Estimating the Phillips Curve Function

EC207 Empirical Project 2 (Term Paper)

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Abstract

In this study, I examine whether a naïve Philips curve function can forecast the U.S. inflation rate in the last four months of 2021. I first select and estimate a Philips curve model based on various selection criteria and then perform dynamic forecasting. The results of the Predictive Accuracy test suggest that my model is misspecified. There are two reasons. One is that the underlying Philips curve is oversimplified. And the other reason is the presence of COVID-19 shock on inflation may change the data generation process in the last four months of 2021.

Section 1: Introduction

Founded in 1958 by A. W. Philips, a professor at the London School of Economics at the time, the Phillips curve has played an important role in macroeconomics ever since (Dornbusch and Fischer, 1990). Philips curve shows a negative relationship between the unemployment rate and either the nominal wage growth rate or the inflation rate. Since then, most equations that relate the unemployment rate to the inflation rate are called Phillips curves.

Though economists developed various specifications of the Phillips curves and used them frequently to forecast future inflations—an important index for policymakers, many still questioned the usefulness of such forecasting (Atkeson and Ohanian, 2001). One critical question is whether the statistical relationship between unemployment and inflation documented in the early empirical studies should be expected to remain stable over time. In other words, will the forecast produced from the "naïve" Phillips curves still be reliable in the later periods, especially today?

This study will try to answer this question by forecasting the inflation rates in the last four months of 2021 using monthly data from 1993m1 to 2021m8 via a relatively simple Philips curve specification, and then compare the forecasts with the actual values. The baseline model, Model 1, is shown in Equation (1). It is a typical Phillips curve setting. The unemployment rate is included as the Philips curve states that there is a negative relationship between the unemployment rate and the inflation rate. The squared unemployment rate is also included as the original Philips curve is indeed a quadratic curve. Notice that dynamics are included in Model 1 as well. Given monthly data is used, the past 12 months of inflation rates are included in the model to form the AR component. Current and past 12 months of unemployment rates are also included in order to increase the predictive power of the model. Also, including dynamics is critical for studying the short-run and long-run impacts of the unemployment rate on the inflation rate.

$$INFL_t = \alpha_0 + \sum_{i=0}^{12} \gamma_i UNPL_{t-i} + \sum_{i=0}^{12} \eta_i UNPL_{t-i}^2 + \sum_{j=1}^{12} \rho_j INFL_{t-j} + u_t \qquad (1)$$

where *INFL* is U.S. inflation rate and *UNPL* is U.S. unemployment rate.

The rest of the paper is arranged as the following. Section 2 describes the data by presenting summary information and testing for time trends, Granger causality, and the GARCH process. Section 3 performs various empirical analyses to find a better model, Model 2, from Model 1 and tests for the accuracy of the forecasts from Model 2. And finally, this paper concludes with some final remarks.

Section 2: Data Description

This study uses the monthly inflation rate and unemployment rate from 1993m1 to 2021m8 as the sample data and preserves the observations in 2021m9-2021m12 for the predictive accuracy test. The definition of the variables is presented in Table 1. And Table 2 shows the summary statistics. From Table 2, I can see that the U.S. average monthly inflation rate over the period of 1993m1 to 2021m8 is 0.19% and the U.S. average monthly unemployment rate is 5.84% over the period of 1993m1 to 2021m8. Similarly, the U.S. average monthly squared unemployment rate is 37.17%^{2} over the period of 1993m1 to 2021m8, which has less economic meaning, though.

The results for testing the presence of time trends of inflation rate over the sample period are in Table 3. As shown in Table 3 Column (1), the coefficient on the linear time variable is not statistically significant at 1%. And the p-value for the t-test that the coefficient is zero is 0.398, so I fail to reject the null hypothesis that there is no linear time trend. Similarly, the F-test for the null hypothesis that the coefficients in Table 3 Column (2) are jointly zero is failed to reject at 1% given that the p-value for the F-test is 0.443. So, I also fail to reject that there is no quadratic time trend. To conclude, the inflation rate in the sample period is not trending. Figure 1 shows the inflation rate and the best linear fit in the sample period. And the average monthly inflation rate is 0.19% in the sample period.

Next, Granger causality between the inflation rate and the unemployment rate is tested. Note that Granger causality is different from contemporaneous causality because Granger causality corresponds to a temporal ordering of changes in x and y. Here, the causal relationship between variables relies on the timing of cause and effect, which implies that current and lagged values of x significantly predict future y. Since monthly data is used, I include 12 lags in the models used to test for Granger causality.

As shown in Table 4, the F-tests for the coefficients are jointly zero are failed to reject at 1% in both ways. When testing the Granger causality of the unemployment rate causing changes in the future inflation rate, the p-value is 0.819 and thus fails to be rejected. Similarly, the p-value is 0.548 when testing if the inflation rate causes changes in the future unemployment rate. Hence, I conclude independence and there is no Granger causality between the inflation rate and the unemployment rate.

This result is different from the presumed causality relationship in the model that the unemployment rate will predict the future inflation rate. However, the result of independence of the two variables is acceptable as economists have questioned if the statistical relationship between the variables found in the old days would still hold in the later periods (Atkeson and Ohanian, 2001).

Last but not least, the GARCH process in the inflation rate is identified. I fixed an AR(4) process for the inflation rate and started with a GARCH(4, 4) process. After identifying the lag length for the ARCH and GARCH components, I ended with a GARCH(2, 2) process, which is presented in Table 5. Note that the estimation process is limited to 20 iterations. The predicted values of the conditional GARCH variance are shown in Figure 2. I can see relatively stable variance besides a high peak in 2007-2009, which makes sense given the presence of the 2008 financial crisis.

