

# 树上差分+01bfs

## 人员

阮文璋、褚锦轩、王承周、许睿谦、王毅博 到课

## 上周作业检查

上周作业链接: <https://cppoj.kids123code.com/contest/2331>

#	用户名	姓名	编程分	时间	A	B	C	D
1	xuruiqian	许睿谦	400	36090	100	100	100	100
2	ruanwenzhang	阮文璋	400	36092	100	100	100	100
3	wangchengzhou	王承周	400	37391	100	100	100	100
4	chujinxuan	褚锦轩	300	37480	100	100	100	

## 本周作业

<https://cppoj.kids123code.com/contest/2455> (课上讲了 A ~ C 题, 课后作业是 D 题)

## 课堂表现

今天讲了 树上差分 和 01bfs 这两个小知识点, 同学们课上听课吸收整体都不错。

## 课堂内容

### [蓝桥杯 2022 国 B] 机房 (上周作业)

对每个点的 deg 维护一个前缀和, 然后后续用 LCA 就可以 O(1) 求了

```
#include <bits/stdc++.h>

using namespace std;

const int N = 1e5 + 5, M = 20;
vector<int> vec[N];
int dis[N], f[N][M], deg[N], p[N];

void dfs(int u, int fa, int d) {
    dis[u] = d; f[u][0] = fa; p[u] = p[fa] + deg[u];
    for (int i = 1; i < M; ++i) f[u][i] = f[f[u][i-1]][i-1];
    for (int i : vec[u]) {
        if (i == fa) continue;
        dfs(i, u, d+1);
    }
}
```

```

}

int lca(int a, int b) {
    if (dis[a] < dis[b]) return lca(b, a);

    for (int i = M-1; i >= 0; --i) {
        if (dis[f[a][i]] >= dis[b]) a = f[a][i];
    }

    if (a == b) return a;

    for (int i = M-1; i >= 0; --i) {
        if (f[a][i] != f[b][i]) a = f[a][i], b = f[b][i];
    }
    return f[a][0];
}

int main()
{
    int n, m; cin >> n >> m;
    for (int i = 1; i <= n-1; ++i) {
        int a, b; cin >> a >> b; ++deg[a], ++deg[b];
        vec[a].push_back(b), vec[b].push_back(a);
    }

    dfs(1, 0, 1);

    while (m-- ) {
        int a, b; cin >> a >> b;
        if (a == b) cout << deg[a] << endl;
        else {
            int fa = lca(a, b);
            cout << p[a] + p[b] - 2*p[fa] + deg[fa] << endl;
        }
    }
    return 0;
}

```

## [USACO15DEC] Max Flow P

树上差分, 对树上的  $u \sim v$  路径进行 +1 时, 可以找到它们的最近公共祖先 id 点

对  $w[u]++$ ,  $w[v]++$ ,  $w[id]--$ ,  $w[fa[id][0]]--$  即可

最后做一遍 dfs, 把子节点的 w 信息自底向上维护一下

```

#include <bits/stdc++.h>

using namespace std;

const int N = 5e4 + 5, M = 20;

```

```
vector<int> vec[N];
int dis[N], f[N][M], w[N];

void dfs(int u, int fa) {
    dis[u] = dis[fa]+1, f[u][0] = fa;
    for (int i = 1; i < M; ++i) f[u][i] = f[f[u][i-1]][i-1];

    for (int i : vec[u]) {
        if (i != fa) dfs(i, u);
    }
}

int lca(int a, int b) {
    if (dis[a] < dis[b]) return lca(b, a);

    for (int i = M-1; i >= 0; --i) {
        if (dis[f[a][i]] >= dis[b]) a = f[a][i];
    }

    if (a == b) return a;

    for (int i = M-1; i >= 0; --i) {
        if (f[a][i] != f[b][i]) a = f[a][i], b = f[b][i];
    }

    return f[a][0];
}

void dfs2(int u, int fa) {
    for (int i : vec[u]) {
        if (i == fa) continue;
        dfs2(i, u);
        w[u] += w[i];
    }
}

int main()
{
    int n, m; cin >> n >> m;
    for (int i = 1; i <= n-1; ++i) {
        int a, b; cin >> a >> b;
        vec[a].push_back(b), vec[b].push_back(a);
    }

    dfs(1, 0);

    while (m-- ) {
        int a, b; cin >> a >> b;
        int u = lca(a, b), fu = f[u][0];
        ++w[a], ++w[b], --w[u], --w[fu];
    }

    dfs2(1, 0);
}
```

```

int res = 0;
for (int i = 1; i <= n; ++i) res = max(res, w[i]);
cout << res << endl;
return 0;
}

