Vrije Universiteit Amsterdam

Econometrics for Quantitative Risk Management

Factor Models

Assignment II

Authors
Joris Kreulen (2653365) & Aytek Mutlu (2648232)

VU UNIVERSITY AMSTERDAM

September 30, 2019

1 Introduction

In this assignment we will explore and estimate the Fama–French three-factor model, which is a model designed in 1992 by Eugene Fama and Kenneth French to describe stock returns. The three factors included in the model are (1) market risk ($Mkt-Rf$), (2) the outperformance of small versus big companies (SMB), and (3) the outperformance of high book/market versus small book/market companies (HML). The data used is that of 4 stocks, respectively: American International Group (AIG), Chevron Corporation (CVX), Microsoft (MSFT) and Tesla (TSLA).

2 Assignment

In order to calculate our factor loadings, we use the following:

$$r_t = \alpha + \beta f_t + \epsilon_t \equiv \xi g_t + \epsilon_t$$

Since g_t is known, we must estimate $\xi \equiv [\alpha, \beta]$. The results of which can be found in the table below:

	AIG	CVX	MSFT	TSLA
α	-0.149	-0.740	0.094	0.183
β_1	1.267	1.032	1.055	0.972
β_2	-0.021	-0.253	-0.508	0.660
β_3	0.632	0.520	-0.337	-1.233

Table 1: Factor loadings of stocks

Where α represents a constant, β_1 represents the company's loading on the factor market risk, β_2 represents the loading on SMB and β_3 represents the loading on HML.

To evaluate which stock returns are explained well by our factors we compute the R^2 of each stock. We use the formula:

$$R^2{}_i = 1\text{-}\frac{E'_{.,i}E_{.,i}}{R'_{.,i}R_{.,i}}$$

The following output is returned:

AIG	CVX	MSFT	TSLA
0.490	0.534	0.450	0.156

Table 2: R^2 of stocks

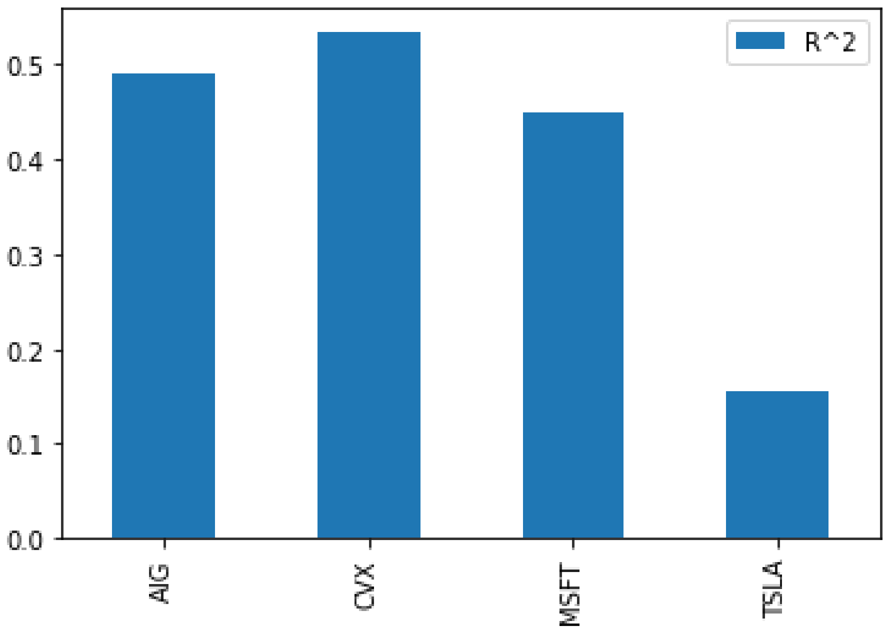

Figure 1: R^2 of stocks visualized

Henseler (2009) proposed a rule of thumb for acceptable R^2 with 0.75, 0.50, and 0.25 being described as substantial, moderate and weak respectively. From our table, we read that the variance in both AIG, CVX and MSFT can be considered to be moderately well described by our factors, this is not the case for TSLA as it's $R^2 < 0.25$, i.e. the variance in it's stock is worse than weakly described by our chosen factors.

