CENG 384 - Signals and Systems for Computer Engineers
Spring 2023
Homework 1

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1. (a) We are given that $z = x + yj$. Then, $\bar{z} = x - yj$, and the equation becomes:
$2(x + yj) + 5 = j - x + yj$
$2x + 2yj + 5 = j + yj - x$
$2x + 5 = -x$ and $2y = y + 1$
$x = \frac{-5}{3}$ and $y = 1$
Then, $z = \frac{-5}{3} + j$
$|z|^2 = x^2 + y^2 = (\frac{-5}{3})^2 + 1^2 = \frac{25}{9} + 1 = \frac{34}{9}$
Plotting $z = \frac{-5}{3} + j$ using x-axis as the real and y-axis as the imaginary axes:

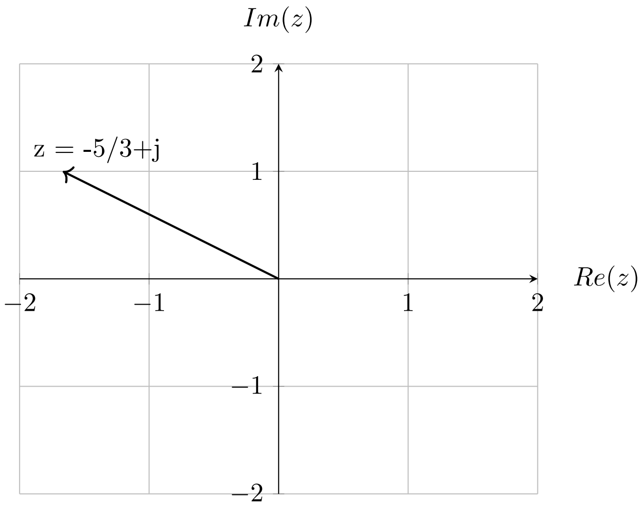

Figure 1: Plot of z on the complex plane.

(b) Given $z = re^{j\theta}$, we can deduce $z^5 = r^5 e^{5\theta j} = r^5 cos(5\theta) + jr^5 sin(5\theta) = 32j$
Then, $r^5 cos(5\theta) = 0$ and $r^5 sin(5\theta) = 32$ from the given information.
Using $r^5 cos(5\theta) = 0$, $5\theta = \frac{(2k+1)\pi}{2}$ and $\theta = \frac{(2k+1)\pi}{10}$ for some integer $k \in \mathbb{Z}$. Using the second equation:
$r^5 sin(5((2k+1)\pi/10)) = r^5 sin((2k + 1/2)\pi) = 32$
If k is even, then $sin((2k + 1/2)\pi) = 1$ and $r^5 = 32$, meaning $r = 2$. Using $k = 2$, the polar form becomes: $2e^{j\pi/2}$
If k is odd, then $sin((2k + 1/2)\pi) = -1$ and $r^5 = -32$, meaning $r = -2$. Using $k = 1$, the polar form becomes: $-2e^{j3\pi/10}$.
So, $z_1 = 2e^{j\pi/2}$ and $z_2 = -2e^{j3\pi/10}$

(c) Simplifying the numerator, we get:
$z = \frac{(1+j)(\frac{1}{2}+\frac{\sqrt{3}j}{2})}{j-1} = \frac{\frac{1-\sqrt{3}}{2}+\frac{(1+\sqrt{3})j}{2}}{j-1}$
Then, multiplying both numerator and denominator by the conjugate of the numerator, we get:
$z = \frac{(\frac{1-\sqrt{3}}{2}+\frac{(1+\sqrt{3})j}{2})(j+1)}{(j-1)(j+1)} = \frac{-\sqrt{3}+j}{-2} = \frac{\sqrt{3}}{2} - \frac{j}{2}$
Magnitude of $z = \frac{\sqrt{3}}{2} - \frac{j}{2}$ is: $|z| = \sqrt{(\frac{\sqrt{3}}{2})^2 + (\frac{1}{2})^2} = \sqrt{\frac{3}{4} + \frac{1}{4}} = 1$
Angle of $z = \frac{\sqrt{3}}{2} - \frac{j}{2}$ is $arctan(\frac{-1/2}{\sqrt{3}/2}) = acrctan(\frac{-1}{3}) = -\frac{\pi}{6}$

(d) We have $z = je^{-j\pi/2}$. Then $r = j$ and $\theta = -\frac{\pi}{2}$. Using the formula for polar form, $z = r(cos\theta + jsin\theta)$, we get:
$z = j(cos(-\frac{\pi}{2} + j\sin(-\frac{\pi}{2}) = j(-j) = 1$

