New Mathematical Models to Calculate Par Scores in Rain-Affected Men's T20 Cricket Games

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Abstract

Par scores for rain-affected limited overs matches of cricket are currently calculated using the Duckworth-Lewis-Stern Method, which was advancement over the Duckworth-Lewis Method that was primarily conceived for 50-over matches. Here I develop two new mathematical models to compute par scores for rain-affected men's T20 matches. The old methods are discussed in Section I, and the key conditions that the new models address are listed in Section II. The static model with fixed conditions is derived in Section III and examples of its usage for T20 matches of the past are presented in Section IV. Section V develops the dynamic model for scenarios where rain halts play in the second innings starting with 20 overs and 10 wickets in hand and the game does not resume. Resource functions and tables for both models are presented in the appendix. Sections VI discusses how to refine these resource functions and selecting the appropriate model subject to scenario.

I. The Duckworth-Lewis and Duckworth-Lewis-Stern Methods

Rain-affected limited overs matches of cricket require the implementation of a statistical model to decide how much the chasing team must score with their available batting resources. Resources for a batting team are namely overs and wickets. If the team batting first is Team 1 and the chasing team is Team 2, The Duckworth-Lewis (DL) Method calculates the par score using the formula,

$$Team \ 2's \ par \ score = Team \ 1's \ score * \frac{Team \ 2's \ used \ resources}{Team \ 1's \ used \ resources}, \tag{1}$$

As intuitively expected, the par score increases proportionally with Team 1's score and Team 2's exhausted resources, and decreases with Team 1's exhausted resources. The key figure which is statistically calculated is the ratio of the two team's resources. Criticisms for the DLS Model include over-importance given to wickets in high-scoring encounters and no distinction given to powerplay and non-powerplay overs. Even though the Duckworth-Lewis-Stern (DLS) Method computes resources based on the target score, it was conceptualized for the 50-over format of the game, and issues compound more heavily for the youngest format of the game, T20 cricket, which is played at a different pace. The new model discussed here after introduces changes which address these issues for men's T20 matches.

II. Key Conditions of New Model

The key changes the new model introduces are listed below:

1. Powerplay (first 6) and non-powerplay (latter 14) overs are weighted differently.

- 2. For both the powerplay and non-powerplay overs, resources should exhaust more quickly towards the end.
- 3. For non-powerplay overs, the first 4 should consume small amount of resource as these are passive overs for batting.
- 4. Wickets play a passive role and loss of first 3-4 wickets does not derail the batting team too heavily, as 20-over cricket is a short format and the onus is on keeping abreast with the required rate.

The principal equation used to find the par score given by Equation (1) is unchanged, and the resources range from 0 (complete exhaustion) to 1 (all resources intact). Resources available equals 1 when 10 wickets and 20 overs are left; resources available equals 0 if all 10 wickets **or** 20 overs are exhausted. The next section uses these ideas to develop the new model.

III. Deriving the New Static Model

Let the resources available to a batting team be given by the function R(x, w), where x is the overs left and w is the wickets left, be given by the product of two functions, F(x) and G(w), such that,

$$R(x, w) = F(x)G(w), \tag{2}$$

$$F(20) = 1, F(0) = 0, (3)$$

$$G(10) = 1, G(0) = 0.$$
 (4)

Equations (3) and (4) ensure that R(x, w) is 0 if either x=0 or w=0, and R(x, w) is 1 if and only if x=20 and w=10.

The next step is to choose a model for F and G which replicate the ideas of Section II. F is developed first, as it is responsive to overs remaining in the innings and should influence R more heavily than G. It is split as a piecewise function, one part for powerplay overs and the other for the non-powerplay phase. A power law model is chosen for the powerplay phase given by Equation (5),

$$F(x) = C(A - b^{-(x-14)}), 14 \le x \le 20. (5)$$

Equation (5) is a power law which increases but plateaus, suitable for satisfying points 1-4 of Section II, hence shall be used for F in the non-powerplay phase and also G. The exponent is offset by 14 to capture the first 6 overs of an (uninterrupted) innings. Using F (14) and F (20) to obtain coefficients we get,

$$F(14) = C(A - 1), (6)$$

$$F(14) = C(A - 1),$$

$$F(20) = 1 = C(A - b^{-6}).$$
(6)

A is chosen to be 2 such that C = F(14) from Equation (6). Using these values of A and C, we get b from Equation (7).

To find F in the non-powerplay phase, use that F(0) = 0, F(14) for continuity from the powerplay phase, and F(10). On the lines of Equation (5), a power law of the type of Equation 8 is chosen,

$$F(x) = C\left(A - e^{-\frac{x}{b}}\right), \qquad 0 \le x \le 14. \tag{8}$$

The choice of base is arbitrary, and was chosen as e for its universality. The appearance of the free parameter in the exponent is to ensure solutions are obtained as the non-powerplay phase does not have an analytical solution. Using F(0) = 0 we get A = 1, and with this result and dividing using the latter two points we get,

$$\frac{1 - e^{-\frac{14}{b}}}{1 - e^{\frac{-10}{b}}} = \frac{F(14)}{F(10)}.$$
(9)

Solving Equation (9) numerically gives b, and consequentially C. Figure 1 plots the function F over all overs remaining for imposed values of Equation (12).

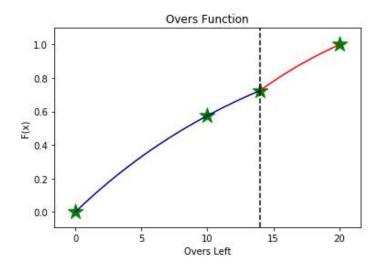


Figure 1: Overs Function F for all overs left. F is one part of the resource function R and denotes how resources for a chasing team with 10 wickets in hand fall. Values are for the conditions of Equation (12). The dashed line marks transition from powerplay to non-powerplay overs. Stars mark the imposed conditions.

Next, let the wickets function G be given by a power law of the same type as Equation (8), with w ranging from 0 to 10 inclusive. Impose that G(0) = 0, and find that A = 1. Impose a value on R (10, 7), such that a team with 10 overs and 7 wickets in hand these many remaining resources. Using this condition with Equation (2) we get,

$$R(10,7) = F(10) * G(7) \leftrightarrow G(7) = \frac{R(10,7)}{F(10)}.$$
 (10)

With the result of Equation (10) and the G(10) = 1 condition, and dividing using them in the same spirit as Equation (9) we get,

$$\frac{1 - e^{-\frac{10}{b}}}{1 - e^{\frac{-7}{b}}} = \frac{1}{G(7)}.$$
 (11)

Solving Equation (11) numerically gives b, and consequentially the value of C. Figure 2 plots the function G over all wickets remaining for imposed values of Equation (12).