We proceed to computing the covariance matrix. We drop our α since this is a constant and doesn't have a variance. We calculate the following:

$$\Sigma_r = \beta \Sigma_f \beta' + D, with D = diag({\sigma^2}_1, ..., {\sigma^2}_k)$$

The output is given:

	AIG	CVX	MSFT	TSLA
AIG	**13.984**	5.455	4.716	4.452
CVX	5.455	**8.135**	3.809	3.239
MSFT	4.716	3.809	**8.967**	4.155
TSLA	4.452	3.239	4.155	**44.001**

Table 3: Factor Estimated Covariance Matrix

We examine the diagonal. We immediately see TSLA has an extremely high variance compared to the other stocks, this explains the 'bad' R^2 value in the previous table. To determine whether the covariance matrix can be considered 'roughly' diagonal we examine the values outside the diagonal, i.e.:

$$\sum_E \approx D$$

These do not equal zero as in a classic diagonal matrix, in fact, the off-diagonal elements of the covariance matrix have non-zero values, indicating a correlation between the dimensions. Further research must be done to examine where this correlation originates, and how this behaves. We are hesitant to confirm even a 'rough' diagonal matrix.

To determine our model specification, the following must hold:

$$R = G\xi' + E$$

To examine the above, we compare covariance matrices with each other, i.e.:

$$Var(r_t|f_t) = \beta\Sigma_f\beta' + D \approx Var(r_t)$$

The covariance matrix of the actual returns is computed, our three-factor model based covariance matrix (table 3) is included for visual comparison reasons:

	AIG	CVX	MSFT	TSLA
AIG	**13.794**	5.224	4.516	2.592
CVX	5.224	**8.013**	3.418	3.107
MSFT	4.516	3.418	**8.805**	4.189
TSLA	2.592	3.107	4.189	**43.619**

Table 4: Actual Returns Based Covariance Matrix

	AIG	CVX	MSFT	TSLA
AIG	**13.984**	5.455	4.716	4.452
CVX	5.455	**8.135**	3.809	3.239
MSFT	4.716	3.809	**8.967**	4.155
TSLA	4.452	3.239	4.155	**44.001**

Table 5: Factor Estimated Covariance Matrix (Replica)

We observe slight differences between the two covariance matrices, however, we consider these negligible and therefore we accept $Var(r_t|f_t) \approx Var(r_t)$.

Now that our model is specified, we shall use it to find Markowitz's Global Minimum Variance Portfolio (GMVP). The GMVP is a portfolio with minimum variance when compared to all possible portfolios of risky assets. This is known as the global minimum-variance portfolio. Since we are only working with 4 assets, our portfolio weights shall be distributed over our four companies in such a way that our variance is minimized. The optimal weights that provides minimum variance are formulated as follows:

$$\hat{w} = argmin_w w' \sum w$$

where;

$$w'\mathbf{1}_k = 1$$

To asses whether our GMVP is consistent between model and stock returns themselves we solved the optimal weights for both and compare the two in Table 6:

	W, factor model	W, stock returns
AIG	0.0954	0.1012
CVX	0.4603	0.4545
MSFT	0.3899	0.3901
TSLA	0.0543	0.0541

Table 6: Portfolio Weights (W), factor model vs stock returns

As expected, the allocated weights are very similar. Both methods compute covariance matrices which are approximately equal to each other, when this variance needs to be minimized the outcome is bound to have commonalities.

References

[1] Henseler, J., Ringle, C. M., & Sinkovics, R. R. *The use of partial least squares path modeling in international marketing. In New challenges to international marketing (pp. 277-319)*. Emerald Group Publishing Limited., 2009.