Target Detection System
Programming assignment - 2

November 12, 2017

Abstract

Design a target detection system for Radar or Sonar in the presence of additive white Gaussian Noise when the signal is a random vector of size N. Evaluate the performance of the system.

1 Introduction

To detect an object by sonar in water or by radar in air, a pulse is transmitted and one looks for a reflected pulse from the object. If there is no target, there will be no reflection, and so the received samples are due to noise only. If a target is present, the received samples would consist of signal added to noise.

It is assumed that the noise signal (W) has a normal distribution $\mathcal{N}(0, \sigma^2)$. Suppose that N samples of signal $X_1, X_2, \ldots, X_n$ are sent, we define two hypothesis, H_W (when target is present) and H_{S+W} (when target is absent)

$$H_W : X_i = W_i \qquad i = 1, 2, \ldots, N$$

$$H_{s+W} : X_i = s_i + W_i \quad i = 1, 2, \ldots, N$$

If no target is present, $X \sim \mathcal{N}(0, \sigma^2)$ and if target is present, $X \sim \mathcal{N}(A, \sigma^2)$ .Given that target is present, a signal is detected if

$$p_X(x; H_{s+W}) > p_X(x; H_W)$$

If a DC signal is sent, $s_i = A$ for $i = 1, 2, \ldots, N$, $A > 0$. Then the following must be satisfied if a target is present

$$\frac{1}{N}\sum_i^n x_i > \frac{1}{2}A$$

performance of the system can be determined for a given A and N by $p_X(X > A/2; H_{s+W})$ which is given by

$$Q\left(\frac{-\sqrt{N}A}{2\sqrt{\sigma^2}}\right) = \int_{\frac{-\sqrt{N}A}{2\sqrt{\sigma^2}}}^{\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

2 Algorithmns

2.1 Estimating performance with different number of samples with fixed amplitude

1. Let the amplitude of DC signal be A and the noise signal (W) has a normal distribution $\mathcal{N}(0, \sigma^2)$.
2. The number of samples taken varies from 1 to 100 (run a loop in N from 1 to 100).
3. For a particular N, in case a target is present sample N points from $X \sim \mathcal{N}(A, \sigma^2)$ using normrnd function.
4. Calculate $Average = \frac{1}{N}\sum_i^n x_i$. If $Average > \frac{1}{2}A$, then a target is present.
5. In case target is absent, sample N points from $X \sim \mathcal{N}(0, \sigma^2)$ using and if $Average <= \frac{1}{2}A$, target is absent.
6. The performance is calculated by cumulative probability $p_X(X > A/2; H_{s+W}) = Q\left(\frac{-\sqrt{N}A}{2\sqrt{\sigma^2}}\right)$.

2.2 Estimating performance with different amplitude and with fixed number of samples

1. Let the number of samples be N (fixed) and the noise signal (W) has a normal distribution $\mathcal{N}(0,\, \sigma^2)$.
2. The amplitude of signal, A varies from 0.5 to 20 (run a loop in N from 1 to 20 in steps of 0.5).
3. For a particular A, in case a target is present sample N points from $X \sim \mathcal{N}(A, \sigma^2)$ using normrnd function.
4. Calculate $Average = \frac{1}{N}\sum_i^n x_i$. If $Average > \frac{1}{2}A$, then a target is present.
5. In case target is absent, sample N points from $X \sim \mathcal{N}(0, \sigma^2)$ using and if $Average <= \frac{1}{2}A$, target is absent.
6. The performance is calculated by cumulative probability $p_X(X > A/2; H_{s+W}) = Q\left(\frac{-\sqrt{N}A}{2\sqrt{\sigma^2}}\right)$.

3 Implementation details

1. Estimating performance with different number of samples with fixed amplitude
   - The amplitude A is taken 1 and σ to be 1 and N is varied from 1 to 100.
2. Estimating performance with different amplitude and with fixed number of samples
   - The number of samples taken is 50 and σ to be 1 and the amplitude, A is varied from 0.5 to 20 in steps of 0.5.

