EECS 16A   Designing Information Devices and Systems I
## Spring 2018 Lecture Notes   Note 19

Golden Rules Revisited

Recall for an ideal op amp, the "golden rules" are

- (1) $I_+ = I_- = 0$. No current can flow into the positive or negative input terminal. **Note that this rule holds regardless of whether there is negative feedback or not.**

- (2) For a circuit with negative feedback, $V_+ = V_-$. One important thing to note now is this rule only holds **when there is negative feedback**.
  $V_+ = V_-$ means that the input voltage $v_{in} = V_+ - V_-$ is $0V$ for an op-amp. In order to understand how v_{out} can be non-zero for $v_{in} = 0$, let's review the block diagram of an op amp in negative feedback.
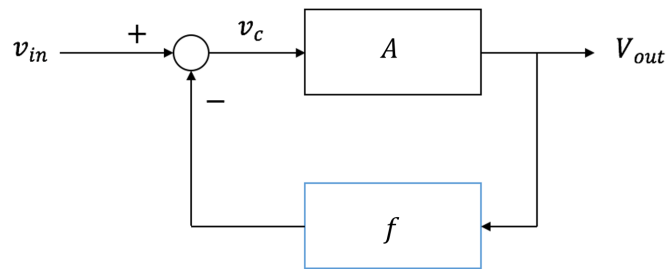
We have negative feedback gain f. For an op-amp in negative feedback, we have

$$v_{in} - f \cdot v_{out} = v_c. \tag{1}$$
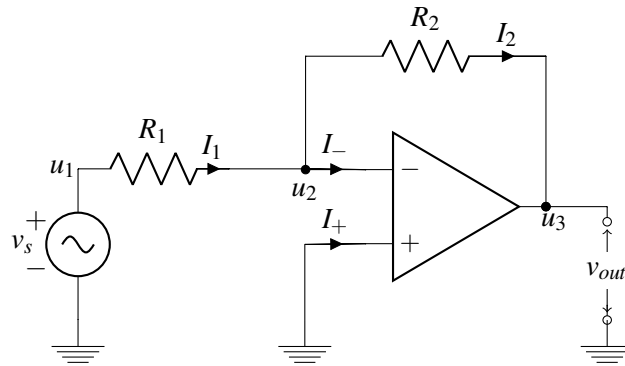
We know $v_{out} = A \cdot v_c$ and we can derive:

$$v_{out} = \frac{A}{1 + A \cdot f} v_{in}. \tag{2}$$

When $v_{in} = 0V$, $v_c = 0V$. Recall mathematically, $\lim_{x \to \infty} k \cdot x \cdot \frac{1}{x} = k$ even though $\lim_{x \to \infty} \frac{1}{x} = 0$. Similarly, for an ideal op-amp, v_{out} can be non-zero even if $v_c = 0$ given $A = \infty$.

Inverting Amplifier & Negative Feedback

Now let's use the golden rules to analyze the circuit we saw earlier:

Let's assume that this circuit is in negative feedback so that we can use golden rule #2, after analyzing the circuit we will show how to check if a circuit is indeed in negative feedback.

We know that by applying KCL, we have

$$I_1 = I_2 + I_-. \tag{3}$$

Using golden rules # 1, we have

$$I_- = 0. \tag{4}$$

Thus, $I_1 = I_2$. Using golden rules # 2, we have

$$v_- = v_+ = 0. \tag{5}$$

The second equation indicates $u_2 = 0$.
Using Ohm's law, we have

$$u_1 - u_2 = I_1 R_1. \tag{6}$$

$$u_2 - u_3 = I_2 R_2. \tag{7}$$

After solving the equations, we get

$$v_{out} = -\frac{R_2}{R_1} v_{in}. \tag{8}$$

This circuit is known as "inverting amplifier". It is called "inverting" because it amplifies v_{in} by some negative factor $-\frac{R_2}{R_1}$.

We have used golden rule #2 to simplify circuit analysis, now we will show how to check if a circuit is in negative feedback:

- **Step 1** Zero out all **independent** sources. For any independent sources that outputs a constant voltage or current, we will zero them out like we did in Thevenin-Norton Equivalencies.

- **Step 2** "Dink" the output, check if the feedback coming from the circuit as a result of "dinking" the output has the opposite direction of "dinking". If the feedback coming from the circuit is in the opposite direction as "dinking" the output, then the circuit is in negative feedback. If not, the circuit is in positive feedback or zero feedback.

