Chapter 8
Factor Analysis

Learning Objectives

After reading this chapter, you should understand:

– The principles of exploratory and confirmatory factor analysis.
– The difference between principal components analysis and principal axis factoring.
– Key terms such as Eigenvalues, communality, factor loadings and factor scores.
– How to determine whether data are suitable for carrying out an exploratory factor analysis.
– How to interpret SPSS factor analysis output.
– The principles of reliability analysis and how to carry it out in SPSS.
– The basic idea behind structural equation modeling.

Keywords Anti-image · Bartlett's test of sphericity · Communality · Exploratory and confirmatory factor analysis · Principal axis factoring · Principal components analysis · Eigenvalue · Factor loadings · Factor scores · Kaiser criterion · Kaiser–Meyer–Olkin (KMO) criterion · Measure of sampling adequacy (MSA) · Scree plot · Orthogonal and oblique rotation · Varimax and direct oblimin rotation · Reliability analysis · Cronbach's Alpha · Structural equation modeling · LISREL · PLS path modeling

The increasingly complex media environment provides media planners with more options to reach their target audiences. However, it also makes media planning more difficult, as consumers react to the abundance of media by being selective and paying less attention to it. In their market research study, Bronner and Neijens (2006) investigate how the media context influences the effects of advertisements. Over 1,000 respondents were asked to rate their experience with advertisements using ten survey items. Factor analysis revealed that five factors underlie these ten items. Using the resulting factor scores, the authors show that, for example, advertising on TV and radio are perceived as irritating, whereas newspaper and cinema ads appear stimulating. Furthermore, advertisements on the Internet are considered informative but somewhat irritating. These factor analysis results help media planners to better target their audiences by choosing the appropriate medium to create awareness and persuasion, or to establish a relationship between the brand and the consumer.

E. Mooi and M. Sarstedt, *A Concise Guide to Market Research*,
DOI 10.1007/978-3-642-12541-6_8,

Introduction

Factor analysis identifies unobserved variables (factors) that explain patterns of correlations within a set of observed variables. It is often used to identify a small number of factors that explain most of the variance embedded in a larger number of variables. Thus, factor analysis is about data reduction. It can also be used to generate hypotheses regarding the composition of factors. Furthermore, factor analysis is often used to screen variables for subsequent analysis (e.g., to identify collinearity prior to performing a linear regression analysis as discussed in Chap. 7).

There are three types of factor analyses we discuss, namely *exploratory factor analysis, confirmatory factor analysis*, and *structural equation modeling*. The first two techniques are identical from a statistical point of view; however, they are used in different ways. Exploratory factor analysis is used to reveal the number of factors and the variables that belong to specific factors. When we conduct a confirmatory factor analysis, we have clear expectations regarding the factor structure (e.g., because we make use of a previously used survey) and we want to test if the expected structure is indeed present. Structural equation modeling differs from those two techniques, both statistically and practically. It is used to evaluate how well variables relate to factors and what the relationships between the factors are. This technique will be briefly discussed at the end of this chapter.

In this chapter, we primarily deal with exploratory factor analysis, as it conveys the principles that underlie all factor analytic procedures. Two exploratory factor analytic procedures are commonly used in market research: *principal components analysis* and *principal axis factoring*. When the purpose is to summarize information (variance) represented by the variables using a small number of factors, principal components analysis is used. Alternatively, if the aim is to identify underlying factors or dimensions that reflect what communalities variables share, principal axis factoring is used. We will discuss this distinction in greater detail later in this chapter. Principal components and principal axis factoring essentially require the same analysis steps and involve the same interpretation. However, as the concept of principal components analysis is easier to understand, we focus on this approach, but briefly highlight differences.

Understanding Principal Components Analysis

In many situations, researchers are faced with the problem of working with large questionnaires that comprise a multitude of questions (i.e., *items*, see Chap. 3). In order to evaluate all aspects of a specific marketing research problem, researchers often have to consider how the questions in these questionnaires are related.

For example, in a survey of a major German soccer club's marketing department, the management was particularly interested in identifying and evaluating performance features that relate to soccer fans' satisfaction. Features that could be of importance include the stadium, the composition of the team and their success, the trainer and the management. The marketing department therefore commissioned a questionnaire comprising 99 items that had been previously identified using literature databases and focus groups with various fans. All the items were measured on Likert scales ranging from 1 ("very dissatisfied") to 7 ("very satisfied"). Table 8.1 shows an overview of selected items considered in the study.

Table 8.1 Items in the soccer fan satisfaction study

Satisfaction with...	
Condition of the stadium	Public appearances of the players
Interior design of the stadium	Number of stars in the team
Outer appearance of the stadium	Interaction of players with fans
Signposting outside the stadium	Volume of the loudspeakers in the stadium
Signposting inside the stadium	Choice of music in the stadium
Roofing inside the stadium	Entertainment program in the stadium
Comfort of the seats	Stadium speaker
Video score boards in the stadium	Newsmagazine of the stadium
Condition of the restrooms	Price of annual season ticket
Tidiness within the stadium	Entry fees
Size of the stadium	Offers of reduced tickets
View onto the playing field	Design of the home jersey
Number of restrooms	Design of the away jersey
Sponsors' advertisements in the stadium	Assortment of merchandise
Location of the stadium	Quality of merchandise
Name of the stadium	Prices of merchandise
Determination and commitment of the players	Pre-sale of tickets
Current success regarding matches	Online-shop
Identification of the players with the club	Opening times of the fan-shops
Quality of the team composition	Accessibility of the fan-shops
Presence of a player with whom fans can identify	Behavior of the sales persons in the fan shops

Imagine you are a market researcher and were asked to evaluate the data from this survey to answer the management's research question which is to identify and evaluate performance features that relate to soccer fans' satisfaction. What would be the first thing to do? Obviously, the first step would be to compute descriptive statistics to gain an overview of the dataset. You could compute the mean values of each item and rank these. You could then identify features with which fans are very dissatisfied and take measures to address these deficiencies. Even though this approach may prove useful to gain a first overview, it is certainly not very helpful in evaluating the research question. The differences in the fans' satisfaction with features might only be marginal and carrying out pairwise t-tests to evaluate whether these differences are significant or not will prove problematic in the light of the large number of items involved (see discussion on the familywise error rate in Chap. 6). In our example, comprising 99 items, we would have to carry out exactly 4,851 pairwise t-tests.[1]

[1]This number is calculated as $k \cdot (k-1)/2$, with k being the number of items to compare.

Furthermore, this approach does not help explain which features contribute most to the respondents' overall satisfaction. In order to address the latter question, you may use the item "Overall, how satisfied are you with your soccer club?" as a criterion variable and regress it on all remaining performance items. However, this is likely to create the collinearity problem discussed in Chap. 7. Just by looking at the formulation of the items, we expect that several of the items are highly correlated. For example, satisfaction with the condition (x_1), outer appearance (x_2), and interior design (x_3) of the stadium cover similar aspects relating to the respondents' satisfaction with the stadium. If a soccer fan is generally very satisfied with the stadium, he/she will most likely answer all three items positively. Conversely, if a respondent is generally dissatisfied with the stadium, he/she is most likely to be rather dissatisfied with all the performance aspects of the stadium, such as the outer appearance and interior design. Consequently, these three items are most likely to be highly correlated, as they cover related aspects of the respondents' overall satisfaction with the stadium. In other words, with regard to their core meaning, these items are to a certain degree redundant. This is where the principal components analysis comes into play.

The basic idea of principal components analysis is to make use of these correlations to summarize sets of items within latent variables. As the name already suggests, these latent variables are not directly observable but each is inferred and based on several items. A latent variable is also called a factor or component – essentially, all three terms mean the same but the more generic term factor is most commonly used in the context of principal components analysis. Figure 8.1 displays

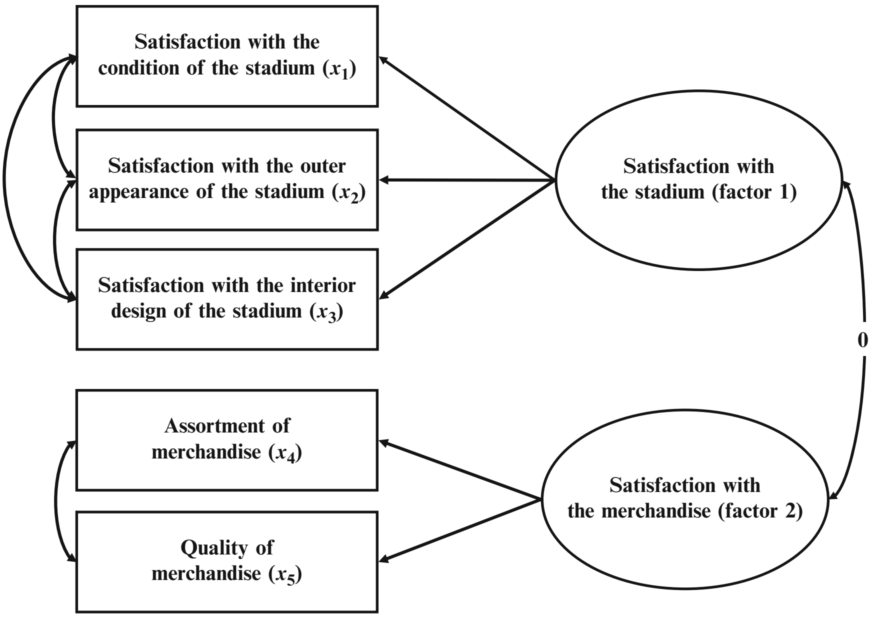

Fig. 8.1 Example of a factor model

the coherence between two factors and a set of items. It is standard practice for factors to be illustrated by circles, whereas rectangles are used for items.

The upper part of the figure suggests that there is one factor that relates directly to the three items x_1, x_2, and x_3. These three items are likely to be highly correlated, as indicated by the arrows between them. We assume that this correlation is caused by an underlying factor, which is indicated by the arrows pointing from the factor to the items. Specifically, the items are reflecting the factor. Consequently, the items can be interpreted as manifestations of the factor with the factor capturing the joint meaning of the items related to it. In our example, the "joint meaning" of the three items could be described as *satisfaction with the stadium*, since the items represent somewhat different, yet related, aspects of this issue. Likewise, we can think of another factor that relates to another set of items as described in Fig. 8.1. This factor relates to two items (x_4 and x_5) which, similarly to the first factor, share a common meaning. This common meaning is captured by the factor which we could label *satisfaction with the merchandise*.

