1 Degradation

$$f(x,y) * w(x,y) \stackrel{?}{=} f(x,y) * w_x(x) * w_y(y) \tag{1}$$

$$w(x,y) = w_x(x) \cdot w_y(y) \tag{2}$$

$$w_x(x) * w_y(y) = w_x(x) \cdot w_y(y) \tag{3}$$

$$w(x,y) = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \end{bmatrix} = w_x(x) \cdot w_y(y) \tag{4}$$

$$w_G(x,y) = \frac{1}{\pi * 2\sigma^2} \cdot \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right) = \underbrace{\left(\frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{x^2}{2\sigma^2}\right)\right)}_{w_x(x)} \cdot \underbrace{\left(\frac{1}{\sigma\sqrt{2\pi}} \cdot \exp\left(-\frac{y^2}{2\sigma^2}\right)\right)}_{w_y(y)} \tag{5}$$

$$10^{-3} \cdot \begin{bmatrix} 3 & 13 & 21 & 13 & 3 \\ 13 & 58 & 96 & 58 & 13 \\ 21 & 96 & 159 & 96 & 21 \\ 13 & 58 & 96 & 58 & 13 \\ 3 & 13 & 21 & 13 & 3 \end{bmatrix} \approx 10^{-6} \cdot \begin{bmatrix} 54 \\ 241 \\ 399 \\ 242 \\ 54 \end{bmatrix} \cdot \begin{bmatrix} 54 & 242 & 399 & 242 & 54 \end{bmatrix} \tag{6}$$

2 Inverse Filterung

FW-Filterung im Frequenzbereich

$$g(x,y)*f(x,y) \quad \circ\!\!-\!\!\bullet \quad G(u,y)\cdot F(u,v) \quad \Rightarrow \quad g(x,y) = IDFT\Big[DFT[f(x,y)]\cdot DFT[h(x,y)]\Big] \tag{7}$$

$$\begin{aligned} \text{diskrete Zeitsequenz} \quad &\circ\!\!-\!\!\bullet \quad \text{periodisches Spektrum} \\ \text{periodische Zeitsequenz} \quad &\circ\!\!-\!\!\bullet \quad \text{diskretes Spektrum} \end{aligned}$$

Inverse Filterung im Frequenzbereich

$$W(u,v) = \frac{1}{H(u,v)} \quad \Rightarrow \quad \hat{F}(u,v) = F(u,v)\cdot H(u,v)\cdot \frac{1}{H(u,v)} = F(u,v) \tag{8}$$

Idealisierte Lösung

$$\begin{aligned} &H_{gauss}(u,v) \\ &\qquad H_{box}(u,v) \\ &\qquad 1/H_{gauss}(u,v) \\ &\qquad 1/H_{box}(u,v) \end{aligned}$$

3 Wiener-Filter

$$J(w(x,y)) = \mathbf{E}\Big\{(f(x,y) - \hat{f}(x,y))^2\Big\} \quad \xrightarrow{\text{Parseval}} \quad J(W(u,v)) = \mathbf{E}\Big\{\big|(F(u,v) - \hat{F}(u,v)\big|^2\Big\} \tag{9}$$

$$\begin{aligned}
J(W) &= \mathbf{E}\Big\{\big|(F - \hat{F}\big|^2\Big\} \\
&= \mathbf{E}\Big\{\big|(F - G \cdot W\big|^2\Big\} \\
&\vdots \\
J(W) &= S_f + WW^\star\Big[S_f|H|^2 + S_n\Big] - S_f W H - S_f W^\star H \\
\frac{\partial J(W)}{\partial W} &= W^\star\Big[S_f|H|^2 + S_n\Big] - S_f H \quad \stackrel{!}{=} 0 \\
&\Updownarrow \\
W^\star &= \frac{S_f H}{S_f|H|^2 + S_n} = \frac{H}{|H|^2 + \frac{S_n}{S_f}} \quad \Leftrightarrow \quad W = \frac{H^\star}{|H|^2 + \frac{S_n}{S_f}} = \frac{1}{H} \cdot \frac{|H|^2}{|H|^2 + \frac{S_n}{S_f}}
\end{aligned}$$

$$W = \frac{1}{H} \cdot \frac{|H|^2}{|H|^2 + K} \tag{10}$$