Section 3: Empirical Analysis

The ordering of empirical analysis is the following. (1) Test for unit root; (2) Test for cointegration; (3) Determine if a linear time trend needs to be added; (4) Determine if seasonal indicators are needed; (5) Test for 1^{st} -order and 4^{th} -order serial correlation; (6) Test for lag length. This ordering is selected because I need first to determine if the variables have unit roots and need to be first differenced because whether the variables are first differenced or not makes such a big difference that if I perform this step later, then I need to redo all the steps before it if the first difference is needed. The immediate next step is testing for cointegration. Then, it is natural to continue with time trend and seasonality because these are all steps trying to identify the deterministic trends in the time series and they should go before selecting the lag length and testing for serial correlations that are concerning the stochastic trend. The reason why testing serial correlation should go before testing for lag length is that I want to start with including sufficient dynamics when testing for lag length, and more importantly, I do not want to use serial correlation tests as the final criteria for determining the lag length.

The results for testing unit root and cointegration are presented in Table 6. I reject the unit root for inflation under the selected lag length of 7 at 1% with a p-value of 0.0000. However, I fail to reject the unit root for unemployment under the selected lag length of 1 at 1% with a p-value of 0.0116. Hence I conclude that inflation is I(0) and unemployment is I(1).

Since inflation is already the first difference of price level and it is I(0), I recover the price level from inflation and see if it has a unit root. As shown in Table 6, I fail to reject the unit root for price level at the selected lag of 8 at 1% with a p-value of 0.9913. So, I test for cointegration between the price level and unemployment, which are both I(1). I fail to reject the unit root for the residuals of regressing the price level on the unemployment rate with a p-value of 0.9337 and thus price level and unemployment are not cointegrated. Therefore, I use the first-differenced unemployment rate ($\Delta UNPL_t$) and its squared ($\Delta UNPL_t^2$) as parameters in the rest of the study.

The test for linear time trend is the same as in Section 2. In Table 3, I fail to reject that there is no linear time trend at 1% with a p-value of 0.398.

The regression results for seasonality are shown in Table 7. I reject the null hypothesis that all coefficients on month dummies are jointly zero at 1%. The p-value for the F-test is 0.000. Therefore, I add seasonality to Model 1.

The next step is to test for serial correlations. The detailed steps for Durbin's Alternative Test can be found in *Appendix 1 5*. The key step here is to first estimate Model 1 with the first-differenced unemployment rate (and its squared) and seasonality to get the residual. Then, use the residual as the dependent variable while adding lagged residual(s) corresponding to the order of serial correlation to be tested to the right-hand side of the modified version of Model 1 to run a regression and test for the coefficient(s) on the lagged residual(s). The p-value for the t-test that the coefficient on the first lag residual equals zero is 0.658, and I fail to reject the null hypothesis. So, there is no 1st order serial correlation at 1%. Similarly, the p-value for the F-test that the coefficients on the first lag to the fourth lag residuals are jointly zero is 0.369, so I fail to reject the null hypothesis at 1%. To conclude, there is no 1st or 4th -order serial correlation in the residuals.

Last but not least, I test for lag length. The detailed steps for the selection process can be found in *Appendix 1 6*. Basically, I start with 12 lags for both *UNPL* and *INFL* and conduct t-tests for the coefficient on *INFL* while conduct F-tests for the coefficients on $\Delta UNPL_t$ and $\Delta UNPL_t^2$ jointly. *INFL* is significant at 1% with 12 lags with a p-value of 0.003 (so reject the null that the coefficient on the 12th lag of *INFL* is zero at 1%) while it is not significant at 13 lags with a p-value of 0.650, so *INFL* is fixed at 12 lags. $\Delta UNPL_t$ and $\Delta UNPL_t^2$ are only jointly significant with 0 lags, and the p-value is 0.0099 (reject the null hypothesis at 1%). The tests give me Model 2 as shown in Equation 2, which includes 12 lags of *INFL* and 0 lag of $\Delta UNPL_t$ (and $\Delta UNPL_t^2$). The regression results for Model 2 are presented in Table 8.

$$INFL_t = \alpha_0 + \gamma \Delta \mathrm{UNPL_t} + \eta \Delta UNPL_t^2 + \sum_{j=1}^{12} \rho_j INFL_{t-j} + \sum_{i=2}^{12} \beta_i Month_i + u_t \quad (2)$$

where $Month_i$ is a dummy for the i^{th} month of the year, $\Delta UNPL_t$ is the first-differenced $UNPL_{t}$, and $\Delta UNPL_t^2 = (\Delta UNPL_t)^2$.

Given the regression results of Model 2, I calculate the short-run impact multiplier and the long-run propensity for the unemployment rate. The short-run impact multiplier for the unemployment rate is -0.0851. It means a temporary, one-period 1 percentage increase in the unemployment rate will lead to a 0.0851 percent decrease in the inflation rate in the same period, on average. Since its standardized coefficient is -0.1545, it is not economically significant. Note that I am using the standardized coefficient here because the mean values of $INFL_t$ and $\Delta UNPL_t$ are not comparable. Similarly, the long-run propensity of unemployment is -0.1069. It means a permanent 1 percent increase in the unemployment rate will lead to a 0.1069 percent decrease in the inflation rate in the long run, on average. Also, since its standardized coefficient is -0.1940, it is still not economically significant, but it is close. And a graph of the actual values of inflation vs the sample regression estimates of Model 2 is Figure 3.

