exercise 4.12 :

a) $$p(D|\mu,\Sigma) \simeq |\Sigma|^{-N/2} \exp\left(-\frac{1}{2} tr(\Sigma^{-1} S_\mu)\right)$$

$$\Rightarrow \log p(D|\mu,\Sigma) = -\frac{N}{2}\log|\hat{\Sigma}| - \frac{N}{2} tr\left(\hat{\Sigma}^{-1}\frac{S_{\bar{x}}}{N}\right)$$

$$\rightarrow \log p(D|\mu,\Sigma) = -\frac{N}{2}\log|\hat{\Sigma}| - \frac{N}{2} tr(\hat{\Sigma}^{-1}\hat{S})$$

The definition of BIC is: $BIC = \log(P(D|\hat{\theta}_{ML})) - \frac{d}{2}\log(N)$

where d is number of parameter in the model.

We have: $S = \frac{1}{N}\sum_{i=1}^{N}(x_i - \hat{\mu})(x_i-\hat{\mu})^T$ is emperical covariance

which is equal to the maximum likelihood so $\frac{N}{2} tr(\hat{\Sigma}^{-1}\hat{S}) = \frac{ND}{2}$.

$$BIC = -\frac{ND}{2} - \frac{N}{2}\log(\hat{\Sigma}) - \frac{D(D+3)}{4}\log(N)$$

b) in this case the number of parameter is 2D. therefore the BIC is:

$$BIC = -\frac{ND}{2} - \frac{N}{2}\log(\hat{\Sigma}) - D\log(N)$$

exercice 5.9 :

$$\rho(a|x) = \int |y-a|\, p(y|x)\,dy = \int_{y>a}(y-a)\,p(y|x)\,dy - \int_{y<a}(y-a)\,p(y|x)\,dy$$

$$\frac{\partial\rho}{\partial a} = -\int_{y>a} p(y|x)\,dy + \int_{y<a} p(y|x)\,dy = 0$$

$$\Leftrightarrow P(y<a|x) = P(y\geq a|x) = 1/2.$$

Exercice 10.2 :

a) All variables that are independent of A given evidence on B : None.

b) All variables independent of A given evidence J : C, F.