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### QUESTION 1:

a) G1 represents the language which includes strings in the form  $0^m 1^m$  or  $1^m 0^m$  such that  $m \geq 0$ .

b) Yes this grammar is ambiguous because for empty string there is actually two different leftmost derivation (which also means 2 different parse trees):

$S \Rightarrow A \Rightarrow e$

$S \Rightarrow B \Rightarrow e$

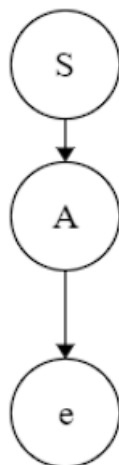


Figure 1: Parse tree for given derivation

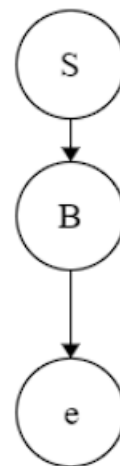


Figure 2: Parse tree for given derivation

### QUESTION 2:

a) To show that this grammar is ambiguous, I will use this string: ab. Because for the string ab there exists 2 different left-most derivation(which also means 2 different parse trees):

**Derivation 1)**  $S \Rightarrow AB \Rightarrow aAB \Rightarrow aB \Rightarrow abB \Rightarrow ab$

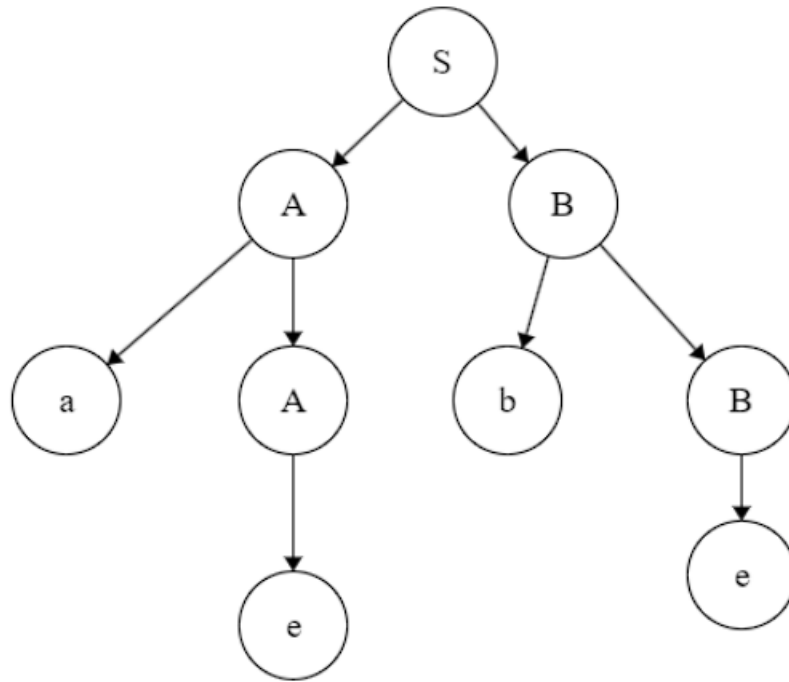


Figure 3: Parse tree for given derivation

**Derivation 2)**  $S \Rightarrow AB \Rightarrow aB \Rightarrow ab$

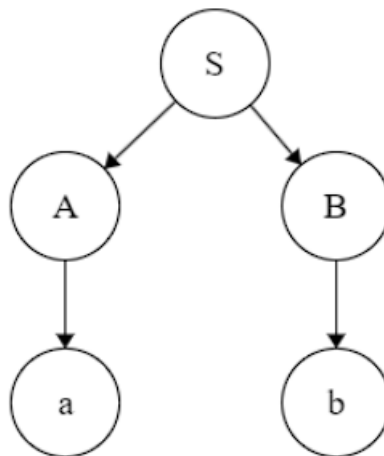


Figure 4: Parse tree for given derivation

**b)** Lets call the unambiguous version of  $G_2$  is called  $G_3$ . And  $G_3$  will be like this:

$G_3 = \{V, \Sigma, R, S\}$  where  $V = \{a, b, S, A, B\}$ ,  $\Sigma = \{a, b\}$  and  $R = \{$

$S \rightarrow AB,$

$A \rightarrow aA \mid e$ ,

$B \rightarrow bB \mid e$

c)

$S \Rightarrow AB \Rightarrow aAB \Rightarrow aB \Rightarrow abB \Rightarrow abbB \Rightarrow abbbB \Rightarrow abbbb$