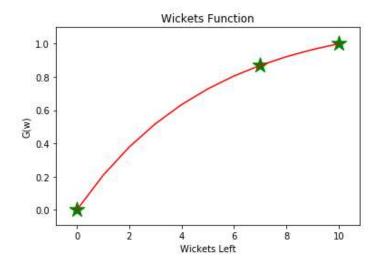


Figure 2: Wickets G for all wickets in hand. G is one part of the resource function R and denotes how resources for a chasing team fall with respect to wickets. Values are for conditions in Equation (12). Stars mark these imposed conditions.

Equations (13) and (14) explicitly represent R (x, w) for the powerplay and non-powerplay phases for the imposed conditions of Equation (12). Figure 3 utilizes them to plot R as a function of overs remaining for constant lines of wickets in hand.

$$F(14) = 0.725, F(10) = 0.575, R(10,7) = 0.50 \leftrightarrow G(7) = 0.869.$$
 (12)

$$R(x, w) = 0.725(2 - 1.082^{-(x - 14)}) * 1.161(1 - e^{-\frac{w}{5.075}}), 14 \le x \le 20$$
 (13)

$$R(x,w) = 0.725 \left(2 - 1.082^{-(x-14)}\right) * 1.161 \left(1 - e^{-\frac{w}{5.075}}\right), 14 \le x \le 20$$

$$R(x,w) = 1.285 \left(1 - e^{-\frac{x}{16.865}}\right) * 1.161 \left(1 - e^{-\frac{w}{5.075}}\right). \quad 0 \le x \le 14$$

$$(13)$$

Of course, this model depends on the conditions imposed which are summarized by Equation (12). These are not statistically produced values, but are very reasonable from learned insight of the game. To see how well this model behaves in match conditions, Section IV shows results of this model in 6 different match situations.

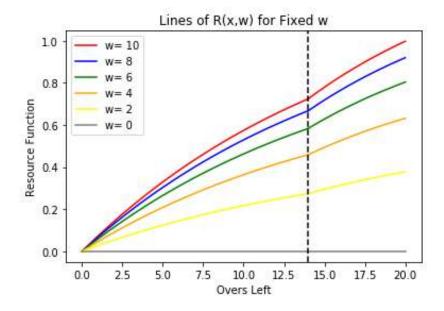


Figure 3: Resource Function R for all overs left and certain values of wickets in hand. R is the complete resource function used for par score calculation. Dashed line marks transition from powerplay to non-powerplay overs. Values are for imposed conditions of Equation (12).

IV. Usage Examples

Below are examples of how to use the new model in certain match situations. For easy reference, values used in this section are encapsulated in **Tables 1 and 2 in the appendix** of this paper.

IV. a. India vs South Africa, March 30, 2012

This game is an example of the team batting first using all of its resources and the chasing team's innings being interrupted by rain. India was 71/0 after 7.5 overs in pursuit of South Africa's 220 and lost the match by 11 runs. According to the new model, India's target score for x=12.167 and w=10 is given by Equations (1) and (14), and is 219*(1-R (12.167, 10)) =75 (rounded up). Below are the target scores for different values of w:

W	10	9	8	7	6	5	4	3	2	1	0
Target	7.5	00	06	0.4	402	444	120	4.45	4.65	400	220
Score	75	80	86	94	103	114	128	145	165	189	220

So according to the new model, India would still have lost though by 3 runs. Notice that if India had come out to bat knowing that they had only 7.5 overs and 10 wickets in hand, then the target score would have been larger. ICC rules state that for a T20 game between 5-8 overs, there are 2 powerplay overs. Thus India's available resource in this case would be R (16, 10)-R (14, 10) + R (5.5, 10) =0.830-0.725+0.375 =0.48, and the target score would have been 219*(0.48) = 106 (rounded up).

IV. b. Kolkata Knight Riders vs Sunrisers Hyderabad, May 17, 2017

This game is an example of the team batting first using all of its resources, and then the chasing team's innings starting with depleted resources. Sunrisers Hyderabad made 128/7 from 20 overs, and Kolkata Knight Riders required 48 runs with 6 overs and 10 wickets left. 2 of these overs were powerplay overs and 4 were non-powerplay overs. To find the total resources available in the powerplay, perform R (16, 10)-R (14, 10) =0.105. Likewise, the total resources available in the non-powerplay phase is R (4, 10) =0.271. So the target score for Kolkata Knight Riders using the new model is 128*(0.105+0.271) = 49 (rounded up), 1 more than the actual target.

IV. c. England vs West Indies, May 3, 2010

This game is an example of the team batting first using all of its resources, and then the chasing team's innings being interrupted by rain midway through their innings. This particular high-stakes World T20 match grabbed eyeballs because of the alleged unfair advantage given to the chasing team. England made 191/5 in their 20 overs, and West Indies were 30/0 in 2.2 overs when play was halted by rain. The revised target was 60 runs from 6 overs. According to the new model, the resources used inside the first 2.2 overs were 1-R (17.687, 10) =0.092, and the resources available in the last 3.4 overs were R (3.4, 10) =0.234. So the target score for West Indies using the new model is 191*(0.092+0.234) =63 (rounded up). Notice that if West Indies had lost wickets inside the first 2.2 overs, then the target score would have differed as follows:

W	10	9	8	7	6	5	4	3	2	1	0
Target											
Score	63	68	73	80	88	98	110	125	143	165	192

Also notice that if West Indies had come out to bat knowing only 6 overs were left, then the target score would have been 191*(R(16, 0)-R(14, 0)+R(4, 0)) = 72 (rounded up).

IV. d. Kolkata Knight Riders vs Kings XI Punjab, April 21, 2018

This is another example of a game where the team batting first utilized all their resources and the chasing team's innings was halted mid-play. Kings XI Punjab were at 96/0 from 8.2 overs, 31 runs ahead of the par score in response to Kolkata Knight Riders' 191/7. After play resumed, the revised target was 125 from 13 overs. According to the new model, the resources consumed before interruption were 1-R (11.668, 10) =0.358, and the resources available to completion were R (4.668, 10) =0.310. So the target score for the chasing team in this regime would be 191*(0.358+0.310) = 128 (rounded up).

IV. e. Delhi Capitals vs Rajasthan Royals, May 2, 2018

This is an example of a game where the team batting first had an interrupted innings, and the chasing team began with depleted resources. Starting with 18 overs and 10 wickets in hand, Delhi Capitals batted 17.1 overs to make 196/6 before rain halted play. The target for

Rajasthan Royals in 12 overs with 10 wickets in hand was 151. The total resources used by Delhi Capitals were [R (19, 10)-R (14, 10) +R (13, 10)]-R (0.835, 4) =0.885. The total resources available to Rajasthan were R (18, 10) – R (14, 10) + R (8, 10) = 0.680. According to Equation (1), the target for Rajasthan Royals is 196*(0.680/0.885) =151 (rounded up), exactly the same.

