Student Information

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Answer 1

a)

To calculate their joint pdf and joint cdf functions, I should use the formulas given in the book:
$F(t_A, t_B)_{(T_A,T_B)} = P\{T_A \leq t_A \cap T_B \leq t_B\}$
Since they are "independent" and uniformly distributed events, I can rewrite it like this:
$F(T_A, T_B)_{(t_A,t_B)} = P\{T_A \leq t_A\} \times P\{T_B \leq t_B\} = F(t_A)_{(T_A)} \times F(t_B)_{(T_B)}$
Then by using the formula given in the book:
$F(t_A)_{(T_A)} = P\{T_A \leq t_A\} = \int_0^{t_A} \frac{1}{100}\, dx = \frac{t_A}{100}$

$F(t_B)_{(T_B)} = P\{T_B \leq t_B\} = \int_0^{t_B} \frac{1}{100}\, dx = \frac{t_B}{100}$

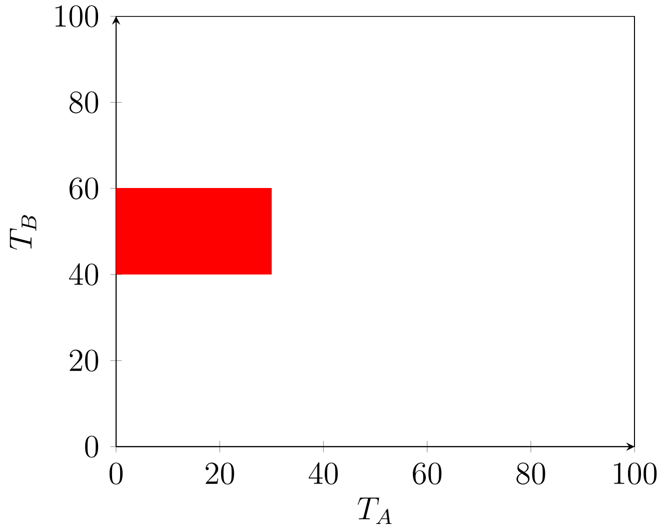
So finally I found the joint cdf funtion of $F(T_A, T_B)_{(t_A,t_B)} = \dfrac{t_A \times t_B}{10000}$

Now I can calculate the joint pdf function by double integrating the cdf function:

$$f(t_A, t_B)_{(T_A,T_B)} = \frac{\partial^2}{\partial t_A \partial t_B} F(T_A, T_B)_{(t_A,t_B)} = \frac{\partial^2}{\partial t_A \partial t_B} \frac{t_A \times t_B}{10000} = \frac{1}{10000}$$

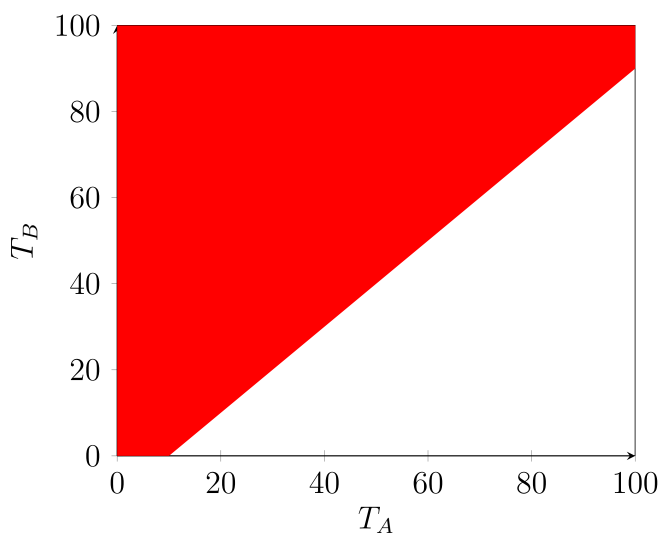
b)

To answer this question I should consider the area as a square with each side has length 100. And I can call the x-axis as T_A and y-axis as T_B. The small area is the area which stays between these values for t_A and t_B:
$t_A \leq 30$
$40 \leq t_B \leq 60$
And the large area is the area of square. So in my this question it asks the ratio of $\dfrac{smallarea}{largearea}$.
And the ratio becomes like this: $\dfrac{20 \times 30}{100 \times 100}$=0.06



c)

We can actually model this question as follows:
$t_A - t_B \leq 10$ which actually equals to $t_A - 10 \leq t_B$. So I can actually model this question as an area which stays at upper part of this line. We can think of T_A is at x-axis and T_B is at y axis. Since this line intersects our square at (10,0) and (100,90) our area becomes a trapezoid area which can be calculated by substracting the triangle area stays at lower-right part of the square which has the area of 4050 from the the bigger square. So if I substract this area from 10000(which is the total area of square) our probability becomes $\frac{smallarea}{largearea} = \frac{5950}{10000} = 0.595$

