

Survival driven deconvolution (DeSurv) reveals prognostic and interpretable cancer subtypes

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Molecular subtyping in cancer is an ongoing problem that relies on the identification of robust and replicable gene signatures. While transcriptomic profiling has revealed recurrent gene expression patterns in various types of cancer, the prognostic value of these signatures is typically evaluated in retrospect. This is due to the reliance on unsupervised learning methods for identifying cell-type-specific signals and clustering patients into molecular subtypes. Here we present a Survival-driven Deconvolution tool (deSurv) that integrates bulk RNA-sequencing data with patient survival information to identify cell-type-enriched gene signatures associated with prognosis. Applying deSurv to various cohorts in pancreatic cancer, we uncover prognostic and biologically interpretable subtypes that reflect the complex interactions between stroma, tumor, and immune cells in the tumor microenvironment. Our approach highlights the value of using patient outcomes during gene signature discovery.

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Molecular subtyping has transformed precision oncology by stratifying patients into biologically and clinically meaningful groups that inform prognosis and guide therapy (1–5). Subtyping relies on the identification of robust biological signals that define subtypes such as transcriptomic signatures. However, the tumor microenvironment (TME) contains mixtures of diverse cell types such as malignant, stromal, immune, and endothelial cells, and disentangling tumor specific signals from this mixture can be challenging. As such, subtyping pipelines typically rely on the deconvolution of bulk transcriptomic data or single-cell analysis to discover distinct cell types and their corresponding signatures. Downstream, the signatures are evaluated for clinical relevance such as overall survival or response to treatment.

Separating discovery from validation can risk overfitting and limits biological and clinical generalizability. Identified cell types may capture dataset-specific noise rather than reproducible biological signals, undermining their utility in downstream analyses or therapeutic targeting (6, 7). Moreover, even when discovered cell types are biologically valid and reproducible, they may not correspond to the cellular programs most relevant for predicting or influencing clinical outcomes (8, 9). Therefore, there is a clear need for integrative methods that jointly uncover biologically meaningful programs while directly incorporating clinical endpoints to ensure prognostic relevance.

However, integrating patient outcomes into the discovery phase is not straightforward with current technology and methodology. Single-cell transcriptomics can resolve programs at the cellular level, but cohort sizes are often too small to support survival analyses. In contrast, large bulk transcriptomic cohorts with clinical annotations are well-suited for outcome modeling (10, 11), yet deconvolution is needed to disentangle overlapping cellular signals. Reference-based deconvolution

methods focus on estimating cell-type proportions from predefined signatures, which limits the utility of these methods for discovery of novel programs (12).

Nonnegative matrix factorization (NMF) is widely used in cancer genomics because its nonnegativity constraints produce biologically interpretable, additive molecular programs (13–16). Although recent extensions have incorporated supervision into the factorization, most target regression or classification rather than time-to-event outcomes. Two studies have proposed survival-aware NMF formulations (17, 18), but both integrate the survival objective through the sample-specific loadings rather than the gene-level programs. This design emphasizes prediction accuracy but limits the model's ability to restructure or refine the underlying molecular programs, reducing its value for biological interpretation and subtype discovery, which are core objectives in cancer transcriptomics. In addition, neither study provides a principled approach for hyperparameter selection or model assessment, and convergence properties are only briefly addressed in one manuscript. Both works remain unpublished and unreviewed, leaving their methodological robustness and reproducibility uncertain. These gaps highlight the need for a rigorously formulated, survival-aware deconvolution method that jointly estimates interpretable molecular programs and their prognostic relevance.

Here we present DeSurv, a Survival-supervised Deconvolution framework that integrates non-negative matrix factor-

Significance Statement

Tumor transcriptomes reflect mixtures of malignant and microenvironmental cell populations, making it challenging to identify the molecular programs that truly drive clinical outcomes. Existing deconvolution and matrix factorization methods discover latent transcriptional programs but do not ensure that these programs are prognostic, while supervised extensions optimized for prediction offer limited biological interpretability. We present DeSurv, a survival-supervised deconvolution framework that integrates nonnegative matrix factorization with Cox modeling to jointly learn biologically coherent gene programs and their associations with patient survival. By embedding outcome information directly into the discovery process and performing automatic model selection, DeSurv reveals clinically relevant transcriptional programs that are reproducible across cohorts. This approach advances the statistical foundations of tumor deconvolution and provides a general tool for identifying actionable molecular drivers of disease progression.

Please provide details of author contributions here.

Please declare any conflict of interest here.

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ization (NMF) with Cox proportional hazards modelling. In contrast to fully unsupervised approaches that evaluate survival associations only after the factorization, and to existing supervised NMF models that link outcomes to the subject-level factor loadings, DeSurv integrates survival information directly into the gene signature matrix. This design ensures that the discovered transcriptional programs are not only biologically interpretable but also intrinsically aligned with patient outcomes. To enhance robustness and reproducibility, DeSurv performs automatic parameter selection via Bayesian optimization, addressing the quintessential challenge of rank determination in matrix factorization.