Again notice, that if Delhi Capitals had more wickets in hand, the target would have been even bigger. The converse is true if they had lost more than 6 wickets.

IV. f. Delhi Capitals vs Rajasthan Royals, April 11, 2018

This situation is very similar to the previous example but for an even shorter second innings. Rajasthan Royals made 153/5 from 17.5 overs and the revised target for Delhi Capitals from 6 overs was 71. The resources used by Rajasthan Royals were 1-R (2.167, 5) = 0.887, and the resources available to Delhi Capitals were R (16, 10)-R (14, 10) +R (4, 10) = 0.376. Again, Equation (1) tells that the target for Delhi Capitals is 153*(0.376/0.887) = 65 (rounded up).

One can argue that the last result gives the chasing team an unfair advantage, but the team batting first did exhaust 88.7 % of resources. This approximates a final target of 173 from 20 overs if it had not rained, so 65 from 6 would seem like a fair target. The glaring challenge is presented by example IV. a, which meant that a team scoring at 9.57 runs per over would be expected to chase at 12 runs per over in the last 12 overs, albeit with 10 wickets in hand. The DLS Model was updated to account for such high scoring scenarios that began to arise in 50 over cricket by factoring in the first innings score to calculate resources used. The next section will follow the same logic to create a dynamic version of the new model.

V. The Dynamic Model for Variable First Innings Totals

The values used in Equation (12) for F (14), F (10), R (10, 7) (and thereby G (7)), are suitable for first innings totals between 140 and 200. However, for high-scoring matches, chasing teams are expected to keep abreast with the required run rate, and number of wickets in hand becomes less crucial. Similarly for low-scoring games, teams can afford to go slowly at the start if they have wickets in hand. The static model uses fixed initial conditions and does not account for the first innings total. To expose this problem, a chasing team with a score of 128 runs for no loss of wickets in pursuit of 300 would be declared the winner in the static model regime. Clearly, the imposed conditions must take into account the first innings total on board, and this calls for the dynamic model.

Let F (14) be a power law functions of the first innings total S, such that it reads as F (14, S). Introduce values on F (14, S \leq 120), F (14, S \leq 250) and F (14, S \geq 300) such that it is a power law that decays slowly and teams must score quicker in the powerplay to chase large totals. Equation (15) fits this description,

$$F(14,S) = C(A + e^{-Sb}). (15)$$

A solution to the above problem would be to numerically solve for b for the following equation,

$$\frac{e^{-120b} - e^{-250b} \left(\frac{F(14, S \le 120)}{F(14, S = 250)}\right)}{\left(\frac{F(14, S \le 120)}{F(14, S = 250)}\right) - 1} + e^{-120b} = \left(\frac{F(14, S \le 120)}{F(14, S = 250)}\right),$$

$$\frac{e^{-120b} - e^{-250b} \left(\frac{F(14, S \le 120)}{F(14, S = 250)}\right) + e^{-300b}}{\left(\frac{F(14, S \le 120)}{F(14, S = 250)}\right) - 1} + e^{-300b}$$

Figure 4 plots F (14, S) for scores between 120 and 300 for imposed values of Equation (20).

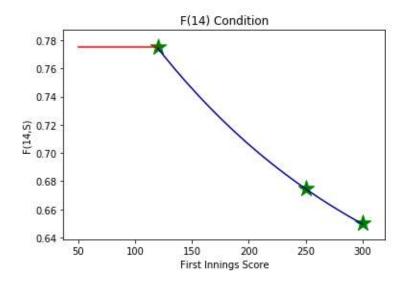


Figure 4: Condition on Overs Function at x=14 for several first innings scores S. In the dynamic regime, the imposed conditions for F (14), F (10) and R (10, 7) depend on S and follow a power law. Stars mark the imposed values of Equation (20).

Similarly, Let F (10) be power law functions of the first innings total S. Introduce values on F (10, S \leq 120), F (10, S \leq 250) and F (10, S \geq 300) such that it is also power law that decays slowly, and that teams have to get closer to half the target for big totals. A solution is again to numerically solve for b for the following equation,

$$\frac{e^{-120b} - e^{-250b} \left(\frac{F(10, S \le 120)}{F(10, S = 250)} \right)}{\left(\frac{F(10, S \le 120)}{F(10, S = 250)} \right) - 1} + e^{-120b} \\
\frac{e^{-120b} - e^{-250b} \left(\frac{F(10, S \le 120)}{F(10, S = 250)} \right) - 1}{\left(\frac{F(10, S \le 120)}{F(10, S = 250)} \right) - 1} + e^{-300b}$$

$$\frac{e^{-120b} - e^{-250b} \left(\frac{F(10, S \le 120)}{F(10, S = 250)} \right) - 1}{\left(\frac{F(10, S \le 120)}{F(10, S = 250)} \right) - 1} + e^{-300b}$$

Figure 5 plots F (10, S) for scores between 120 and 300 for the values in Equation (21).

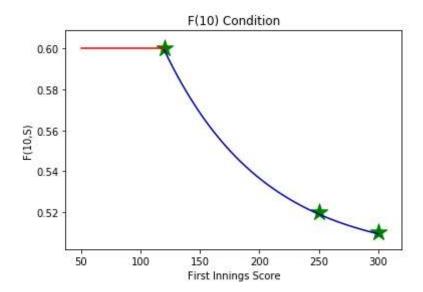


Figure 5: Condition on Overs Function at x=10 for several first innings scores S. In the dynamic regime, the imposed conditions for F (14), F (10) and R (10, 7) depend on S and follow a power law. Stars mark the imposed values of Equation (21).

Again, let R (10, 7) be power law functions of the first innings total S. Introduce values on R (10, 7, $S \le 120$), R (10, 7, S = 150) and R (10, 7, $S \ge 180$) such that it is a power law that decays quickly, so choose the form below,

$$R(10,7,S) = C(A - e^{Sb}). (18)$$

The choice of initial conditions on R (10, 7, S) impacts G (7, S) in the same way as Equation (10), but varies depending on S. A solution for R (10, 7, S) would be to numerically solve for b for the following equation,

$$\frac{e^{150b} \left(\frac{R(10,7,S \le 120)}{R(10,7,S = 150)}\right) - e^{120b}}{\left(\frac{R(10,7,S \le 120)}{R(10,7,S = 150)}\right) - 1} - e^{120b}} - e^{120b} \\
\frac{\left(\frac{R(10,7,S \le 120)}{R(10,7,S = 150)}\right) - 1}{\left(\frac{R(10,7,S \le 120)}{R(10,7,S = 150)}\right) - e^{120b}} - e^{180b}}{\left(\frac{R(10,7,S \le 120)}{R(10,7,S = 150)}\right) - 1}$$

Figure 6 plots R (10, 7, S) for scores between 120 and 180 for the imposed conditions of Equation (22).