4 Results

4.1 Estimating performance with different number of samples with fixed amplitude

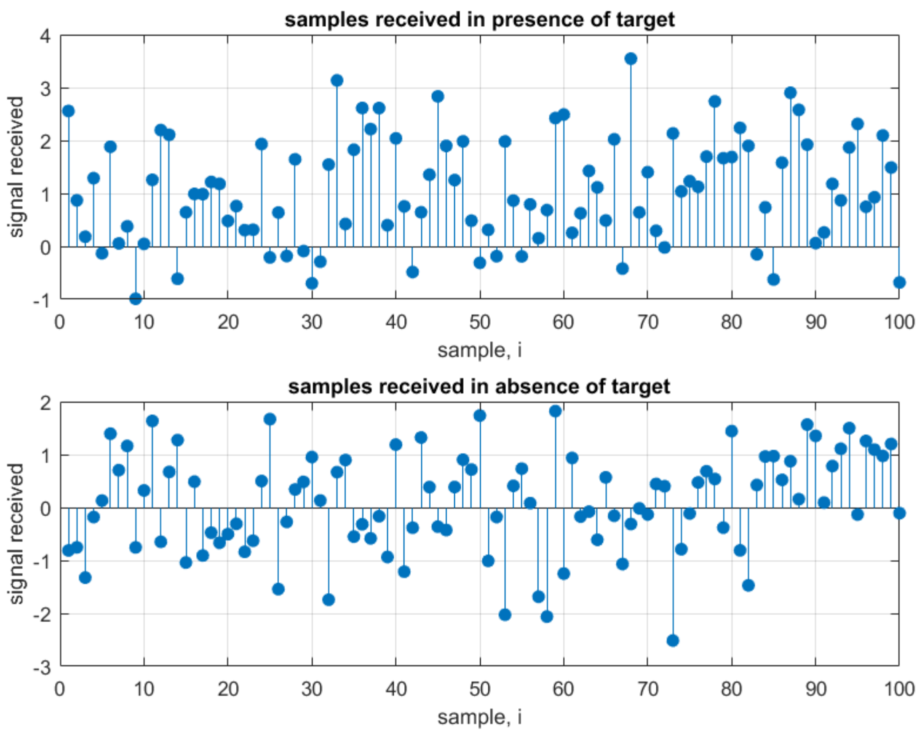

figure 1.1: samples received in presence and absence of target

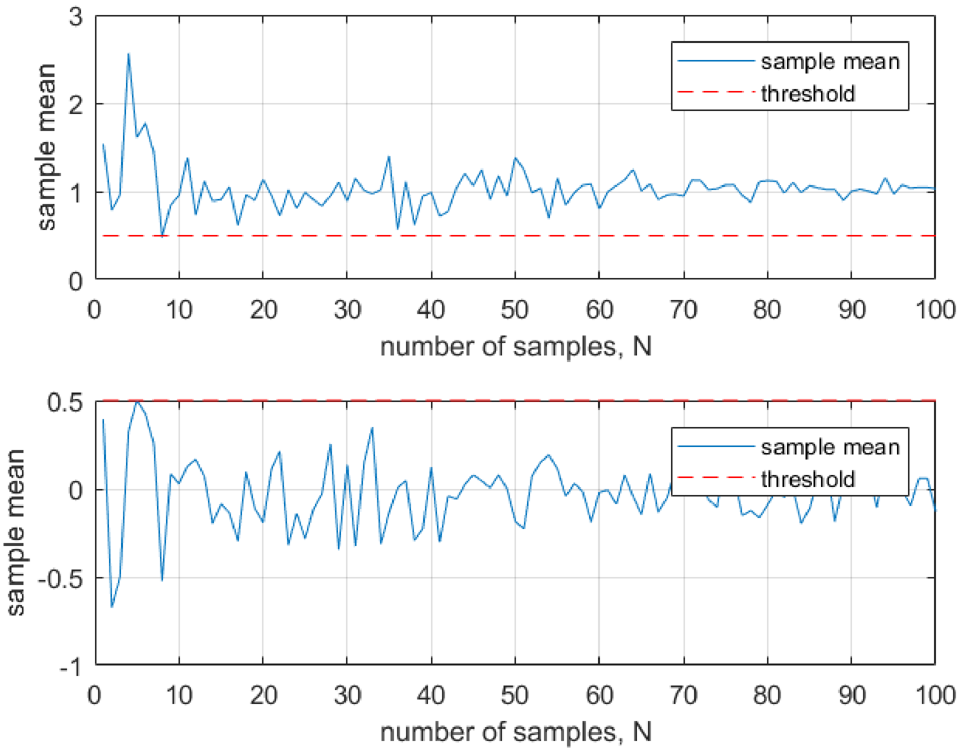

figure 1.2: sample mean X vs N for samples received in presence and absence of target

