USER GROUPING AND TWO LEVEL PRECODING FOR DIMENSIONALITY REDUCTION IN MASSIVE MIMO

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ABSTRACT

We consider a 5G Massive multiple-input multiple-output (MIMO) set-up with very large number of antennas. Large number of antennas results in higher dimensional complexity while performing conventional MIMO processing. In this paper, we focus on fully digital two level beamforming, where the beamformer is divided into outer beamformer that depends on the second order channel statistics and the inner beamformer based on fast channel variations. The users are partitioned into groups based on their channel correlation matrix wherein co-located users forms the outer beamformer thereby efficiently reducing the channel dimension accounting to only group specific users. The main aim of outer beamformer is to reduce the dimensionality of the effective channel by exploiting the near-orthogonal channel covariance eigen spaces of different user groups. A two level precoding weighted sum rate maximization (WSRM) problem is proposed for a single cell downlink (DL) system targeting at minimizing the inter-group and intra-group interferences, respectively. We also discuss different methods to form the outer precoder matrix for each group. Numerical simulations evaluates the performance of different group specific beamforming methods as a function of statistical beams.

1. INTRODUCTION

Massive MIMO is considered to be the future enabling technology for the 5G cellular communication [1–3]. This system consist of large number of antenna elements which supports increased data rate, reliability, diversity, reduced interference and results in degrees of freedom (DoF) gain and beamforming gain. However, the down side of massive MIMO is the increased computational complexity due to the large number of antennas. The conventional MIMO processing involves higher dimensional matrix operations involving precoders and channel matrices. Moreover, to make use of the conventional processing such as minimum mean squared error (MMSE) and zero-forcing (ZF), computation of matrix inverses are based on higher dimensional channel matrix are required. Moreover, these higher dimensional channel matrix requires accurate channel state information (CSI), which accounts to more complexity in CSI acquisition.

Complexity reduction in massive MIMO has gained lot of attention among researchers. In recent years joint spatial division and multiplexing (JSDM) has gained more focus and is a promising method for fully digital two level beamforming [4] which has importance in terms of complexity reduction. The main idea of JSDM lies in the partitioning of the users into groups based on similar transmit correlation matrices so as to reduce the inter group interference with the users geographic location. In [4], author discusses various methods to form the outer beamformer and discusses performance analysis using the techniques of deterministic equivalents for different types of group processing like joint group and per-group processing. In [5–7] JSDM was studied extensively for user grouping. Several other techniques in addition to JSDM for two level precoding is suggested, where [8] the outer and the inner beamformers are used to control the inter and intra-cell interference, respectively. In [9], two level precoding was explored for different heuristic methods and the performance was evaluated as a function of statistical pre-beams.

In our work, we formulate a WSRM problem for a single cell DL massive MIMO system utilizing two level beamforming and user grouping. Following the approaches in [10] we formulate the problem to optimize the inner precoder for the fast channel variations with a fixed outer precoder for each group. The outer precoder is fixed by using the eigen selection method mentioned in [9]. The beamformer design can be classified into group specific beamforming with centralized interference variables and group specific beamforming with fixed inter-group interference constant. Thus the outer beamforming matrix is chosen so as to minimize the interference across different groups and the inner precoder takes care of the interference within one group. The simulation results suggests how the group specific outer precoder dimension can be reduced for each group without impacting the achievable rate.

