

# STA112 ASSIGNMENT 1 SOLUTION

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1. **Solution:** First, count total distinct permutations of "ACKNOWLEDGEMENT":

- Total letters = 13
- Repeats: E(3), C(1), K(1), etc.
- Total permutations =  $\frac{13!}{3!}$  (since E repeats 3 times)

Now count permutations where all 4 vowels (A, E, E, E) come together:

- Treat the vowels as one "super letter"
- Total items to arrange = 10 (9 consonants + 1 vowel block)
- Permutations =  $\frac{10!}{3!}$  (since E repeats within the block)

$$\text{Final answer} = \text{Total permutations} - \text{Vowels together} = \frac{13!}{3!} - \frac{10!}{3!}$$

2. **Solution:** Total 3-digit numbers = 900 (from 100 to 999) Numbers with **no** 7s:

- Hundreds digit: 8 choices (1-9 except 7)
- Tens digit: 9 choices (0-9 except 7)
- Units digit: 9 choices (0-9 except 7)
- Total without 7s =  $8 \times 9 \times 9 = 648$

$$\text{Numbers with at least one 7} = \text{Total} - \text{Without any 7s} = 900 - 648 = 252$$

3. **Solution:** We have:

- Houses with  $\leq 2$  flats: 52
- Houses with  $> 2$  flats: 35

Cases:

- (a) 18 houses with  $\leq 2$  flats and 2 with  $> 2$  flats:  $\binom{52}{18} \binom{35}{2}$
- (b) 19 houses with  $\leq 2$  flats and 1 with  $> 2$  flats:  $\binom{52}{19} \binom{35}{1}$
- (c) 20 houses with  $\leq 2$  flats:  $\binom{52}{20}$

$$\text{Total ways} = \text{Sum of all cases} = \binom{52}{18} \binom{35}{2} + \binom{52}{19} \binom{35}{1} + \binom{52}{20}$$

4. **Solution:**

- Choose 1 Ace from 4:  $\binom{4}{1}$
- Choose 4 non-Aces from remaining 48 cards:  $\binom{48}{4}$
- Total combinations =  $\binom{4}{1} \times \binom{48}{4}$

5. **Solution:** Total members = 7 girls + 4 boys = 11

- (a) No boys (all girls):  $\binom{7}{5}$
- (b) At least 1 girl and 1 boy: Total ways - All girls - All boys =  $\binom{11}{5} - \binom{7}{5} - \binom{4}{5}$  (Note:  $\binom{4}{5} = 0$  since we can't choose 5 boys from 4)
- (c) At least 3 boys:  $\binom{4}{3}\binom{7}{2} + \binom{4}{4}\binom{7}{1}$  (3 boys + 2 girls or 4 boys + 1 girl)