EECS 16B Designing Information Devices and Systems II
Fall 2018 Elad Alon and Miki Lustig

Homework 4

This homework is solely for your own practice. However, everything on it is in scope for midterm 1, and it will be assumed in lab that you have completed the lab-related questions.

1. Mystery Microphone

You are working for Mysterious Miniature Microphone Multinational when your manager asks you to test a batch of the company's new microphones. You grab one of the new microphones off the shelf, use a tone generator [1] to play pure tones of uniform amplitude at various frequencies, and measure the resultant peak-to-peak voltages using an oscilloscope. You collect data, and then plot it (on a logarithmic scale). The plot is shown below:

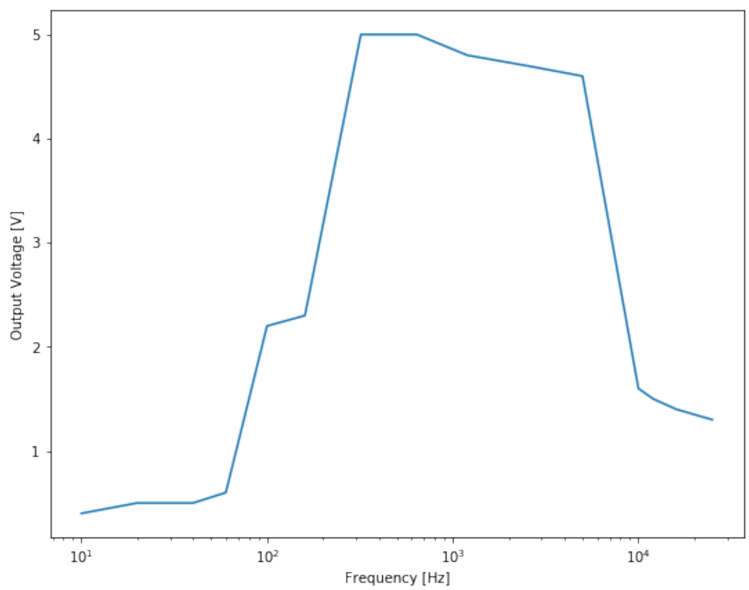


Figure 1: Frequency Response

(a) To which frequencies is the microphone most sensitive, and to which frequencies is the microphone least sensitive?

You report these findings to your manager, who thanks you for the preliminary data and proceeds to co-ordinate some human listener tests. In the meantime, your manager asks you to predict the effects of the microphone recordings on human listeners, and encourages you to start thinking more deeply about the relationships.

[1] Note that soundwaves are simply sinusoids at various frequencies with some amplitude and phase. The microphone's diaphragm oscillates with the sound (pressure) waves, moving the attached wire coil back and forth over an internal magnet, which induces a current in the wire. In this way, a microphone can be modeled as a signal-dependent current source. The output current can be converted to a voltage by simply adding a known resistor to the circuit and measuring the voltage across that resistor.

(b) For testing purposes, you have a song with sub-bass (150 Hz or less), mid-range (≈ 1 kHz), and some high frequency electronic parts (> 12 kHz). Which frequency ranges of the song would you be able to hear easily, and which parts would you have trouble hearing? Why?

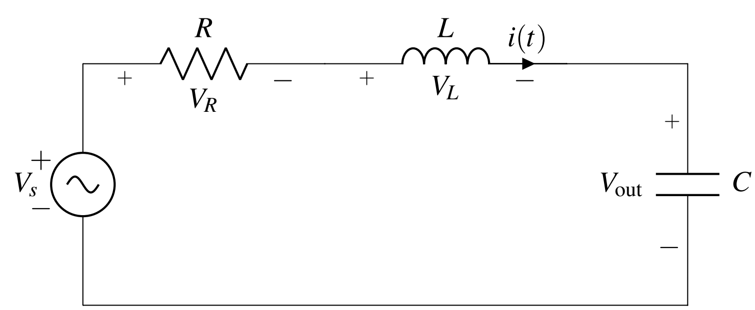
(c) After a few weeks, your manager reports back to you on the findings. Apparently, this microphone causes some people's voices to sound really weird, resulting in users threatening to switch to products from a competing microphone company.

