INTEGRATION QUESTIONS

NUTM Nexus Writing Team

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1 Differentiation of Integrals

1. Use the theorem of integration as inverse of differentiation (FTC) to find $\frac{dy}{dx}$ if:

   (a) $y = \int_a^x (t^3 + 1)\ \mathrm{d}t$

   (b) $y = \int_{1+3x^2}^4 \frac{1}{2 + e^t}\ \mathrm{d}t$

2. Find the derivatives:

   (a) $\frac{d}{dt} \int_0^{t^4} \sqrt{u}\ \mathrm{d}u$

   (b) $\frac{d}{dx} \int_0^{x^3} e^{-t}\ \mathrm{d}t$

2 Evaluation of Definite Integrals

Evaluate the following definite integrals (numbering follows original presentation order):

1. $\int_{-2}^{0} (2x + 5)\ \mathrm{d}x$

2. $\int_{-3}^{1} \left(5 - \frac{x}{2}\right)\ \mathrm{d}x$

3. $\int_0^2 x(x - 3)\ \mathrm{d}x$

4. $\int_{-1}^{1} (x^2 - 2x + 3)\ \mathrm{d}x$

5. $\int_0^4 \left(3x - \frac{x^3}{4}\right)\ \mathrm{d}x$

6. $\int_{-2}^{2} (x^3 - 2x + 3)\ \mathrm{d}x$

7. $\int_0^1 (x^2 + \sqrt{x})\ \mathrm{d}x$

8. $\int_1^{32} x^{-6/5}\ \mathrm{d}x$

9. $\displaystyle\int_{1}^{-1} (r+1)^2 \, \mathrm{d}r$

10. $\displaystyle\int_{-\sqrt{3}}^{\sqrt{3}} (t+1)(t^2+4) \, \mathrm{d}t$

11. $\displaystyle\int_{\sqrt{2}}^{1} \left(\frac{u^7}{2} - \frac{1}{u^5} \right) \, \mathrm{d}u$

12. $\displaystyle\int_{-3}^{-1} \frac{y^5 - 2y}{y^3} \, \mathrm{d}y$

13. $\displaystyle\int_{1}^{\sqrt{2}} \frac{s^2 + \sqrt{s}}{s^2} \, \mathrm{d}s$

14. $\displaystyle\int_{1}^{8} \frac{(x^{1/3}+1)(2-x^{2/3})}{x^{1/3}} \, \mathrm{d}x$