By coupling latent program discovery with direct survival supervision, DeSurv resolves longstanding challenges in disentangling tumor–microenvironment interactions and aligns molecular heterogeneity with clinical outcomes. This unified approach represents a methodological advance in translational cancer genomics and provides a general framework for deriving actionable insights from high-dimensional transcriptomic data.

Results

Model Overview. We have developed an integrated framework, DeSurv, that couples Nonnegative Matrix Factorization (NMF) with Cox proportional hazards regression to identify latent gene-expression programs associated with patient survival (Figure 1A). The model takes as input a bulk expression matrix of p genes by n patients (X_{Train}) together with corresponding survival times (y_{Train}) and censoring indicators (δ_{Train}) (Figure 1A).

DeSurv optimizes a joint objective combining the NMF reconstruction loss and the Cox model’s log-partial likelihood, weighted by a supervision parameter (α) that determines the relative contribution of each term (fig. 1B):

$$(1 - \alpha) \mathcal{L}_{NMF}(X_{Train} \approx WH) - \alpha \mathcal{L}_{Cox}(X_{Train}^T W \beta, y_{Train}, \delta_{Train}) \quad [1]$$

When $\alpha = 0$, the method reduces to standard unsupervised NMF; when $\alpha > 0$, survival information directly guides the learned factors toward prognostic structure.

Within this framework, the product ($X_{Train}^T W$) represents patient-level factor scores - the inferred burden of each latent program across subjects. These factor scores serve as covariates in the Cox model, and their regression coefficients (β) indicate whether higher activity of a given program corresponds to improved or reduced survival.

Model training yields gene weights (\hat{W}), factor loadings (\hat{H}), and Cox coefficients ($\hat{\beta}$) (Figure 1C), where the inner dimension (k) specifies the number of latent factors. Genes with high gene weights in one factor and low gene weights in all others define the factor-specific signature genes (Figure 1D). By integrating survival supervision into the factorization, DeSurv not only reconstructs the underlying expression structure, preserving biological interpretability, but also guides latent factors to be prognostically informative. Subsequent analyses can therefore focus on the survival-associated gene programs (Figure 1E).

Bayesian optimisation selects the DeSurv hyperparameters (k, α).

A. Outcome-guided model selection resolves ambiguity in NMF rank choice. We examined the problem of selecting the number of latent components (k) in nonnegative matrix factorization (NMF) using gene expression data from pancreatic ductal adenocarcinoma (PDAC) cohorts. These heterogeneous tumor transcriptomes provide a representative setting in which to evaluate how commonly used unsupervised rank-selection heuristics behave in practice.

Across a range of candidate ranks, standard NMF diagnostics yielded inconsistent guidance (Fig. 2A-C). Reconstruction residuals decreased smoothly with increasing k and did not exhibit a clear elbow, a pattern consistent with both relatively small solutions ($k \approx 3-4$) and substantially larger ranks ($k \approx 6-8$). The cophenetic correlation coefficient began to decline at low ranks ($k \approx 3-4$) but continued to fluctuate at higher values without a distinct transition point. In contrast, mean silhouette width, evaluated across multiple distance metrics, was highest at very small ranks ($k \approx 2-3$) and decreased monotonically thereafter, favoring low-dimensional solutions that conflicted with recommendations based on reconstruction error or cophenetic correlation. Together, these unsupervised criteria pointed to different and incompatible values of k , highlighting the ambiguity of rank selection in standard NMF when applied to PDAC data.

To resolve this ambiguity, we applied DeSurv, which incorporates survival outcomes directly into the factorization process and evaluates models using survival-based predictive performance. Using the same PDAC gene expression data, we assessed model performance across the joint space of the number of components (k) and supervision strength (α) using cross-validated concordance index (C-index). The resulting C-index surface summarizes expected predictive performance across candidate models and enables direct comparison of solutions that differ in both model complexity and degree of supervision (Fig. 2D).

Model selection was based on standard cross-validation principles. Rather than selecting the single parameter combination with the highest predicted C-index, we selected the smallest value of k whose predicted performance lay within one standard error of the maximum. This criterion yielded a stable and parsimonious choice of model rank in the PDAC data, in contrast to the conflicting recommendations produced by unsupervised NMF heuristics.

To further evaluate rank recovery under controlled conditions, we conducted simulation studies in which the true underlying rank was known ($k = 3$). Across repeated simulation replicates, DeSurv consistently selected the correct rank, producing a concentrated distribution of selected k values centered at the true value. In contrast, standard NMF followed by post hoc Cox modeling ($\alpha = 0$) exhibited substantially greater variability and a systematic tendency toward under-selection. Together, these results indicate that incorporating outcome information during model fitting improves the reliability of rank selection in settings where unsupervised criteria yield conflicting conclusions.

