Chapter 8
Two-Way Analysis of Variance: Examining Influence of Two Factors on Criterion Variable

Learning Objectives

After completing this chapter, you should be able to do the following:

- Explain the importance of two-way analysis of variance (ANOVA) in research.
- Understand different designs where two-way ANOVA can be used.
- Describe the assumptions used in two-way analysis of variance.
- Learn to construct various hypotheses to be tested in two-way analysis of variance.
- Interpret various terms involved in two-way analysis of variance.
- Learn to apply two-way ANOVA manually in your data.
- Understand the procedure of analyzing the interaction between two factors.
- Know the procedure of using SPSS for two-way ANOVA.
- Learn the model way of writing the results in two-way analysis of variance by using the output obtained in the SPSS.
- Interpret the output obtained in two-way analysis of variance.

Introduction

A two-way analysis of variance is a design with two factors where we intend to compare the effect of multiple levels of two factors simultaneously on criterion variable. The two-way ANOVA is applied in two situations: first, where there is one observation per cell and, second, where there is more than one observation per cell. In a situation where there is more than one observation per cell, it is mandatory that the number of observations in each cell must be equal. Using two-way ANOVA with n observations per cell facilitates us to test if there is any interaction between the two factors.

Two-way analysis of variance is in fact an extension of one-way ANOVA. In one-way ANOVA, the effect of one factor is studied on the criterion variable, whereas in two-way ANOVA, the effect of two factors on the criterion variable is

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studied simultaneously. An additional advantage in two-way ANOVA is to study the interaction effect between the two factors. One of the important advantages of two-way analysis of variance design is that there are two sources of assignable causes of variation, and this helps to reduce the error variance and thus making this design more efficient.

Consider an experiment where a personal manger is interested to know whether the job satisfaction of the employees in different age categories is same or not irrespective of an employee being male or female. In testing this hypothesis, 15 employees may be randomly selected in each of three age categories: 20–30, 31–40, and 41–50 years, and one-way ANOVA experiment may be planned. Since in making the groups male and female employees were selected at random, and, therefore, if any difference in the satisfaction level is observed in different age categories, it may not be truly attributed due to the age category only. The variation might be because of their gender difference as well.

Now, the same experiment may be planned in two-way ANOVA with one factor as age and the second as gender. Here, the factor age has 3 levels and gender has 2 levels. By planning this experiment in two-way ANOVA, the total variability may be broken into two assignable causes, that is, age and gender, and, therefore, more variability among the employees' satisfaction level can be explained resulting in reduction of error variance. Thus, an experimenter is in a better position to explain the overall variability in the satisfaction level of the employees. Moreover, interaction effect, if any, between gender and age on the satisfaction level of the employees can also be studied in this design.

There may be several instances where two-way ANOVA experiment can be planned. For example, in studying the effect of three outlet locations on the sale of a particular facial cream, one may select another factor as age because it is assumed that the age is also responsible in the sale of this product besides the variation in the outlet location. In framing this experiment as a two-way ANOVA, the variation in the sale of this product due to difference in the outlet location can be efficiently explained by separating the variation due to age difference. The design has been shown in the following table.

	District			
	1	2	3	4
Outlet 1	B	C	D	A
Outlet 2	D	A	B	C
Outlet 3	C	D	A	B
Outlet 4	A	B	C	D

Principles of ANOVA Experiment

All the three basic principles of design, that is, randomization, replication, and local controls, are used in planning a two-way ANOVA experiment in order to minimize the error variance. In one-way ANOVA experiments only two principles i.e. randomization and replication are used to control the error variance whereas in two-way ANOVA experiments all the three principles i.e. randomization, replication, and local control are used to control the error variance. The very purpose of using these three principles of design is to enable the researcher to conclude with more authority that the variation in the criterion variable is due to the identified level of a particular factor.

In two-way ANOVA experiment, the principle of randomization means that the samples in each group are selected in a random fashion so as to make the groups as homogeneous as possible. The randomization avoids biases and brings control in the experiment and helps in reducing the error variance up to a certain extent.

The principle of replication refers to studying the effect of two factors on more than one subject in each cell. The logic is that one should get the same findings on more than one subject. In two-way ANOVA experiment, the principle of replication allows a researcher to study the significance of interaction between the two factors. Interaction effect cannot be studied if there is only one observation in each cell.

The principle of local control refers to making the groups as homogeneous as possible so that variation due to one or more assignable causes may be segregated from the experimental error. Thus, the application of local control helps us in reducing the error variation and making the design more efficient.

In the example discussed above, in studying the effect of age on job satisfaction if the employees were divided only according to their age, then we would have ignored the effect of gender on job satisfaction which would have increased the experimental error. However, if the researcher feels that instead of gender if the job satisfaction varies as per their salary structure, then the subjects may be selected as per their salary bracket in different age categories. This might further reduce the experimental error. Thus, maximum homogeneity can be ensured among the observations in each cell by including the factor in the design which is known to vary with the criterion variable.

Classification of ANOVA

By using the above-mentioned principles the two-way ANOVA can be used for different designs. Some of the most popular designs where two-way ANOVA can be used are discussed below.

Factorial Analysis of Variance

Two-way ANOVA is the most widely used in factorial designs. The factorial design is used for more than one independent variable. The independent variables are also referred to as factors. In factorial design, there are at least two or more factors. Usually, two-way analysis of variance is used in factorial designs having two factors.

In this design, the effect of two factors (having different levels) is seen on the dependent variable. Consider an experiment where age (A) and gender (B) are taken as two factors whose effect has to be seen on the dependent variability, sincerity. Further, let the factor A has three levels (20–30, 31–40, and 41–50 years) and the factor B has two levels (Male and Female). Thus, in this design, 2×3, that is, six combination of treatment groups need to be taken. This design facilitates in studying the effect of both the factors A and B on the dependent variable. Further, in this design, significance of the interaction effect between the two factors can also be tested. The factorial design is very popular in the behavioral research, social sciences, and humanities. This design has a few advantages over single-factor designs. The most important aspect of the factorial design is that it can provide some information about how factors *interact* or combine in the effect they have on the dependent variable. The factorial design shall be discussed in detail while in solving two-way ANOVA problem later in this chapter.

Repeated Measure Analysis of Variance

Another design where two-way ANOVA is used is the repeated measure design. This design is also known as a within-subject design. In this design, same subject is tested under repeated conditions over a time. The repeated measure design can be considered to be an extension of the paired-samples t-test because in this case, comparison is done between more than two repeated measures. The repeated measure design is used to eliminate the individual differences as a source of between-group differences. This helps to create a more powerful test. The only care to be taken in the repeated measure design is that while testing the same subject repeatedly, no carryover effect should be there.

Multivariate Analysis of Variance (MANOVA)

In this design, effect of two factors is studied on more than one dependent variable. It is similar to the factorial design having two factors, but the only difference is that here we have more than one dependent variable. At times, it makes sense to combine the dependent variables for drawing the conclusion about the effects of two factors on it. For instance, in an experiment, if the effect of teaching methods and hostel facilities have to be seen on the overall academic performance (consisting four subjects: Physics, Chemistry, Math, and English) of the students, then it makes sense to see the effect of these two factors, that is, teaching methods and hostel facilities on all the subjects together. Once the effect of any of these factors is found to be significant, then the two-way ANOVA for each of the dependent variable is applied.

In using two-way MANOVA, the dependent variables should neither be highly correlated among themselves nor should they be totally uncorrelated. The benefit of using MANOVA is that one can study the effect of each factor and their interaction on the whole basket of dependent variables. It makes sense to study the effect of two factors on the group of dependent variables like personality, employees, students, etc. Personality is the sum total of many variables like honesty, sincerity, and positivity; similarly, employees may be classified as male and female, whereas students may be categories as undergraduate and postgraduate. At times, it may be interesting to see the impact of two factors like age and education on the personality of an individual. Once any of this factor's effect on the personality as a whole is significant, then the two-way analysis of variance may be applied for each of these dimensions of the personality separately to see how these dimensions are affected individually by the age and education.

Advantages of Two-Way ANOVA over One-Way ANOVA

Two-way ANOVA design is more efficient over one-way ANOVA because of the following four reasons:

1. Unlike one-way ANOVA, the two-way ANOVA design facilitates us to test the effect of two factors at the same time.
2. Since in two-way ANOVA variation is explained by two assignable causes, it reduces the error variance. Due to this fact, two-way ANOVA design is more efficient than one-way ANOVA.
3. In two-way ANOVA, one can test for independence of the factors provided there is more than one observation per cell. However, number of observations in each cell must be equal. On the other hand, in one-way ANOVA, one may have the unequal number of scores in each group.
4. Besides reducing the error variance, two-way ANOVA also reduces the computation as it includes several one-way ANOVA.

Important Terminologies Used in Two-Way ANOVA

Factors

Independent variables are usually known as *factors*. In two-way ANOVA, the effect of two factors is studied on certain criterion variable. Each of the two factors may have two or more levels. The degrees of freedom for each factor is equal to the number of levels in the factor minus one.

