Thesis -

ADMM method for designing precoders

List of Abbreviations

WSRM weighted-sum-rate-maximization

MIMO multiple-input-multiple-output

ADMM alternative-distributed-method-of-multipliers

SOCP second-order-cone-programming

BS base-station

SINR signal-to-noise-ratio

SOC second-order-cone

KKT karush-khun-tucker

1. Introduction

In this report we have formulated a convex optimization problem to maximize the weighted Sum rate maximization (WSRM) by designing the precoders using primal decomposition and alternative distributed method of multipliers (admm).

1.a Problem Formulation

Consider a system of $\mathcal{B}$ coordinated BSs of N transmit antennas each and K single antenna receivers. The set of all K users is denoted by $\mathcal{U} = \{1, 2, \ldots, K\}$. We assume that data for the k^{th} user is transmitted only from one BS, which is denoted by $b_k \in \mathcal{B}$, where $\mathcal{B} \triangleq \{1, 2, \ldots, \mathcal{B}\}$ is the set of all BSs. The set of all users served by BS b is denoted by $\mathcal{U}_b$. Under flat fading channel conditions, the signal received by the k^{th} user is

$$y_k = \mathbf{h}_{b_k,k}\mathbf{w}_k d_k + \sum_{i=1, i\neq k}^{K} \mathbf{h}_{b_i,k}\mathbf{w}_i d_i + n_k \tag{1}$$

where $\mathbf{h}_{b_i}, k \in \mathbb{C}^{1\times N}$ is the channel (row) vector from BS b_i to user k, $\mathbf{w}_k \in \mathbb{C}^{N\times 1}$ is the beamforming vector (beamformers) from BS b_k to user k, d_k is the normalized complex data symbol, and $n_k \sim \mathcal{CN}(0, \sigma^2)$ is complex circularly symmetric zero mean gaussian noise with variance σ^2. The term $\sum_{i=1,i\neq k}^{K} \mathbf{h}_{b_i,k}\mathbf{w}_i d_i$ in (1) includes both intra- and inter-cell interference. The total power transmitted by BS b is $\sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|^2$. The SINR γ_k of user k is

$$\gamma_k = \frac{\|\mathbf{h}_{b_k,k}\mathbf{w}_k\|^2}{\sigma^2 + \sum_{i=1,i\neq k}^{K} \|\mathbf{h}_{b_i,k}\|} \tag{2}$$

In this report, we are interested in the problem of WSRM under per-BS power constraints, which is formulated as,

$$\max_{\sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|^2 \leq P_b, \forall\, b\in\mathcal{B}} \quad \sum_{k=1}^{K} \alpha_k \log(1+\gamma_k) \tag{3}$$

where α_k's are positive weighting factors which are typically introduced to maintain a certain degree of fairness among users.

2. Existing Formulation for WSRM

To achieve a tractable solution for the Low-Complexity beamfomer design, we note that following monotonicity of logarithmic function, (3) is equivalent to,

$$\max_{\sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|^2 \leq P_b, \forall b\in\mathcal{B}} \quad \prod_k (1+\gamma_k)^{\alpha_k} \tag{4}$$

which can be re-written as,

$$\begin{aligned}
\max_{w_k, t_k} \quad & \prod_k t_k && \text{(5a)}\\
\text{subject to} \quad & \gamma_k \geq {t_k}^{1/\alpha_k} - 1, \forall k \in \mathcal{U}, && \text{(5b)}\\
& \sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|_2^2 \leq P_b, \forall b \in \mathcal{B}. && \text{(5c)}
\end{aligned}$$

In equation (4) and (5) it can be seen that all constraints are active at the optimum, otherwise, we can obtain a larger objective by increasing t_k without violating the constraints. We can reformulate (5) by reintroducing additional slack variable β_k,

$$
\begin{aligned}
\underset{w_k,t_k,\beta_k}{\text{maximize}} \quad & \prod_k t_k && \text{(6a)}\\
\text{subject to} \quad & \mathbf{h}_{b_k,k}\mathbf{w}_k \geq \sqrt{{t_k}^{1/\alpha_k}-1}\beta_k, \forall k \in \mathcal{U} && \text{(6b)}\\
& \mathrm{Im}(\mathbf{h}_{b_k,k}\mathbf{w}_k) = 0, \forall k \in \mathcal{U}, && \text{(6c)}\\
& (\sigma^2 + \sum_{i=1,i\neq k}^{K} \|\mathbf{h}_{b_i,k}\mathbf{w}_k\|^2)^{1/2} \leq \beta_k, \forall k \in \mathcal{U}, && \text{(6d)}\\
& \sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|_2^2 \leq P_b, \forall b \in \mathcal{B}. && \text{(6e)}
\end{aligned}
$$

The relation between (5) and (6) can be seen as, first, by forcing the imaginary part of $\mathbf{h}_{b_k,k}\mathbf{w}_k$ to zero in (6c) does not affect the optimality of (5) since phase rotation on $\mathbf{w}_k$ will result in the same objective while satisfying all constraints. Second we can show that all the constraints in (6d) hold with equality at the optimum.

