

$$I_m = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 5 & 6 & 7 & 8 \\ 9 & 10 & 11 & 12 \\ 13 & 14 & 15 & 16 \end{bmatrix}$$

$$\Rightarrow I = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ \vdots \\ 16 \end{bmatrix}$$

$$K = \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \\ -7 & -8 & -9 \end{bmatrix}$$

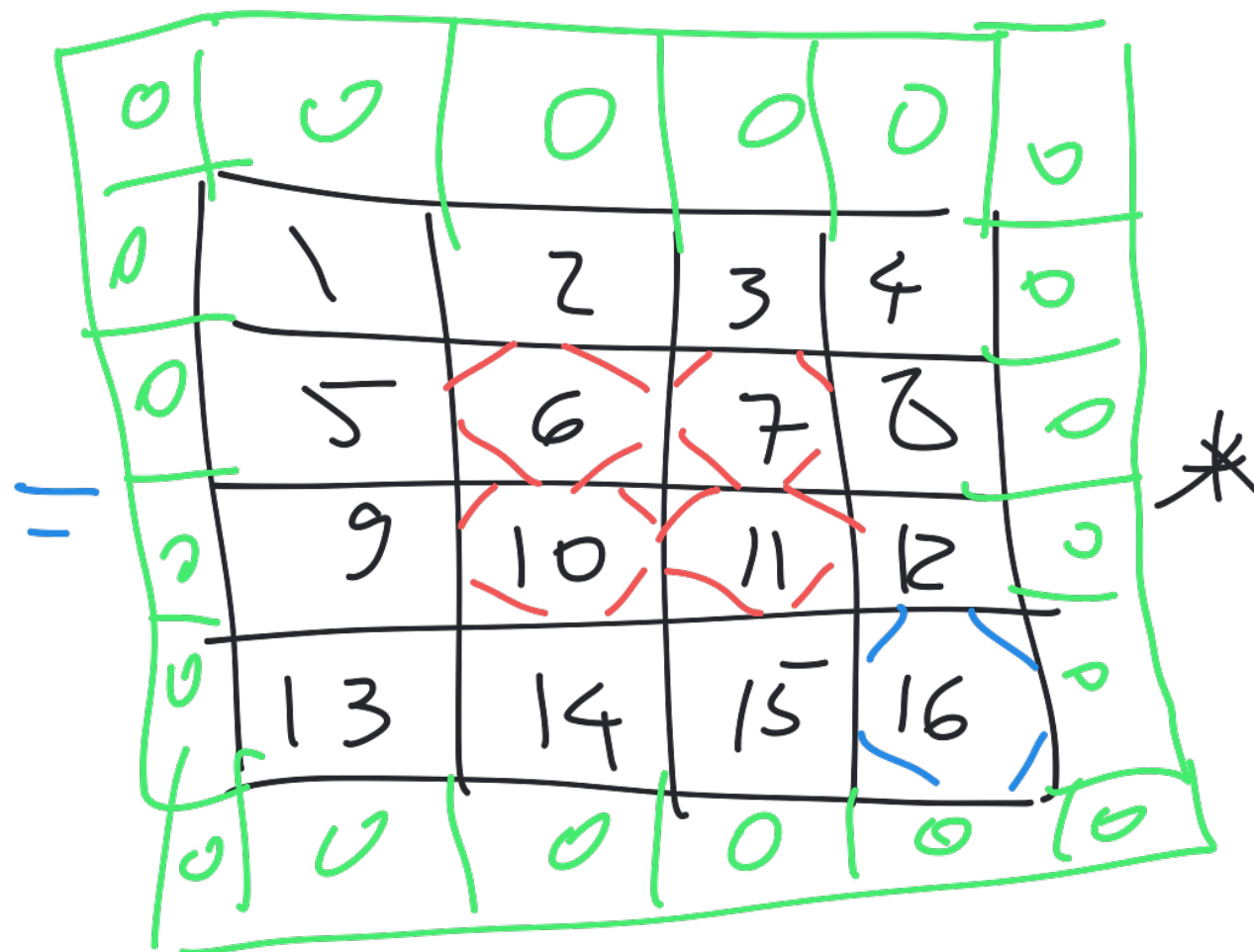
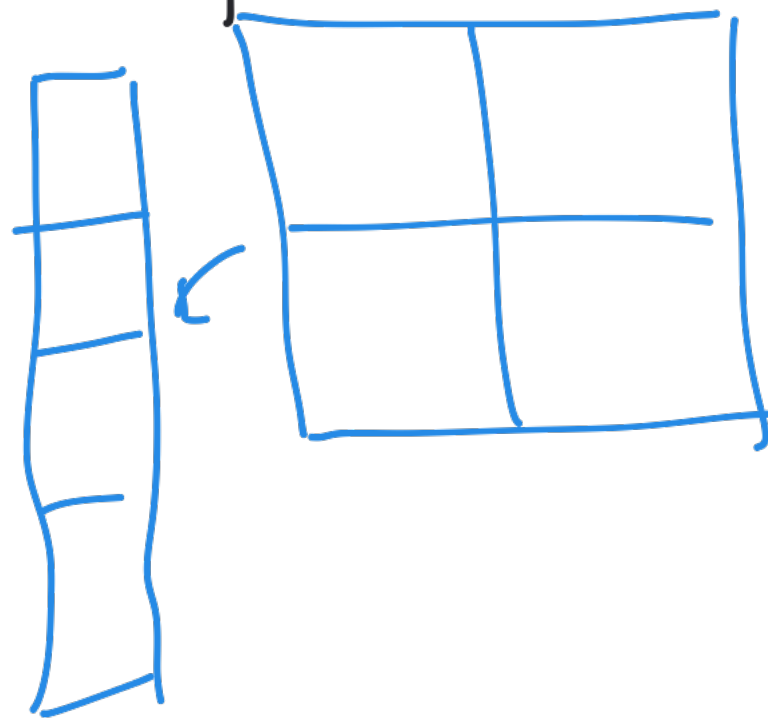
$$I = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ \vdots \\ 16 \end{bmatrix} \quad 16 \times 1$$

