Stochastic Optimization Assignment 1

Aytek Mutlu

September 29th, 2019

1 Introduction

This assignment focuses on revenue management of a flight with dynamic programming where potential ticket prices are discrete and the demand for each ticket is a function of time. During all steps, diversion assumption will be placed so that the total demand will be satisfied with the price of the optimal class.

2 Total Expected Revenue and Optimal Policy

For the mentioned problem, following equation is used:

$$V_t(x) = max_{j=1,...n}\{\sum_{i=1}^{j} \lambda_t(i)[y_i + V_{t+1}(x-1)] + [1 - \sum_{i=1}^{j} \lambda_t(i)]V_{t+1}(x)\}$$

Then, the optimal policy is filled in starting from T=0 where expected revenue is 0. Expected revenue at each time for each capacity level and for each booking class is calculated and total expected revenue is observed at T=600 and capacity=100.

$$V_{600}(100) = \mathbf{30{,}448.32}$$

3 Optimal Policy Plot

Based on the methodology in Part 1, the following optimal policy is obtained in Figure 1:

The optimal policy suggests starting with the cheapest ticket price (3rd class, 200 Euros) and then increasing the price. Most expensive ticket price (1st class, 500 Euros) is only optimal when most of the tickets are sold and only a few left.

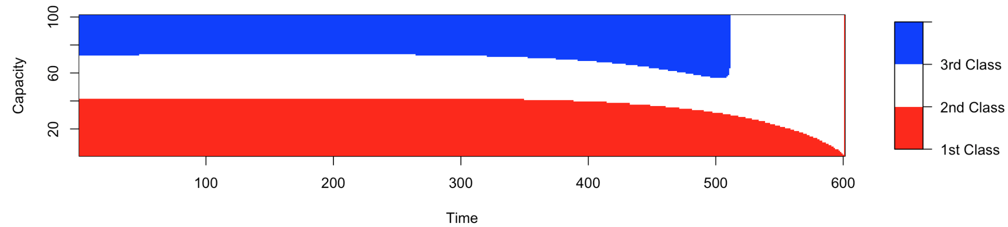

Figure 1: Optimal Policy for Revenue Management Problem for Flight Tickets

4 Simulation

The objective is to observe and analyze the demand over time. For this purpose, 10,000 realizations are performed where change in capacity, its respective booking class and price are simulated at each time. The behaviour of the simulation is illustrated on the optimal policy in order to show how it reacts to optimal booking class changes (Figure 2):

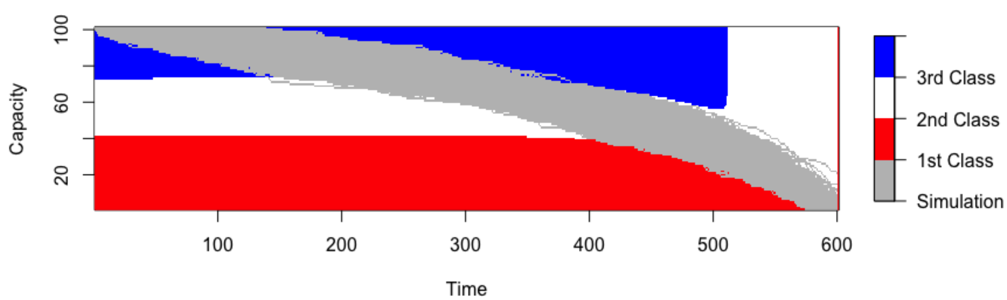

Figure 2: 10,000 Simulations of Demand Based on the Optimal Policy from Part 1

Each of the 10,000 simulations starts from T=600 and capacity=100 and runs until either T=0 or capacity=0. It can be observed from Figure 2 that both cases have realized, meaning that all flight tickets are sold before departure time on some realizations and some flight tickets remained unsold on some other realizations.

Another point is that the realizations roughly followed the border between 2nd class and 1st class and oscillated between zones. This is depicted in Figure 3 which shows the change in booking class captured from one of the realizations. After T=500, the booking class clearly changed between 2nd and 1st multiple times.

