)$

B. $(2, 2)$

C. $(3, 4)$

D. $(3, 2)$

26. For what value of x is the tangent to the curve $y = x^2 - 4x + 3$ parallel to the x-axis?

A. 0

B. 1

C. 2

D. 3

27. Differentiate: $\frac{6x^3 - 5x^2 + 1}{3x^2}$ with respect to x

A. $2 - \frac{2}{3x^3}$

B. $2 + \frac{1}{6x}$

C. $2x - \frac{5}{3}$

D. $2 - \frac{1}{3x^2}$

28. If $y = (1 + x)^2$, find $\frac{dy}{dx}$.

A. $x + 1$

B. $2x - 1$

C. $2 + 2x$

D. $1 + 2x$

29. Differentiate: $3x^3 + 2x^2 + 3x + 1$ with respect to x

A. $9x^2 + 4x + 3$

B. $9x^2 + 4x - 3$

C. $9x^2 - 4x - 3$

D. $9x^2 - 4x + 3$

30. Differentiate: $\frac{2}{3}x^3 - \frac{4}{x}$

A. $2x^2 + \frac{4}{x^2}$

B. $2x^2 - \frac{4}{x}$

C. $3x^2 - \frac{4}{x}$

D. $3x^2 + \frac{4}{x^2}$

31. Find the derivative of $\frac{\sin x}{\cos x}$

A. $\tan x \cos x$

B. $\csc x \sec x$

C. $\sec^2 x$

D. $\cot^2 x$

32. If $y = x^2 - 3x + 4$, find $\frac{dy}{dx}$ at $x = 5$.

A. 9

B. 7

C. 5

D. 3

33. If $y = 2x\cos 2x - \sin 2x$, find $\frac{dy}{dx}$ when $x = \frac{\pi}{2}$.

A. 0

B. $-\pi$

C. π

D. -2π

34. If $y = 3\cos 4x$, find $\frac{dy}{dx}$

A. $-24\sin 4x$

B. $12\sin 4x$

C. $-12\sin 4x$

D. $6\sin 8x$

35. Find the derivative of $(2 + 3x)(1 - x)$ with respect to x.

A. $6x - 1$

B. $1 - 6x$

C. -3

D. 6

36. Find $\frac{dy}{dx}$, if $y = -3x^3 + 2x^2 - 3x + 1$.

A. $-9x^2 + 4x + 3$

B. $-9x^2 + 4x - 3$

C. $-9x^2 - 4x + 3$

D. $-9x^2 - 4x - 3$

37. If $y = 2x^3 + 6x^2 + 6x + 1$, find $\frac{dy}{dx}$.

A. $6x^2 + 12x + 1$

B. $6x^2 - 12x + 1$

C. $6x^2 + 12x + 6$

D. $6x^2 + 6x + 6$

38. Find the derivative of $y = \left(\frac{1}{3}x + 6\right)^2$.

A. $2\left(\frac{1}{3}x + 6\right)$

B. $\frac{2}{3}\left(\frac{1}{3}x + 6\right)$

C. $\frac{1}{3}\left(\frac{1}{3}x + 6\right)^2$

D. $\frac{2}{3}\left(\frac{1}{3}x + 6\right)^2$

39. If $y = x^2 - 3x + 4$, find $\frac{dy}{dx}$ at $x = 2$.

A. -1

B. 1

C. 2

D. -2

40. If $y = x^2 + \sqrt{x}$, find $\frac{dy}{dx}$.

A. $2x - \frac{1}{2}x^{\frac{1}{2}}$

B. $2x - \frac{1}{2}x^{-\frac{1}{2}}$

C. $2x + x^{-\frac{1}{2}}$

D. $2x + \frac{1}{2}x^{-\frac{1}{2}}$

41. Find $\frac{dy}{dx}$, if $y = \frac{2}{3}x^3 - \frac{4}{x}$

A. $3x^2 - \frac{4}{x}$

B. $3x^2 + \frac{4}{x^2}$

C. $2x^2 - \frac{4}{x}$

D. $2x^2 + \frac{4}{x^2}$

42. If $y = \cos 3x$, find $\frac{dy}{dx}$.

A. $\frac{1}{3}\sin 3x$

B. $3\sin 3x$

C. $-\frac{1}{3}\sin 3x$

D. $-3\sin 3x$

43. Find $\frac{dy}{dx}$, if $y = \cos x$

A. $\sin x$

B. $-\sin x$

C. $\tan x$

D. $-\tan x$

44. Find the slope of the curve: $y = 2x^2 + 5x - 3$ at $(1, 4)$.

A. 4

B. 6

C. 7

D. 9

45. Find the derivative of $y = \sin^2(5x)$ with respect to x.

A. $5\sin 5x \cos 5x$

B. $2\sin 5x \cos 5x$

C. $15\sin 5x \cos 5x$

D. $10\sin 5x \cos 5x$

46. The slope of the tangent to the curve: $y = 3x^2 - 2x + 5$ at the point $(1, 6)$ is

A. 1

B. 4

C. 5

D. 6

47. If the gradient of the curve $y = 2kx^2 + x + 1$ at $x = 1$ is 9, find the value of k

A. 2

B. -2

C. 4

D. -4

48. The distance travelled by a particle from a fixed point is given as $s = \left(t^3 - t^2 - t + 5\right)$ cm. Find the minimum distance that the particle can cover from the fixed point (for $t \geq 0$).

A. 2.3 cm

B. 4.0 cm

C. 5.2 cm

D. 6.0 cm

49. Differentiate $(2x+5)^2(x-4)$ with respect to x.

A. $4(2x+5)(x-4)$

B. $4(2x+5)(4x-3)$

C. $(2x+5)(6x-11)$

D. $(2x+5)(2x-13)$

50. Find the rate of change of the volume v of a sphere with respect to its radius r when $r=1$. ($V=\frac{4}{3}\pi r^3$)

A. 24π

B. 12π

C. 4π

D. 8π

51. If $y=2x\cos 2x-\sin 2x$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x=\dfrac{\pi}{4}$

A. $\dfrac{\pi}{4}$

B. $\dfrac{\pi}{2}$

C. $-\pi$

D. $-\dfrac{\pi}{2}$

52. Differentiate $\dfrac{x}{\cos x}$ with respect to x

A. $1+\sec^2 x$

B. $1+x\tan x\sec x$

C. $\cos x+x\tan x$

D. $\sec x+x\tan x\sec x$

53. If $y=243(4x+5)^{-2}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$ when $x=1$

A. $\dfrac{-8}{9}$

B. $\dfrac{9}{8}$

C. $\dfrac{-8}{3}$

D. $\dfrac{3}{8}$

54. What is the derivative of $t^2\sin(3t-5)$ with respect to the variable t?

A. $2t\sin(3t-5)+3t^2\cos(3t-5)$

B. $2t\sin(3t-5)-3t^2\cos(3t-5)$

C. $6t\cos(3t-5)$

D. $2t\sin(3t-5)+t^2\cos(3t)$

55. A circle with radius 5 cm has its radius increasing at the rate of 0.2 cm/s. What will be the corresponding increase in area?