Next, I perform a dynamic forecast of inflation from 2021m9 to 2021m12. I first estimate an AR(4) (Equation (3)) model of *UNPL* in the sample period and use that model to predict *UNPL* from 2021m9 to 2021m12. Then, I get $\Delta UNPL_t$ and $\Delta UNPL_t^2$ from 2021m9 to 2021m12 based on the predictions and replace the actual value in the dataset with these predictions. After this, I perform dynamic forecasts of *INFL* in 2021m9 to 2021m12 using Model 2 (estimate using the sample data only) where all the out-of-sample *INFL*, $\Delta UNPL$, and $\Delta UNPL^2$ s are forecast values. For example, I use predicted 2021m10 $\Delta UNPL$ and $\Delta UNPL^2$ and predicted 2021m9 *INFL* as well as the actual 2020m10 to 2021m8 *INFL* to forecast 2021m10 *INFL*, as shown in Equation (4).

$$UNPL_t = \delta_0 + \sum_{i=1}^{4} \delta_i UNPL_{t-i} + v_t \quad (3)$$

$$\widehat{INFL}_{n+2|n} = \widehat{\alpha_0} + \hat{\gamma}\Delta\widehat{UNPL}_{n+2} + \hat{\eta}\Delta\widehat{UNPL}^2_{n+2} + \hat{\rho}_1\widehat{INFL}_{n+1|n} + \sum_{j=2}^{12}\hat{\rho}_j INFL_{n-j+2} + \sum_{i=2}^{12}\hat{\beta}_i Month_i \ (4)$$

where n represents 2021m8.

In Stata, I use the *Arima* function to re-estimate Model 2 using the sample data and then perform the dynamic forecasts. The actual and predicted values of *INFL* in 2021m9 to 2021m12 are shown in Table 9. As I can see from the last column of Table 9, the model tends to under-predict inflation as the differences between the actual and predicted inflation rates are all greater than zero.

Finally, I carry out the Predictive Accuracy test. The details can be found in *Appendix 1* 9. The results suggest that Model 2 is misspecified. I reject the null hypothesis that Model 2 is correctly specified at 1% with a p-value less than 0.0001. There are two reasons why I reject the null hypothesis. Firstly, the actual model could be more complicated than Model 2. For example, many economists believe that the expected inflation rate plays an important role in the Philips curve (Dornbusch and Fischer, 1990). Also, I can extend the Philips curves to include aggregate demand and aggregate supply. Secondly, it is reasonable to believe that the data generation process is different in the last four months (2021m9 – 2021m12) compared to the sample period (1993m1 – 2021m8). This is likely to be related to COVID-19. The model tends to under-predict inflation because I am expecting a period of high inflation due to the COVID-19 shock and the relevant policies, like the American Rescue Plan Act of 2021 (a.k.a. COVID-19 Stimulus Package).

Conclusion

In this study, I use the historical U.S. inflation rate and unemployment rate from 1993m1 to 2021m8 to forecast the U.S. inflation rate from 2021m9 to 2021m12 using a Philips curve model that I select. And I compare my predictions with the actual values and find that the model fails to forecast the inflation rate accurately which indicates that the model is misspecified. Both the naivety of the model and the presence of COVID-19 shock explain why the model is misspecified. And the results suggest that policymakers should be cautious when relying on the forecasted inflation rate through the naïve Philips curve, especially when there is a shock like COVID-19.

References

Atkeson, A., & Ohanian, L. E. (2001). Are Phillips curves useful for forecasting inflation? *Federal Reserve bank of Minneapolis quarterly review*, 25(1), 2-11.

Dornbusch, R. & Fischer, S. (1990). Macroeconomics, 5th Edition, McGraw-Hill, New York.

Table 1. Definition of Variables

Variable	Definition
INFL	U.S. monthly inflation rate
UNPL	U.S. monthly unemployment rate

Table 2. Summary Statistics

Variable	Mean	p50	SD	Min	Max
INFL	0.19	0.19	0.34	-1.92	1.22
UNPL	5.84	5.40	1.77	3.50	14.70
UNPL2	37.17	29.16	25.32	12.25	216.09

Table 3. Regression Results of Regressing Inflation Rate on Time

	(1)	(2)
VARIABLES	INFL	INFL
Time	-0.0002	-0.0008
	[0.0002]	[0.0007]
Time^2		0.0000
		[0.0000]
Constant	0.2181***	0.2573***
	[0.0363]	[0.0547]
Observations	344	344
R-squared	0.002	0.005

Standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Table 4. Granger Causality Regression Results

VARIABLES	(1) INFL	(2) UNPL
L.INFL	0.5439***	-0.2907**
	[0.0573]	[0.1241]
L2.INFL	-0.2094***	0.0424
	[0.0648]	[0.1404]
L3.INFL	-0.0627	0.0694
	[0.0662]	[0.1434]
L4.INFL	0.0505	-0.1823
	[0.0659]	[0.1429]
L5.INFL	-0.0667	-0.0849
	[0.0658]	[0.1425]
L6.INFL	-0.0702	0.0441
	[0.0659]	[0.1427]
L7.INFL	0.0355	-0.0915
	[0.0661]	[0.1432]
L8.INFL	-0.0989	-0.0647
	[0.0660]	[0.1431]
L9.INFL	-0.0767	0.0553
	[0.0664]	[0.1440]
L10.INFL	0.0648	-0.0981
	[0.0665]	[0.1440]
L11.INFL	0.1103*	-0.0288
	[0.0655]	[0.1419]
L12.INFL	0.0676	0.1799
	[0.0580]	[0.1258]
L.UNPL	-0.0119	0.9692***
	[0.0264]	[0.0573]
L2.UNPL	0.0206	-0.1291
	[0.0368]	[0.0797]
L3.UNPL	-0.0244	0.0994
	[0.0369]	[0.0800]
L4.UNPL	0.0093	-0.1132
	[0.0370]	[0.0802]
L5.UNPL	-0.0276	0.1154
	[0.0371]	[0.0805]
L6.UNPL	0.0126	-0.0710
	[0.0373]	[0.0808]
L7.UNPL	-0.0104	0.0595
	[0.0373]	[0.0809]
L8.UNPL	-0.0038	0.0114
	[0.0372]	[0.0807]
L9.UNPL	0.0212	0.0086

