

Department of Mathematics  
Indian Institute of Technology Guwahati  
**MA 101: Mathematics I**  
**Tutorial Sheet-1**  
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1. Find the supremum and the infimum of the following sets:

(a)  $\left\{ 1 - \frac{(-1)^n}{n} : n \in \mathbb{N} \right\}$

(b)  $\left\{ 1 - \frac{1}{n} : n \in \mathbb{N} \right\}$

(c)  $\left\{ \frac{1}{n} - \frac{1}{m} : n, m \in \mathbb{N} \right\}$

(d)  $\left\{ \frac{m}{m+n} : m, n \in \mathbb{N} \right\}$

2. Let  $A$  and  $B$  be two bounded subsets of  $\mathbb{R}$ . The sum of  $A$  and  $B$  is defined by

$$A + B = \{a + b : a \in A, b \in B\}.$$

Show that

(a)  $\inf(A + B) = \inf(A) + \inf(B)$

(b)  $\sup(A + B) = \sup(A) + \sup(B)$

3. Let  $A$  be a nonempty bounded subset of  $\mathbb{R}$ . For a real number  $x$ , we define

$$xA = \{xa : a \in A\}.$$

Prove that:

(a) If  $x > 0$ , then  $\inf(xA) = x \cdot \inf(A)$  and  $\sup(xA) = x \cdot \sup(A)$ .

(b) If  $x < 0$ , then  $\inf(xA) = x \cdot \sup(A)$  and  $\sup(xA) = x \cdot \inf(A)$ .

4. Let  $q_1$  and  $q_2$  be two distinct real numbers. Then show that there exists an irrational number between them.
5. Use the Archimedean property to show that

$$\bigcap_{n \in \mathbb{N}} \left( -\frac{1}{n}, \frac{1}{n} \right) = \{0\}.$$