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Answer 1

Let's call $6^{2n}-1$=P(n)
1) Base case: n=1
P(n)=35 which is already divisible by 5 and 7.

2)Inductive step: Assume that P(n) is divisible by 5 and 7 when n=k. Then we can say that $5m = 7n = 6^{2k} - 1$ or $5m + 1 = 7n + 1 = 6^{2k}$ as m,n,k$\in \mathbb{N}^+$.

3)Let's consider the case n=k+1. P(k+1)=$6^{2k+2}-1$=$36 * 6^{2k} - 1$. To check the divisiblity for 5:
3.1) Since $6^{2k} = 5m+1$ We can state that: P(k+1)=$36\times(5m+1)-1 = 180m+35 = 5\times(36m+7)$. Since 36m+7 is an integer so P(k+1)=$5\times(36m+7)$ is divisible by 5. So i have proven that $6^{2n}-1$ is divisible by 5.

Let's check divisibility for 7:
3.2) Since $6^{2k} = 7n+1$ We can state that: P(k+1)=$36\times(7n+1)-1 = 252n+35 = 7\times(36n+5)$. Since 36n+5 is an integer so P(k+1)=$7\times(36n+5)$ is divisible by 7. So i have proven that $6^{2n}-1$ is divisible by 7.

Since I have showed that $6^{2n}-1$ is divisible by 5 and 7 for n=k+1, by using mathematical induction I have showed that $6^{2n}-1$ is divisible by 5 and 7 for n $\geq$1

Answer 2

1)Base case n=0:
$H_0 = 1 \leq 9^0 = 1$
Base case n=1:
$H_1 = 5 \leq 9^1 = 9$
Base case n=2:
$H_2 = 7 \leq 9^2 = 81$
Base case n=3, which can be deriven by using upper 3 base case:
$H_3 = 8H_2 + 8H_1 + 9H_0 = 105 \leq 729$

2)Inductive step: This holds for all $3 \leq k \leq n$
$H_n \leq 9^n$
$H_{n-1} \leq 9^{n-1}$
$H_{n-2} \leq 9^{n-2}$

3) Check for n+1:
$H_{n+1} = 8 \times H_n + 8 \times H_{n-1} + 9 \times H_{n-2}$
By using equations in Inductive Step:
$8 \times H_n \leq 8 \times 9^n$
$8 \times H_{n-1} \leq 8 \times 9^{n-1}$
$9 \times H_{n-2} \leq 9 \times 9^{n-2}$ So H_{n+1} becomes:
$H_{n+1} \leq 8 \times 9^n + 8 \times 9^{n-1} + 9 \times 9^{n-2}$ If we organize them:
$H_{n+1} \leq 72 \times 9^{n-1} + 8 \times 9^{n-1} + 9^{n-1} = 81 \times 9^{n-1} = 9^{n+1}$
Since I have showed that $H_{n+1} \leq 9^{n+1}$, by using strong induction I have concluded that $H_n \leq 9^n$ statement is true for all integer n$\geq$ 3.

Answer 3

How many bit strings of length 8 contain either 4 consecutive 0s or 4 consecutive 1s?
We can think of bit strings of length 8 as xxxxxxxx. To calculate the possibilites contain 4 consecutive 0s:
1)We can put 0s in the first 4 bits: 0000xxxx. And 2^4 possible case for the other 4 bits. 16 possible cases
2)We can put 0s between 2^{nd} and 5^{th} bits: 10000xxx. And 2^3 possible case for the other 3 bits. 8 possible cases
3)We can put 0s between 3^{rd} and 6^{th} bits: x10000xx. And 2^3 possible case for the other 3 bits. 8 possible cases
4)We can put 0s between 4^{th} and 7^{th} bits: xx10000x. And 2^3 possible case for the other 3 bits. 8 possible cases
5)We can put 0s between 5^{th} and 8^{th} bits: xxx10000. And 2^3 possible case for the other 3 bits. 8 possible cases
So there is $16 + 4 \times 8 = 48$ possible cases for 4 consecutive 0s.
Note: I added extra 1 to the beginning of 4 0s to avoid counting duplicate bit strings twice.

