# INTEGRATION BY PARTS SOLUTIONS

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April 29, 2025

## 1 Introduction: Integration by Parts (IBP)

**Explanation.** Integration by Parts is a technique specifically designed to find the integral of a product of two functions. It's derived directly from the product rule for differentiation.

Recall the Product Rule: $\frac{d}{dx}[u(x)v(x)] = u'(x)v(x) + u(x)v'(x)$.

Writing this using differentials ($\mathrm{d}u = u'(x)\mathrm{d}x, \mathrm{d}v = v'(x)\mathrm{d}x$): $d(uv) = v\,\mathrm{d}u + u\,\mathrm{d}v$.

If we integrate both sides with respect to the underlying variable (like $x$):

$\int d(uv) = \int v\,\mathrm{d}u + \int u\,\mathrm{d}v$.

Since the integral of a differential is just the function itself, $\int d(uv) = uv$.

So, $uv = \int v\,\mathrm{d}u + \int u\,\mathrm{d}v$.

Rearranging this equation to solve for one of the integrals gives the **Integration by Parts Formula**:

$$\boxed{\int u\ \mathrm{d}v = uv - \int v\ \mathrm{d}u}$$

**The Goal:** The purpose of this formula is to replace one integral, $\int u\mathrm{d}v$, which might be hard to solve directly, with a different integral, $\int v\mathrm{d}u$, which is hopefully easier to solve.

**Choosing $u$ and $\mathrm{d}v$:** This selection is the most critical part of the process.

- You must split the entire original integrand (including the $\mathrm{d}x$) into two parts: $u$ and $\mathrm{d}v$.
- The part chosen as $\mathrm{d}v$ **must** be something you know how to integrate to find $v$.
- The part chosen as $u$ will be differentiated to find $\mathrm{d}u$.
- **Strategic Choice:** The ideal choice makes the **new** integral, $\int v\mathrm{d}u$, simpler than the original one. This often happens when differentiating $u$ makes it significantly simpler (like $x^2 \to 2x$), while integrating $\mathrm{d}v$ doesn't make $v$ overly complicated.
- **LIATE Mnemonic:** A very useful (though not foolproof) guideline for choosing $u$ is the acronym LIATE. Look at the types of functions in your product and choose $u$ based on the type that appears first in this list:
  - **L**: **L**ogarithmic functions (like $\ln x, \log_b x$). These are often good choices for $u$ because their derivatives ($1/x$) are simpler algebraic functions, while their integrals are more complex and often require IBP themselves.

**I**: **I**nverse trigonometric functions (like $\arcsin x, \arctan x$). Similar to logs, their derivatives are algebraic, while their integrals are harder.

**A**: **A**lgebraic functions (polynomials like $x^3, 2x-1$, roots like $\sqrt{x}$). Differentiation reduces their degree, making them simpler. Integration increases their degree.

**T**: **T**rigonometric functions (like $\sin x, \cos x, \sec^2 x$). Differentiation and integration often cycle between sin and cos, or involve other trig functions.

**E**: **E**xponential functions (like $e^x, a^x$). These generally stay similar upon differentiation or integration (up to constants).

Once $u$ is chosen according to LIATE, everything else in the integrand (including $\mathrm{d}x$) automatically becomes $\mathrm{d}v$.

## 2 Solutions: Examples and Class Work

1. **Problem:** *(Example 1)* Solve $\displaystyle\int xe^x \,\mathrm{d}x$.

   *Solution:*

   **Strategy.** We have a product of $x$ (Algebraic) and $e^x$ (Exponential). Following LIATE, Algebraic (A) comes before Exponential (E). Therefore, we choose $u = x$. This is a good choice because its derivative, $\mathrm{d}u = \mathrm{d}x$, is simpler than $u = x$. The remaining part, $e^x\mathrm{d}x$, becomes $\mathrm{d}v$, which is easy to integrate. We expect one application of IBP to solve this.

   **Method 1: Standard IBP**

   *Step 1: Define $u$ and $\mathrm{d}v$.* Let $u = x$. Let $\mathrm{d}v = e^x\mathrm{d}x$.

   *Step 2: Compute $\mathrm{d}u$ and $v$.*

   Differentiate $u$: $\mathrm{d}u = \left(\frac{d}{dx}(x)\right)\mathrm{d}x = (1)\mathrm{d}x = \mathrm{d}x$.

   Integrate $\mathrm{d}v$: $v = \int \mathrm{d}v = \int e^x\mathrm{d}x = e^x$. *(Constant $C$ omitted until the final answer)*

   *Step 3: Apply IBP Formula: $\int u\mathrm{d}v = uv - \int v\mathrm{d}u$.*

$$\int xe^x \,\mathrm{d}x = (x)(e^x) - \int (e^x)(\mathrm{d}x)$$

$$= xe^x - \int e^x\mathrm{d}x$$

   *Step 4: Solve the new integral.* This integral is simpler than the original.

$$\int e^x\mathrm{d}x = e^x$$

   *Step 5: Combine results and add the constant of integration $C$.*

$$\int xe^x \,\mathrm{d}x = xe^x - (e^x) + C = xe^x - e^x + C$$

   *Final Answer (Optional Factoring):* $e^x(x-1) + C$.

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   **Method 2: Tabular Method**

**Explanation.** The Tabular Method is also applicable here, though slightly overkill for a single application. It's useful for verifying the standard method. We set up columns for alternating signs, derivatives of $u$, and integrals of $\mathrm{d}v$.

Let $u = x$, $\mathrm{d}v = e^x \mathrm{d}x$.

| Sign | $u$ & derivatives | $\mathrm{d}v = e^x$ & integrals |
|---|---|---|
| + | $x$ | $e^x$ |
| - | 1 | $e^x$ |
| + | 0 | $e^x$ |

Multiply diagonally (Sign × u-term × v-integral in **next** row):

$$= (+)(x)(e^x) + (-)(1)(e^x) + C$$

$$= xe^x - e^x + C$$

*Result:* $e^x(x-1) + C$. Both methods agree.

2. **Problem:** *(Example 2)* Solve $\displaystyle\int xe^{2x}\, \mathrm{d}x$.

   *Solution:*

   **Strategy.** Product of Algebraic ($x$) and Exponential ($e^{2x}$). LIATE: A before E, so $u = x$. Derivative $\mathrm{d}u = \mathrm{d}x$ is simpler. $\mathrm{d}v = e^{2x}\mathrm{d}x$ is integrable. IBP once.

   **Method 1: Standard IBP**

   *Step 1: Define* $u, \mathrm{d}v$. $u = x$, $\mathrm{d}v = e^{2x}\mathrm{d}x$.

