

APPLICATIONS OF INTEGRATION

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1 Examples

1. Find the area of the region bounded by the x-axis and the graph of $f(x) = x^2 - 1$ for $1 \leq x \leq 2$.
2. Evaluate the definite integral $\int_0^1 (4t + 1)^2 dt$.

2 ClassWork Problems

2.1 Finding Areas

Find the area of the shaded region described by the function and bounds (implicitly or explicitly shown/stated):

3. $y = x - x^2$ (Implied bounds from x-intercepts, likely 0 to 1)
4. $y = 1 - x^4$ (Implied bounds from x-intercepts, likely -1 to 1)
5. $y = \frac{1}{x^2}$ bounded by $x = 1, x = 2$, and the x-axis.
6. $y = \frac{2}{\sqrt{x}}$ bounded by $x = 1, x = 4$, and the x-axis.
7. $y = 3e^{-x/2}$ bounded by $x = 1, x = 4$, and the x-axis.
8. $y = 2e^{x/2}$ bounded by $x = 1, x = 3$, and the x-axis.
9. $y = \frac{x^2 + 4}{x}$ bounded by $x = 1, x = 4$, and the x-axis.
10. $y = \frac{x - 2}{x}$ bounded by $x = 2, x = 4$, and the x-axis.

2.2 Evaluating Definite Integrals

Evaluate the following definite integrals:

11. $\int_0^1 2x \, dx$
12. $\int_2^7 3v \, dv$

13. $\int_{-1}^0 (x - 2) \, dx$

14. $\int_2^5 (-3x + 4) \, dx$

15. $\int_{-1}^1 (2t - 1)^2 \, dt$

16. $\int_0^1 (1 - 2x^2) \, dx$

17. $\int_0^3 (x - 2)^3 \, dx$

18. $\int_2^2 (x - 3)^4 \, dx$

19. $\int_{-1}^1 (\sqrt[3]{t} - 2) \, dt$

20. $\int_1^4 \sqrt{\frac{2}{x}} \, dx$

21. $\int_1^4 \frac{u - 2}{\sqrt{u}} \, du$

22. $\int_0^1 \frac{x - \sqrt{x}}{3} \, dx$

23. $\int_{-1}^0 (t^{1/3} - t^{2/3}) \, dt$

24. $\int_0^4 (x^{1/2} + x^{1/4}) \, dx$

25. $\int_0^1 e^{-2x} \, dx$

26. $\int_1^2 e^{1-x} \, dx$

27. $\int_1^3 \frac{e^{3/x}}{x^2} \, dx$

28. $\int_{-1}^1 (e^x - e^{-x}) \, dx$

29. $\int_0^1 e^{2x} \sqrt{e^{2x} + 1} \, dx$

30. $\int_0^1 \frac{e^{-x}}{\sqrt{e^{-x} + 1}} \, dx$

31. $\int_0^2 \frac{x}{1+4x^2} \, dx$

32. $\int_0^1 \frac{e^{2x}}{e^{2x}+1} \, dx$