The basic idea of principal components analysis is to use large correlations between the items to compute factors that best represent the items in our dataset. Therefore, a certain amount of correlation is necessary to conduct a principal components analysis. Often, many items in a dataset are correlated with one another. Figure 8.1 therefore only indicates that there are high correlations between x_1, x_2, and x_3 on the one hand and x_4 and x_5 on the other. Other items, such as x_1 and x_4, are most likely somewhat correlated but to a lesser degree than the group of items x_1, x_2, and x_3 and the pair x_4 and x_5.

Using the correlations between sets of items, the principal components analysis extracts a number of factors which can be considered salient unobserved variables capturing important aspects of the complete item set. Initially, these factors are, by definition, uncorrelated so that each factor covers distinct and unrelated aspects (indicated by the 0 on the arrow linking factors 1 and 2).[2] This is a very important feature, as it means that factors are uncorrelated and thus, if we use them in regression analysis, collinearity is not an issue. Instead of using many highly correlated items as independent variables in a regression analysis, we can use only a few uncorrelated factors that represent the original item set. However, using only a few factors instead of many items comes at the expense of accuracy. Naturally, these factors cannot represent all the information inherent in the items. Consequently, there is a trade-off between simplicity and accuracy. In order to make the analysis as simple as possible, we want to extract only a few factors. At the same time, we do not want to lose too much information by having too few factors. This trade-off has to be addressed in any principal components analysis when deciding how many factors should be extracted from the data.

[2]Note that this changes when factors are rotated in a oblique way. We will discuss factor rotation later in this chapter.

In the example above, we do not have any prior knowledge of how the items relate to a factor or how many factors underlie the items (that's the basic idea of an exploratory factor analysis). We simply use all the items related to our research question and integrate them into one analysis. However, in practice, we usually have an idea of what the factor structure could look like and, thus, might adjust the factor solution if this is more reasonable. These adjustments might affect the number of factors extracted or the way in which items are assigned to a factor.

After having decided on the number of factors to retain from the data, we can proceed with the interpretation of the factor solution. This requires us to come up with a label for each factor that best characterizes the joint meaning of all the variables associated with it. Often, this step is very challenging but a rotation of the factors can facilitate the interpretation of the factor solution. Lastly, we have to assess how well the factors reproduce the data, which is done by examining the solution's goodness-of-fit. Figure 8.2 illustrates the steps involved in a principal components analysis; we will discuss these in more detail in the following sections.

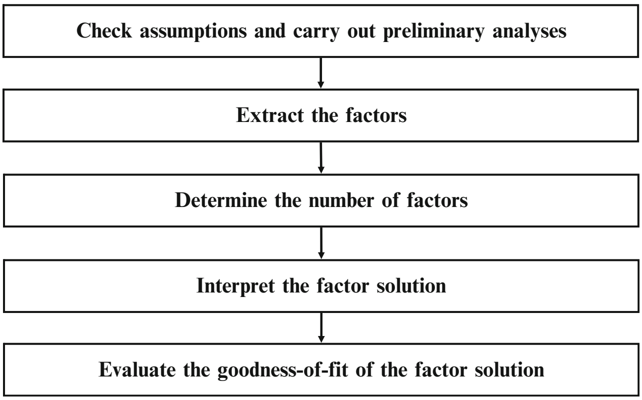

Fig. 8.2 Steps involved in conducting a principal components analysis

Conducting a Principal Components Analysis

Check Assumptions and Carry out Preliminary Analyses

Before carrying out a principal components analysis, we have to consider some basic assumptions that underlie the approach.

To conduct principal components analysis, it is best to have data measured on an interval or ratio scale level. In practical applications, however, it has become common to also use items measured on an ordinal scale level. This procedure usually leads to valid results, particularly if there is a large number of response categories (five or more) for the items. However, using ordinal data requires the items' scale steps to be equidistant. Equidistant means that the difference in the wording between scale steps is the same. Accordingly, the difference in wording between ordinal categories should capture this condition (see Chap. 3). Another point of concern is the sample size. As a rule, the number of (valid) observations should be at least ten times the number of items used for analysis. This also includes a missing value analysis. Since it is generally recommended that cases with missing values should be excluded, this can greatly reduce the number of valid observable variables in our analysis.

Furthermore, we have to ensure that the observations are independent. We discuss this issue in Chap. 3.

As indicated before, principal components analysis is based on correlations between items. Consequently, carrying out a principal components analysis only makes sense if the items are sufficiently correlated. The problem is how to decide what "sufficient" actually means.

An obvious step is to examine the *correlation matrix*. Naturally, we want correlations between different items to be as high as possible.[3] However, this will not always be the case. Regarding our previous example, we expect high correlations between x_1, x_2, and x_3, on the one hand, and x_4 and x_5 on the other. Conversely, we might expect lower correlations between, for example, x_1 and x_4 and between x_3 and x_5. Thus, not all elements of the correlation matrix necessarily have to have high values. Generally, correlations should be above absolute 0.30. However, if single correlations are lower, this is not necessarily problematic. Only when all the correlations are around zero, does principal components analysis stop being useful. In addition, the statistical significance of each correlation coefficient (indicated in the output produced by SPSS) helps decide whether it differs significantly from zero.

There are additional measures to determine whether the items are sufficiently correlated. One is the *anti-image*. The anti-image describes the portion of an item's variance that is independent of another item in the analysis. Obviously, we want all items to be highly correlated, so that an item's anti-images are as small as possible. This issue is captured in the anti-image matrices. Initially, we do not interpret these matrices directly but, instead, revert to two measures based on the concept of anti-image: *The Kaiser–Meyer–Olkin (KMO)* statistic and the *Bartlett's test of sphericity*. The KMO statistic, also called the *measure of sampling adequacy (MSA)*, indicates whether the correlations between variables can be explained by the other variables in the dataset. Kaiser (1974), who introduced the statistic, recommends a set of (very vividly labeled) threshold values for KMO and MSA, as presented in Table 8.2.

The *Bartlett's test of sphericity* can be used to test the null hypothesis that the correlation matrix is a diagonal matrix (i.e., all non-diagonal elements are zero) in the population. Since we need high correlations for principal components

[3]Note that variables should not be perfectly correlated (the correlation is -1 or 1), as this might lead to problems in the analysis.

Table 8.2 Threshold values for KMO and MSA

KMO/MSA value	Adequacy of the correlations
Below 0.50	Unacceptable
0.50–0.59	Miserable
0.60–0.69	Mediocre
0.70–0.79	Middling
0.80–0.89	Meritorious
0.90 and higher	Marvelous

analysis, we want to reject the null hypothesis. A large test statistic value and a small p-value will each favor the rejection of the hypothesis. In practical applications, it is virtually impossible not to reject this null hypothesis, as some variables will always be correlated. Moreover, low test statistic values always go hand in hand with poor KMO values.

As a consequence, the final decision of whether the data are appropriate for principal components analysis should be primarily based on the KMO statistic. Likewise, the correlation matrix with the associated significance levels provides a first insight into the correlation structures. If these measures indicate sufficiently correlated variables, we can continue the analysis of the results.

How do we proceed if this is not the case? In such a situation, we can try to identify items that are only weakly correlated with the remaining items and, thus, affect the results negatively. This can be done by examining the correlation matrix and the significance levels of correlations. A better approach, however, is to take another look at the anti-image correlation matrix. The diagonal elements of this matrix describe the *variable-specific MSA values*, which are interpreted like the overall KMO statistic (see Table 8.2). In fact, the KMO statistic is nothing but the overall mean of all item-specific MSA values. Consequently, all MSA values on the anti-image correlation matrix's diagonal should also lie above the threshold level of 0.50. If this is not the case, we should consider removing this item from the analysis. An item's *communality* (we will discuss this term in the next section) can also serve as a useful indicator of how well an item is represented by the factors extracted. However, communalities are more often considered when evaluating the solution's goodness-of-fit.

Extract the Factors

In a principal components analysis, factors are extracted in such a way that the variables' initial correlation matrix is reproduced in the best possible way, which means that the discrepancy between the initial and reproduced correlation matrix should be as small as possible. Operationally, the principal components analysis selects a number of variables that are highly correlated and relates those to a certain factor. Next, another set of highly correlated variables is chosen and related to a second factor. This is repeated in a step-by-step manner until all the variables have been included.

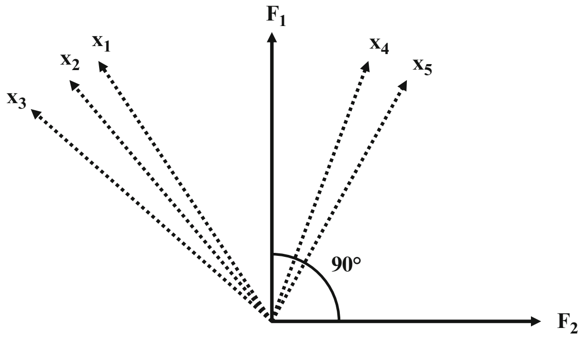

Fig. 8.3 Factor extraction

The first factor is extracted in such a way that it maximizes its variance accounted for. We can easily visualize this by looking at the vector space illustrated in Fig. 8.3. In this example, we have five variables ($x_1,\ldots,x_5$) that are represented by five vectors starting at the zero point (with its length standardized to one). To maximize the variance accounted for, the first factor F_1 is fitted into this vector space in such a way that the sum of all the angles between this factor and the five variables in the vector space is minimized. This is done because we can interpret the angle between two vectors as correlations. For example, if the factor's vector and a variable's vector are congruent, the angle between these two is zero, indicating that the factor and the variable are perfectly correlated. On the other hand, if the factor and the considered variable are uncorrelated, the angle between these two is 90°. This correlation between factor and variables is referred to as *factor loading*.

After this, a second factor (F_2) is extracted and maximizes the remaining variance accounted for. This factor is – by definition – independent from the first one and, thus, is fitted into the vector space at a 90° angle (Fig. 8.3). If we extracted a third factor, it would explain the maximum amount of the variance that has hitherto not been accounted for by factors 1 and 2. This one would also be fitted in a 90° angle from the first two factors. We don't illustrate this third factor in Fig. 8.3, as we would be looking at a three-dimensional space.[4] This procedure continues until as many factors as there are variables (i.e., five) are extracted.