A. π

B. 2π

C. 4π

D. 5π

56. Find the dimensions of the rectangle of greatest area which has a fixed perimeter p.

A. Square of sides $\frac{p}{2}$

B. Square of sides p

C. Square of sides $\frac{p}{4}$

D. Square of sides $2p$

57. The gradient of a curve is $2x+7$ and the curve passes through the point $(2,0)$. Find the equation of the curve.

A. $y=x^2+7x-18$

B. $y=x^2+7x+18$

C. $y=x^2+7x-9$

D. $y=x^2+7x+9$

58. Differentiate $y=\sqrt[3]{x^2}(2x-x^2)$

A. $\dfrac{10x^{\frac{5}{3}}}{3}-\dfrac{8x^{\frac{5}{3}}}{3}$

B. $\dfrac{10x^{\frac{2}{3}}}{3}-\dfrac{8x^{\frac{5}{3}}}{3}$

C. $\dfrac{10x^{\frac{5}{3}}}{3}-\dfrac{8x^{\frac{2}{3}}}{3}$

D. $\dfrac{10x^{\frac{2}{3}}}{3}-\dfrac{8x^{\frac{2}{3}}}{3}$

59. The slope of the tangent to the curve: $y=5x^2-3x+5$ at the point $(1,6)$ is

A. 19

B. 7

C. 4

D. 3

60. Find the derivative of the function $y=2x^2(2x-1)$ at the point $x=-1$

A. 18

B. -4

C. 16

D. -6

61. Find the derivative of $y = \ln(4x^3 - 2x)$ (assuming natural logarithm)

    A. $\dfrac{4x^2 - 2}{7x + 6}$

    B. $\dfrac{12x - 2}{4x^2}$

    C. $\dfrac{43x^2 - 2x}{7x}$

    D. $\dfrac{12x^2 - 2}{4x^3 - 2x}$

62. Find the derivative of $y = e^x \sin x$.

    A. $e^x(\sin x + \cos x)$

    B. $e^x(\sin x - \cos x)$

    C. $e^x \cos x$

    D. $xe^x \sin x$

63. Find the second derivative of $y = 8x^3 - 3x^2 + 7x - 1$

    A. $11x^2 + 6x - 7$

    B. $24x^2 - 6x + 7$

    C. $48x - 6$

    D. $32x + 7$

64. For what value of x is the tangent to the curve $y = x^2 + 6x + 8$ parallel to the x-axis?

    A. -3

    B. 3

    C. 4

    D. -4

65. Find the second derivative of $y = x\sin(x)$

    A. $2\cos(x) - x\sin(x)$

    B. $\sin(x) - x\cos(x)$

    C. $\sin(x) + x\cos(x)$

    D. $x\sin(x) - 2\cos(x)$

66. Differentiate $\dfrac{2x}{\sin(x)}$ with respect to x

    A. $2\cot x \sec x(1 + \tan x)$

    B. $2\csc x - x\cot x$

    C. $2x\csc x + \tan x$

    D. $2\csc x(1 - x\cot x)$

67. Find the point (x, y) on the Euclidean plane where the curve $y = 2x^2 - 2x + 3$ has 2 as gradient

    A. $(1, 3)$

    B. $(2, 7)$

    C. $(3, 15)$

    D. $(0, 3)$

68. Find the equation of the tangent at the point $(2, 0)$ to the curve $y = x^2 - 2x$

    A. $y = 2x - 4$

    B. $y = 2x + 4$

    C. $y = 2x + 2$

    D. $y = 2x - 2$

69. Differentiate $y = 20x^{-4} + 9$.

    A. $-80x^{-5}$

    B. $-80x^5$

    C. $80x^{-5}$

    D. $80x^5$

70. Differentiate $y = x^2 \ln x$.

    A. $x(2\ln x + 1)$

    B. $2x\ln x$

    C. $x + \ln x$

    D. $2x + 1/x$

71. If $f(x) = 3x^3 + 4x^2 + x - 8$, what is the value of $f(-2)$?

    A. -24

    B. 30

    C. -18

    D. -50

72. Find the derivative of $y = \sqrt{1 - x^2}$.

    A. $\dfrac{x}{\sqrt{1 - x^2}}$

    B. $\dfrac{-x}{\sqrt{1 - x^2}}$

    C. $\dfrac{1}{2\sqrt{1 - x^2}}$

    D. $\dfrac{-1}{\sqrt{1 - x^2}}$

73. If $y = \arctan(x)$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

    A. $\dfrac{1}{1 + x^2}$

    B. $\dfrac{-1}{1 + x^2}$

    C. $\sec^2 x$

    D. $\dfrac{1}{\sqrt{1 - x^2}}$

74. Differentiate $y = \dfrac{e^x}{x}$ with respect to x.

A. $\dfrac{e^x(x-1)}{x^2}$

B. $\dfrac{e^x(x+1)}{x^2}$

C. e^x

D. $\dfrac{e^x}{x^2}$

75. The slope of the normal to the curve $y = x^2 - 5x + 2$ at $x = 1$ is:

A. -3

B. $1/3$

C. 3

D. $-1/3$

76. Find $\frac{\mathrm{dy}}{\mathrm{dx}}$ if $y = (x^2+1)^3$.

A. $3x(x^2+1)^2$

B. $6x(x^2+1)^2$

C. $2x(x^2+1)^3$

D. $(x^2+1)^2$

77. Given $f(x) = \dfrac{1}{x}$, find $f''(x)$.

A. $-\dfrac{1}{x^2}$

B. $\dfrac{1}{x^3}$

C. $\dfrac{2}{x^3}$

D. $-\dfrac{2}{x^3}$

78. A function $f(x) = x^3 - 6x^2 + 5$. Find the interval where the function is decreasing.

A. $x < 0$ or $x > 4$

B. $0 < x < 4$

C. $x < 2$ or $x > 6$

D. $2 < x < 6$

79. Find the derivative of $y = \sin(\sqrt{x})$.

A. $\dfrac{\cos(\sqrt{x})}{2\sqrt{x}}$

B. $\cos(\sqrt{x})$

C. $2\sqrt{x}\cos(\sqrt{x})$

D. $\dfrac{-\cos(\sqrt{x})}{2\sqrt{x}}$

80. If $y = \sec x$, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

A. $\tan^2 x$

B. $\sec x \tan x$

C. $-\sec x \tan x$

D. $\csc x \cot x$

81. Find the gradient of the curve $y = \ln(x^2)$ at $x = 2$.

A. $1/2$

B. 1

C. 2

D. $\ln 4$

82. The position of a particle is given by $s(t) = t^3 - 3t^2 + 3t + 7$. What is its acceleration when velocity is zero?

A. 0

B. 6

C. -6

D. 1

83. Differentiate $y = 5^{2x}$.

A. $5^{2x}\ln 5$

B. $2 \cdot 5^{2x}\ln 5$

C. $2x \cdot 5^{2x-1}$

D. $2 \cdot 5^{2x}$

84. Find the critical points of $f(x) = x + \dfrac{1}{x}$.

A. $x = 0, x = 1$

B. $x = 1, x = -1$

C. $x = 0$ only

D. $x = 1$ only

85. If $y = \cos^2(3x)$, find $\frac{\mathrm{dy}}{\mathrm{dx}}$.

A. $-3\sin(6x)$

B. $6\cos(3x)\sin(3x)$

C. $-2\sin(3x)$

D. $-6\sin(3x)$

86. The derivative of $y = \arcsin(2x)$ is:

A. $\dfrac{1}{\sqrt{1-4x^2}}$

B. $\dfrac{2}{\sqrt{1-4x^2}}$

C. $\dfrac{2}{\sqrt{1-x^2}}$

D. $\dfrac{1}{1+4x^2}$

87. Find the equation of the tangent to $y = e^x$ at $x = 0$.

A. $y = x + 1$

B. $y = x - 1$

C. $y = x$

D. $y = ex$

88. If $f(x) = (x + 1)^2(x - 1)$, find $f'(0)$.

A. -1

B. 1

C. 0

D. 2

89. Differentiate $y = \log_{10}(x)$.

A. $\dfrac{1}{x}$

B. $\dfrac{\ln 10}{x}$

C. $\dfrac{1}{x \ln 10}$

D. $\dfrac{x}{\ln 10}$

90. For the curve $y = x^2 e^{-x}$, find the x-coordinate (s) of the turning points.

A. $(0, -2)$

B. $(0, 2)$

C. $(1, 2)$

D. $(-1, 0)$

91. If $y = \dfrac{\sin x}{1 + \cos x}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

A. $\dfrac{1}{1 + \cos x}$

B. $\dfrac{\cos x}{(1 + \cos x)^2}$

C. $\dfrac{1}{(1 + \cos x)^2}$

D. $\dfrac{-\sin x}{(1 + \cos x)^2}$

92. Find the derivative of $y = \sqrt{x^2 + a^2}$, where a is a constant.

A. $\dfrac{x}{\sqrt{x^2 + a^2}}$

B. $\dfrac{1}{2\sqrt{x^2 + a^2}}$

C. $\dfrac{2x}{\sqrt{x^2 + a^2}}$

D. $\dfrac{ax}{\sqrt{x^2 + a^2}}$

93. The minimum value of $f(x) = x^2 + \dfrac{16}{x}$ for $x > 0$ is:

A. 8

B. 12

C. 16

D. 4

94. Differentiate $y = x^x$ with respect to x.

A. xx^{x-1}

B. $x^x \ln x$

C. $x^x(1 + \ln x)$

D. x^x

95. If $y = \dfrac{1}{x^n}$, find $\frac{\mathrm{d}y}{\mathrm{d}x}$.

A. $\dfrac{n}{x^{n+1}}$

B. $\dfrac{-n}{x^{n-1}}$

C. $\dfrac{-n}{x^{n+1}}$

D. nx^{n-1}

96. What is the slope of the tangent to the curve $y = \sqrt{x}$ at $x = 4$?

A. 1/4

B. 1/2

C. 2

D. 4

97. Find the second derivative of $y = \ln(x)$.

A. $1/x^2$

B. $-1/x^2$

C. $1/x$

D. $-1/x$

98. If the radius of a sphere is increasing at 2 cm/s, find the rate of increase of its volume when the radius is 3 cm. ($V = \frac{4}{3}\pi r^3$)

A. 24π cm^3/s

B. 36π cm^3/s

C. 72π cm^3/s

D. 12π cm^3/s

99. The function $y = |x - 2|$ is not differentiable at:

A. $x = 0$

B. $x = 1$

C. $x = 2$

D. All points

100. Differentiate $y = \tan(3x + 2)$ with respect to x.

A. $3\sec^2(3x + 2)$

B. $\sec^2(3x + 2)$

C. $3\cot(3x + 2)$

D. $-3\sec^2(3x + 2)$

4.1.2 Solutions

1. To find the minimum point of $y = x^2 - 6x + 5$, we first find the derivative, which gives the slope of the curve. $\frac{dy}{dx} = \frac{d}{dx}(x^2) - \frac{d}{dx}(6x) + \frac{d}{dx}(5)$ Using the power rule ($\frac{d}{dx}(x^n) = nx^{n-1}$) and that the derivative of a constant is 0: $\frac{dy}{dx} = 2x - 6 + 0 = 2x - 6$. At a minimum (or maximum) point, the slope is 0. So, set $\frac{dy}{dx} = 0$: $2x - 6 = 0 \implies 2x = 6 \implies x = 3$. Substitute $x = 3$ back into the original equation for y: $y = (3)^2 - 6(3) + 5 = 9 - 18 + 5 = -4$. The point is $(3, -4)$. To confirm it's a minimum, check the second derivative: $\frac{d^2y}{dx^2}2 = \frac{d}{dx}(2x - 6) = 2$. Since $2 > 0$, it's a minimum. **Answer: E**

2. For $y = x^2 + x + 1$, find the value of x where it's minimum. First derivative: $\frac{dy}{dx} = \frac{d}{dx}(x^2) + \frac{d}{dx}(x) + \frac{d}{dx}(1) = 2x + 1 + 0 = 2x + 1$. Set $\frac{dy}{dx} = 0$: $2x + 1 = 0 \implies 2x = -1 \implies x = -\frac{1}{2}$. Second derivative: $\frac{d^2y}{dx^2}2 = \frac{d}{dx}(2x + 1) = 2$. Since $2 > 0$, this x value corresponds to a minimum. **Answer: B**

3. For $y = x^2 - 2x - 3$, find the value of x where it's minimum. First derivative: $\frac{dy}{dx} = 2x - 2$. Set $\frac{dy}{dx} = 0$: $2x - 2 = 0 \implies 2x = 2 \implies x = 1$. Second derivative: $\frac{d^2y}{dx^2}2 = 2$. Since $2 > 0$, this x value corresponds to a minimum. **Answer: A**

4. For $y = -x^2 + 2x + 3$, find the maximum value. First derivative: $\frac{dy}{dx} = -2x + 2$. Set $\frac{dy}{dx} = 0$: $-2x + 2 = 0 \implies -2x = -2 \implies x = 1$. Substitute $x = 1$ into y: $y = -(1)^2 + 2(1) + 3 = -1 + 2 + 3 = 4$. Second derivative: $\frac{d^2y}{dx^2}2 = -2$. Since $-2 < 0$, this is a maximum. The maximum value is 4. **Answer: D**

5. For $y = 3x^2 - x^3$, find the maximum value. First derivative: $\frac{dy}{dx} = 6x - 3x^2$. Set $\frac{dy}{dx} = 0$: $6x - 3x^2 = 0 \implies 3x(2 - x) = 0$. This gives $x = 0$ or $x = 2$. Second derivative: $\frac{d^2y}{dx^2}2 = 6 - 6x$. At $x = 0$: $\frac{d^2y}{dx^2}2 = 6 - 6(0) = 6 > 0$ (minimum). Value: $y(0) = 0$. At $x = 2$: $\frac{d^2y}{dx^2}2 = 6 - 6(2) = 6 - 12 = -6 < 0$ (maximum). Value: $y(2) = 3(2)^2 - (2)^3 = 3(4) - 8 = 12 - 8 = 4$. The maximum value is 4. **Answer: B**

6. For $y = x^3 - 3x + 1$, find the minimum value. First derivative: $\frac{dy}{dx} = 3x^2 - 3$. Set $\frac{dy}{dx} = 0$: $3x^2 - 3 = 0 \implies 3x^2 = 3 \implies x^2 = 1 \implies x = \pm 1$. Second derivative: $\frac{d^2y}{dx^2}2 = 6x$. At $x = 1$: $\frac{d^2y}{dx^2}2 = 6(1) = 6 > 0$ (minimum). Value: $y(1) = (1)^3 - 3(1) + 1 = 1 - 3 + 1 = -1$. At $x = -1$: $\frac{d^2y}{dx^2}2 = 6(-1) = -6 < 0$ (maximum). Value: $y(-1) = (-1)^3 - 3(-1) + 1 = -1 + 3 + 1 = 3$. The minimum value is -1. **Answer: A**

7. For $f(x) = 2x^3 - x^2 - 4x + 4$, find x for maximum value. First derivative: $f'(x) = 6x^2 - 2x - 4$. Set $f'(x) = 0$: $6x^2 - 2x - 4 = 0$. Divide by 2: $3x^2 - x - 2 = 0$. Factor: $(3x + 2)(x - 1) = 0$. So $x = -\frac{2}{3}$ or $x = 1$. Second derivative: $f''(x) = 12x - 2$. At $x = -\frac{2}{3}$: $f''(-\frac{2}{3}) = 12(-\frac{2}{3}) - 2 = -8 - 2 = -10 < 0$ (maximum). At $x = 1$: $f''(1) = 12(1) - 2 = 10 > 0$ (minimum). The value of x for maximum is $-\frac{2}{3}$. **Answer: D**

8. For $f(x) = 3x^3 - 9x^2$, find x for minimum value. First derivative: $f'(x) = 9x^2 - 18x$. Set $f'(x) = 0$: $9x^2 - 18x = 0 \implies 9x(x - 2) = 0$. So $x = 0$ or $x = 2$. Second derivative: $f''(x) = 18x - 18$. At $x = 0$: $f''(0) = 18(0) - 18 = -18 < 0$ (maximum). At $x = 2$: $f''(2) = 18(2) - 18 = 36 - 18 = 18 > 0$ (minimum). The value of x for minimum is 2. **Answer: A**

9. For $f(x) = 2 + x - x^2$, find the maximum value. First derivative: $f'(x) = 1 - 2x$. Set $f'(x) = 0$: $1 - 2x = 0 \implies 2x = 1 \implies x = \frac{1}{2}$. Substitute $x = \frac{1}{2}$ into $f(x)$: $f(\frac{1}{2}) = 2 + \frac{1}{2} - (\frac{1}{2})^2 = 2 + \frac{1}{2} - \frac{1}{4} = \frac{8+2-1}{4} = \frac{9}{4}$. Second derivative: $f''(x) = -2$. Since $-2 < 0$, this is a maximum. The maximum value is $\frac{9}{4}$. **Answer: A**

10. For $y = 1 - 2x - 3x^2$, find the maximum value. First derivative: $\frac{dy}{dx} = -2 - 6x$. Set $\frac{dy}{dx} = 0$: $-2 - 6x = 0 \implies -6x = 2 \implies x = -\frac{2}{6} = -\frac{1}{3}$. Substitute $x = -\frac{1}{3}$ into y: $y = 1 - 2(-\frac{1}{3}) - 3(-\frac{1}{3})^2 = 1 + \frac{2}{3} - 3(\frac{1}{9}) = 1 + \frac{2}{3} - \frac{1}{3} = 1 + \frac{1}{3} = \frac{4}{3}$. Second derivative: $\frac{d^2y}{dx^2}2 = -6$. Since $-6 < 0$, this is a maximum. The maximum value is $\frac{4}{3}$. **Answer: A**

11. For $y = x^2 - 6x + 8$, find the minimum value. First derivative: $\frac{dy}{dx} = 2x - 6$. Set $\frac{dy}{dx} = 0$: $2x - 6 = 0 \implies x = 3$. Substitute $x = 3$ into y: $y = (3)^2 - 6(3) + 8 = 9 - 18 + 8 = -1$. Second derivative: $\frac{d^2y}{dx^2}2 = 2$. Since $2 > 0$, this is a minimum. The minimum value is -1. **Answer: D**

12. For $f(x) = x^3 - 12x + 11$, find the maximum value. First derivative: $f'(x) = 3x^2 - 12$. Set $f'(x) = 0$: $3x^2 - 12 = 0 \implies 3x^2 = 12 \implies x^2 = 4 \implies x = \pm 2$. Second derivative: $f''(x) = 6x$. At $x = 2$: $f''(2) = 6(2) = 12 > 0$ (minimum). Value: $f(2) = (2)^3 - 12(2) + 11 = 8 - 24 + 11 = -5$. At $x = -2$: $f''(-2) = 6(-2) = -12 < 0$ (maximum). Value: $f(-2) = (-2)^3 - 12(-2) + 11 = -8 + 24 + 11 = 16 + 11 = 27$. The maximum value is 27. **Answer: D**

13. For $y = 1 + hx - 3x^2$, the maximum value is 13. Find h. First derivative: $\frac{dy}{dx} = h - 6x$. Set $\frac{dy}{dx} = 0$: $h - 6x = 0 \implies x = \frac{h}{6}$. This x value gives the maximum. Substitute it into y and set $y = 13$: $13 = 1 + h(\frac{h}{6}) - 3(\frac{h}{6})^2$ $13 = 1 + \frac{h^2}{6} - 3(\frac{h^2}{36}) = 1 + \frac{h^2}{6} - \frac{h^2}{12}$ $12 = \frac{2h^2 - h^2}{12} = \frac{h^2}{12}$. $h^2 = 12 \times 12 = 144$. $h = \pm\sqrt{144} = \pm 12$. Second derivative: $\frac{d^2y}{dx^2} 2 = -6$. This is negative, confirming a maximum for any $h \neq 0$. Given options are positive. **Answer: C** (assuming h is positive, or options mean $|h|$)