	[0.0371]	[0.0804]
L10.UNPL	0.0109	-0.0219
	[0.0370]	[0.0802]
L11.UNPL	0.0212	0.0313
	[0.0368]	[0.0798]
L12.UNPL	-0.0134	-0.0108
	[0.0265]	[0.0573]
Constant	0.1130	0.3740**
	[0.0731]	[0.1584]
Granger Causality F-tests		
p-value	0.8595	0.5098
Observations	336	336
R-squared	0.362	0.890

Standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Table 5. Regression Results for GARCH(2, 2) of Inflation Rate

	(1)	(2)
VARIABLES	INFL	ARCH
L.INFL	0.5317***	
	[0.0652]	
L2.INFL	-0.1848**	
	[0.0723]	
L3.INFL	-0.0691	
	[0.0659]	
L4.INFL	-0.0266	
	[0.0604]	
L.arch		0.2349***
		[0.0551]
L2.arch		0.2362***
		[0.0535]
L.garch		-0.2786***
		[0.0659]
L2.garch		0.7426***
		[0.0623]
Constant	0.1365***	0.0098*
	[0.0220]	[0.0054]
Observations	340	340

Standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Table 6. Results for Unit Root Tests and Cointegration Test

Variable	Selected lag	p-value	Result at 1%
INFL	7	0.0000	Reject unit root
UNPL	1	0.0116	Fail to reject unit root
Price Level	8	0.9913	Fail to reject unit root
Residual of Price Level on UNPL	0	0.9337	Fail to reject unit root, so *Price Level* and *UNPL* are not cointegrated

Table 7. Regression Results for Seasonality Test

VARIABLES	(1) INFL
2.month	0.0337
	[0.0730]
3.month	0.1145
	[0.0730]
4.month	-0.0389
	[0.0730]
5.month	-0.1157
	[0.0730]
6.month	-0.1214*
	[0.0730]
7.month	-0.2635***
	[0.0730]
8.month	-0.2052***
	[0.0730]
9.month	-0.1460**
	[0.0737]
10.month	-0.3266***
	[0.0737]
11.month	-0.5175***
	[0.0737]
12.month	-0.5319***
	[0.0737]
Constant	0.3657***

	[0.0516]
F-test for Seasonality	
p-value	0.0000
Observations	344
R-squared	0.337

Standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Table 8. Regression Results for Model 2

VARIABLES	(1) INFL
L.INFL	0.5270***
	[0.0560]
L2.INFL	-0.1858***
	[0.0633]
L3.INFL	0.0153
	[0.0640]
L4.INFL	0.0309
	[0.0640]
L5.INFL	-0.0910
	[0.0639]
L6.INFL	0.0171
	[0.0641]
L7.INFL	-0.0329
	[0.0642]
L8.INFL	-0.0585
	[0.0640]
L9.INFL	0.0145
	[0.0642]
L10.INFL	0.0128
	[0.0642]
L11.INFL	0.1365**
	[0.0630]
L12.INFL	-0.1820***
	[0.0561]
Δ UNPL	-0.0851*
	[0.0491]
Δ $UNPL^2$	0.0021
	[0.0052]
2.month	-0.2397***
	[0.0727]
3.month	-0.0504

	[0.0763]
4.month	-0.2565***
	[0.0801]
5.month	-0.2954***
	[0.0776]
6.month	-0.2407***
	[0.0760]
7.month	-0.4470***
	[0.0776]
8.month	-0.2839***
	[0.0760]
9.month	-0.2128***
	[0.0785]
10.month	-0.4438***
	[0.0796]
11.month	-0.5491***
	[0.0766]
12.month	-0.6027***
	[0.0746]
Constant	0.4492***
	[0.0591]
Observations	332
R-squared	0.544

Standard errors in brackets
*** p<0.01, ** p<0.05, * p<0.1

Table 9. Actual VS Predicted Inflation Rate in 2021m8 to 2021m12

Date	Actual INFL	Predicted INFL	Actual – Predicted
2021m9	0.27152	0.18046	0.09106
2021m10	0.83095	-0.02863	0.85958
2021m11	0.49135	-0.23429	0.72564
2021m12	0.30735	-0.30825	0.61560

