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Answer 1

Let's call A(x)=$a_0 * x^0 + a_1 * x^1 + a_2 * x^2 + a_3 * x^3 + ...$ Also I can write equation like this $\Sigma_{n=2}^{\infty}(a_n x^n) = 3\Sigma_{n=2}^{\infty}(a_{n-1}x^n) + 4\Sigma_{n=2}^{\infty}(a_{n-2}x^n)$. Hence $A(x) - (a_0 + a_1 x) = 3x(A(x) - a_0) + 4x^2 A(x)$ then I can find that $A(x)(1 - 3x - 4x^2) = a_0(1 - 3x) + a_1 x = 1 - 2x$. So $A(x) = \dfrac{(1-2x)}{(1-4x)(1+x)}$. So By using partial fractions:$A(x) = \dfrac{B}{1-4x} + \dfrac{C}{1+x}$ and B(1+x)+C(1-4x)=1-2x. So B-4C=-2, B+C=1 which corresponds to B=$\dfrac{2}{5} andC = \dfrac{3}{5}$ $A(x) = \dfrac{2}{5(1-4x)} + \dfrac{3}{5(1+x)}$. Hence $A(x) = \Sigma_{n=0}^{\infty}(\dfrac{2}{5}4^n.x^n) + \Sigma_{n=0}^{\infty}(\dfrac{3}{5}(-1)^n.x^n)$. So $a_n = \dfrac{2}{5}4^n + \dfrac{3}{5}(-1)^n$

Answer 2

a)

Let's create another sequence like $< 3, 5, 11, 29, 83, 245... >$ by just increasing the initial term by 1. For this sequence we can find a recursive relation like this: $b_0 = 3$ and $b_n = 3b_{n-1} - 4$ for $n \geq 1$. And let's say that we represent the generating function given the question by A(x) and for the one I created as B(x). And by definition we can easily say that A(x)=B(x)-1. I can also say that $B(x) = \Sigma_{n=0}^{\infty}(b_n.x^n) = b_0 + \Sigma_{n=1}^{\infty}(b_n.x^n) = 3 + \Sigma_{n=1}^{\infty}(b_n.x^n)$. Also I can replace the term b_n with the one I have already found. So $B(x) = 3 + \Sigma_{n=1}^{\infty}((3b_{n-1} - 4).x^n)$ =$3 + \Sigma_{n=1}^{\infty}(3b_{n-1}.x^n) - 4\Sigma_{n=1}^{\infty}.x^n$ Let's change the starting indexes of them to compatible with the defined functions A(x) and B(x). B(x)=$3 + \Sigma_{n=0}^{\infty}(3b_n.x^{n+1}) - 4(-1 + \Sigma_{n=0}^{\infty}.x^n)$. Also I can take out the excessive x term: B(x)=$3 + 3.x.\Sigma_{n=0}^{\infty}(b_n.x^n) + 4 - 4\Sigma_{n=0}^{\infty}.x^n)$. So I can replace the $\Sigma_{n=0}^{\infty}.x^n)$ term with their original one B(x) and $\Sigma_{n=0}^{\infty}.x^n)$ with $\dfrac{1}{1-x}$. So i got: $B(x) = 3 + 3.x.B(x) + 4 + \dfrac{-4}{1-x}$. I can put B(x) terms in left: $B(x)(1-3x) = 7 - \dfrac{-4}{1-x}$ And if I divide both sides by (1-3x): $B(x) = \dfrac{3-7x}{(1-x)(1-3x)}$
And finally A(x)=$B(x) - 1 = \dfrac{3-7x}{(1-x)(1-3x)} - 1 = \dfrac{-3x^2 - 3x + 2}{(3x^2 - 4x + 1)}$

b)

I can write $G(x) = \dfrac{7-9x}{(2x-1)(x-1)}$ Also by using partial fractions, $G(x) = \dfrac{A}{2x-1} + \dfrac{B}{x-1}$ I can say that B(2x-1)+A(x-1)=7-9x. So 2B+A=-9 and A+B=-7 so I can find that B=-2 and A=-5. So I can write $G(x) = \dfrac{-5}{2x-1} + \dfrac{-2}{x-1} = \dfrac{5}{1-2x} + \dfrac{2}{1-x}$. So I have found that $G(x) = \Sigma_{n=0}^{\infty}(5(2)^n.x^n + 2.x^n)$. So the sequence is like $< 7, 12, 22, 42, 82, ... >$

Answer 3

a)

Let's check the reflexivity of this relation. Let's say our 2 edges have length equals to a so for it to be reflexive aRa should be true. On the other hand, according to the Pythagorean theorem, hypotenuse(let's say has length b) should have the length $b = 2^{1/2}.a$ which is not possible in the domain of integer numbers. For instance, Let's take a=4 then the hypotenuse should be $4.2^{1/2}$, which is not possible since also n should be an integer number. It is possible for the domain of real numbers not for integer domain. Since this relation is not reflexive in integer domain I have showed that it is not an equivalence relation, and there is no need to show whether it is transitive or not.

b)

1)Let's check reflexivity. It is reflexive because:
For any pair in form of a (a,b), (a,b)R(a,b) is correct because 2a+b=2a+b always.

2)Let's check whether it is symmetric or not: Yes it is symmetric too, because:
Let's take two pair of (x_1, y_1) and (x_2, y_2). If $2x_1 + y_1 = 2x_2 + y_2$ then for sure $2x_2 + y_2 = 2x_1 + y_1$ is also correct. So I have showed that it is symmetric.

3)Let's check whether it is transitive or not: Yes it is transitive, too. Because:
Let's take the pairs as these: $(x_1, y_1), (x_2, y_2), (x_3, y_3)$. If I show that $(x_1, y_1)R(x_2, y_2)$ and $(x_2, y_2)R(x_3, y_3)$.
1^{st} eqn:$2x_1 + y_1 = 2x_2 + y_2$
2^{nd} eqn:$2x_2 + y_2 = 2x_3 + y_3$. Since $2x_1 + y_1 = 2x_2 + y_2$ I can deduce that $2x_1 + y_1 = 2x_3 + y_3$ or $(x_1, y_1)R(x_3, y_3)$ in other words. So this actually proves that it is transitive. So finally since I have showed that it is symmetric, transitive and reflexive, i have showed that it is an equivalence relation.

4)Determination of equivalence class of (1,-2). Let's take a point (x, y). For this point our relation will give the equation 2x+y=2.1+(-2)=0 which actually equals to the equation y=-2x. And this line represents a line passing through origin with the slope equals to (-2) in the coordinate system.

Answer 4

a)

$$\begin{tikzcd}[column sep=small, row sep=small]
 & 60 \arrow[dr, dash] & & \\
18 \arrow[dr, dash] & & 10 \arrow[dl, dash] \arrow[dr, dash] & \\
 & 2 & & 5
\end{tikzcd}$$

b)

Note: Top left is for 2 and bottom left, top right is for 60.

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

c)

Note: Top left is for 2 and bottom left, top right is for 60.

$$\begin{bmatrix} 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 1 \end{bmatrix}$$

(10,2), (10,5), (18,2), (60,2), (60,5), (60,10)

d)

It is not possible to get a total ordering by just changing one element. For example 18 creates a problem and if we change it with 120 the relation is not total ordering again because there are 2 alternative way to go 60, starting from 2 or 5. In addition if we change 5 with 120 there are 2 way we can go when we start from 2. But if we are allowed to remove 2 elements and add one, I would remove 18 and 5 and I would add 120. That's way I could get a total ordering because there would be just one chain from 2 to 120.