   *Step 2: Compute* $\mathrm{d}u, v$. $\mathrm{d}u = \mathrm{d}x$.

   $v = \int e^{2x}\, \mathrm{d}x$. (Use substitution $w = 2x, \mathrm{d}w = 2\mathrm{d}x \implies \mathrm{d}x = \frac{1}{2}\mathrm{d}w$.

   $\int e^w \left(\frac{1}{2}\mathrm{d}w\right) = \frac{1}{2}e^w = \frac{1}{2}e^{2x}$).

   So, $v = \frac{1}{2}e^{2x}$.

   *Step 3: Apply Formula.* $\int xe^{2x}\mathrm{d}x = x\left(\frac{1}{2}e^{2x}\right) - \int \left(\frac{1}{2}e^{2x}\right)\mathrm{d}x = \frac{1}{2}xe^{2x} - \frac{1}{2}\int e^{2x}\mathrm{d}x$.

   *Step 4: Solve new integral.* $\int e^{2x}\mathrm{d}x = \frac{1}{2}e^{2x}$.

   *Step 5: Combine + C.* $\int xe^{2x}\mathrm{d}x = \frac{1}{2}xe^{2x} - \frac{1}{2}\left(\frac{1}{2}e^{2x}\right) + C = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$.

   *Factored:* $\dfrac{e^{2x}}{4}(2x-1) + C$.

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   **Method 2: Tabular Method** Let $u = x, \mathrm{d}v = e^{2x}\mathrm{d}x$.

| Sign | $u$ & derivatives | $\mathrm{d}v = e^{2x}$ & integrals |
|---|---|---|
| + | $x$ | $e^{2x}$ |
| - | 1 | $\frac{1}{2}e^{2x}$ |
| + | 0 | $\frac{1}{4}e^{2x}$ |

$$= (+)(x)\left(\frac{1}{2}e^{2x}\right) + (-)(1)\left(\frac{1}{4}e^{2x}\right) + C = \frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x} + C$$

*Factored:* $\frac{e^{2x}}{4}(2x-1)+C$. Both methods agree.

3. **Problem:** *(ClassWork 1)* Solve $\int x^2 \ln x \, \mathrm{d}x$.

   *Solution:*

   **Strategy.** Product of Algebraic ($x^2$) and Logarithmic ($\ln x$). LIATE: L before A, so $u = \ln x$. This is good because $\mathrm{d}u = (1/x)\,\mathrm{d}x$ is simpler, and $\mathrm{d}v = x^2\mathrm{d}x$ is easy to integrate. Tabular method is not suitable here because the derivatives of $\ln x$ do not terminate at zero.

   **Method 1: Standard IBP**

   *Step 1: Define* $u, \mathrm{d}v$. $u = \ln x$, $\mathrm{d}v = x^2\mathrm{d}x$.

   *Step 2: Compute* $\mathrm{d}u, v$. $\mathrm{d}u = \frac{1}{x}\mathrm{d}x$, $v = \int x^2\mathrm{d}x = \frac{x^3}{3}$.

   *Step 3: Apply Formula.* $\int x^2 \ln x\mathrm{d}x = (\ln x)(\frac{x^3}{3}) - \int(\frac{x^3}{3})(\frac{1}{x}\mathrm{d}x) = \frac{x^3\ln x}{3} - \int \frac{x^2}{3}\mathrm{d}x$.

   *Step 4: Solve new integral.* $\int \frac{x^2}{3}\mathrm{d}x = \frac{1}{3}\int x^2\mathrm{d}x = \frac{1}{3}(\frac{x^3}{3}) = \frac{x^3}{9}$.

   *Step 5: Combine + C.* $\int x^2\ln x\mathrm{d}x = \frac{x^3\ln x}{3} - \frac{x^3}{9} + C$.

   *Factored:* $\frac{x^3}{9}(3\ln x - 1) + C$.

4. **Problem:** *(ClassWork 2)* Solve $\int x^2 e^x \, \mathrm{d}x$.

   *Solution:*

   **Strategy.** Product of Algebraic ($x^2$) and Exponential ($e^x$). LIATE: A before E, $u = x^2$. Since $u$ is $x^2$, its derivative $2x$ is simpler but still contains $x$. This indicates we'll likely need to apply IBP twice. Both methods are suitable.

   **Method 1: Repeated IBP**

   *First IBP Application:*

   Let $u_1 = x^2$, $\mathrm{d}v_1 = e^x\mathrm{d}x$. Then $\mathrm{d}u_1 = 2x\mathrm{d}x$, $v_1 = e^x$.

   $$I = \int x^2 e^x \, \mathrm{d}x = (x^2)(e^x) - \int (e^x)(2x\mathrm{d}x) = x^2e^x - 2\int xe^x\mathrm{d}x \quad (*)$$

   The new integral $\int xe^x\mathrm{d}x$ still requires IBP.

   *Second IBP Application (for* $\int xe^x\mathrm{d}x$*):*

   Let $u_2 = x$, $\mathrm{d}v_2 = e^x\mathrm{d}x$. Then $\mathrm{d}u_2 = \mathrm{d}x$, $v_2 = e^x$.

   $$\int xe^x\mathrm{d}x = (x)(e^x) - \int (e^x)(\mathrm{d}x) = xe^x - e^x$$

   *Combine Results:* Substitute the result of the second IBP back into equation $(*)$:

   $$I = x^2e^x - 2(xe^x - e^x) + C$$

Distribute the $-2$:

$$I = x^2e^x - 2xe^x + 2e^x + C$$

*Factored:* $e^x(x^2 - 2x + 2) + C$.

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**Method 2: Tabular Method** Let $u = x^2, \mathrm{d}v = e^x\mathrm{d}x$.

| Sign | $u$ & derivatives | $\mathrm{d}v = e^x$ & integrals |
|---|---|---|
| + | $x^2$ | $e^x$ |
| - | $2x$ | $e^x$ |
| + | $2$ | $e^x$ |
| - | $0$ | $e^x$ |

Multiply diagonally (Sign × u-term × v-integral in next row) and sum:

$$= (+)(x^2)(e^x) + (-)(2x)(e^x) + (+)(2)(e^x) + C$$

$$= x^2e^x - 2xe^x + 2e^x + C$$

*Factored:* $e^x(x^2 - 2x + 2) + C$. Both methods agree.

## 3 Solutions: Practice Problems

5. **Problem:** Solve $\displaystyle\int xe^{3x}\,\mathrm{d}x$.

   *Analysis:* Algebraic × Exponential. LIATE: $u = x$. IBP once.