Figure 1. Monthly Inflation Rate and the Best Linear Fit from 1993m1-2021m8

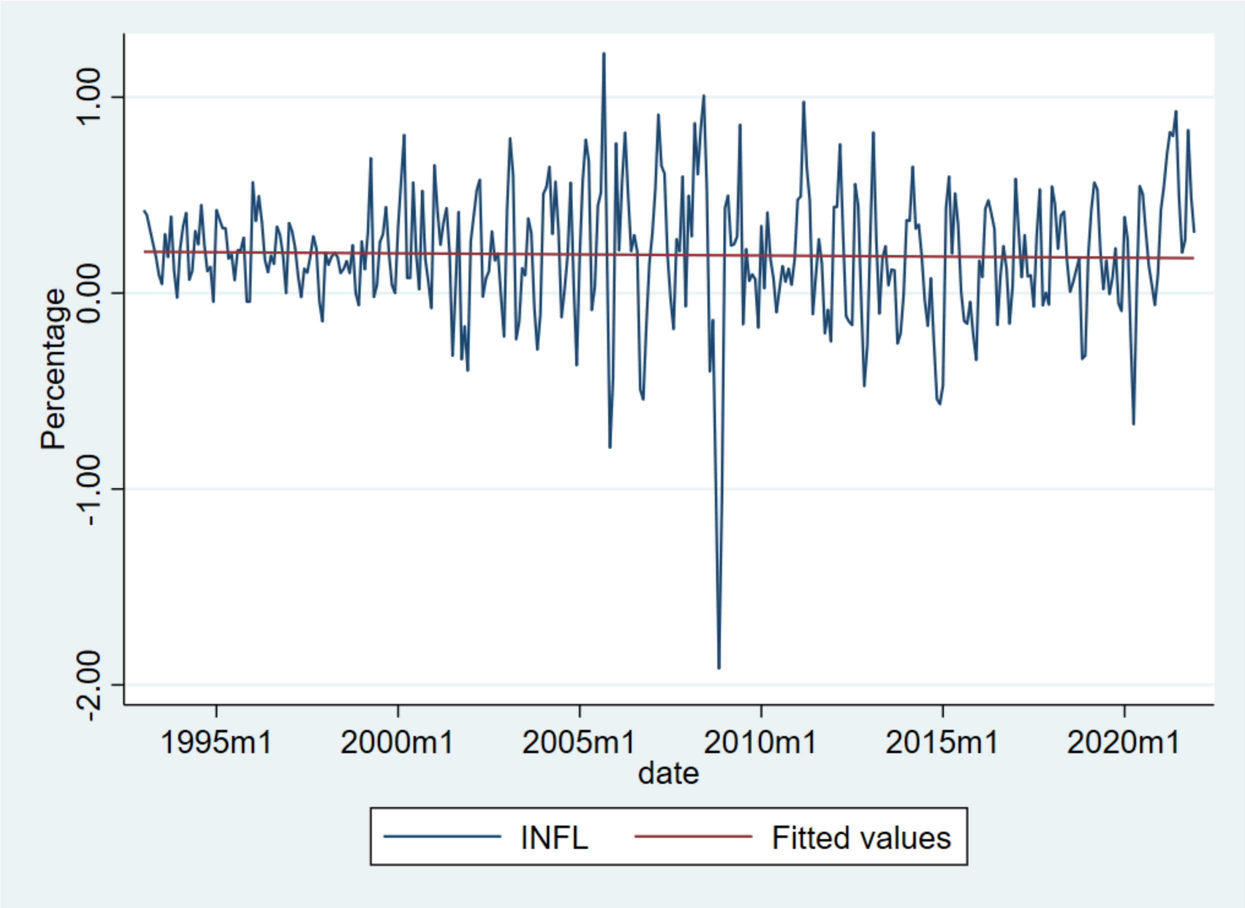


Figure 2. Predicted Values of the Conditional GARCH Variance of Inflation Rate from 1993m1-2021m8