One important feature of the principal components analysis is that it works with standardized variables (see Chap. 5 for an explanation of what standardized

[4]Note that in Fig. 8.3, we consider a special case as the five variables are scaled down into a two-dimensional space. Actually, in this set-up, it would be possible to explain all five variables by means of the two factors. However, in real-life, the five variables span a five-dimensional vector space.

variables are). The standardization of the variables is done automatically by SPSS, which means we do not have to bother with this. However, this characteristic has important implications for our analysis, since it helps us assess how much information a factor captures. This information is incorporated in a factor's *Eigenvalue*. An Eigenvalue describes how much variance is accounted for by a certain factor. Likewise, the standardization enables us to assess how much of each variable's variance is captured or reproduced by the factors extracted. This is referred to as *communality*.

How can we interpret these two measures adequately? To answer this question, think of the soccer fan satisfaction study (Fig. 8.1). In the example, there are five variables, each with a variance of one. In a simplified way, we could say that the overall information (i.e., variance) that we want to reproduce by means of factor extraction is five. Let us assume that we extract the two factors presented above.

The first factor's Eigenvalue indicates how much variance of the total variance (i.e., five units) this factor accounts for. If this factor has an Eigenvalue of, let's say 2.10, it covers the information of 2.10 variables or, put differently, accounts for 2.10/5.00 = 42% of the overall variance (Fig. 8.4).

Extracting a second factor will allow us to explain another part of the remaining variance (i.e., 5.00 − 2.10 = 2.90 units, Fig. 8.4). However, the Eigenvalue of the second factor will always be smaller than that of the first factor. Assume that the second factor has an Eigenvalue of 1.30 units. The second factor then accounts for 1.30/5.00 = 26% of the overall variance. Together, these two factors explain (2.10 + 1.30)/5.00 = 68% of the overall variance.

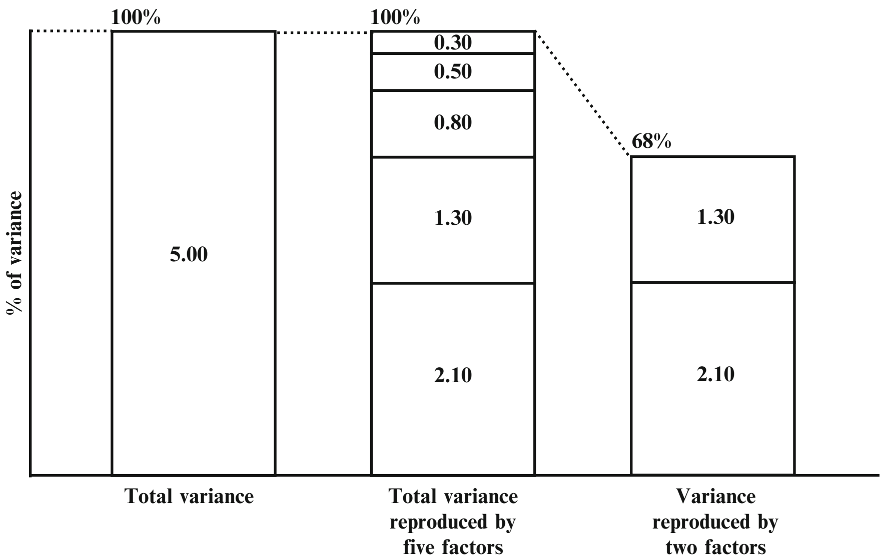

Fig. 8.4 Total variance explained by variables and factors

Every additional factor extracted, increases the variance accounted for until we have extracted as many factors as there are variables. In this case, the overall variance accounted for by the factors is 100%, which means that the complete variance is reproduced by the factors (Fig. 8.4).

Following the principal components approach, we assume that each variable's entire variance can be reproduced by means of factor extraction. In other words, we assume that each variable's variance is common, that is, it is shared with other variables. As a consequence, variables do not have any unique variance. A different, but also popular, approach to factor analysis is *principal axis factoring* (also called common factor analysis or principal factor analysis), which we briefly discuss in Box 8.1.

Box 8.1 Principal components analysis vs. principal axis factoring

Unlike with principal components analysis, the principal axis factoring approach assumes that each variable has some common as well as unique variance. However, only the variance shared with all other variables in your analysis (i.e., the communality) can be reproduced by means of factor extraction. This variance shared is usually the R^2 obtained from a regression of that variable on all remaining ones. Consequently, each variable's initial communality, which indicates the amount of variance that can be explained, is lower than one (unlike in the principal components approach in which the initial communality is always 1). The extraction of factors follows the same principles as in principal components analysis and the interpretation of statistical measures such as KMO, Eigenvalues, or factor loadings are analogous.

From a theoretical perspective, the assumption that there is a unique variance which cannot be fully accounted for by the factors is generally more realistic but, at the same time, more restrictive. Although theoretically sound, this restriction can sometimes lead to complications in the analysis which have contributed to the widespread use of principal components analysis, especially in market research practice.

Usually, researchers suggest using the principal components analysis when data reduction is the primary concern; that is, when the focus is to extract a minimum of factors that account for a maximum proportion of the variables' total variance. On the contrary, if the primary concern is to identify latent dimensions represented in the variables, principal axis factoring should be applied.[5] However, as both approaches often yield very similar results, we generally suggest using principal components analysis.

(*continued*)

[5]Researchers often argue along the lines of measurement error when distinguishing between principal components analysis and principal axis factoring (e.g., Hair et al. 2010). However, as this distinction does not really have implications for market research studies, we omitted this argument.

If you are interested in finding out more about the differences between principal components analysis and principal axis factoring, just visit Garson's Statnotes page on factor analysis:

http://faculty.chass.ncsu.edu/garson/PA765/factor.htm

Lastly, additional methods for carrying out factor analyses are available, such as the unweighted least squares, generalized least squares, or maximum likelihood approaches. However, these are statistically complex and inexperienced users should therefore not consider them. Hair et al. (2010) provide an excellent introduction to these advanced techniques.

Whereas the Eigenvalue tells us how much variance is accounted for by each factor, the *communality* indicates how much variance of each variable can be reproduced through factor extraction. There is no commonly agreed threshold for a variable's communality, as this depends strongly on the complexity of the analysis at hand. However, generally, at least 30% of a variable's variance should be accounted for through the extracted factors. Thus, the communalities should lie above 0.30. Every additional factor extracted will increase this variance and if we extract as many factors as there are items (in our example five), the communality of each variable would be 1.00. The variable will then be entirely explained by the factors extracted; that is, a certain amount of its variance will be explained by the first factor, another part by the second factor, and so on.

However, as our overall objective is to reduce the number of variables through factor extraction, we should rather extract only a few factors that account for a high degree of the overall variation. This raises the question of how to decide on the number of factors to extract from the data, which we discuss in the following section.

Determine the Number of Factors

The intuitive way to decide on the number of factors is to extract all factors with an Eigenvalue greater than one. The reason is that every factor with an Eigenvalue greater than one accounts for more variance than a single variable (remember, we are looking at standardized variables, which is why each variable's variance is exactly one). As the overall objective of principal components analysis is to reduce the overall number of variables, each factor should, of course, account for more variance than a single variable can. If this occurs, then this factor is useful for reducing the set of variables. Extracting all factors with an Eigenvalue greater than one is frequently called the *Kaiser criterion* and is by far the most frequently used criterion to decide the number of factors. However, in some situations, the inclusion (exclusion) of additional factors with a smaller (higher) Eigenvalue than one might prove beneficial for interpreting the solution in a meaningful way. Ultimately, we should not entirely rely on the data but keep in mind that the research results should be interpretable and actionable for market research practice.

There are also alternative approaches to decide on the number of factors to extract. SPSS offers the possibility to plot each factor's Eigenvalue (y-axis) against the factor with which it is associated (x-axis). This results in a so-called *scree plot* that typically has a distinct break in it, thereby showing the "correct" number of factors. This distinct break is referred to as the "elbow" and it is generally recommended that all factors should be retained above this break, as they contribute most to the explanation of the variance in the dataset. Thus, we select one factor less than indicated by elbow. In some situations, however, the distinct break is not clear-cut and we should instead rely on the Kaiser criterion.

In other situations, we might have a priori information regarding the number of factors we want to find, which might be the case in a situation in which we want to replicate a previous market research study. In this case, we should also be guided by the findings of this previous study. Thus, if we find four factors and a number of previous studies suggest that five are present, we should try to select five factors. However, strictly speaking, this is a confirmatory approach to factor analysis. Another approach is to split the dataset into two halves and carry out separate factor analyses on each dataset. Only factors with a high correspondence of factor loadings across the two samples are extracted (split-half reliability).[6] Whatever approach is used to determine the number of factors, the factors extracted should account for at least 50% of the total variance explained (75% or more is recommended).

After having decided on the number of factors to retain from the data, we can proceed with the interpretation of the factor solution.

Interpret the Factor Solution

To interpret the solution of the principal components analysis, we have to determine which variables strongly relate to each of the factors extracted. This is done by

---

[6]Note that there are further approaches to determine the number of factors such as the parallel analysis or the minimum average partial test which we discuss in the Web Appendix (Web Appendix → Chapter 8).

examining the factor loadings, which represent the correlations between the factors and the variables and, thus, can take values from −1 to +1. High factor loadings indicate that a variable is well represented by a certain factor. Subsequently, we always look for high absolute values as the correlation between a variable and a factor can also be negative. Using the highest absolute factor loadings, we "assign" each variable to a certain factor and then try to come up with a label for each factor that best characterizes the joint meaning of all the variables associated with it. This labeling is subjective, but nevertheless a key step in principal components analysis. An example of a label is the respondents' satisfaction with the stadium that represents the items referring to its condition, outer appearance, and interior design.

To facilitate the interpretation of the factors, we can make use of *factor rotation*. We do not have to rotate the factor solution, but it will facilitate interpreting findings, particularly if we have a reasonably large number of items (say six or more). To understand what factor rotation is all about, reconsider the factor structure described in Fig. 8.3. Here, we see that both factors relate to the variables in the set. However, the first factor appears to be generally more strongly correlated with the variables, whereas the second factor is only weakly correlated with the variables (to clarify, we look for small angles between the factors and variables). This implies that we "assign" all variables to the first factor without taking the second one into consideration. This does not appear to be very meaningful, as we want both factors to represent certain facets of the variable set. This problem can be resolved by means of factor rotation. By rotating the factor axes, we can bring about a situation in which a set of variables is loaded highly on only one specific factor, whereas another set loads highly on another. Figure 8.5 illustrates the factor rotation graphically.