Treatment Groups

The number of treatments in two-way ANOVA experiment is equal to the number of combinations of the levels of the two factors. For example, if the factor A has 2 levels, A_1 and A_2, and the factor B has 3 levels, B_1, B_2, and B_3, then there will be $2 \times 3 = 6$ different treatment groups A_1B_1, A_1B_2, A_1B_3, A_2B_1, A_2B_2, and A_2B_3.

Main Effect

The main effect is the effect of one independent variable (or factor) on the dependent variable across all the levels of the other variable. The interaction is ignored for this part. Just the rows or just the columns are used, not mixed. This is the part which is similar to one-way analysis of variance. Each of the variances calculated to analyze the main effects (rows and columns) is like between variances. The degrees of freedom for the main effect are one less than its number of levels. For example, if the factor A has r levels and factor B has c levels, then the degrees of freedom for the factor A and B would be $r - 1$ and $c - 1$, respectively.

Interaction Effect

The joint effect of two factors on the dependent variable is known as interaction effect. It can also be defined as the effect that one factor has on the other factor. The degrees of freedom for the interaction is the product of degrees of freedom of both the factors. If the factors A and B have levels r and c, respectively, then the degrees of freedom for the interaction would be $(r - 1) \times (c - 1)$.

Within-Group Variation

The within-group variation is the sum of squares within each treatment groups. In two-way ANOVA, all treatment groups must have the same sample size. The total number of treatment groups is the product of the number of levels for each factor. The within variance is equal to within variation divided by its degrees of freedom. The within group is also denoted as error. The within-group variation is often denoted by SSE.

Two-Way ANOVA Model and Hypotheses Testing

Let us suppose that there are two factors A and B whose effects have to be tested on the criterion variable X, and let the factors A and B have levels r and c, respectively, with n units per cell, then these scores can be written as follows:

		Factor B					
		1	..	j	..	c	
	1	X_{111}		X_{1j1}		X_{1c1}	
		X_{112}	..	X_{1j2}	..	X_{1c2}	
		.		.		.	
		X_{11n}		X_{1jn}		X_{1cn}	R_1
		T_{11}		T_{1j}		T_{1c}	
Factor A	i	X_{i11}		X_{ij1}		X_{ic1}	
		X_{i12}	..	X_{ij2}	..	X_{ic2}	
		.		.		.	
		X_{i1n}		X_{ijn}		X_{icn}	R_i
		T_{i1}		T_{ij}		T_{ic}	
	r	X_{r11}		X_{rj1}		X_{rc1}	
		X_{r12}	..	X_{rj2}	..	X_{rc2}	
		.		.		.	
		X_{r1n}		X_{rjn}		X_{rcn}	R_r
		T_{r1}		T_{rj}		T_{rc}	
		C_1		C_j		C_c	$G = \sum R_i = \sum C_j$

where

X_{ijk} represents the kth score in the (i,j)th cell
T_{ij} represents the total of all the n scores in the (i,j)th cell
G is the grand total of all the scores
R_i is the total of all the scores in ith level of the factor A
C_j is the total of all the scores in jth level of the factor B
N is the total of all the scores and is equal to $r \times c \times n$

In two-way ANOVA, the total variability among the above-mentioned N scores can be attributed to the variability due to row (or factor A), due to column (or factor B), due to interaction (row $\times$ column ($A \times B$)), and due to error. Thus, the total variability can be broken into the following four components:

$$\begin{aligned}\text{Total variability} &= \text{Variability due to Row(SSR)} + \text{Variability due to Column(SSC)} \\ &\quad + \text{Variability due to Interaction(SSI)} + \text{Variability due to Error} \\ \text{or} \qquad & \text{TSS} = \text{SSR} + \text{SSC} + \text{SSI} + \text{SSE} \qquad (8.1)\end{aligned}$$

Remark: SSE is the variability within group which was represented by SS_w in one-way ANOVA.

The above-mentioned model is a two-way ANOVA model where it is assumed that the variability among the scores may be due to row factor, column factor, interaction due to row and column, and the error factor. Since the variation in the group has been explained by the two factors instead of one factor in one-way ANOVA, reduction of error variance is more in two-way ANOVA in comparison to that of one-way ANOVA design. This makes this design more efficient than one-way ANOVA. After developing the above-mentioned model of variability, it is required to test whether the effects of these factors are significant or not in explaining the variation in the data. The significance of these components is tested by means of using F-test.

(a) *Hypotheses construction*: The hypotheses which are being tested in two-way ANOVA are as follows:

   (i) $H_0 : \mu_{A_1} = \mu_{A_2} = \ldots = \mu_{A_r}$
   (The population means of all the levels of the factor A are equal. This is like the one-way ANOVA for the row factor.)
   (ii) $H_0 : \mu_{B_1} = \mu_{B_2} = \ldots = \mu_{B_c}$
   (The population means of all the levels of the factor B are equal. This is like the one-way ANOVA for the column factor.)
   (iii) H_0: There is no interaction between factors A and B.
   (This is similar to performing a test for independence with contingency table.)

   The above-mentioned null hypotheses are tested against the alternative hypothesis that at least one mean is different.

   If the null hypotheses mentioned in (i) and (ii) are rejected, then it is concluded that the variability due to factor is significant, and it is inferred that the means of all the groups in that factor is not same. On the other hand, if the null hypothesis is failed to be rejected, one may draw the inference that all group means are equal. If the hypothesis mentioned in (iii) is rejected, then one may conclude that there is a significant interaction between the factors A and B. In other words, it may be concluded that the pattern of differences of group means in factor A is not the same in different levels of factor B. This fact shall be discussed in detail while solving the Example 8.1. Thus, if the effects of factors A and B having levels m and n, respectively, are to be seen on the criterion variable, then the following steps will explain the procedure of testing the hypotheses:
(b) *Level of significance*: The level of significance may be chosen beforehand. Usually, it is taken as .05 or .01.
(c) *Statistical test*: The F-test is used to compare the variability between levels of a factor with that of variability within groups. If F-value is significant, it indicates that variability between levels of the factor groups is significantly higher than the variability within groups; in that case, the null hypothesis of no difference between the group means is rejected. Before computing F-value, it is required to compute the total sum of squares (TSS), sum of squares due to row factor

A (SSR), sum of squares due to column factor B (SSC), sum of squares due to interaction $A \times B$ (SSI), and sum of squares due to error (SSE).

(i) Total sum of squares (TSS): It represents the total variation present in the data set and is usually represented by TSS. It is defined as the sum of the squared deviations of all the scores in the data set from their grand mean. The TSS is computed by the following formula:

$$\text{TSS} = \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n}\left(X_{ijk} - \frac{G}{N}\right)^2$$

after solving

$$= \sum_{i}\sum_{j}\sum_{k} X_{ijk}^2 - \frac{G^2}{N} \tag{8.2}$$

Since the degrees of freedom for the TSS are $N - 1$, therefore mean sum of squares is computed by dividing the TSS by $N - 1$.

(ii) Sum of squares due to row factor (SSR): It is the measure of variation between the row group means and is usually denoted by SSR. This is also known as the variation due to row factor (one of the assignable causes). The sum of squares due to row is computed as follows:

$$\text{SSR} = \sum_{i=1}^{r}\frac{R_i^2}{nc} - \frac{G^2}{N} \tag{8.3}$$

Since r levels of row factor A is compared in two-way ANOVA, the degrees of freedom for SSR are given by $r - 1$. Thus, mean sum of squares for row factor is obtained by dividing the SSR by its degrees of freedom $r - 1$.

(iii) Sum of squares due to column factor (SSC): It is the measure of variation between the column group means and is usually denoted by SSC. This explains the variation due to column factor (one of the assignable causes). The sum of squares due to column factor is computed as

$$\text{SSC} = \sum_{j=1}^{c}\frac{C_j^2}{nr} - \frac{G^2}{N} \tag{8.4}$$

The degrees of freedom for SSC are given by $c - 1$ as there are c column group means that are required to be compared. The mean sum of square for column factor is obtained by dividing SSC by $c - 1$.

(iv) Sum of squares due to interaction (SSI): It is the measure of variation due to the interaction of both the factors A and B. It facilitates us to test whether or not there is a significant interaction effect. The sum of squares due to interaction is usually denoted by SSI. The SSI is computed as follows:

$$\mathrm{SSI} = \sum_{i=1}^{r}\sum_{j=1}^{c}\frac{T_{ij}^2}{n} - \frac{G^2}{N} - \mathrm{SSR} - \mathrm{SSC} \tag{8.5}$$

The degrees of freedom for interaction are obtained by $(r-1)\times(c-1)$. The mean sum of squares due to interaction is obtained by dividing SSI by its degrees of freedom $(r-1)\times(c-1)$.

(v) *Sum of squares due to error (SSE)*: It is the measure of unexplained portion of the variation in the data and is denoted by SSE. This error is minimized in two-way ANOVA model because here the variability in the data is explained by the two factors besides interaction, in contrast to that of one factor in one-way ANOVA model. The degrees of freedom are given by $N - rc$. The SSE is computed as

$$\mathrm{SSE} = \mathrm{TSS} - \mathrm{SSR} - \mathrm{SSC} - \mathrm{SSI} \tag{8.6}$$

The mean sum of square due to error is obtained by dividing SSE by its degrees of freedom $N - rc$.

