

Week 2 Notes and Exercises

1 Notes

Matrix operations

Calculate $A + B$ and $A - B$ or state why you can't.

a)

$$A = \begin{bmatrix} 1 & 0 & 4 \\ -1 & 2 & 2 \\ 0 & -2 & -3 \end{bmatrix}, B = \begin{bmatrix} -1 & 3 & 5 \\ 2 & 2 & -3 \\ 2 & -3 & 0 \end{bmatrix}$$
$$A + B = \begin{bmatrix} 0 & 3 & 9 \\ 1 & 4 & -1 \\ 2 & -5 & -3 \end{bmatrix}, A - B = \begin{bmatrix} 2 & -3 & -1 \\ -3 & 0 & 5 \\ -2 & 1 & -3 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} -1 & 0 & 5 & 6 \\ -4 & -3 & 5 & -2 \end{bmatrix}, B = \begin{bmatrix} -3 & 9 & -3 & 4 \\ 0 & -2 & -1 & 2 \end{bmatrix}$$
$$A + B = \begin{bmatrix} -4 & 9 & 2 & 10 \\ -4 & -5 & 4 & 0 \end{bmatrix}$$

Exercises find AB or state why it doesn't exist.

a)

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & -1 & -1 \\ -1 & 1 & 0 \end{bmatrix}, B = \begin{bmatrix} 0 & 1 & -1 \\ 1 & -1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

b)

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -1 \\ 0 & -2 \\ -1 & 5 \end{bmatrix}, B = \begin{bmatrix} -1 & 3 & 2 & 2 \\ 0 & -1 & 4 & 3 \end{bmatrix}$$

2 Exercise 2.1

Questions 1,2,3,7,8,9,10,12

In Exercise 1 and 2, compute each matrix sum or product if it is defined. If an expression is undefined, explain why. Let

$$A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & -5 & 2 \end{bmatrix}, B = \begin{bmatrix} 7 & -5 & 1 \\ 1 & -4 & -3 \end{bmatrix}, C = \begin{bmatrix} 1 & 2 \\ -2 & 1 \end{bmatrix}, D = \begin{bmatrix} 3 & 5 \\ -1 & 4 \end{bmatrix}, E = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$$

Sum

$$A + B = \begin{bmatrix} 2+7 & 0+(-5) & -1+1 \\ 4+1 & -5+(-4) & 2+(-3) \end{bmatrix} = \begin{bmatrix} 9 & -5 & 0 \\ 5 & -9 & -1 \end{bmatrix}$$
$$C + D = \begin{bmatrix} 1+3 & 2+5 \\ -2+(-1) & 1+4 \end{bmatrix} = \begin{bmatrix} 4 & 2 \\ -3 & 5 \end{bmatrix}$$

Rest of matrices can't be added together since they don't have the same dimensions.

Product

$$C_{2 \times 2} \cdot A_{2 \times 3} = \begin{bmatrix} 1 \cdot 2 + 2 \cdot 4 & 1 \cdot 0 + 2 \cdot -5 & 1 \cdot -1 + 2 \cdot 2 \\ -2 \cdot 2 + 1 \cdot 4 & -2 \cdot 0 + 1 \cdot -5 & -2 \cdot -1 + 1 \cdot 2 \end{bmatrix} = \begin{bmatrix} 10 & -10 & 3 \\ 0 & -5 & 4 \end{bmatrix}$$

$$C_{2 \times 2} \cdot B_{2 \times 3} = \begin{bmatrix} 1 \cdot 7 + 2 \cdot 1 & 1 \cdot -5 + 2 \cdot -4 & 1 \cdot 1 + 2 \cdot -3 \\ -2 \cdot 7 + 1 \cdot 1 & -2 \cdot -5 + 1 \cdot -4 & -2 \cdot 1 + 1 \cdot -3 \end{bmatrix} = \begin{bmatrix} 9 & -13 & -5 \\ -13 & 6 & -5 \end{bmatrix}$$

$$C_{2 \times 2} \cdot D_{2 \times 2} = \begin{bmatrix} 1 \cdot 3 + 2 \cdot 5 & 1 \cdot 5 + 2 \cdot 4 \\ -2 \cdot 3 + 1 \cdot -1 & -2 \cdot 5 + 1 \cdot 4 \end{bmatrix} = \begin{bmatrix} 13 & 13 \\ -7 & -5 \end{bmatrix}$$

$$C_{2 \times 2} \cdot E_{2 \times 1} = \begin{bmatrix} 1 \cdot -5 + 2 \cdot 3 \\ -2 \cdot -5 + 1 \cdot 3 \end{bmatrix} = \begin{bmatrix} 1 \\ 13 \end{bmatrix}$$

$$D_{2 \times 2} \cdot A_{2 \times 3} = \begin{bmatrix} 3 \cdot 2 + 5 \cdot 4 & 3 \cdot 0 + 5 \cdot -5 & 3 \cdot -1 + 5 \cdot 2 \\ -1 \cdot 2 + 4 \cdot 4 & -1 \cdot 0 + 4 \cdot -5 & -1 \cdot -1 + 4 \cdot 2 \end{bmatrix} = \begin{bmatrix} 26 & -25 & 7 \\ 6 & -20 & 9 \end{bmatrix}$$

$$D_{2 \times 2} \cdot B_{2 \times 3} = \begin{bmatrix} 3 \cdot 7 + 5 \cdot 1 & 3 \cdot -5 + 5 \cdot -4 & 3 \cdot 1 + 5 \cdot -3 \\ -1 \cdot 7 + 4 \cdot 1 & -1 \cdot -5 + 4 \cdot -4 & -1 \cdot 1 + 4 \cdot -3 \end{bmatrix} = \begin{bmatrix} 26 & -35 & -12 \\ -3 & -11 & -13 \end{bmatrix}$$

$$D_{2 \times 2} \cdot E_{2 \times 1} = \begin{bmatrix} 3 \cdot -5 + 5 \cdot 3 \\ -1 \cdot -5 + 4 \cdot 3 \end{bmatrix} = \begin{bmatrix} 0 \\ 17 \end{bmatrix}$$

Rest of matrices can't have a dot product since they don't have the required dimensions.

3 Exercise 2.2

Questions 1,2,3,4,5,6,7a,8,29-32