$$h = G \cdot I$$

4×1 4×16 16×1

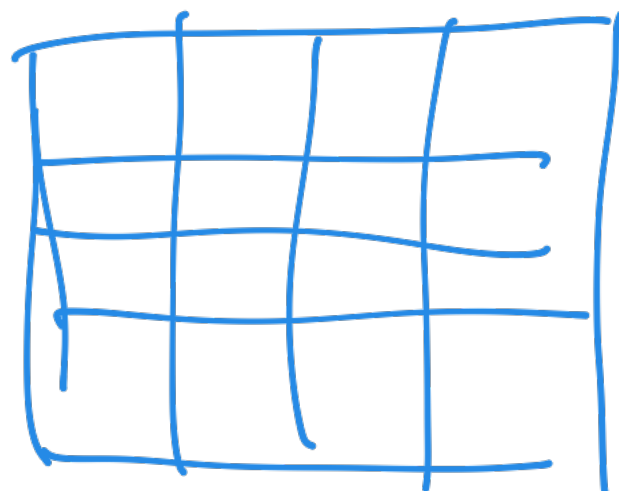
$$G = ?$$

without padding



-1	-2	-3
-4	-5	-6
-7	-8	-9

with padding



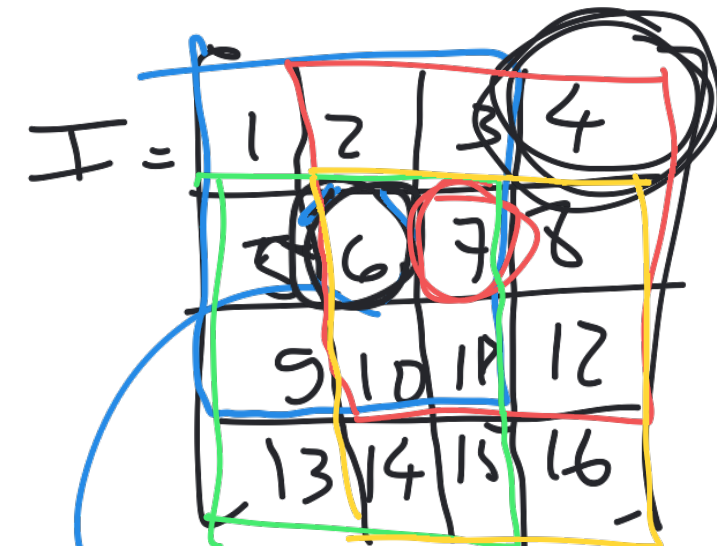
G : doubly ^{block}circulant matrix

$$\begin{bmatrix} h_{11} \\ h_{12} \\ h_{21} \\ h_{22} \end{bmatrix} = \begin{bmatrix} \boxed{1} & \boxed{-2} & \boxed{-3} & 0 & \boxed{-4} & \boxed{-5} & \boxed{-6} & 0 & \boxed{-7} & \boxed{-8} & \boxed{-9} & 0 & 0 & 0 & 0 \\ 0 & \boxed{1} & \boxed{-2} & \boxed{-3} & 0 & \boxed{-4} & \boxed{-5} & \boxed{-6} & 0 & \boxed{-7} & \boxed{-8} & \boxed{-9} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & \boxed{1} & \boxed{-2} & \boxed{-3} & 0 & \boxed{-4} & \boxed{-5} & \boxed{-6} & 0 & \boxed{-7} & \boxed{-8} & \boxed{-9} \\ 0 & 0 & 0 & 0 & \boxed{1} & \boxed{-2} & \boxed{-3} & 0 & \boxed{-4} & \boxed{-5} & \boxed{-6} & 0 & \boxed{-7} & \boxed{-8} & \boxed{-9} \end{bmatrix}$$

output image

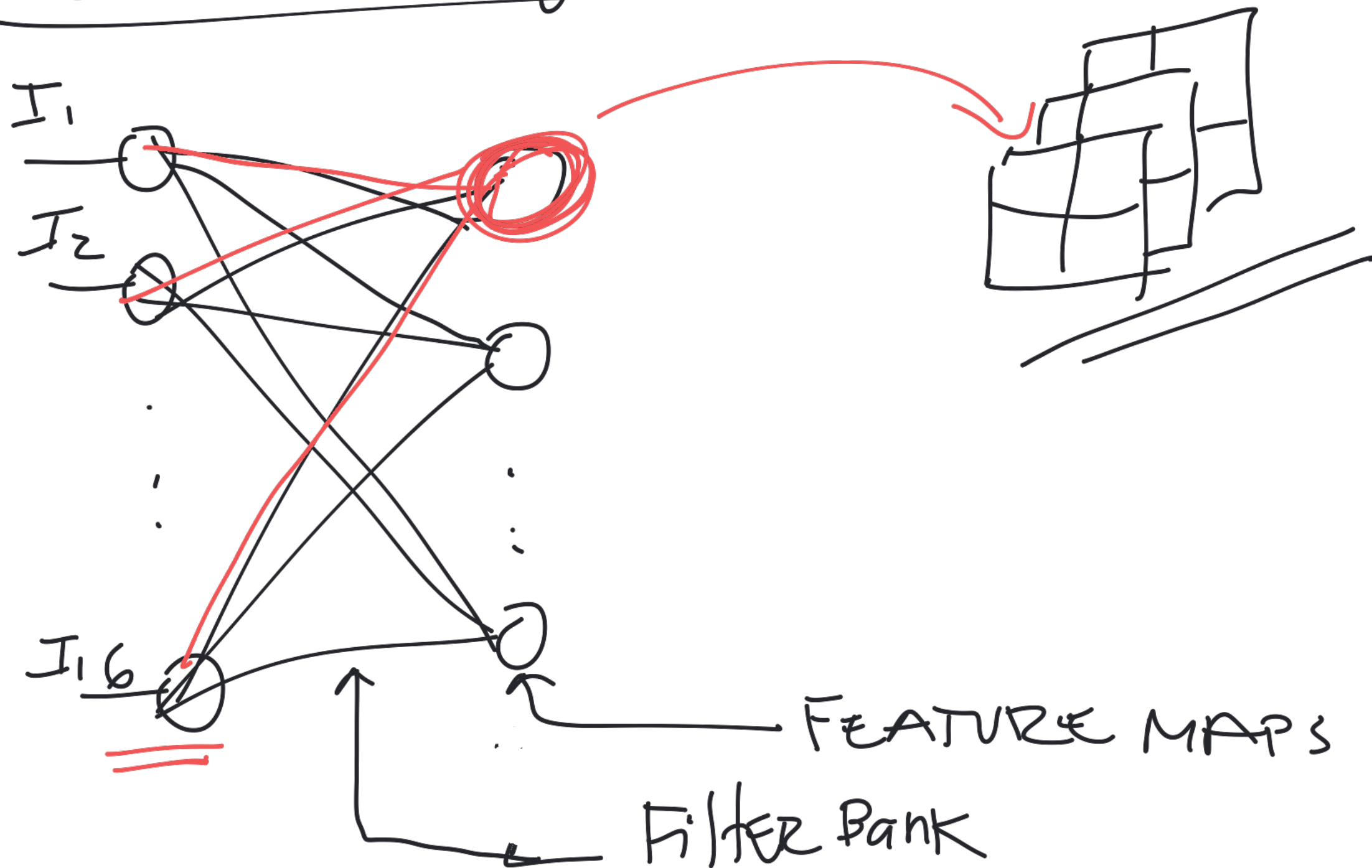
$$H = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix}$$