14. Profit $P(x) = 10x - x^2$. Maximize profit. First derivative: $P'(x) = 10 - 2x$. Set $P'(x) = 0$: $10 - 2x = 0 \implies 2x = 10 \implies x = 5$. Second derivative: $P''(x) = -2$. Since $-2 < 0$, this is a maximum. Number of bags for maximum profit is 5. **Answer: B**

15. For $y = x^3 - x$, find x for minimum value. First derivative: $\frac{dy}{dx} = 3x^2 - 1$. Set $\frac{dy}{dx} = 0$: $3x^2 - 1 = 0 \implies 3x^2 = 1 \implies x^2 = \frac{1}{3} \implies x = \pm\frac{1}{\sqrt{3}} = \pm\frac{\sqrt{3}}{3}$. Second derivative: $\frac{d^2y}{dx^2} 2 = 6x$. At $x = \frac{\sqrt{3}}{3}$: $\frac{d^2y}{dx^2} 2 = 6(\frac{\sqrt{3}}{3}) = 2\sqrt{3} > 0$ (minimum). At $x = -\frac{\sqrt{3}}{3}$: $\frac{d^2y}{dx^2} 2 = 6(-\frac{\sqrt{3}}{3}) = -2\sqrt{3} < 0$ (maximum). The value of x for minimum is $\frac{\sqrt{3}}{3}$. **Answer: A**

16. For $f(x) = x^2 - 2x - 3$, find the least value and corresponding x. First derivative: $f'(x) = 2x - 2$. Set $f'(x) = 0$: $2x - 2 = 0 \implies x = 1$. Substitute $x = 1$ into $f(x)$: $f(1) = (1)^2 - 2(1) - 3 = 1 - 2 - 3 = -4$. Second derivative: $f''(x) = 2$. Since $2 > 0$, this is a minimum (least value). Least value is $f(x) = -4$ at $x = 1$. **Answer: C**

17. If $y = 3\cos\left(\frac{x}{3}\right)$, find $\frac{dy}{dx}$ when $x = \frac{3\pi}{2}$. Use chain rule. Let $u = \frac{x}{3}$, so $\frac{du}{dx} = \frac{1}{3}$. Then $y = 3\cos(u)$, so $\frac{dy}{du} = -3\sin(u)$. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = -3\sin(u) \cdot \frac{1}{3} = -\sin(u) = -\sin\left(\frac{x}{3}\right)$. When $x = \frac{3\pi}{2}$: $\frac{dy}{dx} = -\sin\left(\frac{1}{3} \cdot \frac{3\pi}{2}\right) = -\sin\left(\frac{\pi}{2}\right)$. Since $\sin\left(\frac{\pi}{2}\right) = 1$, then $\frac{dy}{dx} = -1$. **Answer: A**

18. Rate of change of volume $V = \frac{2}{3}\pi r^3$ of a hemisphere with respect to its radius r, when $r = 2$. This is $\frac{dV}{dr}$. $\frac{dV}{dr} = \frac{d}{dr}\left(\frac{2}{3}\pi r^3\right) = \frac{2}{3}\pi \cdot (3r^2) = 2\pi r^2$. When $r = 2$: $\frac{dV}{dr} = 2\pi(2)^2 = 2\pi(4) = 8\pi$. **Answer: C**

19. If $y = (1 - 2x)^3$, find $\frac{dy}{dx}$ at $x = -1$. Use chain rule. Let $u = 1 - 2x$, so $\frac{du}{dx} = -2$. Then $y = u^3$, so $\frac{dy}{du} = 3u^2$. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 3u^2 \cdot (-2) = -6u^2 = -6(1 - 2x)^2$. At $x = -1$: $\frac{dy}{dx} = -6(1 - 2(-1))^2 = -6(1 + 2)^2 = -6(3)^2 = -6(9) = -54$. **Answer: D**

20. Find the derivative of $y = \sin(2x^3 + 3x - 4)$. Use chain rule. Let $u = 2x^3 + 3x - 4$, so $\frac{du}{dx} = 6x^2 + 3$. Then $y = \sin(u)$, so $\frac{dy}{du} = \cos(u)$. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = \cos(u) \cdot (6x^2 + 3) = (6x^2 + 3)\cos(2x^3 + 3x - 4)$. **Answer: D**

21. Radius r increases at 0.5 cm/sec ($\frac{dr}{dt} = 0.5$). Rate of area A increase ($\frac{dA}{dt}$) when $r = 6$ cm. Area of disc $A = \pi r^2$. $\frac{dA}{dr} = 2\pi r$. Using chain rule: $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = (2\pi r) \cdot (0.5) = \pi r$. When $r = 6$: $\frac{dA}{dt} = \pi(6) = 6\pi$ cm^2/sec. **Answer: C**

22. If $y = \cos x$, find $\frac{dy}{dx}$. This is a standard derivative. $\frac{dy}{dx} = -\sin x$. **Answer: B**

23. Differentiate $y = (\cos\theta - \sin\theta)^2$ with respect to θ. Expand first: $y = \cos^2\theta - 2\cos\theta\sin\theta + \sin^2\theta$. Since $\cos^2\theta + \sin^2\theta = 1$ and $2\cos\theta\sin\theta = \sin 2\theta$: $y = 1 - \sin 2\theta$. Now differentiate: $\frac{dy}{d\theta} = \frac{d}{d\theta}(1) - \frac{d}{d\theta}(\sin 2\theta)$. $\frac{dy}{d\theta} = 0 - (2\cos 2\theta)$ (using chain rule for $\sin 2\theta$). $\frac{dy}{d\theta} = -2\cos 2\theta$. **Answer: C**

24. Differentiate $y = \left(x^2 - \frac{1}{x}\right)^2$ with respect to x. Expand first: $y = (x^2)^2 - 2(x^2)\left(\frac{1}{x}\right) + \left(\frac{1}{x}\right)^2 = x^4 - 2x + \frac{1}{x^2} = x^4 - 2x + x^{-2}$. Differentiate: $\frac{dy}{dx} = \frac{d}{dx}(x^4) - \frac{d}{dx}(2x) + \frac{d}{dx}(x^{-2})$. $\frac{dy}{dx} = 4x^3 - 2 + (-2)x^{-3} = 4x^3 - 2 - \frac{2}{x^3}$. **Answer: B**

25. Curve $y = 2x^2 - 2x + 3$ has gradient (slope) 2. Find point (x, y). Gradient is $\frac{dy}{dx} = \frac{d}{dx}(2x^2 - 2x + 3) = 4x - 2$. Set gradient to 2: $4x - 2 = 2 \implies 4x = 4 \implies x = 1$. Substitute $x = 1$ into y: $y = 2(1)^2 - 2(1) + 3 = 2 - 2 + 3 = 3$. The point is $(1, 3)$. **Answer: A**

26. For $y = x^2 - 4x + 3$, find x where tangent is parallel to x-axis (slope is 0). $\frac{dy}{dx} = 2x - 4$. Set $\frac{dy}{dx} = 0$: $2x - 4 = 0 \implies 2x = 4 \implies x = 2$. **Answer: C**

27. Differentiate $y = \frac{6x^3 - 5x^2 + 1}{3x^2}$. Simplify first: $y = \frac{6x^3}{3x^2} - \frac{5x^2}{3x^2} + \frac{1}{3x^2} = 2x - \frac{5}{3} + \frac{1}{3}x^{-2}$. $\frac{dy}{dx} = \frac{d}{dx}(2x) - \frac{d}{dx}\left(\frac{5}{3}\right) + \frac{d}{dx}\left(\frac{1}{3}x^{-2}\right)$. $\frac{dy}{dx} = 2 - 0 + \frac{1}{3}(-2)x^{-3} = 2 - \frac{2}{3x^3}$. **Answer: A**

28. If $y = (1+x)^2$, find $\frac{dy}{dx}$. Method 1 (Chain Rule): Let $u = 1+x$, $\frac{du}{dx} = 1$. $y = u^2$, $\frac{dy}{du} = 2u$. $\frac{dy}{dx} = 2u \cdot 1 = 2(1+x) = 2+2x$. Method 2 (Expand): $y = 1 + 2x + x^2$. $\frac{dy}{dx} = 0 + 2 + 2x = 2 + 2x$. **Answer: C**

29. Differentiate $y = 3x^3+2x^2+3x+1$. $\frac{dy}{dx} = \frac{d}{dx}(3x^3)+\frac{d}{dx}(2x^2)+\frac{d}{dx}(3x)+\frac{d}{dx}(1)$. $\frac{dy}{dx} = 3(3x^2)+2(2x)+3(1)+0 = 9x^2+4x+3$. **Answer: A**

30. Differentiate $y = \frac{2}{3}x^3 - \frac{4}{x} = \frac{2}{3}x^3 - 4x^{-1}$. $\frac{dy}{dx} = \frac{d}{dx}\left(\frac{2}{3}x^3\right) - \frac{d}{dx}(4x^{-1})$. $\frac{dy}{dx} = \frac{2}{3}(3x^2) - 4(-1)x^{-2} = 2x^2 + 4x^{-2} = 2x^2 + \frac{4}{x^2}$. **Answer: A**

31. Find the derivative of $y = \frac{\sin x}{\cos x}$. This is $y = \tan x$. The derivative of $\tan x$ is $\sec^2 x$. Alternatively, use quotient rule: $u = \sin x, u' = \cos x$; $v = \cos x, v' = -\sin x$. $\frac{dy}{dx} = \frac{(\cos x)(\cos x) - (\sin x)(-\sin x)}{(\cos x)^2} = \frac{\cos^2 x + \sin^2 x}{\cos^2 x} = \frac{1}{\cos^2 x} = \sec^2 x$. **Answer: C**

32. If $y = x^2 - 3x + 4$, find $\frac{dy}{dx}$ at $x = 5$. $\frac{dy}{dx} = 2x - 3$. At $x = 5$: $\frac{dy}{dx} = 2(5) - 3 = 10 - 3 = 7$. **Answer: B**

33. If $y = 2x\cos(2x) - \sin(2x)$, find $\frac{dy}{dx}$ when $x = \frac{\pi}{2}$. For the term $2x\cos(2x)$, use product rule: $u = 2x, u' = 2$; $v = \cos(2x), v' = -2\sin(2x)$. Derivative of $2x\cos(2x)$ is $(2)\cos(2x) + (2x)(-2\sin(2x)) = 2\cos(2x) - 4x\sin(2x)$. Derivative of $\sin(2x)$ is $2\cos(2x)$. So, $\frac{dy}{dx} = (2\cos(2x) - 4x\sin(2x)) - 2\cos(2x) = -4x\sin(2x)$. When $x = \frac{\pi}{2}$: $\frac{dy}{dx} = -4\left(\frac{\pi}{2}\right)\sin\left(2 \cdot \frac{\pi}{2}\right) = -2\pi\sin(\pi)$. Since $\sin(\pi) = 0$, $\frac{dy}{dx} = -2\pi(0) = 0$. **Answer: A**

34. If $y = 3\cos(4x)$, find $\frac{dy}{dx}$. Use chain rule: Let $u = 4x$, $\frac{du}{dx} = 4$. $y = 3\cos u$, $\frac{dy}{du} = -3\sin u$. $\frac{dy}{dx} = (-3\sin u)(4) = -12\sin(4x)$. **Answer: C**

35. Find the derivative of $y = (2 + 3x)(1 - x)$. Method 1 (Product Rule): $u = 2 + 3x, u' = 3$; $v = 1 - x, v' = -1$. $\frac{dy}{dx} = u'v + uv' = (3)(1-x) + (2+3x)(-1) = 3 - 3x - 2 - 3x = 1 - 6x$. Method 2 (Expand): $y = 2(1-x) + 3x(1-x) = 2 - 2x + 3x - 3x^2 = 2 + x - 3x^2$. $\frac{dy}{dx} = 0 + 1 - 6x = 1 - 6x$. **Answer: B**

36. Find $\frac{dy}{dx}$ if $y = -3x^3 + 2x^2 - 3x + 1$. $\frac{dy}{dx} = -3(3x^2) + 2(2x) - 3(1) + 0 = -9x^2 + 4x - 3$. **Answer: B**

37. If $y = 2x^3 + 6x^2 + 6x + 1$, find $\frac{dy}{dx}$. $\frac{dy}{dx} = 2(3x^2) + 6(2x) + 6(1) + 0 = 6x^2 + 12x + 6$. **Answer: C**

38. Find the derivative of $y = \left(\frac{1}{3}x + 6\right)^2$. Use chain rule: Let $u = \frac{1}{3}x + 6$, $\frac{du}{dx} = \frac{1}{3}$. $y = u^2$, $\frac{dy}{du} = 2u$. $\frac{dy}{dx} = (2u)\left(\frac{1}{3}\right) = \frac{2}{3}u = \frac{2}{3}\left(\frac{1}{3}x + 6\right)$. **Answer: B**

39. If $y = x^2 - 3x + 4$, find $\frac{dy}{dx}$ at $x = 2$. $\frac{dy}{dx} = 2x - 3$. At $x = 2$: $\frac{dy}{dx} = 2(2) - 3 = 4 - 3 = 1$. **Answer: B**