(vi) ANOVA table: This is a summary table showing different sum of squares and mean sum of squares for all the components of variation. The computation of F-values is shown in this table. This table is popularly known as two-way ANOVA table. After computing all the sum of squares, the ANOVA table is prepared for further analysis which is shown as follows:

(vii) *F-statistic*: Under the normality assumptions, the F-value obtained in the ANOVA table, say, for row, follows a F-distribution with $(r-1, N-r)$ degrees of freedom. F-statistic is computed for each source of variation.

Two-way ANOVA table

Sources of variation	SS	df	MSS	F
Main effect A (row)	SSR	$r-1$	$S_R^2 = \mathrm{SSR}/(r-1)$	F for row effect $= S_R^2/S_E^2$
Main effect B (column)	SSC	$c-1$	$S_C^2 = \mathrm{SSC}/(c-1)$	F for column effect $= S_C^2/S_E^2$
Interaction effect ($A \times B$)	SSI	$(r-1)\times(c-1)$	$S_I^2 = \mathrm{SSI}/(r-1)\times(c-1)$	F for interaction effect $= S_I^2/S_E^2$
Error	SSE	$N-rc$	$S_E^2 = \mathrm{SSE}/(N-rc)$	
Total	TSS	$N-1$		

Remark: *The t*otal sum of squares is additive in nature

This test statistic F is used to test the null hypothesis of no difference among the group means.

(d) *Decision criteria*: The tabulated value of F at .05 or .01 level of significance with different degrees of freedom may be obtained from Tables A.4 or A.5, respectively in the Appendix. If the calculated value of F is greater than the tabulated F, the null hypothesis is rejected, and in that case, at least one of the means is different than others. Since ANOVA does not tell us where the difference lies, post hoc test is used to get the clear picture. There are several post hoc tests which can be used but least significant difference (LSD) test is generally used in equal sample sizes. However, one may use other post hoc tests like Scheffe's, Tukey, Bonferroni, or Duncan as well.

In all the post hoc tests, a critical difference is computed at a particular level of significance, and if the difference of any pair of observed means is higher than the critical difference, it is inferred that the mean difference is significant otherwise insignificant. By comparing all pair of group means, conclusion is drawn as to which group mean is the highest. The detail procedure of applying the post hoc test has been discussed in the solved Example 8.1.

In LSD test, the critical difference (CD) is computed as

$$\mathrm{CD} = t_{.05}(N - rc) \times \sqrt{\frac{2\mathrm{MSSE}}{n}} \tag{8.7}$$

where the symbols have their usual meanings.

This critical difference (CD) is used for comparing differences in all the pair of means.

The SPSS output provides the significance value (p-value) for each of the F-statistics computed in two-way ANOVA table. If p-value is less than .05, F would be significant. Post hoc test for comparing means is applied for those factors and interaction whose F-value is significant. The SPSS also provides p-values (significant value) for each pair of means in row, column, and interaction to test the significance of difference between them. If p-value for any pair of means is less than .05, it is concluded that means are significantly different otherwise not.

Assumptions in Two-Way Analysis of Variance

Following assumptions need to be satisfied while using the two-way ANOVA design:

(a) The population from which the samples are drawn must be normally distributed.
(b) The samples must be independent.
(c) The population variances must be equal.
(d) The sample size must be same in each cell.

Situation Where Two-Way ANOVA Can Be Used

In two-way ANOVA, we investigate the effect of main effects along with the interaction effect of two factors on dependent variable. The following situation shall develop an insight among the researchers to appreciate the use of this analysis.

Consider a situation where a mobile company is interested to examine the effect of gender and age of the customers on the frequency of short messaging service (sms) sent per week. Each person may be classified according to gender (men and women) and age category (16–25, 26–35, and 36–45 years). Thus, there will be six groups, one for each combination of gender and age. Random sample of equal size in each group may be drawn, and each person may be asked about the number of sms he or she sends per week. In this situation, there are three main research questions that can be answered:

(a) Whether the number of sms sent depends on gender
(b) Whether the number of sms sent depends on age
(c) Whether the number of sms sent depends on gender differently for different age categories of age, and vice versa

All these questions can be answered through testing of hypothesis in two-way ANOVA model. The first two questions simply ask whether sending sms depends on age and gender. On the other hand, the third question asks whether sending sms depends on gender differently for people in different age category, or whether sending sms depends on age differently for men and women. This is because one may think that men send more sms than women in 18–25 years age category, but women send more sms than men in 26–55 years age category. After applying the two-way ANOVA model, one may be able to explain the above-mentioned research questions in the following manner:

whether the factor gender has a significant impact on the number of sms sent irrespective of their age categories. And if it is so, one may come to know whether men send more sms than women or vice versa, irrespective of their age categories.

Similarly, one can test whether the factor age has a significant impact on the number of sms sent irrespective of gender. And if age factor is significant, one can know that in which age category people send more sms irrespective of their gender.

The most important aspect of two-way ANOVA is to know the presence of interaction effect of gender and age on sending the sms. One may test whether these two factors, that is, gender and age, are independent to each other in deciding the number of sms sent. The interaction analysis allows us to compare the average sms sent in different age categories in each of the men and women groups separately. Similarly, it also provides the comparison of the average sms sent by the men and women in different age categories separately.

The information provided through this analysis may be used by the marketing department to chalk out their promotional strategy for men and women separately in different age categories for the mobile users.

Table 8.1 Data on toothpaste sales in different incentive groups

		Incentive		
		I	II	III
Gender of sales manager	Male	15	15	18
		13	14	16
		14	10	10
		11	9	12
		9	8	16
	Female	10	13	11
		7	14	10
		9	16	13
		7	17	12
		8	14	11

Example 8.1 An experiment was conducted by a utility company to study the effects of three sales incentives – toothpaste with 20% extra in the same price (incentive I), toothpaste along with traveling toothpaste (incentive II), and toothpaste along with bath soap (incentive III) – and to study the effects of gender of the sales manager (male and female) on the daily sales of a toothpaste. The daily sale of the toothpaste was recorded in each of the six groups for five continuous days. The data so obtained are shown in Table 8.1.

Apply two-way ANOVA and discuss your findings at 5% level.

Solution Here, the two factors that need to be investigated are gender and incentive and

Number of row factor (gender) $= r = 2$
Number of column factor (incentive) $= c = 3$
Number of scores in each cell $= n = 5$
Total number of scores $= N = n \times c \times r = 30$

In order to apply the two-way ANOVA, the following steps shall be performed:

(a) *Hypotheses construction*: The hypotheses that need to be tested are:

(i) $H_0 : \mu_{\text{Male}} = \mu_{\text{Female}}$
(The sales performance given by male and female sales manager is equal irrespective of the incentives.)

(ii) $H_0 : \mu_{\text{Incentive\_I}} = \mu_{\text{Incentive\_II}} = \mu_{\text{Incentive\_III}}$
(The sales performance under the three incentive schemes is equal irrespective of the gender of the sales manager.)

(iii) H_0: There is no interaction between the gender of the sales manager and the types of sales incentives.
(It is immaterial whether male or female are offering the sales incentives.)

Table 8.2 Computation for two-way ANOVA

		Incentive				
		I	II	III	Row total (R_i)	Row mean
	Male	15	15	18		
		13	14	16		
		14	10	10		
		11	9	12		
		9	8	16		
		$T_{11} = 62$	$T_{12} = 56$	$T_{13} = 72$	$R_1 = 190$	12.67
Gender of sales manager	Female	10	13	11		
		7	14	10		
		9	16	13		
		7	17	12		
		8	14	11		
		$T_{21} = 41$	$T_{22} = 74$	$T_{23} = 57$	$R_2 = 172$	11.47
	Column total	$C_1 = 103$	$C_2 = 130$	$C_3 = 129$	$G = 362$	
	Column mean	10.3	13.0	12.9		

(b) *Level of significance*: .05

(c) *Test statistic*: In order to test the hypotheses, F-statistic shall be computed for row factor, column factor, and interaction in order to test their significance. After computing different sum of squares, the ANOVA table shall be prepared. The following computation shall be done for completing the ANOVA table:

Computation

Before computing components of different sum of squares, let us first compute the row, column, and cell total along with the grand total (Table 8.2).