“(A-D) Analyses based on real pancreatic ductal adenocarcinoma (PDAC) gene expression data illustrate the ambiguity of rank selection in standard nonnegative matrix factorization (NMF) and the use of outcome supervision in DeSurv. (A-C) Commonly used unsupervised heuristics for selecting the number

of components (k) yield inconsistent conclusions. (A) Reconstruction residuals decrease smoothly with increasing k and do not exhibit a clear elbow; diminishing returns could be inferred at intermediate ($k \approx 3$ -4) or larger ($k \approx 6$ -8) ranks. (B) The cophenetic correlation coefficient, often used to select the largest k prior to a marked loss of clustering stability, begins to decline at low ranks ($k \approx 3$ -4) but continues to fluctuate thereafter, providing no unambiguous selection criterion. (C) Mean silhouette width across multiple distance metrics is highest at small ranks ($k \approx 2$ -3) and decreases monotonically with increasing k , favoring lower-dimensional solutions that conflict with the other criteria. (D) Heatmap of the Gaussian process predicted mean cross-validated concordance index (C-index) from Bayesian optimization over the joint space of the number of components (k) and supervision strength (α), computed on the same PDAC data. The predicted performance surface summarizes survival prediction accuracy across parameter settings and illustrates how DeSurv uses outcome information to inform model selection. (E) Results from simulation studies with a known underlying rank ($k = 3$) showing the distribution of selected k values across repeated replicates. DeSurv more consistently recovers the true rank, yielding a concentrated distribution centered at $k = 3$, whereas standard NMF with post hoc Cox modeling ($\alpha = 0$) exhibits greater variability and a tendency toward under-selection. \label{fig:bo}”, fig.env=’figure’, fig.pos=’t’, out.height= “5.5in”, out.width=“6in”}

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tar_load(fig_bo_heat_tcgacptac)
tar_load(fig_residuals_tcgacptac)
tar_load(fig_cophenetic_tcgacptac)
tar_load(fig_silhouette_tcgacptac)
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1. upper = plot_grid(fig_bo_cvk_tcgacptac,fig_bo_cvalpha_tcgacptac,labels=c(“A”,“B”))

