$$\frac{d}{dx}\sqrt{(x+6)^2+25}$$

$$=\frac{d}{dx}\left((x+6)^2+25\right)^{1/2}$$

$$=\frac{1}{2}\left[(x+6)^2+25\right)^{1/2-1}\cdot\frac{d}{dx}\left[(x+6)^2+25\right]$$

$$=\frac{\frac{d}{dx}(x+6)^2+\frac{d}{dx}(25)}{2\sqrt{(x+6)^2+25}}$$

$$=\frac{\left[2(x+6)\cdot\frac{d}{dx}(x+6)\right]+0}{2\sqrt{(x+6)^2+25}}$$

$$=\frac{2(x+6)\cdot\left[\frac{d}{dx}(x)+\frac{d}{dx}(6)\right]}{2\sqrt{(x+6)^2+25}}$$

$$=\frac{2(x+6)\cdot(1+0)}{2\sqrt{(x+6)^2+25}}$$

$$=\frac{2(x+6)}{2\sqrt{(x+6)^2+25}}$$

$$=\frac{x+6}{\sqrt{(x+6)^2+25}}$$

$$\frac{d}{dx}\left[\frac{x+6}{\sqrt{(x+6)^2+25}}\right]$$

$$= \frac{\frac{d}{dx}[x+6]\cdot\sqrt{(x+6)^2+25}-(x+6)\cdot\frac{d}{dx}\left[\sqrt{(x+6)^2+25}\right]}{\left(\sqrt{(x+6)^2+25}\right)^2}$$

$$= \frac{\left(\frac{d}{dx}[x]+\frac{d}{dx}[6]\right)\sqrt{(x+6)^2+25}-(x+6)\cdot\frac{1}{2}\left((x+6)^2+25\right)^{\frac{1}{2}-1}\cdot\frac{d}{dx}\left[(x+6)^2+25\right]}{(x+6)^2+25}$$

$$= \frac{(1+0)\sqrt{(x+6)^2+25}-\frac{(x+6)\left(\frac{d}{dx}\left[(x+6)^2\right]+\frac{d}{dx}[25]\right)}{2\sqrt{(x+6)^2+25}}}{(x+6)^2+25}$$

$$= \frac{\sqrt{(x+6)^2+25}-\frac{(x+6)\left(2(x+6)\cdot\frac{d}{dx}[x+6]+0\right)}{\sqrt{(x+6)^2+25}}}{(x+6)^2+25}$$

$$= \frac{\sqrt{(x+6)^2+25}-\left(\frac{\left(\frac{d}{dx}[x]+\frac{d}{dx}[6]\right)}{\sqrt{(x+6)^2+25}}\right)(x+6)^2}{(x+6)^2+25} = \frac{\sqrt{(x+6)^2+25}-\frac{(1+0)(x+6)^2}{\sqrt{(x+6)^2+25}}}{(x+6)^2+25}$$

$$= \frac{\sqrt{(x+6)^2+25}-\frac{(x+6)^2}{\sqrt{(x+6)^2+25}}}{(x+6)^2+25}$$

$$= \frac{1}{\sqrt{(x+6)^2+25}}-\frac{(x+6)^2}{\sqrt{\left((x+6)^2+25\right)^3}}$$

$$= \frac{25}{\left((x+6)^2+25\right)^{3/2}}$$

$$\frac{d}{dx}\sqrt{(x-6)^2+121}$$

$$=\frac{d}{dx}\left((x-6)^2+121\right)^{1/2}$$

$$=\frac{1}{2}\left[(x-6)^2+121\right]^{1/2-1}\cdot\frac{d}{dx}\left[(x-6)^2+121\right]$$

$$=\frac{\frac{d}{dx}(x-6)^2+\frac{d}{dx}(121)}{2\sqrt{(x-6)^2+121}}$$

$$=\frac{\left[2(x-6)\cdot\frac{d}{dx}(x-6)\right]+0}{2\sqrt{(x-6)^2+121}}$$

$$=\frac{2(x-6)\cdot\frac{d}{dx}(x-6)}{2\sqrt{(x-6)^2+121}}$$

$$=\frac{2(x-6)\cdot\left[\frac{d}{dx}(x)-\frac{d}{dx}(6)\right]}{2\sqrt{(x-6)^2+121}}$$

