EECS 16B Designing Information Devices and Systems II
Fall 2018 Elad Alon and Miki Lustig

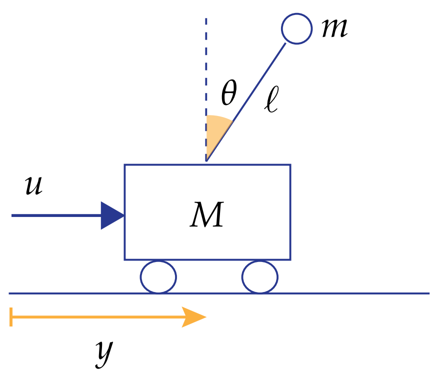
Homework 6

This homework is due on Wednesday, October 10, 2018, at 11:59PM.
Self-grades are due on Monday, October 15, 2018, at 11:59PM.

1. **Inverted Pendulum on a Rolling Cart (Mechanical)**

   Consider the inverted pendulum depicted below, which is placed on a rolling cart and whose equations of motion are given by:

$$\ddot{y} = \frac{1}{\frac{M}{m} + \sin^2\theta}\left(\frac{u}{m} + \dot{\theta}^2 \ell \sin\theta - g\sin\theta\cos\theta\right)$$
$$\ddot{\theta} = \frac{1}{\ell(\frac{M}{m} + \sin^2\theta)}\left(-\frac{u}{m}\cos\theta - \dot{\theta}^2\ell\cos\theta\sin\theta + \frac{M+m}{m}g\sin\theta\right).$$



   (a) Write the state model using the variables $x_1(t) = \theta(t)$, $x_2(t) = \dot{\theta}(t)$, and $x_3(t) = \dot{y}(t)$. We do not include $y(t)$ as a state variable because we are interested in stabilizing at the point $\theta = 0$, $\dot{\theta} = 0$, $\dot{y} = 0$, and we are not concerned about the final value of the position $y(t)$.

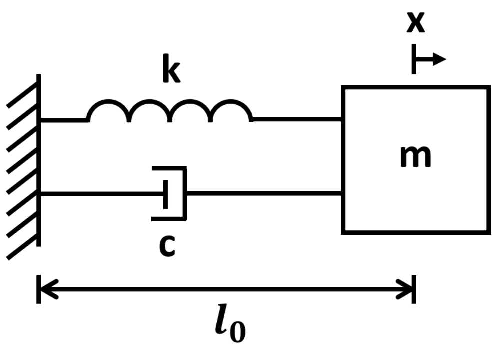
   (b) Linearize this model at the equilibrium $x_1 = 0$, $x_2 = 0$, $x_3 = 0$, and $u = 0$, and indicate the resulting A and B matrices.

2. **Spring and mass (Mechanical)**

   Lets look at a mechanical spring-mass system governed by differential equations similar to those of electrical circuits.

   The total force F acting on a mass can be expressed as $F = ma$, where $a = \frac{\mathrm{d}v}{\mathrm{d}t}$ and $v = \frac{\mathrm{d}x}{\mathrm{d}t}$. Springs generate force according to $F_k = -k\Delta x$ where k is the spring's stiffness and Δx is the displacement of mass from its resting position. We also have a damper which creates a force $F_c = -cv$. We set x to be 0 when the spring is at its rest length l_0 so that $\Delta x = x$. Ignoring gravity, the differential equation describing the motion of the mass is:

$$m\frac{d^2x}{dt^2} + c\frac{dx}{dt} + kx = 0$$



(a) Write the state space model for this system as $\frac{d\vec{x}}{dt} = A\vec{x}$. What is your state vector?

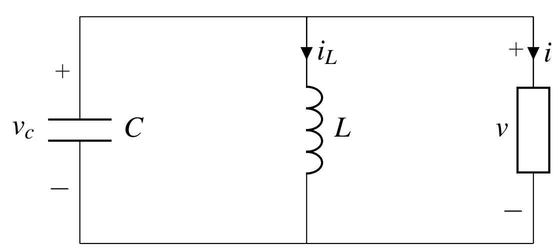
(b) Find the eigenvalues of this system. Is this system stable? Use values $k = 30$N/m, $c = 40$kg/s, and $m = 10$kg. Remember that the standard unit of mass is kg (use the value 10 when plugging in for m, not 10×10^3).

