EECS 16B Designing Information Devices and Systems II
Fall 2018 Elad Alon and Miki Lustig **Homework 4**

This homework is solely for your own practice. However, everything on it is in scope for midterm 1, and it will be assumed in lab that you have completed the lab-related questions.

1. Mystery Microphone

You are working for Mysterious Miniature Microphone Multinational when your manager asks you to test a batch of the company's new microphones. You grab one of the new microphones off the shelf, use a tone generator [1] to play pure tones of uniform amplitude at various frequencies, and measure the resultant peak-to-peak voltages using an oscilloscope. You collect data, and then plot it (on a logarithmic scale). The plot is shown below:

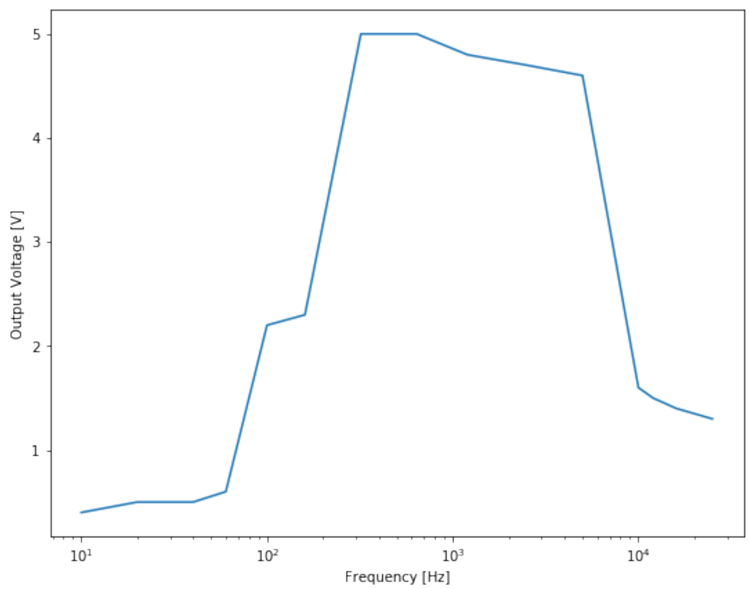


Figure 1: Frequency Response

(a) To which frequencies is the microphone most sensitive, and to which frequencies is the microphone least sensitive?

Solution:

The microphone is most sensitive to frequencies in the range of 320 Hz to 5 kHz, and least sensitive below $\approx$ 100 Hz or so.

You report these findings to your manager, who thanks you for the preliminary data and proceeds to co-ordinate some human listener tests. In the meantime, your manager asks you to predict the effects

[1] Note that soundwaves are simply sinusoids at various frequencies with some amplitude and phase. The microphone's diaphragm oscillates with the sound (pressure) waves, moving the attached wire coil back and forth over an internal magnet, which induces a current in the wire. In this way, a microphone can be modeled as a signal-dependent current source. The output current can be converted to a voltage by simply adding a known resistor to the circuit and measuring the voltage across that resistor.

of the microphone recordings on human listeners, and encourages you to start thinking more deeply about the relationships.

(b) For testing purposes, you have a song with sub-bass (150 Hz or less), mid-range ($\approx$ 1 kHz), and some high frequency electronic parts ($>$ 12 kHz). Which frequency ranges of the song would you be able to hear easily, and which parts would you have trouble hearing? Why?

Solution:

The mid-range would be most audible since the amplitude is the highest at these frequencies. The high frequency electronic parts are the next loudest. The sub-bass parts would be the parts you have trouble hearing since the output amplitude is so low.

(c) After a few weeks, your manager reports back to you on the findings. Apparently, this microphone causes some people's voices to sound really weird, resulting in users threatening to switch to products from a competing microphone company.

It turns out that we can design some filters to "fix" the frequency response so that the different frequencies can be recorded more equally, thus avoiding distortion. Imagine that you have a few (say up to 4 or so) blocks. Each of these blocks detects a set range of frequencies, and if the signal is within this range, it will switch on a op-amp circuit of your choice. For example, it can be configured to switch on an op-amp filter to double the voltage for signals between 100 Hz and 200 Hz.

