

Image Restoration Methods as Preprocessing Tools in Digital Stereo Matching

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TABLE OF CONTENTS

Section		page
ABSTRACT		1
1. INTRODUCTION		1
2. BASIC DEFINITIONS		2
3. MODELLING	1	3 4
4. IMAGE RESTORATION 4.1 Inverse Filtering 4.1.1 Algebraic Approach 4.1.2 Fourier Approach 4.2 Wiener Filtering 4.3 Constrained Filtering 4.4 Geometric Mean Filtering		6 6 9
5. DISCUSSION		12
ACKNOWLEDGEMENT		14
REFERENCES		15

ABSTRACT

In this paper basic restoration techniques for the purpose of increasing matching accuracy in digital stereophotos are investigated. Inverse filter, Wiener filter, constrained filter and geometric mean filter are discussed. These traditional restoration techniques are based on stationary random fields or spatial invariant point-spread-functions or both these criteria. The result shows that other methods based on the actual physical background are required, which means that these methods must be able to process nonstationary images and spatial variant point-spread-functions.

1. INTRODUCTION

During recent years much effort has been invested in digital methods in analytical photogrammetry with the focus on digital image processing methods. The goal of photogrammetry is to describe properties of objects, such as shape and position in an image. The extraction of shapes belongs to the field of pattern recognition and feature extraction. The determination of object position may be performed using digital matching techniques.

An investigation on the geometric accuracy of the matching of objects in simulated SPOT stereo images has been performed at the Department of Photogrammetry at the Royal Institute of Technology, Stockholm (Rosenholm, 1985). He reports an expected accuracy of 0.1 pixels and experimental values of 0.3 pixels. The question is, if there is any possibility of increasing the matching accuracy by preprocessing the images in a suitable manner. The answer may be found by considering the problem as a restoration problem. If the undegraded image can be well recovered, the matching accuracy may increase.

For the purpose of improving digitized aerial photography, a number of basic digital image restoration methods are described in this report. By image restoration we here mean the removal or reduction of degradations that where incurred during the imaging process. Typical degradation factors include camera motion, lens aberrations, film grain noise, low pass filtering due to the electro-optical systems and atmospheric turbulence.

Earlier work in the area of preprocessing due to stereo image matching has been performed by Förstner (1982) and Ehlers (1982). Förstner derived a method of optimizing digital image correlation. By minimizing the variance of the estimated difference of image translation, Förstner designed an ideal low pass filter with an optimum cut-off frequency. It was pointed out in the paper that the method requires that one of the images should be restored. Ehlers

(1982) tested matching accuracy by applying different filters to homogenous satellite imagery with low signal-to-noise ratio (S/N). The results show an increase in matching accuracy when one of the images is preprocessed by suitable low pass filters.

In optical and electrical science, image/signal restoration is well known and well developed. Early work in digital image restoration was, for the most part, based on those early developed methods (linear methods). This work will be based on those basic digital image restoration methods. The investigation has been performed as a literature survey of methods for image restoration.

2. BASIC DEFINITIONS

2.1 Linear System

A system is defined mathematically as a unique transformation or operator (Fig 2.1) that maps an input sequence f into an output sequence g (Oppenheim, 1975). Let L define a linear operator. If $g_1 = L(f_1)$ and $g_2 = L(f_2)$ then the system is linear if and only if

$$L(af_1 + bf_2) = aL(f_1) + bL(f_2) = ag_1 + bg_2$$

for arbitrary constants a and b. A linear operator (or function) is said to be shift invariant or position invariant when its effect on a point does not depend on the position of the point in the image i.e. L(f(t-x)) = g(t-x) for all x.