   **Method 1: Standard IBP**

   Let $u = x, \mathrm{d}v = e^{3x}\mathrm{d}x$. $\mathrm{d}u = \mathrm{d}x, v = \int e^{3x}\mathrm{d}x = \frac{1}{3}e^{3x}$.

$$\int xe^{3x}\mathrm{d}x = x\left(\frac{1}{3}e^{3x}\right) - \int\left(\frac{1}{3}e^{3x}\right)\mathrm{d}x = \frac{1}{3}xe^{3x} - \frac{1}{3}\int e^{3x}\mathrm{d}x$$

$$= \frac{1}{3}xe^{3x} - \frac{1}{3}\left(\frac{1}{3}e^{3x}\right) + C = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

   *Factored:* $\frac{e^{3x}}{9}(3x - 1) + C$.

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   **Method 2: Tabular Method** Let $u = x, \mathrm{d}v = e^{3x}\mathrm{d}x$.

| Sign | $u$ | $\mathrm{d}v$ |
|---|---|---|
| + | $x$ | $e^{3x}$ |
| - | $1$ | $\frac{1}{3}e^{3x}$ |
| + | $0$ | $\frac{1}{9}e^{3x}$ |

$$= (+)(x)\left(\frac{1}{3}e^{3x}\right) + (-)(1)\left(\frac{1}{9}e^{3x}\right) + C = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

6. **Problem:** Solve $\int x^2 e^{3x}\ \mathrm{d}x$.

   *Analysis:* Algebraic ($x^2$) $\times$ Exponential ($e^{3x}$). LIATE: $u = x^2$. IBP twice.

   **Method 1: Repeated IBP**

   *IBP 1:* $u_1 = x^2, \mathrm{d}v_1 = e^{3x}\mathrm{d}x \implies \mathrm{d}u_1 = 2x\mathrm{d}x, v_1 = \frac{1}{3}e^{3x}$.

   $$I = x^2\left(\frac{1}{3}e^{3x}\right) - \int \frac{1}{3}e^{3x}(2x\mathrm{d}x) = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\int xe^{3x}\mathrm{d}x \quad (*)$$

   *IBP 2 (for $\int xe^{3x}\mathrm{d}x$):* From Problem 5, result is $\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}$.

   *Combine:* Substitute into (*):

   $I = \frac{1}{3}x^2e^{3x} - \frac{2}{3}\left(\frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x}\right) + C = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$.

   *Factored:* $\frac{e^{3x}}{27}(9x^2 - 6x + 2) + C$.

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   **Method 2: Tabular Method** Let $u = x^2, \mathrm{d}v = e^{3x}\mathrm{d}x$.

| Sign | $u$ | $\mathrm{d}v$ |
|---|---|---|
| + | $x^2$ | $e^{3x}$ |
| - | $2x$ | $\frac{1}{3}e^{3x}$ |
| + | $2$ | $\frac{1}{9}e^{3x}$ |
| - | $0$ | $\frac{1}{27}e^{3x}$ |

   $$= x^2\left(\frac{1}{3}e^{3x}\right) - 2x\left(\frac{1}{9}e^{3x}\right) + 2\left(\frac{1}{27}e^{3x}\right) + C = \frac{1}{3}x^2e^{3x} - \frac{2}{9}xe^{3x} + \frac{2}{27}e^{3x} + C$$

7. **Problem:** Solve $\int x\ln(2x)\ \mathrm{d}x$.

   *Analysis:* Algebraic $\times$ Logarithmic. LIATE: $u = \ln(2x)$. Standard IBP only.

   **Method 1: Standard IBP**

   Let $u = \ln(2x), \mathrm{d}v = x\mathrm{d}x \implies \mathrm{d}u = \left(\frac{1}{x}\right)\mathrm{d}x, v = \frac{x^2}{2}$.

   $$\int x\ln(2x)\mathrm{d}x = \frac{x^2\ln(2x)}{2} - \int\left(\frac{x^2}{2}\right)\left(\frac{1}{x}\right)\mathrm{d}x = \frac{x^2\ln(2x)}{2} - \int\frac{x}{2}\mathrm{d}x$$

   $$= \frac{x^2\ln(2x)}{2} - \frac{x^2}{4} + C = \frac{x^2}{4}(2\ln(2x) - 1) + C$$

8. **Problem:** Solve $\int \ln(4x)\ \mathrm{d}x$.

   *Analysis:* Logarithmic only. LIATE: $u = \ln(4x)$, $\mathrm{d}v = \mathrm{d}x$. Standard IBP only.

   **Method 1: Standard IBP**

   Let $u = \ln(4x), \mathrm{d}v = \mathrm{d}x \implies \mathrm{d}u = \frac{1}{x}\mathrm{d}x, v = x$.

$$\int \ln(4x)\mathrm{d}x = x\ln(4x) - \int x\left(\frac{1}{x}\right)\mathrm{d}x = x\ln(4x) - \int 1\mathrm{d}x$$

$$= x\ln(4x) - x + C = x(\ln(4x) - 1) + C$$

9. **Problem:** Solve $\int xe^{-3x}\ \mathrm{d}x$.

   *Analysis:* Algebraic × Exponential. LIATE: $u = x$. IBP once.

   **Method 1: Standard IBP**

   Let $u = x, \mathrm{d}v = e^{-3x}\mathrm{d}x \implies \mathrm{d}u = \mathrm{d}x, v = -\frac{1}{3}e^{-3x}$.

$$\int xe^{-3x}\mathrm{d}x = x\left(-\frac{1}{3}e^{-3x}\right) - \int\left(-\frac{1}{3}e^{-3x}\right)\mathrm{d}x = -\frac{1}{3}xe^{-3x} + \frac{1}{3}\int e^{-3x}\mathrm{d}x$$
$$= -\frac{1}{3}xe^{-3x} + \frac{1}{3}\left(-\frac{1}{3}e^{-3x}\right) + C = -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + C$$

   *Factored:* $-\dfrac{e^{-3x}}{9}(3x+1) + C$.

