TD - Piecewise Decision Rules in Robust Optimization

Exercice 1. Uncertainty set partitioning for lot-sizing

We consider a lot-sizing problem in which all of the uncertain customer demand must be satisfied by using three different means : (i) we produce the necessary products at a unit cost of 50, (ii) we order a lot of 25 products at a unit cost of 60 (total cost of 1500) (iii) we order a lot of 25 products at a unit cost of 75 (total cost of 1875). Since production takes some time, the production decisions should be taken before the realization of the demand while the ordering decisions can be taken once the demand realization is known. However, each lot of size 25 can only be ordered only once. If there is any leftover stock once the demand is satisfied then a holding cost of 65 per unit should be paid. The uncertain demand is estimated to be in the interval $\Xi = [5, 95]$. We write an adjustable robust optimization model for this problem as follows :

$$
\begin{aligned}
\min \quad & \theta \\
\text{s.t.} \quad & \theta \geq 50x + 65(x + 25y_1(\xi) + 25y_2(\xi) - \xi) + 1500y_1(\xi) + 1875y_2(\xi) && \forall \xi \in \Xi \\
& (x + 25y_1(\xi) + 25y_2(\xi) - \xi) \geq 0 && \forall \xi \in \Xi \\
& x \geq 0 \\
& y_1(\xi), y_2(\xi) \in \{0, 1\} && \forall \xi \in \Xi
\end{aligned}
$$

1. Write the static robust version of this problem and find its optimal solution using a mixed-integer linear programming solver.
2. Starting from this static robust solution perform three iterations of the uncertainty set partitioning algorithm. At each iteration, state :
   - The identified uncertainty realizations and the partitions obtained.
   - The optimal adjustable solution corresponding to your partitions and its value.
   - The optimality gap.