2.2 Convolution

Shift invariant operations can be carried out either directly in the original spatial domain of the image or in the frequency domain. In the first case, any linear shift invariant operation can be expressed as the convolution of two functions i.e.

$$g(x) = \int_{-\infty}^{+\infty} f(x)h(x-t)dt$$
 (2-1)

or

$$q = f * h$$

where f(x) is the input function of the system, h(x-t) the linear operator and g(x) the output of the system. If f(x) is the original undistorted image and g(x) the distorted output image then h(x-t) is the point-spread-function (psf) of the system. The psf can be expressed as a general function of both x and t:

$$g(x) = \int_{-\infty}^{+\infty} f(x)h(x,t)dt$$
 (2-2)

This equation, known as a Fredholm integral with a twodimensional kernel, is a general description of a linear system in which the operator h(x,t) maps an object f(x) into an image g(x). If the operator h(x,t) is shift variant the shape of the operator changes as a function of its position. If, however, the operator h maintains its functional form independent of its position (shift invariant) then the equation (2-2) can be expressed as equation (2-1).

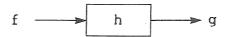


Fig 2.1 A Linear System.

2.3 The Fourier Transform

A one-dimensional forward Fourier transform of a function g(x) is defined by

$$G(s) = F(g(x)) = \int_{-\infty}^{+\infty} g(x) e^{-2\pi j x s} dx$$

where j = $\sqrt{-1}$. The operator F defines the Fourier operation. The inverse Fourier transform is defined by

$$g(x) = F^{-1}(G(s)) = \int_{-\infty}^{+\infty} G(s) e^{2\pi j x s} ds$$

The power spectrum of a function g(x) is defined as

$$P(s) = abs(F(g(x))^2$$

which is a description of the distribution of the energy of the function at certain frequencies. The convolution in the spatial domain corresponds to multiplication in the frequency domain (and vice versa). This means that

$$F(g(x)*h(x)) = G(s)H(s)$$
and
$$F(G(s)*H(s)) = g(x)h(x)$$

which in linear system analysis is called the convolution theorem.

3. MODELLING

3.1 Image Models

There are several models for describing a digital image (Rosenfeld, 1981). Two common models will be described here: statistical and deterministic (Andrews and Hunt, 1977).

3.1.1 Statistical Image Model

In a statistical model of an image we assume that each component (pixel) in the image results from a random variable. Thus, each image is the result of a family of

random variables, a random field. A random field is a two-dimensional case of a random process. Consider the random field $f(r, w_i)$ where r is a vector in the xy-plane. For a given outcome w_i , $f(r,w_i)$ is a function over the xy-plane. A sequence of such functions is an ensemble. For a given value of r, $f(r, w_i)$ is a random variable. The expectation of this random variable $m_f(r) = E(f(r))$ is an ensemble average and is called the mean of the random field at the position r. The autocorrelation of the random field f is defined as $R_{ff}(r_1,r_2) = E(r_1r_2)$. The random field is said to be stationary (homogenous, invariant) if the mean $m_f(r)$ is a constant independent of its position, and the autocorrelation of the random field is independent of its position, varying only as a function of the Euclidian distance between r₁ and r₂. The random field is said to be ergodic if the ensemble mean is constant and equal to the spatial average (time average in one dimension).

3.1.2 Deterministic Image Model

In a deterministic image model it is assumed we know the fundamental nature of the image. This may be described in parametric or nonparametric form. The former requires a functional description of the image. This may be achieved by segmenting the image into primitive regions, which can be described functionally as image primitives (Andrews and Hunt, 1977). The nonparametric form can be described by a set of constraints on the image (ex nonnegative grey levels).

