Homework 12

This homework is due on Friday, December 7 at 11:59 pm. Self grades are due on Monday, December 10 at 11:59 pm.

1. **DFT**

   (a) Compute the DFT coefficients of $x_1[n] = \cos(\frac{2\pi}{6}n)$ where $n \in \{0, 1, \ldots, 5\}$.

   (b) Plot the time domain representation of $\vec{x}_1$. Plot the magnitude, $|X[n]|$, and plot the phase, $\angle X[n]$, for its DFT-basis representation.

   (c) Compute the DFT coefficients of $x_2[n] = \cos(\frac{4\pi}{6}n)$ where $n \in \{0, 1, \ldots, 5\}$.

   (d) Plot the time-domain representation and the magnitude and phase for the DFT-basis representation of $\vec{x}_2$.

   (e) How about the general case, $x_p[n] = \cos(\frac{2\pi}{6}pn)$, where $n \in \{0, 1, \ldots, 5\}$?

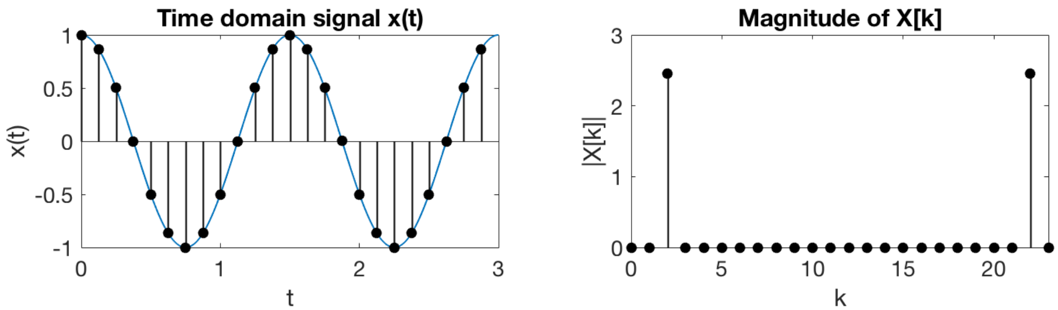
   (f) Compute the DFT coefficients of $\vec{s} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix}^T$.

   (g) Compute the DFT coefficients of $y_1[n] = \cos(\frac{2\pi}{6}n - \pi)$ where $n \in \{0, 1, \ldots, 5\}$.

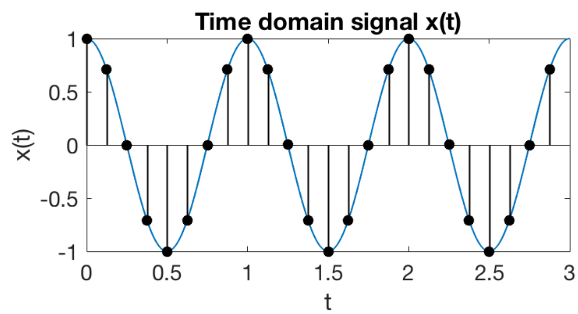
2. DFT Sampling Matching

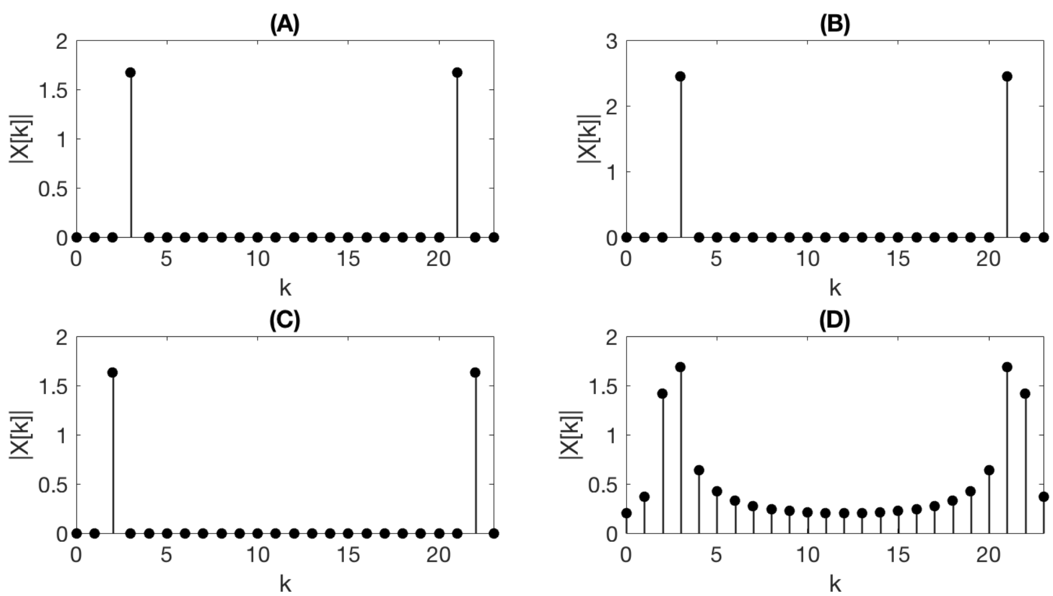
Select the correct answer from the multiple choice options provided and give some justification.