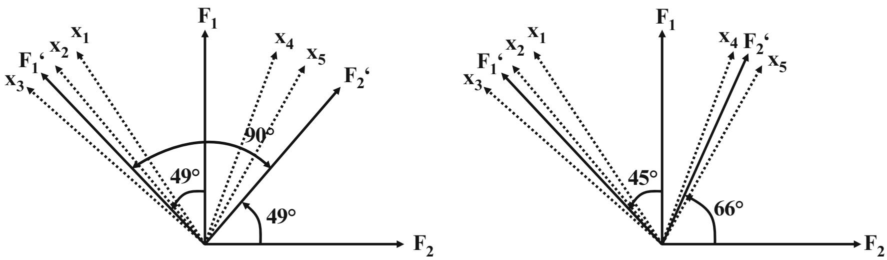

Fig. 8.5 Orthogonal and oblique factor rotation

On the left hand side of the figure, we can see that both factors are rotated 49° in an orthogonal way, meaning that the 90° angle between the factors is sustained during the rotation procedure. Consequently, the factors remain uncorrelated, which is in line with the initial objective of the principal components analysis. By rotating the first factor from F_1 to $F_{1'}$, it is now strongly related to variables x_1, x_2, and x_3 but weakly related to x_4 and x_5. Conversely, by rotating the second factor from F_2 to $F_{2'}$, it is now strongly related to x_4 and x_5 weakly related to the remaining variables. The assignment of the variables is now much clearer, which facilitates the interpretation of the factors significantly. Various orthogonal rotation methods exist which differ with regard to their treatment of the loading structure. The *varimax* procedure is the most prominent one; this procedure aims at maximizing the dispersion of loadings within factors. Other orthogonal rotation methods include the quartimax or equamax procedures. See the SPSS help option for more information on these procedures' properties.

Alternatively, we can choose an oblique rotation technique. In this approach, the 90° angle between the factors is not maintained during rotation and the resulting factors are therefore correlated. Figure 8.5 (right hand side) illustrates an example of an oblique factor rotation. *Direct oblimin* is the most commonly used oblique rotation technique. Here, researchers have to specify a constant δ (pronounced as *delta*) which determines the level of correlation allowed between the factors. The default value of zero ensures that the factors are – if at all – only moderately correlated, which is acceptable for most analyses. For example, it is very likely that the respondents' satisfaction with the stadium is unrelated to their satisfaction with other aspects of a soccer club, such as the number of stars in the team or the quality of merchandise. However, giving up the initial objective of extracting uncorrelated factors can diminish the interpretability of the factors.

We recommend using the varimax rotation to enhance the interpretability of the results. Only if the results are difficult to interpret, should an oblique rotation be applied.

Despite the factor rotation, the interpretation of the factors is usually the most challenging step in a principal components analysis, as the assignment of variables is often not clear-cut. Sometimes it is reasonable to assign a variable to another factor even though it does not load highly on this specific factor. While this can help to increase the results' face validity, we should make sure that the variable's factor loading with the designated factor still lies above an acceptable level. With very few factors extracted, the loading should be at least 0.50, but with a high number of factors, lower loadings of above 0.30 are feasible. In other situations, it might even be worthwhile eliminating a certain variable. In the end, the results should be interpretable and actionable, but keep in mind that this is first and foremost exploratory.

Following the rotation and interpretation of the factors, we can consider another important element of the analysis, the *factor scores*. These represent the degree to which each respondent exhibits the characteristic of a particular factor. While factor loadings describe the association between variables and factors, factor scores characterize the relation between observations and factors. Each factor's scores are standardized to a mean of zero. This means that if a respondent has a value greater than zero for a certain factor, he or she exhibits the characteristic described by the factor above the average. Conversely, if a factor score is below zero, then this respondent exhibits the characteristic below average. There are different procedures to produce factor scores but we suggest using the regression method, as this is the most commonly used and easily understood approach.[7]

Factor scores are used in several ways: If the objective of the principal components analysis is to reduce the number of variables, subsequent analyses can simply be based upon the factor scores. For example, instead of calculating the average for x_1, x_2, and x_3 and using this as a construct value, we can use the factor scores of *satisfaction with the stadium* in follow-up analyses. The scores of the uncorrelated factors, instead of highly correlated predictor variables, can specifically be used if the objective of the principal components analysis was to overcome collinearity problems in an OLS regression.

Evaluate the Goodness-of-fit of the Factor Solution

The overall objective of principal components analysis is to extract factors in such a way that the factors and their loadings reproduce the data in the best possible way. Consequently, we can make use of the differences between the correlations in the data and those implied by the factors to assess the obtained solution's goodness-of-fit. We require the absolute difference between the observed and reproduced correlation coefficients (the residuals) to be as small as possible. In practice, we check the proportion of the correlation matrices' residuals with an absolute value higher than 0.05. Even though there is no rule of thumb regarding the maximum proportion, a value of more than 50% should raise concern. However, low correlations and an unsatisfactory KMO measure usually go hand in hand with high residuals; consequently, this problem already emerges in the testing of assumptions.

Furthermore, we should consider each variable's communality, which should, of course, be as high as possible. However, if several communalities exhibit rather low values, we should consider removing these variables. To help make that decision, we could also take the variable-specific MSA measures into account. There is no minimum value for a variable's communality, as the values usually depend on the number of variables considered. If there are more variables in your dataset, communalities usually become smaller, however, if your factor solution accounts for less than 30% (i.e., the variable's communality is less than 0.30) of a variable's variance, it is worthwhile reconsidering your set-up.

In Table 8.3 we summarize the main steps that need to be taken when conducting a factor analysis using SPSS.

[7]Alternative procedures include the Bartlett method and the Anderson–Rubin method, which are designed to overcome potential problems associated with the regression technique. However, these problems are of rather theoretical nature and of little importance to market research practice.

Table 8.3 Steps involved in carrying out a factor analysis in SPSS

Theory	SPSS
Research problem	
Select variables that should be reduced to a set of underlying factors (principal factor analysis) or that should be used to identify underlying dimensions (principal axis factoring)	Enter these into the Variables box in the Factor Dialog Box: ► Analyze ► Dimension Reduction ► Factor.
Assumptions	
Are the variables interval or ratio scaled?	Determine the measurement level of your variables (see Chap. 3). If ordinal variables are used, make sure that the scale steps are equidistant.
Missing value analysis	Check descriptive statistics output for the number of valid cases out of the total number of cases: ► Analyze ► Dimension Reduction ► Factor ► Descriptives ► Univariate descriptives
Is the sample size sufficiently large?	Check that the number of valid observations is at least ten times the number of items.
Are the observations independent?	Determine whether the observations are dependent or independent (see Chap. 3).
Are the variables sufficiently correlated?	► Analyze ► Dimension Reduction ► Factor ► Descriptives ► Coefficients | Significance levels | KMO and Bartlett's test of sphericity | Anti-image. – Are correlation coefficients ≥ 0.30? – Are correlation coefficients' p-values ≤ 0.05? – Is the KMO ≥ 0.50? – Is the p-value of the Bartlett's test ≤ 0.05? – Are the variable-specific MSA values ≥ 0.50?
Specification	
Handle missing values	Delete missing values listwise: ► Analyze ► Dimension Reduction ► Factor ► Options ► Exclude cases listwise
Choose the method of factor analysis	If the scope of the analysis is to reduce the number of variables: ► Analyze ► Dimension Reduction ► Factor ► Extraction ► Principal components. If the scope of the analysis is to identify underlying dimensions: ► Analyze ► Dimension Reduction ► Factor ► Extraction ► Principal axis factoring
Determine the number of factors	Extract all factors with an Eigenvalue greater than one (default) and create a scree plot ► Analyze ► Dimension Reduction ► Factor ► Extraction ► Scree plot Alternatives: – Pre-specify the number of factors based on a priori information: ► Analyze ► Data Reduction ► Factor ► Extraction ► Factors to extract

(*continued*)

Table 8.3 (continued)

Theory	SPSS
	– Extract factors that jointly account for at least 50% (75% recommended) of the total variance: Create plot Total Variance Explained. – Split-half reliability: Split up the dataset in two halves of equal size and carry out separate factor analyses.
Rotate the factors	Use the varimax procedure or, if necessary, choose the direct oblimin procedure with $\delta = 0$: ▸ Analyze ▸ Dimension Reduction ▸ Factor ▸ Rotation
Assign variables to factors	Use the rotated solution to assign each variable to a certain factor based on the highest absolute loading. To facilitate interpretation, you may also assign a variable to a different factor but check that the loading lies at a passable level (0.50 if only few factors are extracted, 0.30 if many factors are extracted).
Compute factor scores	Save factor scores as new variables using the regression method: ▸ Analyze ▸ Dimension Reduction ▸ Factor ▸ Scores ▸ Save as variables: Regression
Checking results	
Determine the number of factors	Check the factors' Eigenvalues, the % of variance and the cumulative % explained: Examine the scree plot.
Interpret the factors	Consider the rotated component matrix and find an umbrella term for pairs of items assigned to each factor.
Checking goodness-of-fit	
Check the congruence of the initial and reproduced correlations	Create reproduced correlation matrix: ▸ Analyze ▸ Dimension Reduction ▸ Factor ▸ Descriptives ▸ Reproduced Is the proportion of residuals greater than $0.05 \leq 50\%$?
Check how much of each variable's variance is reproduced by means of factor extraction	Examine the communalities. Check if communalities are greater than 0.30.

Confirmatory Factor Analysis and Reliability Analysis

Many researchers and practitioners acknowledge the prominent role that factor analysis plays in exploring data structures. Data can be analyzed without preconceived ideas regarding the number of factors or how they relate to the variables under consideration. Whereas this approach is exploratory in nature, the *confirmatory factor analysis* allows for testing hypothesized structures underlying a set of variables.

Consequently, in a confirmatory factor analysis, the researcher needs to first specify the factors and their associations with variables, which should be based on previous measurements or theoretical considerations. Instead of allowing the procedure to determine the number of factors, as is done in an exploratory factor analysis, the confirmatory factor analysis tells us how well the actual data fit the pre-specified structure. Reverting to our introductory example, we could, for example, assume that the construct *satisfaction with the stadium* can be measured using the three items x_1, x_2, and x_3 (without having carried out an exploratory factor analysis beforehand!). Likewise, we could hypothesize that *satisfaction with the merchandise* can be adequately measured using the items x_4 and x_5. In a confirmatory factor analysis, we set up a theoretical model (also referred to as measurement model) linking the items with the respective factor (or construct – note that in confirmatory factor analysis, we use the term construct rather than factor).