Equations (23) through (25) explicitly mention the functional forms of these initial conditions, for the imposed conditions mentioned in Equations (20) through (22).

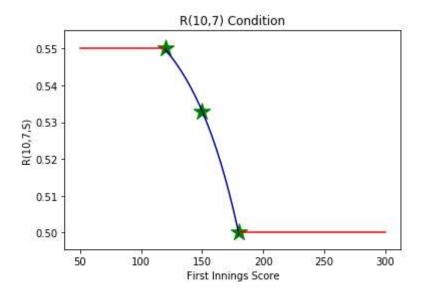


Figure 6: Condition on Resource Function at x=10, w=7 for several first innings scores S. In the dynamic regime, the imposed conditions for F (14), F (10) and R (10, 7) depend on S and follow a power law. Stars mark the imposed values of Equation (22).

$$F(14, \le 120) = 0.775, F(14, 250) = 0.675, F(14, \ge 300) = 0.650, \tag{20}$$

$$F(10, \le 120) = 0.600, F(10, 250) = 0.520, F(10, \ge 300) = 0.510, \tag{21}$$

$$R(10, 7, \le 120) = 0.550, R(10, 7, 150) = 0.533, R(10, 7, \ge 180) = 0.500.$$
 (22)

$$F(14,S) = 0.385(1.438 + e^{-0.00464S}), (23)$$

$$F(10,S) = 0.429(1.160 + e^{-0.012S}), (24)$$

$$R(10,7,S) = 0.00106(532.883 - e^{0.023S}). (25)$$

With these formulae, resource tables for each first innings total S is calculated under the dynamic regime, by repeating the work of the static regime for a specific S value. Resource tables and functional forms of R (x, w, S) are given for few values of S in the appendix. Much like in the static case, the results depend on imposed conditions of the dynamic model. The values adopted in Equations (20) through (22) are not statistically produced, but are very reasonable from learned insight.

As a check, consider example IV. a again. The target score for India using the dynamic model after about 8 overs and 10 wickets in hand in pursuit of 220 is 85 using Table 14 in the appendix, so South Africa would have won the match by a healthier margin of 13 runs.

VI. Comments on Functional Forms, Refining Resource Values and Model Selection

The values for the resource function R are ultimately dependent on the imposed conditions such as those of Equations (12) and (20) through (22), and the functional forms of Equations (5), (8), (15) and (18). There is no good justification for using classes such as trigonometric, and logarithmic functions don't cover the entire number line. The other competition to power laws would seem like algebraic functions, but power laws outclass them with a small number

of free parameters (three used for every function in this treatment) and high degree of controllability on concavity.

The traditional method to find the power laws of the static model would be to look at completed first innings data for several T20 matches (in spite of this model having its use in determining the par score for a run chase, data from a completed first innings is used as this ensures all resources are utilized), and fit these power laws to a given (x, w) pair using the formula below using non-linear regression,

$$R(x,w) = 1 - \frac{score(x,w)}{final\ score}.$$
 (26)

This would automatically set the conditions of Equation (12) without having to impose them. However as remarked, first innings scores that are unnaturally low or high do not get proper representation with such a method and the concepts of the dynamic model must be employed. The author suggests grouping first innings totals into ranges and performing different functional fits for these groupings. This removes the need for Equations (20) through (22).

The dynamic regime faces some ambiguity when the first innings is not completed as in examples IV. e and IV. f, as the resource table to use depends on S. Also consider the case when the second innings begins with lesser overs remaining in pursuit of a large total as in examples IV. c and IV. d. Use of the dynamic model results in a smaller target, because the imposed conditions of this regime weight the middle overs stronger than the static model, and hence reduces end over resource value. For cases such as these the author suggests to use the static model, as it weights overs according to the intuitive flow of a T20 game. The dynamic model was conceived to fix the case when rain halts play after the chasing team began with full resources in hand, so it is best reserved for situations such as example IV. a.

VII. Conclusion

The Duckworth-Lewis-Stern Method calculates par scores in rain-affected limited overs games of cricket based on the resources used by both teams and the first innings score. This was advancement over a method used for 50-over matches, and does not capture the pace of a T20-game. The new static model developed here weights powerplay and non-powerplay overs separately and emphasizes on chasing team's keeping abreast with the required run rate. The model was tested for some past T20 matches and produced very satisfactory results, except for the case when the first innings score was unnaturally high and the second innings did not resume after having begun with 20 overs and 10 wickets in hand. The dynamic model was conceived to fix this problem and passes conditions to the static model based on the first innings score. The key change the dynamic model made was to weight powerplay and and middle overs higher than the static model for larger targets and vice versa for smaller targets.

The values adopted for both models here are not statistically produced, but are chosen based on learned insight of the game. As remarked in Section VI, the most comprehensive but bulky method to produce resource functions would be to perform a non-linear fit on data from past T20 games with a completed first innings (or in the rarer case, a second innings

which goes to the last ball and was a victorious result). The discussion here does not perform this rigorous method, but introduces parameterization techniques which can better improve the DLS Method for T20 games.

Appendix

A. 1. The Static Model

$$F(14) = 0.725, F(10) = 0.575, R(10,7) = 0.50 \leftrightarrow G(7) = 0.869.$$
 (12)

$$R(x, w) = 0.725(2 - 1.082^{-(x-14)}) * 1.161(1 - e^{-\frac{w}{5.075}}), 14 \le x \le 20$$
 (13)

$$R(x,w) = 0.725(2 - 1.082^{-(x-14)}) * 1.161(1 - e^{-\frac{w}{5.075}}), 14 \le x \le 20$$

$$R(x,w) = 1.285(1 - e^{-\frac{x}{16.865}}) * 1.161(1 - e^{-\frac{w}{5.075}}). \quad 0 \le x \le 14$$
(13)

Table 1: R(x,w) in the powerplay phase for the static model.

x/w	10	9	8	7	6	5	4	3	2	1	0
6	1.000	0.962	0.919	0.867	0.804	0.726	0.632	0.517	0.377	0.207	0.000
5	0.960	0.926	0.885	0.835	0.774	0.699	0.609	0.498	0.363	0.200	0.000
4	0.920	0.888	0.848	0.800	0.741	0.670	0.583	0.477	0.348	0.191	0.000
3	0.877	0.846	0.808	0.762	0.707	0.639	0.556	0.455	0.332	0.182	0.000
2	0.830	0.801	0.765	0.722	0.669	0.604	0.526	0.430	0.314	0.172	0.000
1	0.779	0.752	0.718	0.678	0.628	0.567	0.494	0.404	0.295	0.162	0.000
0	0.725	0.699	0.668	0.630	0.584	0.527	0.459	0.376	0.274	0.151	0.000

