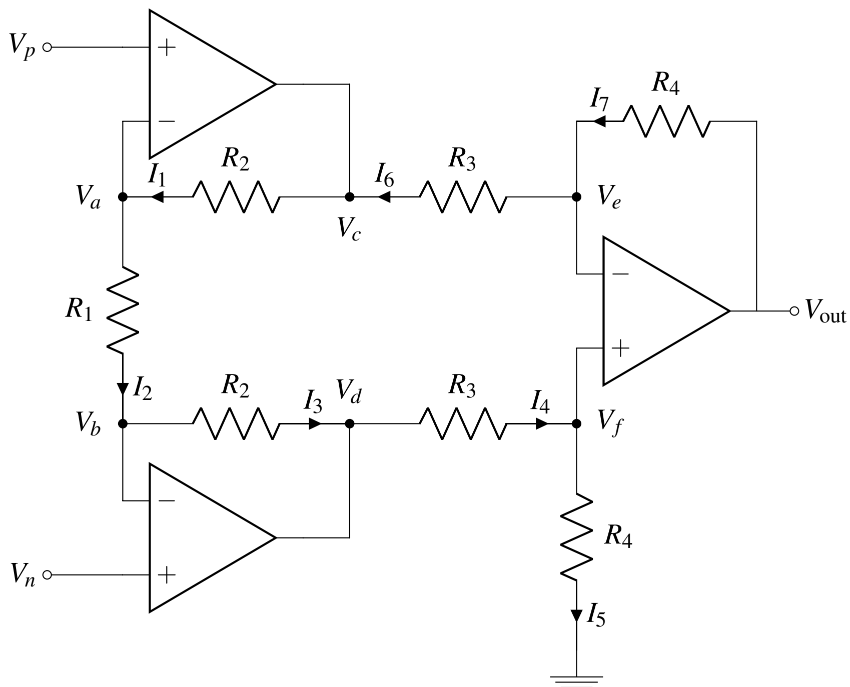
This homework is due on Wednesday, August 29, 2018, at 11:59PM
Self-grades are due on Monday, September 03, 2018, at 11:59PM

Mechanical:

1. Op-Amp Review

Consider the circuit below:



(a) Write the KCL equations at each node (you can skip the nodes V_c and V_d). Use Ohm's law and the golden rules of op-amps to express I_1 through I_7 in terms of voltages and resistances.

Solution:

For an op-amp in negative feedback, the Golden Rules are (1) the voltage difference between the two inputs is zero ($V^+ = V^-$), and (2) no current goes into the inputs of an op-amp.

According to the first Golden Rule, we can write down:

$$V_a = V_p$$
$$V_b = V_n$$
$$V_e = V_f$$

Then we write down the node current equations based on Kirchhoff's Current Law (KCL) (the sum of total currents flowing into one node is the same as the sum of currents flowing out of that same node.):

$$
\begin{aligned}
I_1 = I_2 &\implies \frac{V_c - V_p}{R_2} = \frac{V_a - V_b}{R_1} = \frac{V_p - V_n}{R_1} \\
I_2 = I_3 &\implies \frac{V_a - V_b}{R_1} = \frac{V_p - V_n}{R_1} = \frac{V_n - V_d}{R_2} \\
I_4 = I_5 &\implies \frac{V_d - V_f}{R_3} = \frac{V_f}{R_4} \\
I_6 = I_7 &\implies \frac{V_e - V_c}{R_3} = \frac{V_{\text{out}} - V_e}{R_4}
\end{aligned}
$$

(b) In part (a), we used a general circuit analysis procedure to develop a full set of equations that we could then solve (as we could for any circuit).

However, with this specific circuit, we can make some observations to reduce the amount of necessary calculations. Notice that there exists a symmetry between the two op-amps at the first stage of this circuit. What is the relationship between I_1 and I_3? How do I_1 and I_3 influence I_2?

Solution:

According to the branch current equations: $I_1 = I_2$ and $I_2 = I_3$, so the currents going through the two R_2's are the same but with opposite directions: one is from the output of the op-amp to the inverting input, while the other is from the inverting input to the output. The current through R_1 is the same as the current through the R_2's.