Let's use the rules to check some example circuits.
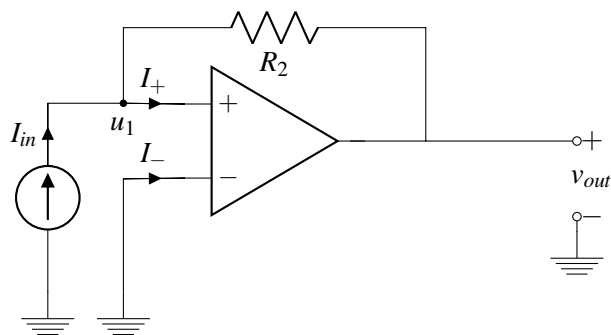**Example circuit 1**
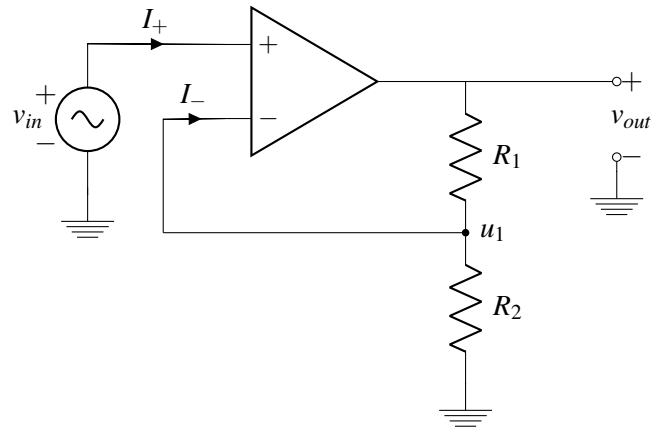
Step 1: zero-out all independent sources We can zero-out the independent source v_{in} in the circuit, and the circuit diagram becomes:
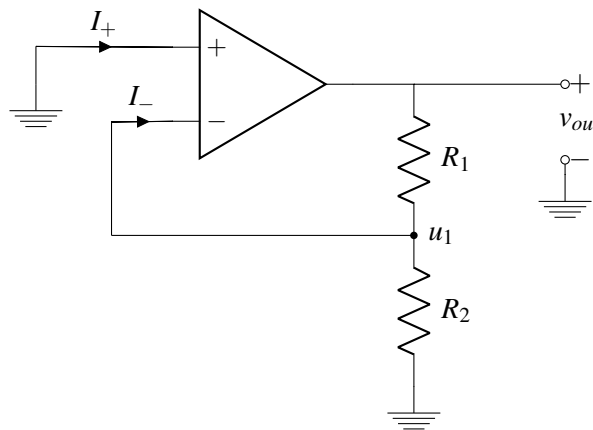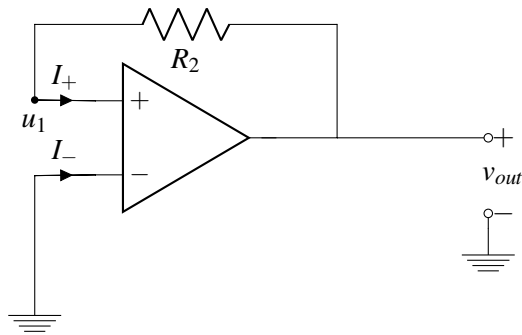

Step 2 "dink v_{out}" If we increase v_{out}, we know u_1 is going to increase because R_1 and R_2 form a voltage divider circuit. u_1 is connected to v_-, so v_- increases, therefore $v_{in} = v_+ - v_-$ decreases, and thus v_{out} decreases. So the feedback from the circuit is in the opposite direction, this circuit is in negative feedback.
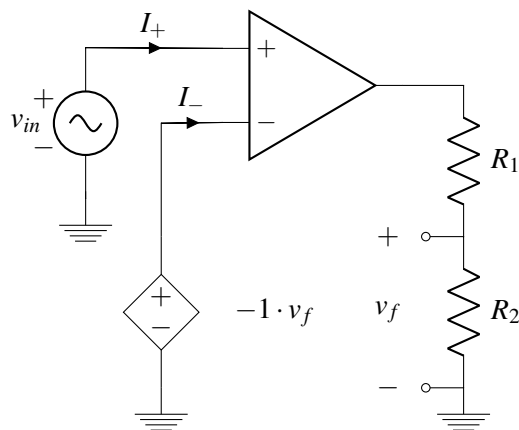**Example circuit 2**

Step 1 Zero out all **independent** sources.

Step 2 "dink v_{out}" If we increase v_{out}, we know u_1 is going to increase because $u_1 = v_{out}$. u_1 is connected to v_+, so v_+ increases, therefore $v_{in} = v_+ - v_-$ increases, and thus v_{out} increases. So the feedback from the circuit is in the same direction, this circuit is in positive feedback.