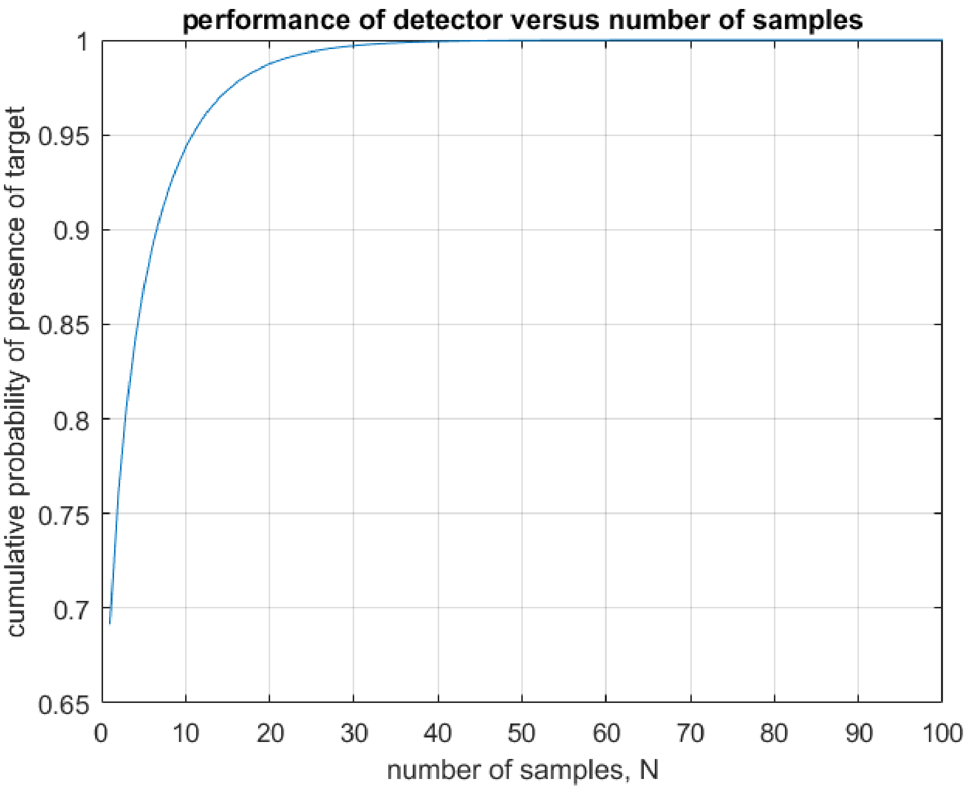

figure 14: performance of detector versus number of samples

- from figure 1.2, we observe that as the number of samples increase, the sample mean tends towards the expected value which is 1 in case the target is present and 0 in case it is absent.
- from figure 1.3, we observe the probability of detection increases with the number of samples taken and almost becomes 1 after taking 40 samples.