It turns out that we can design some filters to "fix" the frequency response so that the different frequencies can be recorded more equally, thus avoiding distortion. Imagine that you have a few (say up to 4 or so) blocks. Each of these blocks detects a set range of frequencies, and if the signal is within this range, it will switch on a op-amp circuit of your choice. For example, it can be configured to switch on an op-amp filter to double the voltage for signals between 100 Hz and 200 Hz.

What ranges of signals would require such a block, and what gain would you apply to each block such that the resulting peak-to-peak voltage is about 5 V for all frequencies?

2. RLC Circuit

In this question, we will take a look at an electrical system described by a second order differential equations and analyze it using the phasor domain. Consider the circuit below, where $R = 8\,\text{k}\Omega$, $L = 1\,\text{mH}$, $C = 200\,\text{nF}$, and $V_s = 2\cos\left(2000t + \frac{\pi}{4}\right)$.



(a) What are the impedances of the resistor Z_R, inductor Z_L, and capacitor Z_C?

(b) Solve for $\widetilde{V}_{\text{out}}$ in phasor form.

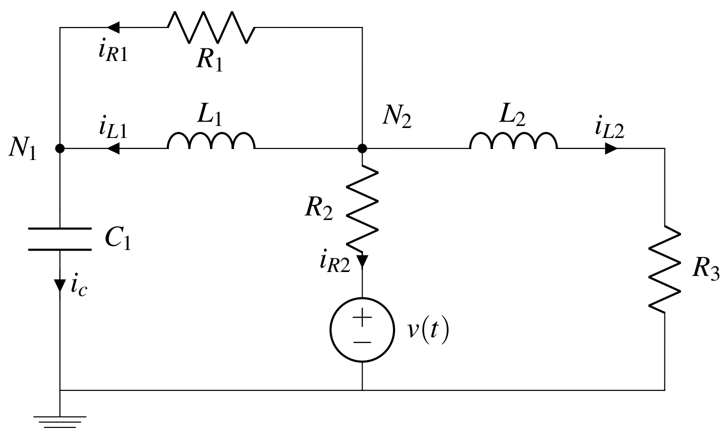
(c) What is V_{out} in the time domain?

(d) Solve for the current $i(t)$.

(e) Solve for the transfer function $H(\omega) = \frac{\widetilde{V}_{\text{out}}}{\widetilde{V}_s}$
Leave your answer in terms of R, L, C, and ω.

3. Phasor-Domain Circuit Analysis

The analysis techniques you learned previously for resistive circuits are equally applicable for analyzing AC circuits (circuits driven by sinusoidal inputs) in the phasor domain. In this problem, we will walk you through the steps with a concrete example. Consider the circuit below.



The components in this circuit are given by:

Voltage source:

$$v(t) = 10\sqrt{2}\cos\left(100t - \frac{\pi}{4}\right)$$

Resistors:

$$R_1 = 5\,\Omega, \quad R_2 = 5\,\Omega, \quad R_3 = 1\,\Omega$$

Inductors:

$$L_1 = 50\,\text{mH}, \quad L_2 = 20\,\text{mH}$$

Capacitor:

$$C_1 = 2\,\text{mF}$$

(a) Transform the given circuit to the phasor domain (components and sources).

(b) Write out KCL for node N_1 and N_2 in the phasor domain in terms of the currents provided.

(c) Find expressions for each current in terms of node voltages in the phasor domain. The node voltages $\widetilde{V}_1$ and $\widetilde{V}_2$ are the voltage drops from N_1 and N_2 to the ground.

(d) Write the equations you derived in part (c) in a matrix form, i.e., $\mathbf{A}\begin{bmatrix}\widetilde{V}_1\\ \widetilde{V}_2\end{bmatrix} = \vec{b}$. Write out $\mathbf{A}$ and $\vec{b}$ numerically.

(e) Solve the systems of linear equations you derived in part (d) with any method you prefer and then find $i_c(t)$.

4. Analyzing Mic Board Circuit

In this problem, we will work up to analyzing a simplified version of the mic board circuit. In lab, we will address the minor differences between the final circuit in this problem and the actual mic board circuit.