3. Decentralized precoder design using ADMM for WSRM

In the decentralized precoder design, the precoders are designed independently at each base station. This design requires less information exchange. The decentralized approach is explained with the alternating method of multipliers.

3.a Alternating Direction Method of Multipliers (ADMM)

In contrary with the primal decomposition method, dual composition method reduces the constraint by considering the objective functions in the sub problem. In this method we hold a local and a global copy of the signal interference. At optimality the copies remain equal.

ADMM method can be formulated as following,

$$
\begin{aligned}
\underset{\substack{w_k,t_k,\beta_k,\\ \delta_{b,k}^{b_k},\delta_{b_k,i}^{b_k}}}{\text{maximize}} \quad & \sum_k \log t_k + \sum_{k\in\mathcal{U}_{b_k}} \sum_{b\in\bar{\mathcal{B}}_{b_k}} \lambda_{b,k}^{b_k}(\delta_{b,k}^{b_k} - \delta_{b,k}^{G}) + \sum_{i\in\bar{\mathcal{U}}_{b_k}} \lambda_{b_k,i}^{b_k}(\delta_{b_k,i}^{b_k} - \delta_{b_k,i}^{G}) - \\
& \frac{\rho}{2} \sum_{k\in\mathcal{U}_{b_k}} \sum_{b\in\bar{\mathcal{B}}_{b_k}} \|\delta_{b,k}^{b_k} - \delta_{b,k}^{G}\|_2^2 - \frac{\rho}{2} \sum_{i\in\bar{\mathcal{U}}_{b_k}} \|\delta_{b_k,i}^{b_k} - \delta_{b_k,i}^{G}\|_2^2 && \text{(7a)}\\
\text{subject to} \quad & \mathbf{h}_{b_k,k}\mathbf{w}_k \geq \beta_k\sqrt{({t_k}^{1/\alpha_k}-1)}, \forall k \in \mathcal{U}_{b_k} && \text{(7b)}\\
& \mathrm{Im}(\mathbf{h}_{b_k,k}\mathbf{w}_k) = 0, \forall k \in \mathcal{U}_{b_k}, && \text{(7c)}\\
& \sigma^2 + \sum_{\substack{i\in\mathcal{U}_{b_k}\\ i\neq k}} \|\mathbf{h}_{b_k,k}\mathbf{w}_i\|^2 + \sum_{b\in\bar{\mathcal{B}}_{b_k}} \delta_{b,k}^{(b_k)} \leq \beta_k, \forall k \in \mathcal{U}_{b_k}, && \text{(7d)}\\
& \delta_{b,j}^{(b_k)} \geq \sum_{i\in\mathcal{U}_b} \|\mathbf{h}_{b_k,k}\mathbf{w}_i\|^2, \forall j \in \bar{\mathcal{U}}_{b_k} && \text{(7e)}\\
& \sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|_2^2 \leq P_b, \forall k \in \mathcal{U}_{b_k}. && \text{(7f)}
\end{aligned}
$$

In the equation above the objective function of the precoder design is to maximize the sum rate (WSRM)problem. The ADMM method uses the sub gradient update method, where we assume the interference term from the adjascent BS. The $\delta^{(b_k)}$ is the local interference term taken into account from the adjascent BS, similarly the $\delta^{(G)}$ is the global interference term taken into account from the adjascent BS.At optimality the local and the global interference terms becomes same. The equation (7b) and the equation (7c) remains the same as the previous formulation. In the equation (7d) we observe that there is the local interference term coming from the adjascent BS adding into the noise plus the interference from the same BS. Similarly equation (7e) shows the sum of all interefrence terms from the adjascent BS.

