EECS 16B Designing Information Devices and Systems II
Fall 2018 Elad Alon and Miki Lustig **Homework 5**

This homework is due on Wednesday, October 3, 2018, at 11:59PM. Self-grades are due on Monday, October 8, 2018, at 11:59PM.

1. Dirty Dishes

I am a trip planner who lodges travellers at Bob's Bed and Breakfast. At the beginning of each day, Bob will do half of the dirty dishes in the sink. During the day, each of his guests will use 4 pounds of dishes minus an eighth pound of dishes for each pound of dishes already in the sink at the beginning of the day (as Bob's kitchen gets too messy).

(a) What is the state vector for Bob's kitchen sink system? What are the input? Write out the state space model.

Solution:

The dishes in the sink are the state variable x. The number of guests are the input u.

$$x[t+1] = \frac{1}{2}x[t] + \left(4 - \frac{1}{8}x[t]\right)u[t]$$

(b) Is Bob's kitchen sink a linear system? If it is, write it in the form $\frac{d}{dt}\vec{x} = A\vec{x} + B\vec{u}$. If it isn't, write out why it is not.

Solution:

No, the state variable is multiplied by the input.

(c) On Wednesday morning (before Bob gets up), there are 8 pounds of dishes in the sink. On Wednesday Bob has 8 guests and on Thursday he has 4 guests. How many pounds of dishes are in the sink after Thursday?

Solution:

$$x[1] = \frac{1}{2}(8) + \left(4 - \frac{1}{8}(8)\right)(8) = 28$$
$$x[2] = \frac{1}{2}(28) + \left(4 - \frac{1}{8}(28)\right)(4) = 16$$

(d) I am a very eccentric trip planner and I want Bob to have exactly 24 pounds of dishes in his sink. He has 16 pounds of dishes in his sink. How many guests should I lodge at Bob's Bed and Breakfast today? How many guests should I lodge tomorrow?

Solution:

$$24 = \frac{1}{2}(16) + \left(4 - \frac{1}{8}(16)\right)u[0]$$
$$u[0] = 8$$

$$24 = \frac{1}{2}(24) + \left(4 - \frac{1}{8}(24)\right)u[1]$$
$$u[1] = 12$$

2. **Eigenvalues and State Space Systems**

(a) Let's assume we have a system whose dynamics are described by

$$\begin{bmatrix} x_0[k+1] \\ x_1[k+1] \end{bmatrix} = A \begin{bmatrix} x_0[k] \\ x_1[k] \end{bmatrix}$$
$$A = \begin{bmatrix} 0.7 & 0.96 \\ 1 & 0.3 \end{bmatrix}$$

Find the eigenvalues and eigenvectors of the transition matrix A.

Solution:

$$\det(\lambda I - A) = (\lambda - 0.7)(\lambda - 0.3) - (1)(0.96) = 0$$
$$\lambda^2 - 1\lambda - 0.75 = 0$$

Solve with the quadratic formula:

$$\lambda = \frac{1 \pm \sqrt{1^2 - 4(1)(-0.75)}}{2} = \lambda = 0.5 \pm 1$$
$$\lambda_1 = 1.5$$
$$\lambda_2 = -0.5$$

Next, we find the eigenvector for λ_1:

$$Av_1 = \lambda_1 v_1$$
$$\begin{bmatrix} 0.7 & 0.96 \\ 1 & 0.3 \end{bmatrix} \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix} = 1.5 \begin{bmatrix} v_{11} \\ v_{12} \end{bmatrix}$$

Looking at the bottom row:

$$v_{11} + 0.3v_{12} = 1.5v_{12}$$
$$v_{11} = 1.2v_{12}$$
$$v_1 = \begin{bmatrix} 1.2 \\ 1 \end{bmatrix}$$

Solving for the eigenvector of λ_2:

$$Av_2 = \lambda_2 v_2$$
$$\begin{bmatrix} 0.7 & 0.96 \\ 1 & 0.3 \end{bmatrix} \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix} = -0.5 \begin{bmatrix} v_{21} \\ v_{22} \end{bmatrix}$$

Looking at the bottom row:

$$v_{21} + 0.3v_{22} = -0.5v_{22}$$
$$v_{21} = -0.8v_{22}$$
$$v_2 = \begin{bmatrix} -0.8 \\ 1 \end{bmatrix}$$

