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QUESTION 1:

- i. 1954
- ii. Enigma
- iii. Turing Test
- iv. The Chemical Basis of Morphogenesis
- v. The Imitation Game

QUESTION 2:

a)

Formal Definition for it:

$K = \{q_0, q_1, q_2, q_3, q_4, q_5\}$

$\Sigma = \{a, b, \sqcup, \triangleright\}$

$S = q_0$

$H = q_4$

$\delta(q_0, a) = (q_2, \rightarrow), \delta(q_0, b) = (q_2, \rightarrow), \delta(q_0, \sqcup) = (q_1, \rightarrow),$

$\delta(q_1, a) = (q_1, \rightarrow), \delta(q_1, b) = (q_3, \rightarrow), \delta(q_1, \sqcup) = (q_2, \sqcup),$

$\delta(q_2, a) = (q_2, a), \delta(q_2, b) = (q_2, b), \delta(q_2, \sqcup) = (q_2, \sqcup),$

$\delta(q_3, a) = (q_3, \rightarrow), \delta(q_3, b) = (q_4, \rightarrow), \delta(q_3, \sqcup) = (q_2, \sqcup),$

$\delta(q_4, a) = (q_2, a), \delta(q_4, b) = (q_2, b), \delta(q_4, \sqcup) = (q_5, \sqcup)$

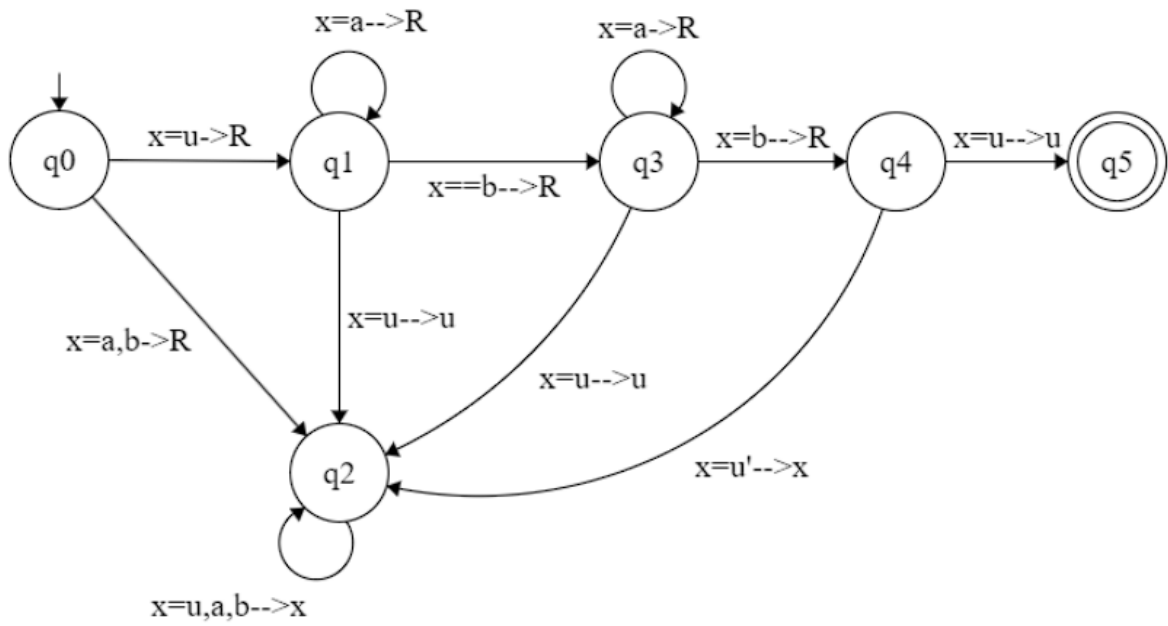


Figure 1: Turing Machine

b)

