Answers of problems on Problem Set 09

1. Taking $\varepsilon = 0.001$,

$$
\begin{aligned}
&P\left(X \leq x | 5-\varepsilon < Y \leq 5+\varepsilon\right)\\
&= \begin{cases}
0 & \text{if } x \leq 0\\
\dfrac{15}{A}\displaystyle\int_0^x \int_{5-\varepsilon}^{5+\varepsilon} e^{-(2t+3s)} ds dt & \text{if } 0 < x < 5-\varepsilon\\
\dfrac{15}{A}\left[\displaystyle\int_0^{5-\varepsilon} \int_{5-\varepsilon}^{5+\varepsilon} e^{-(2t+3s)} ds dt + \int_{5-\varepsilon}^{x} \int_{x}^{5+\varepsilon} e^{-(2t+3s)} ds dt\right] & \text{if } 5-\varepsilon < x < 5+\varepsilon\\
\dfrac{15}{A}\displaystyle\int_{5-\varepsilon}^{5+\varepsilon} \int_0^{s} e^{-(2t+3s)} dt ds & \text{if } x > 5+\varepsilon,
\end{cases}
\end{aligned}
$$

where $A = P\left(5-\varepsilon < Y \leq 5+\varepsilon\right) = 15\displaystyle\int_{5-\varepsilon}^{5+\varepsilon} \int_0^{y} e^{-(2x+3y)} dx dy$.

2. The conditional PMF of Y given $X = 5$ is

$$
f_{Y|X}(y|5) = \begin{cases}
\frac{2}{21} & \text{if } y = 1\\
\frac{2}{7} & \text{if } y = 2\\
\frac{1}{2} & \text{if } y = 3\\
\frac{5}{42} & \text{if } y = 4\\
0 & \text{otherwise.}
\end{cases}
$$

3. For $y = 0,\, 1,\, \ldots$,

$$
f_{X|Y}(k|y) = \begin{cases}
\dfrac{(1-p)^{k-y}\lambda^{k-y}e^{-\lambda(1-p)}}{(k-y)!} & \text{if } k = y,\, y+1,\, \ldots\\
0 & \text{otherwise.}
\end{cases}
$$

4. For $x = 0,\, 1,\, 2,\, \ldots$, the CPMF of Y given $X = x$ is

$$
f_{Y|X}(y|x) = \begin{cases}
\dbinom{x+y+k-1}{x+k-1}\theta_2^y\left(1-\theta_2\right)^{x+k} & \text{if } y = 0,\, 1,\, 2,\, \ldots\\
0 & \text{otherwise.}
\end{cases}
$$

For $y = 0,\, 1,\, 2,\, \ldots$, the CPMF of X given $Y = y$ is

$$
f_{X|Y}(x|y) = \begin{cases}
\dbinom{x+y+k-1}{y+k-1}\theta_1^x\left(1-\theta_1\right)^{y+k} & \text{if } x = 0,\, 1,\, 2,\, \ldots\\
0 & \text{otherwise.}
\end{cases}
$$

5. $P\left(X_1 + X_2 \geq 1\right) = 1 - \ln 2$ and $E\left(X_1 | X_2 = x_2\right) = \frac{x_2 - 1}{\ln x_2}$.

6. The CPDFs are

$$f_{X|Y}(x|y) = \begin{cases} \dfrac{\Gamma(\theta_1+\theta_3)}{\Gamma(\theta_1)\Gamma(\theta_3)} \dfrac{x^{\theta_1-1}(1-x-y)^{\theta_3-1}}{(1-y)^{\theta_1+\theta_3-1}} & \text{if } 0 < x < 1-y \\ 0 & \text{otherwise.} \end{cases}$$

$$f_{Y|X}(y|x) = \begin{cases} \dfrac{\Gamma(\theta_2+\theta_3)}{\Gamma(\theta_2)\Gamma(\theta_3)} \dfrac{y^{\theta_2-1}(1-x-y)^{\theta_3-1}}{(1-x)^{\theta_2+\theta_3-1}} & \text{if } 0 < y < 1-x \\ 0 & \text{otherwise.} \end{cases}$$

7. Expected number of injuries in a week is 8.

8. $\frac{n-2}{n-1}$.

9. 10.

10. $\dfrac{1-p^k}{p^k(1-p)}$.

11. $\dfrac{n+1}{2^{n+2}}$.

12. For $n = 0, 1, \ldots$ and $m = 0, 1, \ldots$, $P(N = n, M = m) = \dfrac{e^{\lambda p}(\lambda p)^n}{n!} \times \dfrac{e^{\lambda(1-p)}(\lambda(1-p))^m}{m!}$.