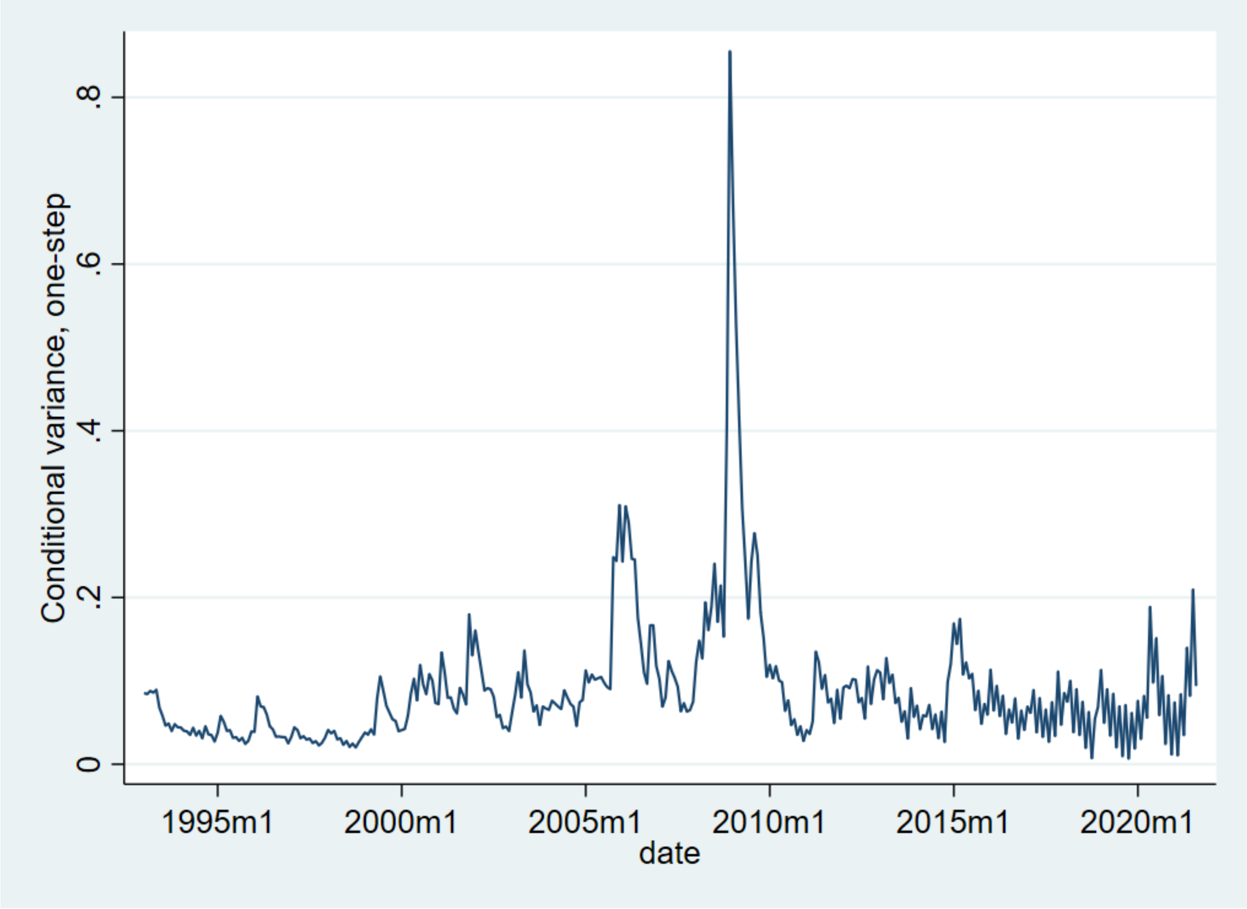
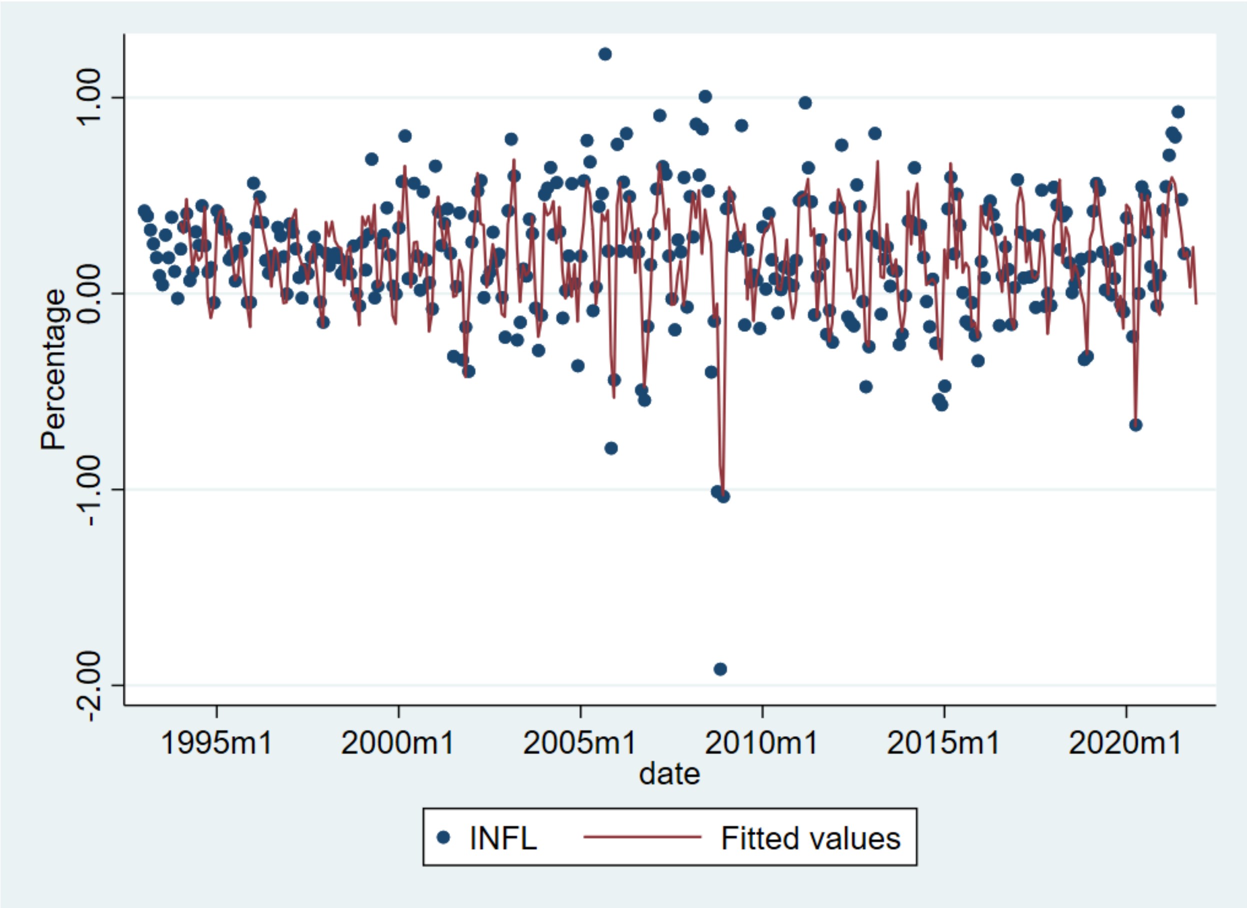


Figure 3. Actual Inflation VS the Sample Regression Line of Model 2 (1993m1 to 2021m8)



Appendix 1: Calculations for Hypothesis Tests in Section 3

1. Unit Root Tests

(a) INFL

First, select the lag length p:

$$INFL_t = \alpha_0 + \alpha_1 INFL_{t-1} + \sum_{i=1}^{p} \beta_i \Delta INFL_{t-p} + u_t$$

Start with p = 12 and use t-test to see if β_{12} is significant at 1%:

$$\left.\begin{matrix} H_0: \beta_{12} = 0 \\ H_A: \beta_{12} \neq 0 \end{matrix}\right\} \; Reject\ H_0\ if\ t = \left|\frac{\hat{\beta}_{12} - 0}{se(\hat{\beta}_{12})}\right| > Z_{0.005}$$

Result: $t = 1.40 < Z_{0.005} = 2.576$ and p-value = 0.162. So, fail to reject H_0 at 1%.

Therefore, I continue to perform the same test for p = 11, which gives a p-value of 0.256, so I fail to reject H_0 again. The p-values are 0.013, 0.025, and 0.395 when p = 10, 9, 8. And I continue this process until p = 7 when I reject H_0 at 1% with a p-value of 0.003.

Second, unit root test for *INFL* with lag = 7 (Augmented Dicky-Fuller Test):

$$\left.\begin{matrix} H_0: \alpha_1 = 0 \\ H_A: \alpha_1 < 0 \end{matrix}\right\} \; Reject\ H_0\ if\ t_0 = \frac{\hat{\alpha}_1 - 0}{se(\hat{\alpha}_1)} < DF_{0.01}$$

Result: $t_0 = -8.919 < DF_{0.01} = -3.453$ and p-value = 0.0000. So, reject H_0 at 1%

Therefore, I reject unit root for INFL.