Table 2: R(x,w) in the non-powerplay phase for the static phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
14	0.725	0.699	0.667	0.630	0.583	0.527	0.459	0.376	0.274	0.150	0.000
13	0.690	0.666	0.636	0.600	0.556	0.502	0.437	0.358	0.261	0.143	0.000
12	0.654	0.631	0.603	0.568	0.527	0.476	0.414	0.339	0.247	0.136	0.000
11	0.615	0.593	0.567	0.535	0.496	0.448	0.390	0.319	0.233	0.128	0.000
10	0.575	0.554	0.529	0.499	0.463	0.418	0.364	0.298	0.217	0.119	0.000
9	0.531	0.512	0.489	0.462	0.428	0.387	0.336	0.275	0.201	0.110	0.000
8	0.485	0.468	0.447	0.422	0.391	0.353	0.307	0.251	0.184	0.101	0.000
7	0.436	0.421	0.402	0.379	0.351	0.318	0.276	0.226	0.165	0.091	0.000
6	0.384	0.371	0.354	0.334	0.310	0.280	0.244	0.199	0.145	0.080	0.000
5	0.329	0.318	0.304	0.286	0.265	0.240	0.209	0.171	0.125	0.068	0.000
4	0.271	0.262	0.250	0.236	0.218	0.197	0.172	0.141	0.103	0.056	0.000
3	0.209	0.202	0.193	0.182	0.169	0.152	0.133	0.109	0.079	0.043	0.000
2	0.144	0.139	0.132	0.125	0.116	0.105	0.091	0.074	0.054	0.030	0.000
1	0.074	0.071	0.068	0.064	0.060	0.054	0.047	0.038	0.028	0.015	0.000
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

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A. 2. The Dynamic Model

$$F(14, \le 120) = 0.775, F(14, 250) = 0.675, F(14, \ge 300) = 0.650, \tag{20}$$

$$F(10, \le 120) = 0.600, F(10, 250) = 0.520, F(10, \ge 300) = 0.510, \tag{21}$$

$$R(10, 7, \le 120) = 0.550, R(10, 7, 150) = 0.533, R(10, 7, \ge 180) = 0.500.$$
 (22)

$$F(14,S) = 0.385(1.438 + e^{-0.00464S}), (23)$$

$$F(10,S) = 0.429(1.160 + e^{-0.012S}), (24)$$

$$R(10,7,S) = 0.00106(532.883 - e^{0.023S}). (25)$$

$$R(x, w, 120) = 0.775(2 - 1.059^{-(x-14)}) * 1.069(1 - e^{-\frac{w}{3.645}}), 14 \le x \le 20$$
 (27)

$$R(x, w, 120) = 1.680 \left(1 - e^{-\frac{x}{22.651}}\right) * 1.069 \left(1 - e^{-\frac{w}{3.645}}\right). \quad 0 \le x \le 14$$
 (28)

Table 3: R(x, w, 120) in the powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
6	1.000	0.978	0.949	0.912	0.862	0.797	0.712	0.599	0.451	0.256	0.000
5	0.967	0.946	0.918	0.882	0.834	0.771	0.689	0.580	0.436	0.248	0.000
4	0.933	0.913	0.886	0.851	0.805	0.744	0.664	0.559	0.421	0.239	0.000
3	0.896	0.877	0.851	0.818	0.773	0.715	0.638	0.537	0.405	0.230	0.000
2	0.858	0.839	0.815	0.783	0.740	0.684	0.611	0.514	0.387	0.220	0.000
1	0.817	0.800	0.776	0.745	0.705	0.652	0.582	0.490	0.369	0.210	0.000
0	0.775	0.757	0.735	0.706	0.668	0.618	0.551	0.464	0.349	0.199	0.000

Table 4: R(x, w, 120) in the non-powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
14	0.775	0.758	0.736	0.707	0.668	0.618	0.552	0.464	0.350	0.199	0.000
13	0.734	0.718	0.697	0.669	0.633	0.585	0.523	0.440	0.331	0.188	0.000
12	0.691	0.676	0.656	0.630	0.596	0.551	0.492	0.414	0.312	0.177	0.000
11	0.646	0.632	0.614	0.590	0.558	0.516	0.460	0.388	0.292	0.166	0.000
10	0.600	0.587	0.570	0.550	0.517	0.478	0.427	0.360	0.271	0.154	0.000
9	0.551	0.539	0.523	0.503	0.475	0.439	0.392	0.330	0.249	0.141	0.000
8	0.500	0.489	0.475	0.456	0.431	0.399	0.356	0.300	0.226	0.128	0.000
7	0.447	0.437	0.424	0.407	0.385	0.356	0.318	0.268	0.202	0.115	0.000
6	0.391	0.383	0.371	0.357	0.337	0.312	0.278	0.234	0.176	0.100	0.000
5	0.333	0.326	0.316	0.304	0.287	0.265	0.237	0.200	0.150	0.085	0.000
4	0.272	0.266	0.258	0.248	0.235	0.217	0.194	0.163	0.123	0.070	0.000
3	0.208	0.204	0.198	0.190	0.180	0.166	0.148	0.125	0.094	0.053	0.000
2	0.142	0.139	0.135	0.130	0.123	0.113	0.101	0.085	0.064	0.036	0.000
1	0.073	0.071	0.069	0.066	0.063	0.058	0.052	0.044	0.033	0.019	0.000
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

$$R(x, w, 140) = 0.755(2 - 1.068^{-(x-14)}) * 1.045(1 - e^{-\frac{w}{3.185}}), 14 \le x \le 20$$

$$R(x, w, 140) = 1.850(1 - e^{-\frac{x}{26.696}}) * 1.045(1 - e^{-\frac{w}{3.185}}). \quad 0 \le x \le 14$$
(30)

$$R(x, w, 140) = 1.850 \left(1 - e^{-\frac{x}{26.696}} \right) * 1.045 \left(1 - e^{-\frac{w}{3.185}} \right). \quad 0 \le x \le 14$$
 (30)

Table 5: R(x, w, 140) in the powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
6	1.000	0.984	0.961	0.930	0.887	0.829	0.748	0.638	0.488	0.282	0.000
5	0.966	0.950	0.928	0.898	0.857	0.800	0.722	0.616	0.471	0.272	0.000
4	0.929	0.914	0.893	0.864	0.824	0.769	0.695	0.593	0.453	0.262	0.000
3	0.890	0.875	0.855	0.827	0.789	0.737	0.665	0.568	0.434	0.251	0.000
2	0.848	0.834	0.814	0.788	0.752	0.702	0.634	0.541	0.413	0.239	0.000
1	0.803	0.789	0.771	0.746	0.712	0.665	0.600	0.512	0.391	0.226	0.000
0	0.755	0.742	0.725	0.701	0.669	0.625	0.564	0.481	0.368	0.213	0.000