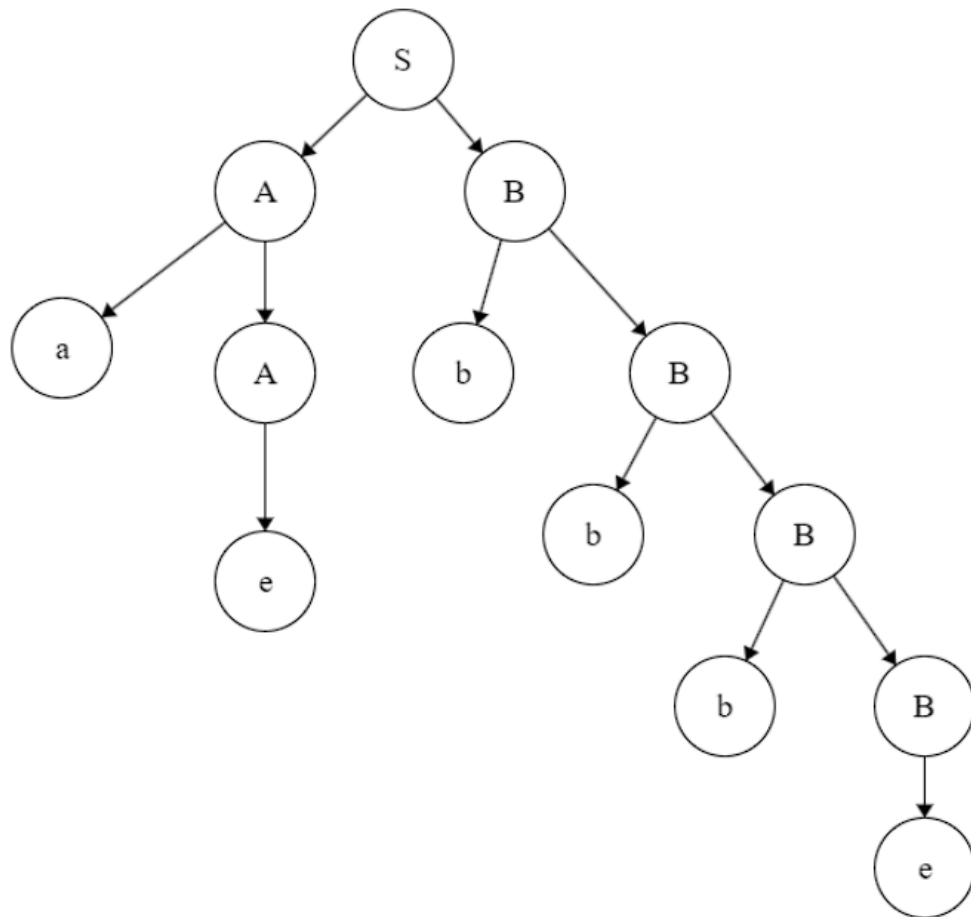


Figure 5: Parse tree for given derivation

### QUESTION 3:

a)

a.1)

- $L_1 = \{ca^m b^n \mid m \neq n\} \cup \{da^m b^{2m} \mid m \geq 0\}$  is deterministic context free language because  $L_1\$$  is accepted by this DPDA:
- For this DPDA, # represents the bottom of stack symbol and \$ represents the end of input symbol.
- States  $q_1, q_3, q_4, q_5, q_8$  represents the  $\{ca^m b^n \mid m \neq n\}$  part of the language  $L_1$ .

- States  $q_2, q_6, q_7$  represents the  $\{da^mb^{2m} \mid m \geq 0\}$  part of the language  $L_1$ .
- We should accept the strings for  $\{ca^mb^n \mid m \neq n\}$  part of the language  $L_1$ , by using two different accepting states. Because either  $m \geq n$  or  $m \leq n$  can be true.
- $Q_5$  accepts the strings that are in this form  $\{ca^mb^n \mid m \geq n\}$ .
- $Q_8$  accepts the strings that are in this form  $\{ca^mb^n \mid m \leq n\}$ .
- $Q_7$  accepts the strings that are in this form  $\{da^mb^{2m} \mid m \geq 0\}$ .
- Also it is guaranteed that there is no two transitions that are compatible. (I checked each pair of transitions.)