1. Raw sum of square (sum of squares of all the scores in the study)

$$\begin{aligned}\text{RSS} &= \sum_{i=1}^{r}\sum_{j=1}^{c}\sum_{k=1}^{n} X_{ijk}^2 \\ &= \left(15^2 + 13^2 + \ldots 9^2\right) + \left(15^2 + 14^2 + \ldots 8^2\right) + \left(18^2 + 16^2 + \ldots 16^2\right) \\ &\quad + \left(10^2 + 7^2 + \ldots 8^2\right) + \left(13^2 + 14^2 + \ldots 14^2\right) + \left(11^2 + 10^2 + \ldots 11^2\right) \\ &= 792 + 666 + 1080 + 343 + 1106 + 655 \\ &= 4642\end{aligned}$$

2. Correction factor = CF

$$= \frac{G^2}{N} = \frac{362^2}{30} = 4368.13$$

3. Total sum of squares = TSS

$$= \sum_i \sum_j \sum_k X_{ijk}^2 - \frac{G^2}{N} = \text{RSS} - \text{CF}$$
$$= 4642 - 4368.13 = 273.87$$

4. Sum of squares due to row factor(gender) = SSR

$$= \sum_{i=1}^{r} \frac{R_i^2}{nc} - \frac{G^2}{N} = \frac{190^2 + 172^2}{5 \times 3} - 4368.13$$
$$= 10.80$$

5. Sum of squares due to column factor(incentives) = SSC

$$= \sum_{j=1}^{c} \frac{C_j^2}{nr} - \frac{G^2}{N}$$
$$= \frac{103^2 + 130^2 + 129^2}{5 \times 2}$$
$$- 4368.13 = 46.87$$

6. Sum of squares due to interaction = SSI

$$= \sum_{i=1}^{r} \sum_{j=1}^{c} \frac{T_{ij}^2}{n} - \frac{G^2}{N} - \text{SSR} - \text{SSC}$$
$$= \frac{62^2 + 56^2 + 72^2 + 41^2 + 74^2 + 57^2}{5}$$
$$- 4368.13 - 10.80 - 46.87$$
$$= 4514 - 4425.8 = 88.20$$

7. Sum of squares due to error = SSE

$$= \text{TSS} - \text{SSR} - \text{SSC} - \text{SSI}$$
$$= 273.87 - 10.80 - 46.87 - 88.20$$
$$= 128$$

Tabulated value of F can be seen from Table A.4 in the Appendix. Thus, from Table A.4, the value of $F_{.05}\,(1,24) = 4.26$ and $F_{.05}\,(2,24) = 3.40$.

In Table 8.3, since the calculated value of F for incentives and interaction is greater than their corresponding tabulated value of F, these two F-ratios are significant. However, F-value for gender is not significant.

Table 8.3 Two-way ANOVA table for the data on sales of toothpaste

Source of variation	Sum of squares (SS)	df	Mean sum of squares (MSS)	F	Tab. F at 5% level
Gender	10.80	$r - 1 = 1$	10.80	2.03	4.26
Incentives	46.87	$c - 1 = 2$	23.44	4.398*	3.40
Interaction (gender* incentives)	88.20	$(r - 1) \times (c - 1) = 2$	44.10	8.27*	3.40
Error	128.00	$N - rc = 24$	5.33		
Corrected total	273.87	$N - 1 = 29$			

*Significant at 5% level

Table 8.4 Mean sale in different incentive groups (both gender combined)

Incentives			CD at 5% level
II	III	I	
13.0	12.9	10.3	1.48

"_____/" Denotes no difference between the means at .05 level

Post hoc test shall be used to further analyze the column factor (incentives) and the interaction effect.

Post Hoc Test for Column Factor (Incentives)

LSD test shall be used to find the critical difference for comparing the means of the groups in the column factor. The critical difference (CD) is given by

$$\text{CD at .05 significance level} = t_{.05}(24) \times \sqrt{\frac{2\text{MSSE}}{n \times r}}$$

$$= 2.064 \times \sqrt{\frac{2 \times 5.33}{5 \times 2}} \text{[From Table A.2 in the Appendix, } t_{.05}(24) = 2.064\text{]}$$

$$= 2.064 \times 1.03$$

$$= 2.13$$

Table 8.4 shows that the mean difference between II and III incentive groups is less than the critical difference (=2.13); hence, there is no difference between these two incentive groups. To show this, a line has been drawn below these two group means. On the other hand, there is a significant difference between the means of II and I as well as III and I incentive groups. Thus, it may be concluded that the II and III incentives are equally effective and better than Ist incentive in enhancing the sale of the toothpaste irrespective of the gender of the sales manager.

Table 8.5 Comparison of mean sale of toothpaste between male and female groups in each of the three incentive groups

Gender / Incentives	Male	Female	Mean *diff.*	CD at 5% level
I	12.4	8.2	4.2*	2.10
II	11.2	14.8	3.6*	2.10
III	14.4	11.4	3.0*	2.10

*Significant at 5% level

Table 8.6 Comparison of mean sale of toothpaste among different incentive groups in each of the two gender groups

Gender	Incentives			CD at 5% level
Male	14.4 (III)	12.4 (I)	11.2 (II)	2.10
Female	14.8 (II)	11.4 (III)	8.2 (I)	2.10

Remark: Arrange means of the groups in descending order

Post Hoc Test for Interaction (Gender × Incentives)

Since interaction effect is significant, comparison shall be made among the means of each incentive groups in each gender. Similarly, mean comparison shall also be made between male and female groups in each of the incentive groups. Since the cell size is similar, the critical difference for row comparison in each column and for column comparison in each row shall be same. The CD using LSD test shall be obtained by

$$\begin{aligned}\text{CD at .05 significance level} &= t_{.05}(24) \times \sqrt{\frac{2\text{MSSE}}{n}}\\ &= 2.064 \times \sqrt{\frac{2 \times 5.33}{5}}\\ &= 2.064 \times 1.46\\ &= 3.01\end{aligned}$$

Table 8.5 provides the post hoc comparison of means of male and female groups in each of the three incentive groups. Since the mean difference between male and female in each of the three incentive groups is higher than the critical difference, these differences are significant at 5% level. Further, it may be concluded that the average sales of male group in I and III incentives groups are higher than that of female group whereas average sales of female is higher than that of male in II incentive group.

Table 8.6 shows the comparison of different incentive groups in each of the gender group. It can be seen from this table that in male section, average sales are significantly different in III and II incentive groups, whereas average sales in the III and I incentive groups as well as I and II incentive groups are same. On the other hand, in female section, the average sales in all the three incentive groups are significantly different from each other.

Table 8.7 Data on sale of chocolates of different flavours and colours

Chocolate color			
Sweetness	White	Milk	Dark
Semisweet chocolate	25	20	35
	20	15	32
	22	17	31
	28	21	42
	23	25	30
Bittersweet chocolate	28	32	15
	24	35	22
	32	37	25
	32	38	12
	23	29	20
Unsweetened chocolate	26	20	15
	24	15	12
	32	17	10
	26	21	22
	31	25	13

Thus, on the basis of the given data, the result suggests that IIIrd incentive should be preferred if it is promoted by the male sales manager whereas the sales of the toothpaste would increase if it is promoted by the female sales manager using IInd incentive.

Solved Example of Two-Way ANOVA Using SPSS

Example 8.2 A chocolate manufacturing company wanted to know the impact of color and sweetness of its chocolates on buying decision of the customers. Data in Table 8.7 shows the number of units sold per day in a city for five consecutive days. Apply two-way analysis of variance to see whether the factors sweetness and color have significant effect on the sale of chocolates. Also test the significance of interaction between these two factors and discuss your findings at 5% level.

Solution Here, two main factors, namely, sweetness and color as well as interaction between sweetness and color, need to be investigated. Thus, following three hypotheses shall be tested:

(i) $H_0 : \mu_{Semi\_Sweet} = \mu_{Bitter\_Sweet} = \mu_{Un\_Sweet}$
(The sales of semisweet, bittersweet, and unsweetened chocolates are same irrespective of the color of the chocolate.)

(ii) $H_0 : \mu_{White} = \mu_{Milk} = \mu_{Dark}$
(The sales of white, milk, and dark chocolates are same irrespective of the sweetness of the chocolate.)

(iii) H_0: There is no interaction between the sweetness and color of the chocolate.

(It is immaterial whether any color of the chocolates is semisweet, bittersweet, or unsweetened.)

The SPSS output for two-way analysis of variance provides F-values for the sweetness factor, color factor, and interaction (sweetness $\times$ color) along with their significant values (p-values). The F-values for these factors and interaction shall be significant if their p-values are less than .05. For any factor or interaction if the F-value is significant, then a post hoc test shall be applied to compare the paired means. SPSS provides option to choose any of the post hoc tests for comparing the significance of mean difference.

In this example, since the sample sizes are equal, LSD test shall be used as a post hoc test for comparing group means. The SPSS output provides the significance values (p-values) for the difference of each pair of group means. The significance of difference between pair of group means is tested by using these p-values rather than computing the critical differences as is done in case of solving two-way ANOVA manually.

Computation in Two-Way ANOVA Using SPSS

(a) ***Preparing data file***

(i) *Starting the SPSS*: Use the below-mentioned command sequence to start SPSS on your system:

Start → Programs → IBM SPSS Statistics → IBM SPSS Statistics 20

After clicking the **Type in Data**, you will be taken to the **Variable View** option for defining the variables in the study.

(ii) *Defining variables*: There are three variables in this example, namely, sales, sweetness, and color. Since sales were measured on ratio scale, it has been defined as a scale variable, whereas sweetness and color were measured on nominal scale hence they have been defined as nominal variables. The procedure of defining the variables and their characteristics in SPSS is as follows:

1. Click **Variable View** to define variables and their properties.
2. Write short name of the variables as *Sale*, *Sweetness*, and *Colour* under the column heading **Name**.
3. Under the column heading **Label**, define full name of these variables as *Chocolate_Sale*, *Chocolate_Sweetness*, and *Chocolate_Colour*. Alternate names may also be chosen for describing these variables.
4. Under the column heading **Measure**, select the option “Scale” for the variable *Sale* and “Nominal” for the variables *Sweetness* and *Colour*.