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upper = plot_grid(fig_residuals_tcgacptac,
fig_cophenetic_tcgacptac, fig_silhouette_tcgacptac, ncol =
3, labels = c(“A”,“B”,“C”))
k_hist = plot_grid(alt_plots$k_hist,NULL,nrow=2,rel_heights
= c(4,1))
lower = plot_grid(fig_bo_heat_tcgacptac,
k_hist,ncol=2,labels=c(“D”,“E”),rel_widths = c(3,2))
plot_grid(upper,lower,nrow=2,rel_heights = c(2,3))
```

DeSurv improves selection of prognostic gene signatures and supervised survival analysis. To test whether supervision improves the selection of prognostic gene signatures, we used simulations with known lethal

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{\centering \includegraphics[width=\textwidth]{/work/users/a/y/a/figures/figure10.pdf}}
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\caption{Performance comparison between DeSurv and unsupervised NMF ($\alpha = 0$) in simulation. (A) Distribution of c

Survival-informed factorization reorganizes transcriptomic data to reveal prognostic structure. To assess the biological structure captured by standard nonnegative matrix factorization (NMF) and DeSurv, we examined

In the standard NMF solution, factors largely reflected dominant

By contrast, DeSurv produced a factorization that emphasized axo

These differences were reflected quantitatively by contrasting t

To further characterize how DeSurv reorganizes the transcription

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\caption{Survival-informed factorization reorganizes transcriptomic data to reveal prognostic structure.

DeSurv-derived latent structure generalizes to independent data. To assess generalization of DeSurv latent factors, the gene-level

For each method, we focused on the factor showing the largest in

When validation samples were pooled and stratified into high- and

Together, these results indicate that DeSurv identifies latent factors

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\caption{Figure X. DeSurv learns prognostic structure that generalizes to independent data.

Bladder cancer analysis {.unnumbered}

We will report a focused bladder cancer analysis here, including

Discussion {.unnumbered}

We present DeSurv, a survival-driven deconvolution framework that

Bladder cancer analysis {.unnumbered}

In pancreatic ductal adenocarcinoma (PDAC), DeSurv identified the

To test whether supervision improves the selection of prognostic gene signatures, we used simulations with known lethal

Importantly, DeSurv outperformed unsupervised NMF in cross-validation

By analyzing PDAC using DeSurv, we provide a paper for publication in the journal of the American Association of Cancer Research.

Materials and methods {.unnumbered}

Problem formulation and notation

Let $\mathbf{X} \in \mathbb{R}^{p \times n}$, $\mathbf{y} \in \mathbb{R}^n$ denote the nonnegative

The DeSurv Model

Personalized cancer factorization (NMF) with personalized supervision. To assess the biological structure captured by standard nonnegative matrix factorization (NMF) and DeSurv, we examined

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The joint objective is
\begin{equation}
\label{eqn:desurv}
\mathcal{L}(W,H,\beta) =
(1-\alpha)\|\mathcal{L}_{\mathrm{NMF}}(W,H)
- \alpha\|\mathcal{L}_{\mathrm{Cox}}(W,\beta),
\end{equation}
where  $\mathcal{L}_{\mathrm{NMF}}(W,H)$  is the NMF reconstruction loss, and  $\mathcal{L}_{\mathrm{Cox}}(W,\beta)$  is the Cox log-likelihood.

## Hyperparameter selection and cross-validation
Hyperparameters  $(k,\alpha,\lambda_H,\lambda,\xi)$  were selected by maximizing the log-likelihood on the training data and minimizing the cross-validation error on the test data.

## Simulation studies
Simulation studies were conducted to assess recovery of parameters from simulated data. Survival times were generated from an exponential proportional hazards model in which risk depended on marker gene expression.

## Evaluation metrics
We used the following metrics to evaluate the performance of the methods: the Frobenius norm of the difference between the estimated and true matrices, the Cox log-likelihood, and the area under the ROC curve.

## Real-world datasets
We analyzed publicly available RNA-seq and microarray data from pancreatic ductal adenocarcinoma (PDAC) and bladder cancer datasets.

## Simulations, Benchmarking, and Availability
Code and processed data used in this study are available at the GitHub repository link.

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\showacknow

\subsection{Model details}

Let  $X \in \mathbb{R}_{\geq 0}^{p \times n}$  denote the nonnegative expression matrix with  $p$  genes and  $n$  subjects. DeSurv approximates  $X \approx WH$  with  $W \in \mathbb{R}_{\geq 0}^{p \times k}$  and  $H \in \mathbb{R}_{\geq 0}^{k \times n}$ , and links the shared parameters  $W$  to survival via an elastic-net-penalized Cox model.

Let  $y \in \mathbb{R}^n$  be the observed survival times,  $\delta \in \{0,1\}^n$  the event indicators, and  $Z \in \mathbb{R}^{n \times k}$  the matrix of basis functions. We write  $Z_i^{\text{top}}$  for the  $i$ th row of  $Z$ , and  $n_{\text{event}} = \sum_{i=1}^n \delta_i$ . For convenience we define  $\lambda_H = \lambda_H / n_{\text{event}}$  and  $\lambda = \lambda / n_{\text{event}}$ .

\begin{align}
\mathcal{L}(W,H,\beta) &= \text{nonnumber} \left( (1-\alpha) \left( \frac{1}{2np} \|X - WH\|_F^2 + \frac{1}{2k} \|\beta\|_1 \right) + \alpha \ell(W,\beta) \right)
\end{align}
where  $\ell(W,\beta) = -\sum_{i=1}^n \log \left( \sum_{j=1}^k Z_{ij} \exp(Z_j^{\text{top}} \beta) \right)$  is the Cox log-likelihood.