2. SYSTEM MODEL

We consider a DL single-cell multi-user (MU) massive MIMO system where a single base station (BS) is equipped with M transmit antennas serving K single-antenna user terminal (UT) with $M > K$. The set of all users is represented as $\mathcal{U}_k = \{1, \ldots, K\}$ and the users are grouped into G groups, where $G = M/K$ and the number of users belonging to a group is $k_g = K/G$. The set of all groups is represented by $\mathcal{U}_g = \{1, \ldots, G\}$ and the set of users belonging to a group is represented as $\mathcal{U}_{k_g} = \{1, \ldots, K_g\}$. As a result of the user grouping, the received signal y_{k_g} of user $k_g \in g$, can be expressed as

$$\begin{aligned} y_{k_g} &= \underbrace{\mathbf{h}_{k_g}^H \mathbf{v}_{k_g} x_{k_g}}_{desired\ signal} + \underbrace{\sum_{i \in g, \backslash k_g} \mathbf{h}_{k_g}^H \mathbf{v}_{k_i} x_{k_i}}_{intra-group\, interference} \\ &+ \underbrace{\sum_{j \in G \backslash g} \mathbf{h}_{k_g}^H \mathbf{v}_{k_j} x_{k_j}}_{inter-group\, interference} + \mathbf{n}_{k_g} \end{aligned} \tag{1}$$

where the first term in (1) is the desired signal while the second and third term represents the intra-group and inter-group interference. We assume the channel between the BS and a user in group $\mathcal{U}_{k_g}$ to be denoted as $\mathbf{h}_{k_g} \in \mathbb{C}^{M \times 1}$ and assume the $M \times K_g$ dimensional channel matrix with $\mathbf{H_g} = [\mathbf{h}_{1,g}, \ldots, \mathbf{h}_{k,g}]$, while $\mathbf{v}_{k_g} \in \mathbb{C}^{M \times 1}$ denotes the precoding vector for user k_g. The transmitted data symbol for user $k_g \in \mathcal{U}_{k_g}$ is denoted by x_{k_g} with $\mathbb{E}[|x_{k_g}|^2] \leq 1, \forall k_g \in \mathcal{U}_{k_g}$. $\mathbf{n}_{k_g}$ represents the zero mean white Gaussian noise at the receiver with variance N_0. We can represent the data symbol $\mathbf{x}_{k_g} = \mathbf{V_g d_{k_g}}$, where $\mathbf{d}_{k_g}$ is expressed by the $S_g \times 1$ vector of transmitted user data symbols and $S_g = S/G$ is the design parameter describing the amount of statistical pre-beams used for a group $\mathcal{U}_{k_g}$.

We also assume that the precoding is applied in two levels as $\mathbf{V} = \mathbf{BW}$ where $\mathbf{V} \in \mathbb{C}^{M \times K}$ is the total precoding matrix of all the users in $\sum_{\mathcal{U}_g} G$, where $\mathbf{B} \in \mathbb{C}^{M \times S}$ is the outer precoder matrix of all users based on slow-varying channel statistics and $\mathbf{W} \in \mathbb{C}^{S \times K}$ is the inner precoder matrix of all users applying multi-user processing based on effective channel $\bar{H} = \mathbf{H}^H \mathbf{B}$ of dimensions $K \times S$.

2.1. Channel Model

The general channel vector $\mathbf{h}_k$ for K users is modelled as the classical multipath model for uniform linear array (ULA) [11]:

$$\mathbf{h}_k = \frac{\beta_k}{\sqrt{L}} \sum_{l=1}^{L} \mathbf{a}(\theta_{k,l}) e^{j\phi_{k,l}}, \tag{2}$$

where β_k denotes the path loss between the BS and the user k, L denotes the number of independent (and independent and identically distributed (i.i.d)) paths, $\phi_{k,l}$ is a random phase caused by the channel for path l, i.i.d between different paths, and $\mathbf{a}(\theta)$ is the array signature vector given by

$$\mathbf{a}(\theta) = \begin{bmatrix} 1 \\ e^{j2\phi \frac{D}{\lambda} \cos(\theta)} \\ . \\ . \\ . \\ e^{j2\phi \frac{(M-1)D}{\lambda} \cos(\theta)} \end{bmatrix}, \tag{3}$$

where D is the BS antenna spacing, λ is the carrier wavelength and θ is the angle of departure (AoD). The user specific channel correlation matrix can be defined as $\mathbf{R}_k = \mathbb{E}[\mathbf{h}_k \mathbf{h}_k^H]$, and the sum of all correlation matrices determine the total channel correlation matrix $\mathbf{R} = \sum_k \mathbf{R}_k$.

