1. Consider the recursion:

$$U_{i+1} = (U_{i-17} - U_{i-5}).$$

   In the event that $U_i < 0$, set $U_i = U_i + 1$.

   (a) Use linear congruence generator to generate the first 17 values of U_i.

   (b) Then generate the values of $U_{18}, U_{19}, \ldots, U_N$ for $N = 1000$, 10000, and 100000 based on the recursion above.

   (c) For each N, plot histogram. What are your observations?

   (d) For each N, plot (U_i, U_{i+1}). What are your observations?

2. Consider the exponential distribution with CDF

$$F(x) = 1 - e^{-x/\theta}\ ,\ x \geq 0,$$

   where $\theta > 0$.

   (a) Generate $X_1, X_2, \ldots, X_N$ from the above distribution for $N = 10$, 100, 1000, 10000, 100000.

   (b) For each value of N, plot the empirical CDF (definition is given below) of these generated values, and the actual distribution function (using the above formula).
   Defn: Let $x_1, x_2, \ldots, x_N$ be sample observations. Then the empirical CDF at $x \in \mathbb{R}$ is defined by

$$F_N(x) = \frac{\#\{i : x_i \leq x\}}{N}.$$

   (c) Provide the corresponding values of the sample mean and variance. Compare the values of mean and variance to see whether they converge to actual values.

3. Consider the Arcsin law with the distribution:

$$F(x) = \frac{2}{\pi} \text{ arc sin } \sqrt{x}\ ,\ 0 \leq x \leq 1.$$

   (a) Generate $X_1, X_2, \ldots, X_N$ from the above distribution for $N = 10$, 100, 1000, 10000, 100000.

   (b) For each value of N, plot the empirical CDF of these generated values, and the actual distribution function (using the above formula).

   (c) Provide the corresponding values of the sample mean and variance.

4. Using the algorithm to generate random variables from a discrete distribution, generate 100000 random numbers from a discrete uniform distribution on $\{1, 3, 5, \ldots, 9999\}$. Tabulate the frequency of each observed values.

Submission Deadline: August 16, 2022, 11:50 AM