2. Question 2:

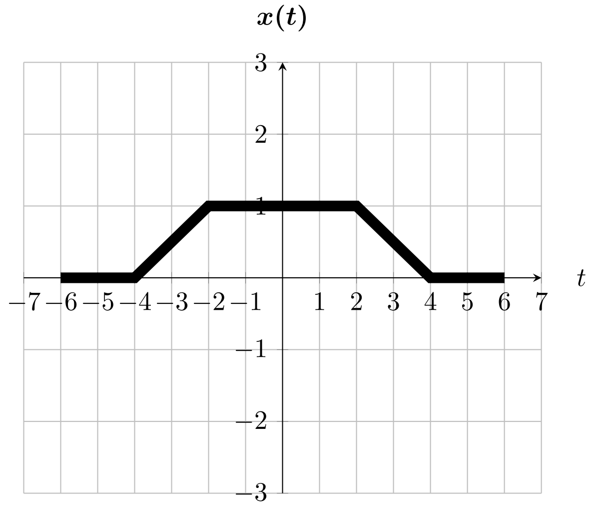

Figure 2: t vs. $x(t/2)$.

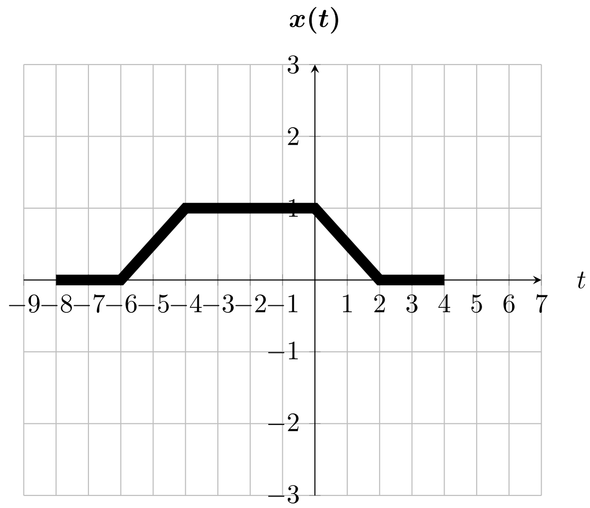

Figure 3: t vs. $x(t/2 + 1)$.

3. (a) $x[-n]$ is the reflection of $x[n]$ over y-axis, so we get the following result if we plot $x[-n]$:

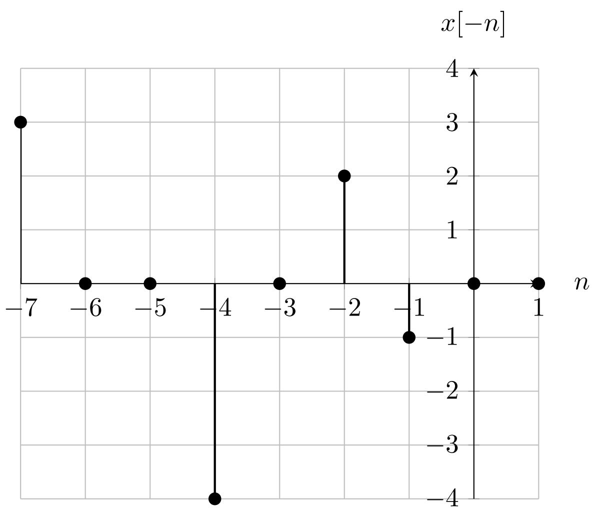

Figure 4: n vs. $x[-n]$.

Similarly, $x[2n-1]$ means shrinking $x[n]$ by 2 and then shifting it to right by $\frac{1}{2}$. Since it represents a discrete-time signal, we should only consider the integer values of n. We get:

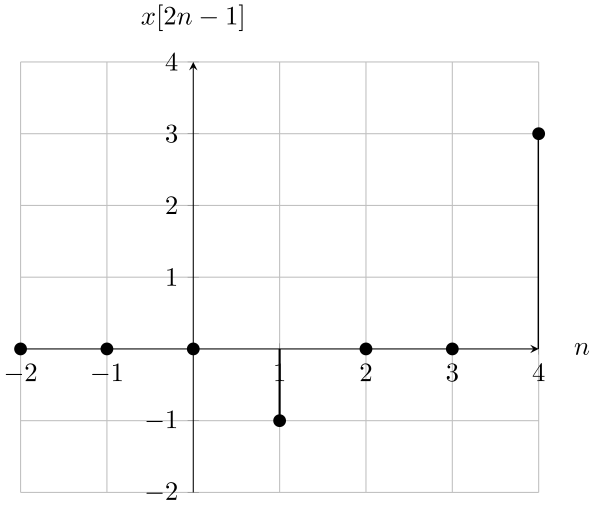

Figure 5: n vs. $x[2n-1]$.