Table 6: R(x, w, 140) in the non-powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
14	0.755	0.742	0.725	0.701	0.669	0.625	0.564	0.481	0.368	0.213	0.000
13	0.713	0.701	0.685	0.663	0.632	0.590	0.533	0.455	0.348	0.201	0.000
12	0.670	0.658	0.643	0.622	0.594	0.554	0.501	0.427	0.326	0.189	0.000
11	0.625	0.614	0.600	0.580	0.554	0.517	0.467	0.398	0.304	0.176	0.000
10	0.578	0.568	0.555	0.537	0.512	0.478	0.432	0.369	0.282	0.163	0.000
9	0.529	0.520	0.508	0.492	0.469	0.438	0.396	0.338	0.258	0.149	0.000
8	0.479	0.471	0.460	0.445	0.424	0.396	0.358	0.305	0.233	0.135	0.000
7	0.427	0.419	0.410	0.396	0.378	0.353	0.319	0.272	0.208	0.120	0.000
6	0.372	0.366	0.358	0.346	0.330	0.308	0.278	0.237	0.181	0.105	0.000
5	0.316	0.311	0.303	0.294	0.280	0.261	0.236	0.201	0.154	0.089	0.000
4	0.257	0.253	0.247	0.239	0.228	0.213	0.192	0.164	0.125	0.072	0.000
3	0.197	0.193	0.189	0.183	0.174	0.163	0.147	0.125	0.096	0.055	0.000
2	0.134	0.131	0.128	0.124	0.118	0.111	0.100	0.085	0.065	0.038	0.000
1	0.068	0.067	0.065	0.063	0.060	0.056	0.051	0.043	0.033	0.019	0.000
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

$$R(x, w, 160) = 0.737(2 - 1.076^{-(x-14)}) * 1.044(1 - e^{-\frac{w}{3.165}}), 14 \le x \le 20$$
 (31)

$$R(x, w, 160) = 0.737(2 - 1.076^{-(x-14)}) * 1.044(1 - e^{-\frac{w}{3.165}}), 14 \le x \le 20$$

$$R(x, w, 160) = 1.941(1 - e^{-\frac{x}{29.304}}) * 1.044(1 - e^{-\frac{w}{3.165}}). \quad 0 \le x \le 14$$
(32)

Table 7: R(x, w, 160) in the powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
6	1.000	0.982	0.960	0.929	0.886	0.828	0.748	0.639	0.489	0.283	0.000
5	0.963	0.947	0.925	0.895	0.854	0.798	0.721	0.616	0.471	0.272	0.000
4	0.924	0.909	0.888	0.859	0.820	0.766	0.692	0.591	0.452	0.261	0.000
3	0.882	0.868	0.848	0.820	0.783	0.731	0.661	0.564	0.432	0.250	0.000
2	0.837	0.823	0.804	0.779	0.743	0.694	0.627	0.535	0.410	0.237	0.000
1	0.789	0.776	0.758	0.734	0.700	0.654	0.591	0.505	0.386	0.223	0.000
0	0.737	0.725	0.708	0.685	0.654	0.611	0.552	0.471	0.360	0.208	0.000

Table 8: R(x, w, 160) in the non-powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
14	0.737	0.725	0.708	0.685	0.654	0.611	0.552	0.471	0.361	0.209	0.000
13	0.695	0.684	0.668	0.647	0.617	0.576	0.521	0.445	0.340	0.197	0.000
12	0.652	0.641	0.627	0.606	0.579	0.541	0.489	0.417	0.319	0.184	0.000
11	0.607	0.597	0.584	0.565	0.539	0.504	0.455	0.388	0.297	0.172	0.000
10	0.561	0.552	0.539	0.522	0.498	0.465	0.420	0.359	0.274	0.159	0.000
9	0.513	0.505	0.493	0.477	0.455	0.425	0.384	0.328	0.251	0.145	0.000
8	0.464	0.456	0.445	0.431	0.411	0.384	0.347	0.296	0.227	0.131	0.000
7	0.412	0.406	0.396	0.383	0.366	0.342	0.309	0.264	0.202	0.117	0.000
6	0.359	0.353	0.345	0.334	0.319	0.298	0.269	0.230	0.176	0.102	0.000
5	0.304	0.299	0.292	0.283	0.270	0.252	0.228	0.195	0.149	0.086	0.000
4	0.248	0.244	0.238	0.230	0.220	0.205	0.185	0.158	0.121	0.070	0.000
3	0.189	0.186	0.181	0.176	0.168	0.157	0.141	0.121	0.092	0.053	0.000
2	0.128	0.126	0.123	0.119	0.114	0.106	0.096	0.082	0.063	0.036	0.000
1	0.065	0.064	0.063	0.061	0.058	0.054	0.049	0.042	0.032	0.018	0.000
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

$$R(x, w, 180) = 0.721(2 - 1.085^{-(x-14)}) * 1.074(1 - e^{-\frac{w}{3.735}}), 14 \le x \le 20$$
 (33)

$$R(x, w, 180) = 0.721(2 - 1.085^{-(x-14)}) * 1.074(1 - e^{-\frac{w}{3.735}}), 14 \le x \le 20$$

$$R(x, w, 180) = 1.986(1 - e^{-\frac{x}{31.045}}) * 1.074(1 - e^{-\frac{w}{3.735}}). \quad 0 \le x \le 14$$
(34)

Table 9: R(x, w, 180) in the powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
6	1.000	0.978	0.948	0.909	0.859	0.792	0.706	0.593	0.445	0.252	0.000
5	0.963	0.941	0.912	0.875	0.826	0.763	0.679	0.571	0.429	0.243	0.000
4	0.922	0.901	0.874	0.838	0.791	0.730	0.651	0.547	0.410	0.233	0.000
3	0.878	0.858	0.832	0.798	0.753	0.695	0.619	0.520	0.391	0.221	0.000
2	0.830	0.811	0.786	0.754	0.712	0.657	0.586	0.492	0.369	0.209	0.000
1	0.778	0.760	0.737	0.707	0.668	0.616	0.549	0.461	0.346	0.196	0.000
0	0.721	0.705	0.683	0.656	0.619	0.571	0.509	0.428	0.321	0.182	0.000

