TD - Adjustable Robust Optimization

Exercice 1. Robust location-transportation problem (adapted from Zeng and Zhao, 2013)

We revisit the location-transportation problem. A mixed-integer programming formulation for the deterministic version of the problem is given as follows :

$$\begin{aligned}
\min \quad & \sum_{i\in[m]} f_i x_i + \sum_{i\in[m]} a_i z_i + \sum_{i\in[m]}\sum_{j\in[n]} c_{ij} y_{ij} \\
\text{s.t.} \quad & z_i \le C_i x_i && \forall i \in [m] \\
& \sum_{j\in[n]} y_{ij} \le z_i && \forall i \in [m] \\
& \sum_{i\in[m]} y_{ij} \ge d_j && \forall j \in [n] \\
& x \in \{0,1\}^m, z \in \mathbb{R}_+^m, y \in \mathbb{R}_+^{m\times n}.
\end{aligned}$$

In reality, when decisions to build warehouses and determine their capacities are made, the demand of customers is unknown. To capture this uncertainty, assume that the demands d_j are replaced by the estimations $\bar{d}_j$ and deviations $\hat{d}_j$ in such a way that the uncertain demand $\tilde{d}_j$ is assumed to be in the interval $[\bar{d}_j, \bar{d}_j + \hat{d}_j]$. In order to avoid being overly conservative, we assume that at most Γ customers will have their demand deviate from its estimated value, giving rise to the following uncertainty set :

$$D^\Gamma = \left\{ \tilde{d} \in \mathbb{R}^n \mid \sum_{j\in[n]} \frac{\tilde{d}_j - \bar{d}_j}{\hat{d}_j} \le \Gamma, \tilde{d}_j \in [\bar{d}_j, \bar{d}_j + \hat{d}_j] \text{ for } j \in [n] \right\}.$$

We will then assume that $\sum_{i\in[m]} C_i \ge \max_{\tilde{d}\in D^\Gamma} \sum_{j\in[n]} \tilde{d}_j$ so that the problem always has a feasible solution. We consider the following two-stage robust optimization problem with continuous recourse :

$$\begin{aligned}
\min_{\substack{x\in\{0,1\}^m, z\in\mathbb{R}_+^m \\ z_i \le C_i x_i \quad \forall i\in[m]}} f^\top x + a^\top z + \max_{\tilde{d}\in D^\Gamma} \min_{y\ge 0} \quad & c^\top y \\
\text{s.t.} \quad & \sum_{j\in[n]} y_{ij} \le z_i && \forall i \in [m] \\
& \sum_{i\in[m]} y_{ij} \ge \tilde{d}_j && \forall j \in [n].
\end{aligned}$$

This model considers that the decisions concerning the location and capacity of each warehouse will be made without knowing what the demand realization will be while the decisions concerning the transportation of goods from warehouses to customers will be made after observing the actual realization of the demand.

Question 1. Write a deterministic equivalent optimization problem using the affine decision rule approximation.

Question 2. Derive a constraint that can be added to the first-stage feasible region in order to ensure relatively complete recourse, *i.e.*, by adding this constraint any first-stage solution (x, z) yields a feasible solution

y for $\tilde{d} \in D^{\Gamma}$.

Question 3. Sketch the details of the constraint generation algorithm applied to this problem. Specifically,
— Write an initial constraint-generation relaxation.
— Derive a mixed-integer linear programming formulation of the subproblem.
— Specify the form of the optimality cuts to be added to the relaxation at each iteration.

Question 4. Sketch the details of the constraint-and-column generation algorithm applied to this problem. Specifically,
— Write an initial constraint-and-column generation relaxation.
— Derive a mixed-integer linear programming formulation of the subproblem.
— Specify the form of the variables and constraints to be added to the relaxation at each iteration.

Question 5. For both the constraint and constraint-and-column generation approaches, specify how to calculate an upper and a lower bound on the optimal value of the problem at each iteration and provide a stopping criteria based on these bounds.