Islamic Contribution to the Abstract Science of Number, Quantity, and Space

Maya Ayoubi

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*The Qur’an* is a religious text that promotes the act of acquiring knowledge in a variety of ways. One of these ways included the act of recitation and writing. In Surah 96, “Recite! Your Lord is most bountiful. He taught you with the pen. He taught man what he knew not (96:4,5).” The mentioning of the pen is to promote what Allah has given us, the object which purpose revolves around both the acquisition and dissemination of knowledge.

One of the oldest and sophisticated academic fields in history is mathematics. Mathematics has an expansive number of branches that one can analyze, but for the sake of time and articulation, this specific paper will deal with only one branch of mathematics; Algebra, mainly the concept of *al-jabr* and *al-muqabala*. In order to be able to begin to delve into how Islam had a huge contribution to this specific branch of mathematics, one must define what Algebra is. Dictionary.com defines Algebra as the branch of mathematics that deals with widespread statements of relations, employing the use of letters and other symbols to represent specific sets of numbers, values, and vectors and so on, in the explanation of such relations.

A Greek mathematician by the name of Diophantus is often recognized as “the father of Algebra,” however the title is better suited for Persian mathematician from Baghdad by the name of Muhammad ibn Musa al-Khwarizmi due to the basic fact that his work is on a more elementary and rhetorical level than that of Diophantus (Overbay). The first Islamic author to write “on the solution of problems by *al-jabr* and *al-muqabala*” was al-Khwarizmi. The meaning of *jabr* in a mathematical context is the act of adding equal terms to both sides of an equation in order to eliminate negative terms. A less common meaning of the term is the act of multiplying both sides of an equation by one and the same number in order to eliminate fractions. The literal meaning of the word *muqabala* translates into English as comparing, or posing opposite (van der Waerden). Mathematically speaking, *muqabala* is the reduction of positive terms by subtracting equal amounts from both sides of an equation. When combining both *al-jabr* and *muqabala*, we have the act of *al-jabr wal-muqabala*, or performing algebraic operations.

An example on the uses of *al-jabr wal-muqabala* and how Khwarizmi rhetorically writes out equations is seen on page 35 of Rosen’s translation of the *Algebra of Mohammed ben Musa*. The following problem reads:

I have divided ten into two portions. I have multiplied the one of the two portions by the other. After this I have multiplied one of the two by itself, and the product of the multiplication by itself is for times as much as that of one of the portions by the other.

Because we have two portions being discussed, Khwarizmi calls one of the portions “thing” (*shay*) and the other portion as “ten minus thing.” The author uses the word *mal* (“wealth” or “property”) to describe the square of the unknown “thing.” Finally, the following equation is obtained:

“A square, which is equal to forty things minus four squares.” In present day algebraic notation this would read as the following:

.

The operation *al-jabr* is then used by the author. Let be added to both sides, thus obtaining:

or

implying.

Page 40 in Rosen’s translation, Khwarizmi has the equation

and once reduced by *al-muqabla*, we have

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In AD 825, Islamic contributions to mathematics began when Khwarizmi wrote his famous disquisition *al-Kitab al-mukhtasar fi hisab al-jabr wa’l-muqabala* or *The Compendious Book on Calculation by Completion or Balancing* (Morgan). His book was considered the foundational text of modern algebra although he did not present his work in present day algebraic notation, but rather employed his works through a completely rhetorical way. In other words, Khwarizmi would write out equations in full sentences. Khwarizmi’s book is elementary and was not concerned with the complexity of Algebraic problems in indeterminate analysis; this means he concerns himself with straightforward and elementary interpretations of the solutions to equations (Al-Khwarizmi). This book consisted of three parts.

In the first part of the book, Khwarizmi described the six types of linear and quadratic equations that could be reduced:

(1)

(2)

(3)

(4)

(5)

(6)

For all .

In order to solve these equations, Khwarizmi gives us rules, demonstrations of the rules, and represents them by worked examples.