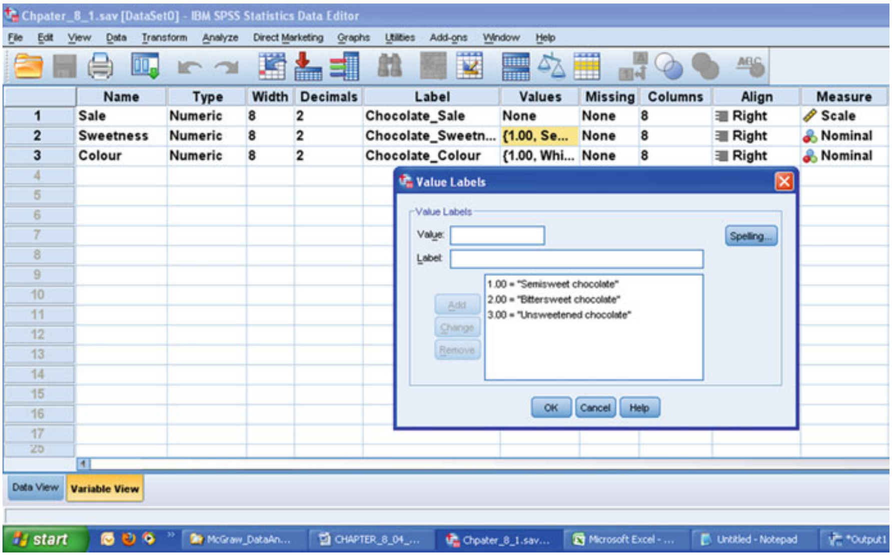


Fig. 8.1 Defining variables along with their characteristics

5. For the variable *Sweetness*, double-click the cell under the column **Values** and add following values to different labels:

Value	Label
1	Semisweet chocolate
2	Bittersweet chocolate
3	Unsweetened chocolate

6. Similarly, for the variable *Colour*, add the following values to different labels:

Value	Label
1	White
2	Milk
3	Dark

7. Use default entries in rest of the columns.
   After defining the variables in variable view, the screen shall look like Fig. 8.1.

(iii) *Entering data*: After defining these variables in the **Variable View**, click **Data View** on the left bottom of the screen to enter the data. One should note the procedure of data feeding carefully in this example. First 15 sales data of semisweet chocolate of Table 8.6 are entered in the column of *Sales* after which next 15 data of bittersweet chocolate are entered in the same column, and thereafter the remaining 15 data of unsweetened chocolate are entered in the same column. Thus, in the column of *Sales* variable, there will be 45 data. Under the column *Sweetness*, first 15 scores are entered as 1 (denotes semisweet chocolate), the next 15 scores are entered as 2 (denotes bittersweet chocolate), and the remaining 15 scores are entered as 3 (denotes unsweetened chocolate). Under the column *Colour*, first five scores are entered as 1 (denotes white color chocolate), next five scores as 2 (denotes milk color chocolate), and subsequent five scores as 3 (denotes dark color chocolate). These 15 data belong to semisweet chocolate group. Similarly, next 15 scores of bittersweet chocolate group and unsweetened chocolate groups can be just the repetition of the semisweet chocolate group. Thus, after feeding the first 15 data in the *Colour* column, repeat this set of 15 data twice in the same column.

After entering the data, the screen will look like Fig. 8.2. Save the data file in the desired location before further processing.

(b) **SPSS *commands for two-way ANOVA***
After entering the data in data view as per above-mentioned scheme, follow the below-mentioned steps for two-way analysis of variance:

(i) *Initiating the SPSS commands for two-way ANOVA*: In D**ata View**, click the following commands in sequence:

Analyze → General Linear Model → Univariate

The screen shall look like Fig. 8.3.

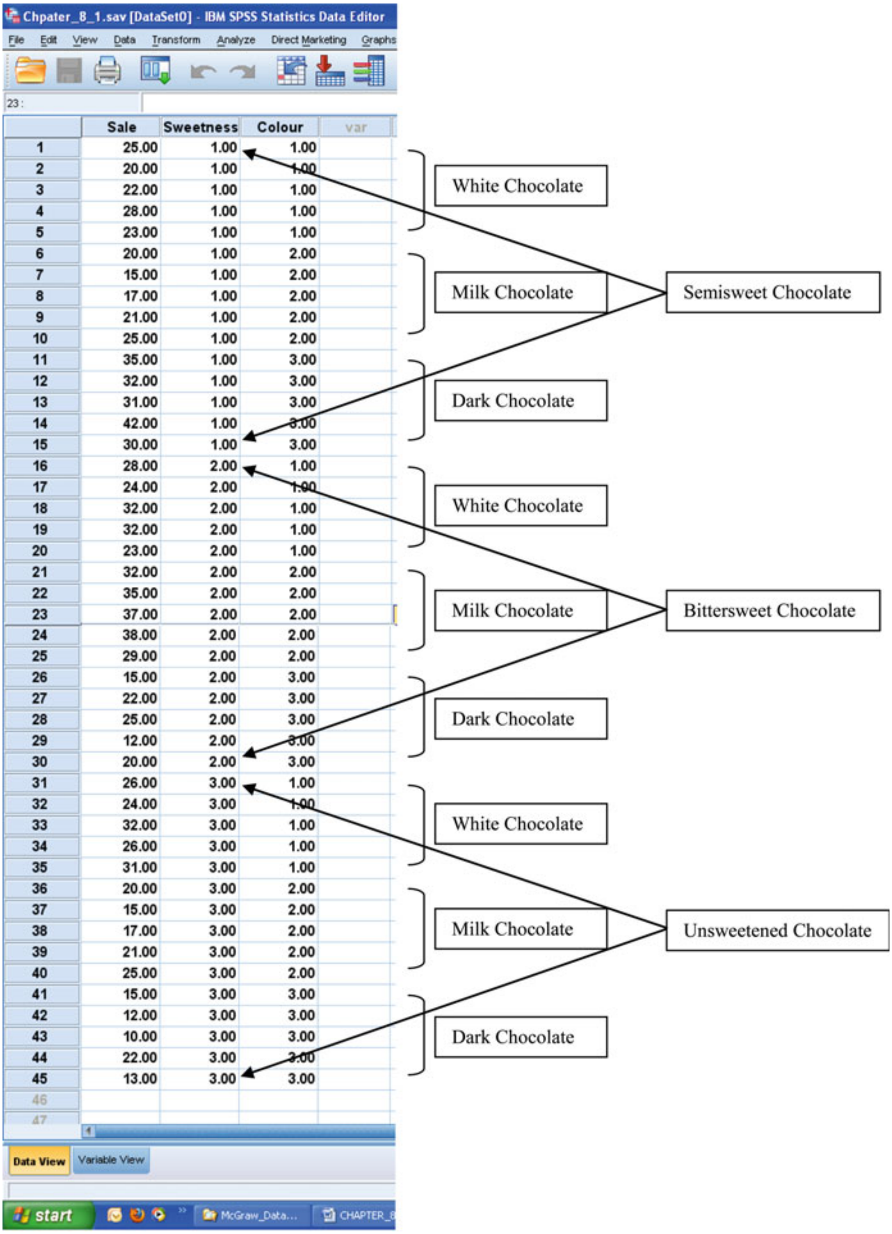
(ii) *Selecting variables for two-way ANOVA*: After clicking the **Univariate** option, you will be taken to the next screen for selecting variables. Select the variables *Chocolate_Sale* from left panel to the "Dependent variable" section of the right panel. Similarly, select the variables *Chocolate_Sweetness* and *Chocolate_Colour* from left panel to the "Fixed Factor(s)" section of the right panel. The screen will look like Fig. 8.4.

(iii) *Selecting the option for computation*: After selecting the variables, various options need to be defined for generating the output in two-way ANOVA. Do the following:

- Click the tag **Post Hoc** in the screen shown in Fig. 8.4. Then,
  - Select the factors *Sweetness* and *Colour* from the left panel to the "Post Hoc Tests for" panel on the right side by using the arrow key.
  - Check the option "LSD." LSD test is selected as a post hoc because the sample sizes are equal in each cell.

The screen will look like Fig. 8.5.

- Click ***Continue***. This will again take you back to the screen as shown in Fig. 8.4.
  - Now click the tag **Options** on the screen and do the following steps:
    - Check the option "Descriptive."



