Quantum Animations Discrete Hamiltonian Spectrum

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We want to find the eigen values and eigen vectors of a finite-dimensional space of dimension d. The Hamiltonian is given by,

$$(H\psi)_n = \psi_{n+1} + \psi_{n-1} + V_n \psi_n, \tag{1}$$

where V_n is some random potential. The random potential can either be equal to zero, in which case $H = \Delta$, or be sampled from some normal distribution with $\mu = 0$, $\sigma = 1$. The dimension is chosen to be in the range $20 \leq d \leq 200$.

The Hamiltonian can be visualized as (almost) a tridiagonal matrix. For the case $d = 5$, it looks like,

$$\begin{pmatrix} V_1 & 1 & 0 & 0 & 1 \\ 1 & V_2 & 1 & 0 & 0 \\ 0 & 1 & V_3 & 1 & 0 \\ 0 & 0 & 1 & V_4 & 1 \\ 1 & 0 & 0 & 1 & V_5 \end{pmatrix}. \tag{2}$$

Periodic boundary conditions have been applied in the elements that are most off-diagonal.

And that is all; the expectation is to obtain plane waves for the case $V_n = 0$ and that the spectrum is symmetric, with the symmetric quality improving as d is made large.