Quantum Animations PDEs

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1 Heat Equation

The heat equation is,

$$\partial_t u = \partial_{xx} u, \tag{1}$$

and we want to solve it for the initial and boundary conditions,

$$u(0,t) = u(1,t) = 0, \quad u(x,0) = sin(n\pi x), \quad x \in [0,1]. \tag{2}$$

This is a heated rod between two infinitely cold reservoirs. After separation of variables, the solution is simply,

$$u(x,t) = e^{-n^2\pi^2 t} sin(n\pi x). \tag{3}$$

2 Wave Equation

The wave equation is,

$$\partial_{tt} f = \partial_{xx} f, \tag{4}$$

and we want to solve it for the initial and boundary conditions,

$$f(0,t) = f(1,t) = 0, \quad f(x,0) = sin(n\pi x), \quad \partial_t f(x,0) = 0, \quad x \in [0,1]. \tag{5}$$

The solution is now,

$$f(x,t) = cos(n\pi t) sin(n\pi x). \tag{6}$$

3 Schrödinger Equation

Using $m = \hbar = 1$, the expression is,

$$i\partial_t \psi = -\frac{1}{2}\partial_{xx}\psi, \tag{7}$$

and we want to solve it for the initial and boundary conditions,

$$\psi(0,t) = \psi(1,t) = 0, \quad \psi(x,0) = sin(n\pi x), \quad x \in [0,1]. \tag{8}$$

This is an infinite square well. The solution is now,

$$\psi(x,t) = e^{-i\frac{n^2\pi^2 t}{2}} sin(n\pi x). \tag{9}$$