Indian Institute of Technology Guwahati
Probability Theory (MA590)
Problem Set 03

1. You roll a fair four-sided die. If the result is 1 or 2, you roll once more but otherwise, you stop. What is the probability that the sum total of your rolls is at least 4? (Ans: 9/16.)

2. A student is taking a probability course and at the end of each week, she can be either up-to-date or she may have fallen behind. If she is up-to-date in a given week, the probability that she will be up-to-date in the next week is 0.8. If she is behind in a week, the probability that she will be up-to-date in the next week is 0.4. She is up-to-date when she starts the class. Find the probability that she is up-to-date after three weeks. (Ans: 0.688.)

3. (The Monty Hall problem) Suppose you're on a game show, and you're given the choice of three doors: Behind one door is a car; behind the others are goats. You pick a door, say No. 1, and the host, who knows what's behind the doors, opens another door, which has a goat. He then asks to you, "Do you want to pick the other closed door?" What should be your answer? (Ans: Yes, as the probability of wining the car is $\dfrac{2}{3}$ if I pick the other closed door.)

4. Consider four coding machines M_1, M_2, M_3, and M_4 producing binary codes 0 and 1. The machine M_1 produces codes 0 and 1 with respective probabilities $\frac{1}{4}$ and $\frac{3}{4}$. The code produced by machine M_k is fed into machine M_{k+1}, ($k = 1, 2, 3$), which may either leave the received code unchanged or may change it. Suppose that each of the machines M_2, M_3, and M_4 change the code with probability $\frac{3}{4}$. Given that the machine M_4 has produced code 1, find the conditional probability that the machine M_1 produced code 0. (Ans: 3/10.)

5. A locality has n houses numbered $1, \ldots, n$ and a terrorist is hiding in one of these houses. Let H_j denote the event that the terrorist is hiding in house numbered j, $j = 1, \ldots, n$, and let $P(H_j) = p_j \in (0, 1)$, $j = 1, \ldots, n$. During a search operation, let F_j denote the event that search of the house number j will fail to nab the terrorist there and let $P(F_j|H_j) = r_j \in (0, 1)$, $j = 1, \ldots, n$. For each $i, j \in \{1, \ldots, n\}$, $i \neq j$, show that H_j and F_j are negatively associated but H_i and F_j are positively associated. (Ans: $P(H_j|F_j) = \dfrac{r_j p_j}{1 - p_j + r_j p_j}$, and $P(H_i|F_j) = \dfrac{p_i}{1 - p_j + r_j p_j}$.)

6. A laboratory blood test is 95% effective in detecting a certain disease when it is, in fact, present. However, the test also yields a "false positive" result for 1% of the healthy persons tested. If 0.5% of the population actually has the disease, what is the probability a person has the disease given his test result is positive? (Ans: 95/294)

7. If an aircraft is present in a certain area, a radar detects it and generates an alarm signal with probability 0.99. If an aircraft is not present, the radar generates a (false) alarm, with probability 0.10. We assume that an aircraft is present with probability 0.05. What is the probability of no aircraft presence and a false alarm? What is the probability of aircraft presence and no detection? If the radar generates a alarm, what is the probability of the presence of an aircraft?

8. (The False-Positive Puzzle) A test for a certain rare disease is assumed to be correct 95% of the time. A random person drawn from a certain population has probability 0.001 of having the disease. Given that the person just tested positive, what is the probability of having the disease?