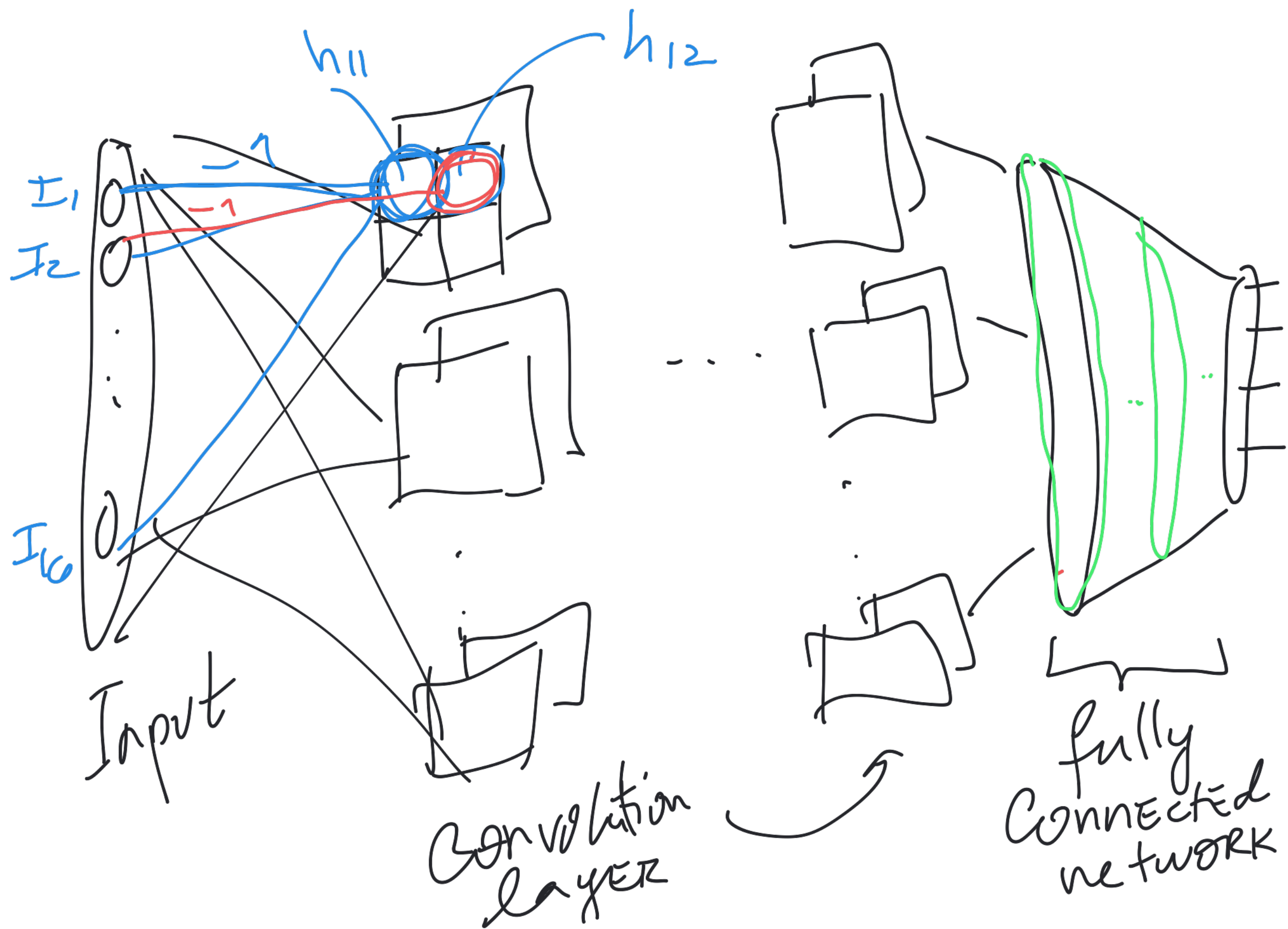
$$K = \begin{bmatrix} -1 & -2 & -3 \\ -4 & -5 & -6 \\ -7 & -8 & -9 \end{bmatrix}$$



$$\begin{aligned} & 1 \times (-1) + 2 \times (-2) \\ & + 3 \times (-3) + 5 \times (-4) \\ & + 6 \times (-5) + 7 \times (-6) \\ & + 9 \times (-7) + 10 \times (-8) \end{aligned}$$