$$=\frac{2(x-6)\cdot(1-0)}{2\sqrt{(x-6)^2+121}}$$

$$=\frac{2(x-6)}{2\sqrt{(x-6)^2+121}}$$

$$=\frac{x-6}{\sqrt{(x-6)^2+121}}$$

$$\frac{d}{dx}\left[\frac{x-6}{\sqrt{(x+6)^2+121}}\right]$$

$$= \frac{\frac{d}{dx}[x-6]\cdot\sqrt{(x-6)^2+121} - (x-6)\cdot\frac{d}{dx}\left[\sqrt{(x-6)^2+121}\right]}{\left(\sqrt{(x-6)^2+121}\right)^2}$$

$$= \frac{\left(\frac{d}{dx}[x]+\frac{d}{dx}[-6]\right)\sqrt{(x-6)^2+121} - (x-6)\cdot\frac{1}{2}\left((x-6)^2+121\right)^{\frac{1}{2}-1}\cdot\frac{d}{dx}\left[(x-6)^2+121\right]}{(x-6)^2+121}$$

$$= \frac{(1+0)\sqrt{(x-6)^2+121} - \frac{(x-6)\left(\frac{d}{dx}[(x-6)^2]+\frac{d}{dx}[121]\right)}{2\sqrt{(x-6)^2+121}}}{(x+6)^2+121}$$

$$= \frac{\sqrt{(x-6)^2+121} - \frac{(x-6)\left(2(x-6)\cdot\frac{d}{dx}[x+6]+0\right)}{2\sqrt{(x+6)^2+121}}}{(x+6)^2+121}$$

$$= \frac{\sqrt{(x+6)^2+121} - \frac{\left(\frac{d}{dx}[x]+\frac{d}{dx}[-6]\right)(x-6)^2}{\sqrt{(x+6)^2+121}}}{(x+6)^2+121}$$

$$= \frac{\sqrt{(x+6)^2+121} - \frac{(1+0)(x-6)^2}{\sqrt{(x-6)^2+121}}}{(x+6)^2+121}$$

$$= \frac{\sqrt{(x-6)^2+121} - \frac{(x+6)^2}{\sqrt{(x-6)^2+121}}}{\sqrt{(x-6)^2+121}}$$

$$= \frac{1}{\sqrt{(x-6)^2+121}} - \frac{(x-6)^2}{\left((x-6)^2+121\right)^{3/2}}$$

$$= \frac{121}{\left((x-6)^2+121\right)^{3/2}}$$

$$y = \sqrt{(x+6)^2+25} + \sqrt{(x-6)^2+121}$$

$$y' = \frac{dy}{dx} = \frac{d}{dx}\left(\sqrt{(x+6)^2+25}\right) + \frac{d}{dx}\left(\sqrt{(x-6)^2+121}\right)$$

$$y' = \frac{dy}{dx} = \frac{x+6}{\sqrt{(x+6)^2+25}} + \frac{x-6}{\sqrt{(x-6)^2+121}}$$

Equating y' to zero.

$$\frac{x+6}{\sqrt{(x+6)^2+25}} + \frac{x-6}{\sqrt{(x-6)^2+121}} = 0$$

Square both sides.

$$\frac{(x+6)^2}{\left(\sqrt{(x+6)^2+25}\right)^2} = \frac{(x-6)^2}{\left(\sqrt{(x-6)^2+121}\right)^2}$$

$$(x+6)^2(x-6)^2 + 121(x+6)^2 = (x+6)^2(x-6)^2 + 25(x-6)^2$$

$$121(x+6)^2 = 25(x-6)^2$$

$$121x^2 + 1452x + 4356 = 25x^2 - 300x + 900$$

collecting like terms

$$96x^2 + 1752x + 3456 = 0.$$

solving the quadratic equation

$$x = -16, -2.25$$

In order to determine the minimum of the roots, let's substitute into y''

$$y'' = \frac{25}{((x+6)^2+25)^{3/2}} + \frac{121}{((x-6)^2+121)^{3/2}}$$

at $x = -16$, $y'' = 0.02602$; at $x = -2.25$, $y'' = 0.1489$

Hence, $x = -16$ is the minimum value for $\underline{y}$.

Scanned by TapScanner