40. If $y = x^2 + \sqrt{x} = x^2 + x^{1/2}$, find $\frac{dy}{dx}$. $\frac{dy}{dx} = 2x + \frac{1}{2}x^{1/2-1} = 2x + \frac{1}{2}x^{-1/2}$. **Answer: D**

41. Find $\frac{dy}{dx}$ if $y = \frac{2}{3}x^3 - \frac{4}{x} = \frac{2}{3}x^3 - 4x^{-1}$. $\frac{dy}{dx} = \frac{2}{3}(3x^2) - 4(-1)x^{-2} = 2x^2 + 4x^{-2} = 2x^2 + \frac{4}{x^2}$. (This is a repeat of question 30, already answered as A based on options there. The options here are different, matching this solution.) The options for question 30 were: A. $2x^2 + \frac{4}{x^2}$, B. $2x^2 - \frac{4}{x}$, C. $3x^2 - \frac{4}{x}$, D. $3x^2 + \frac{4}{x^2}$. The options for this question 41 are: A. $3x^2 - \frac{4}{x}$, B. $3x^2 + \frac{4}{x^2}$, C. $2x^2 - \frac{4}{x}$, D. $2x^2 + \frac{4}{x^2}$. The correct derivative is $2x^2 + \frac{4}{x^2}$. **Answer: D** (for Q41 based on its options)

42. If $y = \cos(3x)$, find $\frac{dy}{dx}$. Chain rule: Let $u = 3x$, $\frac{du}{dx} = 3$. $y = \cos u$, $\frac{dy}{du} = -\sin u$. $\frac{dy}{dx} = (-\sin u)(3) = -3\sin(3x)$. **Answer: D**

43. Find $\frac{dy}{dx}$ if $y = \cos x$. (Repeat of Q22) $\frac{dy}{dx} = -\sin x$. **Answer: B**

44. Find the slope of $y = 2x^2 + 5x - 3$ at $(1, 4)$. Slope is $\frac{dy}{dx} = 4x + 5$. At $x = 1$ (from the point $(1, 4)$): slope $= 4(1) + 5 = 9$. (Check y-coordinate: $y(1) = 2(1)^2 + 5(1) - 3 = 2 + 5 - 3 = 4$. Point is on curve.) **Answer: D**

45. Derivative of $y = \sin^2(5x) = (\sin(5x))^2$. Chain rule: Let $u = \sin(5x)$. Then $y = u^2$, so $\frac{dy}{du} = 2u$. For $u = \sin(5x)$, let $v = 5x$, $\frac{dv}{dx} = 5$. $u = \sin v$, $\frac{du}{dv} = \cos v$. So $\frac{du}{dx} = \cos(5x) \cdot 5 = 5\cos(5x)$. $\frac{dy}{dx} = \frac{dy}{du} \cdot \frac{du}{dx} = 2u \cdot 5\cos(5x) = 2\sin(5x) \cdot 5\cos(5x) = 10\sin(5x)\cos(5x)$. (This can also be written as $5\sin(10x)$). **Answer: D**

46. Slope of tangent to $y = 3x^2 - 2x + 5$ at $(1, 6)$. $\frac{dy}{dx} = 6x - 2$. At $x = 1$: slope $= 6(1) - 2 = 4$. (Check y-coordinate: $y(1) = 3(1)^2 - 2(1) + 5 = 3 - 2 + 5 = 6$. Point is on curve.) **Answer: B**

47. Gradient of $y = 2kx^2 + x + 1$ at $x = 1$ is 9. Find k. Gradient $\frac{dy}{dx} = 4kx + 1$. At $x = 1$, gradient is $4k(1) + 1 = 4k + 1$. Given this is 9: $4k + 1 = 9 \implies 4k = 8 \implies k = 2$. **Answer: A**

48. Distance $s = t^3 - t^2 - t + 5$. Minimum distance for $t \geq 0$. Velocity $\frac{ds}{dt} = 3t^2 - 2t - 1$. Set $\frac{ds}{dt} = 0$: $3t^2 - 2t - 1 = 0 \implies (3t + 1)(t - 1) = 0$. Solutions are $t = -\frac{1}{3}$ or $t = 1$. Since $t \geq 0$, consider $t = 1$. Acceleration $\frac{d^2s}{dt^2}2 = 6t - 2$. At $t = 1$, $\frac{d^2s}{dt^2}2 = 6(1) - 2 = 4 > 0$, so $t = 1$ is a local minimum. Value at $t = 1$: $s(1) = 1^3 - 1^2 - 1 + 5 = 1 - 1 - 1 + 5 = 4$. Check boundary $t = 0$: $s(0) = 0^3 - 0^2 - 0 + 5 = 5$. The minimum distance is 4 cm. **Answer: B**

49. Differentiate $y = (2x + 5)^2(x - 4)$. Product rule: $u = (2x + 5)^2$, $v = x - 4$. For u: use chain rule. Let $w = 2x + 5$, $w' = 2$. $u = w^2, u'_w = 2w$. So $u' = 2(2x + 5)(2) = 4(2x + 5)$. For v: $v' = 1$. $\frac{dy}{dx} = u'v + uv' = 4(2x + 5)(x - 4) + (2x + 5)^2(1)$. Factor out $(2x + 5)$: $(2x + 5)[4(x - 4) + (2x + 5)]$. $= (2x + 5)[4x - 16 + 2x + 5] = (2x + 5)(6x - 11)$. **Answer: C**

50. Rate of change of volume $V = \frac{4}{3}\pi r^3$ of a sphere wrt radius r, when $r = 1$. This is $\frac{dV}{dr}$. $\frac{dV}{dr} = \frac{4}{3}\pi(3r^2) = 4\pi r^2$. When $r = 1$: $\frac{dV}{dr} = 4\pi(1)^2 = 4\pi$. **Answer: C**

51. If $y = 2x\cos(2x) - \sin(2x)$, find $\frac{dy}{dx}$ when $x = \frac{\pi}{4}$. We found in Q33 that $\frac{dy}{dx} = -4x\sin(2x)$. When $x = \frac{\pi}{4}$: $\frac{dy}{dx} = -4\left(\frac{\pi}{4}\right)\sin\left(2 \cdot \frac{\pi}{4}\right) = -\pi\sin\left(\frac{\pi}{2}\right)$. Since $\sin\left(\frac{\pi}{2}\right) = 1$, $\frac{dy}{dx} = -\pi(1) = -\pi$. **Answer: C**

52. Differentiate $y = \frac{x}{\cos x}$. Quotient rule: $u = x, u' = 1$; $v = \cos x, v' = -\sin x$. $\frac{dy}{dx} = \frac{(1)(\cos x) - (x)(-\sin x)}{(\cos x)^2} = \frac{\cos x + x\sin x}{\cos^2 x}$. Split the fraction: $\frac{\cos x}{\cos^2 x} + \frac{x\sin x}{\cos^2 x} = \frac{1}{\cos x} + x\frac{\sin x}{\cos x}\frac{1}{\cos x}$. $= \sec x + x\tan x\sec x$. **Answer: D**

53. If $y = 243(4x+5)^{-2}$, find $\frac{dy}{dx}$ when $x = 1$. Chain rule: Let $u = 4x+5$, $\frac{du}{dx} = 4$. $y = 243u^{-2}$, $\frac{dy}{du} = 243(-2)u^{-3} = -486u^{-3}$. $\frac{dy}{dx} = (-486u^{-3})(4) = -1944(4x + 5)^{-3} = \frac{-1944}{(4x + 5)^3}$. When $x = 1$: $\frac{dy}{dx} = \frac{-1944}{(4(1) + 5)^3} = \frac{-1944}{(9)^3} = \frac{-1944}{729}$. $1944/729 = (8 \times 243)/(3 \times 243) = 8/3$. So $\frac{dy}{dx} = -\frac{8}{3}$. **Answer: C**

54. Derivative of $y = t^2\sin(3t - 5)$ wrt t. Product rule: $u = t^2, u' = 2t$; $v = \sin(3t - 5)$. For v, chain rule: let $w = 3t-5, w' = 3$. $v = \sin w, v'_w = \cos w$. So $v' = 3\cos(3t-5)$. $\frac{dy}{dt} = u'v+uv' = (2t)\sin(3t-5)+(t^2)(3\cos(3t-5))$. $= 2t\sin(3t - 5) + 3t^2\cos(3t - 5)$. **Answer: A**

55. Circle radius $r = 5$ cm. $\frac{dr}{dt} = 0.2$ cm/s. Increase in area $\frac{dA}{dt}$. $A = \pi r^2$. $\frac{dA}{dr} = 2\pi r$. $\frac{dA}{dt} = \frac{dA}{dr} \cdot \frac{dr}{dt} = (2\pi r)(0.2)$. When $r = 5$: $\frac{dA}{dt} = 2\pi(5)(0.2) = 10\pi(0.2) = 2\pi$. **Answer: B**

56. Rectangle of greatest area with fixed perimeter p. Let length be l and width be w. Perimeter $p = 2(l + w)$, so $l + w = p/2$. Thus $w = p/2 - l$. Area $A = lw = l(p/2 - l) = \frac{p}{2}l - l^2$. To maximize area, $\frac{dA}{dl} = \frac{p}{2} - 2l$. Set $\frac{dA}{dl} = 0$: $\frac{p}{2} - 2l = 0 \implies 2l = \frac{p}{2} \implies l = \frac{p}{4}$. Then $w = \frac{p}{2} - l = \frac{p}{2} - \frac{p}{4} = \frac{p}{4}$. So it's a square with sides $\frac{p}{4}$. $\frac{d^2A}{dl^2} = -2 < 0$, confirming maximum. **Answer: C**

57. Gradient is $2x + 7$. Curve passes through $(2, 0)$. Find equation. Gradient $\frac{dy}{dx} = 2x + 7$. Integrate to find y: $y = \int(2x + 7)dx = x^2 + 7x + C$, where C is constant of integration. Curve passes through $(2, 0)$, so substitute $x = 2, y = 0$: $0 = (2)^2 + 7(2) + C \implies 0 = 4 + 14 + C \implies 0 = 18 + C \implies C = -18$. Equation is $y = x^2 + 7x - 18$. **Answer: A**

58. Differentiate $y = \sqrt[3]{x^2}(2x - x^2) = x^{2/3}(2x - x^2)$. Expand: $y = 2x^{2/3}x^1 - x^{2/3}x^2 = 2x^{2/3+1} - x^{2/3+2} = 2x^{5/3} - x^{8/3}$. Differentiate: $\frac{dy}{dx} = 2\left(\frac{5}{3}\right)x^{5/3-1} - \left(\frac{8}{3}\right)x^{8/3-1}$. $= \frac{10}{3}x^{2/3} - \frac{8}{3}x^{5/3}$. **Answer: B**

59. Slope of tangent to $y = 5x^2 - 3x + 5$ at $(1, 6)$. $\frac{dy}{dx} = 10x - 3$. At $x = 1$: slope $= 10(1) - 3 = 7$. (Check y-coordinate: $y(1) = 5(1)^2 - 3(1) + 5 = 5 - 3 + 5 = 7$. Point in question is $(1, 6)$, my calculated $y(1) = 7$. There is a mismatch. Assuming the question implies x=1 for the curve $y = 5x^2 - 3x + 5$, then point would be $(1, 7)$. If the point $(1, 6)$ must be used, it means it's not on the curve, and the question is ill-posed for a tangent *to the curve at that point*. Assuming it means "at $x = 1$ for the given curve":) Slope is 7. **Answer: B**

60. Derivative of $y = 2x^2(2x - 1)$ at $x = -1$. Expand: $y = 4x^3 - 2x^2$. $\frac{dy}{dx} = 12x^2 - 4x$. At $x = -1$: $\frac{dy}{dx} = 12(-1)^2 - 4(-1) = 12(1) + 4 = 12 + 4 = 16$. **Answer: C**

61. Derivative of $y = \ln(4x^3 - 2x)$. Chain rule: Let $u = 4x^3 - 2x$, $\frac{du}{dx} = 12x^2 - 2$. $y = \ln u$, $\frac{dy}{du} = \frac{1}{u}$. $\frac{dy}{dx} = \frac{1}{u} \cdot (12x^2 - 2) = \frac{12x^2 - 2}{4x^3 - 2x}$. **Answer: D**

62. Derivative of $y = e^x \sin x$. (Repeat of Q64's structure) Product rule: $u = e^x, u' = e^x$; $v = \sin x, v' = \cos x$. $\frac{dy}{dx} = u'v + uv' = e^x \sin x + e^x \cos x = e^x(\sin x + \cos x)$. **Answer: A**

63. Second derivative of $y = 8x^3 - 3x^2 + 7x - 1$. First derivative: $\frac{dy}{dx} = 24x^2 - 6x + 7$. Second derivative: $\frac{d^2y}{dx^2}2 = \frac{d}{dx}(24x^2 - 6x + 7) = 48x - 6$. **Answer: C**

64. For $y = x^2 + 6x + 8$, find x where tangent is parallel to x-axis (slope=0). $\frac{dy}{dx} = 2x + 6$. Set $\frac{dy}{dx} = 0$: $2x + 6 = 0 \implies 2x = -6 \implies x = -3$. **Answer: A**