Summing up these two graphs, we get the desired result $x[-n]+x[2n-1]$

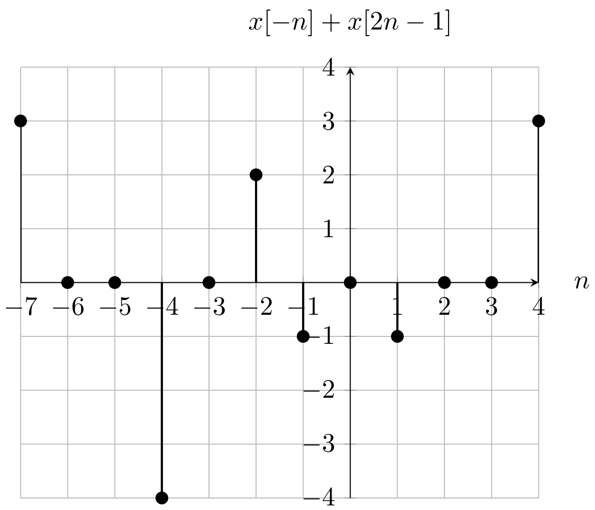

Figure 6: n vs. $x[-n]+x[2n-1]$.

(b) Using the graph in Figure 6, we could represent $x[-n]+x[2n-1]$ in terms of the unit impulse function as follows: $x[-n]+x[2n-1] = 3\delta[n+7] - 4\delta[n+4] + 2\delta[n+2] - \delta[n+1] - \delta[n-1] + 3\delta[n-4]$.

4. (a) $x(n) = 5sin(3t - \frac{\pi}{4})$
$w_0 = 3$
$T = \frac{2\pi}{3}$
YES, it is periodic.

(b) $x[n] = cos[\frac{13\pi}{10}n] + sin[\frac{7\pi}{10}n]$

For cosine part, $w_0 = \frac{13\pi}{10}$
Fundamental period is $T_1 = \frac{2\pi k}{w_0} = \frac{20k}{13}$, where T and k $\in \mathbb{Z}^+$.
Smallest k = 13, then $T_1 = 20$.

For sine part, $w_0 = \frac{7\pi}{10}$
Fundamental period is $T_2 = \frac{2\pi k}{w_0} = \frac{20k}{7}$, where T and k $\in \mathbb{Z}^+$.
Smallest k = 7, then $T_2 = 20$.

T = LCM(20,20) = 20. The fundamental period is 20.
YES, it is periodic.

(c) $x[n] = \frac{1}{2}cos[7n-5]$
$w_0 = 7$

Fundamental period is $T = 2\pi k/w_0 = 2\pi k/7$, where $\{T, k\} \in \mathbb{Z}^+$
Since there is no integer that satisfies this condition, this signal is not periodic.
NO, it is not periodic.

5. (a) $x(t) = u(t-1) - 3u(t-3) + u(t-4)$
   (b) $\frac{dx(t)}{t} = \delta(t-1) - 3\delta(t-3) + \delta(t-4)$

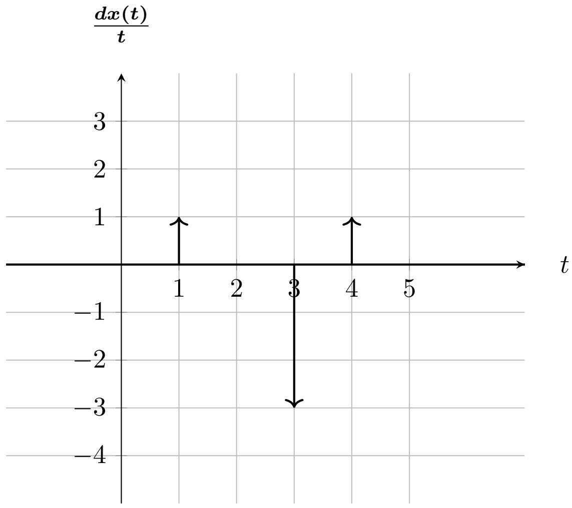

Figure 7: t vs. $\frac{dx(t)}{t}$.