	Sale	Sweetness	Colour	var
1	25.00	1.00	1.00	
2	20.00	1.00	1.00	
3	22.00	1.00	1.00	
4	28.00	1.00	1.00	
5	23.00	1.00	1.00	
6	20.00	1.00	2.00	
7	15.00	1.00	2.00	
8	17.00	1.00	2.00	
9	21.00	1.00	2.00	
10	25.00	1.00	2.00	
11	35.00	1.00	3.00	
12	32.00	1.00	3.00	
13	31.00	1.00	3.00	
14	42.00	1.00	3.00	
15	30.00	1.00	3.00	
16	28.00	2.00	1.00	
17	24.00	2.00	1.00	
18	32.00	2.00	1.00	
19	32.00	2.00	1.00	
20	23.00	2.00	1.00	
21	32.00	2.00	2.00	
22	35.00	2.00	2.00	
23	37.00	2.00	2.00	
24	38.00	2.00	2.00	
25	29.00	2.00	2.00	
26	15.00	2.00	3.00	
27	22.00	2.00	3.00	
28	25.00	2.00	3.00	
29	12.00	2.00	3.00	
30	20.00	2.00	3.00	
31	26.00	3.00	1.00	
32	24.00	3.00	1.00	
33	32.00	3.00	1.00	
34	26.00	3.00	1.00	
35	31.00	3.00	1.00	
36	20.00	3.00	2.00	
37	15.00	3.00	2.00	
38	17.00	3.00	2.00	
39	21.00	3.00	2.00	
40	25.00	3.00	2.00	
41	15.00	3.00	3.00	
42	12.00	3.00	3.00	
43	10.00	3.00	3.00	
44	22.00	3.00	3.00	
45	13.00	3.00	3.00	
46				
47				

Fig. 8.2 Screen showing data entry for different variables in the data view

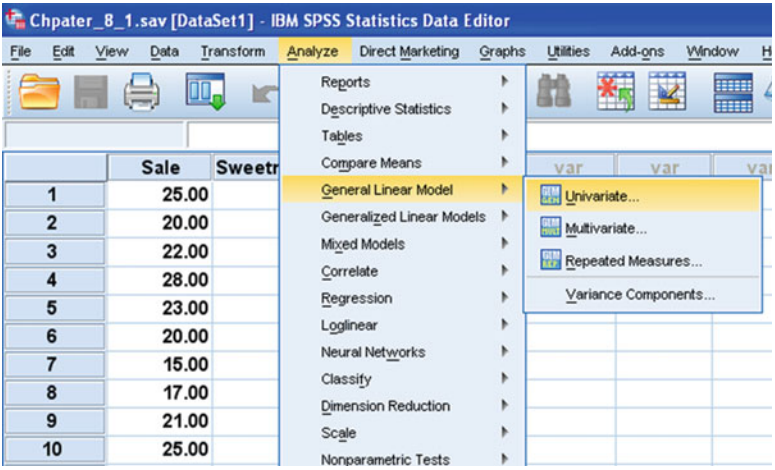


Fig. 8.3 Screen showing SPSS commands for two-way ANOVA

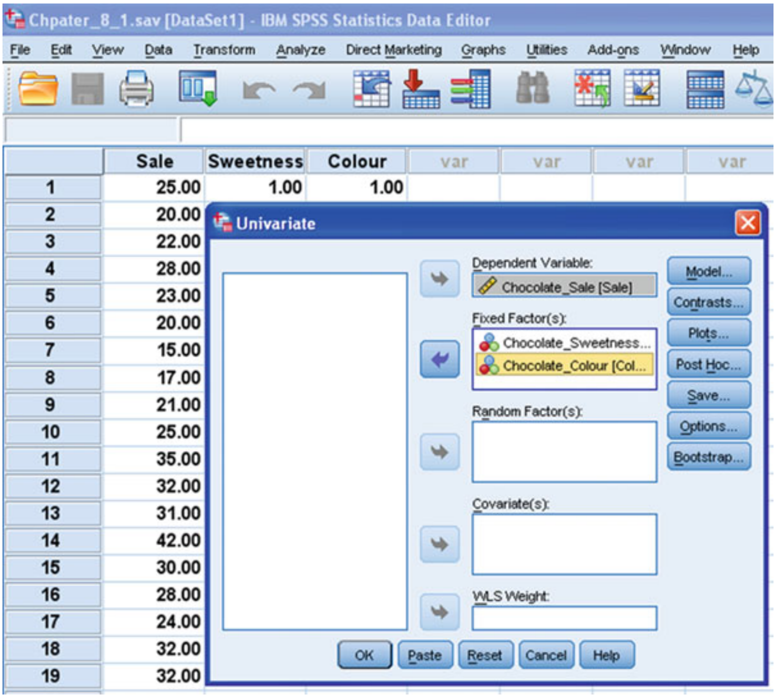


Fig. 8.4 Screen showing selection of variables for two-way ANOVA

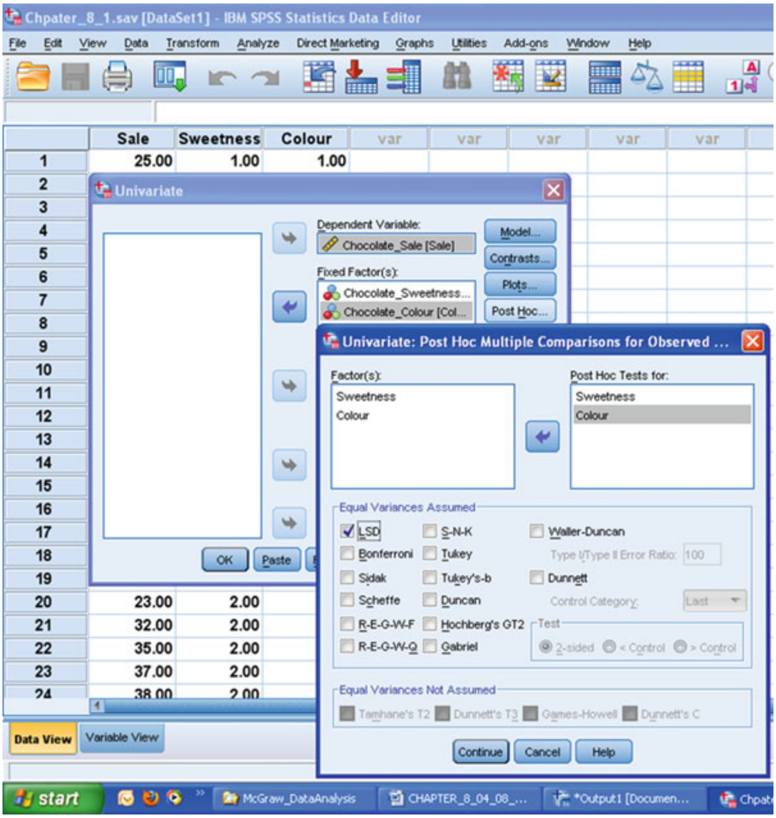


Fig. 8.5 Screen showing options for post hoc test

– Select the variables *OVERALL*, *Sweetness*, *Colour*, and *Sweetness* × *Colour* from the left panel and bring them into the "Display Means for" section of the right panel.
– Check the option "Compare main effects."
– Ensure the value of Significance level as .05 in the box. The screen for these options shall look like as shown in Fig. 8.6.
– Click ***Continue*** to go back to the main screen.

After selecting the options, the screen shown in Fig. 8.4 shall be restored.

– Press **OK**.

(c) ***Getting the output***
After clicking **OK** in the screen shown in Fig. 8.4, various outputs shall be generated in the output window. Relevant outputs may be selected by using

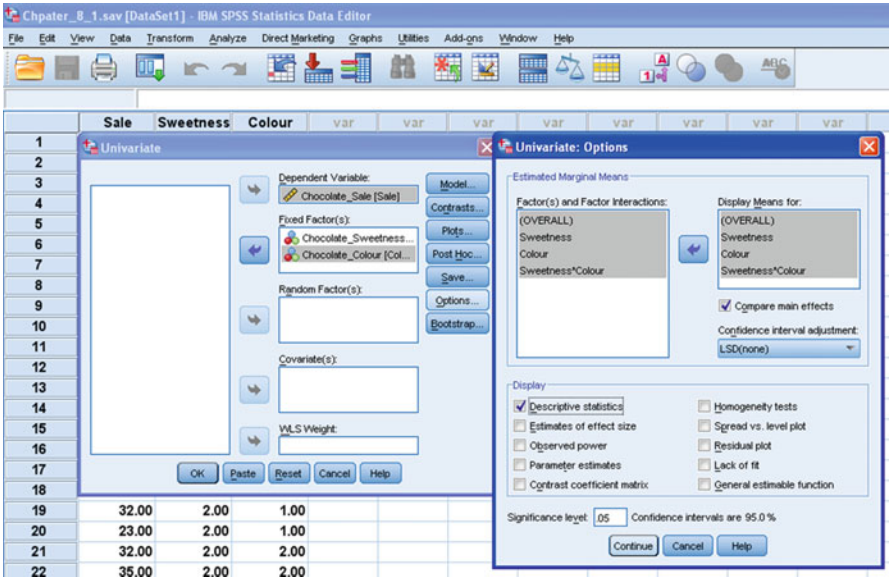


Fig. 8.6 Screen showing options for descriptive statistics and comparison of main effects

right click of the mouse and may be copied in the word file. Here, the following outputs shall be selected:

1. Descriptive statistics
2. Two-way ANOVA table
3. Pairwise comparisons of sweetness groups (all color groups combined)
4. Pairwise comparisons of different color groups (all sweetness groups combined)

In this example, all the identified outputs so generated by the SPSS will look like as shown in Tables 8.8, 8.9, 8.10, and 8.11.
In order to interpret the findings, these outputs may be rearranged so that it can directly be used in your project. These rearranged formatted tables have been shown under the heading "**Model Way of Writing the Results**" in the next section.

