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Answer 1

a)degree of b=3
degree of a=3
degree of c=3
degree of e=3
degree of d=2. So sum of degrees of all nodes is 14. Which is also twice of number of edges.

b) $\begin{bmatrix} 0 & 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$

Since this is a adjacency matrix of graph G, there are 14 non-zero entries.

c)$Edge_1$ is between a and b vertices.
$Edge_2$ is between b and e vertices.
$Edge_3$ is between b and c vertices.
$Edge_4$ is between a and e vertices.
$Edge_5$ is between a and c vertices.
$Edge_6$ is between c and d vertices.
$Edge_7$ is between e and d vertices. And rows are edges(starting from $Edge_1$ and ending at $Edge_7$), columns are vertices(a,b,c,d,e).

$$\begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

So number of zero entries is actually 21.

d) Since a complete graph is a graph that has an edge between every single vertex in the graph. There is no complete subgraph of this graph with at least 4 vertices.

e) We should check whether this graph is 2-colorable or not. For example if we color node-a with red we can colour b with blue, which is adjacent to a. But in this case we can't colour c with neither blue nor red. So this graph is not bipartite.

f) Since every edge can have 2 possible direction. There is actually $2^7 = 128$ possible directed graphs that have G as their underlying undirected graph.

g)Length of simple longest path is actually 4 (there is lots of path which has length 4 but one of them is this). a-b-c-d-e.

h)1. Because connected component of a graph G is maximal connected subgraph of G so connected component of graph G is itself.

i) By using theorem-1 given in the book for euler circuit. Since some of edges of G has degree 3 which is not even, the graph G has no euler circuit.

j) By using theorem-2 given in the book for euler circuit. ıf a graph doesn't have eule circuit then it must have exactly 2 vertices with odd degree, otherwise it has no euleriean path. Since graph G has 4 vertices with odd degree, there is actually no euler path in graph G.

k) Yes. a-b-e-d-c-a

l) Yes. a-c-b-e-d

Answer 2

1) Graph G has 5 nodes. Graph-H has also 5 nodes.
2) Graph-G has 5 edges. Graph H has also 5 edges.
3) All nodes in graph-g has degree of 2. All graphs in graph-H has also degree of 2.
4) Now we have to form 1-1 correspondence between these 2 graph. If I can form it, it means that they are isomorphic.

Step-1) Let's choose "a" as a starting point. Since they all have same degrees I can choose a' as a correspondent term to a.
Step-2) Let's take now b as adjacent node to a. And since all nodes have same degree I can choose e' or b' as a correspondence to b.
Step-3) Let's take now c as adjacent node to b. And I choose the c' as a correspondent node to c (no option left.).
Step-4) Let's take now d as adjacent node to c. And I choose the d' as a correspondent node to d (no option left.).
Step-5) Let's take now e as adjacent node to d. And I choose the e' as a correspondent node to e (no option left.).
Step-6) Let's form the adjacency matrix for these graphs.

Step-7) Adjaceny matrix for graph-G:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Step-8) Adjaceny matrix for graph-H:

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Since these matrices are same, this means that Graph-G and graph-H are isomorphic.

Answer 3

In this question, I have to go look for the shortest path to reach the adjacent vertices of each node while reaching at node-t from node-s.

1)Shortest path to reach the closest node from s is s-w which has length 3. So the closest vertex from vertex-s is w. Since I visited s and w, I add them to a set called visited and I will denote it by V={s,w}.

2)Now I will look for the shortest path which starts from s and visit a vertex in set of visited and goes to the one which is not included in set of visited. (One of the alternative is s-v but it is not the shortest. So i dont choose it.) Path s-u has length=3 so this means that u is the second closest node to s. So now I add the vertex-u to the set of V={s,w,u}

3)Now I will look for the shortest path which starts from s and visit a vertex in set of visited and goes to the one which is not included in set of visited. Even though path s-w-v has length 6, it is not the shortest path. Path s-v has length=5 so this means that v is the third closest vertex to s. So now I add the vertex-v to the set of V={s,w,u,v}

4)Now I will look for the shortest path which starts from s and visit a vertex in set of visited and goes to the one which is not included in set of visited. Even though path s-w-x has length 11, it is not the shortest path. Path s-v-x has length=7 so this means that x is the fourth closest vertex to s. So now I add the vertex-x to the set of V={s,w,u,v,x}