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   **Method 2: Tabular Method**

   Let $u = x, \mathrm{d}v = e^{-3x}\mathrm{d}x$.

| Sign | $u$ | $\mathrm{d}v$ |
|---|---|---|
| + | $x$ | $e^{-3x}$ |
| - | 1 | $-\frac{1}{3}e^{-3x}$ |
| + | 0 | $\frac{1}{9}e^{-3x}$ |

$$= (+)(x)(-\frac{1}{3}e^{-3x}) + (-)(1)(\frac{1}{9}e^{-3x}) + C = -\frac{1}{3}xe^{-3x} - \frac{1}{9}e^{-3x} + C$$

10. **Problem (10):** Solve $\int x^2e^{-x}\ \mathrm{d}x$. *Analysis:* Algebraic ($x^2$) × Exponential ($e^{-x}$). LIATE: $u = x^2$. IBP twice. **Method 1: Repeated IBP** *IBP 1:* $u_1 = x^2, \mathrm{d}v_1 = e^{-x}\mathrm{d}x \implies \mathrm{d}u_1 = 2x\mathrm{d}x, v_1 = -e^{-x}$.

$$I = x^2(-e^{-x}) - \int(-e^{-x})(2x\mathrm{d}x) = -x^2e^{-x} + 2\int xe^{-x}\mathrm{d}x$$

   *IBP 2 (for $\int xe^{-x}\mathrm{d}x$):* $u_2 = x, \mathrm{d}v_2 = e^{-x}\mathrm{d}x \implies \mathrm{d}u_2 = \mathrm{d}x, v_2 = -e^{-x}$.

$$\int xe^{-x}\mathrm{d}x = x(-e^{-x}) - \int(-e^{-x})\mathrm{d}x = -xe^{-x} - e^{-x}$$

   *Combine:* $I = -x^2e^{-x} + 2(-xe^{-x} - e^{-x}) + C = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$. *Factored:* $-e^{-x}(x^2 + 2x + 2) + C$.

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   **Method 2: Tabular Method** Let $u = x^2, \mathrm{d}v = e^{-x}\mathrm{d}x$.

| Sign | $u$ | d$v$ |
|---|---|---|
| + | $x^2$ | $e^{-x}$ |
| - | $2x$ | $-e^{-x}$ |
| + | 2 | $e^{-x}$ |
| - | 0 | $-e^{-x}$ |

$$= (+)(x^2)(-e^{-x}) + (-)(2x)(e^{-x}) + (+)(2)(-e^{-x}) + C = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$$

11. **Problem (11):** Solve $\int x^2e^{-2x}\,\mathrm{d}x$. *Analysis:* Algebraic ($x^2$) × Exponential ($e^{-2x}$). LIATE: $u = x^2$. IBP twice. **Method 1: Repeated IBP** *IBP 1:* $u_1 = x^2, \mathrm{d}v_1 = e^{-2x}\mathrm{d}x \implies \mathrm{d}u_1 = 2x\mathrm{d}x, v_1 = -\frac{1}{2}e^{-2x}$.

$$I = x^2(-\frac{1}{2}e^{-2x}) - \int(-\frac{1}{2}e^{-2x})(2x\mathrm{d}x) = -\frac{1}{2}x^2e^{-2x} + \int xe^{-2x}\mathrm{d}x$$

*IBP 2 (for $\int xe^{-2x}\mathrm{d}x$):* $u_2 = x, \mathrm{d}v_2 = e^{-2x}\mathrm{d}x \implies \mathrm{d}u_2 = \mathrm{d}x, v_2 = -\frac{1}{2}e^{-2x}$.

$$\int xe^{-2x}\mathrm{d}x = x(-\frac{1}{2}e^{-2x}) - \int(-\frac{1}{2}e^{-2x})\mathrm{d}x = -\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}$$

*Combine:* $I = -\frac{1}{2}x^2e^{-2x} + (-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}) + C$. *Factored:* $-\frac{e^{-2x}}{4}(2x^2 + 2x + 1) + C$.

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**Method 2: Tabular Method** Let $u = x^2, \mathrm{d}v = e^{-2x}\mathrm{d}x$.

| Sign | $u$ | d$v$ |
|---|---|---|
| + | $x^2$ | $e^{-2x}$ |
| - | $2x$ | $-\frac{1}{2}e^{-2x}$ |
| + | 2 | $\frac{1}{4}e^{-2x}$ |
| - | 0 | $-\frac{1}{8}e^{-2x}$ |

$$= (+)(x^2)(-\frac{1}{2}e^{-2x}) + (-)(2x)(\frac{1}{4}e^{-2x}) + (+)(2)(-\frac{1}{8}e^{-2x}) + C$$
$$= -\frac{1}{2}x^2e^{-2x} - \frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x} + C$$

12. **Problem (12):** Solve $\int x^2e^{2x}\,\mathrm{d}x$. *Analysis:* Algebraic ($x^2$) × Exponential ($e^{2x}$). LIATE: $u = x^2$. IBP twice. **Method 1: Repeated IBP** *IBP 1:* $u_1 = x^2, \mathrm{d}v_1 = e^{2x}\mathrm{d}x \implies \mathrm{d}u_1 = 2x\mathrm{d}x, v_1 = \frac{1}{2}e^{2x}$.

$$I = x^2(\frac{1}{2}e^{2x}) - \int(\frac{1}{2}e^{2x})(2x\mathrm{d}x) = \frac{1}{2}x^2e^{2x} - \int xe^{2x}\mathrm{d}x$$

*IBP 2 (for $\int xe^{2x}\mathrm{d}x$):* From Problem 2, result is $\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}$. *Combine:* $I = \frac{1}{2}x^2e^{2x} - (\frac{1}{2}xe^{2x} - \frac{1}{4}e^{2x}) + C = \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$. *Factored:* $\frac{e^{2x}}{4}(2x^2 - 2x + 1) + C$.

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**Method 2: Tabular Method** Let $u = x^2, \mathrm{d}v = e^{2x}\mathrm{d}x$.

| Sign | $u$ | $dv$ |
|---|---|---|
| + | $x^2$ | $e^{2x}$ |
| - | $2x$ | $\frac{1}{2}e^{2x}$ |
| + | $2$ | $\frac{1}{4}e^{2x}$ |
| - | $0$ | $\frac{1}{8}e^{2x}$ |

$$= (+)(x^2)(\frac{1}{2}e^{2x}) + (-)(2x)(\frac{1}{4}e^{2x}) + (+)(2)(\frac{1}{8}e^{2x}) + C$$
$$= \frac{1}{2}x^2e^{2x} - \frac{1}{2}xe^{2x} + \frac{1}{4}e^{2x} + C$$

13. **Problem (13):** Solve $\int \ln(2x)\ dx$. *Analysis:* Logarithmic only. Standard IBP only ($u = \ln(2x)$). (Same as Problem 8) **Method 1: Standard IBP** Let $u = \ln(2x), dv = dx$. $du = \frac{1}{x}dx, v = x$.

$$\int \ln(2x)dx = x\ln(2x) - \int x(\frac{1}{x}dx) = x\ln(2x) - \int 1dx = x\ln(2x) - x + C$$

*Final Answer:* $x(\ln(2x) - 1) + C$.