65. Second derivative of $y = x \sin x$. First derivative (product rule: $u = x, u' = 1; v = \sin x, v' = \cos x$): $\frac{dy}{dx} = (1)\sin x + (x)\cos x = \sin x + x\cos x$. Second derivative (apply product rule to $x\cos x$): $\frac{d^2y}{dx^2}2 = \frac{d}{dx}(\sin x) + \frac{d}{dx}(x\cos x)$. $\frac{d}{dx}(x\cos x) = (1)\cos x + (x)(-\sin x) = \cos x - x\sin x$. So, $\frac{d^2y}{dx^2}2 = \cos x + (\cos x - x\sin x) = 2\cos x - x\sin x$. **Answer: A**

66. Differentiate $y = \frac{2x}{\sin x}$. Quotient rule: $u = 2x, u' = 2$; $v = \sin x, v' = \cos x$. $\frac{dy}{dx} = \frac{(2)(\sin x) - (2x)(\cos x)}{(\sin x)^2} = \frac{2\sin x - 2x\cos x}{\sin^2 x}$. Split fraction: $\frac{2\sin x}{\sin^2 x} - \frac{2x\cos x}{\sin^2 x} = \frac{2}{\sin x} - 2x\frac{\cos x}{\sin x}\frac{1}{\sin x}$. $= 2\csc x - 2x\cot x\csc x = 2\csc x(1 - x\cot x)$. **Answer: D**

67. Curve $y = 2x^2 - 2x + 3$ has gradient 2. Find point (x, y). (Repeat of Q25) Gradient $\frac{dy}{dx} = 4x - 2$. Set gradient to 2: $4x - 2 = 2 \implies 4x = 4 \implies x = 1$. $y = 2(1)^2 - 2(1) + 3 = 3$. Point is $(1, 3)$. **Answer: A**

68. Equation of tangent at $(2, 0)$ to $y = x^2 - 2x$. Check point: $y(2) = (2)^2 - 2(2) = 4 - 4 = 0$. Point is on curve. Slope $\frac{dy}{dx} = 2x - 2$. At $x = 2$, slope $m = 2(2) - 2 = 4 - 2 = 2$. Equation: $y - y_1 = m(x - x_1) \implies y - 0 = 2(x - 2)$. $y = 2x - 4$. **Answer: A**

69. Differentiate $y = 20x^{-4} + 9$. $\frac{dy}{dx} = 20(-4)x^{-4-1} + 0 = -80x^{-5}$. **Answer: A**

70. Differentiate $y = x^2 \ln x$. Product rule: $u = x^2, u' = 2x$; $v = \ln x, v' = 1/x$. $\frac{dy}{dx} = (2x)(\ln x) + (x^2)(1/x) = 2x\ln x + x = x(2\ln x + 1)$. **Answer: A**

71. If $f(x) = 3x^3 + 4x^2 + x - 8$, value of $f(-2)$. (Not differentiation, direct substitution) $f(-2) = 3(-2)^3 + 4(-2)^2 + (-2) - 8$. $= 3(-8) + 4(4) - 2 - 8 = -24 + 16 - 2 - 8 = -8 - 2 - 8 = -18$. **Answer: C**

72. Derivative of $y = \sqrt{1 - x^2} = (1 - x^2)^{1/2}$. Chain rule: Let $u = 1 - x^2, u' = -2x$. $y = u^{1/2}$, $\frac{dy}{du} = \frac{1}{2}u^{-1/2}$. $\frac{dy}{dx} = \frac{1}{2}(1 - x^2)^{-1/2}(-2x) = \frac{-x}{\sqrt{1 - x^2}}$. **Answer: B**

73. If $y = \arctan(x)$ (or $\tan^{-1} x$), find $\frac{dy}{dx}$. Standard derivative. $\frac{dy}{dx} = \frac{1}{1 + x^2}$. **Answer: A**

74. Differentiate $y = \frac{e^x}{x}$. Quotient rule: $u = e^x, u' = e^x$; $v = x, v' = 1$. $\frac{dy}{dx} = \frac{e^x(x) - e^x(1)}{x^2} = \frac{e^x(x - 1)}{x^2}$. **Answer: A**

75. Slope of normal to $y = x^2 - 5x + 2$ at $x = 1$. Slope of tangent $\frac{dy}{dx} = 2x - 5$. At $x = 1$, slope of tangent $m_t = 2(1) - 5 = -3$. Slope of normal $m_n = -1/m_t = -1/(-3) = 1/3$. **Answer: B**

76. Find $\frac{dy}{dx}$ if $y = (x^2 + 1)^3$. Chain rule: Let $u = x^2 + 1, u' = 2x$. $y = u^3, \frac{dy}{du} = 3u^2$. $\frac{dy}{dx} = 3(x^2 + 1)^2(2x) = 6x(x^2 + 1)^2$. **Answer: B**

77. Given $f(x) = \dfrac{1}{x} = x^{-1}$, find $f''(x)$. $f'(x) = -1x^{-2} = -x^{-2}$. $f''(x) = -(-2)x^{-3} = 2x^{-3} = \dfrac{2}{x^3}$. **Answer: C**

78. Function $f(x) = x^3 - 6x^2 + 5$. Interval where decreasing ($f'(x) < 0$). $f'(x) = 3x^2 - 12x = 3x(x - 4)$. We need $3x(x - 4) < 0$. Critical points at $x = 0, x = 4$. Test intervals: If $x < 0$ (e.g., $x = -1$): $3(-1)(-1 - 4) = (-3)(-5) = 15 > 0$. If $0 < x < 4$ (e.g., $x = 1$): $3(1)(1 - 4) = (3)(-3) = -9 < 0$. (Decreasing) If $x > 4$ (e.g., $x = 5$): $3(5)(5 - 4) = (15)(1) = 15 > 0$. Decreasing for $0 < x < 4$. **Answer: B**

79. Derivative of $y = \sin(\sqrt{x})$. Chain rule: Let $u = \sqrt{x} = x^{1/2}, u' = \frac{1}{2}x^{-1/2} = \frac{1}{2\sqrt{x}}$. $y = \sin u, \frac{dy}{du} = \cos u$. $\frac{dy}{dx} = \cos(\sqrt{x}) \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{\cos(\sqrt{x})}{2\sqrt{x}}$. **Answer: A**

80. If $y = \sec x$, find $\frac{dy}{dx}$. Standard derivative. $\frac{dy}{dx} = \sec x \tan x$. **Answer: B**

81. Gradient of $y = \ln(x^2)$ at $x = 2$. $y = \ln(x^2) = 2\ln x$ (for $x > 0$). $\frac{dy}{dx} = 2 \cdot \dfrac{1}{x} = \dfrac{2}{x}$. At $x = 2$, gradient $= \dfrac{2}{2} = 1$. **Answer: B**

82. Position $s(t) = t^3 - 3t^2 + 3t + 7$. Acceleration when velocity is zero. Velocity $v(t) = s'(t) = 3t^2 - 6t + 3$. Acceleration $a(t) = v'(t) = 6t - 6$. Set $v(t) = 0$: $3t^2 - 6t + 3 = 0 \implies 3(t^2 - 2t + 1) = 0 \implies 3(t - 1)^2 = 0 \implies t = 1$. Acceleration at $t = 1$: $a(1) = 6(1) - 6 = 0$. **Answer: A**

83. Differentiate $y = 5^{2x}$. Chain rule ($\frac{d}{dx}a^u = a^u \ln a \cdot \frac{du}{dx}$): Let $u = 2x, u' = 2$. $\frac{dy}{dx} = 5^{2x}(\ln 5)(2) = 2 \cdot 5^{2x} \ln 5$. **Answer: B**

84. Critical points of $f(x) = x + \dfrac{1}{x} = x + x^{-1}$. $f'(x) = 1 - x^{-2} = 1 - \dfrac{1}{x^2}$. Set $f'(x) = 0$: $1 - \dfrac{1}{x^2} = 0 \implies 1 = \dfrac{1}{x^2} \implies x^2 = 1 \implies x = \pm 1$. (Note: $f(x)$ and $f'(x)$ are undefined at $x = 0$, but $x = 0$ is not in the domain of f.) **Answer: B**

85. If $y = \cos^2(3x) = (\cos(3x))^2$. Chain rule: Outermost is u^2, middle is $\cos v$, innermost is $3x$. $\frac{dy}{dx} = 2(\cos(3x))^1 \cdot \frac{d}{ddx}(\cos(3x))$. $\frac{d}{ddx}(\cos(3x)) = -\sin(3x) \cdot \frac{d}{ddx}(3x) = -\sin(3x) \cdot 3 = -3\sin(3x)$. So, $\frac{dy}{dx} = 2\cos(3x) \cdot (-3\sin(3x)) = -6\sin(3x)\cos(3x)$. Using $2\sin A \cos A = \sin 2A$, this is $-3(2\sin(3x)\cos(3x)) = -3\sin(2 \cdot 3x) = -3\sin(6x)$. **Answer: A**

86. Derivative of $y = \arcsin(2x)$. Chain rule ($\frac{d}{dx}\arcsin u = \frac{1}{\sqrt{1-u^2}}\frac{du}{dx}$): Let $u = 2x, u' = 2$. $\frac{dy}{dx} = \dfrac{1}{\sqrt{1 - (2x)^2}} \cdot 2 = \dfrac{2}{\sqrt{1 - 4x^2}}$. **Answer: B**

87. Equation of tangent to $y = e^x$ at $x = 0$. Point: At $x = 0, y = e^0 = 1$. Point is $(0, 1)$. Slope: $\frac{dy}{dx} = e^x$. At $x = 0$, slope $m = e^0 = 1$. Equation: $y - 1 = 1(x - 0) \implies y - 1 = x \implies y = x + 1$. **Answer: A**

88. If $f(x) = (x + 1)^2(x - 1)$, find $f'(0)$. Product rule: $u = (x + 1)^2, u' = 2(x + 1)(1) = 2(x + 1)$; $v = x - 1, v' = 1$. $f'(x) = 2(x+1)(x-1)+(x+1)^2(1)$. At $x = 0$: $f'(0) = 2(0+1)(0-1)+(0+1)^2(1) = 2(1)(-1)+(1)^2(1) = -2+1 = -1$. **Answer: A**

89. Differentiate $y = \log_{10}(x)$. Standard derivative. $\frac{dy}{dx} = \dfrac{1}{x \ln 10}$ ($\ln 10$ is the natural logarithm of 10). **Answer: C**

90. Curve $y = x^2e^{-x}$ x-coordinates of turning points ($\frac{dy}{dx} = 0$). Product rule: $u = x^2, u' = 2x$; $v = e^{-x}, v' = -e^{-x}$. $\frac{dy}{dx} = (2x)(e^{-x}) + (x^2)(-e^{-x}) = 2xe^{-x} - x^2e^{-x} = xe^{-x}(2 - x)$. Set $\frac{dy}{dx} = 0$: $xe^{-x}(2 - x) = 0$. Since e^{-x} is never 0, then $x = 0$ or $2 - x = 0 \implies x = 2$. **Answer: B**

91. If $y = \dfrac{\sin x}{1 + \cos x}$ Quotient rule: $u = \sin x, u' = \cos x$; $v = 1+\cos x, v' = -\sin x$. $\frac{dy}{dx} = \dfrac{(\cos x)(1 + \cos x) - (\sin x)(-\sin x)}{(1 + \cos x)^2} = \dfrac{\cos x + \cos^2 x + \sin^2 x}{(1 + \cos x)^2}$. Since $\cos^2 x + \sin^2 x = 1$: $\frac{dy}{dx} = \dfrac{\cos x + 1}{(1 + \cos x)^2} = \dfrac{1}{1 + \cos x}$ for $1 + \cos x \neq 0$. **Answer: A**

92. Derivative of $y = \sqrt{x^2 + a^2} = (x^2 + a^2)^{1/2}$. Chain rule: Let $u = x^2 + a^2, u' = 2x$ (since a is constant, $\frac{\mathrm{d}}{\mathrm{d}x}(a^2) = 0$). $y = u^{1/2}$, $\frac{\mathrm{d}y}{\mathrm{d}u} = \frac{1}{2}u^{-1/2}$. $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{2}(x^2 + a^2)^{-1/2}(2x) = \dfrac{x}{\sqrt{x^2 + a^2}}$. **Answer: A**

93. Minimum value of $f(x) = x^2 + \dfrac{16}{x}$ for $x > 0$. $f(x) = x^2 + 16x^{-1}$. $f'(x) = 2x - 16x^{-2} = 2x - \dfrac{16}{x^2}$. Set $f'(x) = 0$: $2x - \dfrac{16}{x^2} = 0 \implies 2x = \dfrac{16}{x^2} \implies 2x^3 = 16 \implies x^3 = 8 \implies x = 2$. (Valid since $x > 0$). $f''(x) = 2 - 16(-2)x^{-3} = 2 + \dfrac{32}{x^3}$. At $x = 2$, $f''(2) = 2 + \dfrac{32}{8} = 2 + 4 = 6 > 0$, so minimum. Value: $f(2) = (2)^2 + \dfrac{16}{2} = 4 + 8 = 12$. **Answer: B**

94. Differentiate $y = x^x$. Use logarithmic differentiation. $\ln y = \ln(x^x) \implies \ln y = x \ln x$. Differentiate both sides wrt x: $\frac{\mathrm{d}}{\mathrm{d}x}(\ln y) = \frac{\mathrm{d}}{\mathrm{d}x}(x \ln x)$. $\dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = (1)(\ln x) + (x)\left(\dfrac{1}{x}\right)$ (Product rule for RHS). $\dfrac{1}{y}\dfrac{\mathrm{d}y}{\mathrm{d}x} = \ln x + 1$. $\frac{\mathrm{d}y}{\mathrm{d}x} = y(\ln x + 1) = x^x(1 + \ln x)$. **Answer: C**