Table 10: R(x, w, 180) in the non-powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
14	0.721	0.705	0.683	0.655	0.619	0.571	0.509	0.427	0.321	0.182	0.000
13	0.680	0.664	0.644	0.618	0.583	0.538	0.480	0.403	0.303	0.171	0.000
12	0.637	0.622	0.604	0.579	0.547	0.505	0.449	0.378	0.284	0.161	0.000
11	0.593	0.579	0.562	0.539	0.509	0.470	0.418	0.351	0.264	0.149	0.000
10	0.547	0.535	0.518	0.500	0.470	0.433	0.386	0.324	0.244	0.138	0.000
9	0.500	0.489	0.474	0.454	0.429	0.396	0.353	0.296	0.223	0.126	0.000
8	0.451	0.441	0.428	0.410	0.387	0.357	0.318	0.268	0.201	0.114	0.000
7	0.401	0.392	0.380	0.364	0.344	0.318	0.283	0.238	0.179	0.101	0.000
6	0.349	0.341	0.331	0.317	0.300	0.277	0.246	0.207	0.155	0.088	0.000
5	0.295	0.289	0.280	0.269	0.254	0.234	0.209	0.175	0.132	0.075	0.000
4	0.240	0.235	0.228	0.218	0.206	0.190	0.169	0.142	0.107	0.061	0.000
3	0.183	0.179	0.173	0.166	0.157	0.145	0.129	0.108	0.081	0.046	0.000
2	0.124	0.121	0.117	0.113	0.106	0.098	0.087	0.073	0.055	0.031	0.000
1	0.063	0.062	0.060	0.057	0.054	0.050	0.044	0.037	0.028	0.016	0.000
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

$$R(x, w, 200) = 0.706(2 - 1.094^{-(x-14)}) * 1.042(1 - e^{-\frac{w}{3.112}}), 14 \le x \le 20$$
 (35)

$$R(x, w, 200) = 0.706 \left(2 - 1.094^{-(x-14)}\right) * 1.042 \left(1 - e^{-\frac{w}{3.112}}\right), 14 \le x \le 20$$

$$R(x, w, 200) = 1.877 \left(1 - e^{-\frac{x}{29.675}}\right) * 1.042 \left(1 - e^{-\frac{w}{3.112}}\right). \quad 0 \le x \le 14$$
(36)

Table 11: R(x, w, 200) in the powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
6	1.000	0.984	0.962	0.932	0.891	0.833	0.754	0.645	0.494	0.286	0.000
5	0.962	0.946	0.925	0.896	0.856	0.801	0.725	0.620	0.475	0.275	0.000
4	0.919	0.905	0.884	0.857	0.818	0.766	0.693	0.592	0.454	0.263	0.000
3	0.873	0.859	0.840	0.814	0.777	0.727	0.658	0.563	0.431	0.250	0.000
2	0.822	0.809	0.791	0.766	0.732	0.685	0.620	0.530	0.406	0.235	0.000
1	0.767	0.755	0.738	0.715	0.683	0.639	0.578	0.494	0.379	0.220	0.000
0	0.706	0.695	0.679	0.658	0.629	0.588	0.532	0.455	0.349	0.202	0.000

Table 12: R(x, w, 200) in the non-powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
14	0.706	0.695	0.679	0.658	0.629	0.588	0.532	0.455	0.349	0.202	0.000
13	0.666	0.655	0.641	0.621	0.593	0.555	0.502	0.429	0.329	0.191	0.000
12	0.624	0.614	0.601	0.582	0.556	0.520	0.471	0.402	0.308	0.179	0.000
11	0.581	0.572	0.559	0.542	0.518	0.484	0.438	0.375	0.287	0.166	0.000
10	0.537	0.528	0.517	0.500	0.478	0.447	0.405	0.346	0.265	0.154	0.000
9	0.491	0.483	0.473	0.458	0.437	0.409	0.370	0.317	0.243	0.141	0.000
8	0.444	0.437	0.427	0.413	0.395	0.369	0.334	0.286	0.219	0.127	0.000
7	0.394	0.388	0.380	0.368	0.351	0.329	0.297	0.254	0.195	0.113	0.000
6	0.344	0.338	0.331	0.320	0.306	0.286	0.259	0.221	0.170	0.098	0.000
5	0.291	0.286	0.280	0.271	0.259	0.242	0.219	0.188	0.144	0.083	0.000
4	0.237	0.233	0.228	0.221	0.211	0.197	0.178	0.153	0.117	0.068	0.000
3	0.180	0.178	0.174	0.168	0.161	0.150	0.136	0.116	0.089	0.052	0.000
2	0.122	0.120	0.118	0.114	0.109	0.102	0.092	0.079	0.060	0.035	0.000
1	0.062	0.061	0.060	0.058	0.055	0.052	0.047	0.040	0.031	0.018	0.000
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

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$$R(x, w, 220) = 0.693(2 - 1.103^{-(x-14)}) * 1.026(1 - e^{-\frac{w}{2.735}}), 14 \le x \le 20$$

$$R(x, w, 220) = 1.801(1 - e^{-\frac{x}{28.845}}) * 1.026(1 - e^{-\frac{w}{2.735}}). \quad 0 \le x \le 14$$
(38)

$$R(x, w, 220) = 1.801 \left(1 - e^{-\frac{x}{28.845}} \right) * 1.026 \left(1 - e^{-\frac{w}{2.735}} \right). \qquad 0 \le x \le 14$$
 (38)

Table 13: R(x, w, 220) in the powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
6	1.000	0.985	0.963	0.933	0.891	0.834	0.755	0.645	0.495	0.287	0.000
5	0.962	0.946	0.925	0.896	0.856	0.801	0.725	0.620	0.475	0.275	0.000
4	0.918	0.903	0.883	0.855	0.817	0.765	0.692	0.592	0.453	0.263	0.000
3	0.870	0.856	0.837	0.811	0.774	0.724	0.656	0.561	0.430	0.249	0.000
2	0.816	0.803	0.786	0.761	0.727	0.680	0.615	0.526	0.403	0.234	0.000
1	0.758	0.746	0.729	0.706	0.675	0.631	0.571	0.488	0.374	0.217	0.000
0	0.693	0.682	0.667	0.646	0.617	0.577	0.522	0.447	0.342	0.198	0.000

Table 14: R(x, w, 220) in the non-powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
14	0.692	0.684	0.672	0.656	0.631	0.596	0.546	0.473	0.369	0.218	0.000
13	0.653	0.645	0.634	0.619	0.596	0.563	0.515	0.447	0.348	0.205	0.000
12	0.613	0.605	0.595	0.580	0.559	0.528	0.483	0.419	0.326	0.193	0.000
11	0.571	0.564	0.554	0.541	0.521	0.492	0.450	0.390	0.304	0.179	0.000
10	0.527	0.521	0.512	0.500	0.481	0.454	0.416	0.361	0.281	0.166	0.000
9	0.482	0.477	0.469	0.457	0.440	0.416	0.381	0.330	0.257	0.152	0.000
8	0.436	0.431	0.424	0.413	0.398	0.376	0.344	0.298	0.232	0.137	0.000
7	0.388	0.383	0.377	0.367	0.354	0.334	0.306	0.265	0.207	0.122	0.000
6	0.338	0.334	0.328	0.320	0.308	0.291	0.267	0.231	0.180	0.106	0.000
5	0.286	0.283	0.278	0.271	0.261	0.247	0.226	0.196	0.153	0.090	0.000
4	0.233	0.230	0.226	0.221	0.213	0.201	0.184	0.159	0.124	0.073	0.000
3	0.178	0.176	0.173	0.168	0.162	0.153	0.140	0.122	0.095	0.056	0.000
2	0.121	0.119	0.117	0.114	0.110	0.104	0.095	0.082	0.064	0.038	0.000
1	0.061	0.061	0.060	0.058	0.056	0.053	0.048	0.042	0.033	0.019	0.000
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