G is a weight matrix







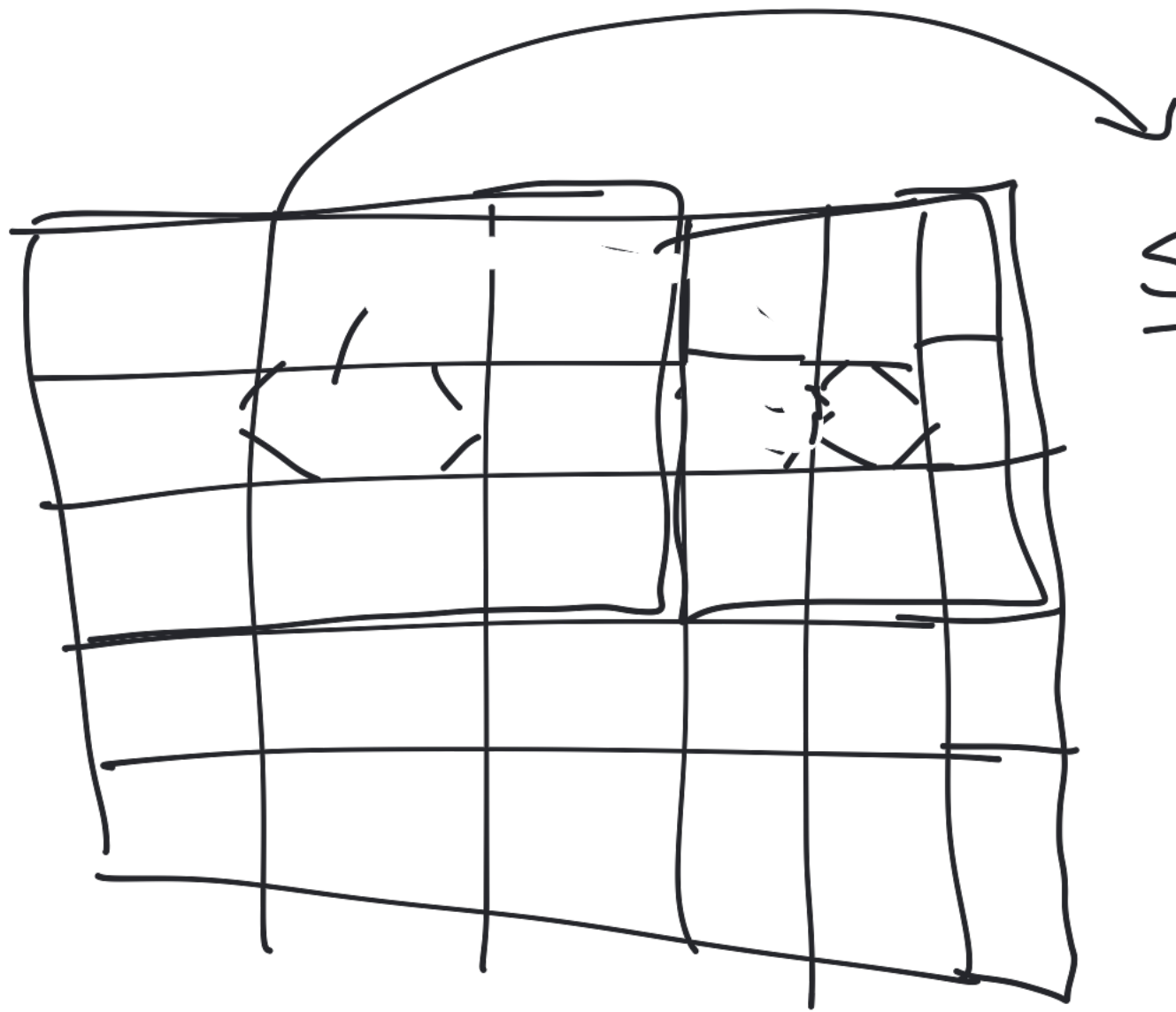
pooling

$$\left[\begin{array}{c} \max(h_{11}, h_{12}, h_{21}, h_{22}) \\ \hline \end{array} \right]$$



Reduce
parameter
space
(# weights
needed to
learn)

pooling



Stride of 2

CNN