3. Nonlinear circuit component

This is a problem adapted from a past midterm problem (Spring 2017 midterm 2).

Consider the circuit below that consists of a capacitor, inductor, and a third element with a nonlinear voltage-current characteristic:

$$i = 2v - v^2 + 4v^3$$



(a) Write a state space model of the form

$$\frac{dx_1(t)}{dt} = f_1(x_1(t), x_2(t))$$

$$\frac{dx_2(t)}{dt} = f_2(x_1(t), x_2(t))$$

Where $x_1(t) = v_c(t)$ and $x_2(t) = i_L(t)$.

(b) Linearize the state model at the equilibrium point $x_1 = x_2 = 0$ and specify the resulting A matrix.

(c) Is the linearized system stable?

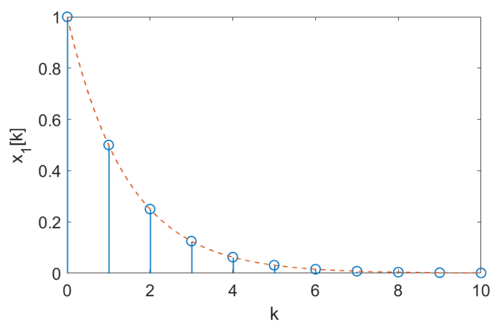
4. Discrete system responses

In many problems, we have an unknown system, and would like to characterize it. One of the ways of doing so is to observe the system response with different initial conditions (or inputs). This problem is also called sytem identification.

As an example, we have an unknown discrete system. The general form of the system can be written as:

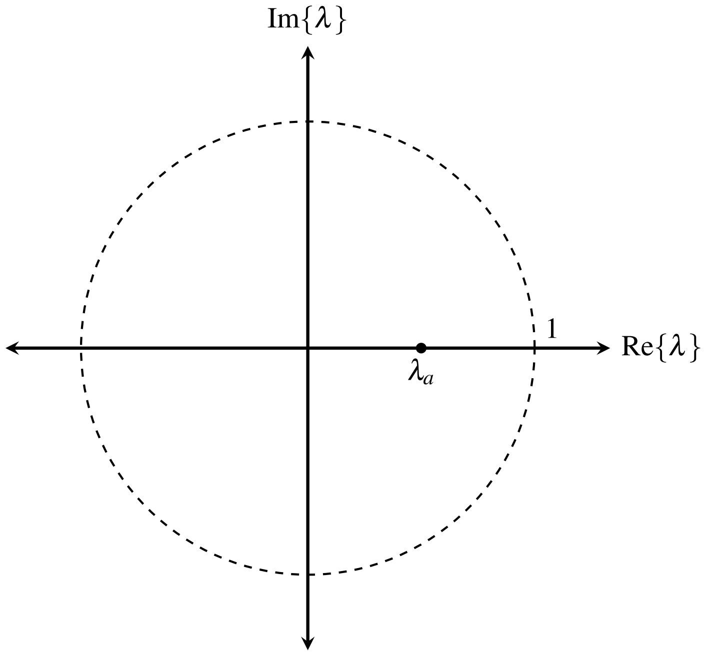
$$\vec{x}[k+1] = A\vec{x}$$

We apply an initial condition $\vec{x}[0] = \vec{x}_a$, where $\vec{x}_a$ is some constant vector. We measure the response of $x_1[k]$ for $k > 0$ which is the first element in $\vec{x}[k]$ and get the following graph:

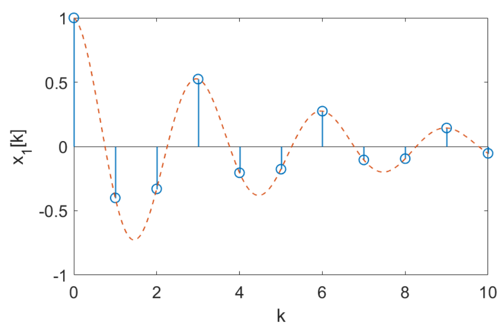


Based on this response, we can estimate the minimum order of the system and approximate the location of the eigenvalue(s) on the real-imaginary plane.