Let us consider a two base station scenario such that the global interference term can be updated as follows,

$$\delta_{b,k}^{G} = \frac{1}{2}(\delta_{b,k}^{(b)} + \delta_{b,k}^{(b_k)}) \tag{8}$$

where $\delta_{b,k}^{(b)}$ refers to the actual interference caused by BS b and $\delta_{b,k}^{(b_k)}$ refers to the local interference caused by the BS b_k

The update for the local interference term is made through iteration of the objective function in each BS. Once the local interference iterations are done then the global interference and the dual variable update are made in the main problem. The dual variable corresponds to the interference terms in the BS b_k and is updated with the subgradient method. *rho* gives the dual update step length.

$$\lambda_{b,k}^{(n+1)} = \lambda_{b,k}^{(n)} - \rho(\delta_{b,k}^{(b_k)^*} - \delta_{b,k}^{(G)^*}) \tag{9}$$

3.b NCVX

$$\underset{w_k,\gamma_k,\beta_k}{\text{maximize}} \quad \sum_k \log(1+\gamma_k) \tag{10a}$$

$$\text{subject to} \quad \|\mathbf{h}_{b_k,k}\,\mathbf{w}_k\|^2 \geq \frac{1}{2\phi_k}\gamma_k^2 + \frac{\phi_k}{2}\beta_k^2, \forall k \in \mathcal{U}, \tag{10b}$$

$$\sigma^2 + \sum_{\substack{i=1,\\ i\neq k}}^{K} \|\mathbf{h}_{b_i,k}\mathbf{w}_i\|^2 \leq \beta_k, \forall i \in \bar{\mathcal{U}}_{b_k}, \tag{10c}$$

$$\sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|_2^2 \leq P_b, \forall b \in \mathcal{B}. \tag{10d}$$

The above problem turns out to be non convex due to constraint in the equation (10b). Inorder to have a convex problem we need to linearize the equation (10b) by taking the first order approximation of the expression $\|\mathbf{h}_{b_k,k}\,\mathbf{w}_k\|^2$ in the LHS.

$$\|\mathbf{h}_{b_k,k}\,\mathbf{w}_k\|^2 = \|\mathbf{h}_{b_k,k}\,\bar{\mathbf{w}}_k\|^2 + 2\bar{\mathbf{w}}_k^H \mathbf{h}_{b_k,k}{}^H \mathbf{h}_{b_k,k}(\mathbf{w}_k - \bar{\mathbf{w}}_k) \tag{11}$$

now the problem can be solved with the KKT condition. The closed form solutions are obtained for the primal variables.

$$\gamma_k = -0.5 \pm 0.5\sqrt{1+\frac{4\phi_k}{\lambda_1}} \tag{12a}$$

$$\beta_k = \frac{\lambda_2}{\lambda_1\phi_k} \tag{12b}$$

$$\mathbf{w}_k = (2\lambda_2^{(i)}\mathbf{h}_{b_i}\mathbf{h}_{b_i}{}^H + 2\lambda_3)^{-1}\lambda_1\mathbf{h}_{b_k} \tag{12c}$$

From the above equations we can find the dual variables λ_1 and λ_2 as follows,

$$\lambda_1 = \frac{\phi_k}{(1+\gamma_k)\gamma_k} \tag{13a}$$

$$\lambda_2 = \lambda_1\phi_k\beta_k \tag{13b}$$

From the above equations we can see that the dual variables the dual variables λ_1 and λ_2 is obtained by the sub gradient search, whereas, λ_3 can be obtained by the bisectin search.

3.c CVX

$$\begin{aligned}
\underset{w_k,\gamma_k,\beta_k}{\text{maximize}} \quad & \sum_k \log(1+\gamma_k) && (14a)\\
\text{subject to} \quad & \text{real}(\mathbf{h}_{b_k,k}\,\mathbf{w}_k) \geq \frac{1}{2\phi_k}\gamma_k + \frac{\phi_k}{2}\beta_k, \forall k \in \mathcal{U}, && (14b)\\
& \text{Im}(\mathbf{h}_{b_k,k}\,\mathbf{w}_k) = 0 && (14c)\\
& \sigma^2 + \sum_{\substack{i=1,\\ i\neq k}}^{K} \|\mathbf{h}_{b_i,k}\mathbf{w}_i\|^2 \leq \beta_k, \forall i \in \bar{\mathcal{U}}_{b_k}, && (14d)\\
& \sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|_2^2 \leq P_b, \forall b \in \mathcal{B}. && (14e)
\end{aligned}$$