To caculate the possibilites contain 4 consecutive 1s:
1)We can put 1s in the first 4 bits: 1111xxxx. And 2^4 possible case for the other 4 bits. 16 possible cases
2)We can put 1s between 2^{nd} and 5^{th} bits: 01111xxx. And 2^3 possible case for the other 3 bits. 8 possible cases
3)We can put 1s between 3^{rd} and 6^{th} bits: x01111xx. And 2^3 possible case for the other 3 bits. 8 possible cases
4)We can put 1s between 4^{th} and 7^{th} bits: xx01111x. And 2^3 possible case for the other 3 bits. 8 possible cases
5)We can put 1s between 5^{th} and 8^{th} bits: xxx01111. And 2^3 possible case for the other 3 bits. 8 possible cases
So there is $16 + 4 \times 8 = 48$ possible cases for 4 consecutive 1s.

Note: I added extra 0 to the beginning of 4 1s to avoid counting duplicate bit strings twice.
But we should find the intersection of these cases:
1)11110000
2)00001111 these 2 cases are actually 2 cases which I have counted twice. So there is actually 48+48-2=94 possible bit strings which contain either 4 consecutive 0s or 4 consecutive 1s.

Answer 4

Since there is 10 distinct stars there is actually 10 different options to selecting a star. Selecting 2 habitable planets out of 20 habitable planets is $C(20, 2)$. Selecting 8 unhabitable planets out of 20 habitable planets is $C(80, 8)$. If we place 8 Non habitable planets one by one there will be 9 space, which we can place 2 habitable planets in, between them. -N-N-N-N-N-N-N-N- . We can place first planet only in the first 3 space. If we place it in first space, then there is 3 option for second habitable planet. If it is in the second space, then there is 2 options for second planet. And if it is in the third space, then there is 1 option for the second planet. So for chosing the placing of planets there is 6 possibility. But since the planets are distinct, their placements can be changed between each other, so we have to multiply with 2!.8!.
Final Result: $2! \times 8! \times C(80, 8) \times C(20, 2) \times 6 \times C(10, 1)$

Answer 5

a) Let me define a_n to be the number of possible ways for robot to reach n cell away. If robot jumps 1 cell at first, there is (n-1) cell to jump which corresponds to a_{n-1}. Also if robot jumps 2 cell at first, there is (n-2) to jump, which corresponds to a_{n-2}. Also if robot jumps 3 cell at first, there is (n-3) to jump, which corresponds to a_{n-3}. So my recurrence relation end up being like this:
$a_n = a_{n-1} + a_{n-2} + a_{n-3}$

b) 1) For robot to jump 1 cell away there is actually 1 way of doing this (1). $a_1 = 1$
2) For robot to jump 2 cell away there is actually 2 way of doing this (1+1 or 2). $a_2 = 2$
3) For robot to jump 3 cell away there is actually 4 way of doing this (1+1+1, 1+2, 2+1, 3). $a_3 = 4$

c) Since recurrence relation is of the form: $a_n = a_{n-1} + a_{n-2} + a_{n-3}$. We can rewrite it like this: $a_n - a_{n-1} - a_{n-2} - a_{n-3} = 0$ Characteristic equation is: $x^3 - x^2 - x - 1 = 0$ And we can write it like this: $x(x^2 - 1) - x^2 - 1 = (x + 1)(x^2 - 1) = 0$ Since finding root of this is very complicated I will just give the answer by going backwards:
$a_4 = a_3 + a_2 + a_1 = 7$
$a_5 = a_4 + a_3 + a_2 = 13$
$a_6 = a_5 + a_4 + a_3 = 24$
$a_7 = a_6 + a_5 + a_4 = 44$
$a_8 = a_7 + a_6 + a_5 = 81$

$a_9 = a_8 + a_7 + a_6 = 149$
a_9=149.