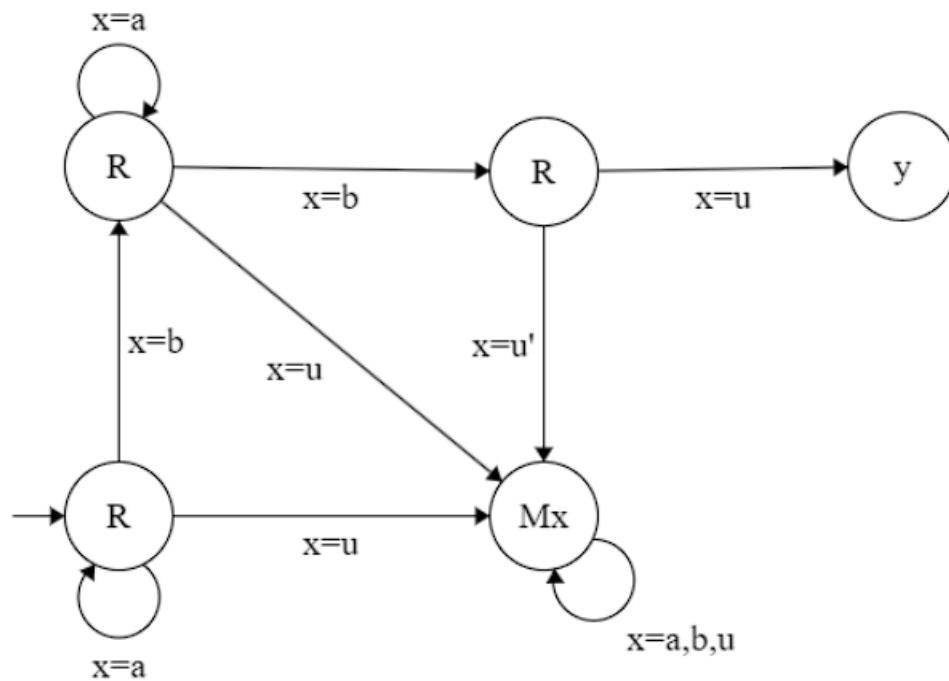


Figure 2: Turing Machine

Note: Both in figure-1 and figure-2 “x” corresponds to the currently read symbol in tape. And u corresponds to “ \sqcup ”. \sqcup' corresponds to the \sqcup complement. Mx corresponds to the basic machine that overwrites the currently read symbol to the symbol that head is on.

QUESTION 3:

Hocam, I will first formally define what each machine do, then I will make assumptions about the machines to make the final Turing Machine easier to draw. Otherwise my formal Turing Machine becomes so large to draw.

Let me first mention about the algorithm to calculate the a^b :

1) First of all, I should clarify what these tapes will represent.

- First tape will be in the structure of $\triangleright \sqcup a, b \sqcup$ such that a and $b \in \{0,1\}^*$.
- I will use the second tape for the intermediate purposes (like holding the intermediate results) related with each multiplication, subtraction and multiplication process.
- I will use the third tape for the purposes to hold the final result by adding the results of intermediate steps to this tape.

2) My algorithm will consists of these steps:

1. First write 1 on the tape-3.
2. Copy/write the number a into the first tape after the second " \sqcup " symbol using copy algorithm. And copy the number in third tape, after " x ," using another copy algorithm (denoted as " y "). After the copy process first tape will be like this $\triangleright \sqcup a, b \sqcup x, y \sqcup$ ($x=a$),
Note: I used this representation to refer to the a, b, x and y separately. But this doesn't mean that there is a difference between a and x , they are totally same.
Aim: Is to hold the value of a same to the end of exponentiation process.
3. Multiply a with the number in tape-3 using the machine " M_x ". The result will be hold in the third tape.
4. Subtract 1 from the b using the machine " M_- ". Result will be stored again in place of b .
5. In first tape erase the part " $x, y \sqcup$ ".
6. If b is 0 then terminate the process. Otherwise go back to the step-2.

3) Now let me clarify what will be the definition for the machines M_x, M_-, M_+ .

- Steps for M_x :
 - 1) If the rightmost digit of " x " is 1, write the " y " on the second tape. Otherwise, write 0 on the second tape.
 - 2) Follow the number just written with a ";".

- 3) Copy the number from the third tape to immediately after the ";".
 - 4) Erase the number from the third tape.
 - 5) Call the addition TM to add the two numbers on the second tape and store the result on the third tape.
 - 6) Erase the (garbage) contents of the second tape.
 - 7) Erase the last digit of "x", overwriting it with a blank.
 - 8) Append a 0 to the right end of "y".
 - 9) If the x becomes zero, terminate. Otherwise, go back to the top of the loop.
- Steps for M_+ (since I just use it as a auxiliary machine for M_x , I will redesign it in a way that it just do the task I expressed.) :
 - 1) First copies the first binary integer in its tape-2 to the tape-3, writing zeros in its place (and in the place of the ";" separating the two integers) in the first tape; this way the first tape contains the second integer, with zeros added in front.
 - 2) The machine goes to the end by R_{\perp} . Then it starts this loop:
 - 3) Go left (L).
 - 4) starting from the least significant bit of both integers, adding the corresponding bits, writing the result in the tape-2, and "remembering the carry" in its state.
 - 5) Go back to Step-3, until the addition ends.
 - 6) Then write the result from the second tape, into the third tape.
 - Steps for M_- (since I just use it as a auxiliary machine for my total machine, I will redesign it in a way that it just do the task I expressed) :
 - 1) First copies the "b" to the tape-2 using a copy machine. That's way, tape-2 will become like this: $\triangleright \perp b \perp$.
 - 2) The machine goes to the end by R_{\perp} . Then it starts this loop:
 - 3) Go left (L).
 - 4) If the number is 1, rewrite 0 on it and stop the loop, write the result in place of b in the first tape, erase the tape-2 content. If the number is 0, make it 0 and go back to Step-3.