This process is also called operationalization (see Chap. 3) and usually involves drawing a visual representation (called a *path diagram*) indicating the hypothesized relationships. This path diagram is very similar to the model presented in Fig. 8.1. In a path diagram, it is common to use a notation in which constructs are represented by Greek characters, whereas measured variables are represented by Latin characters. In a confirmatory factor analysis, constructs (ξ, pronounced as *xi*) are presented as circles or ovals, and measured variables (x) are presented as square or rectangular. Other elements include the relationships between the constructs and respective items (λ, pronounced as *lambda*), the error terms (σ, pronounced as *sigma*) that capture the extent to which a construct does not explain the item, and the correlations between the constructs of interest (Φ, pronounced *theta*). Figure 8.6 shows the hypothesized relationship between the observed items and constructs.

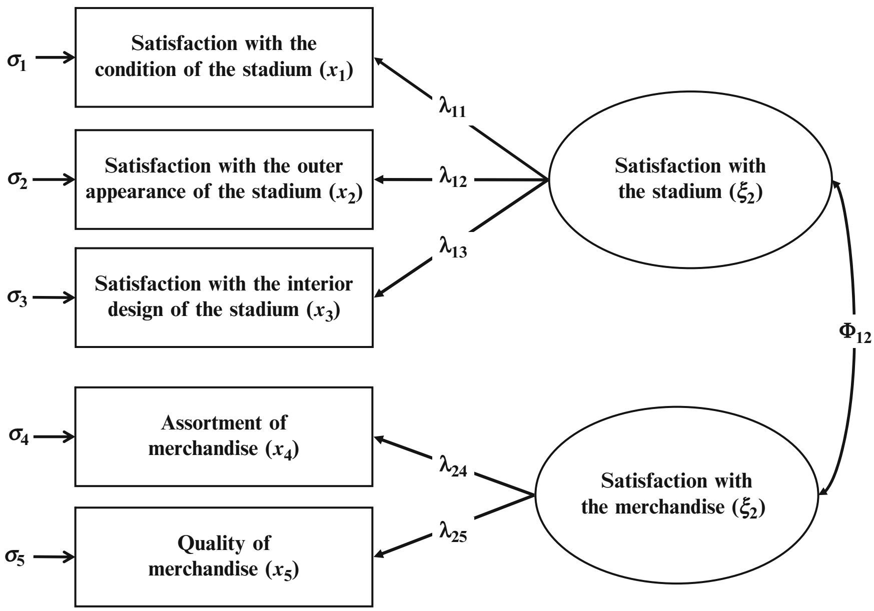

Fig. 8.6 Example of a path diagram

Having defined the individual constructs and developed the overall measurement model, the researcher needs to gather data. This allows the model parameters to be estimated. In this respect, the relationships between the constructs and items, that is, the factor loading λ, are of particular interest. There are several software programs that carry out confirmatory factor analyses (e.g., LISREL or EQS). In addition, the SPSS division of IBM offers AMOS for this task. The parameter estimates allow the researcher to assess the items' ability to measure a specific construct. This is referred to as construct validity and includes evaluating whether a measure correlates with another established scale of the same construct (convergent validity), or whether a construct is truly distinct from others in the model (discriminant validity). Furthermore, it is essential to assess each construct's reliability.

Carrying out a confirmatory factor analysis is rather demanding and requires a thorough understanding of the principles of measurement. An important element of a confirmatory factor analysis, which is crucial when working with scales, is the reliability analysis. Unlike the confirmatory factor analysis, a reliability analysis (see Chap. 3 for a discussion of reliability) can easily be carried out in SPSS. The preferred way to evaluate reliability is to make two independent measurements (using the same subjects) and to compare them by means of correlation analyses. This is also referred to as test-retest reliability (see Chap. 3).

However, in practice, researchers have often not had the opportunity to recapture their subjects for a second survey. This difficulty has been circumvented by the introduction of the split-half approach in which scale items are divided into halves and the scores of the halves are correlated to obtain an estimate of reliability. As all items should be consistent in what they indicate about the construct, the halves can be considered approximations of alternative forms of the same scale. Consequently, instead of looking at the scale's stability by means of test-retest reliability, researchers consider the scale's equivalence, which shows to which extent two measures of the same general trait agree. We call this type of reliability the *internal consistency reliability*.

In the example of "satisfaction with the stadium," we could compute this scale's split-half reliability manually by, for example, splitting up the scale into x_1 on the one side, and x_2 and x_3 on the other. We then compute the sum of x_2 and x_3 (or calculate the items' average) to form a total score and correlate this score with x_1. A high correlation indicates that the two subsets of items are measuring related aspects of the same underlying construct and, thus, a high degree of internal consistency. However, this example indicates that the computation result strongly depends on how the items are split. Cronbach (1951) proposed calculating the average of all possible split-half coefficients resulting from different ways of splitting the sample's scale items. His *Cronbach's Alpha* coefficient has become by far the most popular measure of internal consistency. In the example above, this would comprise calculating the average of the correlations between (1) x_1 and x_2+x_3, (2) x_2 and x_1+x_3, as well as (3) x_3 and x_1+x_2. The Cronbach's Alpha coefficient varies from 0 to 1, whereas a generally agreed lower limit for the coefficient is 0.70. However, in exploratory studies, a value of 0.60 is acceptable, while in the more advanced stages of research, values of 0.80 or higher are regarded as satisfactory.

SPSS not only reports one Cronbach's Alpha value for the entire scale, but also indicates how this value would be altered if a particular item was deleted. This information provides valuable suggestions on how to improve a scale's reliability if the initial estimate implies a low degree of internal consistency.

When calculating Cronbach's Alpha, ensure that all items are formulated identically. For example, in psychological measurement, it is common to use both negatively and positively worded items in a questionnaire. These need to be reversed prior to the reliability analysis. In SPSS, this can be achieved using the **Recode** option discussed in Chap. 5. Furthermore, we have to be aware of potential subscales in our item set. Some multi-item scales comprise subsets of items that measure different facets of a multidimensional construct. For example, soccer fan satisfaction is a multidimensional construct that includes aspects such as satisfaction with the stadium, the merchandise (as described above), the team, and the coach. Each of these dimensions is measured by means of different sets of items, which have to be evaluated separately with regard to internal consistency. Calculating one total Cronbach's Alpha value as a function of all 99 items in the study would certainly be inappropriate. Cronbach's Alpha is always calculated over the items belonging to one construct and not all items in the dataset!

One issue of assessing Cronbach's Alpha is its tendency to increase as the number of items in the scale increases. Consequently, researchers have to impose more stringent requirements (i.e., higher threshold values for Cronbach's Alpha) for scales with a large number of items. In particular, scales with more than ten items, should have a Cronbach's Alpha of least 0.80.

We will illustrate a reliability analysis using the standard SPSS module in the example at the end of this chapter.

Structural Equation Modeling

Whereas a confirmatory factor analysis involves testing if and how items relate to specific constructs, structural equation modeling involves the estimation of relations between these constructs. It has become one of the most important methods in the social sciences, including marketing research.

There are two approaches for estimating structural models. The covariance-based approach, which is also referred to as linear structural relations (LISREL) owing to its corresponding software application, and the variance-based approaches, of which partial least squares (PLS) path modeling is currently the most prominent procedure. Both estimation methods are based on the idea of an underlying model that allows the researcher to model, verify, and measure causal relationships between multiple items and constructs.

Figure 8.7 shows an example path model with four constructs and their respective items (note that we omitted the error terms for clarity's sake). A path model incorporates two types of constructs (1) Exogenous constructs (indicated with ξ, and pronounced as *ksi*) that do not depend on other constructs and (2) endogenous constructs (indicated with η, and pronounced as *eta*) that depend on one or more exogenous (or other endogenous) constructs. The relations between the constructs (indicated with γ, and pronounced as *gamma*) are called inner relations, while the relations between the constructs and their respective items (indicated with λ, and pronounced as *lambda*) are called outer relations. One can distinguish between the structural model (also inner model) that incorporates the relations between the constructs and the (exogenous and endogenous) measurement models (also outer models) that represent the relations between the constructs and their dedicated items. Items that measure exogenous (endogenous) constructs are labeled x (y).

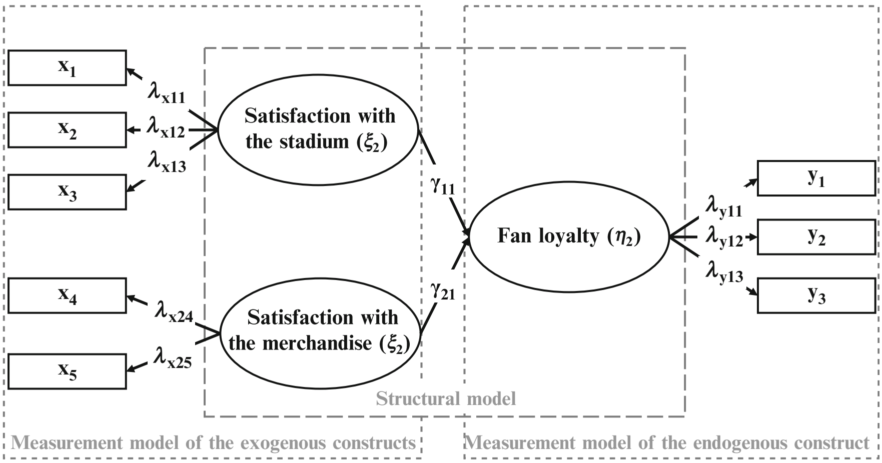

Fig. 8.7 Path model

In this model, we assume that the two exogenous constructs *satisfaction with the stadium* and *satisfaction with the merchandise* relate to the endogenous construct *fan loyalty*, which is operationalized by means of three items y_1, y_2, and y_3. Depending on the research question, we could, of course, incorporate additional exogenous and endogenous constructs. Using data from an empirical survey, we could test this model and, thus, evaluate the exogenous constructs' influence on the endogenous construct. By doing so, we could assess which of the two constructs ξ_1 or ξ_2 exerts the greater influence on η_1. This could guide us in developing marketing plans in order to increase fan loyalty by answering the research question whether we should rather concentrate on increasing the fans' satisfaction with the stadium or with the merchandise.

The results evaluation of a path model analysis requires several steps that include the assessment of both measurement models and the structural model. Janssens et al. (2008) describe covariance-based structural equation modeling. Likewise, Diamantopoulos and Siguaw (2000) and Hair et al. (2010) provide a thorough description of this approach and its application. Chin (1998), Haenlein and Kaplan (2004), as well as Henseler et al. (2009) and Hair et al. (2011) illustrate the PLS path modeling methodology and criteria for assessing results, while Hulland (1999) provides a review of sample PLS path modeling applications.