3.2 Image Degradation Model

Normally, most analysis models for image processing systems are formulated under the assumptions that these systems have a linear psf (Andrews, 1975). A common degradation process may then be modelled by a linear shift invariant operator h(x), which, together with an additive noise term n(x), operates on an input image (or signal) f(x) to produce a degraded output image g(x) (Fig 3.1). This may be expressed as

$$g(x) = h(x)*f(x) + n(x)$$
 (3-1)

The operator h is commonly referred to as the point-spreadfunction (psf) of the process. This function, however, is for many imaging systems, a product of several psf components, where each component is assumed to correspond to a linear shift invariant system. Such components can be caused by for example diffraction, abberation, defocusing, motion blur and atmospheric turbulence (Castleman, 1979). The noise term n(x) is assumed to be a random variable. Images orignally recorded on photographic film are subject to degradation due to film grain noise. In addition to this, the digitizing or sampling process generates noise due to quantization, sampling rate and signal variation. In equation (3-1) the noise is assumed to be signal independent. Unfortunately, in reality this is seldom the case. Much of the noise in the process is signal dependent,

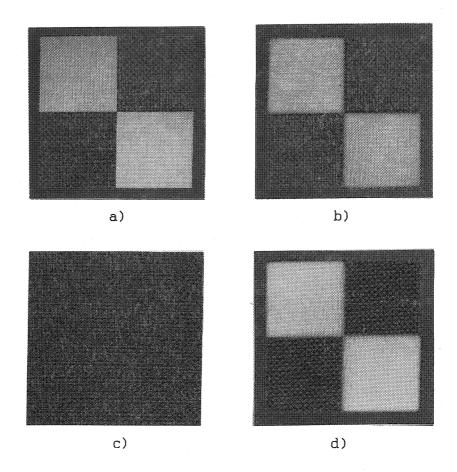


Fig 3.1 Image degradation model. a) original image, b) original deblurred by a Gaussian psf, c) random noise, d) deblurred plus random noise

film grain noise for example (Andrews and Hunt 1979, Sondhi 1972, Pratt 1978). However, to simplify the description the independent model for the noise will be used.

3.3 Determination of Degradation Parameters

The aim of image restoration is to estimate the original image f by using the recorded image g and knowledge concerning the degradation process. Such estimation, however, requires some form of knowledge concerning the degradation function h. An example of a priori determination is an experiment by McGlamery (1967) where determination of the turbulence psf was made. A posteriori determination of the psf may be performed by measuring the density of a sharp point in the degraded image. It is then assumed that the point in the original image is an approximation of the impulse function (Fig 3.2). In the same way the psf may be determined a posteriori from lines and edges (Andrews and Hunt 1977, Rosenfeld and Kak 1976, Pratt 1978). presence of noise it is also desirable to have some knowledge about the statistical properties of the noise. noise is usually assumed to be white i.e. the expectation of the power spectrum of the noise is constant. assumption is convenient but somewhat inaccurate (Rosenfeld

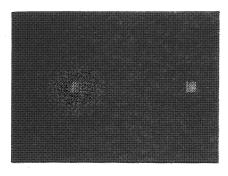


Fig 3.2 Determination of the psf. The original sharp point to the right is deblurred by the psf (left image).

and Kak, 1976). Different restoration techniques require different amounts of a priori information about noise. Wiener filtering requires knowledge about the noise power spectrum. In constrained restoration, however, knowledge of the variance of the noise is nesessary.

4. IMAGE RESTORATION

Consider the signal independent analytical degradation model

$$g(x) = h(x)*f(x) + n(x)$$
 (4-1)

where the shift invariant linear psf h(x) operates on the original input function f(x) and together with the additative noise function n(x) produces the output function g(x).

The goal of image restoration is to recover (deblur) the original function f(x) (image) as well as possible. There are several methods of recovering the image both linear and non linear. Non linear methods are beyond the scope of this paper. Linear restoration methods are usually mentioned collectively as deconvolution. Depending on the underlying assumed image model, the restoration techniques in the following sections are based on either statistical or deterministic image models, or both. The inverse filter will mainly be treated in the deterministic approach. Further on, the Wiener smoothing filter will be derived in the least square sense based on a statistical approach. It is possible to combine some elements of the deterministic and statistic approach. Constrained least square restoration techniqe is an example of that mixed approach.