(a) A sampled time domain signal and its DFT coefficients are given below:

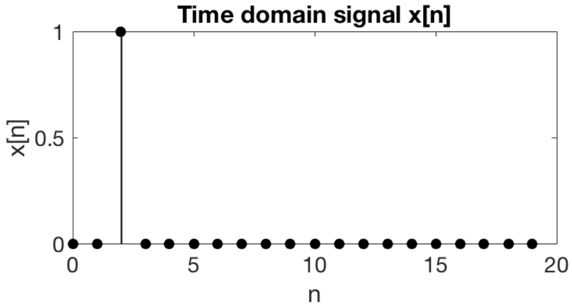


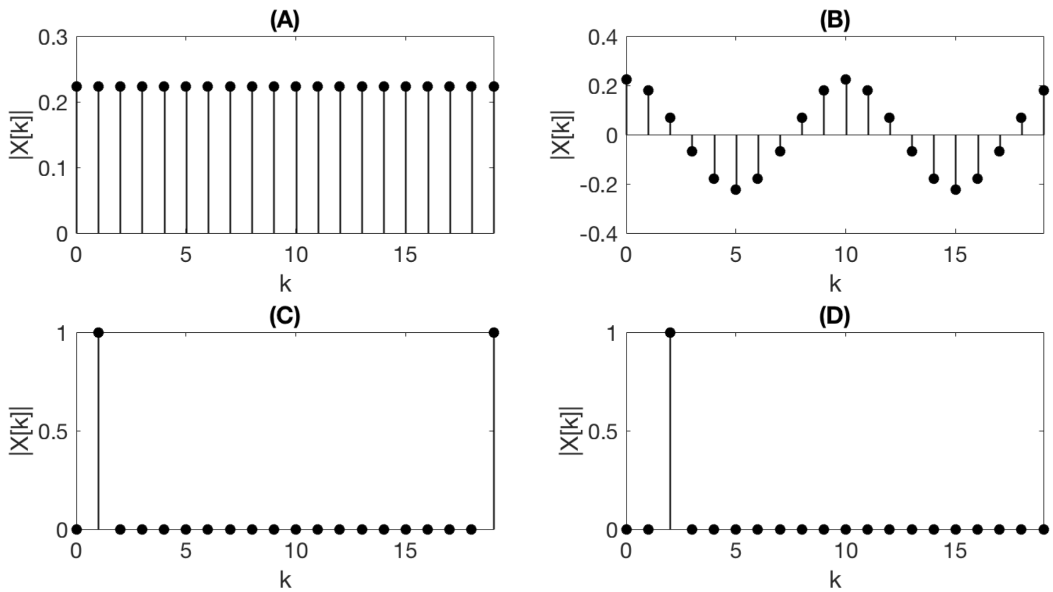
Now given the following time domain signal, which of the options below shows the correct DFT coefficient magnitudes?



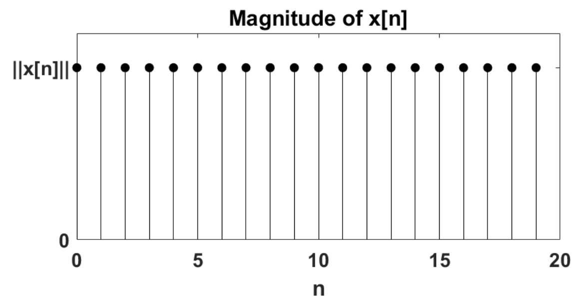


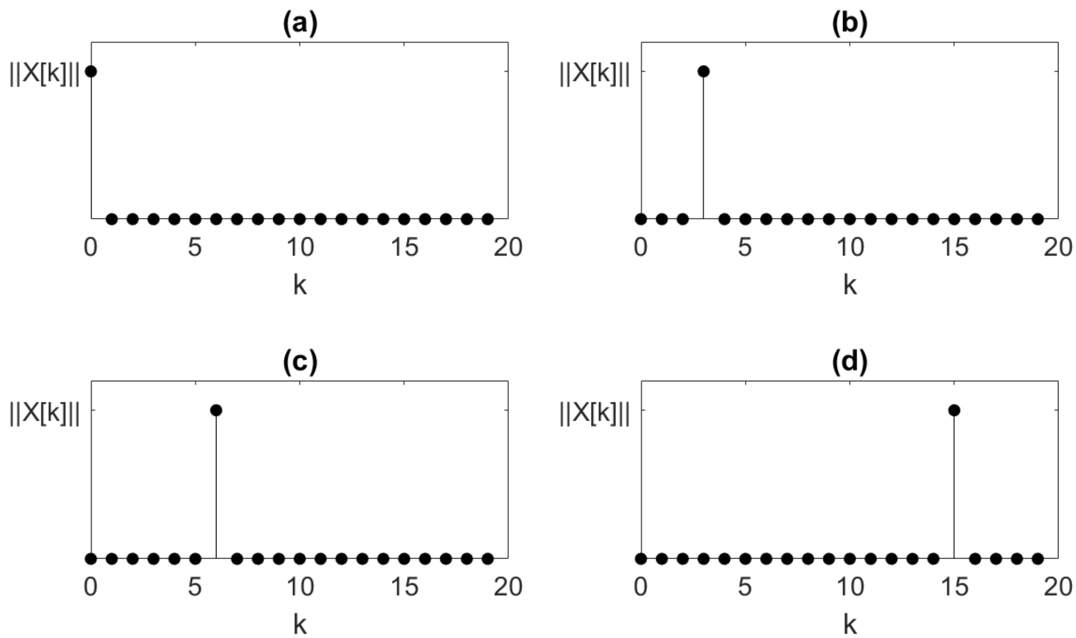
(b) Given the time domain signal below, which of the options below shows the correct DFT coefficient magnitudes?