Lastly, it should be noted that confirmatory factor analysis and structural equation modeling primarily involve the testing of a hypothesized model based on theoretical reasoning, which is more the domain of academic research than business practice.[8] Consequently, these procedures currently play a limited role in market research practice, although this is expected to grow as the investigation of complex model set-ups becomes increasingly important to gain consumer insights and competitive advantages.

Example: Principal Components Analysis and Reliability Analysis

In this example, we take a closer look at some of the items from our soccer fan satisfaction study (*soccer_fan_satisfaction.sav*, Web Appendix → Chap. 8). For each of the following items, the respondents had to rate their degree of satisfaction from 1 (“very unsatisfied”) to 7 (“very satisfied”). The performance features comprise the respondents’ satisfaction with the following features (variable names in parentheses):

- Condition of the stadium (x_1),
- Outer appearance of the stadium (x_2),
- Interior design of the stadium (x_3),
- Quality of the team composition (x_4),
- Number of stars in the team (x_5),
- Price of annual season ticket (x_6), and
- Entry fees (x_7).

The complete item set was presented to various clubs’ soccer fans in July 2007. The selection of the respondents comprised fans of several German soccer clubs. Invitations to participate in the study were posted on various Internet fan forums not affiliated with any particular club. Of the 953 people who participated in the study, 495 completed the survey. Since the link to the survey appeared in several club-affiliated Internet forums, the database had to be reduced further to avoid having too many fans from the same clubs. In total, the market research agency selected 251 observations for further analysis. Let us use that dataset to run a principal components analysis.

To run the principal components analysis, simply click on ► Analyze ► Dimension Reduction ► Factor, which will open a dialog box similar to Fig. 8.8. Next, enter all seven variables into the **Variables** box.

---

[8] In some cases, path analysis can also be used in an exploratory way, especially when using PLS path modeling. Asparouhov and Muthén (2009) provide a rather technical discussion of this subject in the context of covariance-based structural equation modeling. However, please note that using structural equation modeling in an exploratory way is still an exception.

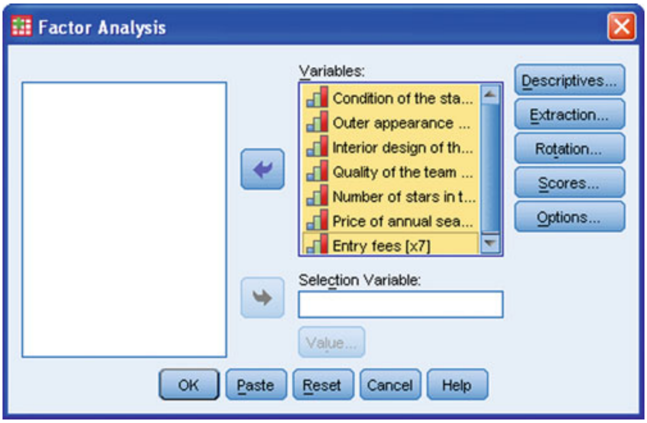

Fig. 8.8 Factor analysis dialog box

SPSS provides several options that relate to Table 8.3. Under **Descriptives**, you can choose to display univariate descriptive statistics, which is a useful way of gaining an overview of the extent of the missing values in your dataset. Furthermore, you can choose from several outputs and statistical measures that relate to the correlation matrix and help assess the data's appropriateness for the analysis. Be sure to check the **Coefficients**, **Significance levels**, **Reproduced** (which refers to the reproduced correlation matrix), **Anti-image**, as well as the **KMO and Bartlett's test of sphericity** boxes to check the assumptions previously discussed. All other options are of minor importance, so skip these and click **Continue**.

The **Extraction** option allows you to specify the analysis method. By default, SPSS sets this as the principal components; however, you can also set it to different types in the **Method** drop-down menu. Under **Extract**, you can determine the rule for factor extraction: By default, all factors with an Eigenvalue greater than one will be extracted. This default option is acceptable – except when you have previous information on the factor structure. If so, you should specify the number of factors manually.

Under **Display**, you should check the scree plot option, which provides additional help in determining the number of factors to extract. Now click **Continue** to access the main menu again.

Under **Rotation**, you can choose between several orthogonal and oblique rotation methods. Always use the **Varimax** procedure – unless your initial solution is difficult to interpret and you have strong theoretical grounds for assuming (moderately) correlated factors. Click **Continue**.

If you wish to use factor scores in subsequent analyses, you can save these in the **Scores** option. Since we strongly recommend using the Regression method, choose **Save as variables** and **Regression**, followed by **Continue**.

Lastly, under **Options**, you can decide how missing values should be handled and specify the display format of the coefficients in the component matrix. In general, we recommend excluding cases that have missing values in any of the variables used in any of the analyses (Option: **Exclude cases listwise**). Avoid replacing missing values with the mean as this will diminish the variation in the data, especially if there are many missing values in your dataset. You should always check the menu **Sorted by size** (under **Coefficient Display Format**), as this greatly increases the clarity of the display of results. If you wish, you can suppress loadings less than 0.10; however, in this example, we ignore this option.

After having specified all the options, you can proceed by clicking the **OK** button.

The descriptive statistics in Table 8.4 reveal that there are several observations with missing values in the dataset. According to our rule of thumb, we would need 7 $\times$ 10 $=$ 70 observations. As there are 195 observations without any missing values in the dataset (last column on the right in Table 8.4), we can proceed with checking the variables' correlations. The correlation matrix in Table 8.5 indicates that there are several pairs of variables that are highly correlated. For example, condition of the stadium (x_1) is highly correlated with the outer appearance (x_2, correlation $=$ 0.783), as well as the interior design (x_3, correlation $=$ 0.754) of the stadium. Likewise, x_2 and x_3 are also highly correlated (correlation $=$ 0.762). As these variables' correlations with the remaining ones are less pronounced, we may suspect that these three variables constitute one factor. As you can see, by just looking at the correlation matrix, we can already hypothesize about a potential factor structure.

Table 8.4 Descriptive statistics

Descriptive Statistics

	Mean	Std. Deviation	Analysis N
x1	5.39	1.962	195
x2	5.50	1.795	195
x3	5.02	1.905	195
x4	4.65	1.718	195
x5	4.56	1.864	195
x6	4.43	1.614	195
x7	4.22	1.509	195

However, at this point of the analysis, we are more interested in checking whether the variables are sufficiently correlated to conduct a principal components analysis. Most correlation coefficients lie above the threshold value of 0.30. When we look at the lower part of Table 8.5, we see that the p-values are extremely low. These results indicate that the variables are sufficiently correlated. However, for a concluding evaluation, we need to take the anti-image and related statistical measures into account. These are presented in Tables 8.6 and 8.7.

The analysis results in Table 8.6 reveal that the KMO value is 0.721, which is middling (see Table 8.2) but still satisfactory. Likewise, the variable-specific MSA values (Table 8.7) on the diagonal of the anti-image correlation matrix are all above the threshold value of 0.50. Furthermore, the Bartlett's test is significant ($p < 0.05$), which means that we can reject the null hypothesis that, in the population, all

Table 8.5 Correlation matrix

Correlation Matrix

		x1	x2	x3	x4	x5	x6	x7
Correlation	x1	1.000	.783	.754	.395	.338	.246	.172
	x2	.783	1.000	.762	.445	.350	.238	.224
	x3	.754	.762	1.000	.445	.329	.287	.235
	x4	.395	.445	.445	1.000	.829	.311	.241
	x5	.338	.350	.329	.829	1.000	.304	.214
	x6	.246	.238	.287	.311	.304	1.000	.696
	x7	.172	.224	.235	.241	.214	.696	1.000
Sig. (1-tailed)	x1		.000	.000	.000	.000	.000	.008
	x2	.000		.000	.000	.000	.000	.001
	x3	.000	.000		.000	.000	.000	.000
	x4	.000	.000	.000		.000	.000	.000
	x5	.000	.000	.000	.000		.000	.001
	x6	.000	.000	.000	.000	.000		.000
	x7	.008	.001	.000	.000	.001	.000	

Table 8.6 KMO and Bartlett's test measures

KMO and Bartlett's Test

Kaiser-Meyer-Olkin Measure of Sampling Adequacy.		.721
Bartlett's Test of Sphericity	Approx. Chi-Square	809.353
	df	21
	Sig.	.000

variables are uncorrelated. Consequently, we know that the data are appropriate for principal components analysis.

We can now take a look at the factor extraction process table, shown in Table 8.8.

Table 8.8 lists the Eigenvalues associated with each factor before extraction, after extraction, and after rotation. In the columns labeled **Initial Eigenvalues**, we see the results before extraction. SPSS lists all seven factors (we know that there are potentially as many factors as there are variables) in this column. Most of these factors are of course only of minor importance. This is reflected in each factor's Eigenvalue, which is displayed in the table's second column. Here, we see that the first factor has an Eigenvalue of 3.520. As there are seven variables in our dataset, this factor accounts for 3.520/7.00 = 50.290% of the overall variance, which is indicated in the third column. It is quite remarkable that by using only one factor instead of seven variables, we can account for over 50% of the overall variance! The second factor has an Eigenvalue of 1.415 and, thus, still covers more variance than a single variable (remember: since we are looking at standardized variables,

Table 8.7 Anti-image matrices

Anti-image Matrices

		x1	x2	x3	x4	x5	x6	x7
Anti-image Covariance	x1	.321	-.153	-.127	.023	-.032	-.032	.039
	x2	-.153	.308	-.120	-.033	.007	.033	-.042
	x3	-.127	-.120	.337	-.054	.038	-.031	-.008
	x4	.023	-.033	-.054	.275	-.229	-.007	-.011
	x5	-.032	.007	.038	-.229	.304	-.036	.012
	x6	-.032	.033	-.031	-.007	-.036	.479	-.329
	x7	.039	-.042	-.008	-.011	.012	-.329	.507
Anti-image Correlation	x1	.787[a]	-.488	-.385	.077	-.101	-.082	.096
	x2	-.488	.798[a]	-.373	-.113	.024	.086	-.105
	x3	-.385	-.373	.824[a]	-.177	.119	-.076	-.018
	x4	.077	-.113	-.177	.672[a]	-.793	-.018	-.028
	x5	-.101	.024	.119	-.793	.638[a]	-.094	.031
	x6	-.082	.086	-.076	-.018	-.094	.647[a]	-.668
	x7	.096	-.105	-.018	-.028	.031	-.668	.607[a]

a. Measures of Sampling Adequacy(MSA)

Table 8.8 Results of factor extraction

Total Variance Explained

Component	Initial Eigenvalues			Extraction Sums of Squared Loadings			Rotation Sums of Squared Loadings		
	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %	Total	% of Variance	Cumulative %
1	3.520	50.290	50.290	3.520	50.290	50.290	2.527	36.096	36.096
2	1.415	20.217	70.507	1.415	20.217	70.507	1.831	26.162	62.258
3	1.135	16.220	86.727	1.135	16.220	86.727	1.713	24.469	86.727
4	.312	4.450	91.177						
5	.257	3.666	94.844						
6	.208	2.969	97.812						
7	.153	2.188	100.000						

Extraction Method: Principal Component Analysis.

each variable has a variance of 1). The same holds for the third factor whose Eigenvalue lies at 1.135. Factors 4–7, however, only marginally account for the total variance explained, as their Eigenvalues are considerably smaller than 1.

