Linear Algebra: Week 3 Notes and Exercises

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May 23, 2018

1 Notes

Exercises

1) Find the length of the vectors $a = \begin{bmatrix} -10 \\ 5 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$||\vec{a}|| = \sqrt{(-10)^2 + 5^2}$
$||\vec{a}|| = \sqrt{125}$
$||\vec{b}|| = \sqrt{(3)^2 + (3)^2}$
$||\vec{b}|| = \sqrt{18}$

2) Find the magnitude of the vectors $r = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$ and $q = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$

$||\vec{r}|| = \sqrt{(7)^2 + (-3)^2}$
$||\vec{r}|| = \sqrt{58}$
$||\vec{q}|| = \sqrt{(-3)^2 + (7)^2}$
$||\vec{q}|| = \sqrt{58}$

3) Normalize the vectors in $\mathbb{R}^2$ $a = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

$||\vec{a}|| = \sqrt{(4)^2 + (5)^2}$
$||\vec{a}|| = \sqrt{41}$
$\frac{\vec{a}}{||\vec{a}||} = \frac{\begin{bmatrix} 4 & 5 \end{bmatrix}}{\sqrt{41}} = \begin{bmatrix} \frac{4}{\sqrt{41}} & \frac{5}{\sqrt{41}} \end{bmatrix}$
$||\vec{b}|| = \sqrt{(5)^2 + (-4)^2}$
$||\vec{b}|| = \sqrt{41}$
$\frac{\vec{b}}{||\vec{b}||} = \frac{\begin{bmatrix} 5 & -4 \end{bmatrix}}{\sqrt{41}} = \begin{bmatrix} \frac{5}{\sqrt{41}} & \frac{-4}{\sqrt{41}} \end{bmatrix}$

4) Normalize the vectors in $\mathbb{R}^2$ $q = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$ and $p = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$||\vec{q}|| = \sqrt{(-3)^2 + (6)^2}$
$||\vec{q}|| = \sqrt{41}$
$\frac{\vec{q}}{||\vec{q}||} = \begin{bmatrix} \frac{-3}{\sqrt{41}} & \frac{6}{\sqrt{41}} \end{bmatrix}$
$||\vec{p}|| = \sqrt{(-1)^2 + (-1)^2}$
$||\vec{p}|| = \sqrt{2}$
$\frac{\vec{p}}{||\vec{p}||} = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$

5) Find the distance between the points $a = \begin{bmatrix} -10 \\ 5 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$\vec{a} - \vec{b} = \begin{bmatrix} -13 \\ 2 \end{bmatrix}$
$||\vec{a} - \vec{b}|| = \sqrt{(-13)^2 + (2)^2}$
$||\vec{a} - \vec{b}|| = \sqrt{173}$

6) Find the distance between the points $q = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ and $r = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$

$\vec{q} - \vec{r} = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$
$||\vec{q} - \vec{r}|| = \sqrt{(-10)^2 + (10)^2}$
$||\vec{q} - \vec{r}|| = \sqrt{200}$

10) Find the dot product of the two vectors. Are the two vectors perpendicular?
a)

$$v_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} v_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$v_1 \cdot v_2 = \begin{bmatrix} -1 \times 2 + (-1) \times -2 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$
$||\vec{v_1}|| = \sqrt{(-1)^2 + (-1)^2}$
$||\vec{v_1}|| = \sqrt{2}$
$||\vec{v_2}|| = \sqrt{(2)^2 + (-2)^2}$
$||\vec{v_2}|| = \sqrt{8}$
$\cos\theta = \frac{\vec{v_1} \cdot \vec{v_2}}{||\vec{v_1}|| \cdot ||\vec{v_2}||} = \frac{0}{\sqrt{2} \cdot \sqrt{8}} = \frac{0}{\sqrt{16}} = 0$ Therefore, it is perpendicular.