EECS 16B Designing Information Devices and Systems II
Fall 2018 Elad Alon and Miki Lustig **Discussion 6B**

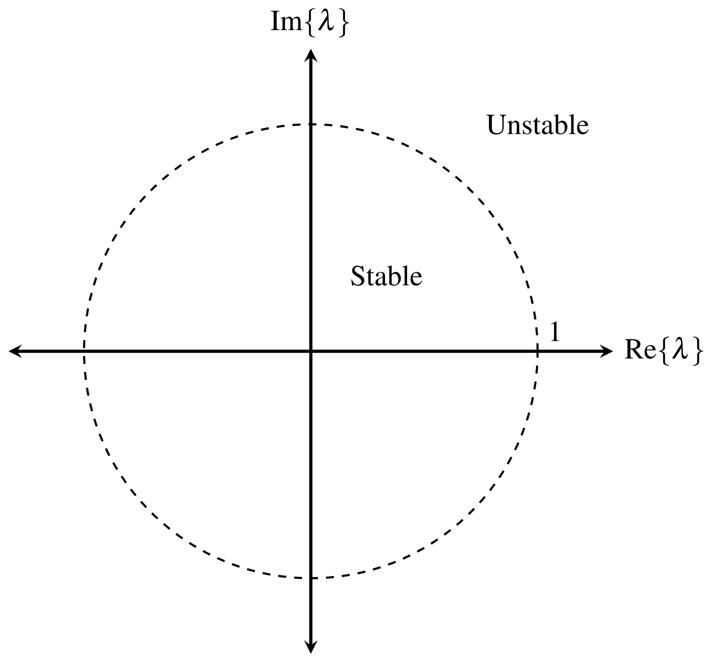
1 Stability

1.1 Discrete time systems

A discrete time system is of the form:

$$\vec{x}[t+1] = A\vec{x}[t] + B\vec{u}[t]$$

Let λ be any particular eigenvalue of A. This system is stable if $|\lambda| < 1$ for all λ. If we plot all λ for A on the real-imaginary axis, if all λ lie within (not on) the unit circle, then the system is stable.

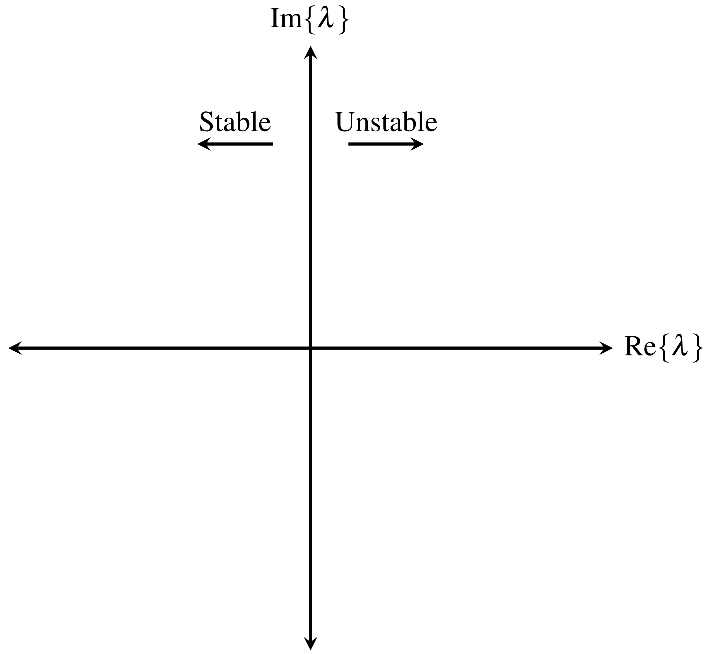


1.2 Continuous time systems

A continuous time system is of the form:

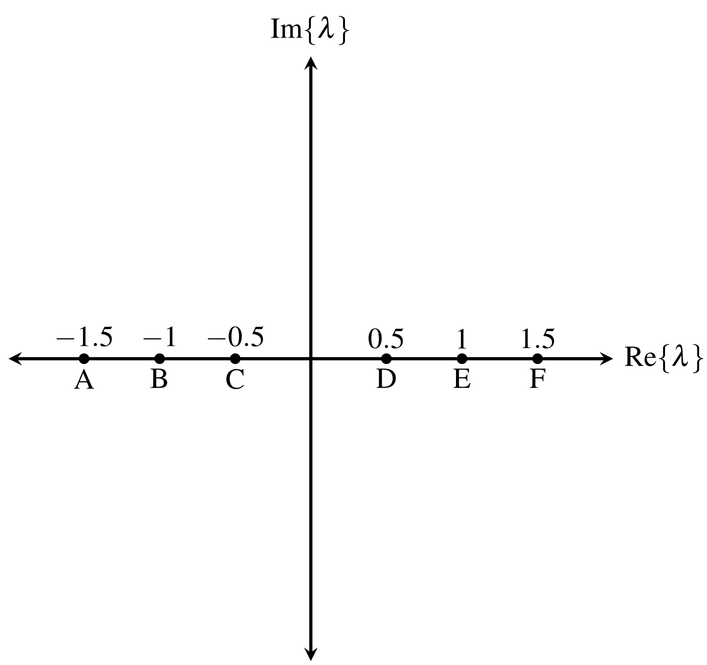
$$\frac{\mathrm{d}\vec{x}}{\mathrm{d}t}(t) = A\vec{x}(t) + B\vec{u}(t)$$

Let λ be any particular eigenvalue of A. This system is stable if $\mathrm{Re}\{\lambda\} < 0$ for all λ. If we plot all λ for A on the real-imaginary axis, if all λ lie to the left of $\mathrm{Re}\lambda = 0$, then the system is stable.



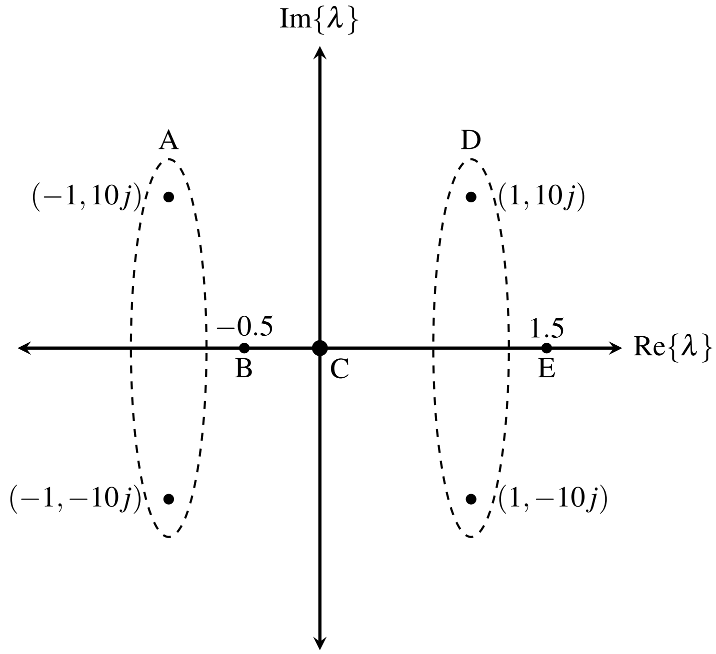
1. Discrete time system responses

We have a system $x[k+1] = \lambda x[k]$. For each λ value plotted on the real-imaginary axis, sketch $x[k]$ with an initial condition of $x[0] = 1$. Determine if each system is stable.



2. Continuous time system responses

We have a system $\frac{d\vec{x}}{dt} = A\vec{x}$ with eigenvalues λ. For each set of λ values plotted on the real-imaginary axis, sketch $\vec{x}(t)$ with an initial condition of $x(0) = 1$. Determine if each system is stable.



3. Discrete-Time Stability

Determine which values of α and β will make the following discrete-time state space models stable:

(a)
$$x[t+1] = \alpha x[t]$$

(b)
$$\vec{x}[t+1] = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix} \vec{x}[t]$$

(c)
$$\vec{x}[t+1] = \begin{bmatrix} 1 & \alpha \\ 0 & 1 \end{bmatrix} \vec{x}[t]$$

4. Linearization and Stability

We have a system:

$$\frac{dx_1(t)}{dt} = x_1(t)x_2(t) - 3$$
$$\frac{dx_2(t)}{dt} = u(t)x_2(t) + 8x_1(t) - x_2(t)x_1(t) - 5$$

(a) Find the equilibrium point of this system when $u(t) = 0$.

(b) Linearize the system around its equilibrium point.

(c) Is the linearized system stable?