(c) Compute I_2.

Solution:

The current through R_1 is $I_2 = \frac{V_p - V_n}{R_1}$. If $V_p - V_n$ is negative, the current will flow in the opposite direction of what is drawn in the diagram.

(d) Compute V_c and V_d.

Solution:

For the upper op-amp, $V_c = V_a + I_1 R_2$, where $I_1 = \frac{V_p - V_n}{R_1}$, and $V_a = V_p$. Therefore, the output voltage V_c of the upper op-amp is $V_p + \frac{(V_p - V_n)}{R_1} R_2$.

For the lower op-amp, $V_d = V_b - I_3 R_2$, where $I_3 = \frac{V_p - V_n}{R_1}$, and $V_b = V_n$. Therefore, the output voltage V_d of the lower op-amp is $V_n - \frac{(V_p - V_n)}{R_1} R_2$.

(e) Compute V_f.

Solution: From part (a), we know that $\frac{(V_d - V_f)}{R_3} = \frac{V_f}{R_4}$. Hence, we could express V_f with V_d as follows:

$$V_f = \frac{R_4}{R_3 + R_4} V_d$$

and plug in the value of V_d we computed in (d), $V_d = V_n - \frac{(V_p - V_n)}{R_1} R_2$:

$$V_f = \frac{R_4}{R_3 + R_4} \left(V_n - \frac{(V_p - V_n)}{R_1} R_2 \right).$$

(f) Compute V_{out}.

Solution:

There are two ways to compute V_{out}: (1) use all known values to derive the answer, or (2) start with V_c and V_d as inputs (free variables) first, and then plug in the values of V_c and V_d in the end of computation. Here we will show you (1), and in part (h), you will see (2).

From part (a) (Golden Rules), we know that $V_e = V_f$, and from part (e), we derived $V_f = \frac{R_4}{R_3+R_4}\left(V_n - \frac{(V_p-V_n)}{R_1}R_2\right)$.

Also, from part (d), we showed that $V_c = V_p + \frac{(V_p-V_n)}{R_1}R_2$.

From part (a), we know that $\frac{V_e-V_c}{R_3} = \frac{V_{\text{out}}-V_e}{R_4}$. We can express V_{out} in terms of V_e and V_c:

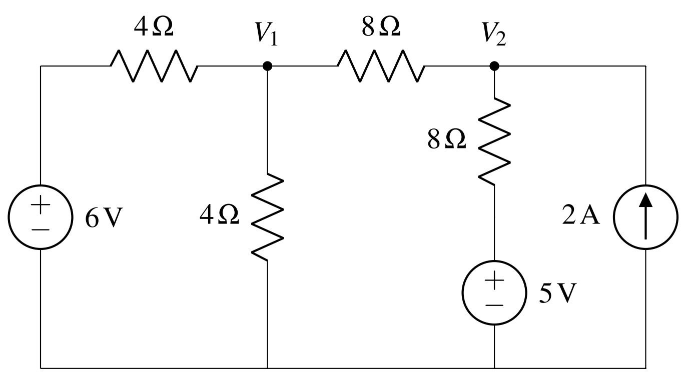
$$V_{\text{out}} = \left(1+\frac{R_4}{R_3}\right)V_e - \frac{R_4}{R_3}V_c$$

After plugging in the values for V_c and V_e, we get V_{out}:

$$\begin{aligned}
V_{\text{out}} =& \left(1+\frac{R_4}{R_3}\right)\frac{R_4}{R_3+R_4}\left(V_n - \frac{(V_p-V_n)}{R_1}R_2\right) - \frac{R_4}{R_3}\left(V_p + \frac{(V_p-V_n)}{R_1}R_2\right)\\
=&\frac{R_4}{R_3}\left(V_n - V_p - \frac{2R_2(V_p-V_n)}{R_1}\right)\\
=&(V_n-V_p)\frac{R_4}{R_3}\left(1+\frac{2R_2}{R_1}\right)
\end{aligned}$$

2. More practice

Consider the circuit shown below:



Compute V_1 and V_2.