What ranges of signals would require such a block, and what gain would you apply to each block such that the resulting peak-to-peak voltage is about 5 V for all frequencies?

Solution:

The output amplitude for $<$ 100 Hz is $\approx$ 0.5 V, so it needs a gain of 10.

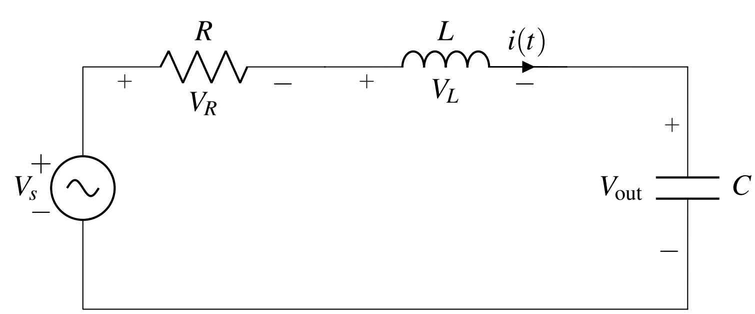
For 100-160Hz, the amplitude is $\approx$ 2.5 V, so it needs a gain of 2.

320-5000Hz already has an amplitude of 5V, so no gain is needed.

10000-20000Hz has an amplitude of $\approx$ 1.5 V, so it needs a gain of 3.33.

2. RLC Circuit

In this question, we will take a look at an electrical system described by a second order differential equations and analyze it using the phasor domain. Consider the circuit below, where $R = 8\,\mathrm{k}\Omega$, $L = 1\,\mathrm{mH}$, $C = 200\,\mathrm{nF}$, and $V_s = 2\cos\left(2000t + \frac{\pi}{4}\right)$.



(a) What are the impedances of the resistor Z_R, inductor Z_L, and capacitor Z_C?

Solution:

The impedance of a resistor is the same as its resistance.

$$Z_R = 8000\,\Omega$$

We can find the frequency of the circuit by looking at V_s. The form of a cosine function is $A\cos(\omega t+\phi)$, where A is the amplitude, ω is the frequency, and ϕ is the phase. In this case, the frequency is $2000\,\frac{\text{rad}}{\text{s}}$.

$$Z_L = j\omega L = j2000\cdot 10^{-3} = j2\Omega$$
$$Z_C = \frac{1}{j\omega C} = \frac{1}{j2000\cdot 2\cdot 10^{-7}} = -j25\cdot 10^2\Omega$$

(b) Solve for $\widetilde{V}_{\text{out}}$ in phasor form.

Solution:

Converting V_s into phasor form, we have

$$\widetilde{V}_s = |A|e^{j\phi} = 2e^{j\frac{\pi}{4}}$$

The circuit given is a voltage divider. Since impedances act like resistors, we can use the same equation as that for a resistive voltage divider.

$$\widetilde{V}_{\text{out}} = \widetilde{V}_s \frac{Z_C}{Z_R+Z_L+Z_C} = 2e^{j\frac{\pi}{4}}\frac{-j*2.5*10^3}{8*10^3+j*2-j*25*10^2} = 2e^{j\frac{\pi}{4}}\frac{-j*2500}{8000-j*2498}$$

We can solve for the magnitude and angle of the divider using

$$\left|2e^{j\frac{\pi}{4}}\frac{-j*2500}{8000-j*2498}\right| = 2\frac{2500}{\sqrt{8000^2+(-2498)^2}} = 0.597$$

$$\angle\left(2e^{j\frac{\pi}{4}}\frac{-j*2500}{8000-j*2498}\right) = \angle(2e^{j\frac{\pi}{4}}) + \angle(-j*2500) - \angle(8000-j*2498)$$
$$= \frac{\pi}{4} + \frac{-\pi}{2} - \text{atan2}(-2498, 8000) = -0.4827\,\text{rad}$$
$$\widetilde{V}_{\text{out}} = 0.597e^{-j0.4827}$$

(c) What is V_{out} in the time domain?