The microphone can be modeled as a frequency-dependent current source, $I_{MIC} = k\sin(\omega t) + I_{DC}$, where I_{MIC} is the current generated by the mic (which flows from VDD to VSS), I_{DC} is some constant current, k is the force [2] to current conversion ratio, and ω is the signal's frequency (in $\frac{\text{rad}}{\text{s}}$). VDD and VSS are $5\,\text{V}$ and $-5\,\text{V}$, respectively.

[2] The force is exerted by the soundwaves on the mic's diaphragm.

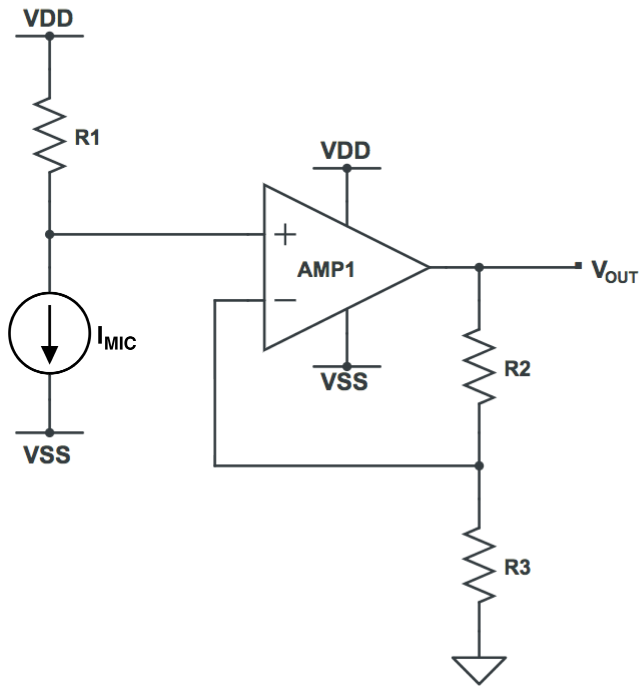


Figure 2: Step 1. The microphone is modeled as a DC current source.

(a) **DC Analysis** Assume for now that $k = 0$ (so that we can examine just the "DC" response of the circuit), find V_{OUT} in terms of I_{DC}, R_1, R_2, and R_3 (Hint: You do not need to worry about V_{ss} in your calculations).

(b) Now, let's include the sinusoidal part of I_{MIC} as well. We can model this situation as shown below, with I_{MIC} split into two current sources so that we can analyze the whole circuit using superposition. Let $I_{\mathrm{AC}} = k\sin(\omega t)$. Find and plot the function $V_{\mathrm{OUT}(t)}$.

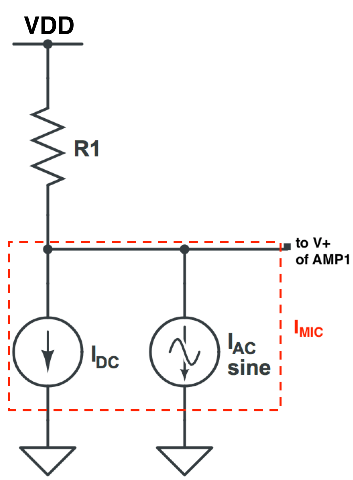


Figure 3: Step 2. The microphone is modeled as the superposition of a a DC and a sinusoidal ("AC") current source.

(c) Given that $V_{DD} = 5\,\mathrm{V}$, $V_{SS} = -5\,\mathrm{V}$, $R_1 = 10\,\mathrm{k}\Omega$, and $I_{\mathrm{DC}} = 10\,\mu\mathrm{A}$, find the maximum value of the gain G of the noninverting amplifier circuit for which the op-amp would not need to produce voltages greater than VDD or less than VSS (i.e, find the maximum gain G we can use without causing the op-amp to clip).

(d) We have modified the circuit as shown below to include a high-pass filter so that the term related to I_{DC} is removed before we apply gain to the signal. Provide a symbolic expression for V_{OUT} given that that $\text{VDD}_0 = 5\,\text{V}$, $\text{VSS}_0 = -5\,\text{V}$, $\text{VDD}_1 = 3.3\,\text{V}$, $\text{VSS}_1 = 0\,\text{V}$. Show your work.

