This homework is due Wednesday, November 14 at 11:59pm
Self grades are due Monday, November 19 at 11:59pm

1. Aliasing

The concept of "aliasing" is intuitively about having a signal of interest whose samples look identical to a different signal of interest — creating an ambiguity as to which signal is actually present.

While the concept of aliasing is quite general, it is easiest to understand in the context of sinusoidal signals.

(a) Consider two signals,

$$x_1(t) = a\cos(2\pi f_0 t + \phi)$$

and

$$x_2(t) = a\cos(2\pi(-f_0 + m f_s)t - \phi)$$

where $f_s = 1/T_s$. Are these two signals the same or different when viewed as functions of continuous time t?

Solution: They are different in the continuous time. x_1 has a frequency of f_0, which is different from the frequency of x_2, which is $f_0 - m f_s$. They would only be the same if $m = 0$, in which case,

$$x_2(t) = a\cos(-2\pi f_0 t - \phi) = a\cos(2\pi f_0 t + \phi) = x_1(t)$$

(b) Consider the two signals from the previous part. These will both be sampled with the sampling interval T_s. For which values of m will the corresponding discrete-time signals $x_{d,1}[n]$ and $x_{d,2}[n]$ be identical? (The $[n]$ refers to the nth sample taken — this is the sample taken at real time nT_s.)?

Solution: Using the fact that the nth sample is taken at $t = nT_s$, we can write out:

$$x_{d,1}[n] = x_1(nT_s) = a\cos(2\pi f_0 n T_s + \phi)$$

and

$$\begin{aligned}
x_{d,2}[n] &= x_2(nT_s) \\
&= a\cos(2\pi(-f_0 + m f_s)nT_s - \phi) \\
&= a\cos(2\pi(-f_0 + m\frac{1}{T_s})nT_s - \phi) \\
&= a\cos(2\pi(-f_0 n T_s + mn) - \phi) \\
&= a\cos(-2\pi f_0 n T_s + 2\pi mn - \phi)
\end{aligned}$$

If m is an integer, then the $2\pi mn$ term can be dropped from the cosine:

$$\begin{aligned}
&= a\cos(2\pi(-f_0)nT_s - \phi) \\
&= a\cos(2\pi f_0 n T_s + \phi) = x_{d,1}[n]
\end{aligned}$$

Therefore, if m is any integer, then for any $t = nT_s$, $x_{d,2}[n] = x_2(nT_s) = x_1(nT_s) = x_{d,1}[n]$

(c) How could you find the sinusoid $a\cos(\omega t + \phi)$ that has the smallest $\omega \geq 0$ but still agrees at all of its samples (taken every T_s seconds) with $x_1(t)$ above?

Solution: As we can see from (b), to match frequencies we can choose any integer value of m for $x_2(t)$ and the corresponding sampled signal will have the same frequency as $x_1(t)$, under the same sampling frequency, f_s. Note however, that since we are free to select only ω and not ϕ, we cannot use the absolute value as in part b. To find the smallest $\omega \geq 0$, we need to find m such that $\omega = 2\pi(f_0 - mf_s)$ is the smallest while still greater than or equal to 0. Notice that we assume f_s and f_0 follow the sampling theorem, which implies $\frac{f_s}{2} > |f_0|$.

(d) Watch the following video: `https://www.youtube.com/watch?v=jQDjJRYmeWg`.

Assume the video camera running at 30 frames per second. That is to say, the camera takes 30 photos within a second, with the time between photos being constant.

Given that the main rotor has 5 blades, list *all* the possible rates at which the main rotor is spinning in revolutions per second assuming no physical limitations.

Hint: Your answer should depend on k where k can be any integer.

Solution: Let the main rotor have a frequency of $\frac{2\pi}{T}$ radians per second. That is to say, it completes one revolution in T seconds, where T is the period of the revolution.

Let $\Delta = \frac{1}{30}s$ be the sample period. Since there are 5 blades, the rotor will look like itself after it finishes a fifth of a revolution, which takes $\frac{T}{5}$ seconds.