$K=\{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7, q_8\}$

$\Sigma=\{a, b, c, d, \$\}$

$\Gamma=\{a, \#\}$

$S=q_0$

$F=\{q_5, q_7, q_8\}$

$\Delta=\{((q_0, c, e), (q_1, \#)), ((q_0, d, e), (q_2, \#)),$

$((q_1, a, e), (q_1, a)), ((q_1, b, a), (q_3, e)),$

$((q_1, b, \#), (q_4, e)), ((q_1, \$, a), (q_5, e)),$

$((q_3, b, a), (q_3, e)), ((q_3, b, \#), (q_4, e)),$

$((q_4, b, e), (q_4, e)), ((q_4, \$, e), (q_8, e)),$

$((q_3, \$, a), (q_5, e)), ((q_5, e, a), (q_5, e)),$

$((q_2, a, e), (q_2, aa)), ((q_2, \$, \#), (q_7, e)),$

$((q_2, b, a), (q_6, e)), ((q_6, b, a), (q_6, e)),$

$((q_6, \$, \#), (q_7, e)), ((q_5, e, \#), (q_5, e))\}$

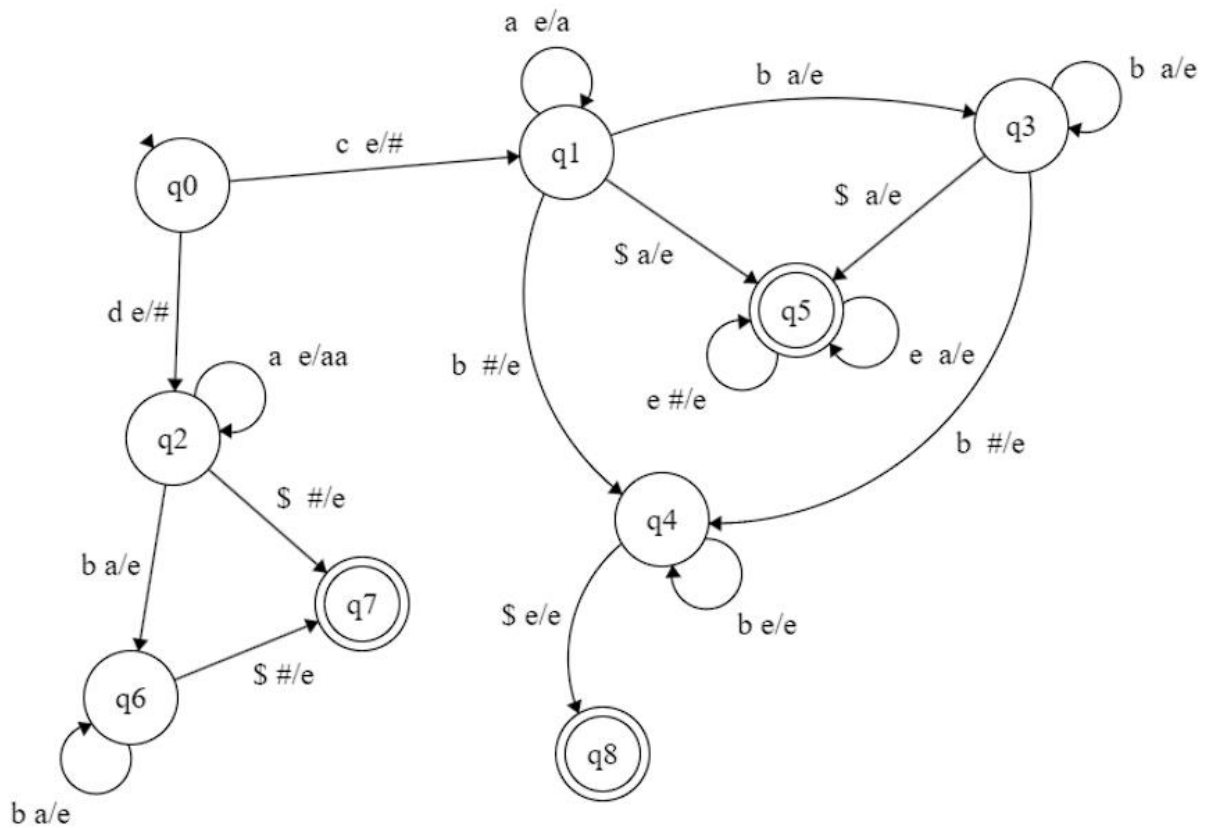


Figure 6: DPDA for  $L_1$

## a.2)

- $L_2 = \{a^m c b^n \mid m \neq n\} \cup \{a^m d b^{2m} \mid m \geq 0\}$  is deterministic context free language because  $L_2 \$$  is accepted by this DPDA:
- For this DPDA, # represents the bottom of stack symbol and \$ represents the end of input symbol.
- In this question since the strings do not start with the identifier symbols as we see on the  $L_1$  like "c" and "d". We should push the bottom of stack symbol before recognizing them.
- States q1, q3, q7 represents the  $\{a^m d b^{2m} \mid m \geq 0\}$  part of the language  $L_2$ .
- States q1, q2, q4, q5 q6 represents the  $\{a^m c b^n \mid m \neq n\}$  part of the language  $L_2$ .

