

Linear Algebra: Week 3 Notes and Exercises

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1 Notes

Exercises

1) Find the length of the vectors $a = \begin{bmatrix} -10 \\ 5 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$$||\vec{a}|| = \sqrt{(-10)^2 + 5^2}$$

$$||\vec{a}|| = \sqrt{125}$$

$$||\vec{b}|| = \sqrt{(3)^2 + (3)^2}$$

$$||\vec{b}|| = \sqrt{18}$$

2) Find the magnitude of the vectors $r = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$ and $q = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$

$$||\vec{r}|| = \sqrt{(7)^2 + (-3)^2}$$

$$||\vec{r}|| = \sqrt{58}$$

$$||\vec{q}|| = \sqrt{(-3)^2 + (7)^2}$$

$$||\vec{q}|| = \sqrt{58}$$

3) Normalize the vectors in \mathbb{R}^2 $a = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$ and $b = \begin{bmatrix} 5 \\ -4 \end{bmatrix}$

$$||\vec{a}|| = \sqrt{(4)^2 + (5)^2}$$

$$||\vec{a}|| = \sqrt{41}$$

$$\frac{\vec{a}}{||\vec{a}||} = \frac{\begin{bmatrix} 4 & 5 \end{bmatrix}}{\sqrt{41}} = \begin{bmatrix} \frac{4}{\sqrt{41}} & \frac{5}{\sqrt{41}} \end{bmatrix}$$

$$||\vec{b}|| = \sqrt{(5)^2 + (-4)^2}$$

$$||\vec{b}|| = \sqrt{41}$$

$$\frac{\vec{b}}{||\vec{b}||} = \frac{\begin{bmatrix} 5 & -4 \end{bmatrix}}{\sqrt{41}} = \begin{bmatrix} \frac{5}{\sqrt{41}} & \frac{-4}{\sqrt{41}} \end{bmatrix}$$

4) Normalize the vectors in \mathbb{R}^2 $q = \begin{bmatrix} -3 \\ 6 \end{bmatrix}$ and $p = \begin{bmatrix} -1 \\ -1 \end{bmatrix}$

$$||\vec{q}|| = \sqrt{(-3)^2 + (6)^2}$$

$$||\vec{q}|| = \sqrt{41}$$

$$\frac{\vec{q}}{||\vec{q}||} = \begin{bmatrix} \frac{-3}{\sqrt{41}} & \frac{6}{\sqrt{41}} \end{bmatrix}$$

$$||\vec{p}|| = \sqrt{(-1)^2 + (-1)^2}$$

$$||\vec{p}|| = \sqrt{2}$$

$$\frac{\vec{p}}{||\vec{p}||} = \begin{bmatrix} \frac{-1}{\sqrt{2}} & \frac{-1}{\sqrt{2}} \end{bmatrix}$$

5) Find the distance between the points $a = \begin{bmatrix} -10 \\ 5 \end{bmatrix}$ and $b = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$

$$\vec{a} - \vec{b} = \begin{bmatrix} -13 \\ 2 \end{bmatrix}$$

$$||\vec{a} - \vec{b}|| = \sqrt{(-13)^2 + (2)^2}$$

$$||\vec{a} - \vec{b}|| = \sqrt{173}$$

6) Find the distance between the points $q = \begin{bmatrix} -3 \\ 7 \end{bmatrix}$ and $r = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$

$$\vec{q} - \vec{r} = \begin{bmatrix} -10 \\ 10 \end{bmatrix}$$

$$||\vec{q} - \vec{r}|| = \sqrt{(-10)^2 + (10)^2}$$

$$||\vec{q} - \vec{r}|| = \sqrt{200}$$

10) Find the dot product of the two vectors. Are the two vectors perpendicular?

a)

$$v_1 = \begin{bmatrix} -1 \\ -1 \end{bmatrix} \quad v_2 = \begin{bmatrix} 2 \\ -2 \end{bmatrix}$$

$$v_1 \cdot v_2 = [-1 \times 2 + (-1) \times -2] = [0]$$

$$||\vec{v}_1|| = \sqrt{(-1)^2 + (-1)^2}$$

$$||\vec{v}_1|| = \sqrt{2}$$

$$||\vec{v}_2|| = \sqrt{(2)^2 + (-2)^2}$$

$$||\vec{v}_2|| = \sqrt{8}$$

$$\cos \theta = \frac{\vec{v}_1 \cdot \vec{v}_2}{||\vec{v}_1|| \cdot ||\vec{v}_2||} = \frac{0}{\sqrt{2} \cdot \sqrt{8}} = \frac{0}{\sqrt{16}} = 0$$