Example circuit 3

Step 1 Zero out all **independent** sources. We cannot zero out controlled voltage source $-1 \cdot v_f$ because this is not an independent voltage source.
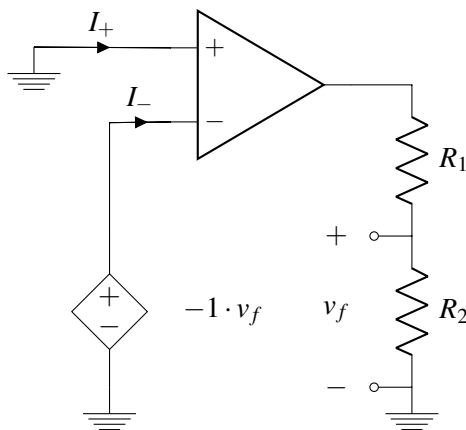
Step 2 "dink v_{out}" If we increase v_{out}, v_f increases which then decreases $v_- = -1 \cdot v_f$, which then increases $v_{in} = V_+ - V_-$, thus v_{out} increases. The circuit response to the increase in v_{out} is in the same direction as we "dinked" v_{out}. So this circuit is not in negative feedback.

Non-inverting & Inverting Amplifiers

There are two important categories of amplifier circuits that you should be familiar with, they are non-inverting amplifier and inverting amplifier:
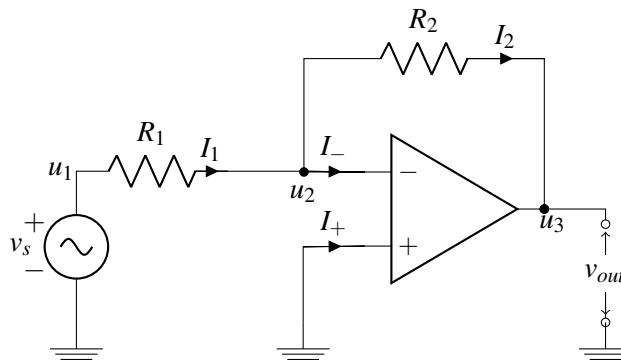
- **Non-inverting Amplifier**
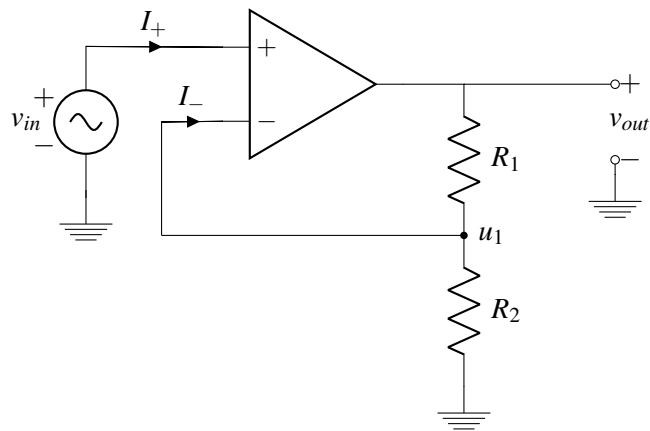
For the non-inverting amplifier circuit, we have

$$v_{out} = (1 + \frac{R_1}{R_2}) \cdot v_{in}. \tag{9}$$

- **Inverting Amplifier**

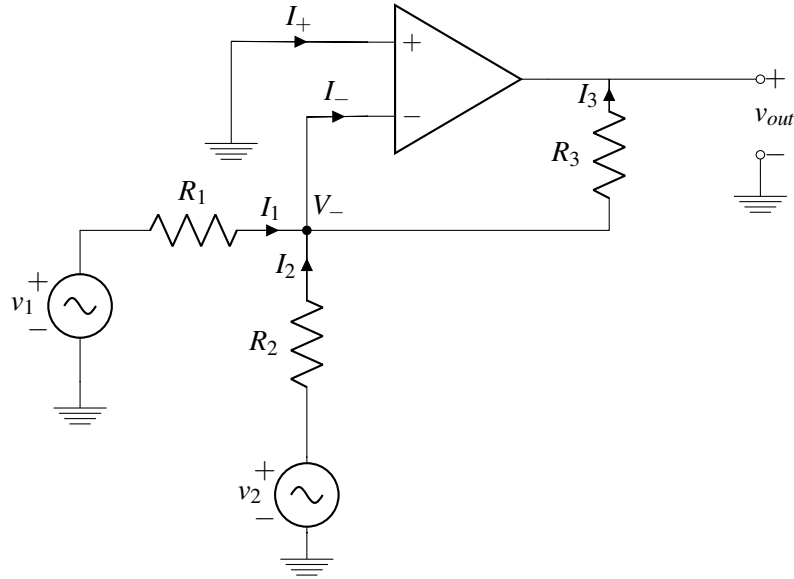For the inverting amplifier circuit, we have

$$v_{out} = -\frac{R_2}{R_1} \cdot v_{in}. \tag{10}$$

Artificial Neuron

The underlying mathematical operations for artificial intelligence is matrix multiplication, which in essence is vector multiplication:

$$\begin{bmatrix} a_1 & a_2 \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = a_1 \cdot x_1 + a_2 \cdot x_2 \tag{11}$$