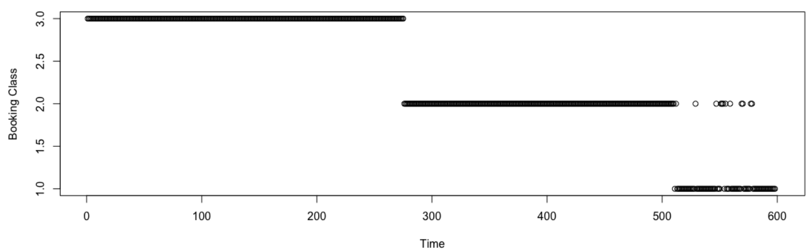

Figure 3: Booking Class of a realization of Simulation at Each Time

The total revenue for the realizations are recorded for each of the simulations and plotted with the following histogram in Figure 4:

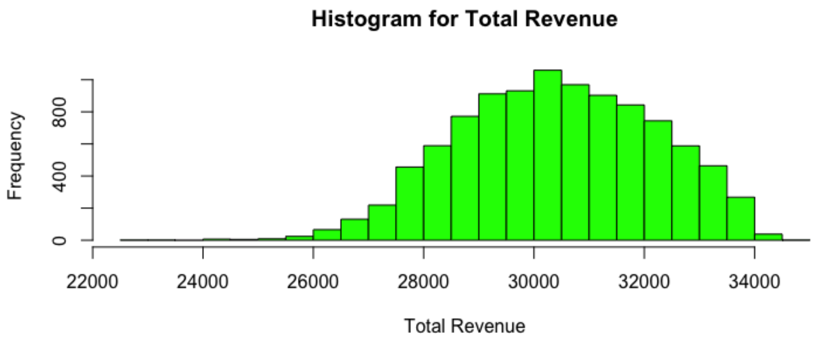

Figure 4: Total Revenue Distribution among 10,000 Simulations

The distribution is roughly normal and the average revenue among 10,000 distributions are **30,463.45**. This revenue is almost identical with the total expected revenue calculated in Part 1.

Similarly,number of tickets sold per class are also plotted with histograms in Figure 5, Figure 6 and Figure 7.