2.2. User Grouping

In real life scenario, users cannot be naturally partitioned into groups. Here, we discuss how the users are grouped to form the outer precoding matrix $\mathbf{B}$. K users are partitioned into G groups based on the similarity of their channel correlation matrices. The channel vector for k_g^{th} user in group g is denoted as $\mathbf{h}_{k_g} = \frac{\beta_{k_g}}{\sqrt{L}} \sum_{l=1}^{L} \mathbf{a}(\theta_{k_g,l}) e^{j\phi_{k_g,l}}$ and for the overall $M \times K$ system, channel matrix is denoted as $\mathbf{H} = [\mathbf{H}_1, \ldots, \mathbf{H}_G]$. Since, we assume two level precoding for all groups $\mathcal{U}_g$, we denote the outer precoder matrix of group $\mathcal{U}_{k_g}$ as $\mathbf{B}_g$ of dimensions $M \times S_g$, such that $\sum_{g=1}^{G} S_g = S$, hence we have $\mathbf{B} = [\mathbf{B}_1, \ldots, \mathbf{B}_g]$ and the group specific channel correlation matrix as R_{k_g}.

2.2.1. DFT based Fixed Quantized User Grouping

In this method, the groups are predefined based on the geographical location and their channel scattering as discussed in [5]. Since, our approach of user grouping is for massive MIMO, which is based on large number of antennas, the channel eigen spaces are considered to be mutually orthogonal as mentioned in [4]. Hence, the groups are chosen in a way such that the angle of arrival (AoA)s θ_g and angular spread (AS) are fixed, so that the resulting groups are disjoint. This helps us in computing the eigen space corresponding to the newly constructed covariance matrix.

As a result of user grouping and two level precoding, the received signal can be expressed with the outer and inner group specific precoder as

$$\begin{aligned} y_{k_g} &= \underbrace{\mathbf{h}_{k_g}^H \mathbf{B}_{k_g} \mathbf{w}_{k_g} x_{k_g}}_{desiredsignal} + \underbrace{\sum_{i \in g, \backslash k_g} \mathbf{h}_{k_g}^H \mathbf{B}_{k_i} \mathbf{w}_{k_i} x_{k_i}}_{intra-group\, interference} \\ &+ \underbrace{\sum_{j \in G \backslash g} \mathbf{h}_{k_g}^H \mathbf{B}_{k_j} \mathbf{w}_{k_j} x_{k_j}}_{inter-group\, interference} + \mathbf{n}_{k_g}. \end{aligned} \tag{4}$$

Thus, the signal-to-interference-plus-noise-ratio (SINR) of user k_g in group g can be expressed as

$$\gamma_{k_g} = \frac{|\mathbf{h}_{k_g}^H \mathbf{B}_{k_g} \mathbf{w}_{k_g}|^2}{|\sum_{i \in \mathcal{U}_g, \setminus K_g} \mathbf{h}_{k_g}^H \mathbf{B}_{k_i} \mathbf{w}_{k_i}|^2 + |\sum_{j \in G \setminus g} \mathbf{h}_{k_g}^H \mathbf{B}_{k_j} \mathbf{w}_{k_j}|^2 + N_0}. \tag{5}$$

Utilizing the SINR expression in equation (5), we can determine the weighted sum-rate of the given system as

$$R = \sum_g \sum_{k_g} \alpha_{k_g} \log_2(1 + \gamma_{k_g}) \approxeq \alpha_k \sum_k \log_2(1 + \gamma_k), \tag{6}$$

where $\alpha_{k_g} \geq 0$ is a user specific weight coefficient that can be determined with user scheduling. We use approximately symbol in (6) since weighted sum rate of each user is equal to the weighted sum rate of group specific users.