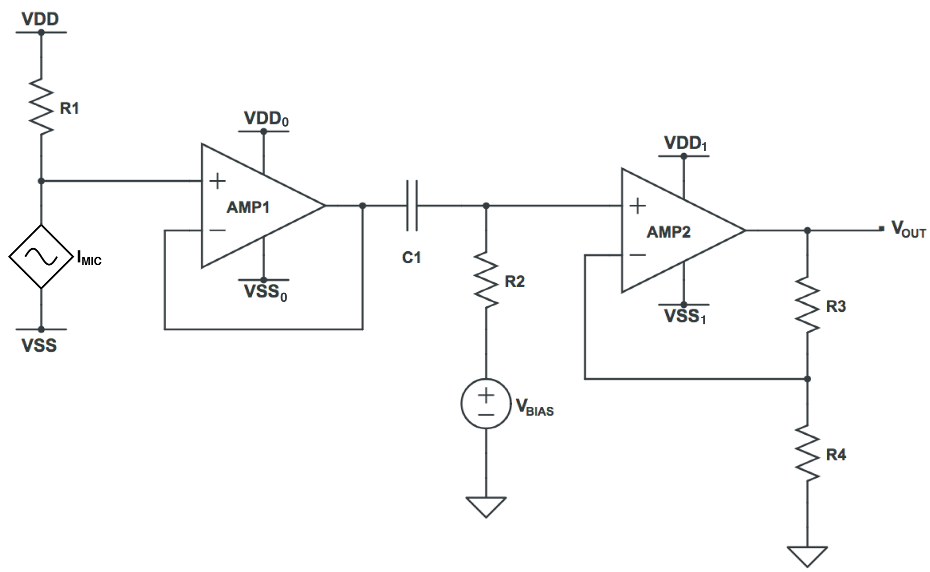


Figure 4: Step 3. Approaching the real mic board circuit. The microphone is still modeled as the superposition of a a DC and a sinusoidal ("AC") current source.

(e) We would now like to choose V_{BIAS} so that we can get as much gain G out of the non-inverting amplifier circuit (AMP2) as possible without causing AMP2 to clip (i.e, the output of AMP2 must stay between 0V and 3.3V). What value of V_{BIAS} will achieve this goal? If $k = 10^{-5}$ and $R_1 = 10\,\text{k}\Omega$, what is the maximum value of G you can use without having AMP2 clip?

5. Color Organ Filter Design

In the fourth lab, we will design low-pass, band-pass, and high-pass filters for a color organ. There are red, green, and blue LEDs. Each color will correspond to a specified frequency range of the input audio signal. The intensity of the light emitted will correspond to the amplitude of the audio signal.

(a) First, you realize that you can build simple filters using a resistor and a capacitor. Design the first-order **passive** low and high pass filters with following frequency ranges for each filter using 1 µF capacitors. ("Passive" means that the filter does not require any power supply.)

- Low pass filter – 3-dB frequency at 2400 Hz = $2\pi \cdot 2400 \frac{rad}{sec}$
- High pass filter – 3-dB frequency at 100 Hz = $2\pi \cdot 100 \frac{rad}{sec}$

Draw the schematic-level representation of your designs and show your work finding the resistor values. Also, please mark V_{in}, V_{out}, and ground nodes in your schematic. Round your results to two significant figures.

(b) You decide to build a bandpass filter by simply cascading the first-order low-pass and high-pass filters you designed in part (a). Connect the V_{out} node of your low-pass filter directly to the V_{in} node of your high pass filter. The V_{in} of your new band-pass filter is the V_{in} of your old low-pass filter, and the V_{out} of the new filter is the V_{out} of your old high-pass filter. What is H_{BPF}, the transfer function of your new band-pass filter? Use R_L, C_L, R_H, and C_H for low-pass filter and high-pass filter components, respectively. Show your work.

(c) Plug the component values you found in (a) into the transfer function H_{BPF}. Using MATLAB or IPython, draw a Bode plot from 0.1 Hz to 1 GHz. If you use iPython, you may find the function `scipy.signal.bode` useful. What are the frequencies of the poles and zeros? What is the maximum magnitude of H_{BPF} in dB? Is that something that you want? If not, explain why not and suggest a simple way (either adding passive or active components) to fix it.