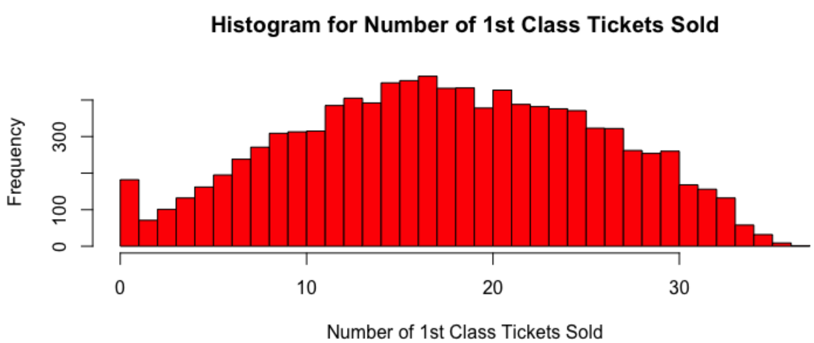

Figure 5: Distribution of number of 1st Class Tickets Sold

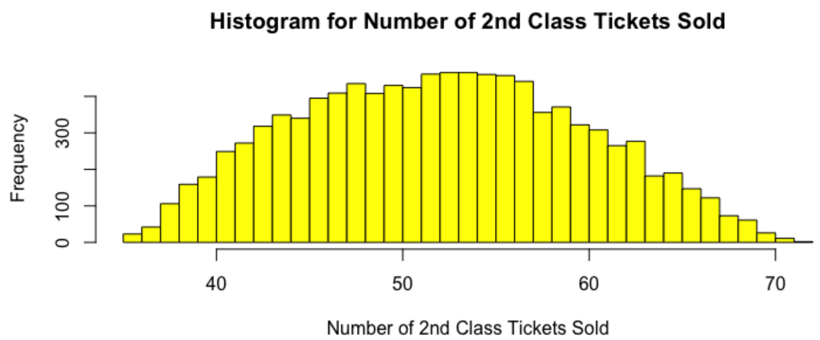

Figure 6: Distribution of number of 2nd Class Tickets Sold

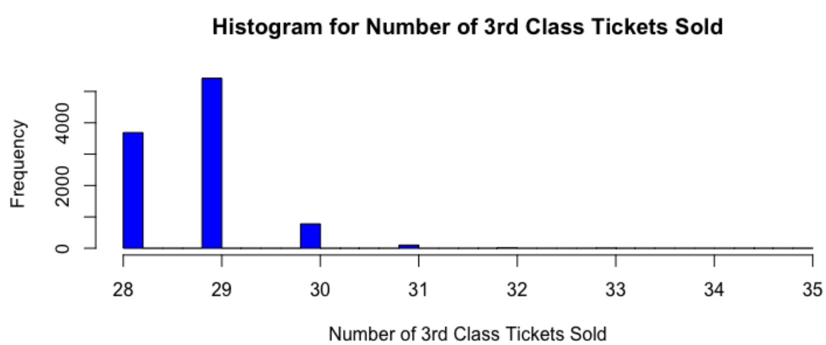

Figure 7: Distribution of number of 3rd Class Tickets Sold

Table 1 summarizes the average number of tickets sold per booking class. The simulation realizations sold slightly more than half of the tickets from 2nd class.

	Average Number of Tickets Sold
1st Class	17.92
2nd Class	52.52
3rd Class	28.73
TOTAL	99.17

Table 1: Average Number of Tickets Sold per Booking Class

5 Scenario where prices cannot go down

All of the actions in Part 2 to 4 is repeated with the addition of constraint in which prices cannot go down at any time. This has changed the total expected revenue to the following:

$$V_{600}(100) = \mathbf{28{,}682.03}$$

Not surprisingly, the constraint pulled the expected revenue down by around 6%. Moreover, the optimal policy has also changed and depicted in Figure 8.

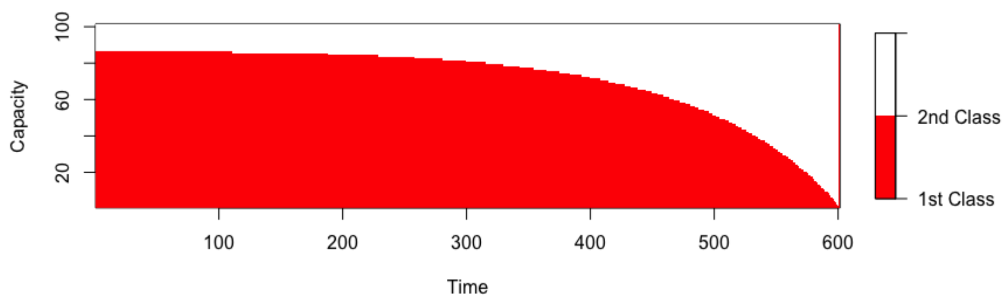

Figure 8: Optimal Policy for Revenue Management Problem for Flight Tickets with constraint of avoiding prices to go down

With the constraint, 3rd class has disappeared from the policy and the 1st class region has increased significantly. Similar to Part 4, 10,000 simulations were run on this optimal policy and following path is obtained (Figure 9):

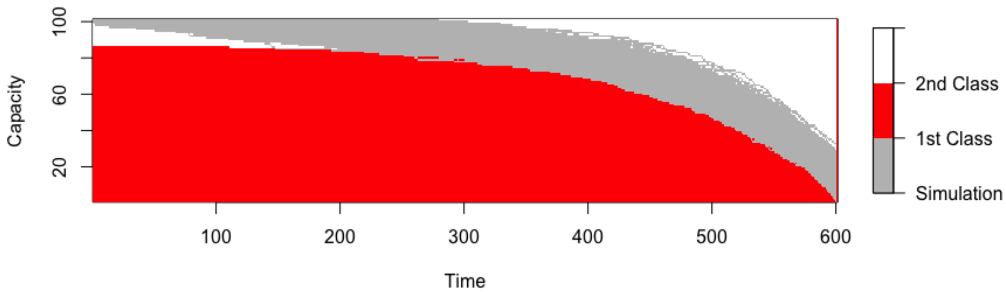

Figure 9: 10,000 Simulations of Demand Based on the Optimal Policy with constraint of avoiding price drops

Again, it is obvious that the realizations followed the border between 1st and 2nd classes and ended up with unsold tickets at some realizations.