This vector multiplication can be realized by the following circuit model:

First, let's apply the first golden rule, $I_- = I_+ = 0$. Applying KCL at the node labelled V_-, we have

$$I_1 + I_2 = I_- + I_3 = 0 + I_3 = I_3. \tag{12}$$

Hence, we have

$$I_1 + I_2 = I_3. \tag{13}$$

Now by the second golden rule and the fact that the positive input terminal is connected to ground, we have

$$v_+ = v_- = 0. \tag{14}$$

Applying Ohm's law at each of the three resistors, we have

$$I_1 = \frac{v_1 - v_-}{R_1} = \frac{v_1}{R_1} \tag{15}$$

$$I_2 = \frac{v_2 - v_-}{R_2} = \frac{v_2}{R_2} \tag{16}$$

$$I_3 = \frac{v_- - v_{out}}{R_3} = -\frac{v_{out}}{R_3} \tag{17}$$

$$\tag{18}$$

Plugging in the above result to the KCL equation $I_1 + I_2 = I_3$ derived previously, we have

$$\frac{v_1}{R_1} + \frac{v_2}{R_2} = -\frac{v_{out}}{R_3}. \tag{19}$$

Multiplying both sides by R_3, we have

$$v_{out} = -\frac{R_3}{R_1} v_1 - \frac{R_3}{R_2} v_2, \tag{20}$$

relating the final output with the two inputs v_1 and v_2, which are x_1 and x_2 in equation (13). Now, $a_1 = \frac{R_3}{R_1}$, $a_2 = \frac{R_3}{R_2}$. This circuit is named "inverting summing amplifier".

Because R_1, R_2, R_3 are resistors and all have positive values, the coefficients a_1 and a_2 are all negative. How do we make a_1 and a_2 positive instead? By adding another inverter amplifier circuit at v_{out}, we can get positive coefficients.
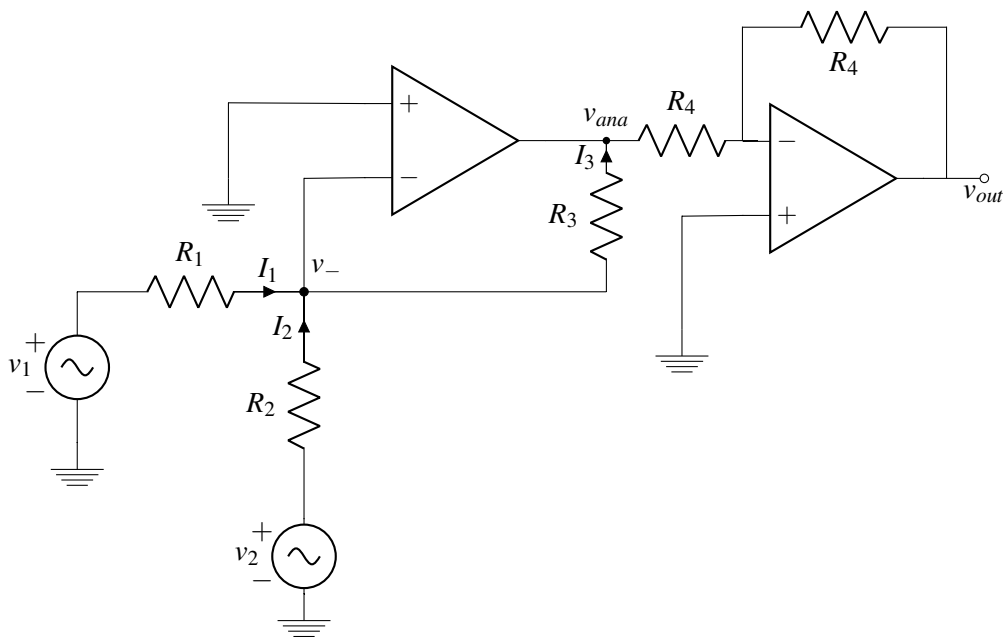
Now we have

$$v_{ana} = -\frac{R_3}{R_1} v_1 - \frac{R_3}{R_2} v_2. \tag{21}$$

For the inverting amplifier just added

$$v_{out} = -\frac{R_4}{R_4} v_{ana}. \tag{22}$$

Hence,

$$v_{out} = \frac{R_3}{R_1} v_1 + \frac{R_3}{R_2} v_2. \tag{23}$$

Now we have positive coefficients $a_1 = \frac{R_3}{R_1}, a_2 = \frac{R_3}{R_2}$.