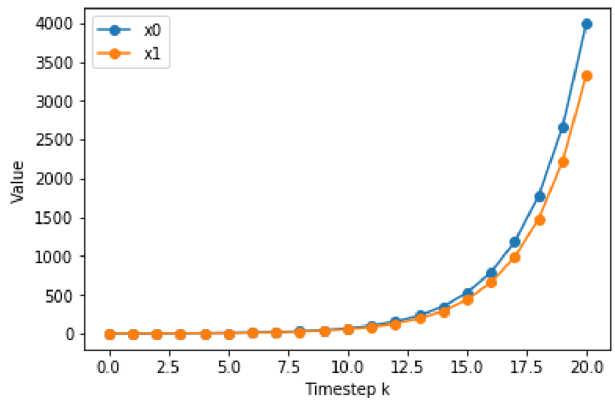
(b) Use IPython to simulate the system from part (a) with an initial condition of:

$$\begin{bmatrix} x_0[0] \\ x_1[0] \end{bmatrix} = \vec{v}_1,$$

where $\vec{v}_1$ is the first eigenvector from part (a). Plot $x_0[k]$ and $x_1[k]$ vs. k for $k = 0$ to 20.

Solution:

For code, see the IPython notebook.



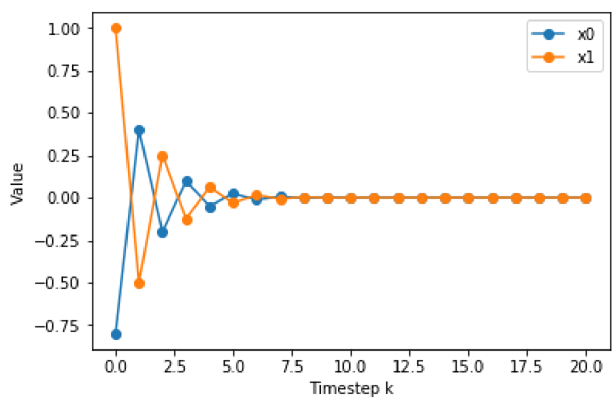
(c) Use IPython to simulate the system from part (a) with an initial condition of:

$$\begin{bmatrix} x_0[0] \\ x_1[0] \end{bmatrix} = \vec{v}_2,$$

where $\vec{v}_2$ is the second eigenvector from part (a). Plot $x_0[k]$ and $x_1[k]$ vs. k for $k = 0$ to 20.

Solution:

For code, see the IPython notebook.



(d) Why does the system act so differently in part (c) compared to part (b) when the only thing that changed was the initial condition of x? How does the system response change with respect to the eigenvalue of the input eigenvector?

Solution:

$x_0[k+1]$ and $x_1[k+1]$ can be represented as a linear combination of the one-dimensional systems defined by λ_1 and λ_2:

$$z_1[k+1] = \lambda_1 z_1[k]$$

$$z_2[k+1] = \lambda_2 z_2[k]$$

$$x_0[k+1] = a_0 z_1[k+1] + b_0 z_2[k+1]$$

$$x_1[k+1] = a_1 z_1[k+1] + b_1 z_2[k+1]$$

In part (b), our initial condition lay on the eigenvector of λ_1. Since eigenvectors are perpendicular to each other, $x_0[k+1]$ and $x_1[k+1]$ are solely related to the λ_1 system ($b_0 = b_1 = 0$).
Similarly in part (c), our initial condition lay on the eigenvector of λ_2, which means $x_0[k+1]$ and $x_1[k+1]$ are solely related to the λ_2 system ($a_0 = a_1 = 0$).

3. Write Your Own Question And Provide a Thorough Solution.

Writing your own problems is a very important way to really learn material. The famous "Bloom's Taxonomy" that lists the levels of learning is: Remember, Understand, Apply, Analyze, Evaluate, and Create. Using what you know to create is the top level. We rarely ask you any homework questions about the lowest level of straight-up remembering, expecting you to be able to do that yourself (e.g. making flashcards). But we don't want the same to be true about the highest level. As a practical matter, having some practice at trying to create problems helps you study for exams much better than simply counting on solving existing practice problems. This is because thinking about how to create an interesting problem forces you to really look at the material from the perspective of those who are going to create the exams. Besides, this is fun. If you want to make a boring problem, go ahead. That is your prerogative. But it is more fun to really engage with the material, discover something interesting, and then come up with a problem that walks others down a journey that lets them share your discovery. You don't have to achieve this every week. But unless you try every week, it probably won't ever happen.

4. Redo problem 1 of the midterm. (Optional. Required to be eligible for clobber policy)

(a)

(b)

(c)

(d)

5. Redo problem 2 of the midterm. (Optional. Required to be eligible for clobber policy)

(a)

(b)

(c)

6. Redo problem 3 of the midterm. (Optional. Required to be eligible for clobber policy)

7. Redo problem 4 of the midterm. (Optional. Required to be eligible for clobber policy)

(a)

(b)

(c)

8. Redo problem 5 of the midterm. (Optional. Required to be eligible for clobber policy)

(a)

(b)

(c)