In this problem the constraints are in convex form so it can be solved normally. On solving the equations by the KKT condition, we obtain,

$$\gamma_k = \frac{2\phi_k}{\lambda_1} - 1 \tag{15a}$$

$$\beta_k = \text{equation}(14b) \tag{15b}$$

$$\mathbf{w}_k = (2\lambda_2^{(i)}\mathbf{h}_{b_i}\mathbf{h}_{b_i}{}^H + 2\lambda_3)^{-1}\lambda_1\mathbf{h}_{b_k} \tag{15c}$$

From the above equations we observe that β_k is same equal to the equation (28c) because constraint is active since the dual variable λ_2 is positive by the condition of complementary slackness. And the dual variables λ_1 and λ_2 can be obtained as,

$$\lambda_1 = \frac{2\phi_k}{1+\gamma_k} \tag{16a}$$

$$\lambda_2 = \frac{\lambda_1\phi_k}{2} \tag{16b}$$

From the above equations we can see that the dual variables λ_1 can be solved by the subgradient search λ_3 can be obtained by the bisection search, whereas the dual variable λ_2 can be directly solved once λ_1 is known.

3.d Method 1

The problem,

$$\begin{aligned}
\underset{w_k,\gamma_k,\beta_k}{\text{maximize}} \quad & \sum_k \log(1+\gamma_k) && \text{(17a)}\\
\text{subject to} \quad & a_k : \Re(\mathbf{h}_{b_k,k}\,\mathbf{w}_k) \geq \frac{1}{2\phi_k}\gamma_k^2 + \frac{\phi_k}{2}\beta_k^2, \forall k \in \mathcal{U}, && \text{(17b)}\\
& b_k : \sigma^2 + \sum_{\substack{i=1,\\ i\neq k}}^{K} \|\mathbf{h}_{b_i,k}\mathbf{w}_i\|^2 \leq \beta_k, \forall i \in \bar{\mathcal{U}_{b_k}}, && \text{(17c)}\\
& c_b : \sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|_2^2 \leq P_b, \forall b \in \mathcal{B}. && \text{(17d)}
\end{aligned}$$

The lagrangian,

$$L(\gamma_k,\beta_k,w_k,a_k,b_k,c_b) = -\sum_k \log(1+\gamma_k) + \tag{18}$$

$$a_k\left[\frac{1}{2\phi_k}\gamma_k^2 + \frac{\phi_k}{2}\beta_k^2 - \|\mathbf{h}_{b_k,k}\,\mathbf{w}_k\|\right] + \tag{19}$$

$$b_k\left[\sigma^2 + \sum_{\substack{i=1,\\ i\neq k}}^{K} \|\mathbf{h}_{b_i,k}\mathbf{w}_i\|^2 - \beta_k\right] + \tag{20}$$

$$c_b\left[\sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|_2^2 - P_b\right] \tag{21}$$

Difference of Lagrangian

$$\nabla\gamma_k : \frac{-1}{(1+\gamma_k)} + \frac{a_k}{\phi_k}\gamma_k = 0 \tag{22}$$

$$\nabla\beta_k : a_k\phi_k\beta_k - b_k = 0 \tag{23}$$

$$\nabla w_k : -a_k h_{b_k} + 2\sum_{i\neq K} b_i h_{b_i} {h_{b_i}}^H w_k + 2c_b \sum_k w_k = 0 \tag{24}$$

Slackness

$$\left(\frac{1}{2\phi_k}\gamma_k^2 + \frac{\phi_k}{2}\beta_k^2 - \|\mathbf{h}_{b_k,k}\,\mathbf{w}_k\| = 0\right) a_k = 0 \tag{25}$$

$$\left(\sigma^2 + \sum_{\substack{i=1,\\ i\neq k}}^{K} \|\mathbf{h}_{b_i,k}\mathbf{w}_i\|^2 - \beta_k = 0\right) b_k = 0 \tag{26}$$

$$\left(\sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|_2^2 - P_b = 0\right) c_b = 0 \tag{27}$$

3.e Method 2

The problem,

$$\begin{aligned} \underset{w_k,\gamma_k,\beta_k}{\text{maximize}} \quad & \sum_k \log(1+\gamma_k) && \text{(28a)}\\ \text{subject to} \quad & a_k : (\|\mathbf{h}_{b_k,k}\,\mathbf{w}_k\|^2) \geq \gamma_k\beta_k, \forall k \in \mathcal{U}, && \text{(28b)}\\ & b_k : \sigma^2 + \sum_{\substack{i=1,\\ i\neq k}}^{K} \|\mathbf{h}_{b_i,k}\mathbf{w}_i\|^2 \leq \beta_k, \forall i \in \bar{\mathcal{U}}_{b_k}, && \text{(28c)}\\ & c_b : \sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|_2^2 \leq P_b, \forall b \in \mathcal{B}. && \text{(28d)} \end{aligned}$$