4.1 Inverse Filtering

The inverse filter derived in this section is based on the deterministic image model. This approach reduces eq.(4-1) to the problem of solving a system of linear equations (Andrews and Hunt, 1977).

4.1.1 Algebraic Approach

In matrix formulation Eq. (4-1) may be written as

$$g = H f + n \tag{4-2}$$

where g, f and n are column vectors and H is a matrix. For

a shift invariant system matrix H is symmetric with respect to the diagonal. Each row is the same as the row above except that it is shifted one element to the right. Under this condition the psf H is called a Toeplitz matrix. The image restoration problem is to estimate the object f given samples of the recorded image g. An approximate solution of eq. (4-2) is

$$\overline{\mathbf{f}} = \mathbf{H}^{-1} \mathbf{q} \tag{4-3}$$

By substituting eq. (4-2) in eq. (4-3) the solution can be expressed as

$$\overline{f} = f + H^{-1} n \tag{4-4}$$

Thus, the estimate of the object \overline{f} consists of two parts: the actual object distribution and the term involving the inverse acting on the noise. If S/N is small (noisy image), the second term in eq. (4-4), the error term, is very large. The reason for this is that H, which represents the psf, has small eigenvalues, and causes H^{-1} to have very large elements.

Difficulties can also arise in connection with the inversion of the matrix H. Andrews and Hunt (1977) and Jansson (1984) mentioned problem as i) limitations in primary computer memory when H is very large, ii) if the inverse exists and is unique, it may be ill conditioned which may cause the term $H^{-1}n$ in (4-4) to dominate the result, iii) roundoff error may become a serious problem. To illustrate the computer storage problem, consider the two-dimensional case. If g has the dimension N x N then the matrix H has the dimension $N^2 \times N^2$. Thus, for a rather small image the H-1 matrix becomes difficult to solve with conventional methods. Andrews and Hunt (1977) illustrate the problem with noise by deriving S/N between data g and solution f. If S/N for the data is alpha and snr for the solution is beta then the ratio is defined as alpha/beta = $norm(H^{-1})$ which is the Euclidian norm of the matrix H^{-1} . Usually norm $(H^{-1}) >> 1$ which leads to the solution being dominated by noise. Again, this arises from the difficulties of solving H-1 with regard to the above mentioned ill conditioning.

Equation (4-2) can be solved using least square techniques. We have

$$n = g - H f (4-5)$$

Minimize the norm of n. If H is square and has full rank

$$f = (H^T H)^{-1} H^T q = H^{-1} q$$
 (4-6)

which has the same solution as the approximate inverse solution eq. (4-3).

4.1.2 Fourier Approach

The Fourier transform of eq. (4-2) gives

$$G(s) = H(s)F(s) + N(s)$$
 (4-7)

As in the matrix approach, an approximate solution of the unknown undegraded image f may be obtained by an inverse computation. By multiplying G with a filter transfer function Y(s) = 1/H(s) we obtain

$$F(s) = Y(s)G(s) = F(s) + Y(s)N(s)$$
 (4-8)

which is an approximation of the original function. In optical science literature it is common to describe the inverse filtering procedure with the true solution on the left hand side of the equation. Equation (4-7) would then form F(s)=Y(s)G(s)-Y(s)N(s) =written in the G(s)/H(s)-N(s)/H(s). For frequencies where H approaches zero the filter Y in eq. (4-8) becomes very large. usually the case for higher frequencies. Since the expected amplitude of the noise is independent of the frequency it follows that the signal dominates the spectrum at low frequencies and the noise dominates at high frequencies (Fig 4.1). That is one reason for using a filter which only restores frequencies with high S/N (Castleman 1979, Rosenfeld and Kak 1976) so that the second term in eq. (4-8) will never dominate the result. This suggests a filter similar to the filter Y but with a given cut-off frequency. On the basis of resolution it is desirable to have the cut-off frequency $f_{\mathbb{C}}$ as large as possible. On the other hand as f_c increases the filter Y yields a large additional noise. Assume that the object image f and the noise n are modelled as stationary random fields and that noise and object are uncorrelated. With a psf in the Fourier frequency domain described as linear decreasing ($sinc^2$ in the spatial domain), Frieden (1975) shows that the optimum cut-off frequency f_c that minimizes the mean square error,i.e., minimum $abs(F(s)-F(s))^2$ is