14. **Problem (14):** Solve $\int \ln(x^2)\ dx$. *Analysis:* Logarithmic. Standard IBP only. Can simplify first. **Method 1: Simplify First** Assume $x > 0$. $\int \ln(x^2)dx = \int 2\ln x dx$. Use IBP: $u = \ln x, dv = 2dx$. $du = \frac{1}{x}dx, v = 2x$.

$$\int 2\ln x dx = (\ln x)(2x) - \int 2x(\frac{1}{x}dx) = 2x\ln x - \int 2dx = 2x\ln x - 2x + C$$

*Final Answer:* $2x(\ln x - 1) + C$.

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**Method 2: Direct IBP** Let $u = \ln(x^2), dv = dx$. $du = \frac{2x}{x^2}dx = \frac{2}{x}dx, v = x$.

$$\int \ln(x^2)dx = x\ln(x^2) - \int x(\frac{2}{x}dx) = x\ln(x^2) - \int 2dx$$
$$= x\ln(x^2) - 2x + C$$

*(Equivalent result)*

15. **Problem (15):** Solve $\int x\ln x\ dx$. *Analysis:* Algebraic × Logarithmic. LIATE: $u = \ln x$. Standard IBP only. **Method 1: Standard IBP** Let $u = \ln x, dv = xdx$. $du = \frac{1}{x}dx, v = \frac{x^2}{2}$.

$$\int x\ln x dx = (\ln x)(\frac{x^2}{2}) - \int \frac{x^2}{2}\frac{1}{x}dx = \frac{x^2\ln x}{2} - \int \frac{x}{2}dx$$
$$= \frac{x^2\ln x}{2} - \frac{1}{2}(\frac{x^2}{2}) + C = \frac{x^2\ln x}{2} - \frac{x^2}{4} + C$$

*Final Answer:* $\frac{x^2}{4}(2\ln x - 1) + C$.

16. **Problem (16):** Solve $\int x^3 \ln x \, \mathrm{d}x$. *Analysis:* Algebraic × Logarithmic. LIATE: $u = \ln x$. Standard IBP only. **Method 1: Standard IBP** Let $u = \ln x, \mathrm{d}v = x^3 \mathrm{d}x$. $\mathrm{d}u = \frac{1}{x}\mathrm{d}x, v = \frac{x^4}{4}$.

$$\int x^3 \ln x \mathrm{d}x = (\ln x)(\frac{x^4}{4}) - \int \frac{x^4}{4}\frac{1}{x}\mathrm{d}x = \frac{x^4 \ln x}{4} - \int \frac{x^3}{4}\mathrm{d}x$$
$$= \frac{x^4 \ln x}{4} - \frac{1}{4}(\frac{x^4}{4}) + C = \frac{x^4 \ln x}{4} - \frac{x^4}{16} + C$$

*Final Answer:* $\frac{x^4}{16}(4\ln x - 1) + C$.

17. **Problem (17):** Solve $\int xe^x \, \mathrm{d}x$. *Analysis:* Algebraic × Exponential. LIATE: $u = x$. IBP once. (Same as Problem 1) **Method 1: Standard IBP** Let $u = x, \mathrm{d}v = e^x \mathrm{d}x$. $\mathrm{d}u = \mathrm{d}x, v = e^x$.

$$\int xe^x \mathrm{d}x = xe^x - \int e^x \mathrm{d}x = xe^x - e^x + C$$

*Factored:* $e^x(x - 1) + C$.

---

**Method 2: Tabular Method** Let $u = x, \mathrm{d}v = e^x \mathrm{d}x$.

| Sign | $u$ | $\mathrm{d}v$ |
|---|---|---|
| + | $x$ | $e^x$ |
| - | 1 | $e^x$ |
| + | 0 | $e^x$ |

$$= (+)(x)(e^x) + (-)(1)(e^x) + C = xe^x - e^x + C$$

18. **Problem (18):** Solve $\int xe^{3x} \, \mathrm{d}x$. *Analysis:* Algebraic × Exponential. LIATE: $u = x$. IBP once. (Same as Problem 5) **Method 1: Standard IBP** Let $u = x, \mathrm{d}v = e^{3x}\mathrm{d}x$. $\mathrm{d}u = \mathrm{d}x, v = \frac{1}{3}e^{3x}$.

$$\int xe^{3x}\mathrm{d}x = x(\frac{1}{3}e^{3x}) - \int \frac{1}{3}e^{3x}\mathrm{d}x = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

*Factored:* $\frac{e^{3x}}{9}(3x - 1) + C$.

---

**Method 2: Tabular Method** Let $u = x, \mathrm{d}v = e^{3x}\mathrm{d}x$.

| Sign | $u$ | $\mathrm{d}v$ |
|---|---|---|
| + | $x$ | $e^{3x}$ |
| - | 1 | $\frac{1}{3}e^{3x}$ |
| + | 0 | $\frac{1}{9}e^{3x}$ |

$$= (+)(x)(\frac{1}{3}e^{3x}) + (-)(1)(\frac{1}{9}e^{3x}) + C = \frac{1}{3}xe^{3x} - \frac{1}{9}e^{3x} + C$$

19. **Problem (19):** Solve $\int x^2 e^{-x}\,\mathrm{d}x$. *Analysis:* Algebraic ($x^2$) $\times$ Exponential ($e^{-x}$). LIATE: $u = x^2$. IBP twice. (Same as Problem 10) **Method 1: Repeated IBP** *IBP 1:* $u_1 = x^2, \mathrm{d}v_1 = e^{-x}\mathrm{d}x \implies \mathrm{d}u_1 = 2x\mathrm{d}x, v_1 = -e^{-x}$.

$$I = x^2(-e^{-x}) - \int(-e^{-x})(2x\mathrm{d}x) = -x^2e^{-x} + 2\int xe^{-x}\mathrm{d}x$$

*IBP 2 (for $\int xe^{-x}\mathrm{d}x$):* $u_2 = x, \mathrm{d}v_2 = e^{-x}\mathrm{d}x \implies \mathrm{d}u_2 = \mathrm{d}x, v_2 = -e^{-x}$.

$$\int xe^{-x}\mathrm{d}x = x(-e^{-x}) - \int(-e^{-x})\mathrm{d}x = -xe^{-x} - e^{-x}$$

*Combine:* $I = -x^2e^{-x} + 2(-xe^{-x} - e^{-x}) + C = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$. *Factored:* $-e^{-x}(x^2 + 2x + 2) + C$.