Model Way of Writing the Results of Two-Way ANOVA and Its Interpretations

The outputs so generated in the SPSS may be presented in user-friendly format by selecting the relevant details from Tables 8.8, 8.9, 8.10, and 8.11 and making some slight modifications. Further, if the interaction is significant, it is not possible to compare the cell means by using the outputs of SPSS. However, critical difference can be computed by using these outputs for testing the significance of mean

Table 8.8 Descriptive statistics

Sweetness	Color	Mean	Std. dev.	*N*
Semisweet chocolate	White	23.60	3.04959	5
	Milk	19.60	3.84708	5
	Dark	34.00	4.84768	5
	Total	25.73	7.28469	15
Bittersweet chocolate	White	27.80	4.26615	5
	Milk	34.20	3.70135	5
	Dark	18.80	5.26308	5
	Total	26.93	7.73181	15
Unsweetened chocolate	White	27.80	3.49285	5
	Milk	19.60	3.84708	5
	Dark	14.40	4.61519	5
	Total	20.60	6.81175	15
Total	White	26.40	3.94244	15
	Milk	24.47	7.94505	15
	Dark	22.40	9.81107	15
	Total	24.42	7.64106	45

Dependent variable: Chocolate_Sale

Table 8.9 Two-way ANOVA table generated by the SPSS

Source	Type III sum	df	Mean square of squares	*F*	Sig.
Corrected model	1946.978[a]	8	243.372	14.086	.000
Intercept	26840.022	1	26840.022	1553.44	.000
Sweetness	339.511	2	169.756	9.825	.000
Color	120.044	2	60.022	3.474	.042
Sweetness * color	1487.422	4	371.856	21.522	.000
Error	622.000	36	17.278		
Total	29409.000	45			
Corrected total	2568.977	44			

Dependent variable: Chocolate_Sale
[a]R squared = .758 (adjusted R squared = .704)

difference among different groups. The procedure of comparing group means has been discussed later in this section.

The first important table consisting F-values for the factors and interaction can be reproduced by deleting some of the contents of Table 8.9. The information so reduced is shown in Table 8.12.

The p-values for the Sweetness, Color, and Interaction (Sweetness × Color) in Table 8.12 are less than .05; hence, all the three F-values are significant at 5% level. Thus, the null hypothesis for the Sweetness factor, Color factor, and Interaction (Sweetness × Color) may be rejected at .05 level of significance. Now the post hoc comparison analysis shall be done for these factors and interaction. These analyses are shown below.

Table 8.10 Pairwise comparison of different sweetness groups

(*I*) Chocolate_Sweetness	(*J*) Chocolate_Sweetness	Mean diff. (*I* – *J*)	Std. error	Sig.[a] (*p*-value)	95% Confidence interval for difference[a] Lower bound	Upper bound
Semisweet chocolate	Bittersweet chocolate	−1.200	1.518	.434	−4.278	1.878
	Unsweetened chocolate	5.133*	1.518	.002	2.055	8.212
Bittersweet chocolate	Semisweet chocolate	1.200	1.518	.434	−1.878	4.278
	Unsweetened chocolate	6.333*	1.518	.000	3.255	9.412
Unsweetened chocolate	Semisweet chocolate	−5.133*	1.518	.002	−8.212	−2.055
	Bittersweet chocolate	−6.333*	1.518	.000	−9.412	−3.255

Dependent variable: Chocolate_Sale
Based on estimated marginal means
*The mean difference is significant at the .05 level
[a]Adjustment for multiple comparisons: least significant difference (equivalent to no adjustments)

Table 8.11 Pairwise comparison of different color groups

(*I*) Chocolate_Colour	(*J*) Chocolate_Colour	Mean diff. (*I* – *J*)	Std. error	Sig.[a]	95% Confidence interval for difference[a] Lower bound	Upper bound
White	Milk	1.933	1.518	.211	−1.145	5.012
	Dark	4.000*	1.518	.012	.922	7.078
Milk	White	−1.933	1.518	.211	−5.012	1.145
	Dark	2.067	1.518	.182	−1.012	5.145
Dark	White	−4.000*	1.518	.012	−7.078	-.922
	Milk	−2.067	1.518	.182	−5.145	1.012

Dependent variable: Chocolate_Sale
Based on estimated marginal means
*The mean difference is significant at the .05 level
[a]Adjustment for multiple comparisons: least significant difference (equivalent to no adjustments)

Table 8.12 Two-way ANOVA table for the data on chocolate sale

Source of variation	Sum of squares (SS)	df	Mean sum of squares (MSS)	*F*	*p*-value (sig.)
Sweetness	339.51	2	169.76	9.83	.000
Color	120.04	2	60.02	3.47	.042
Sweetness × color	1,487.42	4	371.86	21.52	.000
Error	622.00	36	17.28		
Corrected total	2,568.977	44			

Table 8.13 Comparison of mean chocolate sale in all the three sweetness groups (all colors combined)

Bittersweet chocolate	Semisweet chocolate	Unsweetened chocolate	CD at 5% level
26.93	25.73	20.60	3.08

"__________/"Denotes no difference between the means at 5% level
Remark: Arrange means of the group in descending order

Row (Sweetness) Analysis

For row analysis, critical difference has been obtained by using the LSD test. The value of "*t*" at .05 level and 36 df (error degrees of freedom) can be obtained from Table A.2 in Appendix.

Thus,

$$\text{CD for row} = t_{.05}(36) \times \sqrt{\frac{2(\text{MSS})_\text{E}}{nc}} \quad [n = \text{number of scores in each cell} = 5]$$
$$[c = \text{number of column(colour groups)} = 3]$$
$$= 2.03 \times \sqrt{\frac{2 \times 17.28}{5 \times 3}} = 3.08$$

Table 8.13 has been obtained by using the contents from the Tables 8.8 and 8.10. Readers are advised to note the way this table has been generated.

If difference of any two group means is higher than the critical difference, the difference is said to be significant. Owing to this principle from Table 8.13, two conclusions can be drawn.

(a) Average sale of chocolate in bittersweet and semisweet categories is significantly higher than that of unsweetened category.
(b) Average sale of chocolate in bittersweet and semisweet categories is equal.

It may thus be inferred that bittersweet and semisweet chocolates are more preferred than unsweetened chocolates irrespective of the color of the chocolate.

Remark By looking at the *p*-values in Table 8.10, you can infer as to which group means differ significantly. If for any mean difference, significance value (*p*-value) is less than .05, then the difference is considered to be significant. In using this table, you can test the significance of mean difference, but it is difficult to find out as to which group mean is higher until and unless the results of Table 8.8 are combined. Hence, it is advised to construct Table 8.13 for post hoc analysis so as to get the clear picture in the analysis.

Table 8.14 Comparison of mean chocolate sale in all the three Color groups (all sweetness types combined)

White chocolate	Milk chocolate	Dark chocolate	CD at 5% level
26.40	24.47	22.40	3.08

"_______/" Denotes no difference between the means at .05 level

Column (Color) Analysis

For column analysis, critical difference has been obtained by using the LSD test as there are equal numbers of samples in each column.

Thus,

$$\text{CD for column} = t_{.05}(36) \times \sqrt{\frac{2(\text{MSS})_{\text{E}}}{nr}} \quad [n = \text{number of scores in each cell} = 5]$$

$$[r = \text{number of row(sweetness groups)} = 3]$$

$$= 2.03 \times \sqrt{\frac{2 \times 17.28}{5 \times 3}} = 3.08$$

Table 8.14 has been obtained from the contents in Tables 8.8 and 8.11.

From Table 8.14, the following two conclusions can be drawn:

(a) There is no difference in the average sale of white and milk chocolate. Similarly average sale of milk and dark chocolate is also same.
(b) Average sale of white chocolates is significantly higher than that of dark chocolates.

Thus, it may be inferred, in general, that the mean sale of the white chocolates is more in comparison to that of dark chocolate irrespective of the type of sweetness.

Remark You may note that critical difference for row and column analysis is same. It is so because the number of rows is equal to the number of columns.

Interaction Analysis

Since F-value for the interaction is significant, it indicates that there is a joint effect of the chocolate sweetness and chocolate colors on the sale of chocolates. In other words, there is an association between sweetness and color of the chocolates. Thus, to compare the average chocolate sale among the three levels of sweetness in each of the color groups and to compare the average sales in all the three types of colored

Table 8.15 Comparison of mean chocolate sale among different sweetness groups in each of the three colour groups

Color	Sweetness			CD at 5% level
White	27.80 (Bittersweet)	27.80 (Unsweetened)	23.60 (Semisweet)	5.34
Milk	34.20 (Bittersweet)	19.60 (Unsweetened)	19.60 (Semisweet)	5.34
Dark	34.00 (Semisweet)	18.80 (Bittersweet)	14.40 (Unsweetened)	5.34

"_____/" Denotes no difference between the means at .05 level

chocolates in each of the sweetness group, a critical difference (CD) has to be computed:

$$\text{CD for Interaction} = t_{.05}(36) \times \sqrt{\frac{2(\text{MSS})_E}{n}}$$
$$= 2.03 \times \sqrt{\frac{2 \times 17.28}{5}} = 5.34$$

Tables 8.15 and 8.16 have been generated with the help of the contents of Table 8.8. Readers are advised to note that CD is same for comparing all the three types of sweetness groups in each color group as well as for comparing all the colored groups in each of the sweetness group. It is so because the number of samples (n) in each cell is equal.