Solution: We know for $V_{out}(t) = A\cos(\omega t+\phi)$, we have $\tilde{V}_{out} = A(\cos(\phi)+j\sin(\phi)) = Ae^{j\phi}$. Thus, we have $A = 0.597, \phi = -0.4827$, which gives us $V_{out}(t) = 0.597\cos(\omega t - 0.4827)$

(d) Solve for the current $i(t)$.

Solution:

$$\widetilde{i} = \frac{\widetilde{V}_s}{Z_R+Z_L+Z_C} = \frac{|\widetilde{V}_s|}{|Z_R+Z_L+Z_C|}e^{j\left(\angle\widetilde{V}_s - \angle(Z_R+Z_L+Z_C)\right)} = 2.38\cdot 10^{-4}e^{j1.088}$$

Going back to the time domain:

$$i(t) = 2.38\cdot 10^{-4}\cos(2000t+1.088)$$

(e) Solve for the transfer function $H(\omega) = \frac{\widetilde{V}_{\text{out}}}{\widetilde{V}_s}$

Leave your answer in terms of R, L, C, and ω.

Solution:

Looking back at part (b),

$$\widetilde{V}_{\text{out}} = \widetilde{V}_{\text{s}} \frac{Z_C}{Z_R + Z_L + Z_C}$$

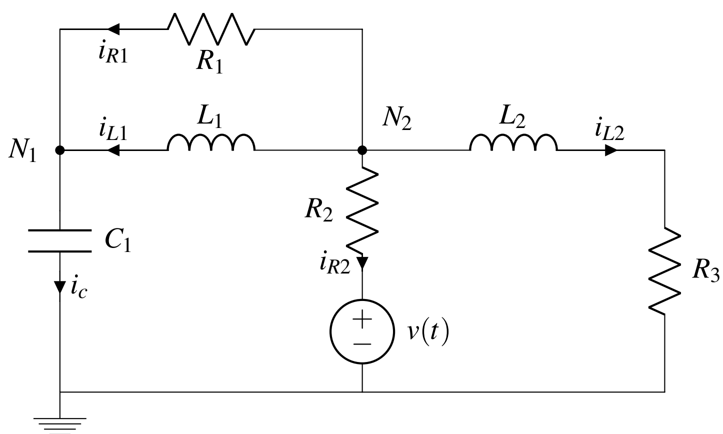
Rearranging, we get

$$H(\omega) = \frac{\widetilde{V}_{\text{out}}}{\widetilde{V}_{\text{s}}} = \frac{Z_C}{Z_R + Z_L + Z_C} = \frac{\frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$H(\omega) = \frac{1}{1 + j\omega RC + (j\omega)^2 LC}$$

3. Phasor-Domain Circuit Analysis

The analysis techniques you learned previously for resistive circuits are equally applicable for analyzing AC circuits (circuits driven by sinusoidal inputs) in the phasor domain. In this problem, we will walk you through the steps with a concrete example. Consider the circuit below.



The components in this circuit are given by:

Voltage source:

$$v(t) = 10\sqrt{2}\cos\left(100t - \frac{\pi}{4}\right)$$

Resistors:

$$R_1 = 5\,\Omega, \quad R_2 = 5\,\Omega, \quad R_3 = 1\,\Omega$$

Inductors:

$$L_1 = 50\,\text{mH}, \quad L_2 = 20\,\text{mH}$$

Capacitor:

$$C_1 = 2\,\text{mF}$$

(a) Transform the given circuit to the phasor domain (components and sources).

Solution:

$$
\begin{aligned}
Z_{L1} &= j\omega L = j100 \times 50 \times 10^{-3} = j5\ \Omega \\
Z_{L2} &= j\omega L = j100 \times 20 \times 10^{-3} = j2\ \Omega \\
Z_C &= \frac{1}{j\omega C} = \frac{1}{j100 \times 2 \times 10^{-3}} = -j5\ \Omega \\
\widetilde{v} &= |v|e^{j\angle v} = 10\sqrt{2}e^{-j\frac{\pi}{4}}
\end{aligned}
$$

(b) Write out KCL for node N_1 and N_2 in the phasor domain in terms of the currents provided.

