TECHNIQUES OF DIFFERENTIATION: QUOTIENT & CHAIN RULES - WEEK 6

NUTM Nexus Writing Team

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1 Quotient Rule

1.1 Example

1. Find the derivative of $y = \dfrac{x-1}{2x+3}$.

1.2 ClassWork Problems

Differentiate the following functions:

2. $y = \dfrac{2x+5}{3x-2}$
3. $z = \dfrac{4-3x}{3x^2+x}$
4. $g(x) = \dfrac{x^2-4}{x+0.5}$
5. $f(t) = \dfrac{t^2-1}{t^2+t-2}$
6. $v = (1-t)(1+t^2)^{-1}$
7. $w = (2x-7)^{-1}(x+5)$
8. $f(s) = \dfrac{\sqrt{s}-1}{\sqrt{s}+1}$
9. $u = \dfrac{5x+1}{2\sqrt{x}}$
10. $v = \dfrac{1+x-4\sqrt{x}}{x}$
11. $r = 2\left(\dfrac{1}{\sqrt{\theta}} + \sqrt{\theta}\right)$
12. $y = \dfrac{1}{(x^2-1)(x^2+x+1)}$
13. $y = \dfrac{(x+1)(x+2)}{(x-1)(x-2)}$

14. $y = 2e^{-x} + e^{3x}$
15. $y = \dfrac{x^2 + 3e^x}{2e^x - x}$
16. $s = \dfrac{t^2 + 5t - 1}{t^2}$
17. $u = \dfrac{(x^2 + x)(x^2 - x + 1)}{x^4}$
18. $y = \dfrac{x^3 + 7}{x}$
19. $p = \dfrac{q^2 + 3}{(q-1)^3 + (q+1)^3}$
20. $r = \dfrac{(\theta - 1)(\theta^2 + \theta + 1)}{\theta^3}$
21. $w = \left(1 + \dfrac{1}{z}\right)(3 - z)$

2 Chain Rule

2.1 Example

21. Find the derivative of:
    (a) $y = \dfrac{1}{x+1}$
    (b) $y = \sqrt{3x^2 - x + 1}$

2.2 ClassWork Problems

Differentiate the following functions:

22. $f(x) = (3x - 2x^2)^3$
23. $y = (x^2 + 3x)^4$
24. $y = (x^2 + 1)^3$
25. $y = (x^3 + 1)^2$
26. $y = \sqrt[3]{(x^2 + 4)^2}$
27. $y = \dfrac{3}{x^2 + 1}$
28. $y = \dfrac{4}{2x + 1}$
29. $y = \dfrac{2}{(x-1)^3}$

30. $y = x^2\sqrt{1-x^2}$

31. $y = \dfrac{3}{(x+1)^2}$

32. $f(x) = \left(\dfrac{x+1}{x-5}\right)^2$

33. $f(x) = \left(\dfrac{3x-1}{x^2+3}\right)^2$

34. $y = x^2\sqrt{x^2+1}$

35. $\dfrac{5}{(1-5x)^{2/3}}$

36. $(2x-1)^{3/4}$

37. $(4x^2+1)^{-1/2}$

38. $(x-6)^{-1/3}$

39. $\dfrac{x^{1/2}}{(1-2x)^{1/3}}$

40. $\dfrac{(3-7x)^{3/2}}{2x}$