Solution:

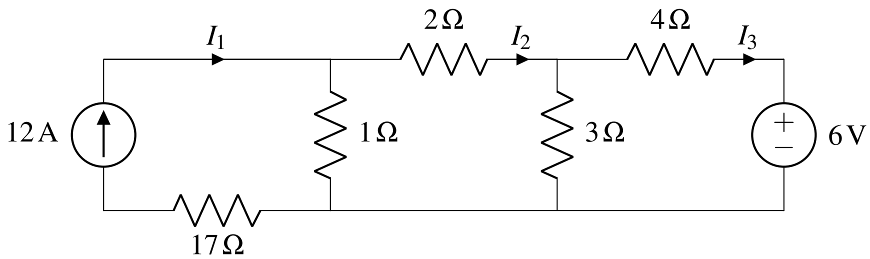
Using KCL on V_1, V_2 yields the equations

$$\begin{aligned}
0 &= \frac{V_1-6}{4} + \frac{V_1}{4} + \frac{V_1-V_2}{8}\\
0 &= \frac{V_2-V_1}{8} + \frac{V_2-5}{8} - 2
\end{aligned}$$

Solving for the two variables in the two equations yields $V_1 = 5\,\text{V}, V_2 = 13\,\text{V}$

3. **OPTIONAL: Even more practice**

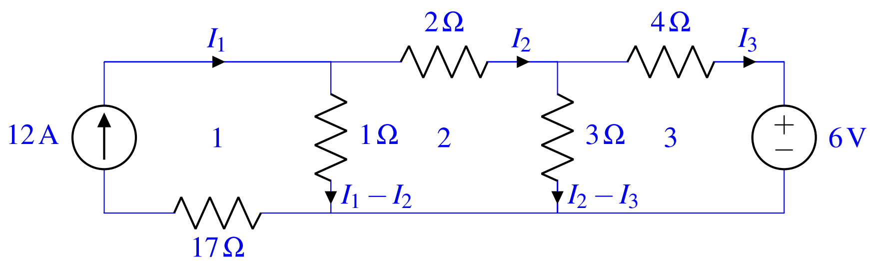
Consider the circuit shown below:



Determine the amount of power supplied by the voltage source. Do not use superposition.

Solution:

We will label the currents I_1, I_2, and I_3 as shown in the following diagram.



$$\begin{cases} \text{Loop 1:} & I_1 = 12\,\mathrm{A} \\ \text{Loop 2:} & (I_2 - I_1) + 2I_2 + 3(I_2 - I_3) = 0 \\ \text{Loop 3:} & 3(I_3 - I_2) + 4I_3 + 6 = 0 \end{cases}$$

Simplification leads to:

$$\begin{cases} 6I_2 - 3I_3 = 12 \\ -3I_2 + 7I_3 = -6 \end{cases}$$

Solving this system of equations gives:

$$I_2 = 2\,\mathrm{A} \qquad I_3 = 0\,\mathrm{A}$$

The power supplied by the voltage source is:

$$P = VI = 6 \cdot (0) = 0\,\mathrm{W}$$

4. **Write Your Own Question And Provide a Thorough Solution.**

Writing your own problems is a very important way to really learn material. The famous "Bloom's Taxonomy" that lists the levels of learning is: Remember, Understand, Apply, Analyze, Evaluate, and Create. Using what you know to create is the top level. We rarely ask you any homework questions about the lowest level of straight-up remembering, expecting you to be able to do that yourself (e.g. making flashcards). But we don't want the same to be true about the highest level. As a practical matter, having some practice at trying to create problems helps you study for exams much better than simply counting on solving existing practice problems. This is because thinking about how to create an interesting problem forces you to really look at the material from the perspective of those who are going to create the exams. Besides, this is fun. If you want to make a boring problem, go ahead. That is your prerogative. But it is more fun to really engage with the material, discover something interesting, and then come up with a problem that walks others down a journey that lets them share your discovery. You don't have to achieve this every week. But unless you try every week, it probably won't ever happen.