- We should accept the strings for  $\{a^mcb^n \mid m \neq n\}$  part of the language  $L_2$ , by using two different accepting states. Because either  $m \geq n$  or  $m \leq n$  can be true.
- $Q_4$  accepts the strings that are in this form  $\{a^mcb^n \mid m \geq n\}$ .
- $Q_6$  accepts the strings that are in this form  $\{a^mcb^n \mid m \leq n\}$ .
- $Q_7$  accepts the strings that are in this form  $\{a^mdb^{2m} \mid m \geq 0\}$ .
- Also it is guaranteed that there is no two transitions that are compatible. (I checked each pair of transitions.)

$K = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$

$\Sigma = \{a, b, c, d, \$\}$

$\Gamma = \{a, \#\}$

$S = q_0$

$F = \{q_4, q_6, q_7\}$

$K = \{q_0, q_1, q_2, q_3, q_4, q_5, q_6, q_7\}$

$\Delta = \{((q_0, e, e), (q_1, \#)), ((q_1, a, e), (q_1, aa)),$   
 $((q_1, c, e), (q_2, e)), ((q_1, d, e), (q_3, e)),$   
 $((q_2, b, aa), (q_2, e)), ((q_2, \$, aa), (q_4, e)),$   
 $((q_4, e, aa), (q_4, e)), ((q_2, b, \#), (q_5, e)),$   
 $((q_5, b, e), (q_5, e)), ((q_5, \$, e), (q_6, e)),$   
 $((q_3, b, a), (q_3, e)), ((q_3, \$, \#), (q_7, e)),$   
 $((q_4, e, \#), (q_4, e))\}$