(b) UNPL

Similarly, I first select lag = 1 for *UNPL* because $t = |-3.38| > Z_{0.005} = 2.576$ and p-value = 0.001 so I reject that the coefficient on lag 1 for *UNPL* is 0.

Next, for the unit root test, I fail to reject unit root at 1% because $t_0 = -3.382 > DF_{0.01} = -3.453$ and p-value = 0.0116.

(c) Price Level

Similarly, I select lag = 8 for Price Level because $t = |-2.73| > Z_{0.005} = 2.576$ and p-value = 0.007 so I reject that the coefficient on lag 8 for Price Level is 0.

For the unit root test, I fail to reject unit root at 1% because $t_0 = 0.782 > DF_{0.01} = -3.453$ and p-value = 0.9913.

2. Cointegration Test

First, regress Price Level on UNPL:

$$Price\ Level_t = \beta_0 + \beta_1 UNPL_t + u_t$$

Then perform a unit root test for the residual at lag = 0 since I am interested in the contemporary effect only.

$$\Delta\hat{u}_t = \alpha_0 + \beta\hat{u}_{t-1} + e_t$$

$$\left.\begin{array}{r} H_0: \beta = 0 \\ H_A: \beta < 0 \end{array}\right\} Reject\ H_0\ if\ t_0 = \frac{\hat{\beta} - 0}{se(\hat{\beta})} < DF_{0.01}$$

Result: $t_0 = -0.239 > DF_{0.01} = -3.453$ and p-value = 0.9337. So, fail to reject H_0 at 1%.

Therefore, there is no evidence of cointegration between *Price Level* and *UNPL*.

3. Linear Time Trend Test

Define t = 1 for 1993m1, t = 2 for 1993m2, … And carry out the t-test:

$$INFL_t = \beta_0 + \beta_1 t + u_t$$

$$\left.\begin{array}{r} H_0: \beta_1 = 0 \\ H_A: \beta_1 \neq 0 \end{array}\right\} Reject\ H_0\ if\ t = \left|\frac{\hat{\beta}_1 - 0}{se(\hat{\beta}_1)}\right| > Z_{0.005}$$

Result: $t = |-0.85| < Z_{0.005} = 2.576$ and p-value = 0.398. So, fail to reject H_0 at 1%. No linear trend.

4. Seasonality Test

Define month dummies $Month_i = 1$ if it is the *i*[th] month of the year (*i* = 2, 3, 4, …, 12).

$$INFL_t = \alpha_0 + \sum_{i=2}^{12} \beta_i Month_i + u_t$$

Then, carry out the following F-test:

$$\left.\begin{array}{r} H_0: \beta_2 = \beta_3 = \cdots = \beta_{12} = 0 \\ H_A: not\ (\beta_2 = \beta_3 = \cdots = \beta_{12} = 0) \end{array}\right\} Reject\ H_0\ if\ F = \frac{(R_U^2 - R_R^2)/11}{(1 - R_U^2)/(n-k-1)} > F_{11,n-k-1,0.01}$$

```
F( 11,    332) =    15.37
       Prob > F =    0.0000
```

The Stata output indicates I should reject H_0 since p-value = 0.0000 < 0.01. So, there is seasonality.

5. Serial Correlation Tests

First, estimate Model 1 with first-difference unemployment rate and seasonality:

$$INFL_t = \alpha_0 + \sum_{i=0}^{12} \gamma_i \Delta UNPL_{t-i} + \sum_{i=0}^{12} \eta_i \Delta UNPL_{t-i}^2 + \sum_{j=1}^{12} \rho_j INFL_{t-j} + \sum_{k=2}^{12} \beta_k Month_k + u_t$$

Get the residual $\hat{u}_t$.

(a) First-Order Serial Correlation

$$\hat{u}_t = \alpha_0 + \alpha_1 \hat{u}_{t-1} + \sum_{i=0}^{12} \gamma_i \Delta UNPL_{t-i} + \sum_{i=0}^{12} \eta_i \Delta UNPL_{t-i}^2 + \sum_{j=1}^{12} \rho_j INFL_{t-j} + \sum_{k=2}^{12} \beta_k Month_k + v_t$$

$$\left.\begin{matrix} H_0\colon \alpha_1 = 0 \\ H_A\colon \alpha_1 \neq 0 \end{matrix}\right\} \; Reject\ H_0\ if\ t = \left| \frac{\hat{\alpha}_1 - 0}{se(\hat{\alpha}_1)} \right| > Z_{0.005}$$

Result: $t = 0.44 < Z_{0.005} = 2.576$ and p-value = 0.658. So, fail to reject H_0 at 1%.

Therefore, there is no first-order serial correlation.

(a) Fourth-Order Serial Correlation

$$\hat{u}_t = \alpha_0 + \sum_{m=1}^{4} \alpha_m \hat{u}_{t-m} + \sum_{i=0}^{12} \gamma_i \Delta UNPL_{t-i} + \sum_{i=0}^{12} \eta_i \Delta UNPL_{t-i}^2 + \sum_{j=1}^{12} \rho_j INFL_{t-j} + \sum_{k=2}^{12} \beta_k Month_k + v_t$$

$$\left.\begin{matrix} H_0\colon \alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0 \\ H_A\colon not\ (\alpha_1 = \alpha_2 = \alpha_3 = \alpha_4 = 0) \end{matrix}\right\} \; Reject\ H_0\ if\ F = \frac{(R_U^2 - R_R^2)/4}{(1 - R_U^2)/(n-k-1)} > F_{4,n-k-1,0.01}$$

```
F(  4,   273) =    1.08
      Prob > F =    0.3689
```

The Stata output indicates I fail to reject H_0 since p-value = 0.3689 > 0.01.