$$f_c = f_N (1 - (N(s)/F(s))^{1/2})$$

where f_N is the Nyqvist frequency. A serious problem appears if H has zeros at spatial frequencies within the range of interest. Linear uniform motion and defocusing are examples of such degradations (Sondhi, 1972). Philip (1979) proposes using the derivates of H and G in cases where H is zero. This means that if H<>0 then compute G/H else compute G'/H'. If H'=0 compute the second order derivates and so forth.

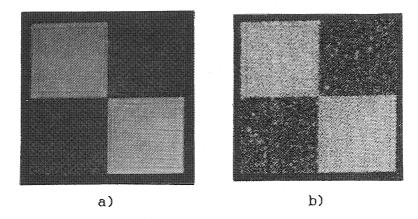


Fig 4.1 Inverse filtering. a) original image fig 3.1b b) original image fig 3.1d

4.2 Wiener Filtering

Assume that the original signal f(x), the corresponding degraded signal g(x) and the noise n(x) belong to different random fields. The degradation model can now be formulated in the following way:

$$g(x) = f(x) + n(x)$$
 (4-9)

Apply a filter function y(x) to eq. (4-9)

$$z(x) = y(x)*g(x) = y(x)*f(x) + y(x)*n(x)$$
 (4-10)

such that the output signal z(x) will be as close as possible to the original function f(x). Define the difference between z(x) and f(x) as the error signal err(x), i.e.,

$$err(x) = f(x) - z(x)$$
 (4-11)

Given the power spectrum $P_f(x)$ of the original signal f(x) and the power spectrum $P_n(x)$ of the noise n(x), determine the impulse response y(x) that minimizes the mean square error

$$MSE = E(err^2(x)) \tag{4-12}$$

where E(.) denotes the expected-value operator. Assume that the signal f(x) and the noise n(x) are uncorrelated. Then the Fourier frequency domain specification of the Wiener filter (Andrews and Hunt, 1977) is

$$Y(s) = \frac{P_f(s)}{P_f(s) + P_n(s)}$$
 (4-13)

The frequency domain expression for the mean square error is

$$MSE = \int P_n(s) Y(s) ds \qquad (4-14)$$

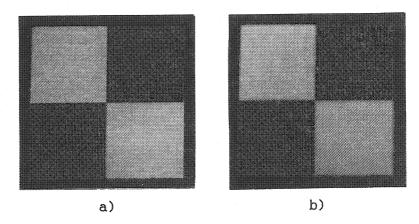


Fig 4.2 Wiener smooth filtering. a) original image plus random noise, b) Wiener smoothed image

If the signal to noise ratio is low (Castleman, 1979) eq. (4-14) reduces to the approximation of sigma i.e., MSE = $\int P_f(s)ds$ which can be approximated with $\int f^2(x)dx$. If the Wiener filter is combined with an ordinary deconvolution filter (inverse in frequency domain), the orginal signal f(x) in the degradation process, eq. (4-2), may be restored by applying the filter

$$Y(s) = \frac{H^{*}(s) P_{f}(s)}{abs(H(s))^{2} P_{f}(s) + P_{n}(s)}$$
(4-15a)

$$= \frac{1}{H(s)} \frac{abs(H(s))^{2}}{abs(H(s))^{2} + P_{n}(s)/P_{f}(s)}$$
(4-15b)

