CENG 280

Formal Languages and Abstract Machines

Spring 2022-2023

Homework I - Sample Solutions

Question 1

a) False. Strings are by definition are of fixed length, hence a countable set, whereas real numbers are uncountably infinite.
Note that answers arguing about the lack of the symbols '$-$' or '.' did not get any grades. Similarly, stating that irrational numbers have infinite digits, hence, cannot be represented got no grades. For example, the set $\{-1, -0.5, \pi, \sqrt{2}\}$ includes all the 'violating' cases, but can be represented with strings over the given alphabet: we can say 00 represents -1, 01 represents -0.5, 10 represents π and 11 represents $\sqrt{2}$. Main root of confusion seems to be that representations are confused with exact constructions.

b) False. The set of all finite representations is a countable set, whereas the set of languages over an alphabet is uncountably infinite.
Similar to **Q1a)**, arguments that do not focus on the countability of the two sets did not get any points.

c) True. Zero repetition of a, followed by two repetition of b, followed by one repetition of a and lastly zero repetition of b.

d) False. Although the regular expression contains all strings that has ab as prefix, it also contains other strings such as aab.

Question 2

a) (20 points) Give each of K, Σ, δ, s, F.
$K : \{q_0, q_1, q_2, q_3\}$
$\Sigma : \{a, b\}$
$s : \{q_0\}$
$F : \{q_0, q_1, q_2\}$
$\delta :$

$$
\begin{aligned}
\delta(q_0, a) &= q_1 \\
\delta(q_0, b) &= q_0 \\
\delta(q_1, a) &= q_1 \\
\delta(q_1, b) &= q_2 \\
\delta(q_2, a) &= q_3 \\
\delta(q_2, b) &= q_0 \\
\delta(q_3, a) &= q_3 \\
\delta(q_3, b) &= q_3
\end{aligned}
$$

b) (20 points) Input is *abbaabab*. States are as following;
$q_0, q_1, q_2, q_0, q_1, q_1, q_2, q_3, q_3$.
Since it ends at q_3 which is not an accept state, the input is rejected by the DFA.

Question 3

This question aims to aid you in understanding the proof of **Theorem 2.2.1** by reusing its notation and walking through the conversion process step by step.

a)

$$
\begin{aligned}
E(q_0) &= \{q_0,\, q_2\} \\
E(q_1) &= \{q_1\} \\
E(q_2) &= \{q_2\} \\
E(q_3) &= \{q_3,\, q_0,\, q_2\} \\
E(q_4) &= \{q_4,\, q_2,\, q_3,\, q_0\}
\end{aligned}
$$

b) The erroneous steps are 4 and 5.

Step 4: The given instruction defines F' as $F' = \{Q \subseteq K \,|\, Q \subseteq F\}$. This is wrong, since we need to include states $Q \in K'$ that include *any of the* states $q \in F$ into the newly formed set F'. In terms of the textbook: $F' = \{Q \subseteq K \,|\, Q \cap F \neq \emptyset\}$.

Step 5: The new transitions should return the unions of the *empty closures* of the states reachable from the other states. In terms of the textbook:

$$
\delta(Q, a) = \bigcup \{E(p) \,|\, p \in K \text{ and } (q, a, p) \in \Delta \text{ for some } q \in Q\}
$$

Step 1: Just a restatement of the textbook's $K' = 2^K$.

Step 2: The two automata use the same alphabet. Note here that e is a special symbol and is not included in the alphabet of the NFA.

Step 3: This would be wrong if we hadn't restricted the set of considered NFAs to those which did not have empty-transition from the starting state. However, with this restriction, $E(s) = \{s\}$ which is the new starting state defined by the instructions.