3. PRECODER DESIGN

In this section, we discuss the joint beamformer design for the system discussed in 2. The joint optimization of outer precoder $\mathbf{B}$ and the inner precoder $\mathbf{W}$ is highly complex due to their time-scale variation. Hence, we design the outer and inner precoders separately, considering various heuristic approaches in constructing the outer precoder matrix $\mathbf{B}$. Furthermore, while optimizing the inner precoder, it is assumed that the outer precoder $\mathbf{B}$ is fixed as it is based on the slow-varying channel statistics.

3.1. Outer Precoder Design

We consider the group specific outer precoder matrix $\mathbf{B}_g, \forall \mathcal{U}_{k_g}$, which is based on the slow-varying channel statistics, *i.e.,* the user covariance matrices. Therefore the obvious solution is as mentioned in [9], where the formulation of $\mathbf{B}_g$ is to decompose the total channel covariance matrix via eigenvalue decomposition (EVD). In EVD, the total channel covariance matrix is represented as $\mathbf{R} = \mathbf{U}\mathbf{\Lambda}\mathbf{U}^H$, and choose S_g channel covariance eigenvectors corresponding to the S_g largest eigenvalues. Thus the resulting outer precoder for a group $\mathcal{U}_{k_g}$ is $\mathbf{B}_g = [\mathbf{u}_{g_1}, \ldots, \mathbf{u}_{g_s}] \in \mathbb{C}^{M \times S_g}$, which effectively forms the pre-beams towards the strongest signal paths.

3.2. Inner Precoder Design

In order to formulate the problem of designing inner precoders with WSRM objective, we consider including the constraint on total transmit power. By doing so, the WSRM problem can be formulated as

$$\underset{\mathbf{w}_{k_g}}{\text{maximize}} \quad \sum_g \prod_{k_g} (1 + \gamma_{k_g})^{\alpha_{k_g}} \tag{7a}$$

$$\text{subject to} \quad \sum_g \sum_{k_g} ||\mathbf{B}_g \mathbf{w}_{k_g}||^2 \leq P_{tot} \tag{7b}$$

where the SINR expression γ_{k_g} is defined in equation (5). The constraint in equation (7b) is used to limit the total transmit power from the BS to be within P_{tot}.

In general the precoder design for the MIMO is difficult due to the non convex nature of the problem formulation [12]. In general, the rate maximizing beamformer designs has an inherent complexity due to existence of optimization variables, i.e., transmit precoders, in both the numerator and in the denominator of the SINR expression as shown in (5). In particular, for a single receive antenna scenario, the goal of precoding is to maximize the received signal power at the intended terminal in a group while minimizing the interference caused to the others in the same group and the ones in other group.

Here, we propose to design the group specific inner precoder with centralized interference variable. We introduce t_{k_g} as an under-estimator for the rate of the user $k_g, \forall g \in \mathcal{U}_g$. We also introduce a slack variable b_{k_g} for the denominator of the SINR expression in (5) and ϵ_g as the total interference caused by the transmission from group $g \in \bar{\mathcal{U}_g}$ to the users in group $\mathcal{U}_g$. Hence we can reformulate the optimization problem as

$$\underset{t_{k_g}, b_{k_g}, \mathbf{w}_{k_g}}{\text{maximize}} \quad \sum_g \prod_{k_g} t_{k_g} \approxeq \prod_k t_k \tag{8a}$$

subject to

$$\frac{|\mathbf{h}_{k_g}^H \mathbf{B}_{k_g} \mathbf{w}_{k_g}|^2}{b_{k_g}} \geq \gamma_{k_g} \tag{8b}$$

$$N_0 + \sum_{i \in \mathcal{U}_{k_g}, \setminus k_g} |\mathbf{h}_{k_g}^H \mathbf{B}_{k_i} \mathbf{w}_{k_i}|^2 + \sum_{g \in \bar{\mathcal{U}_g}} \epsilon_g \leq b_{k_g} \tag{8c}$$