4.2 Estimating performance with different amplitude and with fixed number of samples

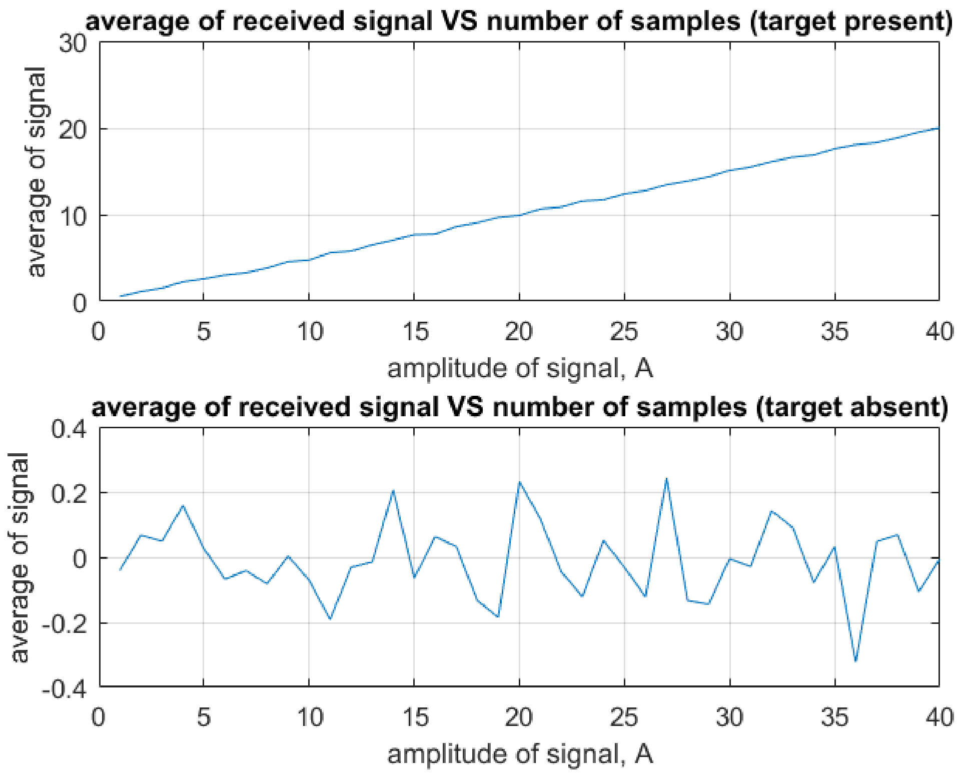

figure 2.1: average of received signal VS number of samples in presence and absence of target

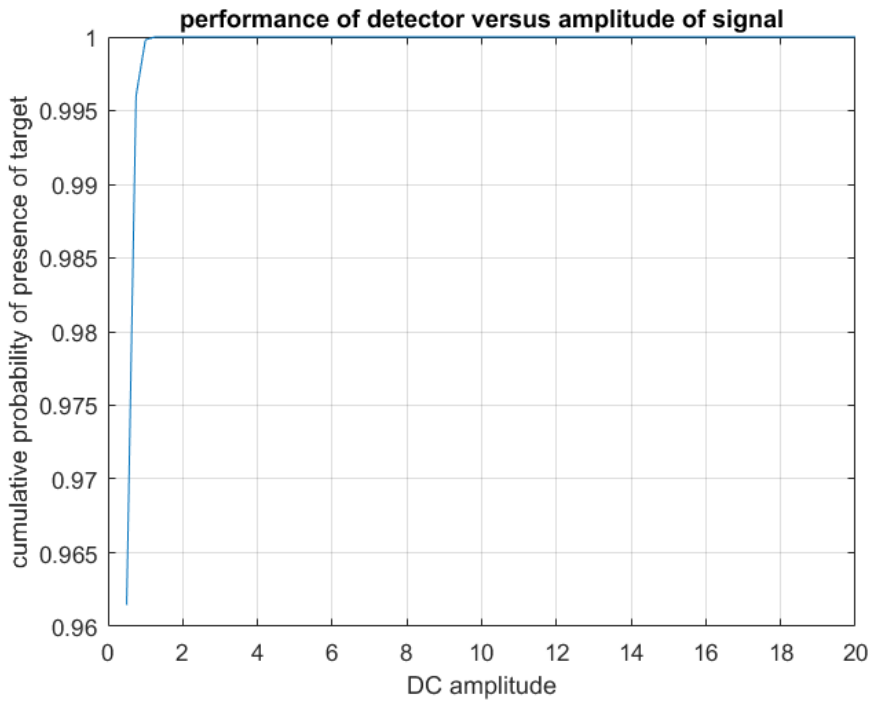

figure 2.2: performance of detector versus amplitude of signal

- From figure 2.1, we observe that the average value of signal is nearly equal to the expectation value in both cases. It happens because we are taking 50 samples for testing.
- from figure 2.2, the probability of detection is almost one after taking 1V amplitude.

References

[1] Steven M. Kay, *Intuitive Probability And Random Processes.*

[2] www.mathworks.com