$$R(x, w, 240) = 0.680(2 - 1.112^{-(x-14)}) * 1.017(1 - e^{-\frac{w}{2.465}}), 14 \le x \le 20$$

$$R(x, w, 240) = 1.614(1 - e^{-\frac{x}{25.585}}) * 1.017(1 - e^{-\frac{w}{2.465}}). \quad 0 \le x \le 14$$

$$(40)$$

$$R(x, w, 240) = 1.614 \left(1 - e^{-\frac{x}{25.585}}\right) * 1.017 \left(1 - e^{-\frac{w}{2.465}}\right). \quad 0 \le x \le 14$$
 (40)

Table 15: R(x, w, 240) in the powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
6	1.000	0.991	0.978	0.958	0.928	0.884	0.817	0.716	0.565	0.339	0.000
5	0.959	0.951	0.938	0.919	0.891	0.848	0.784	0.687	0.543	0.326	0.000
4	0.915	0.907	0.895	0.876	0.849	0.808	0.747	0.655	0.517	0.310	0.000
3	0.865	0.857	0.846	0.829	0.803	0.764	0.706	0.620	0.489	0.294	0.000
2	0.810	0.802	0.792	0.776	0.752	0.715	0.661	0.580	0.458	0.275	0.000
1	0.748	0.741	0.732	0.717	0.694	0.661	0.611	0.536	0.423	0.254	0.000
0	0.680	0.674	0.665	0.651	0.631	0.601	0.555	0.487	0.384	0.231	0.000

Table 16: R(x, w, 240) in the non-powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
14	0.680	0.674	0.665	0.651	0.631	0.601	0.555	0.487	0.384	0.231	0.000
13	0.643	0.637	0.628	0.616	0.597	0.568	0.525	0.460	0.363	0.218	0.000
12	0.604	0.599	0.591	0.579	0.561	0.534	0.493	0.433	0.342	0.205	0.000
11	0.564	0.559	0.551	0.540	0.523	0.498	0.460	0.404	0.319	0.191	0.000
10	0.522	0.517	0.510	0.500	0.484	0.461	0.426	0.374	0.295	0.177	0.000
9	0.478	0.474	0.468	0.458	0.444	0.423	0.391	0.343	0.271	0.162	0.000
8	0.433	0.429	0.424	0.415	0.402	0.383	0.354	0.310	0.245	0.147	0.000
7	0.386	0.383	0.378	0.370	0.358	0.341	0.315	0.277	0.218	0.131	0.000
6	0.337	0.334	0.330	0.323	0.313	0.298	0.275	0.242	0.191	0.114	0.000
5	0.286	0.284	0.280	0.274	0.266	0.253	0.234	0.205	0.162	0.097	0.000
4	0.233	0.231	0.228	0.224	0.217	0.206	0.191	0.167	0.132	0.079	0.000
3	0.178	0.177	0.175	0.171	0.166	0.158	0.146	0.128	0.101	0.061	0.000
2	0.121	0.120	0.119	0.116	0.113	0.107	0.099	0.087	0.069	0.041	0.000
1	0.062	0.061	0.060	0.059	0.057	0.055	0.051	0.044	0.035	0.021	0.000
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

$$R(x, w, 260) = 0.669(2 - 1.120^{-(x-14)}) * 1.011(1 - e^{-\frac{w}{2.220}}), 14 \le x \le 20$$
 (41)

$$R(x, w, 260) = 0.669(2 - 1.120^{-(x-14)}) * 1.011(1 - e^{-\frac{w}{2.220}}), 14 \le x \le 20$$

$$R(x, w, 260) = 1.477(1 - e^{-\frac{x}{23.215}}) * 1.011(1 - e^{-\frac{w}{2.220}}). \quad 0 \le x \le 14$$

$$(42)$$

Table 17: R(x, w, 260) in the powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
6	1.000	0.993	0.983	0.967	0.942	0.904	0.843	0.749	0.600	0.366	0.000
5	0.958	0.952	0.943	0.928	0.904	0.867	0.809	0.718	0.575	0.351	0.000
4	0.913	0.907	0.898	0.883	0.861	0.826	0.771	0.684	0.548	0.335	0.000
3	0.862	0.856	0.848	0.834	0.813	0.780	0.728	0.646	0.517	0.316	0.000
2	0.805	0.799	0.791	0.779	0.759	0.728	0.679	0.603	0.483	0.295	0.000
1	0.741	0.736	0.728	0.717	0.699	0.670	0.625	0.555	0.445	0.272	0.000
0	0.669	0.665	0.658	0.647	0.631	0.605	0.565	0.501	0.402	0.245	0.000

Table 18: R(x, w, 260) in the non-powerplay phase.

x/w	10	9	8	7	6	5	4	3	2	1	0
14	0.669	0.665	0.658	0.647	0.631	0.605	0.565	0.501	0.402	0.245	0.000
13	0.633	0.629	0.623	0.613	0.597	0.573	0.535	0.475	0.380	0.232	0.000
12	0.596	0.592	0.586	0.577	0.562	0.539	0.503	0.447	0.358	0.219	0.000
11	0.557	0.554	0.548	0.539	0.526	0.504	0.471	0.418	0.335	0.204	0.000
10	0.517	0.514	0.508	0.500	0.488	0.468	0.436	0.387	0.310	0.190	0.000
9	0.475	0.472	0.467	0.459	0.448	0.429	0.401	0.356	0.285	0.174	0.000
8	0.430	0.428	0.423	0.417	0.406	0.390	0.363	0.323	0.258	0.158	0.000
7	0.384	0.382	0.378	0.372	0.363	0.348	0.325	0.288	0.231	0.141	0.000
6	0.336	0.334	0.331	0.326	0.317	0.304	0.284	0.252	0.202	0.123	0.000
5	0.286	0.284	0.281	0.277	0.270	0.259	0.242	0.214	0.172	0.105	0.000
4	0.234	0.232	0.230	0.226	0.221	0.211	0.197	0.175	0.140	0.086	0.000
3	0.179	0.178	0.176	0.173	0.169	0.162	0.151	0.134	0.107	0.066	0.000
2	0.122	0.121	0.120	0.118	0.115	0.110	0.103	0.091	0.073	0.045	0.000
1	0.062	0.062	0.061	0.060	0.059	0.056	0.053	0.047	0.037	0.023	0.000
0	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000

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