```

## [USACO12MAR] Tractor S

01bfs, 用 deque 维护即可, 边权为 0 的往前放, 边权为 1 的往后放

```

#include <bits/stdc++.h>

using namespace std;

const int maxn = 1000 + 5;
struct node {
    int x, y;
};
int dis[maxn][maxn], c[maxn][maxn];
bool st[maxn][maxn];
int dx[] = {-1, 1, 0, 0}, dy[] = {0, 0, -1, 1};

int main()
{
    int n, x0, y0; cin >> n >> x0 >> y0;
    while (n -- ) {
        int x, y; cin >> x >> y; ++c[x][y];
    }

    memset(dis, 0x3f, sizeof(dis));
    deque<node> dq; dq.push_back({x0,y0}); dis[x0][y0] = 0;
    while (!dq.empty()) {
        node u = dq.front(); dq.pop_front();
        int x = u.x, y = u.y;
        if (st[x][y]) continue;
        st[x][y] = true;

        for (int i = 0; i < 4; ++i) {
            int nx = x+dx[i], ny = y+dy[i];
            if (nx>=0 && nx<maxn && ny>=0 && ny<maxn && dis[x][y]+c[nx][ny]<dis[nx][ny])
            {
                dis[nx][ny] = dis[x][y]+c[nx][ny];
                if (!c[nx][ny]) dq.push_front({nx,ny});
                else dq.push_back({nx,ny});
            }
        }
    }

    cout << dis[0][0] << endl;
}

```

```

    return 0;
}

```

### [GESP202503 八级] 割裂

树上差分, 把好点对之间的路径都进行 +1 操作, 把坏点对之间的路径都进行 -1 操作, 最后看有哪些点是 -1 即可

```

#include <bits/stdc++.h>

using namespace std;

const int N = 1e6 + 5, M = 20;
vector<int> vec[N];
int dis[N], f[N][M], w[N];

void dfs(int u, int fa) {
    dis[u] = dis[fa]+1, f[u][0] = fa;
    for (int i = 1; i < M; ++i) f[u][i] = f[f[u][i-1]][i-1];

    for (int i : vec[u]) {
        if (i != fa) dfs(i, u);
    }
}

int lca(int a, int b) {
    if (dis[a] < dis[b]) return lca(b, a);

    for (int i = M-1; i >= 0; --i) {
        if (dis[f[a][i]] >= dis[b]) a = f[a][i];
    }

    if (a == b) return a;

    for (int i = M-1; i >= 0; --i) {
        if (f[a][i] != f[b][i]) a = f[a][i], b = f[b][i];
    }
}

return f[a][0];
}

void dfs2(int u, int fa) {
    for (int i : vec[u]) {
        if (i == fa) continue;
        dfs2(i, u);
        w[u] += w[i];
    }
}

int main()
{
    int n, m; cin >> n >> m;

```

```
for (int i = 1; i <= n-1; ++i) {
    int a, b; cin >> a >> b;
    vec[a].push_back(b), vec[b].push_back(a);
}

dfs(1, 0);

for (int i = 1; i <= m+1; ++i) {
    int a, b; cin >> a >> b;
    int u = lca(a, b), fu = f[u][0];
    if (i <= m) ++w[a], ++w[b], --w[u], --w[fu];
    else --w[a], --w[b], ++w[u], ++w[fu];
}

dfs2(1, 0);

int res = 0;
for (int i = 1; i <= n; ++i) {
    if (w[i] < 0) ++res;
}
cout << res << endl;
return 0;
}
```