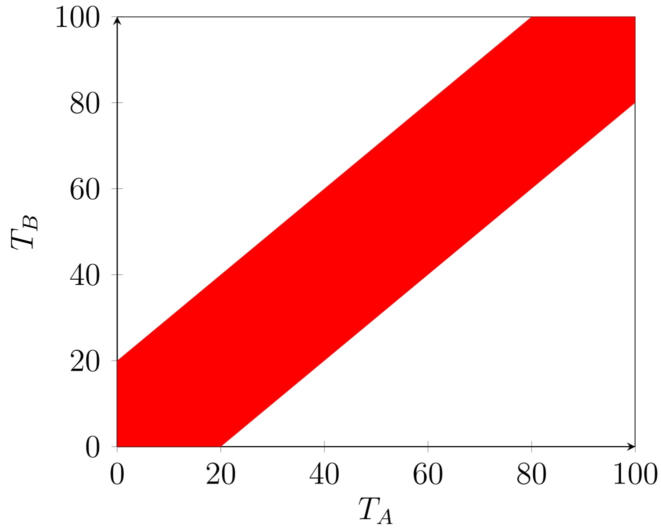


d)

This question is actually similar to the one before. But this has 2 aspects to be considered:
1) $t_B \leq t_A + 20$

2) $t_A \leq t_B + 20$

Actually there is a trick in this question. One should verify that we should take the intersection area of this 2 areas, otherwise we would accept the points which we shouldn't. One area intersects the x-axis at (20,0) and other intersects y-axis at (0,20). And this 2 line are parallel to each other. To find the intersection area, we can actually find the 2 triangle areas which are at upperleft and lower-right corner. Both has the area of $\frac{80 \times 80}{2} = 3200$ so in total these 2 triangle has the area of 6400. So my probability becomes: $\frac{smallarea}{largearea} = \frac{10000 - 6400}{10000} = 0.36$



Answer 2

a)

First of all I can consider all registered customers as a population. So the probability of a single customer to be a frequent shoppers is 0.6. So now I can actually consider the sample of 150 customers as a binomial distribution with n(number of trials)=150 and p=0.6. Since binomial distribution is a sum of individual bernoulli distribution I can apply Central Limit Theorem here, but firstly I need to calculate the expected value and the variance of this distribution.

μ=$0.6 \times 150 = 90$
δ^2=$0.6 \times 150 \times 0.4 = 36$
$\delta = 6$
Also I can state that 65% of 150 corresponds to 97.5. So I can use this as a continuity correction for the Central Limit Theorem. And this question actually asks me this "The probability that at least 98 customers in the sample are frequent shoppers.". And I can calculate this like this:
$P\{X \geq 98\} = 1 - P\{X < 98\} = 1 - P\{X < 97.5\} = 1 - P\{\frac{X - \mu}{\delta} < \frac{97.5 - 90}{6}\} = 1 - \phi(1.25) = 1 - 0.8944 = 0.1056$ By looking at the table for Standard Normal Distribution

b)

This question is really similar to the a) one. The probability of a single customer to be a rare shoppers is 0.1. So now I can actually consider the sample of 150 customers as a binomial distribution with n(number of trials)=150 and p=0.1. Since binomial distribution is a sum of individual bernoulli distribution I can apply Central Limit Theorem here, but firstly I need to calculate the expected value and the variance of this distribution.

μ=$0.1 \times 150 = 15$
δ^2=$0.1 \times 150 \times 0.9 = 13.5$
$\delta \approx 3.67$
Also I can state that 15% of 150 corresponds to 22.5 So I can use this as a continuity correction for the Central Limit Theorem. And this question actually asks me this "The probability that no more than 22 customers in the sample are rare shoppers.". And I can calculate this like this: $P\{X \leq 22\} = P\{X < 22.5\} = P\{\frac{X-\mu}{\delta} < \frac{22.5-15}{3.67}\} = \phi(2.04) = 0.9793$ By looking at the table for Standard Normal Distribution

Answer 3

Let me define the random variable "H" as the height of the adults. Also $\mu = 175$cm and $\sigma = 7cm$ So the question asks me this probability:

$P\{170 < H < 180\}$=$P\{\frac{170-175}{7} < \frac{H-\mu}{\delta} < \frac{180-175}{7}\}$=$P\{-0.71 < Z < 0.71\}$=$\phi(0.71)$-$\phi(-0.71)$