Figure 10 plots the histogram of total revenue with the constraint:

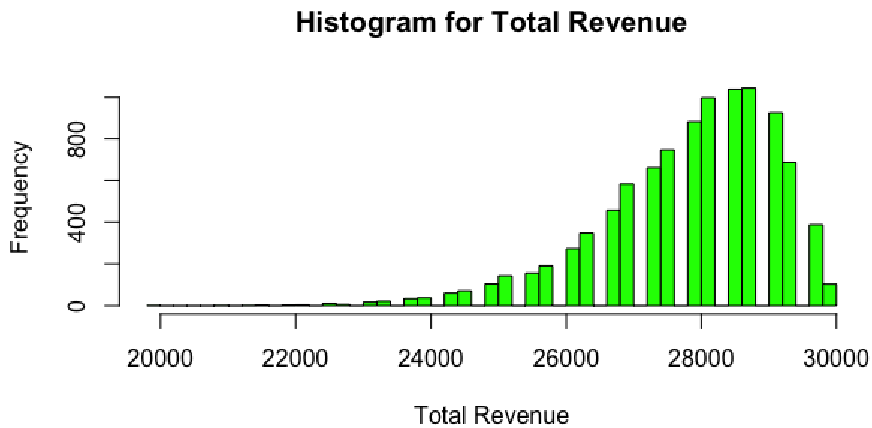

Figure 10: Total Revenue Distribution among 10,000 Simulations with constraint of avoiding price drops

The average revenue among 10,000 distributions are again calculated. Result is **27,864.30**. The revenue is this time significantly less than the maximum total expected revenue. A possible reason for this may well be that the optimal policy does not allow all of the tickets to be sold. Figure 9 gives a clue since almost all of the realizations end up with unsold seats. To justify this clue, histogram of number of 2nd class tickets sold is shown in Figure 11 and the averages are summarized in Table 2. Indeed, average number of total tickets sold of 92.88 supports the idea that optimal policy and the constraint does not allow to sell all available tickets. As a matter of fact, only 2nd class tickets are sold.

	Average Number of Tickets Sold
1st Class	0
2nd Class	92.88
3rd Class	0
TOTAL	92.88

Table 2: Average Number of Tickets Sold per Booking Class with constraint of avoiding price drops

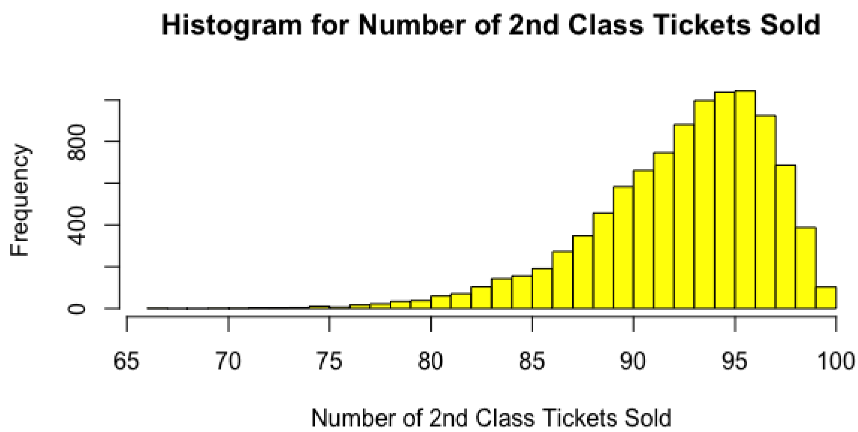

Figure 11: Distribution of number of 2nd Class Tickets Sold with constraint of avoiding price drops

6 Conclusion

This assignment focused on a dynamic programming on revenue management of a flight. During the process, total expected revenues are calculated with and without the constraint of avoiding price drops. It is observed that the expected revenue decreases with the introduction of the constraint.

Moreover, simulations are run on the optimal policies in order to observe general behavior, again with and without the constraint of avoiding price drops. It is observed that the average number of tickets sold in total dropped by around 7 with the introduction of the constraint and the average revenue dropped by around 2600 Euros.