6. (a) $y(t) = tx(2t+3)$
   Memory : Yes, it has memory because it depends on the future value of the input, for example:
   $y(1) = x(2+3) = x(5)$
   Stability : No, the system is not stable, because taking a bounded input, we do not get a bounded output when $t- > \infty$
   Causality : No, the system is not causal because it depends on the future value of the input, as shown in the Memory example.
   Linearity : Yes, the system is linear, because the superposition property holds. If we take two signals $x_1(t)$ and $x_2(t)$, multiply them by constants a_1 and a_2, and then add them, we get:
   $a_1 * tx_1(2t+3) + a2 * tx_2(2t+3)$
   Similarly, if we take $x_3(t) = a_1 * x_1(t) + a_2 * x_2(t)$ and plug it in the equation:
   $t * x_3(2t+3) = a_1 * tx_1(2t+3) + a2 * tx_2(2t+3)$.
   So, the superposition property holds, and the system is linear.
   Invertability : No, since there is no one-to-one correspondence. For instance, if we take two different inputs $x_1(t)$ and $x_2(t)$ where $x_1(3) = 1$ and $x_2(3) = 2$, and we take the value $t = 0$, we get $y(0) = 0 * x_1(3) = 0$ and $y(0) = 0 * x_2(3) = 0$. So, we get the same output for both inputs. Therefore, the system is not invertible.
   Time-invariance : No, the system is not time-invariant, because time shift in input does not result in the same time shift in output, as follows: $y(t-t_0) = (t-t_0)x(2(t-t_0)+3)$, which is not equal to $tx(2(t-t_0)+3)$

   (b) $y[n] = \sum_{k=1}^{\infty} x[n-k]$
   Memory : Yes, the system has memory, because it depends on past values as well. For $k > 1$, $n-k$ is not equal to n.
   Stability : No, the system is not stable because its sum is infinity, so the bounded input does not result in a bounded output.
   Causality : Yes, it is causal, because the output at present depends on the past values of the input, as $n-k < n$
   Linearity : Yes, because the superposition property holds, the system is linear. If we take x_1 and x_2: $\Sigma_{k=1}^{\infty} x_1[n-k]$ and $\Sigma_{k=1}^{\infty} x_2[n-k]$, and multiply them by some constants a_1 and a_2, and add them, we get:
   $a_1 * \Sigma_{k=1}^{\infty} x_1[n-k] + a_2 * \Sigma_{k=1}^{\infty} x_2[n-k]$.
   Similarly, if we take $x_3[n] = a_1 * x_1[n] + a_2 * x_2[n]$ and then plug it in, we get the same result:
   $\Sigma_{k=1}^{\infty} x_3[n-k] = a_1 * \Sigma_{k=1}^{\infty} x_1[n-k] + a_2 * \Sigma_{k=1}^{\infty} x_2[n-k]$.
   So, the superposition property holds and the system is linear.
   Invertability : Yes, the system is invertible, since we can find the inverse as follows: $h^-1(y[n]) = x[n] = y[n+1] - y[n]$
   Time-invariance : Yes, it is time-invariant, because the time-delay on the input is the same as the time-delay of the output function as shown in the below equation:
   $y[n-n_0] = \sum_{k=1}^{\infty} x[(n-n_0)-k]$

7. (a)

```
import matplotlib.pyplot as plt
from sys import argv

def plot_even_odd_parts(x, si):


    x_original = x
    #Reverse the list to get x[-n]
    x_reverse = x[::-1]

    #Append zeros to the beginning because of the starting index
    x = ([0] * abs(si) + x)
    x_reverse = ([0] * abs(si) + x_reverse)


    #Calculate even (xe) and odd (xo) parts of the signal x
    xe = [(x[n] + x_reverse[n])/2 for n in range(abs(si), abs(si)+len(x_original))]
    xo = [(x[n] - x_reverse[n])/2 for n in range(abs(si), abs(si)+len(x_original))]

    xe = [0] * 2 + xe + [0] * 2
    xo = [0] * 2 + xo + [0] * 2

    # Plotting the even part
    plt.stem(range(si-2, si+len(x_original)+2), xe, linefmt='b-', markerfmt='bo', label='Even
    part')

    # Plotting the odd part
    plt.stem(range(si-2, si+len(x_original)+2), xo, linefmt='r-', markerfmt='ro', label='Odd
    part')

    #Adding title and labels for the graph
    plt.title('Even and Odd Parts of Signal a Discrete Signal')
    plt.xlabel('n')
    plt.ylabel('Odd and Even Parts')
    plt.legend()
    plt.show()


# Reading the file name, parsing it and getting the si and x values from
the given input
file_name = argv[1]

f = open(file_name, "r")

lines = f.read()

x = lines.split(',')

si = int(x[0])

x = x[1::]

#Converting string values to float
for i in range(len(x)):
    x[i] = float(x[i])


plot_even_odd_parts(x, si)
```

