Notes

Supplemental Video: https://www.youtube.com/watch?v=3twLwF2F6CY

Nonhomogeneous Differential Equations

The following differential equation is a nonhomogeneous differential equation:

$$\frac{d^2y}{dt^2}+a_1\frac{dy}{dt}+a_0y=b$$

where b is a constant.

Even though this expression isn't equal to 0, we can still solve it using our method for homogeneous differential equations. If we substitute y with $\tilde{y}=y-\frac{b}{a_0}$, then we end up with a new differential equation that is homogeneous:

$$\frac{d^2\tilde{y}}{dt^2}+a_1\frac{d\tilde{y}}{dt}+a_0\tilde{y}=0$$

Now we can solve for $\tilde{y}$ and then reverse our substitution to get y.

Questions

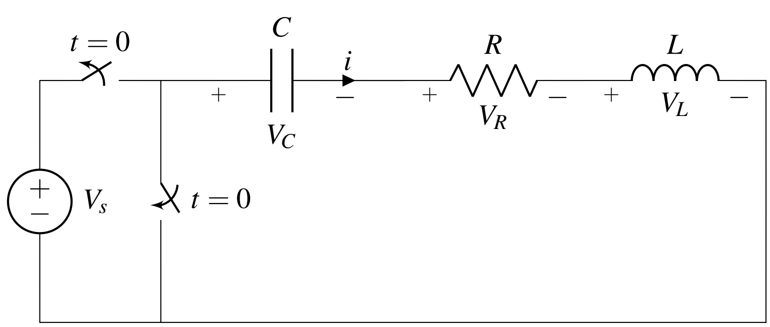
1. **Differential Equations**

   Solve the following second-order differential equation.

   (a) $\frac{d^2y}{dt^2}-4\frac{dy}{dt}+13y=13$, where $y(0)=3$ and $\frac{dy}{dt}(0)=7$

2. **RLC circuit**

   Consider the following circuit:



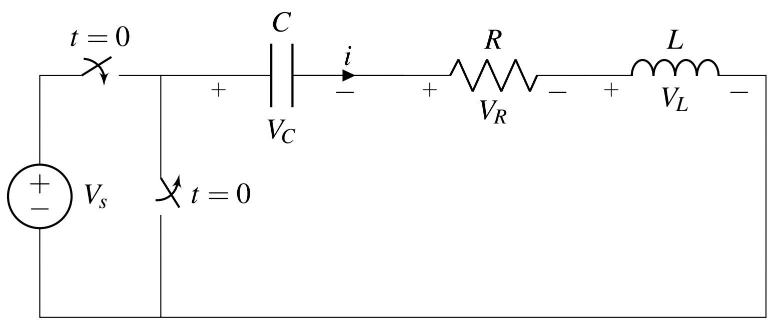
(a) Draw the circuit corresponding to $t < 0$. What are the values of V_C, V_R, V_L, and i at $t = 0_-$, the time right before the switches close. Assume this circuit has been in this state for a long time.

(b) Now draw the circuit corresponding to $t \geq 0$. Using your results from the previous part, what are V_C, V_R, V_L, and i at $t = 0_+$.

(c) Define your state variables as $V_c(t)$ and $i_c(t)$. Find the equation for $V_c(t)$ for $t \geq 0$. Use component values $V_s = 4$V, $C = 2$fF, $R = 60$kΩ, and $L = 1\mu$H.

3. Charging RLC Circuit

Consider the following circuit:



(a) Write out the differential equation describing this circuit for $t \geq 0$ in the form:

$$\frac{d^2V_c}{dt^2} + a_1\frac{dV_c}{dt} + a_0V_c = b$$

(b) Find a $\tilde{V_c}$ and substitute it to the previous equation such that

$$\frac{d^2\tilde{V_c}}{dt^2} + a_1\frac{d\tilde{V_c}}{dt} + a_0\tilde{V_c} = 0$$

(c) Solve for $V_c(t)$ for $t \geq 0$. Use component values $V_s = 4$V, $C = 2$fF, $R = 60$kΩ, and $L = 1\mu$H.

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