5)Now I will look for the shortest path which starts from s and visit a vertex in set of visited and goes to the one which is not included in set of visited. Even though path s-u-y has length 15, it is not the shortest path. Path s-v-x-y has length=8 so this means that y is the fifth closest vertex to s. So now I add the vertex-y to the set of V={s,w,u,v,x,y}

6)Now I will look for the shortest path which starts from s and visit a vertex in set of visited and goes to the one which is not included in set of visited. Even though path s-w-z has length 15, it is not the shortest path. Path s-v-x-y-z has length=12 so this means that z is the sixth closest vertex to s. So now I add the vertex-z to the set of V={s,w,u,v,x,y,z}

7)Now I will look for the shortest path which starts from s and visit a vertex in set of visited and goes to the one which is not included in set of visited. Even though path s-v-x-y-t has length 17, it is not the shortest path. Path s-v-x-y-z-t has length=15 so this means that t is the seventh closest vertex to s. So now I add the vertex-t to the set of V={s,w,u,v,x,y,z,t}. So now I actually found the shortest path from s-t which is like s-v-x-y-z-t and has length 15.

Answer 4

I will use Kruskal's Algorithm.

Step 1) For that I first choose the edge between d and k as a lowest weight.

Step 2) Secondly i continue with edges which have weight of 2. Since they dont form any circuit, I can add them. {b, c}, {c, f} and {h-i}.

Step 3) Since I added all edges which has weight 2. I will continue with edges which have weight 3. Since adding edges with weight 3 doesn't create any circuit as in the case of edges with weight 2, I can add all three edges which have weight 3.

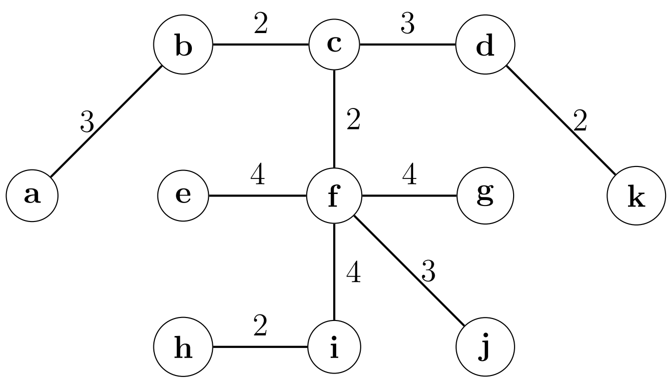
Step 4) Now I should look at edges who have weight=4. I firstly will add the edge f-i. Because it doesn't create any circuit. Now I can add the edge {e, f} and {f, g} because they also don't create any circuit. But since I add the {f, g} edge I can't add the {g, j} edge because it creates circuit.

Step 5) Since I have connected all the nodes, I actually found the minimum spanning tree.

a)

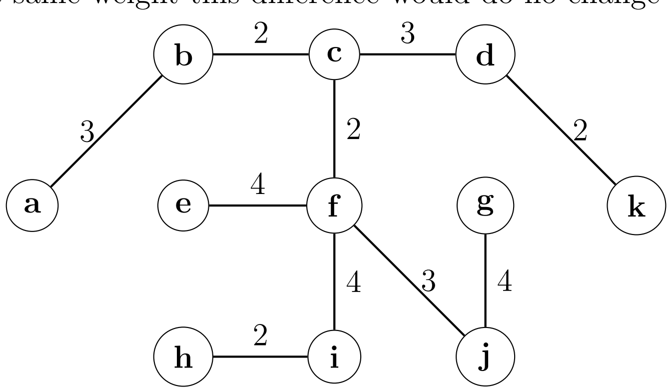
Choice	Edge	weight
1	{d,k}	2
2	{b,c}	2
3	{c,f}	2
4	{h,i}	2
5	{c,d}	3
6	{a,b}	3
7	{f,j}	3
8	{f,g}	4
9	{e,f}	4
10	{f,i}	4

b)



So this is my spanning tree.

c) Actually this spanning tree is not unique because I can get different spanning tree if I've chosen the edge between node g and j instead of the edge between node f and g. Since they have the same weight this difference would do no change in terms of spanning term.



For example this is an another spanning tree.