Listing 1: Python code for Part a

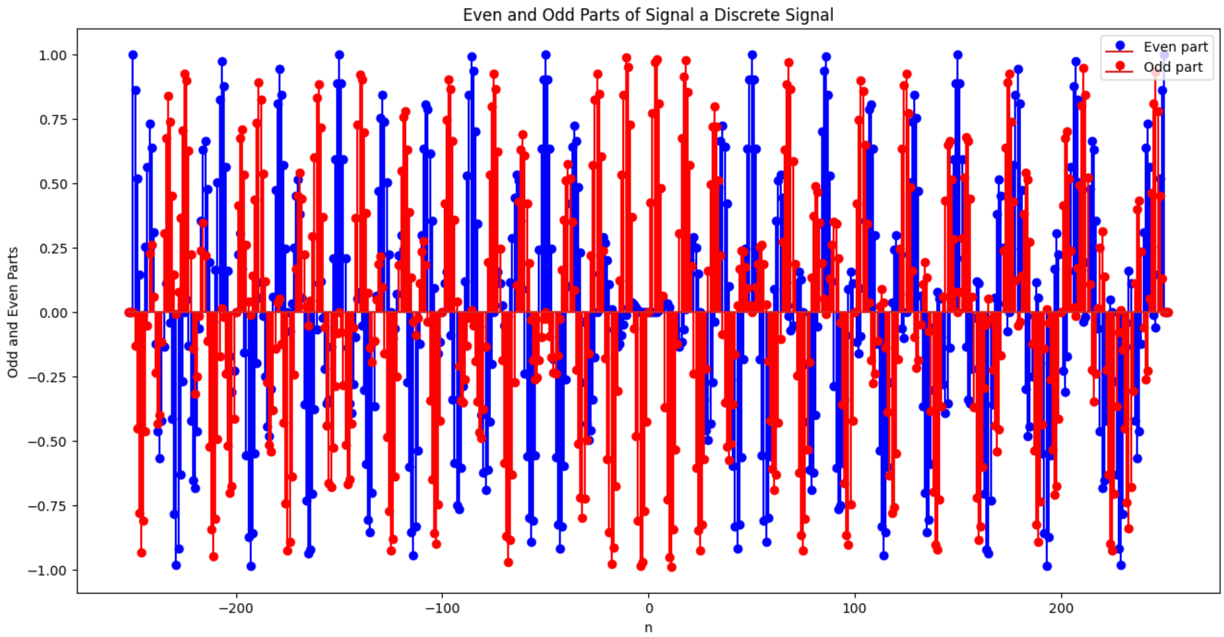

Figure 8: Chirp part (a)