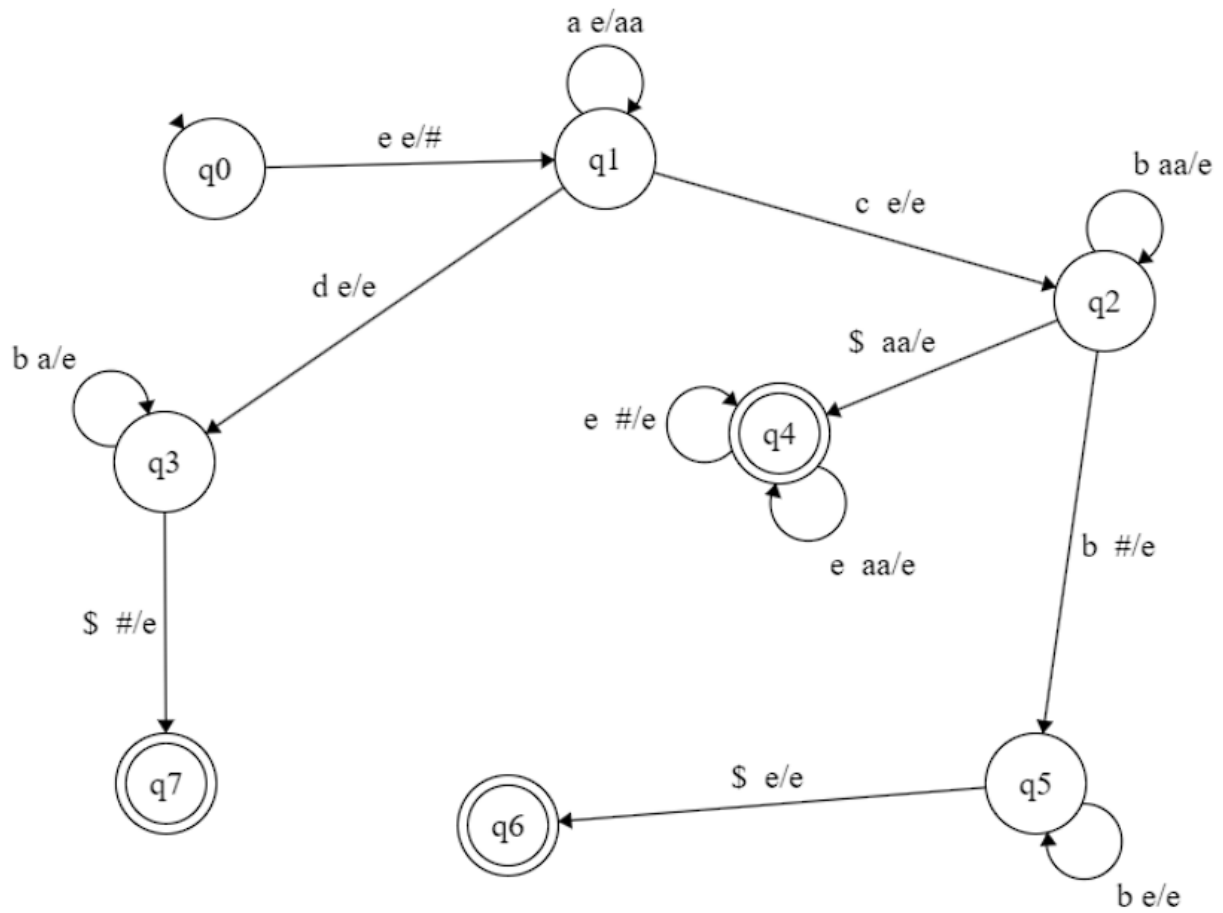


Figure 7: DPDA for  $L_2$

**b)**

- 1) Regular
- 2) Context-free
- 3) The class of the complements of context-free languages

#### 4) Deterministic context-free

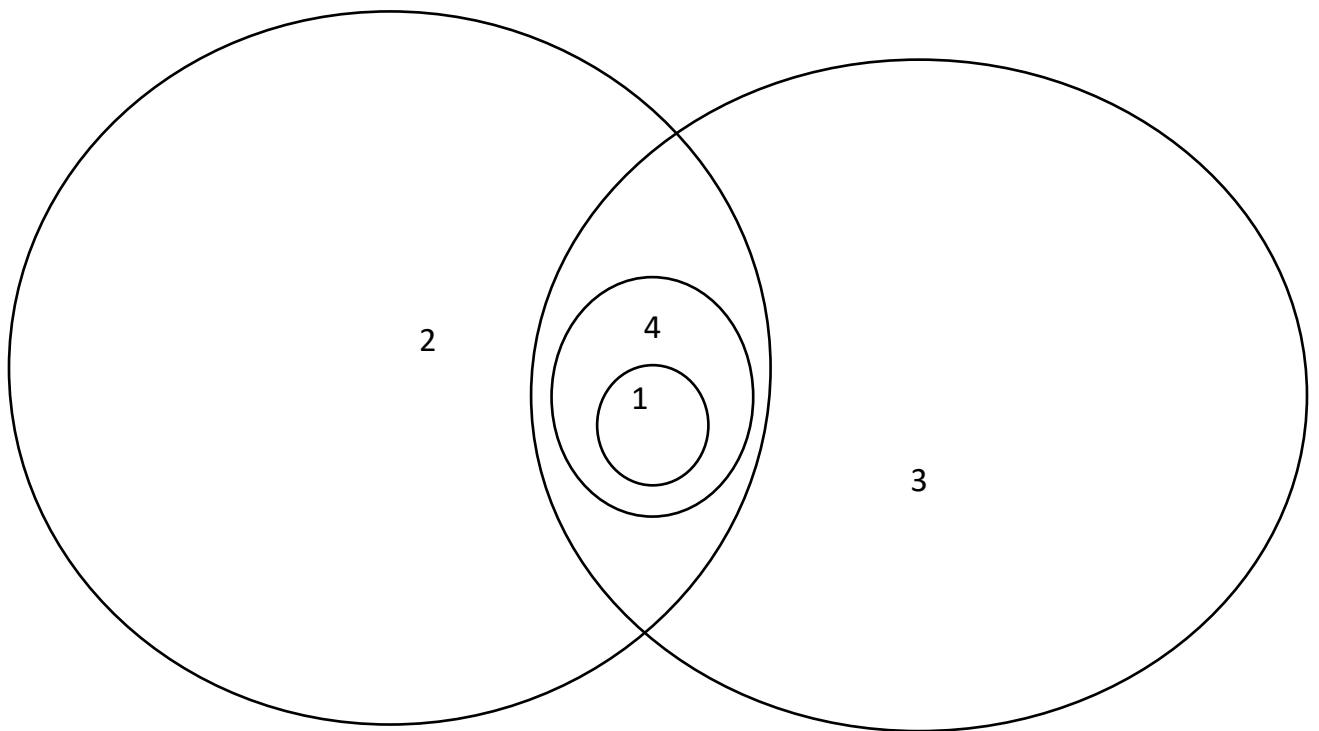


Figure 8: Venn diagram