The second set of columns, labeled **Extraction Sums of Squared Loadings**, contains the factor solutions after extraction. Since we chose the default option, SPSS extracts all factors with an Eigenvalue greater than 1 (the Kaiser criterion discussed previously). This leads to a solution in which three factors are extracted, which account for 86.727% of the overall variance. The final part of the table, labeled **Rotation Sums of Squared Loadings**, displays the factors after rotation. Rotation is carried out to optimize the factor structure in order to facilitate the interpretation of the factor solution. Rotation usually alters the factors' Eigenvalues, but will not change the total variance explained. For example, the third factor accounted for 16.220% of the overall variance before rotation; however, after rotation, it accounts for 24.469%.

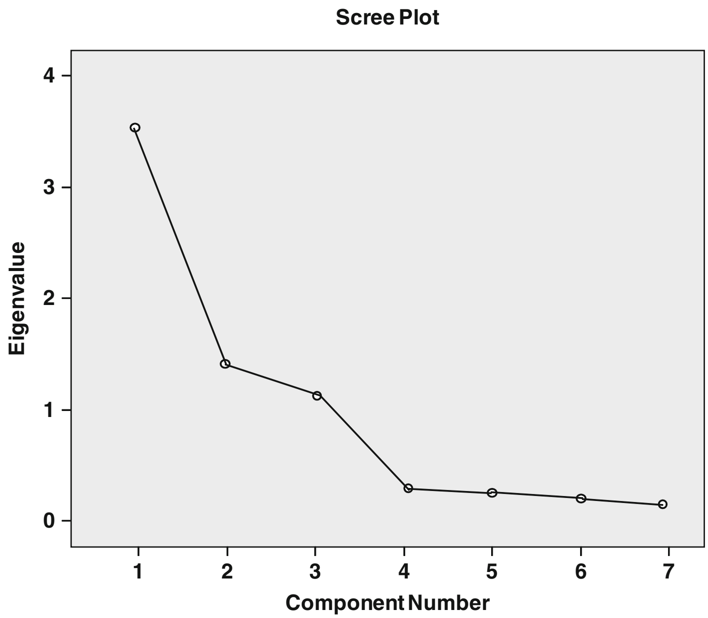

Fig. 8.9 Scree plot

Whereas the Kaiser criterion offers one possibility to determine the number of factors to extract, we can also use the scree plot (Fig. 8.9) to make that decision. In the scree plot, the slope of the curve diminishes and becomes almost horizontal for four factors. Since we always extract one factor less than indicated by the elbow, a three-factor solution is deemed appropriate.[9] As this is in accordance with the Kaiser criterion, we can continue evaluating the results by interpreting the factors.

To do so, take a look at the initial component matrix (Table 8.9), that is, the loadings matrix before factor rotation. In order to interpret the factors, we first "assign" each variable to a certain factor based on its maximum absolute factor loading. After that, we have to find an umbrella term for each factor that best describes the set of variables associated with that factor. Looking at Table 8.9, we see that x_1 through x_5 show the highest loadings for the first factor, whereas the other variables load highly on the second factor. However, what about the third factor? Should it be excluded? It probably should not, as this is a typical example of how an unrotated solution can be misleading. If you take a look at the rotated solution (Table 8.10), a different picture emerges. In this case, only x_1, x_2, and x_3 load highly on the first factor, whereas x_4 and x_5 load on the second, and x_6 and x_7 on the third factor. Comparing the loadings in the unrotated and rotated solution, reveals that the differences in the loadings are quite remarkable. That is why one

[9]In the Web Appendix (→Chapter 8), we illustrate the use of the parallel analysis and the minimum average partial test for determining the number of factors using this dataset.

Table 8.9 Unrotated factor loadings matrix

Component Matrix[a]

	Component		
	1	2	3
x2	.814	−.369	.230
x3	.813	−.326	.256
x1	.792	−.398	.253
x4	.750	.078	−.586
x5	.680	.135	−.665
x7	.485	.718	.326
x6	.555	.688	.248

Extraction Method: Principal Component Analysis.
a. 3 components extracted.

Table 8.10 Rotated factor loadings matrix

Rotated Component Matrix[a]

	Component		
	1	2	3
x1	.903	.167	.083
x2	.895	.201	.107
x3	.880	.185	.152
x5	.166	.937	.132
x4	.280	.902	.142
x7	.102	.077	.917
x6	.139	.177	.890

Extraction Method: Principal Component Analysis.
Rotation Method: Varimax with Kaiser Normalization.
a. Rotation converged in 4 iterations.

should never use the unrotated solution when interpreting factors! The unrotated solution is only used to determine the number of factors to extract from the dataset. Since the rotation changes the factors' Eigenvalues, the unrotated solution might indicate another number of factors to retain from the data than those indicated by the rotated solution. You see this by comparing the first two and the third set of columns in Table 8.8 (**Initial Eigenvalues** and **Extraction Sums of Squared Loadings** rather than **Rotation Sums of Squared Loadings**).

Having identified which variables load highly on which factor in the rotated solution, we should now try to identify labeling terms for each factor. Variables condition of the stadium (x_1), outer appearance of the stadium (x_2), and interior design of the stadium (x_3) clearly relate to the stadium as such. This seems to be the factor that we mentioned in the introduction of this chapter. Therefore, we can label this *satisfaction with the stadium*. Quality of the team composition (x_4) and number of stars in the team (x_5) describe traits of the soccer team that the respondents evaluated. Even though there are certainly more facets to it, we could label this factor *satisfaction with the team*. The remaining variables, i.e. price of annual season ticket (x_6), and entry fees (x_7) relate to ticket prices. Consequently, we can call the third factor *satisfaction with ticket prices*. Of course, the labeling of factors is subjective and you could provide different descriptions.

The last step involves assessing the analysis's goodness-of-fit. To do so, we first look at the residuals (i.e., the differences between observed and reproduced correlations) in the reproduced correlation matrix (Table 8.11). If we examine the lower part of the table, we see that there are several residuals with absolute values larger than 0.50. Nevertheless, we do not have to count every single value in the matrix (this could be quite exhausting if there are over 100 variables in the dataset). Instead, SPSS counts the proportion of residuals with high residuals, which is reported in the first part of the table. As we can see in point **b**. beneath the table, 23.0% of the residuals have absolute values greater than 0.50. Therefore, we can presume a good model fit. This is also illustrated by the variables' communalities, which are displayed in Table 8.12. Over 80% of each variable's variance is explained by the factors, which is a highly satisfactory result.

Table 8.11 Reproduced correlations and residual matrices

Reproduced Correlations

		x1	x2	x3	x4	x5	x6	x7
Reproduced Correlation	x1	.850[a]	.850	.838	.415	.317	.229	.181
	x2	.850	.852[a]	.841	.447	.351	.255	.205
	x3	.838	.841	.832[a]	.434	.339	.290	.243
	x4	.415	.447	.434	.912[a]	.910	.325	.229
	x5	.317	.351	.339	.910	.923[a]	.306	.210
	x6	.229	.255	.290	.325	.306	.843[a]	.845
	x7	.181	.205	.243	.229	.210	.845	.858[a]
Residual[b]	x1		–.067	-.085	–.020	.021	.017	–.009
	x2	–.067		-.080	–.002	–.001	–.017	.019
	x3	–.085	–.080		.010	–.010	–.003	–.008
	x4	–.020	–.002	.010		–.081	–.014	.012
	x5	.021	–.001	–.010	–.081		–.003	.004
	x6	.017	–.017	–.003	–.014	–.003		–.149
	x7	–.009	.019	–.008	.012	.004	–.149	

Extraction Method: Principal Component Analysis.
a. Reproduced communalities
b. Residuals are computed between observed and reproduced correlations. There are 5 (23,0%) nonredundant residuals with absolute values greater than 0.05.

Table 8.12 Communalities

Communalities

	Initial	Extraction
x1	1.000	.850
x2	1.000	.852
x3	1.000	.832
x4	1.000	.912
x5	1.000	.923
x6	1.000	.843
x7	1.000	.858

Extraction Method: Principal Component Analysis.

	x6	x7	FAC1_1	FAC2_1	FAC3_1
1	5	5	-.34006	.55138	.45799
2	8	1	.	.	.
3	8	6	.	.	.
4	8	6	.	.	.
5	4	5	.84722	.61924	-.04384
6	4	6	-1.18989	-1.46760	.89205
7	4	4	.83536	.08754	-.35509
8	8	7	.	.	.
9	8	4	.	.	.
10	4	3	-.00081	.34037	-.67535
11	8	4	.	.	.
12	6	4	.73211	.04437	.33035
13	6	6	.64652	1.14657	.95279
14	6	6	.76819	.50151	1.02641

Fig. 8.10 SPSS data view window

At this point, we have completed the principal components analysis. However, if we wish to continue using the analysis results, we should calculate factor scores. We can save factor scores using the **Scores** option; SPSS creates three new variables, one for each factor in the final solution (Fig. 8.10). Using these variables, we could, for example, evaluate whether male and female fans differ significantly with regard to their satisfaction with the stadium (first factor), the team (second factor), or the ticket prices (third factor). SPSS can only calculate these scores if it has information on all the variables included in the factor analysis. If SPSS does not have all the information, it only shows a "**.**" (dot) in the data view window, indicating system-missing values for a certain observation (as it is the case with observations 2, 3, and 4; Fig. 8.10).