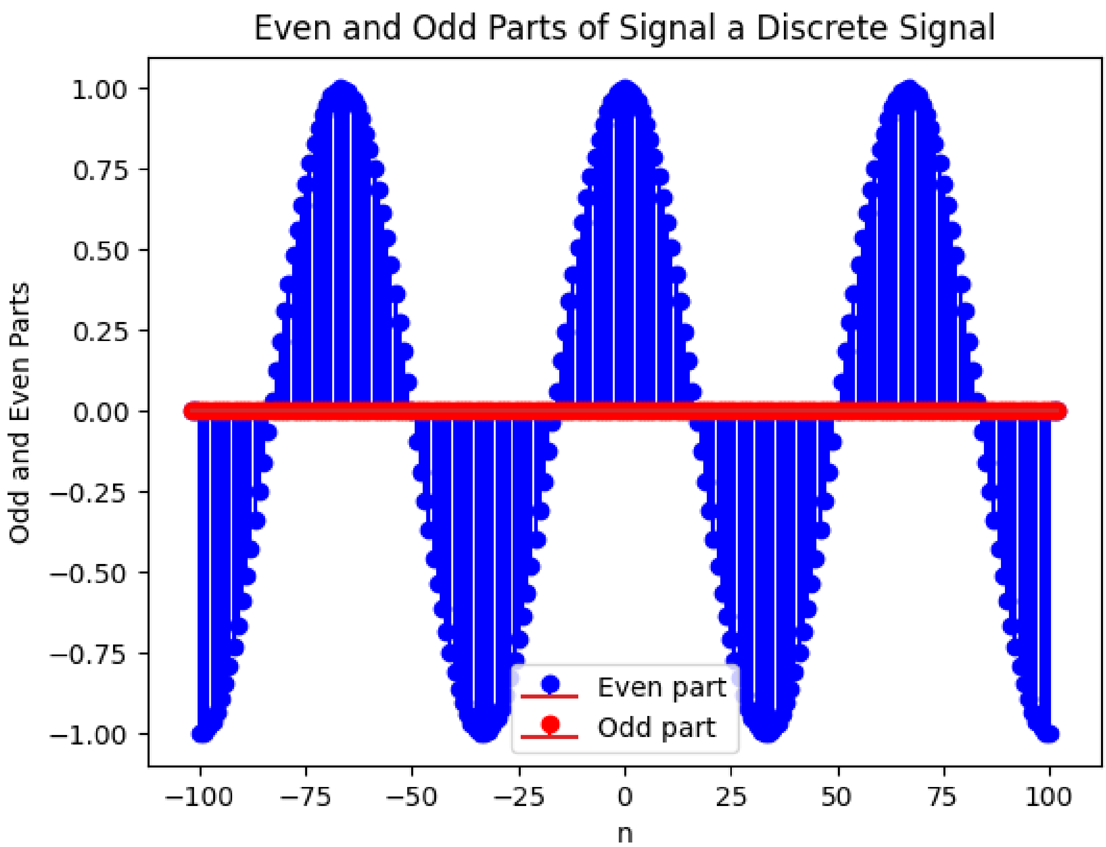

Figure 9: Sine part (a)

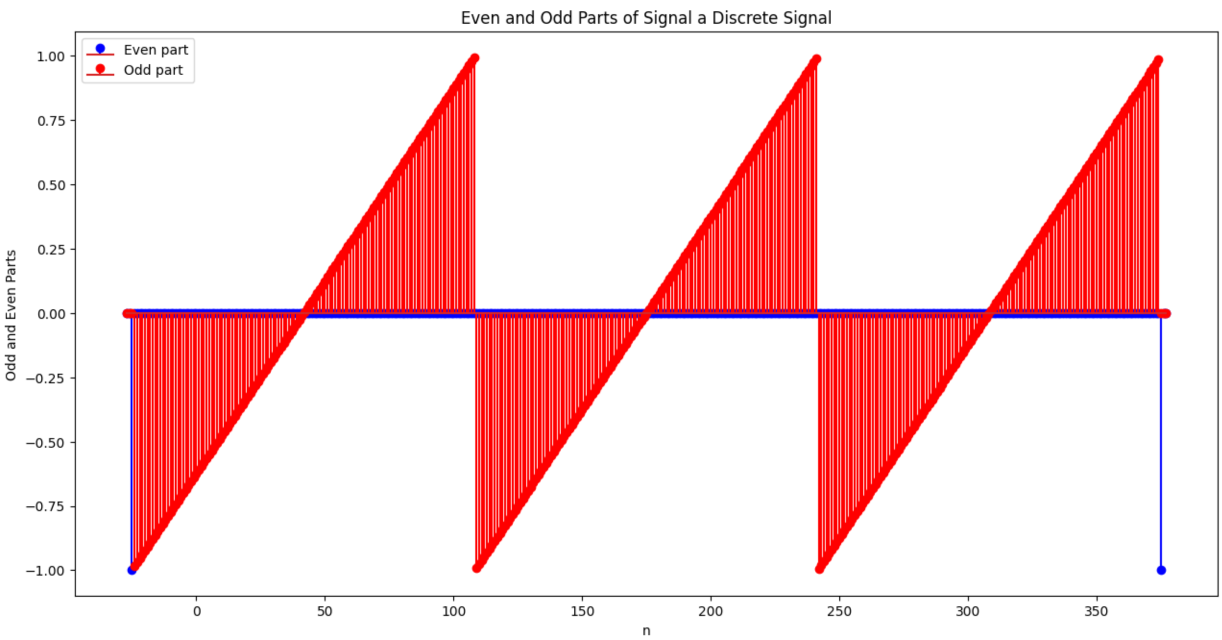

Figure 10: shifted sawtooth part (a)