$$\sum_{g \in \bar{\mathcal{U}_g}} \epsilon_g \geq \sum_{j \in \bar{\mathcal{U}_g}} |\mathbf{h}_{k_g}^H \mathbf{B}_{k_j} \mathbf{w}_{k_j}|^2 \tag{8d}$$

$$\sum_g \sum_{k_g} ||\mathbf{B}_g \mathbf{w}_{k_g}||^2 \leq P_{tot} \tag{8e}$$

since the expression in (5) cannot be handled directly. We assume that the beamformer precision is high enough to guarantee sufficient accuracy. We observe that the equation (8b) gives an under-estimator of γ_{k_g} and equation (8c) gives the over-estimator for the total interference for the users k_g in group $\mathcal{U}_{k_g}$. In spite of relaxing the SINR expression in (5) with (8b) and (8c), even then the problem is not convex due to (8b). Hence, we adopt successive convex approximation (SCA) wherein each nonconvex subset is replaced by convex subset, which can be solved iteratively until convergence [13], [14].

The SINR expression in (8b) can be represented equivalently for user k_g in group $\mathcal{U}_{k_g}$ as

$$\gamma_{k_g} \leq (b_{k_g})^{-1} |\mathbf{h}_{k_g}^H \mathbf{B}_{k_g} \mathbf{w}_{k_g}|^2 \tag{9}$$

However, the fractional term in (9) is of the quadratic-over-linear form, and can be bounded by the linear first order

Taylor's series expansion as follows

$$\frac{|\mathbf{h}_{k_g}^H \mathbf{B}_{k_g} \mathbf{w}_{k_g}|^2}{b_{k_g}} \geq \mathcal{F}_{k_g}^{(i)}(\mathbf{w}_{k_g}, b_{k_g}; \mathbf{w}_{k_g}^{(i)}, b_{k_g}^{(i)}) \triangleq$$
$$2\frac{\mathbf{w}_{k_g}^{(i)} \mathbf{h}_{k_g}^H \mathbf{h}_{k_g}}{b_{k_g}^{(i)}} (\mathbf{w}_{k_g} - \mathbf{w}_{k_g}^{(i)})$$
$$\frac{|\mathbf{h}_{k_g}^H \mathbf{B}_{k_g} \mathbf{w}_{k_g}^{(i)}|^2}{b_{k_g}^{(i)}} \big(1 - \frac{b_{k_g} - b_{k_g}^{(i)}}{b_{k_g}^{(i)}}\big), \quad (10)$$

where $\mathcal{F}_{k_g}^{(i)}(\mathbf{w}_{k_g}, b_{k_g})$ is the linear under-estimator for the right hand side (r.h.s) term in (9) and $\mathbf{w}_{k_g}^{(i)}, b_{k_g}^{(i)}$ are the operating points that are updated in each step by the solution obtained from the previous SCA step.

After linearising the non convex constraints and by replacing (8b) with (10), and fixing the outer precoder $\mathbf{B}$, we can summarize the WSRM problem as

$$\underset{t_k, b_{k_g}, \mathbf{w}_{k_g}}{\text{maximize}} \quad \prod_k t_k \quad (11a)$$
$$\text{subject to}$$
$$\mathcal{F}_{k_g}^{(i)}(\mathbf{w}_{k_g}, b_{k_g}) \geq \gamma_{k_g} \quad (11b)$$
$$\text{(7b) and (8c)} \quad (11c)$$

where γ_{k_g} is positive even after relaxation, since a negative value reduces t_{k_g} for a non zero beamformer entry $\mathbf{w}_{k_g}$, thereby utilizing the power without maximizing the objective.

4. NUMERICAL RESULTS

5. CONCLUSION

6. REFERENCES

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