EECS 16B Designing Information Devices and Systems II
Fall 2018 Elad Alon and Miki Lustig

Discussion 8A

Primer to Complex Linear Algebra

Complex Conjugates and Adjoints

The complex conjugate of a complex number $z = a + bj = re^{j\theta}$ is defined as

$$z^* = z^H = a - bj = re^{-j\theta}$$

Let $\vec{z} \in \mathbb{C}^n$.

$$\vec{z} = \begin{bmatrix} z_1 & z_2 & \dots & z_n \end{bmatrix}^T$$

The conjugate transpose (or adjoint, or Hermitian transpose) of $\vec{z}$ is defined as

$$\vec{z}^* = \vec{z}^H = \begin{bmatrix} z_1^* & z_2^* & \dots & z_n^* \end{bmatrix}$$

Inner Product Properties

An inner product on a complex vector space $\mathbb{C}^n$ is a function such that the following all hold for $\vec{u}, \vec{v}, \vec{w} \in \mathbb{C}^n$

$$\begin{aligned}
\langle \vec{u}, \vec{v} \rangle &= \langle \vec{v}, \vec{u} \rangle^* \\
\langle \vec{u} + \vec{v}, \vec{w} \rangle &= \langle \vec{u}, \vec{w} \rangle + \langle \vec{v}, \vec{w} \rangle \\
\langle \alpha \vec{u}, \vec{v} \rangle &= \alpha \langle \vec{u}, \vec{v} \rangle \\
\langle \vec{u}, \vec{u} \rangle &\geq 0 \\
\langle \vec{u}, \vec{u} \rangle &= 0 \implies \vec{u} = \vec{0}
\end{aligned}$$

Questions

1. Controls

Consider the following system:

$$\begin{aligned}
\frac{dx_1(t)}{dt} &= -x_1(t)^2 + x_2(t)u(t) \\
\frac{dx_2(t)}{dt} &= 2x_1(t) - 2x_2(t)u(t)
\end{aligned}$$

(a) Choose states and write a state space model for the system in the form $\frac{d\vec{x}(t)}{dt} = f(\vec{x}(t), u(t))$.

(b) Find the equilibrium $\vec{x}^*$ and input u^* when $x_2^* = 1$ and $u^* = 1$.

(c) Linearize the system around the equilibrium state and input from the previous part. Your answer should be in the form $\frac{d\vec{\tilde{x}}(t)}{dt} = A\vec{\tilde{x}}(t) + B\tilde{u}(t)$.

(d) Is this system controllable? Is it stable?

(e) Find a state feedback controller K to place both system eigenvalues at $\lambda = -1$, where $\tilde{u}(t) = K\vec{\tilde{x}}(t)$.

Extra Practice

1. Feedback Design

Consider the following system:

$$\vec{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$$

$$\vec{f}(\vec{x}, u) = \begin{bmatrix} f_1(t) \\ f_2(t) \end{bmatrix}$$

$$\frac{dx_1(t)}{dt} = f_1(\vec{x}, u) = x_1(t)^2 x_2(t) - 4x_2(t) + u(t)x_2(t)$$

$$\frac{dx_2(t)}{dt} = f_2(\vec{x}, u) = 2x_2(t) - 3x_1(t) - x_1(t)u(t)$$

(a) Find the equilibrium points of $\vec{x}$ when $u(t) = 0$.

(b) Linearize the system around $\vec{x^*} = \begin{bmatrix} 2 & 3 \end{bmatrix}^T$, and $u^*(t) = 0$.

(c) Is the linearized system stable?

(d) Is the linearized system controllable?

(e) Using state feedback with $\tilde{u} = \begin{bmatrix} k_1 & k_2 \end{bmatrix} \vec{\tilde{x}}$, find k_1 and k_2 to make the system stable with $\lambda = -1, -9$.