(b)

```
import matplotlib.pyplot as plt
from sys import argv


def shifted_scaled(x, si, a, b):
  y = {}

  #Either shrink or expand and then shift the signal
  for i in range(si, si+len(x)):
    val = (i/abs(a))-(b/abs(a))

    if (val == int(val)):
      y[int(val)] = x[i-si]

  #Store the values of keys corresponding to shifted scaled version
  keys = [k for k in y]
  values = [val for val in y.values()]
  #Store initial and final keys to be used in range for plotting
  initial = keys[0]
  final = keys[-1]


  for i in range(len(keys)):
    keys[i] = abs(initial - keys[i])

  #Store the shifted keys
  shifted_indices = [0] * (keys[-1]+1)


  for i in range(len(keys)):
    shifted_indices[keys[i]] = values[i]

  #Plotting the graph depending on value of a

  #NPositive a does not cause reflection
  if (a > 0):
    n_range = range(initial, final+1)
    plt.stem(n_range, shifted_indices, linefmt='red', markerfmt='ro', label='y[n]')
  #If a is negative, the graph is reflected over y-axis
  else:
    n_range = range(-final, -initial+1)
    shifted_indices.reverse()
    plt.stem(n_range, shifted_indices, linefmt='red', markerfmt='ro', label='y[n]')


  #Add title and labels to the graph
  plt.xlabel('n')
  plt.ylabel('x[n] / y[n]')
  plt.title('Shifted and Scaled Signal')
  plt.legend()
  plt.show()



#Read the input file and parse to get si, a, b, and x values
file_name = argv[1]

f = open(file_name, "r")

lines = f.read()

x = lines.split(',')
si = int(x[0])
a = int(x[1])
b = int(x[2])


x = x[3::]

#Convert the string values to float
for i in range(len(x)):
    x[i] = float(x[i])



shifted_scaled(x, si, a, b)
```

Listing 2: Python code for Part b

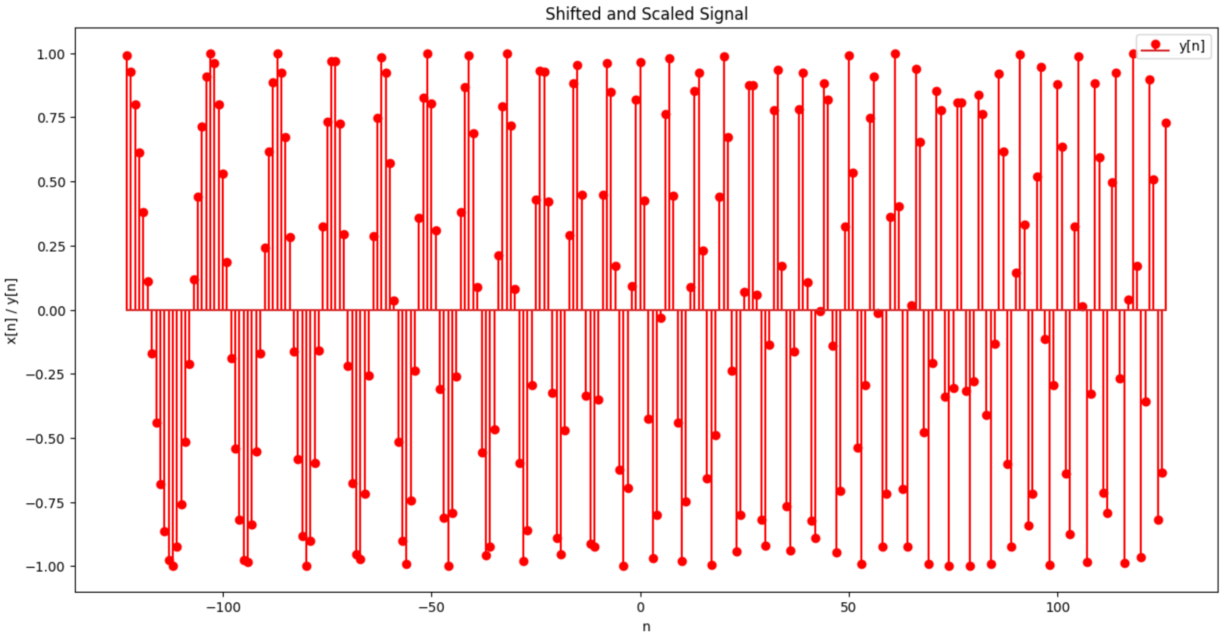

Figure 11: Chirp part (b)