Therefore, there is no fourth-order serial correlation.

6. Tests for Lag Length

Select lags q and l for the following model:

$$INFL_t = \alpha_0 + \sum_{i=0}^{q} \gamma_i \Delta UNPL_{t-i} + \sum_{i=0}^{q} \eta_i \Delta UNPL_{t-i}^2 + \sum_{j=1}^{l} \rho_j INFL_{t-j} + \sum_{k=2}^{12} \beta_k Month_k + u_t$$

Start with $q = l = 12$, and carry out the following tests:

First, t-test for INFL:

$$\left.\begin{array}{l} H_0: \rho_{12} = 0 \\ H_A: \rho_{12} \neq 0 \end{array}\right\} Reject\ H_0\ if\ t = \left|\frac{\hat{\rho}_{12} - 0}{se(\hat{\rho}_{12})}\right| > Z_{0.005}$$

Result: $t = |-2.99| > Z_{0.005} = 2.576$ and p-value = 0.003. So, reject H_0 at 1%.

Since I reject H_0 in the first test, I will increase l and start again with $l = 13$.

Then, F-test for UNPL:

$$\left.\begin{array}{r} H_0: \gamma_{12} = \eta_{12} = 0 \\ H_A: not\ (\gamma_{12} = \eta_{12} = 0) \end{array}\right\} Reject\ H_0\ if\ F = \frac{(R_U^2 - R_R^2)/2}{(1 - R_U^2)/(n-k-1)} > F_{2,n-k-1,0.01}$$

```
F(  2,   281) =    0.02
     Prob > F =    0.9799
```

The Stata output suggests that I fail to reject H_0 since p-value = 0.9799 > 0.01.

Next, I perform the same tests with $q = 12$ and $l = 13$.

I fail to reject both tests. The t-test of ρ_{13} has a test statistic $t = |0.45| < Z_{0.005} = 2.576$ and p-value = 0.650. So, fail to reject H_0 at 1%. Also, the F-test of $\gamma_{12} = \eta_{12} = 0$ has a p-value of 0.9824 > 0.01. So, fail to reject H_0 at 1%.

Therefore, I **fix** $l = 12$ and all the following tests are trying to determine the value of q.

I then test with $l = 12$ and $q = 11$. And the F-test of $\gamma_{11} = \eta_{11} = 0$ has a p-value of 0.1805 > 0.01. So, fail to reject H_0 at 1%.

I repeat the F-tests for q and reduce the value of q by 1 when I fail to reject the F-test.

Finally, I stopped the F-test when $q = 0$ because I reject the null hypothesis with a p-value of 0.0099 < 0.01.

Therefore, I select lag = 12 for *INFL* and lag = 0 for $\Delta UNPL$.

7. Short-run Impact Multiplier

Recall that Model 2 is

$$INFL_t = \alpha_0 + \gamma \Delta \mathrm{UNPL_t} + \eta \Delta UNPL_t^2 + \sum_{j=1}^{12} \rho_j INFL_{t-j} + \sum_{i=2}^{12} \beta_i Month_i + u_t$$

The Short-run Impact Multiplier is

$$\mathrm{SRIM} = \gamma + 2\eta\overline{\Delta UNPL} = -0.0851 + 2 * 0.0021 * (-0.0061) = -0.0851$$

Standardized coefficient is used here since the mean of *INFL* and $\Delta UNPL$ is not comparable:

$$Std.Coeff. = SRIM \times \frac{sd(\Delta UNPL)}{sd(INFL)} = -0.0851 \times \frac{0.6101}{0.3361} = -0.1545$$

It is not economically significant as its absolute value is less than 0.20.

8. Long-run Propensity

The Long-run Propensity is

$$\begin{aligned}\mathrm{LR}P = \frac{\gamma + 2\eta\overline{\Delta UNPL}}{1 - (\sum_{i=1}^{12} \rho_i)} \\ &= [-0.0851 + 2 * 0.0021 * (-0.0061)] \\ &/[1 - 0.52696 + 0.18584 - 0.01528 - 0.03093 + 0.09103 - 0.01714 \\ &+ 0.032926 + 0.058538 - 0.014539 - 0.012799 - 0.13648 + 0.18201] \\ &= -0.1069\end{aligned}$$

Standardized coefficient is used here since the mean of *INFL* and $\Delta UNPL$ is not comparable:

$$Std.Coeff. = LRP \times \frac{sd(\Delta UNPL)}{sd(INFL)} = -0.1069 \times \frac{0.6101}{0.3361} = -0.1940$$

It is not economically significant as its absolute value is less than 0.20. However, it is really close.

9. Predictive Accuracy Test

First get

$$\hat{u}_{n+j|n} = INFL_{n+j} - \widehat{INFL}_{n+j|n}$$

where *n* represents 2021m8 and $j = 1,2,3,4$.

$$\left.\begin{aligned} H_0&: model\ is\ correctly\ specified \\ H_A&: model\ is\ misspecified \end{aligned}\right\} Reject\ H_0\ if\ PA = \frac{\sum_{j=1}^{4} \hat{u}_{n+j|n}^2}{\hat{\sigma}^2} > \chi_{4,0.01}^2$$

$$PA = \frac{\sum_{j=1}^{4} \hat{u}_{n+j|n}^2}{\hat{\sigma}^2} = \frac{0.0083 + 0.7389 + 0.5265 + 0.3790}{0.0574} = 28.7948 > \chi_{4,0.01}^2 = 13.277$$

and p-value is less than 0.0001, so, I reject H_0.