1. Find the supremum and the infimum of the following sets:

    (a) $\left\{ 1 - \dfrac{(-1)^n}{n} : n \in \mathbb{N} \right\}$

    (b) $\left\{ 1 - \dfrac{1}{n} : n \in \mathbb{N} \right\}$

    (c) $\left\{ \dfrac{1}{n} - \dfrac{1}{m} : n, m \in \mathbb{N} \right\}$

    (d) $\left\{ \dfrac{m}{m+n} : m, n \in \mathbb{N} \right\}$

2. Let A and B be two bounded subsets of $\mathbb{R}$. The sum of A and B is defined by

$$A + B = \{ a + b : a \in A, b \in B \}.$$

    Show that

    (a) $\inf(A + B) = \inf(A) + \inf(B)$
    (b) $\sup(A + B) = \sup(A) + \sup(B)$

3. Let A be a nonempty bounded subset of $\mathbb{R}$. For a real number x, we define

$$xA = \{ xa : a \in A \}.$$

    Prove that:

    (a) If $x > 0$, then $\inf(xA) = x \cdot \inf(A)$ and $\sup(xA) = x \cdot \sup(A)$.
    (b) If $x < 0$, then $\inf(xA) = x \cdot \sup(A)$ and $\sup(xA) = x \cdot \inf(A)$.

4. Let q_1 and q_2 be two distinct real numbers. Then show that there exists an irrational number between them.

5. Use the Archimedean property to show that

$$\bigcap_{n \in \mathbb{N}} \left(-\frac{1}{n}, \frac{1}{n} \right) = \{0\}.$$