The lagrangian,

$$L(\gamma_k, \beta_k, w_k, a_k, b_k, c_k) = -\sum_k \log(1+\gamma_k) + \tag{29}$$

$$a_k \left[\sqrt{\gamma_k\beta_k} - h_{b_k} w_k\right] + \tag{30}$$

$$b_k \left[\sigma^2 + \sum_{\substack{i=1,\\ i\neq k}}^{K} \|\mathbf{h}_{b_i,k}\mathbf{w}_i\|^2 - \beta_k\right] + \tag{31}$$

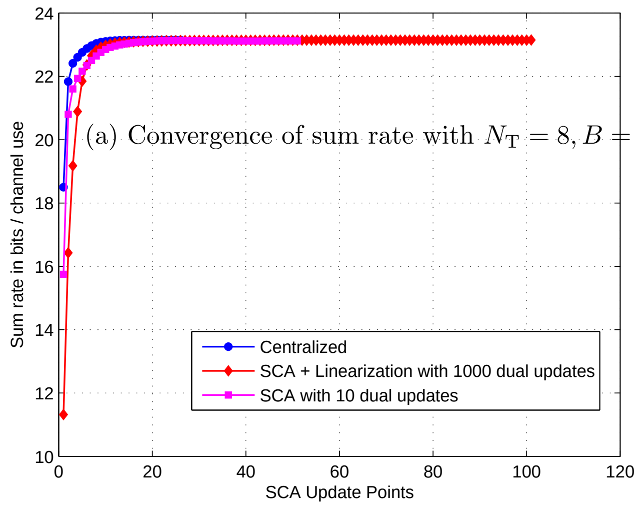
$$c_b \left[\sum_{k\in\mathcal{U}_b} \|\mathbf{w}_k\|_2^2 - P_b\right] \tag{32}$$

Difference of Lagrangian

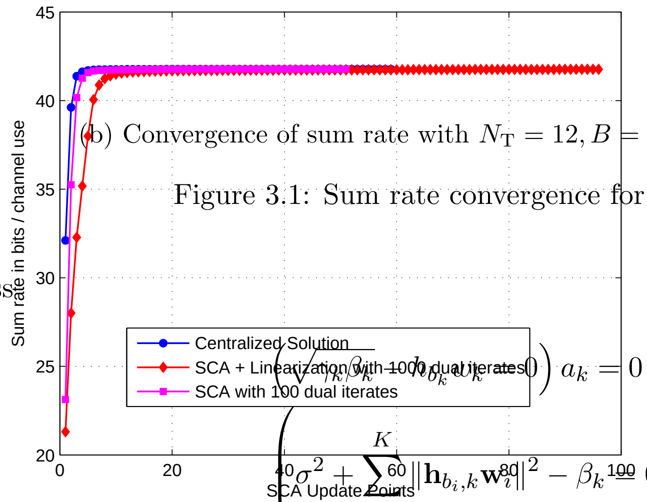
$$\nabla\gamma_k : \frac{-1}{(1+\gamma_k)} + \frac{a_k}{2\sqrt{\gamma_k\beta_k}}\beta_k = 0 \tag{33}$$

$$\nabla\beta_k : \frac{a_k}{2\sqrt{\gamma_k\beta_k} - h_{b_k} w_k} = 0 \tag{34}$$

$$\nabla w_k : -a_k h_{b_k} + 2\sum_{i\neq k} b_i h_{b_i} {h_{b_i}}^H w_k + 2c_b \sum_k w_k = 0 \tag{35}$$



(a) Convergence of sum rate with $N_{\mathrm{T}} = 8, B = 2, K = 6$, Mobile users



(b) Convergence of sum rate with $N_{\mathrm{T}} = 12, B = 3, K = 9$, Mobile users

Figure 3.1: Sum rate convergence for mobile users

Slackness

$$\left(\sqrt{\gamma_k \beta_k} - h_{b_k} \omega_k = 0\right) a_k = 0 \tag{36}$$

$$\left(\sigma^2 + \sum_{\substack{i=1, \\ i \neq k}}^{K} \|\mathbf{h}_{b_i,k} \mathbf{w}_i\|^2 - \beta_k = 0\right) b_k = 0 \tag{37}$$

$$\left(\sum_{k \in \mathcal{U}_b} \|\mathbf{w}_k\|_2^2 - P_b = 0\right) c_b = 0 \tag{38}$$