Hyperparameters satisfy  $\alpha \in [0,1]$ ,  $\lambda_H \geq 0$ ,  $\lambda \geq 0$ , and  $\xi \in [0,1]$ . The constants  $1/(2np)$ ,  $2/n_{\text{event}}$ , and  $1/(2nk)$  are for numerical convenience and do not affect the minimizer.

\subsection{Optimization Algorithm}

\subsubsection{Block coordinate descent scheme}
Algorithm~\ref{alg:desurv} summarizes the block coordinate descent scheme used to minimize  $\mathcal{L}(W,H,\beta)$ . At each outer iteration we update  $H$ , then  $W$ , then  $\beta$  while holding the other blocks fixed.

\begin{algorithm}[H]
\caption{DeSurv block coordinate descent}
\label{alg:desurv}
\begin{algorithmic}[1]
\Require  $X \in \mathbb{R}_{\geq 0}^{p \times n}$ , survival times  $y$ , event indicators  $\delta$ 
\Ensure Fitted DeSurv parameters  $(W,H,\beta)$ 

\State Initialize  $W^{(0)}, H^{(0)}$  with positive entries
\State Initialize  $\beta^{(0)}$  (e.g.,  $\beta^{(0)} = \mathbf{0}$ )
\State  $\text{loss} = \mathcal{L}(W^{(0)}, H^{(0)}, \beta^{(0)})$ 
\State  $\text{seps} = \infty$ ,  $t = 0$ 
\While{ $\text{seps} \geq \text{tol}$  \textbf{and}  $t < \text{maxit}$ }


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\State \textbf{(H-update)} \delta_i
\State \hspace{0.5cm} Update  $H^{(t+1)}$  from  $H^{(t)}$  using the multiplicative rule in Eq.~\ref{eqn:Hupdate}
\State \hspace{0.5cm} holding  $W^{(t)}$  and  $\beta^{(t)}$  fixed.
\State \textbf{(W-update)}  $-\frac{1}{\alpha}$ 
\State \hspace{0.5cm} Update  $W^{(t+1)}$  from  $W^{(t)}$  using the hybrid multiplicative rule in Eq.~\ref{eqn:Wupdate}
\State \hspace{0.5cm} holding  $H^{(t+1)}$  and  $\beta^{(t+1)}$  fixed, and perform backtracking to ensure
\State \hspace{0.5cm}  $\mathcal{L}$  does not increase.
\State \textbf{(\beta-update)}  $\sum_{j: y_j \geq y_i}$ 
\State \hspace{0.5cm} Update  $\beta^{(t+1)}$  from  $\beta^{(t)}$  using a Newton-like step for the Cox loss
\State \hspace{0.5cm} (Eq.~\ref{eqn:beta_update}), holding  $W^{(t+1)}$  and  $H^{(t+1)}$  fixed.
\State  $\text{lossNew} = \mathcal{L}(W^{(t+1)}, H^{(t+1)}, \beta^{(t+1)})$ 
\State  $\text{seps} = |\text{lossNew} - \text{loss}| / |\text{loss}|$  \end{equation}
\State  $\text{loss} = \text{lossNew}$  where  $x_i$  denotes the  $i$ th column of  $X$ .
\State  $t = t + 1$ 
\EndWhile
\State \Return  $\hat{W} = W^{(t+1)}$ ,  $\hat{H} = H^{(t+1)}$ ,  $\hat{\beta} = \beta^{(t+1)}$ 
\end{algorithmic}
\end{algorithm}

\subsubsection{Update for  $W$ }
Conditional on  $W$ , the loss  $\mathcal{L}(W, H, \beta)$  reduces to a convex quadratic function of  $W$ . We adopt the standard NMF multiplicative
update with  $\ell_2$  penalty:
\begin{equation}
\label{eqn:Hupdate}
H \leftarrow \max\left(
\begin{aligned}
& H \odot \frac{W^{\top} X}{W^{\top} W H + \lambda_H H + \epsilon_H} \\
& \epsilon_H
\end{aligned}
\right),
\end{equation}
where  $\odot$  denotes elementwise multiplication, all divisions are
elementwise, and  $\epsilon_H > 0$  is a small floor to prevent  $H$  from
becoming exactly zero. This update is equivalent to a majorization step and guarantees a nonincreasing reconstruction term
[seung2001algorithms; pascualmontano2006nonsmooth]. The
[seung2001algorithms; pascualmontano2006nonsmooth]. The
ensures that iterates remain in the interior of the nonnegative
simplifies the convergence analysis.

\subsubsection{Update for  $W$ }
For  $W$ , we construct a hybrid multiplicative update that
combines contributions of the NMF and Cox gradients. Let

$$\begin{aligned}
& \nabla_W \mathcal{L}_{\text{NMF}}(W, H) \\
& = \frac{1}{n} (W H - X) H^{\top}, \\
& \text{and let} \\
& \nabla_W \mathcal{L}_{\text{Cox}}(W, \beta) \\
& = \frac{2}{n_{\text{event}}} \nabla_W \ell(W, \beta).
\end{aligned}$$

where  $\nabla_W \ell(W, \beta)$  denotes the Cox gradient with respect to  $W$ .
\begin{equation}
\text{nonincrease in the full loss } \mathcal{L}, \text{ we embed it in a back}

line search.
\begin{algorithmic}
[H]

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\caption{$W$ update with backtracking}
\label{alg:backtrack}
\begin{algorithmic}[1]
  \Require Value of $W$ and the previous iteration $\mathbf{W}^{(t)}$, the multiplicative update term at the current
  \Ensure $\mathbf{W}^{(t+1)}$
  \State $\theta = 1$
  \State $b = 1$
  \State $flag\_accept = FALSE$
  \While{$b \leq max\_bt$}
    \State $\mathbf{W}^{(t+1)}_{cand} = \mathbf{W}^{(t)} \odot [(\frac{\mathbf{W}^{(t)} \mathbf{X}^T \mathbf{X} \mathbf{W}^{(t)}}{\mathbf{W}^{(t)} \mathbf{X}^T \mathbf{X} \mathbf{W}^{(t)} + \lambda \mathbf{I}})^{\frac{1}{2}}]$ is the soft
    \State \textbf{bf}{(Column normalization)} operator, and $\mathbf{W}(\tilde{\eta})$, $\mathbf{V}(\tilde{\eta})$ are standard
    \State \hspace{0.5cm} Compute $\mathbf{D} = \text{diag}(\frac{1}{\|\mathbf{W}^{(t+1)}_{cand}\|_2})$ to cancel the $\ell_2$-norms of $\mathbf{W}^{(t+1)}_{cand}$
    \State \hspace{0.5cm} and set $\mathbf{W}^{(t+1)}_{cand} = \mathbf{W}^{(t+1)}_{cand} \mathbf{D}$
    \If{$\mathcal{L}(\mathbf{W}^{(t+1)}_{cand}, \mathbf{H}^{(t+1)}) \leq \mathcal{L}(\mathbf{W}^{(t)}, \mathbf{H}^{(t)})$}
      \State $\mathbf{W}^{(t+1)} = \mathbf{W}^{(t+1)}_{cand}$
      \State $\mathbf{H}^{(t+1)} = \mathbf{H}^{(t+1)}_{cand}$
      \State $\beta^{(t)} = \beta^{(t)}_{cand}$ \subsubsection{Normalization of W}
      \State $flag\_accept = TRUE$
      \State break
    \EndIf
    \State $\theta = \theta * \rho$
    \State $b = b + 1$
  \EndWhile
  \If{$flag\_accept = FALSE$}
    \State $\mathbf{W}^{(t+1)} = \mathbf{W}^{(t)}$
  \EndIf
\end{algorithmic}
\end{algorithm}

\left[
\begin{aligned}
& \mathbf{v}(\tilde{\eta})_i - \sum_{j \neq r} \mathbf{z}_{i,j} \beta_j \\
& \right],
\end{aligned}
\right]

