Department of Mathematics
Indian Institute of Technology Guwahati

MA 101: Mathematics I

Tutorial Sheet-3

July-November 2023

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1. If $x_n = (-1)^n n^2$ for all $n \in \mathbb{N}$, then examine whether the sequence (x_n) has a convergent subsequence?

2. If $x_n = (-1)^n \frac{5n \sin^3 n}{3n-2}$ for all $n \in \mathbb{N}$, then examine whether the sequence (x_n) has a convergent subsequence.

3. Let $a_1 = 1$ and $a_{n+1} = \left(1 + \frac{(-1)^n}{2^n}\right) a_n$ for all $n \in \mathbb{N}$. Prove that (a_n) is a Cauchy sequence.

4. Let $x_1 = 1$ and let $x_{n+1} = \frac{1}{x_n+2}$ for all $n \in \mathbb{N}$. Prove that (x_n) is Cauchy and $\lim\limits_{n\to\infty} x_n = \sqrt{2} - 1$.

5. Given $a, b \in \mathbb{R}$, let $x_1 = a, x_2 = b$ and $x_n = \dfrac{1}{2}(x_{n-1} + x_{n-2})$ for $n \geq 3$. Show that (x_n) is a Cauchy sequence and $\lim x_n = \dfrac{1}{3}(a + 2b)$.

6. Let $x_n = (-1)^n \left(1 + \frac{1}{n}\right)$, $n \geq 1$. Find $\limsup x_n$ and $\liminf x_n$.

7. Let $x_n = (1 + 1/n)^n$ and $y_n = \displaystyle\sum_{k=0}^{n} \frac{1}{k!}$. Prove that $\lim x_n = \lim y_n$.

8. Examine whether the series $\sum\limits_{n=1}^{\infty} \frac{1}{n^{1+\frac{1}{n}}}$ is convergent.

9. Examine whether the following series are convergent.

   (a) $\sum\limits_{n=1}^{\infty} \frac{n!}{n^n}$

   (b) $\sum\limits_{n=1}^{\infty} \frac{1}{n} \sin \frac{1}{n}$

10. Let $x_n > 0$ for all $n \in \mathbb{N}$. Show that the series $\sum\limits_{n=1}^{\infty} x_n$ converges iff the series $\sum\limits_{n=1}^{\infty} \frac{x_n}{1+x_n}$ converges.