\therefore it is perpendicular.

b)

$$a = \begin{bmatrix} -10 \\ 5 \end{bmatrix} \quad b = \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$a \cdot b = [-10 \times 3 + 5 \times 3] = [-15]$$

$$||\vec{a}|| = \sqrt{(-10)^2 + (5)^2}$$

$$||\vec{a}|| = \sqrt{125}$$

$$||\vec{b}|| = \sqrt{(3)^2 + (3)^2}$$

$$||\vec{b}|| = \sqrt{18}$$

$$\cos \theta = \frac{\vec{a} \cdot \vec{b}}{||\vec{a}|| \cdot ||\vec{b}||} = \frac{-15}{\sqrt{125} \cdot \sqrt{18}}$$

$$\cos \theta = -\frac{15}{\sqrt{2250}}$$

\therefore it is not perpendicular.

c)

$$v = \begin{bmatrix} 3 \\ 6 \end{bmatrix} \quad w = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$$

$$v \cdot w = [3 \times 1 + 6 \times 2] = [15]$$

$$||\vec{v}|| = \sqrt{(3)^2 + (6)^2}$$

$$||\vec{v}|| = \sqrt{45}$$

$$||\vec{w}|| = \sqrt{(1)^2 + (2)^2}$$

$$||\vec{w}|| = \sqrt{5}$$

$$\cos \theta = \frac{\vec{v} \cdot \vec{w}}{||\vec{v}|| \cdot ||\vec{w}||} = \frac{15}{\sqrt{45} \cdot \sqrt{5}}$$

$$\cos \theta = \frac{15}{\sqrt{225}} = 1$$

\therefore it is not perpendicular.

11) Compute the cosine of the angle formed by the vectors $r = \begin{bmatrix} 7 \\ -3 \end{bmatrix}$ and $m = \begin{bmatrix} 2 \\ -5 \end{bmatrix}$

$$r \cdot m = [7 \times 2 + (-3) \times -5] = [29]$$

$$||\vec{r}|| = \sqrt{(7)^2 + (-3)^2}$$

$$||\vec{r}|| = \sqrt{58}$$

$$||\vec{m}|| = \sqrt{(2)^2 + (-5)^2}$$

$$||\vec{m}|| = \sqrt{29}$$

$$\cos \theta = \frac{29}{\sqrt{58} \cdot \sqrt{29}} = \frac{29}{1682} = \frac{1}{58} > 0$$

\therefore the angle is going to be acute.

12) Are the following angles acute, obtuse or right angle?

$$\cos \theta_1 = -0.3, \cos \theta_2 = 0, \cos \theta_3 = 0.75$$

θ_1 is an obtuse angle, θ_2 is a right angle, θ_3 is an acute angle.

13) Given the target box $(min_1, min_2) = (4, 3)$, $(max_1, max_2) = (6, 5)$. Find the local point $(\frac{1}{2}, \frac{1}{2})$ in the target box.

$$x_1 = (1 - u_1)min_1 + u_1max_1$$

$$x_1 = (1 - \frac{1}{2})4 + \frac{1}{2}6$$

$$x_1 = 5$$

$$x_2 = (1 - u_2)min_2 + u_2max_2$$

$$x_2 = (1 - \frac{1}{2})3 + \frac{1}{2}5$$

$$x_2 = 4$$

\therefore the local point $(\frac{1}{2}, \frac{1}{2})$ in the target box is $(5, 4)$.

14) Given the target box $(min_1, min_2) = (1, 3)$, $(max_1, max_2) = (6, 8)$. Find the local point $(\frac{1}{2}, \frac{1}{4})$ in the target box.

$$x_1 = (1 - \frac{1}{2})1 + \frac{1}{2}6$$

$$x_1 = 3\frac{1}{2}$$

$$x_2 = (1 - \frac{1}{4})3 + \frac{1}{4}8$$

$$x_2 = 1\frac{1}{2} + 2$$

$$x_2 = 3\frac{1}{2}$$

\therefore the local point $(\frac{1}{2}, \frac{1}{4})$ in the target box is $(3\frac{1}{2}, 3\frac{1}{2})$.