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**Method 2: Tabular Method** Let $u = x^2, \mathrm{d}v = e^{-x}\mathrm{d}x$.

| Sign | $u$ | $\mathrm{d}v$ |
|---|---|---|
| + | $x^2$ | $e^{-x}$ |
| - | $2x$ | $-e^{-x}$ |
| + | $2$ | $e^{-x}$ |
| - | $0$ | $-e^{-x}$ |

$$= (+)(x^2)(-e^{-x}) + (-)(2x)(e^{-x}) + (+)(2)(-e^{-x}) + C = -x^2e^{-x} - 2xe^{-x} - 2e^{-x} + C$$

20. **Problem (20):** Solve $\int (x^2 - 2x + 1)e^{2x}\,\mathrm{d}x$. *Analysis:* Note $x^2 - 2x + 1 = (x-1)^2$. Algebraic $\times$ Exponential. Needs IBP twice. **Method 1: Repeated IBP** Let $u_1 = (x-1)^2, \mathrm{d}v_1 = e^{2x}\mathrm{d}x \implies \mathrm{d}u_1 = 2(x-1)\mathrm{d}x, v_1 = \frac{1}{2}e^{2x}$.

$$I = \frac{1}{2}(x-1)^2e^{2x} - \int \frac{1}{2}e^{2x}\cdot 2(x-1)\mathrm{d}x = \frac{1}{2}(x-1)^2e^{2x} - \int(x-1)e^{2x}\mathrm{d}x$$

Let $u_2 = x - 1, \mathrm{d}v_2 = e^{2x}\mathrm{d}x \implies \mathrm{d}u_2 = \mathrm{d}x, v_2 = \frac{1}{2}e^{2x}$.

$$\int(x-1)e^{2x}\mathrm{d}x = (x-1)(\frac{1}{2}e^{2x}) - \int\frac{1}{2}e^{2x}\mathrm{d}x = \frac{1}{2}(x-1)e^{2x} - \frac{1}{4}e^{2x}$$

Combine: $I = \frac{1}{2}(x-1)^2e^{2x} - [\frac{1}{2}(x-1)e^{2x} - \frac{1}{4}e^{2x}] + C$

$$= \frac{e^{2x}}{4}[2(x-1)^2 - 2(x-1) + 1] + C = \frac{e^{2x}}{4}(2x^2 - 6x + 5) + C$$

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**Method 2: Tabular Method** Let $u = (x-1)^2, \mathrm{d}v = e^{2x}\mathrm{d}x$.

| Sign | $u$ | $\mathrm{d}v$ |
|---|---|---|
| + | $(x-1)^2$ | $e^{2x}$ |
| - | $2(x-1)$ | $\frac{1}{2}e^{2x}$ |
| + | $2$ | $\frac{1}{4}e^{2x}$ |
| - | $0$ | $\frac{1}{8}e^{2x}$ |

$$= (x-1)^2(\frac{1}{2}e^{2x}) - 2(x-1)(\frac{1}{4}e^{2x}) + 2(\frac{1}{8}e^{2x}) + C$$

$$= \frac{1}{2}(x-1)^2e^{2x} - \frac{1}{2}(x-1)e^{2x} + \frac{1}{4}e^{2x} + C = \frac{e^{2x}}{4}(2x^2 - 6x + 5) + C$$

21. **Problem (21):** Solve $\int x^3e^x \, \mathrm{d}x$. *Analysis:* Algebraic ($x^3$) $\times$ Exponential ($e^x$). Needs IBP 3 times. **Method 1: Repeated IBP (Full Detail)** *IBP 1:* Let $u_1 = x^3, \mathrm{d}v_1 = e^x\mathrm{d}x$. Then $\mathrm{d}u_1 = 3x^2\mathrm{d}x, v_1 = e^x$.

$$I = \int x^3e^x\mathrm{d}x = x^3e^x - \int e^x(3x^2\mathrm{d}x) = x^3e^x - 3\int x^2e^x\mathrm{d}x \quad (1)$$

*IBP 2 (for $\int x^2e^x\mathrm{d}x$):* Let $u_2 = x^2, \mathrm{d}v_2 = e^x\mathrm{d}x$. Then $\mathrm{d}u_2 = 2x\mathrm{d}x, v_2 = e^x$.

$$\int x^2e^x\mathrm{d}x = x^2e^x - \int e^x(2x\mathrm{d}x) = x^2e^x - 2\int xe^x\mathrm{d}x \quad (2)$$

*IBP 3 (for $\int xe^x\mathrm{d}x$):* Let $u_3 = x, \mathrm{d}v_3 = e^x\mathrm{d}x$. Then $\mathrm{d}u_3 = \mathrm{d}x, v_3 = e^x$.

$$\int xe^x\mathrm{d}x = xe^x - \int e^x\mathrm{d}x = xe^x - e^x \quad (3)$$

*Combine Backwards:* Substitute (3) into (2):

$$\int x^2e^x\mathrm{d}x = x^2e^x - 2(xe^x - e^x) = x^2e^x - 2xe^x + 2e^x$$

Substitute this result into (1):

$$I = x^3e^x - 3(x^2e^x - 2xe^x + 2e^x) + C$$

$$I = x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C$$

*Factored:* $e^x(x^3 - 3x^2 + 6x - 6) + C$.

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**Method 2: Tabular Method** Let $u = x^3, \mathrm{d}v = e^x\mathrm{d}x$.

| Sign | $u$ | $\mathrm{d}v$ |
|---|---|---|
| + | $x^3$ | $e^x$ |
| - | $3x^2$ | $e^x$ |
| + | $6x$ | $e^x$ |
| - | 6 | $e^x$ |
| + | 0 | $e^x$ |

$$= x^3e^x - 3x^2e^x + 6xe^x - 6e^x + C = e^x(x^3 - 3x^2 + 6x - 6) + C$$

22. **Problem (22):** Solve $\int p^4e^{-p} \, \mathrm{d}p$. *Analysis:* Algebraic ($p^4$) $\times$ Exponential ($e^{-p}$). Needs IBP 4 times. **Method 1: Repeated IBP (Full Detail)** *IBP 1:* Let $u_1 = p^4, \mathrm{d}v_1 =$

$e^{-p}\mathrm{d}p$. Then $\mathrm{d}u_1 = 4p^3\mathrm{d}p, v_1 = -e^{-p}$.

$$I = \int p^4 e^{-p}\mathrm{d}p = p^4(-e^{-p}) - \int (-e^{-p})(4p^3\mathrm{d}p) = -p^4e^{-p} + 4\int p^3e^{-p}\mathrm{d}p \quad (1)$$

*IBP 2 (for $\int p^3e^{-p}\mathrm{d}p$):* Let $u_2 = p^3, \mathrm{d}v_2 = e^{-p}\mathrm{d}p$. Then $\mathrm{d}u_2 = 3p^2\mathrm{d}p, v_2 = -e^{-p}$.