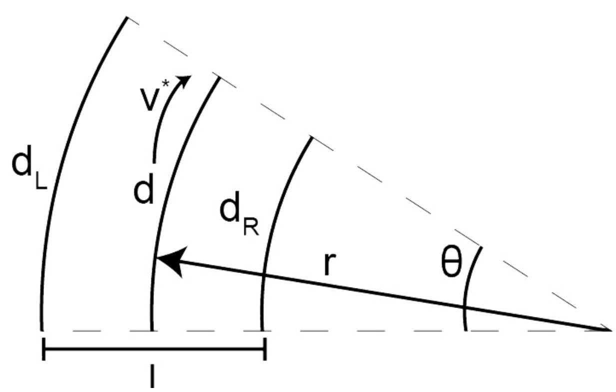
This means that, in one Δ, the rotor could have completed $\frac{T}{5}k$ revolutions, where k is an integer. This means that the rotor could be spinning at $\frac{k}{5\Delta}$ revolutions per second, which is $(6 \times k)Hz$.

2. Turning Via Reference Tracking

We would like the car to turn with a specified radius r and speed v^*. The controller's unit for distance is encoder ticks, but each tick is approximately 1cm of wheel circumference.

To turn, we want δ to change at a particular rate. Without loss of generality, we'll analyze a right turn, corresponding to an increasing δ. For a left turn, we simply negate δ. Our goal is to generate a reference from the desired r and v^* for the controller to follow. This reference will be a function of the controller's time-step.

Inspect the following diagram:



- k - time
- r - turn radius in cm where 1cm $\approx$ 1 encoder tick
- ω - angular velocity
- θ - angle traveled

- d - distance traveled by the center of the car in ticks
- l - distance between the centers of the wheels in cm

From this geometry, can you write $\delta[k]$ in the following form?

$$\delta[k] = f(r, v^*, l, k)$$

Hint: We know from phyiscs (kinematics) that $d[k] = v^* k = \omega r k = r\theta[k]$

Solution:

$$\theta[k] = \frac{v^* k}{r}$$

$$d_L[k] = \left(r + \frac{l}{2}\right)\theta[k]$$

$$d_R[k] = \left(r - \frac{l}{2}\right)\theta[k]$$

$$\delta[k] = d_L[k] - d_R[k] = \left(r + \frac{l}{2} - r + \frac{l}{2}\right)\theta[k]$$

All of this results in:

$$\delta[k] = \frac{v^* l}{r} k$$

This is the desired $\delta[k] = f(r, v^*, l, k)$ for turning the car.

3. **Write Your Own Question And Provide a Thorough Solution.**

   Writing your own problems is a very important way to really learn material. The famous "Bloom's Taxonomy" that lists the levels of learning is: Remember, Understand, Apply, Analyze, Evaluate, and Create. Using what you know to create is the top level. We rarely ask you any homework questions about the lowest level of straight-up remembering, expecting you to be able to do that yourself (e.g. making flashcards). But we don't want the same to be true about the highest level. As a practical matter, having some practice at trying to create problems helps you study for exams much better than simply counting on solving existing practice problems. This is because thinking about how to create an interesting problem forces you to really look at the material from the perspective of those who are going to create the exams. Besides, this is fun. If you want to make a boring problem, go ahead. That is your prerogative. But it is more fun to really engage with the material, discover something interesting, and then come up with a problem that walks others down a journey that lets them share your discovery. You don't have to achieve this every week. But unless you try every week, it probably won't ever happen.

4. **Redo problem 1 of the midterm. (Optional. Required to be eligible for clobber policy)**

   (a)

   (b)

   (c)

   (d)

5. **Redo problem 2 of the midterm. (Optional. Required to be eligible for clobber policy)**

   (a)

(b)

(c)

(d)

(e)

6. Redo problem 3 of the midterm. (Optional. Required to be eligible for clobber policy)

(a)

(b)

7. Redo problem 4 of the midterm. (Optional. Required to be eligible for clobber policy)

(a)

(b)

8. Redo problem 5 of the midterm. (Optional. Required to be eligible for clobber policy)

(a)

(b)