The response shows an exponential decay without oscillation. The minimum order system that could explain this response could be first order. For this case, we can say that the eigenvalue must be real (pure exponential behavior) and has magnitude less than 1 (decaying). Since the decay does not oscillate between positive and negative values, the eigenvalue is also positive. The eigenvalue has been marked at location λ_a

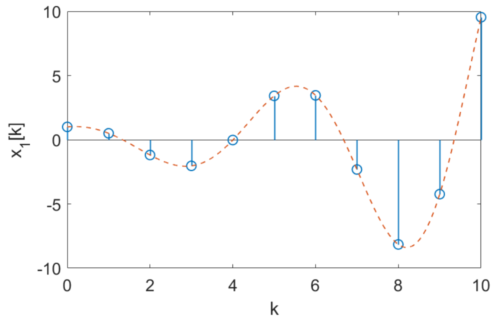


(a) We have a different unknown system, and we apply an initial condition $\vec{x}[0] = \vec{x}_b$. We measure the response $x_1[k]$ for $k > 0$, where $x_1[k]$ is the first element of $\vec{x}[k]$:



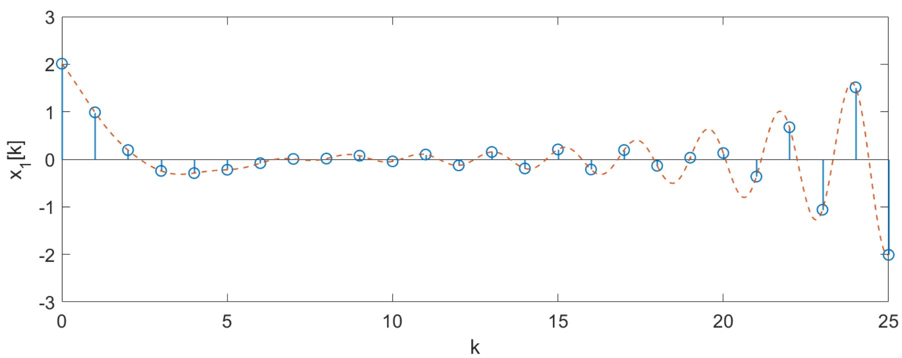
Based on this response, what is the minimum order the system can be? On the real-imaginary plane, plot the approximate location of the eigenvalues that correspond to this response. Your approximation does not need to be exact.

(b) We have an unknown system, and we apply an initial condition $\vec{x}[0] = \vec{x}_c$. We measure the response of $x_1[k]$ for $k > 0$ and get the following graph:



Based on this response, what is the minimum order the system can be? On the real-imaginary plane, plot the approximate location of the eigenvalues that correspond to this response. Your approximation does not need to be exact.

(c) We have an unknown system, and we apply an initial condition $\vec{x}[0] = \vec{x}_d$. We measure the response of $x_1[k]$ for $k > 0$ and get the following graph:



Based on this response, what is the minimum order the system can be? On the real-imaginary plane, plot the approximate location of the eigenvalues that correspond to this response. Your approximation does not need to be exact.

5. Write Your Own Question And Provide a Thorough Solution.

Writing your own problems is a very important way to really learn material. The famous "Bloom's Taxonomy" that lists the levels of learning is: Remember, Understand, Apply, Analyze, Evaluate, and Create. Using what you know to create is the top level. We rarely ask you any homework questions about the lowest level of straight-up remembering, expecting you to be able to do that yourself (e.g. making flashcards). But we don't want the same to be true about the highest level. As a practical matter, having some practice at trying to create problems helps you study for exams much better than simply counting on solving existing practice problems. This is because thinking about how to create an interesting problem forces you to really look at the material from the perspective of those who are going to create the exams. Besides, this is fun. If you want to make a boring problem, go ahead. That is your prerogative. But it is more fun to really engage with the material, discover something interesting, and then come up with a problem that walks others down a journey that lets them share your discovery. You don't have to achieve this every week. But unless you try every week, it probably won't ever happen.