The second part of the book is concentrated around mensuration. For the second part of this book, Solomon Grandz published the full Arabic text with English translations in his book *The Mishnat ha-Middot and the Geometry of Muhammad ibn Musa Al-Khowarizmi*, Quellen and Studien zur Gescichte der Mathematik A2 (Springer-Verlag 1932). The rules for computing areas and volumes are the primary structures of this chapter. Take for example, finding the area of a circle. The area of a circle is found by multiplying half of the diameter by half of the circumference. In order to find the circumference, we must take into account three rules. Let be the diameter and be the periphery, hence we have the following three rules:

(7)

(8)

(9)

Note that rule (7) is true, because it has been proven by Archimedes (and Heron of Alexandria in his *Metrica*, and in *Mishnat ha-Middot*) that . In Chapter XII of the *Brahmasphutasiddhanta* contains rule (8), hence given to be true. Rule (9) is ascribed by Khwarizmi to “the astronomers” because it is the equivalent to the very accurate estimate

(10)

Khwarizmi makes the following statement that in every rectangular triangle the two short sides (let them be called and , respectively), each multiplied by itself and then the addition of both the products, equal the product of the long side (let it be called ) multiplied by itself. Thus, we have the following equation . Note, that this is only true for equilateral cases (). Hence, it is safe to conclude that Khwarizmi’s main source is not a classical Greek work such as *Elements* of Euclid, however this does not mean that classical Greek mathematics is not a source of his works. It is also worth noting that French author Aristide Marre who pusblished a French translation of Khwarizmi’s chapter on mensuration in *Annali di mathematica* *7 (1866)*, says that due to the insufficiency of the proof, there is in no way that the author would have been a member of the Platonic academy.

The third part of the Algebra of Khwarizmi is by far the largest part of the entire book. This part deals with legacies and is entirely made of problems with solutions. Although the solutions of these problems involve strictly simple arithmetic or linear equations, a substantial amount of understanding of the Islamic law of Inheritance is required, hence demonstrating the importance of cultural and religious influence.

Now, a further in depth analysis of Khwarizmi’s solution of the three types of mixed quadratic equations is what will be presented. In Khwarizmi’s own words, the first type is as follows:

Roots and Squares equal to numbers.

For instance: one square which, when increased by ten of its own roots, amounts to thirty-nine?

The solution is: you halve the number of roots, which in the present instance yields five. This you multiply by itself; the product is twenty-five. Add this to thirty-nine; the sum is sixty-four. Now take the root of this, which is eight, and subtract from it half the number of the roots, which is four. The remainder is three. This is the root of the square you thought for; the square itself is nine.

The following in modern notation is

,

which can also take the form of

.

A demonstration of such is what follows. What Khwarizmi does is he draws a square (let’s call it square AB), with a side of a desired root . Next, rectangles having of , or as their breadth are constructed on the four sides. The square with the four rectangles sums up to 39.

D

|  |  |  |
| --- | --- | --- |
|  |  |  |
|  | A B |  |
|  |  |  |

Fig. 1 H

To complete square DH, Khwarizmi says, adding four times the square of is . Hence, the area of the large square is 64, therefore making its side equal to 8. Therefore, the side of the original square is .

Another example that requires a simpler proof than the one in Fig.1, yet leads the same result is having rectangles of breadth 5 that are constructed only on two sides of the square AB (Fig. 2). Yet again, it is safe to assume that Khwarizmi’s main source does not stem from the classical Greek mathematician Euclid, because Khwarizmi’s first proof is much more intricate than Euclid’s proof of proposition II 4,

A

|  |  |
| --- | --- |
|  | B |
|  |  |

Fig. 2

And that says that the square on the line segment is equivalent to the sum of the squares and and twice the rectangle .

Khwarizmi’s sources and inspiration have been at question throughout history and currently still are, in particular his Algebra. There are quite a few different theories that have been proposed such as, he used classical Greek sources, or Hindu sources (due to his essay on Hindu numerals, that two of his estimates of are found in Hindu sources), or popular mathematical writings that belong to the Hellenistic and post-Hellenistic tradition. Personally, I believe much of his influence and sources stem from all three although I do believe that a cultural and religious influence could also have had an influence on his work. As mentioned before, Islam, in particular the *Qur’an* promoted the acquisition of knowledge; hence I believe that the foreground of his discoveries could have been influenced by his religious affiliation to Islam.