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QUESTION 1:

1) At first I think that would be good to write the transitions:

$$\delta(q_0, a) = q_1, \delta(q_0, b) = q_2$$

$$\delta(q_1, a) = q_1, \delta(q_1, b) = q_2$$

$$\delta(q_2, a) = q_2, \delta(q_2, b) = q_3$$

$$\delta(q_3, a) = q_4, \delta(q_3, b) = q_5$$

$$\delta(q_4, a) = q_0, \delta(q_4, b) = q_2$$

$$\delta(q_5, a) = q_2, \delta(q_5, b) = q_3$$

Step-1:

And now I can separate the states to 2 equivalence class: the accepting and non accepting ones: (\equiv_0)

$\{q_0, q_1, q_4, q_3\}$ and $\{q_2, q_5\}$.

Step-2:

Now I can look for whether the states in same classes for \equiv_0 are in same equivalence class for \equiv_1 . To do it I should look at whether the ending of states in same classes are same when they get same input word.

Check for q_0 and q_1 :

Since $\delta(q_0, a) = \delta(q_1, a) = q_1$ and $\delta(q_0, b) = \delta(q_1, b) = q_2$, So q_0 and q_1 must be in the same set for the equivalence class of \equiv_1 .

Check for q_3 and q_4 :

$\delta(q_3, a) = q_4, \delta(q_4, a) = q_0$ and $q_0 \equiv_0 q_4$. $\delta(q_3, b) = q_5, \delta(q_4, b) = q_2$ $q_5 \equiv_0 q_2$. So nothing changes q_3 and q_4 must be in the same equivalence class for \equiv_1 .

Check for q_1 and q_3 :

$\delta(q_1, a) = q_1, \delta(q_3, a) = q_4$ and $q_1 \equiv_0 q_4$. $\delta(q_1, b) = q_2, \delta(q_3, b) = q_5$ and $q_2 \equiv_0 q_5$. So nothing changes q_1 and q_3 must be in the same equivalence class for \equiv_1 .

Check for q_2 and q_5 :

$\delta(q_2, a) = \delta(q_5, a) = q_2, \delta(q_2, b) = \delta(q_5, b) = q_3$ So nothing changes q_2 and q_5 must be in the same equivalence class for \equiv_1 .

Conclusion:

So actually nothing changes between \equiv_1 and \equiv_0 . So my final equivalence classes become $\{q_0, q_1, q_4, q_3\}$ and $\{q_2, q_5\}$. And my transition function change like this:

$$\delta(\{q_0, q_1, q_4, q_3\}, a) = \{q_0, q_1, q_4, q_3\}$$

$$\delta(\{q_0, q_1, q_4, q_3\}, b) = \{q_2, q_5\}$$

$$\delta(\{q_2, q_5\}, a) = \{q_2, q_5\}$$

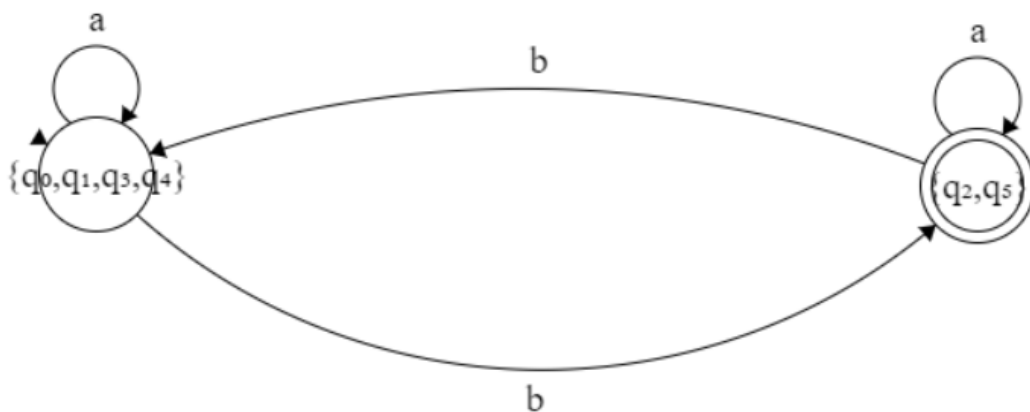
$$\delta(\{q_2, q_5\}, b) = \{q_0, q_1, q_4, q_3\}$$

My final state is: $\{q_2, q_5\}$

My initial state is: $\{q_0, q_1, q_4, q_3\}$

Acceptance state: $\{q_2, q_5\}$

So lets draw the DFA:



2)

$$[e] = Lba^* \cup \{\}$$

$$[b] = L$$

3) Let's say that $n=m=k=4x$ and $u=x$ for a certain string

$$1) w = a^{4x}b^{4x}c^{4x}d^x$$

Let's say that $n=m=k=4x$ and $u=y$ and $x \neq y$ for another string

$$2) q = a^{4x} b^{4x} c^{4x} d^y$$

Then if I concatenate these two strings with the string $z = d^x$

We will see that wz (Because $8x = 4x + 2(x+x)$) is in the language L' but qz is not (Because $4x + 2(x+y) = 6x + 2y$ which is not equal to $8x$ since $x \neq y$)

So I concluded that these two strings are at different equivalence classes.

And I actually proved that the language L' is not a regular language by using Myhill Nerode Theorem since I can produce infinitely many equivalence classes for infinite pair of (x, y) such that $x \neq y$

QUESTION 2:

1) $G = (V, \Sigma, R, S)$ where $V = \{S, B, R\} \cup \Sigma$, $\Sigma = \{a, b\}$ and $R = \{S \rightarrow Bb \mid RS \mid SRB, B \rightarrow Bb, B \rightarrow e, R \rightarrow RR \mid aRb \mid bRa \mid e\}$

- The rule R actually represents the conditions for equal amount of a and b .
- The rule B actually represents the condition for b^*
- The S represents the conditions that we can obtain.