```

After each accepted W update, we normalize the columns of W $\leftarrow W D^{-1}$, and adjust H and β as $H \leftarrow D H$, $\beta \leftarrow D \beta$, where D is the diagonal matrix of column ℓ_2 -norms of W . This preserves the reconstruction WH and the linear predictor leaving both the NMF and Cox terms in the loss unchanged. Normalization prevents degeneracy in the scale-nonidentifiable factorization. The columns of W are comparable in magnitude and interpretable as gene expression levels.

The column normalization preserves both WH and $W\beta$, and therefore leaves the loss \mathcal{L} invariant up to numerical errors. Backtracking of W update from projected coordinate descent guarantees that the accepted W update does not increase the loss. β can be derived directly from the projected coordinate descent update.

Update for β

Recall that the overall loss function is

$$\mathcal{L}(W, H, \beta) = \frac{(1-\alpha)}{2np} \|X - WH\|^2_F + \frac{\alpha}{2np} \sum_{i=1}^n \sum_{j=1}^p \mathbb{I}(\beta_j > 0) \log \frac{\beta_j}{\beta_j + 1}$$

Conditional on (W, H) , the loss in β reduces to a convex elastic-net-penalized Cox problem:

$$\min_{\beta} \sum_{i=1}^n \sum_{j=1}^p \mathbb{I}(\beta_j > 0) \log \frac{\beta_j}{\beta_j + 1} + \frac{\lambda}{2} \sum_{j=1}^p \beta_j^2$$

where $\ell(W, \beta)$ is the log-partial likelihood for the Cox model.

Then the gradient descent update rule at iteration t is

$$\beta^{(t)} = \beta^{(t-1)} - \gamma \left(\frac{\partial \mathcal{L}}{\partial \beta} \right)^T$$

where γ is the step size. γ can be defined as in CITE: $\gamma = \frac{1}{\lambda + \frac{1}{np} \sum_{i=1}^n \sum_{j=1}^p \mathbb{I}(\beta_j > 0) \frac{1}{\beta_j^2}}$.

We solve this subproblem by cyclic coordinate descent following [Simon2011regularization]. Writing $\ell(\beta) = \ell(\beta) + \frac{\lambda}{2} \sum_{j=1}^p \beta_j^2$, the update for coordinate j has a closed form

$$\hat{\beta}_j = \frac{1}{\lambda + \frac{1}{np} \sum_{i=1}^n \sum_{k \neq j} \mathbb{I}(\beta_k > 0) \frac{1}{\beta_k^2}} \left(\sum_{i=1}^n \sum_{k \neq j} \mathbb{I}(\beta_k > 0) \frac{1}{\beta_k^2} \beta_k + \sum_{i=1}^n \sum_{k \neq j} \mathbb{I}(\beta_k > 0) \frac{1}{\beta_k^2} \beta_k \right)$$

Then the update becomes

$$\beta^{(t)} = \beta^{(t-1)} - \frac{np}{\lambda + \frac{1}{np} \sum_{i=1}^n \sum_{j=1}^p \mathbb{I}(\beta_j > 0) \frac{1}{\beta_j^2}} \left(\frac{\partial \mathcal{L}}{\partial \beta} \right)^T$$

Finally, projected coordinate descent projects the W update into the non-negative orthant.

$$W^{\{t\}} = \max \left(W \odot \frac{\frac{1-\alpha}{n_p}}{X^{\{t\}} + \frac{2\alpha}{n_{\text{event}}}} \nabla_W \ell \right) \frac{1-\alpha}{n_p}$$

Since $W \geq 0$ this is equivalent to

$$W^{\{t\}} = W \odot \max \left(\frac{1-\alpha}{n_p}, \frac{1-\alpha}{X^{\{t\}} + \frac{2\alpha}{n_{\text{event}}}} \right)$$

which matches the multiplicative form used in the software implementation.

If no candidate satisfies the Armijo-type condition with number of backtracking steps, the algorithm sets $W^{\{t+1\}} := W^{\{t\}}$.

$\beta^{\{t+1\}} \mapsto \beta^{\{t+1\}}$ is obtained by running the descent method of [Simon2011regularization] subproblem until convergence. Thus

$$\beta^{\{t+1\}} \in \arg\min_{\beta \in \mathbb{R}^k} \mathcal{L}(W^{\{t+1\}}, H^{\{t+1\}}, \beta),$$

and

$$\mathcal{L}(W^{\{t+1\}}, H^{\{t+1\}}, \beta^{\{t+1\}}) \leq \mathcal{L}(W^{\{t+1\}}, H^{\{t+1\}}, \beta^{\{t\}}).$$

For clarity, we first analyze the algorithm without the column normalization of W inside the loop, and then argue in Section~\ref{subsec:conv_normalization} that the normalization preserves stationarity of limit points.

Throughout, let $\theta = (W, H, \beta)$ and denote $\mathcal{L}(\theta) = \mathcal{L}(W, H, \beta)$. The feasible set is

$$\Theta = \{(W, H, \beta) : \begin{aligned} &W \in \mathbb{R}^{p \times k}, \\ &H \in \mathbb{R}^{k \times n}, \\ &\beta \in \mathbb{R}^k. \end{aligned}\}$$

is nonempty, closed, and bounded.