Now when I look at the Standard Normal Distribution Table: I can see the corresponding values as 0.7611 and 0.2389 respectively. So the result is: $0.7611 - 0.2389 \approx 0.52$

Answer 4

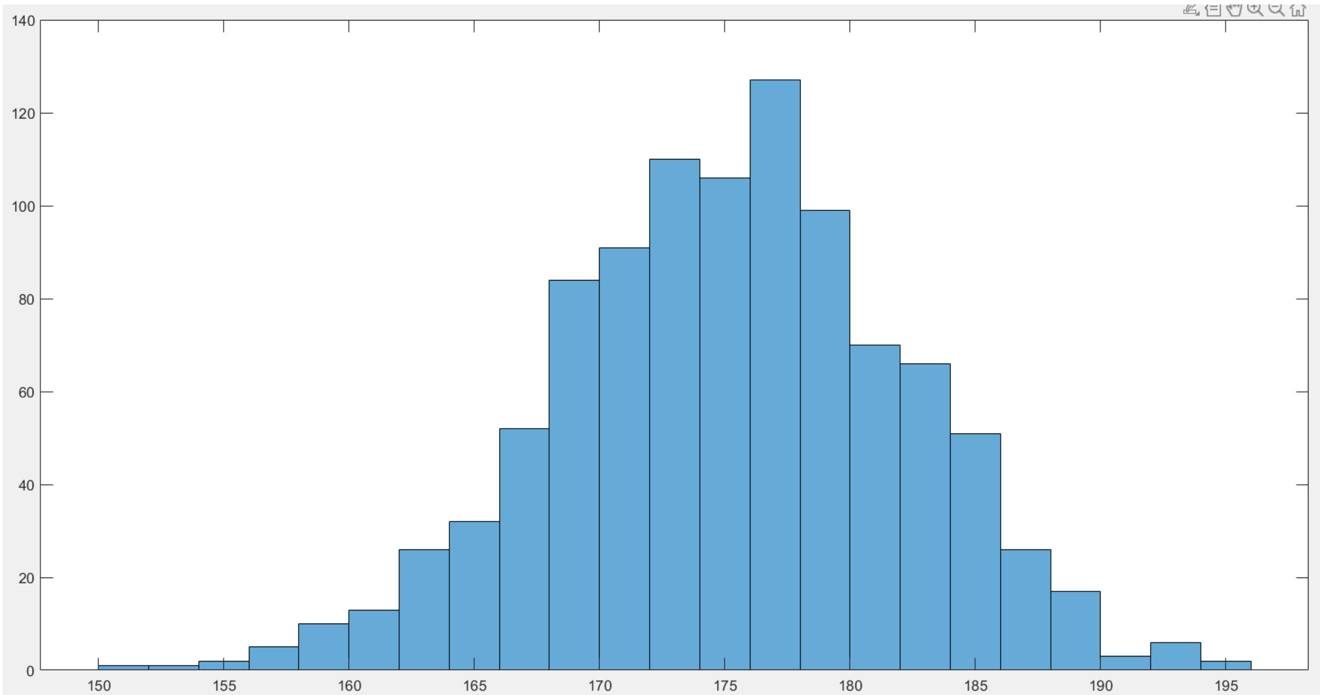
a)

Codes:

```
%datas
mu = 175;
sigma = 7;
n = 1000;
x = normrnd(mu, sigma, n, 1);

%drawing the histogram
figure('Name','Part-a','NumberTitle','off');
```

```
histogram(x);
```

Screenshot:



Brief Comment:

In this code I created a histogram by creating 1000 seperate normal random variables, to resemble the graph of normal distribution. And I can actually see that this histogram resembles to normal distribution pdf function so much.