Instead of using the factor scores in subsequent analyses, we could use an average score, calculated as the mean of the variables related to the respective factor. This is usually done in cases where the researcher already has an idea which variables relate to which factors, or wants to test a hypothesized factor structure (thus carrying out a confirmatory analysis).

A typical application would be to conduct a follow-up study of fan satisfaction. To ensure the comparability of the results, we could replicate the factor structure from the initial analysis, using summated scores obtained from the follow-up study. However, before doing so, we have to carry out a reliability analysis to assess whether the scale's results are consistent. In other words, we have to ensure that the perception of the constructs did not change significantly over time. To illustrate its usage, let's carry out a reliability analysis of the factor *satisfaction with the stadium* by calculating Cronbach's Alpha as a function of the variables satisfaction with the condition of the stadium, satisfaction with the outer appearance of the stadium, and satisfaction with the interior design of the stadium. To run the reliability analysis, simply click on ▸ Analyze ▸ Scale ▸ Reliability Analysis. Next, enter variables x_1, x_2, and x_3 into the **Items** box and type in the scale's name, namely *satisfaction with the stadium*. Check that **Alpha** is selected in the **Model** drop-down list (Fig. 8.11).

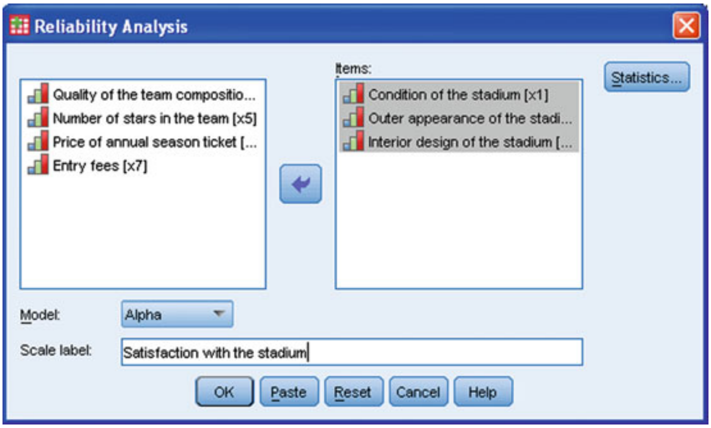

Fig. 8.11 Reliability analysis dialog box

Next, click on **Statistics** and choose **Scale if item deleted** (under **Descriptives for**). If you want, you could also request descriptive statistics for each item and the entire scale or item correlations (submenu **Inter-item**). However, for the sake of simplicity, we will work with the default settings.

The **Reliability Statistics** (Table 8.13) show that the scale exhibits a high degree of reliability. With a value of 0.902, the Cronbach's Alpha coefficient lies well above the commonly suggested threshold of 0.70. This is not surprising, since we are simply testing a scale that has previously been established by means of item correlations. Keep in mind that we usually carry out a reliability analysis to test a scale using a different sample – this example is only for illustration purposes!

Examining the far right column of Table 8.14, we can see how Cronbach's Alpha would develop if a certain item were to be deleted from the scale. When we

Table 8.13 Reliability statistics

Reliability Statistics

Cronbach's Alpha	N of Items
.902	3

Table 8.14 Item-total statistics

Item-Total Statistics

	Scale Mean if Item Deleted	Scale Variance if Item Deleted	Corrected Item-Total Correction	Cronbach's Alpha if them Deleted
x1	10.60	11.082	.808	.860
x2	10.53	11.951	.817	.853
x3	11.00	11.421	.796	.869

compare each of the values to the overall Alpha value, we can see that any change in the scale's set-up would reduce the Alpha value. For example, by removing x_1 from the scale, the Cronbach's Alpha of the new scale comprising only x_2 and x_3 would be reduced to 0.860.

Deleting this item therefore makes little sense. Only if the initial Cronbach's Alpha is below acceptable standards (i.e., below 0.70), we should try to increase it by removing one or more items from the scale. If it is acceptable, we should not attempt to improve it by changing the scale's set-up.

Case Study

Haver & Boecker is one of the world's leading providers of filling and screening systems. The company operates a number of facilities in Germany, as well as production plants in the UK, Belgium, USA, Canada, and Brazil. It is a recognized specialist in the fields of weighing, filling, and material handling technology. Haver & Boecker designs, produces, and markets systems and plants for filling and processing loose bulk materials of every type and, thus, solely operates in industrial markets.

The company's relationships with its customers are usually long-term oriented, and complex. Since the company's philosophy is to assist customers and business partners in solving technical problems and innovating new solutions, their products are often customized to the buyers' needs. Therefore, the customer is no longer a passive buyer, but an active partner. Given this background, the customer's satisfaction plays an important role in establishing, developing, and maintaining successful customer relationships.

http://www.haverboecker.com

Very early on, the company's management realized the importance of customer satisfaction and decided to commission a market research project to identify marketing activities that can positively contribute to the business's overall success. Based on a thorough literature review as well as interviews with experts, the company developed a short survey to explore their customers' satisfaction with specific performance features and their overall satisfaction. All items were measured on 7-point scales with higher scores denoting higher levels of satisfaction. A standardized survey was mailed to customers in 12 countries worldwide, which yielded 281 fully completed questionnaires. The following items (names in parentheses) were listed in the survey:

- Reliability of the machines and systems (s_1)
- Life-time of the machines and systems (s_2)
- Functionality and user-friendliness operation of the machines and systems (s_3)
- Appearance of the machines and systems (s_4)
- Accuracy of the machines and systems (s_5)
- Timely availability of the after-sales service (s_6)
- Local availability of the after-sales service (s_7)
- Fast processing of complaints (s_8)
- Composition of quotations (s_9)
- Transparency of quotations (s_{10})
- Fixed product prize for the machines and systems (s_{11})
- Cost/performance ratio of the machines and systems (s_{12})
- Overall, how satisfied are you with the supplier (*overall*)?

Your task is to analyze the dataset to provide the management of Haver & Boecker with advice for effective customer satisfaction management. The dataset is labeled *haver_and_boecker.sav* (Web Appendix → Chap. 8).

1. Using regression analysis, locate those variables that best explain the customers' overall satisfaction. Evaluate the model fit and assess the impact of each variable on the criterion variable. Remember to consider collinearity diagnostics.

2. Determine the factors that characterize the respondents by means a factor analysis. Use items s_1–s_{12} for this. Run a principal axis factoring with varimax rotation to facilitate interpretation. Consider the following issues:
   (a) Are all assumptions for carrying out a factor analysis met? Pay special attention to the question whether the data are sufficiently correlated.
   (b) How many factors would you extract?
   (c) Try to find suitable labels for the extracted factors.
   (d) Evaluate the solution's goodness-of-fit.
3. Use the factor scores and regress the customers' overall satisfaction (*overall*) on these. Evaluate the strength of the model and compare it with the initial regression. What should Haver & Boecker's management do to increase their customers' satisfaction?
4. Calculate Cronbach's Alpha over items s_1–s_5 and interpret the results.

For further information on the dataset and the study, compare Festge and Schwaiger (2007) as well as Sarstedt et al. (2009).

Questions

1. What is factor analysis? Try to explain what factor analysis is in your own words.
2. Describe the terms Eigenvalue, communality, and factor loading. How do these concepts relate to one another?
3. What is the difference between principal components analysis and principal axis factoring?
4. Describe three approaches used to determine the number of factors.
5. What are the purpose and the characteristic of a varimax rotation? Does a rotation alter Eigenvalues, factor loadings or communalities?
6. Re-run the analysis on soccer fan satisfaction by carrying out principal axis factoring and compare the results with our example analysis.
7. Explain the similarities and differences between exploratory factor analysis and confirmatory factor analysis.
8. Explain the basic principle of structural equation modeling.

Further Readings

Fornell C, Bookstein FL (1982) Two structural equation models: LISREL and PLS applied to consumer exit-voice theory. J Mark Res 19(4):440–452

In this seminal article, the authors compare the statistical principles of covariance- and variance-based structural equation modeling. The illustrations are rather technical and more suited for readers with a strong background in statistics and research methodology.

Nunnally JC, Bernstein IH (1993) Psychometric theory, 3rd edn. McGraw-Hill, New York

Psychometric Theory is a classic text and the most comprehensive introduction to the fundamental principles of measurement. Chapter 7 provides an in-depth discussion of the nature of reliability and its assessment.

Stewart DW (1981) The application and misapplication of factor analysis in marketing research. J Mark Res 18(1):51–62

David Stewart discusses procedures for determining when data are appropriate for factor analysis, as well as guidelines for determining the number of factors to extract, and for rotation.

References

Asparouhov T, Muthén B (2009) Exploratory structural equation modeling. Struct Equ Modeling 16(3):397–438

Bronner F, Neijens P (2006) Audience experiences of media context and embedded advertising. Int J Mark Res 48(1):81–100

Chin WW (1998) The partial least squares approach for structural equation modeling. In: Marcoulides GA (ed) Modern methods for business research. Lawrence Erlbaum Associates, London, pp 295–336

Cronbach LJ (1951) Coefficient alpha and the internal structure of tests. Psychometrika 16(3): 297–334

Diamantopoulos A, Siguaw JA (2000) Introducing LISREL: a guide for the uninitiated. Sage, London

Festge F, Schwaiger M (2007) The drivers of customer satisfaction with industrial goods: an international study. Adv Int Mark 18:179–207

Haenlein M, Kaplan AM (2004) A beginner's guide to partial least squares. Underst Stat 3(4): 283–297

Hair JF, Black WC, Babin BJ, Anderson RE (2010) Multivariate data analysis, 7th edn. Pearson Prentice Hall, Upper Saddle River, NJ

Hair, Joe Jr, Ringle CM, Sarstedt M (2011) PLS-SEM. Indeed a Silver Bullet. Journal of Marketing Theory & Practice, forthcoming

Henseler J, Ringle CM, Sinkovic RR (2009) The use of partial least squares path modeling in international marketing. Adv Int Mark 20:277–320

Hulland J (1999) Use of partial least squares (PLS) in strategic management research: a review of four recent studies. Strateg Manage J 20(2):195–204

Janssens W, Wijnen K, de Pelsmacker P, van Kenhove P (2008) Marketing Research with SPSS, Hanlow, UK: Prentice Hall

Kaiser HF (1974) An index of factorial simplicity. Psychometrika 39(1):31–36

Sarstedt M, Schwaiger M, Ringle CM (2009) Do we fully understand the critical success factors of customer satisfaction with industrial goods?–Festge and Schwaiger's model to account for unobserved heterogeneity. J Bus Mark Manage 3(3):185–206