**Answers of problems on Problem Set 08**

1. The PDF of $\sqrt{X_1^2+X_2^2}$ is

$$f(u)=\begin{cases}2u^3e^{-u^2} & \text{if } u>0\\ 0 & \text{otherwise.}\end{cases}$$

2. The marginal PDFs of $Y_1$, $Y_2$, and $Y_3$, respectively, are

$$f_{Y_1}(y)=\begin{cases}2y & \text{if } 0<y<1\\ 0 & \text{otherwise,}\end{cases}\quad f_{Y_2}(y)=\begin{cases}4y^3 & \text{if } 0<y<1\\ 0 & \text{otherwise,}\end{cases}\quad f_{Y_3}(y)=\begin{cases}6y^5 & \text{if } 0<y<1\\ 0 & \text{otherwise.}\end{cases}$$

3. The JPDF of $Y_1,\ Y_2,$ and $Y_3$ is

$$f(y_1,\,y_2,\,y_3)=\begin{cases}y_3^2e^{-y_3} & \text{if } y_1>0,\ y_2>0,\ y_3>0,\ y_1+y_2<1\\ 0 & \text{otherwise.}\end{cases}$$

The marginal PDFs are

$$f_{Y_1}(y)=f_{Y_2}(y)=\begin{cases}2(1-y) & \text{if } 0<y<1\\ 0 & \text{otherwise,}\end{cases}\qquad f_{Y_3}(y)=\begin{cases}\frac{1}{2}y^2e^{-y} & \text{if } y>0\\ 0 & \text{otherwise.}\end{cases}$$

4. The JPDF of $Y_1,\ Y_2,$ and $Y_3$ is

$$f(y_1,\,y_2,\,y_3)=\begin{cases}y_3^2e^{-y_3} & \text{if } 0<y_1<y_2<1,\ y_3>0\\ 0 & \text{otherwise.}\end{cases}$$

The marginal PDFs are

$$f_{Y_1}(y)=\begin{cases}2(1-y) & \text{if } 0<y<1\\ 0 & \text{otherwise,}\end{cases}\quad f_{Y_2}(y)=\begin{cases}2y & \text{if } 0<y<1\\ 0 & \text{otherwise,}\end{cases}\quad f_{Y_3}(y)=\begin{cases}\frac{1}{2}y^2e^{-y} & \text{if } y>0\\ 0 & \text{otherwise.}\end{cases}$$

5. The PDF of $\frac{X}{X+Y}$ is

$$f(u)=\begin{cases}\frac{1}{B(\alpha_1,\alpha_2)}u^{\alpha_1-1}(1-u)^{\alpha_2-1} & \text{if } 0<u<1\\ 0 & \text{otherwise.}\end{cases}$$

6. The CDF of $X$ is

$$F(x)=\begin{cases}0 & \text{if } x<0\\ \frac{e^{-\lambda}\lambda^{[x]}}{[x]!}(x-[x])+\sum_{k=0}^{[x]-1}\frac{e^{-\lambda}\lambda^k}{k!} & \text{if } x\geq 0.\end{cases}$$

The PDF of $X$ is

$$f(x) = \begin{cases} \frac{e^{-\lambda}\lambda^{[x]}}{[x]!} & \text{if } x > 0 \\ 0 & \text{otherwise.} \end{cases}$$

7. The JCDF of $X_{(1)}$ and $X_{(n)}$ is

$$F_{X_{(1)},\,X_{(n)}}(x,\,y) = \begin{cases} 0 & \text{if } x < 0 \text{ or } y < 0 \\ y^n - (y-x)^n & \text{if } 0 < x < y < 1 \\ 1 - (1-x)^n & \text{if } 0 < x < 1 < y \\ y^n & \text{if } 0 < y < 1,\, x > y \\ 1 & \text{if } x > 1,\, y > 1. \end{cases}$$

The JPDF is

$$f_{X_{(1)},\,X_{(n)}}(x,\,y) = \begin{cases} n(n-1)(y-x)^2 & \text{if } 0 < x < y < 1 \\ 0 & \text{otherwise.} \end{cases}$$

The marginal PDFs are

$$f_{X_{(1)}}(x) = \begin{cases} n(1-x)^{n-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise,} \end{cases} \qquad f_{X_{(n)}}(x) = \begin{cases} ny^{n-1} & \text{if } 0 < x < 1 \\ 0 & \text{otherwise.} \end{cases}$$

8. The PMF of $X_{(2)}$ is

$$f(k) = \begin{cases} \frac{e^{-2\lambda}\lambda^{2k}}{(k!)^2} + \frac{2e^{-\lambda}\lambda^k}{k!}\sum_{i=0}^{k-1}\frac{e^{-\lambda}\lambda^i}{i!} & \text{if } k = 0,\,1,\,2,\,\ldots \\ 0 & \text{otherwise.} \end{cases}$$

9. Perform the calculations

10.

$$P\left(X_1 = k\right) = \begin{cases} \binom{2}{k}\left(\frac{1}{4}\right)^k\left(\frac{3}{4}\right)^{2-k} & \text{if } k = 0,\,1,\,2 \\ 0 & \text{otherwise.} \end{cases}$$

$$P\left(Y = k\right) = \begin{cases} \binom{6}{k}\left(\frac{1}{4}\right)^k\left(\frac{3}{4}\right)^{6-k} & \text{if } k = 0,\,1,\,2,\,\ldots,\,6 \\ 0 & \text{otherwise.} \end{cases}$$