2) $G = (V, \Sigma, R, S)$ where $V = \{S, X, Y\} \cup \Sigma$, $\Sigma = \{0, 1, 2\}$ and $R = \{S \rightarrow XY, X \rightarrow 0X1 \mid e, Y \rightarrow 1Y2 \mid e\}$

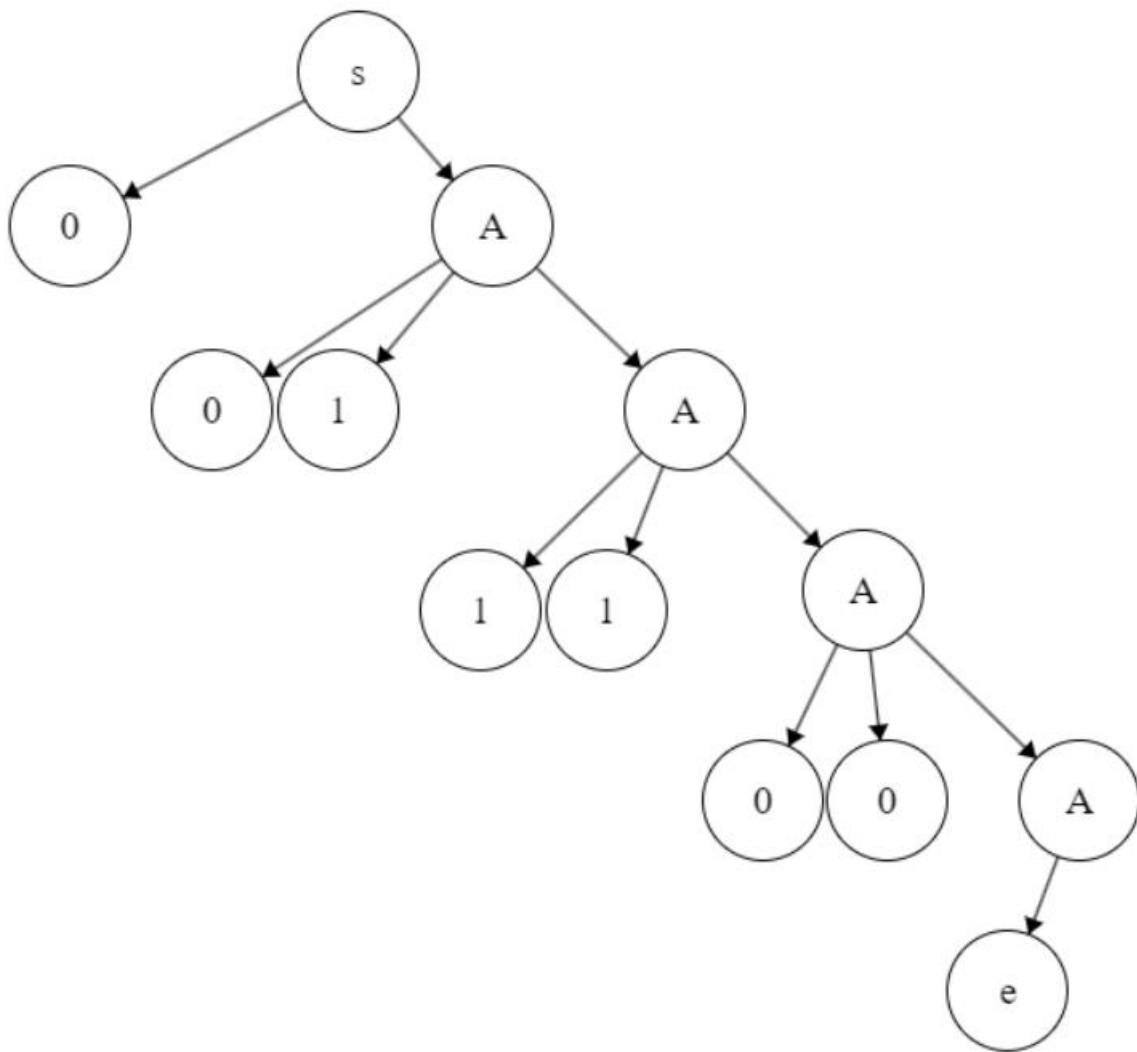
- X actually creates equal amount of 0 and 1.
- Y actually creates equal amount of 1 and 2.

3) $G = (V, \Sigma, R, S)$ where $V = \{S, A\} \cup \Sigma$, $\Sigma = \{0, 1\}$ and $R = \{S \rightarrow 0A \mid 1A, A \rightarrow e \mid 00A \mid 10A \mid 11A \mid 01A\}$

- Rule from A actually shows all possibilities after we get the first digit of the strings.
- And the rules from A actually guarantee at least 1 digit.

Derivation:

$S \Rightarrow 0A \Rightarrow 001A \Rightarrow 00111A \Rightarrow 0011100A \Rightarrow 0011100$



Corresponding Parse Tree

QUESTION 3:

1) $L = \{w \mid w = xzx, |x| = 1 \text{ and } z, x \in \{0, 1\}^* \} \cup \{e\}$

- This x actually represents the same string with length 1 both at the beginning and end.

2) $L = \{w \mid w \text{ includes at least two 1s and } w \in \{0, 1\}^* \}$

- I'm not sure this verbal statement is a correct way of stating the language but I could also state it like this (I mean I am not sure, whether stating languages by using this kind of verbal statement is true or not):

$$L = \{w \mid w = x1y1z \text{ and } x, y, z \in \{0, 1\}^*\}$$