

# LCA

## 人员

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## 上周作业检查

上周作业链接: <https://cppoj.kids123code.com/contest/2238>

The screenshot shows a contest results page with a table of scores for five participants. The table has columns for rank (#), username, name, programming score (编程分), time (时间), and four specific problems (A, B, C, D). The participants and their scores are:

#	用户名	姓名	编程分	时间	A	B	C	D
1	yangjunyan	杨俊彦	400	11042	100	100	100	100
2	liuyichen	刘奕辰	300	4550	100	100	100	
3	yuanchenjun	袁晨峻	253	1904	100	100	53	
4	lijinshu	李锦澍	200	8453	100	100		
5	suitianyi	隋天乙	0	0	0			

## 本周作业

<https://cppoj.kids123code.com/contest/2329> (课上讲了 A ~ C 题, 课后作业是 D 题)

## 课堂表现

今天讲了 LCA 这个知识点, 同学们课上整体理解的都比较好。

## 课堂内容

### Good Tuple Problem (上周作业)

dfs 黑白染色判定二分图

```
#include <bits/stdc++.h>

using namespace std;

const int maxn = 2e5 + 5;
vector<int> vec[maxn];
int a[maxn], b[maxn], col[maxn];
bool flag = true;

void dfs(int u, int value) {
    if (!flag) return;
    if (col[u] == value) return;
    if (col[u] != -1 && col[u] != value) flag = false;
    col[u] = value;
}
```

```

    for (int i : vec[u]) dfs(i, 1-value);
}

int main()
{
    int n, m; cin >> n >> m;
    for (int i = 1; i <= m; ++i) cin >> a[i];
    for (int i = 1; i <= m; ++i) {
        cin >> b[i];
        vec[a[i]].push_back(b[i]), vec[b[i]].push_back(a[i]);
    }

    memset(col, -1, sizeof(col));
    for (int i = 1; i <= n; ++i) {
        if (col[i] != -1) continue;
        dfs(i, 0);
        if (!flag) { cout << "No" << endl; return 0; }
    }
    cout << "Yes" << endl;
    return 0;
}

```

## 【模板】最近公共祖先 (LCA)

倍增法求 最近公共祖先

$f[i][j]$ : i 这个点, 往上  $2^j$  的父亲是谁

(a,b) 的最近公共祖先: 先倍增往上跳, 直到两个点变到一层; 然后倍增往上判, 只要两个点不重合就往上, 最后返回 2 个点的父结点

```

#include <bits/stdc++.h>

using namespace std;

const int N = 5e5 + 5, M = 20;
vector<int> vec[N];
int dis[N], f[N][M];

void dfs(int u, int fa, int d) {
    dis[u] = d; f[u][0] = fa;
    for (int i = 1; i < M; ++i) f[u][i] = f[f[u][i-1]][i-1];
    for (int i : vec[u]) {
        if (i == fa) continue;
        dfs(i, u, d+1);
    }
}

int lca(int a, int b) {
    if (dis[a] < dis[b]) return lca(b, a);

```

```

for (int i = M-1; i >= 0; --i) {
    if (dis[f[a][i]] >= dis[b]) a = f[a][i];
}

if (a == b) return a;

for (int i = M-1; i >= 0; --i) {
    if (f[a][i] != f[b][i]) a = f[a][i], b = f[b][i];
}
return f[a][0];
}

int main()
{
    ios::sync_with_stdio(false);
    cin.tie(0);

    int n, m, root; cin >> n >> m >> root;
    for (int i = 1; i <= n-1; ++i) {
        int a, b; cin >> a >> b;
        vec[a].push_back(b), vec[b].push_back(a);
    }

    dfs(root, 0, 1);

    while (m-- ) {
        int a, b; cin >> a >> b;
        cout << lca(a, b) << "\n";
    }
    return 0;
}

```

## [GESP202312 八级] 大量的工作沟通

dfs 中, 维护 ans[i]: 从根到 i 中最大的编号是多少

找到所有点的 LCA 点 id, 答案即为 ans[id]

```

#include <bits/stdc++.h>

using namespace std;

const int N = 1e5 + 5, M = 20;
vector<int> vec[N];
int dis[N], f[N][M], ans[N];

void dfs(int u, int fa, int d) {
    dis[u] = d; f[u][0] = fa; ans[u] = max(ans[fa], u);
    for (int i = 1; i < M; ++i) f[u][i] = f[f[u][i-1]][i-1];
    for (int i : vec[u]) {
        if (i == fa) continue;

```

```

        dfs(i, u, d+1);
    }
}

int lca(int a, int b) {
    if (dis[a] < dis[b]) return lca(b, a);

    for (int i = M-1; i >= 0; --i) {
        if (dis[f[a][i]] >= dis[b]) a = f[a][i];
    }

    if (a == b) return a;

    for (int i = M-1; i >= 0; --i) {
        if (f[a][i] != f[b][i]) a = f[a][i], b = f[b][i];
    }
    return f[a][0];
}

void solve() {
    int m; cin >> m;
    int fa; cin >> fa; ++fa;
    for (int i = 2; i <= m; ++i) {
        int x; cin >> x; ++x; fa = lca(fa, x);
    }

    cout << ans[fa] - 1 << endl;
}

int main()
{
    int n; cin >> n;
    for (int i = 2; i <= n; ++i) {
        int id; cin >> id; ++id; vec[id].push_back(i);
    }

    dfs(1, 0, 1);

    int T; cin >> T;
    while (T -- ) solve();
    return 0;
}

```

## [蓝桥杯 2025 国 Python A] 巡逻

维护 2 个数组

$dis[i]$ : 从根到  $i$ , 中间的点数

$p[i]$ : 从根到  $i$ , 中间的风险和

树上任意两点  $i j$  之间的距离为:  $dis[i] + dis[j] - 2 * dis[fa]$

树上任意两点 i j 之间的路径和为:  $p[i] + p[j] - 2*p[fa]$

```
#include <bits/stdc++.h>
#define int long long

using namespace std;

const int N = 5000 + 5, M = 20;
struct node {
    int to, value;
};
vector<node> vec[N];
int dis[N], f[N][M], p[N];

void dfs(int u, int fa, int d) {
    dis[u] = d; f[u][0] = fa;
    for (int i = 1; i < M; ++i) f[u][i] = f[f[u][i-1]][i-1];
    for (node it : vec[u]) {
        if (it.to == fa) continue;
        p[it.to] = p[u] + it.value;
        dfs(it.to, u, d+1);
    }
}

int lca(int a, int b) {
    if (dis[a] < dis[b]) return lca(b, a);

    for (int i = M-1; i >= 0; --i) {
        if (dis[f[a][i]] >= dis[b]) a = f[a][i];
    }

    if (a == b) return a;

    for (int i = M-1; i >= 0; --i) {
        if (f[a][i] != f[b][i]) a = f[a][i], b = f[b][i];
    }
    return f[a][0];
}

signed main()
{
    int n, K; cin >> n >> K;
    for (int i = 1; i <= n-1; ++i) {
        int a, b, c; cin >> a >> b >> c;
        vec[a].push_back({b,c}), vec[b].push_back({a,c});
    }

    dfs(1, 0, 1);

    int res = 0;
    for (int i = 1; i <= n; ++i) {
        for (int j = 1; j <= n; ++j) {
            int fa = lca(i, j);
            res += p[i] + p[j] - 2*p[fa];
        }
    }
}
```

```
    if (dis[i]+dis[j]-2*dis[fa] == K) res += p[i]+p[j]-2*p[fa];
}
cout << res << endl;
return 0;
}
```