Case 1: Binomial Trees and Geometric Brownian Motion

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1 Binomial Trees

A commonly used approach to compute the price of an option is the so-called binomial tree method. In this approach, option prices are computed through a well-known backward induction scheme, which was explained in one of the first lectures and can be found in all option pricing literature.

Consider an at-the-money European call option on a non-dividend-paying stock with a maturity of three months. Suppose that the underlying of this option is the S&P-500 index. Let the 3-month annualized interest rate be equal to 1% (with quarterly compounding).

(a) Download historical data of the S&P-500 index. Construct a time series of monthly returns with end date August 31, 2019. The number of months to cover equals the sum of your months of birth times 12 with a minimum of 60 and a maximum of 360. Show a graph of the monthly returns and report a table with the main summary statistics.

(b) Build a binomial tree for the S&P-500 index in MatLab (or package of your preference) with starting date August 31, 2019. The purpose is to price a call-option with strike price equal to USD 3,000 and maturity three months. Build a tree with three steps. Make sure that the volatility in the tree matches with the volatility in your historical data.

Suppose the risk premium on investing in the S&P-500 index is 1% over a period of 3 months.

(c) Calculate the upward probability p that ensures that the expected value of the S&P-500 index is consistent with the assumption on the risk premium.

(d) Calculate the price of a European call option with maturity 3 months and strike price USD 3,000. Compare this price with the output of the Black-Scholes formula.

(e) Study the convergence of the method for the call option of the previous question, increasing the number of steps in the tree (3, 10, 50, 100, 150, 200, 250 steps). Illustrate the convergence by means of graphs of the corresponding quantity vs the number of steps.

(f) Change the code such that it can compute the price of a European put option with strike price USD 3,000 and maturity three months. Calculate the price of the put option for the same parameters as mentioned above and verify whether your result satisfies the put-call parity relationship.

(g) Calculate call and put option prices with maturity 3 months and strikes USD 2,500, USD 2,600, USD 2,700, USD 2,800, USD 2,900, USD 3,100, USD 3,200, USD 3,300, USD 3,400, and USD 3,500. Compare these with the Black-Scholes prices.

(h) Now suppose that the option is American. Change the code such that it can handle early exercise opportunities. What are the values of the American put and call (strike USD 3,000 and maturity 3 months) for the same initial parameters? Which one should be exercised early?

(i) Now consider a European option with an exotic payoff: at maturity, it pays the difference between the terminal stock price and a strike, squared. Adjust your tree code so that it can cope with that and calculate the price of such an option for the strikes: USD 2,500, USD 2,750, USD 3,000, USD 3,250, and USD 3,500.

Final step is to generate 1,000 independent paths for the S&P-500 with a horizon of three months. Suppose that continuously compounded returns are normally distributed on each time interval. Assume that the expectation is the risk-free rate (scaled back to the time step) and the variance equals the variance used above (scaled back to the time step). Take 250 steps for each scenario.

(i) Generate 1,000 independent scenarios for the S&P-500 index with 250 time steps. Calculate in each scenario the payoff of a European call option with maturity three months and strike price USD 3,000. Take the average of all outcomes and discount the result with the risk free rate. Compare the result with the European call option price calculated using the binomial tree and the price that results from the Black-Scholes formula.

General information cases

The two cases of this course should be done in groups of two students. For each case, you have to write a short report where you must include the results, a short discussion/concluding section and the appendix with the codes. The reports should be submitted electronically, before the corresponding deadlines.

The preferred implementation tool is MatLab. During the allocated computer sessions, you will be able to get support and ask questions regarding your MatLab implementations. However, it is advised that you work on the assignments outside teaching hours and not only during the allocated sessions, as they will require significant time and effort.

In principle, you are not bound to MatLab and are free to choose the programming language/environment in which you would like to write your computer programs (so another alternative could be, e.g., R or Python). However, in this case we cannot guarantee implementation support.

The document "Computer tutorials SPF 2019" which is published on Canvas is a good start with MatLab: it gives an introduction into this programming environment and gets you started with some simple exercises. YouTube is an excellent source of MatLab support and information.

Reporting requirements

- Report should be written in English. This means that you also should use English decimal notation in the text and in graphs.
- There is no maximum to the number of words or pages but please try to be concise but please make sure that it is clear to me what you did and what your interpretation of the results is.
- Figures and graphs should have captions such that they can be read independently from the text.
- Use your spelling checker.
- Put names and VU student numbers of all group members on the front page.
- Final reports should be uploaded on Canvas before 1pm CET on the deadline day.

Deadlines

The deadlines for the two cases are as follows:

- Case 1: Friday September 27, 2019 at 1pm CET
- Case 2: Monday October 14, 2019 at 1pm CET

Last but not least:

ENJOY AND GOOD LUCK!!