If the difference of group means is higher than that of the critical difference, it denotes that there is a significant difference between the two means; otherwise, group means are equal. If the mean difference is not significant, an underline is put against both the groups.

From Table 8.15, the following three conclusions can be drawn:

(a) The average sale of chocolates in all the three categories of sweetness groups is same for white chocolates.
(b) In milk chocolates, the average sale in bittersweet category is significantly higher than that of unsweetened and semisweet categories.
(c) In dark chocolates, the average sale in semisweet category is significantly higher than that of bittersweet and unsweetened categories.

It is thus concluded that in white chocolate, it hardly matters which sweetness flavor is being sold, whereas types of sweetness matters in case of milk and dark chocolates.

Table 8.16 Comparison of mean chocolate sale among different colour groups in each of the three sweetness groups

Sweetness	Color			CD at 5% level
Semisweet	34.00 (Dark)	23.60 (White)	19.60 (Milk)	5.34
Bittersweet	34.20 (Milk)	27.80 (White)	18.80 (Dark)	5.34
Unsweetened	27.80 (White)	19.60 (Milk)	14.40 (Dark)	5.34

“_________/” Denotes no difference between the means at .05 level

From Table 8.16, the following three conclusions can be drawn:

(a) In semisweet category, the sale of dark chocolate is significantly higher than that of white and milk chocolates.
(b) In bittersweet category, the sale of milk chocolate is significantly higher than that of white and dark chocolates.
(c) In unsweetened category, the sale of white chocolate is significantly higher than that of milk and dark chocolates.

It may be inferred that in each of the sweetness flavor, it matters as to which color of chocolates is being sold.

Summary of the SPSS Commands for Two-Way ANOVA

(i) Start the SPSS by using the following commands:

Start → Programs → IBM SPSS Statistics → IBM SPSS Statistics 20

(ii) Click **Variable View** tag and define the variables *Sale* as a scale variable and *Sweetness* and *Colour as* nominal variables.
(iii) Under the column heading **Values**, define “1” for semisweet chocolates, “2” for bittersweet chocolates, and “3” for unsweetened chocolates for the variable *Sweetness*.
(iv) Similarly, for the variable *Colour*, under the column heading **Values**, define “1” for white chocolates, “2” for milk chocolates, and “3” for dark chocolates.
(v) Once the variables are defined, type the data for these variables by clicking **Data View**.

(vi) In the data view, follow the below-mentioned command sequence for two-way ANOVA:

Analyze ⇨ General Linear Model ⇨ Univariate

(vii) Select the variable *Chocolate_Sale* from left panel to the "dependent variable" section of the right panel. Similarly, select the variables *Chocolate_Sweetness* and *Chocolate_Colour* from left panel to the "Fixed Factor(s)" section of the right panel.

(viii) Click the tag **Post Hoc** and select the factors *Sweetness* and *Colour* from the left panel to the "Post Hoc test" panel on the right side. Check the option "LSD" and then click ***Continue***.

(ix) Click the tag **Options**, Select the variables *OVERALL, Sweetness, Colour*, and *Sweetness* × *Colour* from left panel to the right panel. Check the "Compare main effects" and "Descriptive" boxes and ensure the value of significance as .05. Click ***Continue***.

(x) Press **OK** for output.

Exercise

Short Answer Questions

Note: Write answer to each of the following questions in not more than 200 words.

Q.1. What do you mean by main effects, interactions effects, and within-group variance? Explain by means of an example.

Q.2. Justify the name "two-way analysis of variance." What are the advantages of using two-way ANOVA design over one-way?

Q.3. While using two-way ANOVA, what assumptions need to be made about the data?

Q.4. Describe an experimental situation where two-way ANOVA can be used. Discuss different types of hypotheses that you would like to test.

Q.5. Discuss a situation where a factorial design can be used in market research. What research questions you would like to investigate?

Q.6. What is repeated measure design? Explain by means of an example. What precaution should be taken in planning such design?

Q.7. Explain MANOVA and discuss any one situation where it can be applied in management studies.

Q.8. Describe Latin square design. Discuss its layout. How is it different than factorial design?

Multiple-Choice Questions

Note: For each of the question, there are four alternative answers. Tick mark the one that you consider the closest to the correct answer.

1. In applying two-way ANOVA in an experiment, where "r" levels of factor A and "c" levels of factor B are studied. What will be the degree of freedom for interaction?

   (a) rc
   (b) $r + c$
   (c) $rc - 1$
   (d) $(r - 1)(c - 1)$

2. In an experiment "r" levels of factor A are compared in "c" levels of factor B. Thus, there are N scores in this experiment. What is the degree of freedom for within group?

   (a) $N - rc$
   (b) $N + rc$
   (c) $N - rc + 1$
   (d) $Nrc - 1$

3. In a two-way ANOVA, if the two factors A and B have levels 3 and 4, respectively, and the number of scores per cell is 3, what would be the degrees of freedom of error?

   (a) 36
   (b) 24
   (c) 12
   (d) 9

4. In order to apply two-way ANOVA

   (a) There should be equal number of observations in each cell.
   (b) There may be unequal number of observations in each cell.
   (c) There should be at least ten observations in each cell.
   (d) There is no restriction on the number of observations per cell.

5. Consider an experiment in which the Satisfaction levels of employees (men and women both) were compared in their plants located in three different cities. Choose the correct statement in defining the three variables Gender, City, and Satisfaction level in SPSS:

   (a) Gender and Satisfaction level are Scale variables and City is Nominal.
   (b) Gender and City are Nominal variables and Satisfaction level is Scale.
   (c) Gender and City are Scale variables and Satisfaction level is Nominal.
   (d) City and Satisfaction level are Scale variables and Gender is Nominal.

6. Command sequence in SPSS for starting two-way ANOVA is

   (a) Analyze -> General Linear Model -> Univariate
   (b) Analyze -> General Linear Model -> Multivariate
   (c) Analyze -> General Linear Model -> Repeated Measures
   (d) Analyze -> Univariate -> General Linear Model

7. While performing two-way ANOVA with SPSS, Fixed Factor(s) refers to

   (a) Dependent variable
   (b) Independent variables
   (c) Both dependent and independent variables
   (d) None of the above

8. If there are N scores in a two-way ANOVA experiment, the total degree of freedom would be

   (a) $N + 1$
   (b) $N - 1$
   (c) N
   (d) $N - 2$

9. If 3 levels of factor A are compared among the 4 levels of factor B, how many treatment groups will have to be created?

   (a) 7
   (b) 1
   (c) 12
   (d) 11

10. In an experiment, motivation level of employees was compared in three different organizations. Employees were categorized as per their gender to see its impact on motivation level. Interaction effect between plants and gender can be investigated only if there are

    (a) Equal number of observations in each cell
    (b) At least one cell must have five or more observations
    (c) Unequal number of observations in each cell
    (d) More than one and equal number of observations in each cell

11. What should be the minimum number of observations in order to perform two-way ANOVA?

    (a) 8
    (b) 6
    (c) 4
    (d) 2

12. What should be the minimum number of observations in order to perform two-way ANOVA with interaction effect?
    (a) 8
    (b) 6
    (c) 4
    (d) 2

Assignments

1. Four salesmen were appointed by a company to sell their products in door-to-door marketing. Their sales were observed in three seasons, summer, rainy, and winter, on month to month basis. The sales data so obtained (in lakhs of rupees) are shown in the following table:

Sales data (in lakhs of rupees) of the sales persons in different season

	Salesmen			
Season	A	B	C	D
Summer	36	36	21	25
	35	32	25	27
	32	30	28	24
	38	33	25	29
Rainy	26	28	29	29
	25	28	32	31
	27	31	33	34
	29	28	38	39
Winter	28	29	31	32
	27	32	35	31
	32	33	31	28
	29	35	41	33

Discuss your findings by applying two-way ANOVA. Test your hypotheses at 5% level.

2. The management of a private bank was interested to know the stress level of their employees in different age categories and gender. A stress questionnaire was administered on the randomly selected employees in different age categories. The scores on their stress level are shown in the following table: Apply two-way ANOVA and discuss your findings at 5% level.

Stress scores of the employees in different age categories

	Age category (years)		
Gender	<35	35–50	>50
Male	28	55	42
	29	51	39
	32	45	41
	25	48	42
	26	53	48
Female	28	51	55
	32	45	58
	35	49	61
	29	43	52
	31	48	50

Answers to Multiple-Choice Questions

Q.1	d	Q.2	a	Q.3	b	Q.4	a
Q.5	b	Q.6	a	Q.7	b	Q.8	b
Q.9	c	Q.10	d	Q.11	c	Q.12	a