95. If $y = \dfrac{1}{x^n} = x^{-n}$. $\frac{\mathrm{d}y}{\mathrm{d}x} = -nx^{-n-1} = -nx^{-(n+1)} = \dfrac{-n}{x^{n+1}}$. **Answer: C**

96. Slope of tangent to $y = \sqrt{x}$ at $x = 4$. $y = x^{1/2}$ $\frac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2}x^{-1/2} = \dfrac{1}{2\sqrt{x}}$. At $x = 4$, slope $m = \dfrac{1}{2\sqrt{4}} = \dfrac{1}{2 \cdot 2} = \dfrac{1}{4}$. **Answer: A**

97. Second derivative of $y = \ln x$. First derivative: $\frac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{x} = x^{-1}$. Second derivative: $\frac{\mathrm{d}^2y}{\mathrm{d}x^2} = -1x^{-2} = -\dfrac{1}{x^2}$. **Answer: B**

98. Radius of sphere $\frac{\mathrm{d}r}{\mathrm{d}t} = 2$ cm/s. Find $\frac{\mathrm{d}V}{\mathrm{d}t}$ when $r = 3$ cm. $V = \frac{4}{3}\pi r^3$. $\frac{\mathrm{d}V}{\mathrm{d}r} = 4\pi r^2$. $\frac{\mathrm{d}V}{\mathrm{d}t} = \frac{\mathrm{d}V}{\mathrm{d}r} \cdot \frac{\mathrm{d}r}{\mathrm{d}t} = (4\pi r^2)(2) = 8\pi r^2$. When $r = 3$: $\frac{\mathrm{d}V}{\mathrm{d}t} = 8\pi(3)^2 = 8\pi(9) = 72\pi$ cm^3/s. **Answer: C**

99. Function $y = |x - 2|$ is not differentiable at? The absolute value function $f(x) = |u|$ is not differentiable where $u = 0$. Here $u = x - 2$. So, not differentiable when $x - 2 = 0 \implies x = 2$. At $x = 2$, the graph has a sharp corner. **Answer: C**

100. Differentiate $y = \tan(3x + 2)$. Chain rule: Let $u = 3x + 2, u' = 3$ $y = \tan u$, $\frac{\mathrm{d}y}{\mathrm{d}u} = \sec^2 u$. $\frac{\mathrm{d}y}{\mathrm{d}x} = (\sec^2 u)(3) = 3\sec^2(3x + 2)$. **Answer: A**

4.2 Integration

4.2.1 Questions

1. Find the integral of $y = 3x^2 - 2x - 1$ with respect to x.

   A. $x^3 - x^2 - x + C$

   B. $x^3 + x^2 - x + C$

   C. $x^3 + x^2 + x + C$

   D. $x^3 - x^2 + x + C$

2. Integrate the expression $6x^2 - 2x + 1$ with respect to x.

   A. $3x^3 - 2x^2 + x + c$

   B. $2x^3 - x^2 + x + c$

   C. $2x^3 - 3x^2 + c$

   D. $x^3 + x^2 - x + c$

3. Integrate $x^{-2} + \cos x$ with respect to x.

   A. $\dfrac{1}{x} + \sin x + k$

   B. $-\dfrac{1}{x} + \sin x + k$

   C. $-\dfrac{1}{x} - \sin x + k$

   D. $\ln|x| + \sin x + k$

4. If the expression $ax^2 + bx + c$ equals 5 at $x = 1$. If its derivative is $2x + 1$, what are the values of a,b, c respectively?

   A. 1, 1, 3

   B. 1, 3, 1

   C. 1, 2, 1

   D. 2, 1, 1

5. Integrate the expression $(2x + 1)^3$ with respect to x.

   A. $\dfrac{(2x+1)^3}{8} + k$

   B. $\dfrac{(2x+1)^4}{8} + k$

   C. $\dfrac{(2x+1)^4}{6} + k$

   D. $\dfrac{(2x+1)^2}{8} + k$

6. Evaluate $\displaystyle\int (4x^{-3} - 7x^2 + 5x - 6)\ \mathrm{d}x$

   A. $-2x^{-2} - \dfrac{7}{3}x^3 + \dfrac{5}{2}x^2 - 6x + C$

   B. $2x^2 + \dfrac{7}{3}x^3 + 5x^2 - 6 + C$

   C. $12x^2 + 14x - 5 + C$

   D. $-12x^{-4} - 14x + 5 + C$

7. Evaluate $\displaystyle\int_{-1}^{2} \left(2x^2 + x\right)\ \mathrm{d}x$

   A. $4\frac{1}{2}$

   B. $3\frac{1}{2}$

   C. $7\frac{1}{2}$

   D. $5\frac{1}{4}$

8. Integrate $\dfrac{x^2 - \sqrt{x}}{x}$ with respect to x.

   A. $\dfrac{x^2}{2} - 2\sqrt{x} + k$

   B. $\dfrac{2(x^2 - x)}{3x} + k$

   C. $\dfrac{x^2}{2} - \sqrt{x} + k$

   D. $\dfrac{x^2 - x}{3x} + k$

9. Evaluate $\displaystyle\int_{-1}^{1} (2x + 1)^2\ \mathrm{d}x$

   A. $3\frac{2}{3}$

   B. 4

   C. $4\frac{1}{3}$

   D. $4\frac{2}{3}$

10. Evaluate $\displaystyle\int (\cos 4x + \sin 3x)\ \mathrm{d}x$

    A. $\sin 4x - \cos 3x + k$

    B. $\sin 4x + \cos 3x + k$

    C. $\dfrac{1}{4}\sin 4x - \dfrac{1}{3}\cos 3x + k$

    D. $\dfrac{1}{4}\sin 4x + \dfrac{1}{3}\cos 3x + k$

11. Evaluate $\displaystyle\int_{0}^{\frac{\pi}{2}} \sin x\ \mathrm{d}x$

    A. -2

    B. -1

    C. 1

    D. 2

12. Evaluate $\int_1^2 \frac{5}{x}\,\mathrm{d}x$

A. 1.47
B. 2.67
C. 3.23
D. 3.47

13. Evaluate the integral $\int_{\frac{\pi}{12}}^{\frac{\pi}{4}} 2\cos 2x\,\mathrm{d}x$

A. $-\frac{1}{2}$
B. -1
C. $\frac{1}{2}$
D. 1

14. Evaluate $\int (2x+3)^{\frac{1}{2}}\,\mathrm{d}x$

A. $\frac{1}{12}(2x+3)^6 + k$
B. $\frac{1}{3}(2x+3)^{\frac{1}{2}} + k$
C. $\frac{1}{3}(2x+3)^{\frac{3}{2}} + k$
D. $\frac{1}{12}(2x+3)^{\frac{3}{4}} + k$

15. Evaluate $\int \left(\sin x - 5x^2\right)\,\mathrm{d}x$

A. $-\cos x - 10x + k$
B. $\cos x - \frac{5x^3}{3} + k$
C. $-\cos x - \frac{5x^3}{3} + k$
D. $\cos x - 10x + k$

16. Evaluate $\int \sin 2x\,\mathrm{d}x$

A. $\cos 2x + k$
B. $\frac{1}{2}\cos 2x + k$
C. $-\frac{1}{2}\cos 2x + k$
D. $-\cos 2x + k$

17. If $y = x(x^4 + x + 1)$, evaluate $\int_0^1 y\,\mathrm{d}x$

A. $\frac{11}{12}$
B. 1
C. $\frac{5}{6}$
D. 0

18. Evaluate $\int_2^{\pi} (\sec^2 x - \tan^2 x)\,\mathrm{d}x$

A. $\frac{\pi}{2}$
B. $\frac{\pi}{3}$
C. $\pi - 2$
D. $\pi + 2$

19. Evaluate $\int_0^{\frac{\pi}{4}} (\sin x - \cos x)\,\mathrm{d}x$

A. $\sqrt{2} + 1$
B. $\sqrt{2} - 1$
C. $1 - \sqrt{2}$
D. $-\sqrt{2} - 1$

20. Evaluate $\int_{-2}^{1} (x-1)^2\,\mathrm{d}x$

A. $-\frac{10}{3}$
B. 7
C. 9
D. 11

21. A function $f(x)$ passes through the origin and its first derivative is $3x + 2$. What is $f(x)$?

A. $f(x) = \frac{3x^2}{2} + 2x$
B. $f(x) = \frac{3x^2}{2} + x$
C. $f(x) = 3x^2 + \frac{x}{2}$
D. $f(x) = 3x^2 + 2x$

22. Evaluate $\int_2^3 \left(x^2 - 2x\right)\,\mathrm{d}x$

A. 4
B. 2
C. $\frac{4}{3}$
D. $\frac{1}{3}$

23. Evaluate $\int_{-4}^{0} (1 - 2x)\,\mathrm{d}x$

A. -20

B. -16

C. 10

D. 20

24. Evaluate $\int_1^2 \left(6x^2 - 2x\right)\,\mathrm{d}x$

A. 11

B. 12

C. 13

D. 16

25. Evaluate $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos x\,\mathrm{d}x$

A. 0

B. 1

C. 2

D. 3

26. Evaluate $\int_0^2 \left(x^3 + x^2\right)\,\mathrm{d}x$

A. $4\frac{5}{6}$

B. $6\frac{2}{3}$

C. $1\frac{5}{6}$

D. $2\frac{5}{6}$

27. Evaluate $\int_1^2 2\,\mathrm{d}x$

A. 3

B. 5

C. 2

D. 6

28. Evaluate $\int_1^2 \left(x^2 - 4x\right)\,\mathrm{d}x$

A. $\dfrac{11}{3}$

B. $\dfrac{3}{11}$

C. $-\dfrac{3}{11}$

D. $-\dfrac{11}{3}$

29. Evaluate $\int (\sin x + 2)\,\mathrm{d}x$

A. $\cos x + x^2 + k$

B. $\cos x + 2x + k$

C. $-\cos x + x^2 + k$

D. $-\cos x + 2x + k$

30. Evaluate $\int \cos 4x\,\mathrm{d}x$

A. $\dfrac{3}{4}\sin 4x + k$

B. $-\dfrac{1}{4}\sin 4x + k$

C. $-\dfrac{3}{4}\sin 4x + k$

D. $\dfrac{1}{4}\sin 4x + k$

31. Integrate $\dfrac{1+x}{x^3}$ with respect to x.

A. $2x^2 - \dfrac{1}{x} + k$

B. $x^2 - \dfrac{1}{x} + k$

C. $-\dfrac{x^2}{2} - \dfrac{1}{x} + k$

D. $-\dfrac{1}{2x^2} - \dfrac{1}{x} + k$

32. Evaluate $\int \left(x^2 + 3x - 5\right)\,\mathrm{d}x$

A. $\dfrac{x^3}{3} - \dfrac{3x^2}{2} - 5x + k$

B. $\dfrac{x^3}{3} - \dfrac{3x^2}{2} + 5x + k$

C. $\dfrac{x^3}{3} + \dfrac{3x^2}{2} - 5x + k$

D. $\dfrac{x^3}{3} + \dfrac{3x^2}{2} + 5x + k$

33. Integrate $\dfrac{2x^3 + 2x}{x}$ with respect to x.

A. $\dfrac{2x^3}{3} - 2x + k$

B. $\dfrac{2x^3}{3} + 2x + k$

C. $x^3 - 2x + k$

D. $x^3 + 2x + k$

34. Evaluate $\int \left(5x^3 + 7x^2 - 2x + 5\right)\,\mathrm{d}x$

A. $\dfrac{5x^4}{4} + \dfrac{7x^3}{3} + 2x + C$

B. $\frac{5x^4}{4} + \frac{7x^3}{3} - x^2 + 5x + C$

C. $\frac{5x^3}{3} + \frac{7x^2}{2} - x + C$

D. $\frac{2x^2}{3} + \frac{x}{5} - C$

35. Find the value of $\int_0^{\pi} \frac{\cos^2\theta - 1}{\sin^2\theta} \mathrm{d}\theta$

A. π

B. $-\pi$

C. $\frac{\pi}{2}$

D. $-\frac{\pi}{2}$

36. The area enclosed by $y = x^2 - 1$, $y = 3$, and $x \geq 0$ is revolved around the y-axis. If the volume is $K\pi$, find K.