Formal definitions should be like as I have written in the above. But now, I will assume somethings related with how the machines work other than the above formal definitions to draw a Turing Machine.

- First let's assume that M_x^3 take "a" and number in tape-3 as inputs, and write the result in tape-3. Let's assume it knows where a is located.
- Let's assume M_- directly subtracts "1" from b. Let's assume where the number b is located.
- State-y means the halting state. And my output is located at tape-3.

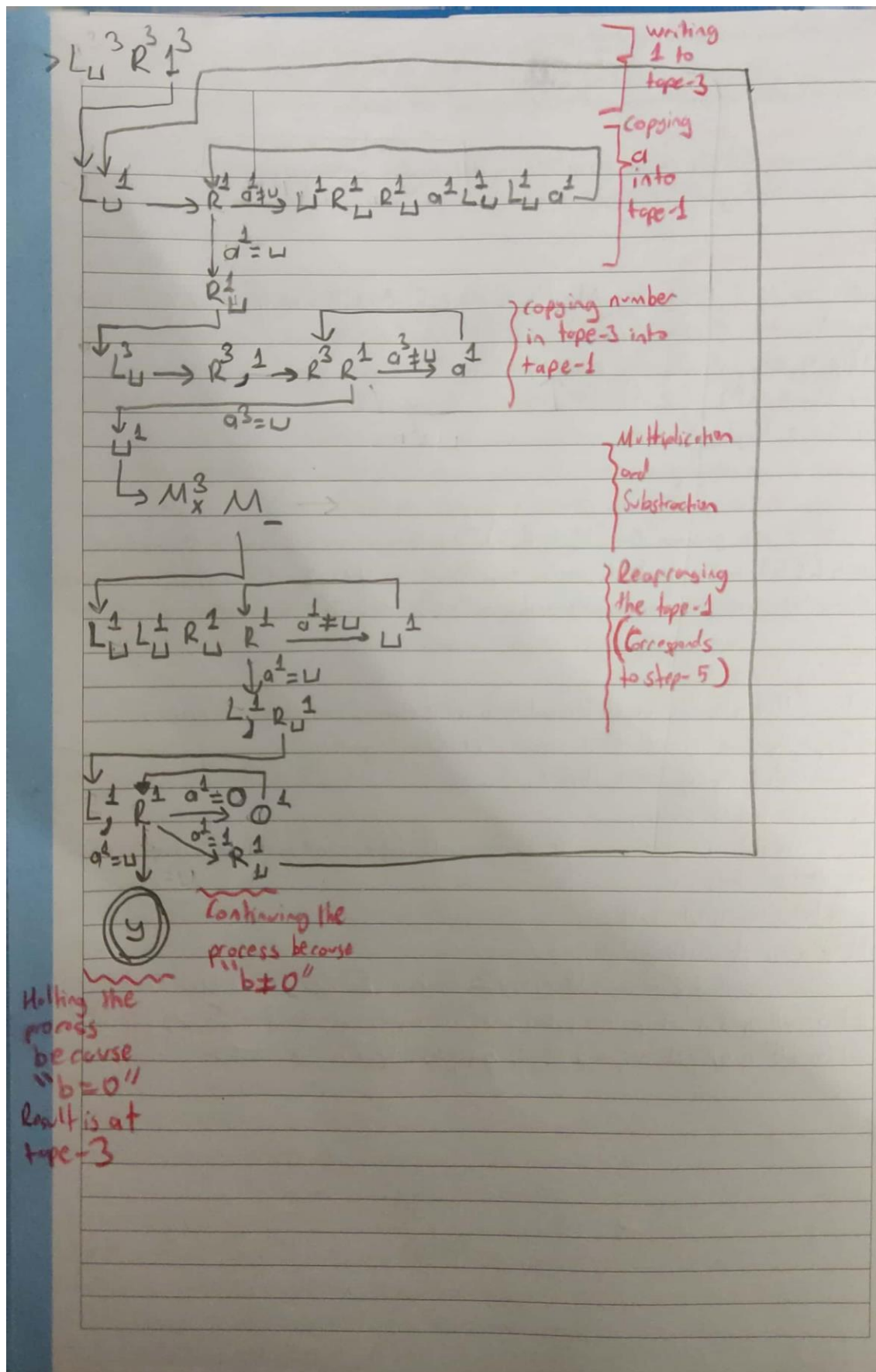


Figure-3: Turing Machine