Note that, in the absence of noise ($P_{\Pi}=0$), the above expression (4-15b) reduces to the ideal filter 1/H(s). The noise-to-signal power ratio term $P_{\Pi}(s)/P_{f}(s)$ in (4-15b) may be regarded as a modification function which smoothes 1/H(s) in the presence of noise. If the statistical properties, i.e., the power spectra of noise and signal are unknown, it is common to approximate the power ratio term with a constant (Rosenfeld and Kak, 1976). Expression (4-15b) will then become

$$Y(s) = \frac{1}{H(s)} \frac{abs(F(s))^2}{abs(F(s))^2 + C}$$
 (4-16)

where C is an arbitrary constant. Wiener filtering provides an optimal method to deconvolve an image blurred by noise. However, there are problems that limit its effectiveness (Castleman, 1979). The Wiener technique requires a spatially invariant psf and also an ergodic stationary random field image model. As to the psf requirement the problem is not so serious. The second condition, however, is serious. Most images are highly nonstationary having large flat areas separated by sharp edges (Fig 4.2). In the

best case the image may be locally stationary (small fluctuations in grey-level). Furthermore, the Wiener filter requires uncorrelated signal and noise which is hardly the case for many noise sources.

4.3 Constrained Filtering

In the Wiener filter discussed in the previous section it was assumed that the undegraded image and noise belonged to two independent and separate stationary random fields and their power spectra were known. In many situations a proiri information is not aviable. Constrained least square filtering requires information about the variance of the noise only and allows the designer additional control over the restoration process (Andrews and Hunt, 1977). Consider

$$g = H f + n \tag{4-17}$$

The problem can be expressed as minimization of some linear operator ${\bf Q}$ on the object f, which is subject to some side-constraint that is known a priori or measurable a posteriori. Suppose that norm(n)², the variance of noise n, satisfies that side constraint of knowledge. Then the constrained least square problem could be formulated as

minimize
$$\operatorname{norm}(Qf)^2 = f^TQ^TQf$$

subject to $\operatorname{norm}(n)^2 = \operatorname{norm}(g - Hf)^2 = n^Tn = (q - Hf)^T(q - Hf)$

which is an optimisation problem with a side constraint. It is also a restoration process that is both deterministic and stochastic using a deterministic criterion function with a side constraint based on statistical assumptions. The solution can be obtained by the method of Lagranges multipliers,

$$U(f) = norm(Qf)^2 - L(norm(g - Hf)^2 - norm(n)^2)$$

Minimizing by taking derivatives of the objective function U with respect to f gives

$$\frac{\partial U(f)}{\partial f} = 2Q^{T}Qf + 2L(H^{T}(g - Hf)) = 0$$

Solving for that f which provides the minimum for U yields

$$\overline{f} = (H^TH + V Q^TQ)^{-1}H^Tg$$
 (4-18)

where V = 1/L. The Lagrangian multiplier V must be adjusted such that the constant norm(g - Hf)² is satisfied. This is often done in an iterative fashion (Andrews and Hunt, 1977). By assigning the linear operator Q different properties the

method allows a variety of possibilities. For instance if Q = I (unit matrix) the solution leads to a general inverse filter $(H^TH + VI)^{-1}H^T$ (Bjerhammar, 1973) called pseudoinverse. Applying the filter to the degraded image g gives the pseudoinverse restored image. If the original image f is known to be a smooth function the criterion could be to minimize the second order derivatives of f. Such a criterion would require that the object estimate does not oscillate too wildly (Andrews and Hunt, 1977).

4.4 Geometric Mean Filter

Suppose it is desirable to de-emphasize the low frequency domination of the Wiener filter while avoiding the early singularity of the inverse filter. That may be performed by a parameterization of the ratio of the inverse filter to Wiener filter (Andrews and Hunt, 1977). Using the Fourier approach one such parameterization might yield an estimate of the object as follows

$$F(s) = (N(s)^a W(s)^{1-a}) G(s)$$
 (4-19)

where $0 \le a \le 1$ and where the linear filter (N^a W^{1-a}) is composed of the inverse filter component N(s) and the Wiener filter component W(s) having the forms

$$N(s) = \frac{H(s)^*}{abs(H(s))^2}$$

and

$$W(s) = \frac{H(s)^*}{abs(H(s))^2 + V P_n(s)/P_f(s)}$$

For V=1 and with "a" varying from 0 to 1 the filter changes continuously from the original Wiener filter to an ideal inverse filter. This filter represents a general class of restoration filters applicable in cases involving linear, space invariant psf and stationary random field image models.