(d) Now that you know how to make filters and amplifiers, we can finally build a system for the color organ circuit below. Before going into the actual schematic design, you must first set specifications for each block. The goal of the circuit is to divide the input signal into three frequency bands and turn the LEDs on based on the input signal's frequency.

In this problem, assume that the mic board is a 3-pole 2-zero system. Poles are located at 10 Hz, 100 Hz, and 10 000 Hz. Zeros are at DC and 200 Hz. This means that the frequency response at the mic board output can be modeled as follows.

$$V_{MIC} = K_{MIC} \frac{j\omega\left(1+\frac{j\omega}{\omega_{z1}}\right)}{\left(1+\frac{j\omega}{\omega_{p1}}\right)\left(1+\frac{j\omega}{\omega_{p2}}\right)\left(1+\frac{j\omega}{\omega_{p3}}\right)}$$

where K_{MIC} is a constant gain, ω_{z1}, ω_{p1}, ω_{p2}, and ω_{p3} are the zero and poles. Note that $j\omega$ term in the numerator denotes the zero at DC. Also note that poles are always in $\frac{rad}{sec}$: for example, $\omega_{p1} = 2\pi \cdot 10\text{Hz}$. The magnitude of the voltage at the mic board output is 1 V peak-to-peak at 40 Hz. (*Hint:* You can use this information to calculate K_{MIC}.)

Suppose that the three filters have transfer functions as below.

- Low pass filter

$$H_{LPF} = \frac{2}{1+\frac{j\omega}{200\pi}}$$

- Band pass filter

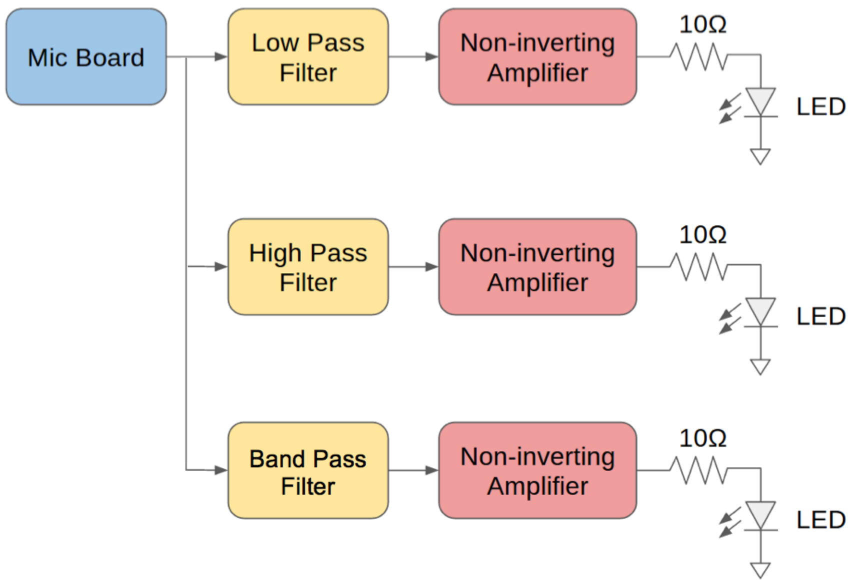
$$H_{BPF} = \frac{4.54 \cdot 10^{-4} j\omega}{\left(1+\frac{j\omega}{400\pi}\right)\left(1+\frac{j\omega}{4000\pi}\right)}$$

- High pass filter

$$H_{HPF} = \frac{\frac{j\omega}{8000\pi}}{1+\frac{j\omega}{8000\pi}}$$

What are the phasor voltages at the output of each filter as a function of ω? To clarify, $\frac{3(1+j\omega(1.5\cdot 10^3))}{1+j\omega(2\cdot 100)}$ would be a valid phasor voltage at the output of some filter. Assume that there are ideal voltage buffers before and after each filter.

(e) For 50 Hz, 1000 Hz, and 8000 Hz, what is the voltage gain required of each non-inverting amplifier such that the output peak to peak voltage measured right before the 10 Ω resistor is 5 V_{pp}?



Mic Board
Low Pass Filter
Non-inverting Amplifier
10Ω
LED
High Pass Filter
Non-inverting Amplifier
10Ω
LED
Band Pass Filter
Non-inverting Amplifier
10Ω
LED