$$I_2 = \int p^3 e^{-p}\mathrm{d}p = p^3(-e^{-p}) - \int (-e^{-p})(3p^2\mathrm{d}p) = -p^3e^{-p} + 3\int p^2e^{-p}\mathrm{d}p \quad (2)$$

*IBP 3 (for $\int p^2e^{-p}\mathrm{d}p$):* Let $u_3 = p^2, \mathrm{d}v_3 = e^{-p}\mathrm{d}p$. Then $\mathrm{d}u_3 = 2p\mathrm{d}p, v_3 = -e^{-p}$.

$$I_3 = \int p^2 e^{-p}\mathrm{d}p = p^2(-e^{-p}) - \int (-e^{-p})(2p\mathrm{d}p) = -p^2e^{-p} + 2\int pe^{-p}\mathrm{d}p \quad (3)$$

*IBP 4 (for $\int pe^{-p}\mathrm{d}p$):* Let $u_4 = p, \mathrm{d}v_4 = e^{-p}\mathrm{d}p$. Then $\mathrm{d}u_4 = \mathrm{d}p, v_4 = -e^{-p}$.

$$I_4 = \int pe^{-p}\mathrm{d}p = p(-e^{-p}) - \int (-e^{-p})\mathrm{d}p = -pe^{-p} + \int e^{-p}\mathrm{d}p = -pe^{-p} - e^{-p} \quad (4)$$

*Combine Backwards:* Substitute (4) into (3): $I_3 = -p^2e^{-p} + 2(-pe^{-p} - e^{-p}) = -p^2e^{-p} - 2pe^{-p} - 2e^{-p} = -e^{-p}(p^2+2p+2)$. Substitute $I_3$ into (2): $I_2 = -p^3e^{-p} + 3[-e^{-p}(p^2+2p+2)] = -p^3e^{-p} - 3p^2e^{-p} - 6pe^{-p} - 6e^{-p} = -e^{-p}(p^3+3p^2+6p+6)$. Substitute $I_2$ into (1): $I = -p^4e^{-p} + 4[-e^{-p}(p^3+3p^2+6p+6)] + C$ $I = -p^4e^{-p} - 4p^3e^{-p} - 12p^2e^{-p} - 24pe^{-p} - 24e^{-p} + C$. *Factored:* $-e^{-p}(p^4 + 4p^3 + 12p^2 + 24p + 24) + C$.

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**Method 2: Tabular Method** Let $u = p^4, \mathrm{d}v = e^{-p}\mathrm{d}p$.

| Sign | $u = p^4$ | $\mathrm{d}v = e^{-p}\mathrm{d}p$ |
|---|---|---|
| + | $p^4$ | $e^{-p}$ |
| - | $4p^3$ | $-e^{-p}$ |
| + | $12p^2$ | $e^{-p}$ |
| - | $24p$ | $-e^{-p}$ |
| + | $24$ | $e^{-p}$ |
| - | $0$ | $-e^{-p}$ |

$$\begin{aligned} &= p^4(-e^{-p}) - (4p^3)(e^{-p}) + (12p^2)(-e^{-p}) - (24p)(e^{-p}) + (24)(-e^{-p}) + C \\ &= -p^4e^{-p} - 4p^3e^{-p} - 12p^2e^{-p} - 24pe^{-p} - 24e^{-p} + C \end{aligned}$$

*Factored:* $-e^{-p}(p^4 + 4p^3 + 12p^2 + 24p + 24) + C$.

23. **Problem (23):** Solve $\int (x^2-5x)e^x \,\mathrm{d}x$. *Analysis:* Algebraic $(x^2-5x)$ × Exponential $(e^x)$. Needs IBP twice. **Method 1: Repeated IBP** *IBP 1:* $u_1 = x^2 - 5x, \mathrm{d}v_1 = e^x\mathrm{d}x \implies \mathrm{d}u_1 = (2x-5)\mathrm{d}x, v_1 = e^x$.

$$I = (x^2 - 5x)e^x - \int e^x(2x-5)\mathrm{d}x$$

*IBP 2 (for $\int (2x-5)e^x\mathrm{d}x$):* $u_2 = 2x - 5, \mathrm{d}v_2 = e^x\mathrm{d}x \implies \mathrm{d}u_2 = 2\mathrm{d}x, v_2 = e^x$.

$$\int (2x-5)e^x\mathrm{d}x = (2x-5)e^x - \int e^x(2\mathrm{d}x) = (2x-5)e^x - 2e^x$$

$$\textit{Combine: } I = (x^2-5x)e^x - [(2x-5)e^x - 2e^x] + C = e^x(x^2-5x-2x+5+2) + C$$
$$= e^x(x^2-7x+7) + C$$

---

**Method 2: Tabular Method** Let $u = x^2-5x, \mathrm{d}v = e^x\mathrm{d}x$.

| Sign | $u$ | $\mathrm{d}v$ |
|---|---|---|
| + | $x^2-5x$ | $e^x$ |
| - | $2x-5$ | $e^x$ |
| + | 2 | $e^x$ |
| - | 0 | $e^x$ |

$$= (x^2-5x)e^x - (2x-5)e^x + 2e^x + C = e^x(x^2-5x-2x+5+2) + C = e^x(x^2-7x+7) + C$$

24. **Problem (24):** Solve $\int (r^2+r+1)e^r \, \mathrm{d}r$. *Analysis:* Algebraic $(r^2+r+1)$ × Exponential $(e^r)$. Needs IBP twice. **Method 1: Repeated IBP** *IBP 1:* $u_1 = r^2+r+1, \mathrm{d}v_1 = e^r\mathrm{d}r \implies \mathrm{d}u_1 = (2r+1)\mathrm{d}r, v_1 = e^r$.

$$I = (r^2+r+1)e^r - \int e^r(2r+1)\mathrm{d}r$$

*IBP 2 (for* $\int(2r+1)e^r\mathrm{d}r$*):* $u_2 = 2r+1, \mathrm{d}v_2 = e^r\mathrm{d}r \implies \mathrm{d}u_2 = 2\mathrm{d}r, v_2 = e^r$.

$$\int (2r+1)e^r\mathrm{d}r = (2r+1)e^r - \int e^r(2\mathrm{d}r) = (2r+1)e^r - 2e^r$$

$$\textit{Combine: } I = (r^2+r+1)e^r - [(2r+1)e^r - 2e^r] + C = e^r(r^2+r+1-2r-1+2) + C$$
$$= e^r(r^2-r+2) + C$$

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**Method 2: Tabular Method** Let $u = r^2+r+1, \mathrm{d}v = e^r\mathrm{d}r$.

