APPLICATIONS OF INTEGRATION

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1 Examples

1. Find the area of the region bounded by the x-axis and the graph of $f(x) = x^2 - 1$ for $1 \leq x \leq 2$.
2. Evaluate the definite integral $\int_0^1 (4t+1)^2 \mathrm{d}t$.

2 ClassWork Problems

2.1 Finding Areas

Find the area of the shaded region described by the function and bounds (implicitly or explicitly shown/stated):

3. $y = x - x^2$ *(Implied bounds from x-intercepts, likely 0 to 1)*
4. $y = 1 - x^4$ *(Implied bounds from x-intercepts, likely -1 to 1)*
5. $y = \dfrac{1}{x^2}$ bounded by $x = 1, x = 2$, and the x-axis.
6. $y = \dfrac{2}{\sqrt{x}}$ bounded by $x = 1, x = 4$, and the x-axis.
7. $y = 3e^{-x/2}$ bounded by $x = 1, x = 4$, and the x-axis.
8. $y = 2e^{x/2}$ bounded by $x = 1, x = 3$, and the x-axis.
9. $y = \dfrac{x^2 + 4}{x}$ bounded by $x = 1, x = 4$, and the x-axis.
10. $y = \dfrac{x - 2}{x}$ bounded by $x = 2, x = 4$, and the x-axis.

2.2 Evaluating Definite Integrals

Evaluate the following definite integrals:

11. $\displaystyle\int_0^1 2x \, \mathrm{d}x$
12. $\displaystyle\int_2^7 3v \, \mathrm{d}v$

13. $\int_{-1}^{0} (x-2)\ \mathrm{d}x$

14. $\int_{2}^{5} (-3x+4)\ \mathrm{d}x$

15. $\int_{-1}^{1} (2t-1)^2\ \mathrm{d}t$

16. $\int_{0}^{1} (1-2x^2)\ \mathrm{d}x$

17. $\int_{0}^{3} (x-2)^3\ \mathrm{d}x$

18. $\int_{2}^{2} (x-3)^4\ \mathrm{d}x$

19. $\int_{-1}^{1} (\sqrt[3]{t}-2)\ \mathrm{d}t$

20. $\int_{1}^{4} \sqrt{\frac{2}{x}}\ \mathrm{d}x$

21. $\int_{1}^{4} \frac{u-2}{\sqrt{u}}\ \mathrm{d}u$

22. $\int_{0}^{1} \frac{x-\sqrt{x}}{3}\ \mathrm{d}x$

23. $\int_{-1}^{0} (t^{1/3}-t^{2/3})\ \mathrm{d}t$

24. $\int_{0}^{4} (x^{1/2}+x^{1/4})\ \mathrm{d}x$

25. $\int_{0}^{1} e^{-2x}\ \mathrm{d}x$

26. $\int_{1}^{2} e^{1-x}\ \mathrm{d}x$

27. $\int_{1}^{3} \frac{e^{3/x}}{x^2}\ \mathrm{d}x$

28. $\int_{-1}^{1} (e^{x}-e^{-x})\ \mathrm{d}x$

29. $\int_{0}^{1} e^{2x}\sqrt{e^{2x}+1}\ \mathrm{d}x$

30. $\int_{0}^{1} \frac{e^{-x}}{\sqrt{e^{-x}+1}}\ \mathrm{d}x$

31. $\displaystyle\int_0^2 \frac{x}{1+4x^2}\,\mathrm{d}x$

32. $\displaystyle\int_0^1 \frac{e^{2x}}{e^{2x}+1}\,\mathrm{d}x$