(c) Given the magnitude of the time domain signal below, which of the options below shows the correct DFT coefficient magnitudes? Hint: this is a trick quesiton!





3. Denoising signals using the DFT

Professor Alon is sad. He just managed to create a beautiful audio clip consisting of a couple pure tones with beats and he wants Professor Lustig to listen to it. He calls Professor Lustig on a noisy phone and plays the message through the phone. Professor Lustig then tells him that the audio is very noisy and that he is unable to truly appreciate the music. Unfortunately, Professor Alon has no other means of letting Professor Lustig listen to the message. Luckily, they have you! You propose to implement a denoiser at Professor Lustig's end.

(a) In the IPython notebook, listen to the noisy message. Plot the time signal and comment on visible structure, if any.

(b) Take the DFT of the signal and plot the magnitude. In a few sentences, describe what the spikes you see in the spectrum are. *Hint: take a look at the documentation for numpy.fft.fft. Note the norm="ortho" option.*

(c) There is a simple method to denoise this signal: Simply threshold in the DFT domain! Threshold the DFT spectrum by keeping the coefficients whose absolute values lie above a certain value. Then take the inverse DFT and listen to the audio. You will be given a range of possible values to test. Write the threshold value you think works best.

Yay, Professor Alon is no longer sad!

4. DFT Exam Question

This question comes from Fall 2017's final exam:

(a) Let $h[n]$, $n = 0, \cdots, 7$ be an $N = 8$ length sequence. Let $H[k]$, $k = 0, \cdots, 7$ be the DFT$_8$ of $h[n]$.

$$H[k] = [1, 0, -1, 0, 1, 0, -1, 0]$$

Use the properties of DFT and compute $X[k]$, which is the DFT$_8$ of x[n], where:

$$x[n] = h[n] + e^{\left(j\frac{2\pi n}{8} + j\frac{\pi}{2}\right)}h[n]$$

(b) Consider a **real** valued sequence $x[n]$, $n = 0, 1, 2, 3$. The norm of this signal: $||x|| = 2$. Let $X[k]$, $k = 0, 1, 2, 3$ be the DFT$_4$ of $x[n]$. We know that $X[0] = 0$, $X[3] = 1 + j$.

Provide one possible set of values for the missing entries $X[1]$, $X[2]$ that satisfy the above constraints. Is the solution unique?

5. Properties of DFT

An N-sample, possibly complex signal $x[n]$ is bounded, so that $|x[n]| < 1$ for all n.

(a) What is the largest value possible for $|X[k]|$, the magnitude of the DFT of $x[n]$?

(b) Find an expression for all the $x[n]$ sequences which achieve this maximum.

6. Inverse DFT

We have a signal $x[n]$, $n = 0, \cdots, 7$. Let $X[k]$,$k = 0, \cdots, 7$ be the DFT$_8$ of $x[n]$.

We know the first five DFT coefficients are $1, 2+2j, j, 6$ and 0. In other words:

$$X[k] = [1, 2+2j, j, 6, 0, ?, ?, ?]$$

If $x[n]$ is purely real, find $x[n]$

7. Write Your Own Question And Provide a Thorough Solution.

Writing your own problems is a very important way to really learn material. The famous "Bloom's Taxonomy" that lists the levels of learning is: Remember, Understand, Apply, Analyze, Evaluate, and Create. Using what you know to create is the top level. We rarely ask you any homework questions about the lowest level of straight-up remembering, expecting you to be able to do that yourself (e.g. making flashcards). But we don't want the same to be true about the highest level. As a practical matter, having some practice at trying to create problems helps you study for exams much better than simply counting on solving existing practice problems. This is because thinking about how to create an interesting problem forces you to really look at the material from the perspective of those who are going to create the exams. Besides, this is fun. If you want to make a boring problem, go ahead. That is your prerogative. But it is more fun to really engage with the material, discover something interesting, and then come up with a problem that walks others down a journey that lets them share your discovery. You don't have to achieve this every week. But unless you try every week, it probably won't ever happen.