A. 7

B. $\frac{15}{2}$

C. 8

D. $\frac{17}{2}$

37. Evaluate $\int (2x-5)^4 \mathrm{d}x$

A. $\frac{(2x-5)^5}{5} + C$

B. $\frac{(2x-5)^5}{10} + C$

C. $8(2x-5)^3 + C$

D. $\frac{(2x-5)^3}{6} + C$

38. Find $\int e^{3x} \mathrm{d}x$

A. $3e^{3x} + C$

B. $e^{3x} + C$

C. $\frac{1}{3}e^{3x} + C$

D. $\frac{1}{3}e^{x} + C$

39. Evaluate $\int_0^1 (x^2 - x + 1) \mathrm{d}x$

A. $\frac{1}{6}$

B. $\frac{5}{6}$

C. 1

D. $\frac{7}{6}$

40. Find $\int \sec^2(3x) \mathrm{d}x$

A. $\tan(3x) + C$

B. $\frac{1}{3}\tan(3x) + C$

C. $3\tan(3x) + C$

D. $\sec(3x)\tan(3x) + C$

41. Evaluate $\int_1^e \frac{1}{x} \mathrm{d}x$

A. 0

B. 1

C. e

D. $e - 1$

42. Find the area under the curve $y = x^2$ from $x = 0$ to $x = 3$.

A. 3

B. 6

C. 9

D. 27

43. Evaluate $\int \frac{2}{x+1} \mathrm{d}x$

A. $2\ln|x+1| + C$

B. $\ln|x+1| + C$

C. $\frac{-2}{(x+1)^2} + C$

D. $2\arctan(x) + C$

44. If $\frac{\mathrm{d}y}{\mathrm{d}x} = 2x - 3$ and $y = 2$ when $x = 1$, find y in terms of x.

A. $y = x^2 - 3x + 2$

B. $y = x^2 - 3x + 4$

C. $y = 2x^2 - 3x + 3$

D. $y = x^2 - 3x$

45. Evaluate $\int_0^{\pi} \cos x \mathrm{d}x$

A. 0

B. 1

C. -1

D. 2

46. Find $\int (3-4x)^{-2} \mathrm{d}x$

A. $\dfrac{(3-4x)^{-1}}{-4}+C$

B. $\dfrac{(3-4x)^{-1}}{4}+C$

C. $\dfrac{(3-4x)^{-3}}{-12}+C$

D. $-4(3-4x)^{-3}+C$

47. What is the area bounded by the curve $y=4-x^2$ and the x-axis?

A. $\dfrac{8}{3}$

B. $\dfrac{16}{3}$

C. $\dfrac{32}{3}$

D. 16

48. Evaluate $\displaystyle\int_0^2 (3x^2+4x-5)\mathrm{d}x$

A. 2

B. 4

C. 6

D. 8

49. Find $\displaystyle\int (x+1)(x-2)\mathrm{d}x$

A. $\dfrac{x^3}{3}-\dfrac{x^2}{2}-2x+C$

B. $\dfrac{x^3}{3}+\dfrac{x^2}{2}-2x+C$

C. $x^2-x-2+C$

D. $\dfrac{(x+1)^2(x-2)^2}{4}+C$

50. Evaluate $\displaystyle\int_1^4 \sqrt{x}\mathrm{d}x$

A. $\dfrac{7}{3}$

B. $\dfrac{14}{3}$

C. 2

D. 3

51. The gradient of a curve is given by $4x+3$. If the curve passes through the point $(1,5)$, find its equation.

A. $y=2x^2+3x$

B. $y=2x^2+3x+5$

C. $y=2x^2+3x-0$

D. $y=4x^2+3x-2$

52. Evaluate $\displaystyle\int e^{-x/2}\mathrm{d}x$

A. $-2e^{-x/2}+C$

B. $-\dfrac{1}{2}e^{-x/2}+C$

C. $2e^{-x/2}+C$

D. $e^{-x/2}+C$

53. Find the area enclosed by the curve $y=x^3$, the x-axis, and the lines $x=1$ and $x=2$.

A. $\dfrac{7}{4}$

B. $\dfrac{15}{4}$

C. 4

D. 7

54. Evaluate $\displaystyle\int_0^{\pi/6} \sec x\tan x\mathrm{d}x$

A. $\dfrac{2\sqrt{3}}{3}-1$

B. $1-\dfrac{2\sqrt{3}}{3}$

C. $\dfrac{\sqrt{3}}{3}$

D. $2-\sqrt{3}$

55. If $\displaystyle\int_0^a (2x+1)\mathrm{d}x=4$, find the positive value of a.

A. 1

B. $\dfrac{3}{2}$

C. 2

D. 3

56. Find $\displaystyle\int \frac{x^2+1}{x^2}\mathrm{d}x$

A. $x-\dfrac{1}{x}+C$

B. $x+\dfrac{1}{x}+C$

C. $1-\dfrac{2}{x^3}+C$

D. $2x+C$

57. Evaluate $\displaystyle\int (1-x)^3\mathrm{d}x$

A. $\dfrac{(1-x)^4}{4}+C$

B. $-\dfrac{(1-x)^4}{4}+C$

C. $-3(1-x)^2+C$

D. $3(1-x)^2 + C$

58. The area bounded by the curve $y = x$, the x-axis, $x = 0$ and $x = 2$ is revolved around the x-axis. Find the volume of the solid generated.

A. $\frac{2\pi}{3}$

B. $\frac{4\pi}{3}$

C. $\frac{8\pi}{3}$

D. 4π

59. Evaluate $\int_0^{\pi/3} \sin(3x)\mathrm{d}x$

A. 0

B. $\frac{1}{3}$

C. $\frac{2}{3}$

D. 1

60. Evaluate $\int (x^2+1)^2 dx$

A. $\frac{x^5}{5} + \frac{2x^3}{3} + x + C$

B. $\frac{(x^2+1)^3}{3} + C$

C. $\frac{x^5}{5} + x^3 + x + C$

D. $\frac{(x^2+1)^3}{6x} + C$

61. If $\int_1^k \frac{1}{x^2} dx = \frac{1}{2}$, find the value of k.

A. 1

B. 2

C. $\frac{1}{2}$

D. 4

62. Find the indefinite integral of $\sec^2 x e^{\tan x}$.

A. $e^{\tan x} + C$

B. $\tan x e^{\tan x} + C$

C. $\sec x e^{\tan x} + C$

D. $e^{\sec^2 x} + C$

63. Evaluate $\int \frac{\ln x}{x} dx$.

A. $\ln|\ln x| + C$

B. $(\ln x)^2 + C$

C. $\frac{1}{2}(\ln x)^2 + C$

D. $\frac{1}{x^2} + C$

64. The area of the region bounded by $y = e^x$, the x-axis, $x = 0$ and $x = 1$ is

A. e

B. $e - 1$

C. $1 - e$

D. 1

65. Evaluate $\int 2^x \mathrm{d}x$

A. $2^x + C$

B. $\frac{2^x}{\ln 2} + C$

C. $2^x \ln 2 + C$

D. $x2^{x-1} + C$

66. Evaluate $\int_0^1 (e^x + e^{-x})\mathrm{d}x$

A. $e - \frac{1}{e}$

B. $e + \frac{1}{e}$

C. $e - \frac{1}{e} - 2$

D. 0

67. Find $\int \frac{1}{2x+3}\mathrm{d}x$

A. $\ln|2x+3| + C$

B. $2\ln|2x+3| + C$

C. $\frac{1}{2}\ln|2x+3| + C$

D. $\frac{-1}{(2x+3)^2} + C$

68. Find the area bounded by $y = \sin x$, the x-axis, from $x = 0$ to $x = \pi$.

A. 0

B. 1

C. 2

D. π

69. Evaluate $\int x e^{x^2} \mathrm{d}x$.

A. $e^{x^2} + C$

B. $x^2 e^{x^2} + C$

C. $\frac{1}{2}e^{x^2} + C$

D. $2e^{x^2} + C$

70. Evaluate $\int_0^{\pi/4} \tan x \sec^2 x \mathrm{d}x$

A. $\frac{1}{4}$

B. $\frac{1}{2}$

C. 1

D. 2

71. Find $\int \cos^2 x \mathrm{d}x$ (Hint: $\cos 2x = 2\cos^2 x - 1$)

A. $\frac{x}{2} + \frac{\sin 2x}{4} + C$

B. $\frac{x}{2} - \frac{\sin 2x}{4} + C$

C. $x + \sin 2x + C$

D. $\frac{\cos^3 x}{3} + C$

72. If $\int_0^b x \mathrm{d}x = 8$, find $b > 0$.

A. 2

B. 4

C. 8

D. 16

73. Evaluate $\int \frac{1}{\sqrt{1-x^2}} \mathrm{d}x$.

A. $\arcsin x + C$

B. $\arccos x + C$

C. $\ln|\sqrt{1-x^2}| + C$

D. $2\sqrt{1-x^2} + C$

74. The volume generated by revolving the area under $y = \sqrt{x}$ from $x = 0$ to $x = 4$ about the x-axis is:

A. 4π

B. 8π

C. 16π

D. $\frac{8\pi}{3}$

75. Evaluate $\int_{-1}^{1} x^3 \mathrm{d}x$.

A. 0

B. $\frac{1}{4}$

C. $\frac{1}{2}$

D. 1

76. Find $\int \frac{e^x}{1+e^x} \mathrm{d}x$.

A. $e^x \ln|1 + e^x| + C$

B. $\ln(1 + e^x) + C$

C. $\frac{e^{2x}}{2 + e^x} + C$

D. $\arctan(e^x) + C$

77. Evaluate $\int_1^2 (x + \frac{1}{x})^2 \mathrm{d}x$.

A. $\frac{29}{6}$

B. $\frac{17}{3}$

C. 5

D. $\frac{10}{3}$

78. If $\frac{\mathrm{d}y}{\mathrm{d}x} = \sin x + x$ and $y(0) = 1$, find y.

A. $y = \cos x + \frac{x^2}{2}$

B. $y = -\cos x + \frac{x^2}{2} + 1$

C. $y = -\cos x + \frac{x^2}{2} + 2$

D. $y = \cos x + \frac{x^2}{2} + 1$

79. Evaluate $\int 5 \mathrm{d}x$.

A. $5 + C$

B. $5x + C$

C. $\frac{x^2}{2} + 5x + C$

D. C

80. Evaluate $\int_0^2 |x - 1| \mathrm{d}x$.

A. 0

B. $\frac{1}{2}$

C. 1

D. 2

81. Find $\int \frac{1}{x \ln x} \mathrm{d}x$.

A. $(\ln x)^2 + C$

B. $\ln|\ln x| + C$

C. $\frac{1}{(\ln x)^2} + C$

D. $\ln x^2 + C$

82. The area bounded by $y = 2x$, the x-axis, $x = 1$ and $x = 3$ is:

A. 4

B. 6

C. 8

D. 10

83. Evaluate $\int \sin^2 x \cos x \mathrm{d}x$.

A. $\frac{\sin^3 x}{3} + C$

B. $\frac{\cos^3 x}{3} + C$

C. $\sin x \cos x + C$

D. $2 \sin x \cos x + C$

84. $\int_e^{e^2} \frac{\mathrm{d}x}{x \ln x}$.

A. 1

B. $\ln 2$

C. 2

D. e

85. Find $\int (x+1)^5 \mathrm{d}x$.

A. $5(x+1)^4 + C$

B. $\frac{(x+1)^6}{6} + C$

C. $(x+1)^6 + C$

D. $\frac{x^6}{6} + x^5 + \cdots + C$

86. Evaluate $\int_0^{\pi/2} \cos^3 x \sin x \mathrm{d}x$.

A. $\frac{1}{4}$

B. $\frac{1}{3}$

C. 0

D. 1

87. The value of $\int_0^1 \frac{dx}{1+x^2}$ is:

A. π

B. $\frac{\pi}{2}$

C. $\frac{\pi}{4}$

D. 1

88. Integrate $\sqrt{ax+b}$ with respect to x.

A. $\frac{1}{2a\sqrt{ax+b}} + C$

B. $\frac{2}{3a}(ax+b)^{3/2} + C$

C. $\frac{1}{a}(ax+b)^{3/2} + C$

D. $\frac{2}{a}(ax+b)^{1/2} + C$

89. If $f'(x) = x - \frac{1}{x^2}$ and $f(1) = \frac{1}{2}$, find $f(x)$.

A. $\frac{x^2}{2} + \frac{1}{x} + 1$

B. $\frac{x^2}{2} - \frac{1}{x} + 1$

C. $\frac{x^2}{2} + \frac{1}{x} - 1$

D. $\frac{x^2}{2} - \frac{1}{x} - \frac{1}{2}$

90. Evaluate $\int_{-2}^{2} (x^3 + \sin x) \mathrm{d}x$.

A. 0

B. 4

C. 8

D. 16/3

91. Find $\int x\sqrt{x^2+1} \mathrm{d}x$.

A. $\frac{1}{2}(x^2+1)^{3/2} + C$

B. $\frac{1}{3}(x^2+1)^{3/2} + C$

C. $(x^2+1)^{1/2} + C$

D. $x^2\sqrt{x^2+1} + C$

92. What is $\int_a^b f(x)\mathrm{d}x + \int_b^c f(x)\mathrm{d}x$?

A. $\int_a^c f(x)\mathrm{d}x$

B. $\int_c^a f(x)\mathrm{d}x$

C. $\int_b^a f(x)\mathrm{d}x + \int_c^b f(x)\mathrm{d}x$

D. 0

93. Evaluate $\int (e^{2x} + e^{-2x})^2 \mathrm{d}x$.

A. $\frac{1}{4}e^{4x} + 2x - \frac{1}{4}e^{-4x} + C$

B. $\frac{1}{2}e^{4x} + 2x - \frac{1}{2}e^{-4x} + C$

C. $e^{4x} + 2 + e^{-4x} + C$

D. $\frac{(e^{2x} + e^{-2x})^3}{3} + C$

94. Find the area between the curves $y = x^2$ and $y = x$.

A. $\frac{1}{3}$

B. $\frac{1}{6}$

C. $\frac{1}{2}$

D. 1

95. $\int \tan^2 x \mathrm{d}x$.