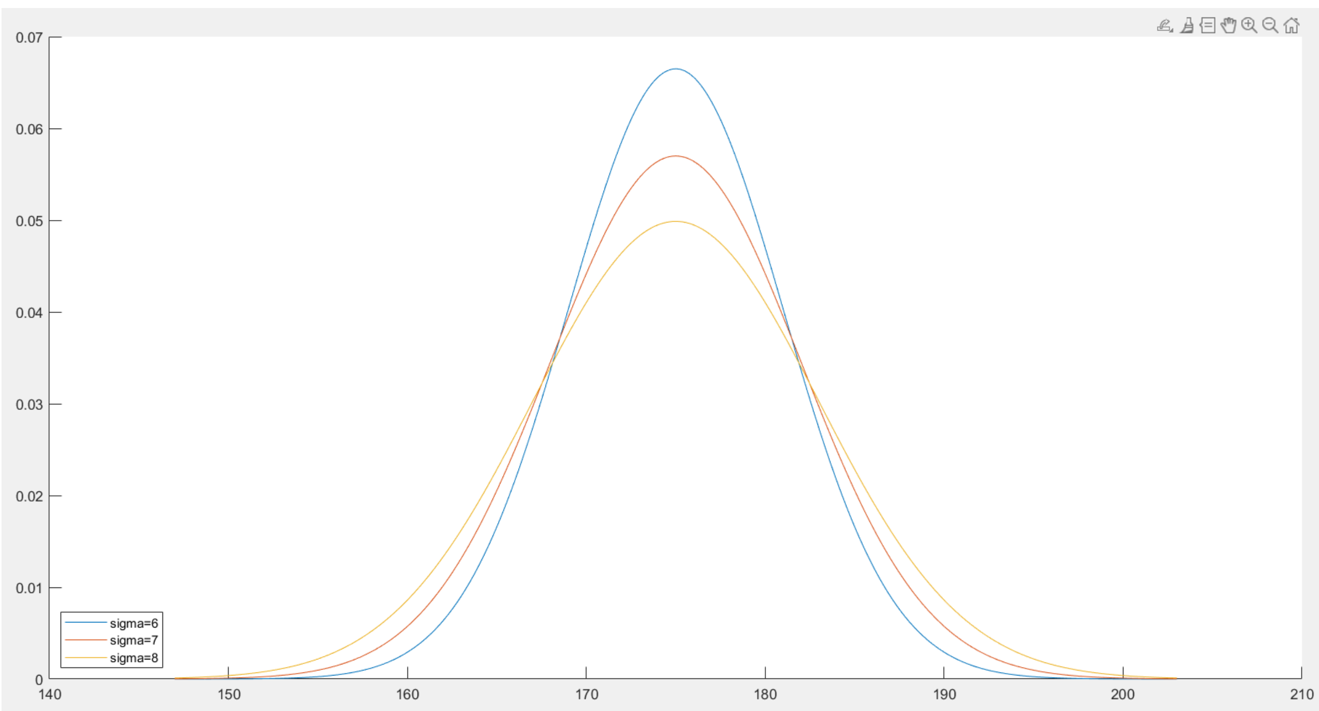
b)

Codes:

```
%datas
mu=175;
sigma_6=6;
sigma_7=7;
sigma_8=8;

figure('Name','Part-b','NumberTitle','off');
hold on
```

```
%finding the corresponding y-values for possible x-values
x_6 = [175-28:.1:175+28];
y_6 = normpdf(x_6,mu,sigma_6);

x_7 = [175-28:.1:175+28];
y_7 = normpdf(x_7,mu,sigma_7);

x_8 = [175-28:.1:175+28];
y_8 = normpdf(x_8,mu,sigma_8);


%drawing the graphs in single plot using hold on-off calls
plot(x_6,y_6);
plot(x_7,y_7);
plot(x_8,y_8);

legend({'sigma=6','sigma=7','sigma=8'},'Location','southwest')
hold off
```

Screenshot:



Brief Comment:

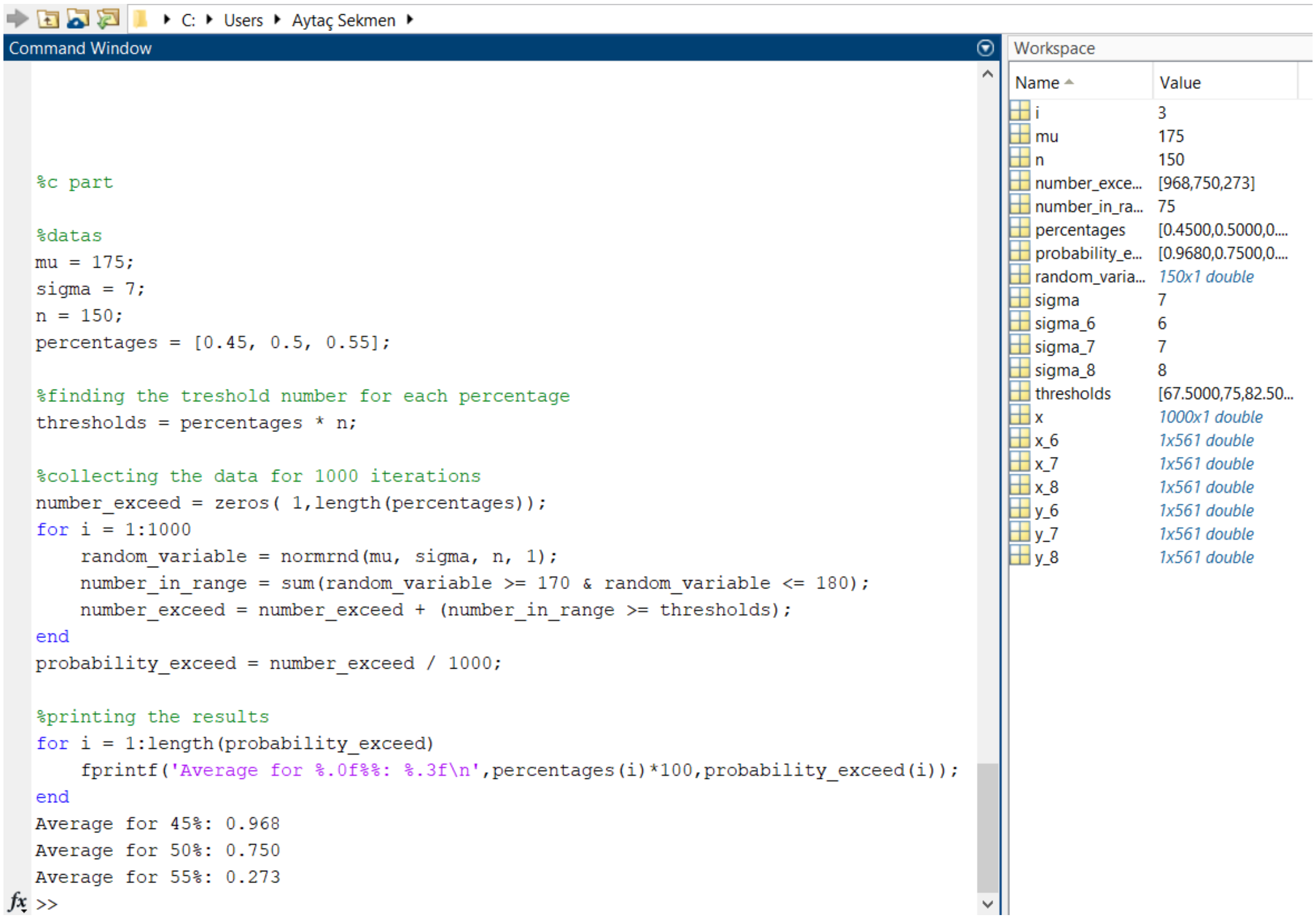
As we can clearly see from the graph, when the σ value increases the graph gets wider and the peak value decreases. But when the σ decreases, the peak value increases and the graph gets narrower.

c)

Codes:

```
%datas
mu = 175;
sigma = 7;
n = 150;
percentages = [0.45, 0.5, 0.55];

%finding the treshold number for each percentage
thresholds = percentages * n;

%collecting the data for 1000 iterations
number_exceed = zeros( 1,length(percentages));
for i = 1:1000
    random_variable = normrnd(mu, sigma, n, 1);
    number_in_range = sum(random_variable >= 170 & random_variable
       <= 180);
    number_exceed = number_exceed + (number_in_range >= thresholds)
       ;
end
probability_exceed = number_exceed / 1000;

%printing the results
for i = 1:length(probability_exceed)
    fprintf('Average for %.0f%%: %.3f\n',percentages(i)*100,
       probability_exceed(i));
end
```

Screenshot:




```
%c part

%datas
mu = 175;
sigma = 7;
n = 150;
percentages = [0.45, 0.5, 0.55];

%finding the treshold number for each percentage
thresholds = percentages * n;

%collecting the data for 1000 iterations
number_exceed = zeros( 1,length(percentages));
for i = 1:1000
    random_variable = normrnd(mu, sigma, n, 1);
    number_in_range = sum(random_variable >= 170 & random_variable <= 180);
    number_exceed = number_exceed + (number_in_range >= thresholds);
end
probability_exceed = number_exceed / 1000;

%printing the results
for i = 1:length(probability_exceed)
    fprintf('Average for %.0f%%: %.3f\n',percentages(i)*100,probability_exceed(i));
end
Average for 45%: 0.968
Average for 50%: 0.750
Average for 55%: 0.273
>>
```

Brief Comment:

It is clear that the probability decreases as the desired percentage increases. And we can clearly see that the difference between average values in the range of: 45%-50% is smaller than the difference between the average values in the range of: 50%-55% even though their percentage differences are same(50-45=55-50). In addition to these, we might have slightly different average values when try to get a different output, because we are not calculating the population probabilites, we are collecting datas to calculate the sample average.