| Sign | $u$ | $\mathrm{d}v$ |
|---|---|---|
| + | $r^2+r+1$ | $e^r$ |
| - | $2r+1$ | $e^r$ |
| + | 2 | $e^r$ |
| - | 0 | $e^r$ |

$$= (r^2+r+1)e^r - (2r+1)e^r + 2e^r + C = e^r(r^2+r+1-2r-1+2) + C = e^r(r^2-r+2) + C$$

25. **Problem (25):** Solve $\int x^5e^x \, \mathrm{d}x$. *Analysis:* Algebraic $(x^5)$ × Exponential $(e^x)$. Needs IBP 5 times. **Method 1: Repeated IBP (Full Detail)** *IBP 1:* $u_1 = x^5, \mathrm{d}v_1 = e^x\mathrm{d}x \implies \mathrm{d}u_1 = 5x^4\mathrm{d}x, v_1 = e^x$.

$$I = x^5e^x - 5\int x^4e^x\mathrm{d}x$$

*IBP 2:* $u_2 = x^4, \mathrm{d}v_2 = e^x \mathrm{d}x \implies \mathrm{d}u_2 = 4x^3 \mathrm{d}x, v_2 = e^x$.

$$\int x^4 e^x \mathrm{d}x = x^4 e^x - 4 \int x^3 e^x \mathrm{d}x$$

$I = x^5 e^x - 5[x^4 e^x - 4 \int x^3 e^x \mathrm{d}x] = x^5 e^x - 5x^4 e^x + 20 \int x^3 e^x \mathrm{d}x$. *IBP 3:* $u_3 = x^3, \mathrm{d}v_3 = e^x \mathrm{d}x \implies \mathrm{d}u_3 = 3x^2 \mathrm{d}x, v_3 = e^x$.

$$\int x^3 e^x \mathrm{d}x = x^3 e^x - 3 \int x^2 e^x \mathrm{d}x$$

$I = x^5 e^x - 5x^4 e^x + 20[x^3 e^x - 3 \int x^2 e^x \mathrm{d}x] = x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60 \int x^2 e^x \mathrm{d}x$. *IBP 4:* $u_4 = x^2, \mathrm{d}v_4 = e^x \mathrm{d}x \implies \mathrm{d}u_4 = 2x \mathrm{d}x, v_4 = e^x$.

$$\int x^2 e^x \mathrm{d}x = x^2 e^x - 2 \int x e^x \mathrm{d}x$$

$I = x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60[x^2 e^x - 2 \int x e^x \mathrm{d}x]$ $I = x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120 \int x e^x \mathrm{d}x$. *IBP 5:* $u_5 = x, \mathrm{d}v_5 = e^x \mathrm{d}x \implies \mathrm{d}u_5 = \mathrm{d}x, v_5 = e^x$.

$$\int x e^x \mathrm{d}x = x e^x - \int e^x \mathrm{d}x = x e^x - e^x$$

*Combine All:* $I = x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120[x e^x - e^x] + C$. $I = x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120x e^x - 120 e^x + C$. *Factored:* $e^x(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120) + C$.

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**Method 2: Tabular Method** Let $u = x^5, \mathrm{d}v = e^x \mathrm{d}x$.

| Sign | $u = x^5$ | $\mathrm{d}v = e^x \mathrm{d}x$ |
|---|---|---|
| + | $x^5$ | $e^x$ |
| - | $5x^4$ | $e^x$ |
| + | $20x^3$ | $e^x$ |
| - | $60x^2$ | $e^x$ |
| + | $120x$ | $e^x$ |
| - | $120$ | $e^x$ |
| + | $0$ | $e^x$ |

$$\begin{aligned} &= x^5 e^x - 5x^4 e^x + 20x^3 e^x - 60x^2 e^x + 120x e^x - 120 e^x + C \\ &= e^x(x^5 - 5x^4 + 20x^3 - 60x^2 + 120x - 120) + C \end{aligned}$$

26. **Problem (26):** Solve $\displaystyle\int t^2 e^{4t}\, \mathrm{d}t$. *Analysis:* Algebraic ($t^2$) $\times$ Exponential ($e^{4t}$). Needs IBP twice. **Method 1: Repeated IBP** *IBP 1:* $u_1 = t^2, \mathrm{d}v_1 = e^{4t} \mathrm{d}t \implies \mathrm{d}u_1 = 2t \mathrm{d}t, v_1 = \frac{1}{4} e^{4t}$.

$$I = t^2(\frac{1}{4} e^{4t}) - \int \frac{1}{4} e^{4t}(2t \mathrm{d}t) = \frac{1}{4} t^2 e^{4t} - \frac{1}{2} \int t e^{4t} \mathrm{d}t$$

*IBP 2 (for $\int t e^{4t} \mathrm{d}t$):* $u_2 = t, \mathrm{d}v_2 = e^{4t} \mathrm{d}t \implies \mathrm{d}u_2 = \mathrm{d}t, v_2 = \frac{1}{4} e^{4t}$.

$$\int t e^{4t} \mathrm{d}t = t(\frac{1}{4} e^{4t}) - \int \frac{1}{4} e^{4t} \mathrm{d}t = \frac{1}{4} t e^{4t} - \frac{1}{16} e^{4t}$$

*Combine:* $I = \frac{1}{4} t^2 e^{4t} - \frac{1}{2}[\frac{1}{4} t e^{4t} - \frac{1}{16} e^{4t}] + C = \frac{1}{4} t^2 e^{4t} - \frac{1}{8} t e^{4t} + \frac{1}{32} e^{4t} + C$. *Factored:* $\frac{e^{4t}}{32}(8t^2 - 4t + 1) + C$.

---

**Method 2: Tabular Method** Let $u = t^2, \mathrm{d}v = e^{4t} \mathrm{d}t$.

| Sign | $u = t^2$ | $\mathrm{d}v = e^{4t}\mathrm{d}t$ |
|---|---|---|
| + | $t^2$ | $e^{4t}$ |
| - | $2t$ | $\frac{1}{4}e^{4t}$ |
| + | $2$ | $\frac{1}{16}e^{4t}$ |
| - | $0$ | $\frac{1}{64}e^{4t}$ |

$$= t^2(\frac{1}{4}e^{4t}) - (2t)(\frac{1}{16}e^{4t}) + 2(\frac{1}{64}e^{4t}) + C$$

$$= \frac{1}{4}t^2e^{4t} - \frac{1}{8}te^{4t} + \frac{1}{32}e^{4t} + C = \frac{e^{4t}}{32}(8t^2 - 4t + 1) + C$$