Lemma 1 (Continuity and bounded level sets). Under Assumption 1, $\mathcal{L} : \Theta \rightarrow \mathbb{R}$ is continuous on the initial sublevel set

$$\mathcal{S}_0 := \{\theta \in \Theta : \mathcal{L}(\theta) \leq \mathcal{L}(\theta^{(0)})\}$$

is nonempty, closed, and bounded.

Assumption 1 (Regularity and parameter space). The NMF term $\|X - WH\|_F^2$ is a polynomial in the entries of W and H , and is continuously differentiable. The penalty $\|H\|_F^2$ is continuous. The Cox partial log-likelihood is a smooth function of the linear coefficients β , and $W, H, \beta \geq 0$. The hyperparameters satisfy $\lambda_H > 0$, $\lambda_W > 0$, and $\lambda_\beta > 0$. The ℓ_2 penalty $\|\beta\|_2^2$ is continuous on Θ .

Assumption 2 (Block updates). The constraints $W, H \geq 0$ define closed convex cones, and $\beta \geq 0$ is unconstrained. Because $\lambda_H > 0$ and the terms $\frac{\lambda_H}{n} \|H\|_F^2$ and $\lambda_\beta \|\beta\|_2^2$ dominate the objective as $\|H\|_F \rightarrow \infty$ or $\|\beta\|_2 \rightarrow \infty$, respectively. Thus \mathcal{S}_0 is bounded in (H, β) . Since $W \geq 0$ and $H \rightarrow H \odot \frac{W^{\text{top}} X}{W^{\text{top}} W + \lambda_H I}$, \mathcal{L} remains below $\mathcal{L}(\theta^{\{t\}})$. Because Θ is closed and \mathcal{L} is continuous, \mathcal{S}_0 is closed. Nonempty \mathcal{S}_0 yields a nonincreasing value of the reconstruction term $\|X - WH\|_F^2$ in \mathcal{S}_0 . $\|W\|_F^2$ and $\|\beta\|_2^2$; see e.g. [Seung2001algorithms; PascualMontano2006nonsmooth].

Lemma 2 (Monotone descent and existence of limit points). Under Assumptions 1 and 2, Algorithm~\ref{alg:desurv} (without backtracking as in Algorithm~\ref{alg:backtrack}), generates a sequence $\{\theta^{\{t\}}\}_{t \geq 0}$ such that

$$\mathcal{L}(\theta^{\{t+1\}}, H^{\{t+1\}}, \beta^{\{t+1\}}) \leq \mathcal{L}(\theta^{\{t+1\}}) \leq \mathcal{L}(\theta^{\{t\}})$$

and $\mathcal{L}(\theta^{\{t\}})$ converges to a finite limit

Moreover, $\|\theta^{(t)}\|$ is bounded and therefore admits a limit point.

Proof.
By Assumption 2(i),

$$\|W^{(t)}, H^{(t+1)}, \beta^{(t)}\| \leq \|W^{(t)}, H^{(t)}, \beta^{(t)}\|.$$
By Assumption 2(ii),

$$\|W^{(t+1)}, H^{(t+1)}, \beta^{(t)}\| \leq \|W^{(t)}, H^{(t+1)}, \beta^{(t)}\|.$$
By Assumption 2(iii),

$$\|W^{(t+1)}, H^{(t+1)}, \beta^{(t+1)}\| \leq \|W^{(t+1)}, H^{(t+1)}, \beta^{(t)}\|.$$
Combining these inequalities gives

$$\|\theta^{(t+1)}\| \leq \|\theta^{(t)}\|.$$
so $\|\theta^{(t)}\|$ is monotonically nonincreasing and bounded below on Θ , the sequence converges to a finite limit θ^* .

Each iterate lies in S_0 by construction, and S_0 is bounded by Lemma 1. Therefore $\theta^{(t)}$ is bounded and has at least one limit point by the Bolzano--Weierstrass theorem. **Lemma 1 (Convergence to a stationary point).**
Under Assumptions 1 and 2, the sequence $\theta^{(t)}$ produced by Algorithm-S (without W normalization) satisfies the following properties:

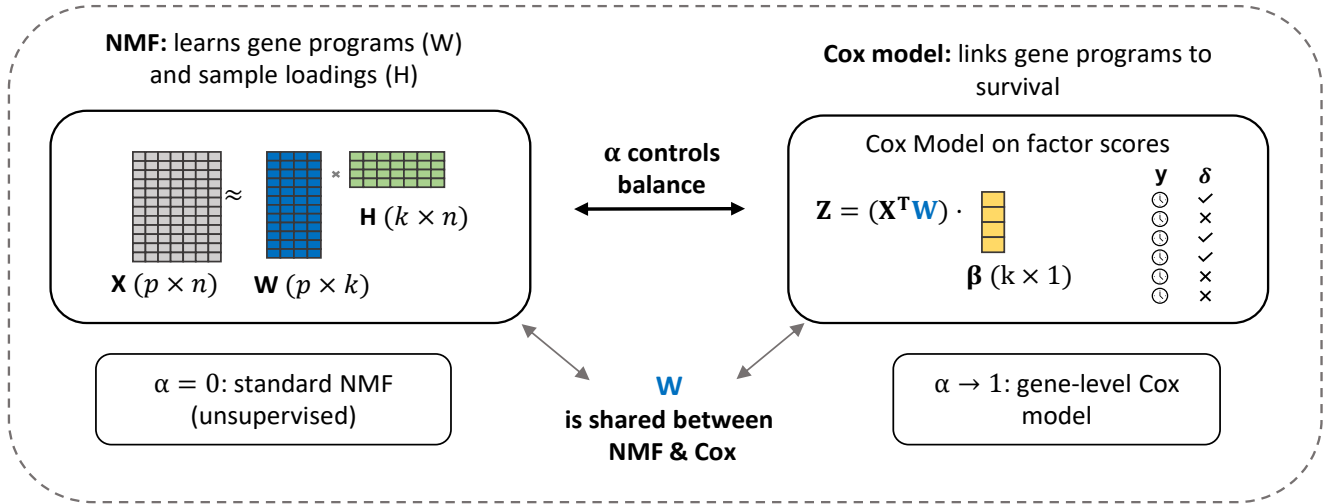
- (a) The sequence of values $\|\theta^{(t)}\|$ are nonincreasing and converge to a finite limit $\|\theta^*\|$.
- (b) Every limit point $\theta^* = (W^*, H^*, \beta^*)$ of $\theta^{(t)}$ is a stationary point of \mathcal{L} on Θ , i.e. the first-order KKT conditions for
$$\min_{\theta \in \Theta} \mathcal{L}(W, H, \beta)$$
are satisfied.

Proof.
Part (a) is Lemma 2. For part (b), Lemma 3 shows that any limit point θ^* is blockwise optimal: each block is optimal (in the KKT sense) for the subproblem with the other blocks fixed. This can be written as
$$\mathcal{L}(\theta) = f(W, H, \beta) + g(\beta) + I_{\{W \geq 0\}}(W) + I_{\{H \geq 0\}}(H),$$
where f is \mathcal{L} with β fixed, $g(\beta) = \lambda \sum_i \xi_i$ is a separable convex (possibly nondifferentiable) penalty, and I_C is the indicator of a closed convex set C . This is the smooth+nonsmooth form considered in block coordinate descent analyses such as [10]. In this setting, any point that is optimal with respect to each

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DRAFT

(A) The DeSurv model



(B) Data-driven model selection via Bayesian optimization

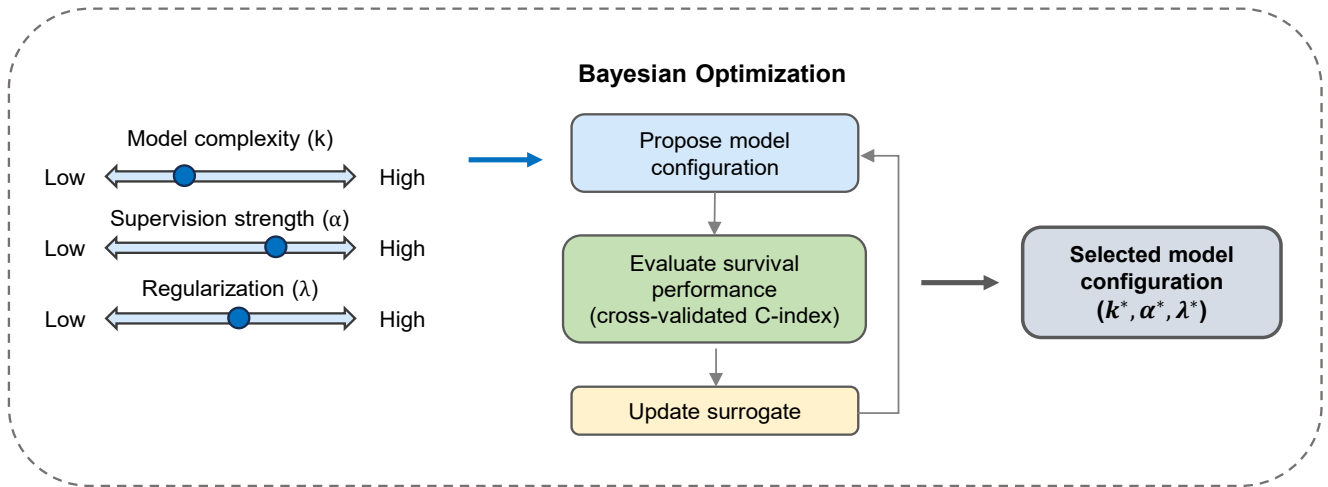


Fig. 1. Overview of the DeSurv framework and data-driven model selection. (A) DeSurv integrates nonnegative matrix factorization (NMF) with survival modeling to learn prognostic gene programs from a gene expression matrix X . NMF decomposes $X \approx WH$, where W represents gene programs and H sample loadings; the learned programs W are shared with a Cox proportional hazards model that links factor-derived scores $Z = X^T W$ to survival outcomes via regression coefficients β . A tuning parameter α controls the balance between unsupervised structure learning ($\alpha = 0$) and supervised survival association ($\alpha = 1$). (B) Model complexity (k), supervision strength (α), and regularization (λ) are selected via Bayesian optimization using cross-validated concordance index.