5. DISCUSSION

Nonstationary Image Restoration

The filters based on statistical models discussed in the previous sections all assumed a stationary random field model. For an image to be stationary, the locally computed power spectrum (the Fourier transform of the autocorrelation function) would have to be approximately equal over the entire image (Castleman, 1979). This condition of position invariant must also be fulfilled for local means. These conditions are seldom satisfied for aerial photos. Such images may be modelled as a collection of homogenous regions separated by boundaries with relatively high gradient. Many noise sources, filmgrain noise for example, cannot be

considered as stationary random fields (Castleman, 1979). Hunt (1981) proposes the following method to process non-stationary images: The image is transformed into a new image which can be described by a stationary random field model. This new image can then be processed by a stationary model algorithm and finally inverse transformed into the non-stationary mode. Even though the images seldom are stationary in a global meaning, they may be stationary in local meaning. Under this assumption Castleman (1979) proposed using the geometric mean filter. Here the power spectra of the object image (and/or the noise) are functions of the position in the image (position variant filtering).

Signal Dependent Noise

The restoration techniques presented in this paper were based on a signal independent degradation model (eq. 4-1). The signal dependence may be ignored for common restoration techniques, but for high levels of accuracy it must be taken into account. The signal dependent property may be handled using homomorphic restoration techniques (Stockham 1972, Oppenheim 1975). Consider the multiplicative noise degradation model

$$g(x) = f(x) n(x)$$
 (5-1)

where the original image f(x) is degraded by a multiplicative noise term n(x). Taking the logarithm of eq. (5-1) yields the additive result

$$\log(g(x)) = \log(f(x)) + \log(n(x)) \tag{5-2}$$

Conventional linear filtering can now be used to estimate log(f(x)) and by exponentiation we obtain the estimate of the original image f(x).

Conclusions

It is reasonable to assume, that the accuracy of matching of stereoscopic aerial photos will increase with better image resolution. Better resolution can be obtained using a suitable restoration technique. The technique should be based on a priori knowledge of the most significant degradation sources. This raises the question which degradation sources are the limiting factors in the restoration process of aerial photos. Three important degradation sources are:

- motion
- film grain noise
- atmosphere

There exist several nonrecursive and recursive methods to remove both space invariant and space variant motion blur (Sawchuk 1972, Frieden 1975, Rosenfeld and Kak 1976, Sondhi 1972, Gonzales and Wintz 1977). Recently, however, new

types of metric aerial cameras (Carl Zeiss Jena, GDR and Zeiss Oberkochen, FRG) have been developed that have built-in compensation for image motion in the flight direction. As a consequence of this, longer exposure time and finer grained film can be used, thus increasing spatial resolution. However, the problem of motion blur may still remain, the reason being that the aircraft with the equipment may be subject to angular motion, the effect of which is in a direction other than the flight direction.

As mentioned in previous chapters film grain noise is signal dependent. It is a limiting factor especially for images with high magnification. A special film grain noise restoration technique into which signal dependent properties have been incorporated has been developed by Naderi and Sawchuk (Pratt, 1978).

Typical atmospheric noise sources are turbulence and haze. In an experiment McGlamery (1967) characterized turbulence by a shift invariant psf. The result showed that a significant improvement of the degraded image can be obtained by processing the image if the optical transfer function of the turbulence is known. It is, however, impossible to know the psf for particular aerial images. In such cases the psf must be determinated a posteriori from the degraded image itself.

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