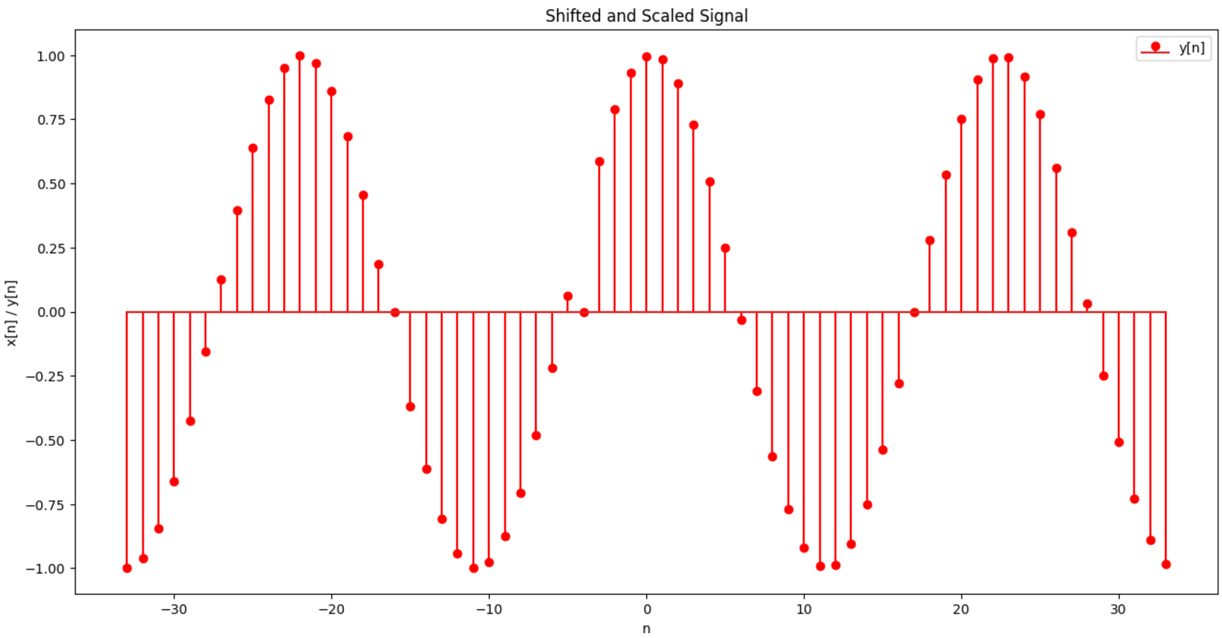

Figure 12: Sine part (b)

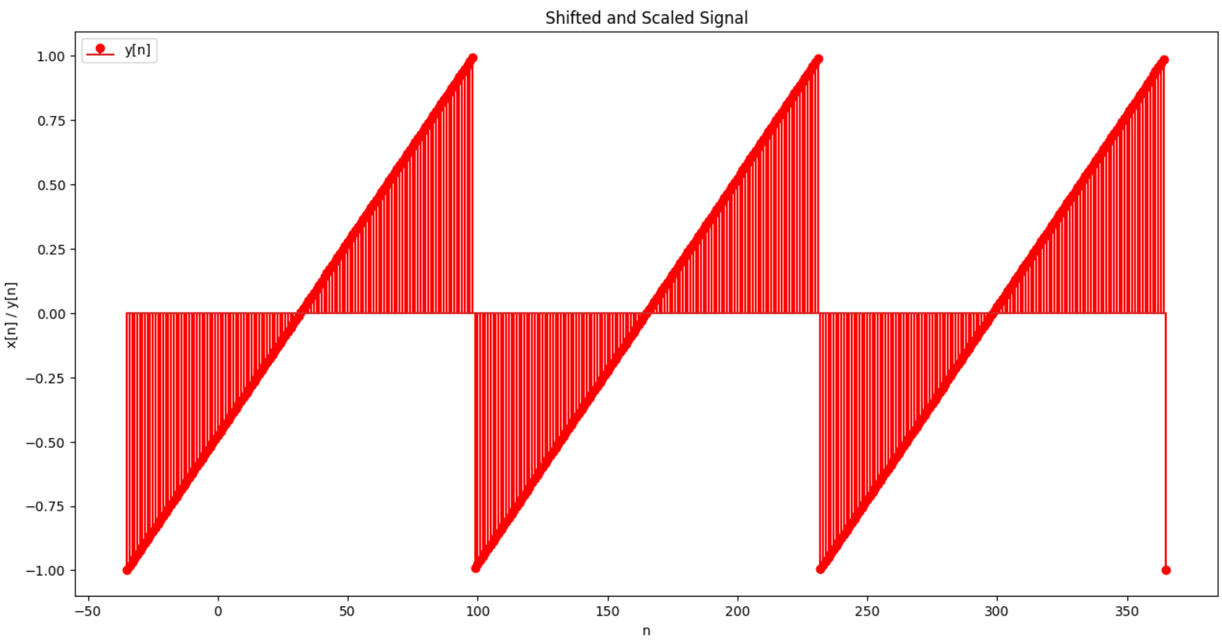

Figure 13: Shifted sawtooth part (b)