A. $\sec^2 x - x + C$

B. $\tan x - x + C$

C. $\frac{\tan^3 x}{3} + C$

D. $\ln|\sec x| + C$

96. Evaluate $\int_0^{\ln 2} e^x \mathrm{d}x$.

A. 1

B. 2

C. $2 - \ln 2$

D. $\ln 2$

97. Find $\int \frac{\cos x}{1 + \sin^2 x} \mathrm{d}x$.

A. $\ln(1 + \sin^2 x) + C$

B. $\arctan(\sin x) + C$

C. $\arcsin(\cos x) + C$

D. $\frac{-\sin x}{(1 + \sin^2 x)^2} + C$

98. Find the volume of the solid generated by revolving the region bounded by $y = \frac{1}{x}$, the x-axis, from $x = 1$ to $x = 2$.

A. π

B. $\frac{\pi}{2}$

C. $\frac{\pi}{3}$

D. 2π

99. Evaluate $\int \frac{2x + 3}{x^2 + 3x + 5} \mathrm{d}x$.

A. $2\ln|x^2 + 3x + 5| + C$

B. $\ln|x^2 + 3x + 5| + C$

C. $\frac{1}{2}\ln|x^2 + 3x + 5| + C$

D. $\arctan(x^2 + 3x + 5) + C$

100. Evaluate $\int_0^1 x(x^2 + 1)^3 \mathrm{d}x$.

A. $\frac{15}{8}$

B. $\frac{7}{4}$

C. 22

D. $\frac{17}{8}$

Chapter 5

Combinatorics

5.1 Combination & Permutation

5.1.1 Questions

1. Ralia has 7 different posters to be hung in her bedroom, living room, and kitchen. Assuming she has plans to plant at least a poster in each of the 3 rooms, how many choices does she have?

   A. 49

   B. 170

   C. 210

   D. 21

2. In how many ways can a committee of 2 women and 3 men be chosen from 6 men and 5 women?

   A. 200

   B. 100

   C. 50

   D. 30

3. In how many ways can the letters of the word MATHEMATICS be arranged?

   A. $\frac{11!}{9!2!}$

   B. $\frac{11!}{9!2!2!}$

   C. $\frac{11!}{2!2!2!}$

   D. $\frac{11!}{2!2!}$

4. In how many ways can the letters of the word ACCEPTANCE be arranged?

   A. $\frac{10!}{2!2!3!}$

   B. $\frac{10!}{2!2!}$

   C. $10!$

   D. $\frac{10!}{2!3!}$

5. Five people are to be arranged in a row for a group photograph. How many arrangements are there if a married couple in the group insist on sitting next to each other?

   A. 48

   B. 12

   C. 7

   D. 10

6. In how many ways can 6 subjects be selected from 10 subjects for an examination

   A. 215

   B. 218

   C. 216

   D. 210

7. In how many ways can a delegation of 3 be chosen from 5 men and 3 women, if atleast 1 man and 1 woman must be included?

   A. 28

   B. 30

   C. 15

   D. 45

8. Find the number of ways of selecting 6 out of 10 subjects for an examination

   A. 218

   B. 216

C. 210

D. 215

9. In how many ways can the letters of the word ELATION be arranged?

   A. 6!

   B. 7!

   C. 5!

   D. 8!

10. In how many ways can the letters of the word CALCULUS be arranged?

    A. 1680

    B. 2100

    C. 5040

    D. 1760

11. In how many ways can the letters of the word COMBINATION be arranged?

    A.

    B.

    C.

    D.

12. In how many ways can 7 directors sit round a table?

    A. 24

    B. 5040

    C. 120

    D. 120

13. In how many ways can the letters of the word TOTALITY be arranged?

    A. 6720

    B. 6270

    C. 6207

    D. 6027

14. How many numbers greater than 1000 can be made from the digits 1, 2, 3, 4, and 5 without repeating any one of them?

    A. 152

    B. 210

    C. 216

    D. 144

15. In how many ways can a team of 3 girls be selected from 7 girls?

    A. $\frac{7!}{5!2!}$

    B. $\frac{7!}{3!}$

    C. $\frac{7!}{3!4!}$

    D. $\frac{7!}{4!}$

16. In how many ways can a student select 2 subjects from 5 subjects?

    A. $\frac{5!}{3!2!}$

    B. $\frac{5!}{2!2!}$

    C. $\frac{5!}{2!3!}$

    D. $\frac{5!}{2!}$

17. In how many ways can five people sit round a circular table?

    A. 24

    B. 60

    C. 12

    D. 120

18. How many two-digit numbers can be formed from the digits 0, 1, 2, and 3 if a digit can be repeated and no number may begin with 0?

    A. 4

    B. 12

    C. 16

    D. 20

19. In how many ways can 9 people be seated if 3 chairs are available?

    A. 720

    B. 504

    C. 336

    D. 210

20. A final examination requires that a student answer any 4 out of 6 questions. In how many ways can this be done?

    A. 15

    B. 20

    C. 45

    D. 30

21. In how many ways can 6 coloured chalks be arranged if 2 are of the same colour?

A. 60

B. 240

C. 120

D. 360

22. How many possible ways are there of seating seven people P,Q,R,S,T,U, and V at a circular table?

A. 720

B. 2520

C. 5040

D. 360

23. A committee of six is to be formed by a state governor from nine state commissioners and three members of the State House of Assembly. In how many ways can the members of the committee be chosen so as to inlcude one member of the House of the Assembly?

A. 924 ways

B. 524 ways

C. 462 ways

D. 378 ways

24. How many two-digit numbers can be formed from the digits 0,1, 2, and 3 if a digit can be repeated and no number may begin with 0?

A. 4

B. 12

C. 16

D. 20

25. Find the number of committees of three that can be formed consisting of two men and one woman from four men and three women

A. 3

B. 6

C. 18

D. 24

26. A.

B.

C.

D.

27. A.

B.

C.

D.

28. A.

B.

C.

D.

29. A.

B.

C.

D.

30. A.

B.

C.

D.

31. A.

B.

C.

D.

32. A.

B.

C.

D.

33. A.

B.

C.

D.

34. A.

B.

C.

D.

35. A.

B.

C.

D.

36. A.

B.

C.

D.

37. A.

B.

C.

D.

38. A.

B.
C.
D.

39. A.
B.
C.
D.

40. A.
B.
C.
D.

41. A.
B.
C.
D.

42. A.
B.
C.
D.

43. A.
B.
C.
D.

44. A.
B.
C.
D.

45. A.
B.
C.
D.

46. A.
B.
C.
D.

47. A.
B.
C.
D.

48. A.
B.
C.
D.

49. A.
B.
C.
D.

50. A.
B.
C.
D.

51. A.
B.
C.
D.

52. A.
B.
C.
D.

53. A.
B.
C.
D.

54. A.
B.
C.
D.

55. A.
B.
C.
D.

56. A.
B.
C.
D.

57. A.
B.
C.
D.

58. A.
B.

C.
D.

59. A.
B.
C.
D.

60. A.
B.
C.
D.

61. A.
B.
C.
D.

62. A.
B.
C.
D.

63. A.
B.
C.
D.

64. A.
B.
C.
D.

65. A.
B.
C.
D.

66. A.
B.
C.
D.

67. A.
B.
C.
D.

68. A.
B.
C.
D.

69. A.
B.
C.
D.

70. A.
B.
C.
D.

71. A.
B.
C.
D.

72. A.
B.
C.
D.

73. A.
B.
C.
D.

74. A.
B.
C.
D.

75. A.
B.
C.
D.

76. A.
B.
C.
D.

77. A.
B.
C.
D.

78. A.
B.
C.

D.

79. A.
B.
C.
D.

80. A.
B.
C.
D.

81. A.
B.
C.
D.

82. A.
B.
C.
D.

83. A.
B.
C.
D.

84. A.
B.
C.
D.

85. A.
B.
C.
D.

86. A.
B.
C.
D.

87. A.
B.
C.
D.

88. A.
B.
C.
D.

89. A.
B.
C.
D.

90. A.
B.
C.
D.

91. A.
B.
C.
D.

92. A.
B.
C.
D.

93. A.
B.
C.
D.

94. A.
B.
C.
D.

95. A.
B.
C.
D.

96. A.
B.
C.
D.

97. A.
B.
C.
D.

98. A.
B.
C.
D.

99. A.
    B.
    C.
    D.

100. A.
     B.
     C.
     D.

5.2 Probability

5.2.1 Questions

1. A.
   B.
   C.
   D.
2. A.
   B.
   C.
   D.
3. A.
   B.
   C.
   D.
4. A.
   B.
   C.
   D.
5. A.
   B.
   C.
   D.
6. A.
   B.
   C.
   D.
7. A.
   B.
   C.
   D.
8. A.
   B.
   C.
   D.
9. A.
   B.
   C.
   D.
10. A.
    B.
    C.
    D.
11. A.
    B.
    C.
    D.
12. A.
    B.
    C.
    D.
13. A.
    B.
    C.
    D.
14. A.
    B.
    C.
    D.
15. A.
    B.
    C.
    D.
16. A.
    B.
    C.
    D.
17. A.
    B.
    C.
    D.
18. A.
    B.
    C.
    D.
19. A.
    B.

C.
D.

20. A.
B.
C.
D.

21. A.
B.
C.
D.

22. A.
B.
C.
D.

23. A.
B.
C.
D.

24. A.
B.
C.
D.

25. A.
B.
C.
D.

26. A.
B.
C.
D.

27. A.
B.
C.
D.

28. A.
B.
C.
D.

29. A.
B.
C.
D.

30. A.
B.
C.
D.

31. A.
B.
C.
D.

32. A.
B.
C.
D.

33. A.
B.
C.
D.

34. A.
B.
C.
D.

35. A.
B.
C.
D.

36. A.
B.
C.
D.

37. A.
B.
C.
D.

38. A.
B.
C.
D.

39. A.
B.
C.

D.

40. A.
B.
C.
D.

41. A.
B.
C.
D.

42. A.
B.
C.
D.

43. A.
B.
C.
D.

44. A.
B.
C.
D.

45. A.
B.
C.
D.

46. A.
B.
C.
D.

47. A.
B.
C.
D.

48. A.
B.
C.
D.

49. A.
B.
C.
D.

50. A.
B.
C.
D.

51. A.
B.
C.
D.

52. A.
B.
C.
D.

53. A.
B.
C.
D.

54. A.
B.
C.
D.

55. A.
B.
C.
D.

56. A.
B.
C.
D.

57. A.
B.
C.
D.

58. A.
B.
C.
D.

59. A.
B.
C.
D.

60. A.
    B.
    C.
    D.
61. A.
    B.
    C.
    D.
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Chapter 6

Statistics

6.1 Measures of Central Tendency

6.1.1 Questions

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Chapter 7

Values To Memorize

7.1 Square Roots

- $\sqrt{1} = 1$
- $\sqrt{2} = 1.4142$
- $\sqrt{3} = 1.7321$
- $\sqrt{4} = 2$
- $\sqrt{5} = 2.2361$
- $\sqrt{6} = 2.4495$
- $\sqrt{7} = 2.6458$
- $\sqrt{8} = 2.8284$
- $\sqrt{9} = 3$
- $\sqrt{10} = 3.1623$

7.2 Squares

- $1^2 = 1$
- $2^2 = 4$
- $3^2 = 9$
- $4^2 = 16$
- $5^2 = 25$
- $6^2 = 36$
- $7^2 = 49$
- $8^2 = 64$
- $9^2 = 81$
- $10^2 = 100$
- $11^2 = 121$
- $12^2 = 144$
- $13^2 = 169$
- $14^2 = 196$
- $15^2 = 225$
- $16^2 = 256$
- $17^2 = 289$
- $18^2 = 324$
- $19^2 = 361$
- $20^2 = 400$
- $21^2 = 441$
- $22^2 = 484$
- $23^2 = 529$
- $24^2 = 576$
- $25^2 = 625$
- $26^2 = 676$
- $27^2 = 729$
- $28^2 = 784$
- $29^2 = 841$
- $30^2 = 900$

7.3 Cubes

- $1^3 = 1$
- $2^3 = 8$
- $3^3 = 27$
- $4^3 = 64$
- $5^3 = 125$
- $6^3 = 216$
- $7^3 = 343$
- $8^3 = 512$
- $9^3 = 729$
- $10^3 = 1000$
- $11^3 = 1331$
- $12^3 = 1728$
- $13^3 = 2197$
- $14^3 = 2744$
- $15^3 = 3375$
- $16^3 = 4096$
- $17^3 = 4913$
- $18^3 = 5832$
- $19^3 = 6859$
- $20^3 = 8000$
- $21^3 = 9261$
- $22^3 = 10648$
- $23^3 = 12167$
- $24^3 = 13824$
- $25^3 = 15625$
- $26^3 = 17576$
- $27^3 = 19683$
- $28^3 = 21952$
- $29^3 = 24389$
- $30^3 = 27000$

7.4 Logarithms

- $\log_{10} 1 = 0$
- $\log_{10} 2 = 0.3010$
- $\log_{10} 3 = 0.4771$
- $\log_{10} 4 = 0.6020$
- $\log_{10} 5 = 0.699$
- $\log_{10} 6 = 0.778$
- $\log_{10} 7 = 0.845$
- $\log_{10} 8 = 0.903$
- $\log_{10} 9 = 0.954$
- $\log_{10} 10 = 1$