Solution:

At node 1:

$$i_{L1} + i_{R1} = i_c$$

At node 2:

$$i_{R1} + i_{L1} + i_{R2} + i_{L2} = 0$$

(c) Find expressions for each current in terms of node voltages in the phasor domain. The node voltages $\widetilde{V}_1$ and $\widetilde{V}_2$ are the voltage drops from N_1 and N_2 to the ground.

Solution:

We have

$$
\begin{aligned}
\frac{\widetilde{V}_2 - \widetilde{V}_1}{Z_{L1}} + \frac{\widetilde{V}_2 - \widetilde{V}_1}{R_1} &= \frac{\widetilde{V}_1}{Z_C} \\
\frac{\widetilde{V}_2 - \widetilde{V}_1}{R_1} + \frac{\widetilde{V}_2 - \widetilde{V}_1}{Z_{L1}} + \frac{\widetilde{V}_2 - \widetilde{v}}{R_2} + \frac{\widetilde{V}_2}{R_3 + Z_{L2}} &= 0
\end{aligned}
$$

Plugging in values from part (a), we get

$$
\begin{aligned}
\frac{\widetilde{V}_2 - \widetilde{V}_1}{j5} + \frac{\widetilde{V}_2 - \widetilde{V}_1}{5} &= \frac{\widetilde{V}_1}{-j5} \\
\frac{\widetilde{V}_2 - \widetilde{V}_1}{5} + \frac{\widetilde{V}_2 - \widetilde{V}_1}{j5} + \frac{\widetilde{V}_2 - 10\sqrt{2}e^{-j\frac{\pi}{4}}}{5} + \frac{\widetilde{V}_2}{1 + j2} &= 0
\end{aligned}
$$

For future parts, we want the denominators of each current to be either purely real or purely imaginary. To put i_{L2} in this form, we can manipulate the expression by multiplying the denominator by its conjugate:

$$\frac{\widetilde{V}_2}{1 + j2}\left(\frac{1 - j2}{1 - j2}\right) = \frac{\widetilde{V}_2(1 - j2)}{1 - (-4)} = \frac{\widetilde{V}_2(1 - j2)}{5}$$

Our final KCL equations at nodes N_1 and N_2 are

$$
\begin{aligned}
\frac{\widetilde{V}_2 - \widetilde{V}_1}{j5} + \frac{\widetilde{V}_2 - \widetilde{V}_1}{5} &= \frac{\widetilde{V}_1}{-j5} \\
\frac{\widetilde{V}_2 - \widetilde{V}_1}{5} + \frac{\widetilde{V}_2 - \widetilde{V}_1}{j5} + \frac{\widetilde{V}_2 - 10\sqrt{2}e^{-j\frac{\pi}{4}}}{5} + \frac{\widetilde{V}_2(1 - j2)}{5} &= 0
\end{aligned}
$$

(d) Write the equations you derived in part (c) in a matrix form, i.e., $\mathbf{A}\begin{bmatrix}\widetilde{V}_1\\ \widetilde{V}_2\end{bmatrix} = \vec{b}$. Write out $\mathbf{A}$ and $\vec{b}$ numerically.

Solution:

From the above two equations, we have

$$\mathbf{A} = \begin{bmatrix} -\frac{1}{5} & \frac{1}{5} - j\frac{1}{5} \\ -\frac{1}{5} + j\frac{1}{5} & \frac{3}{5} - j\frac{3}{5} \end{bmatrix} = \begin{bmatrix} -0.2 & 0.2 - j0.2 \\ -0.2 + j0.2 & 0.6 - j0.6 \end{bmatrix}$$

$$\vec{b} = \begin{bmatrix} 0 \\ \frac{10\sqrt{2}e^{-j\frac{\pi}{4}}}{5} \end{bmatrix} = \begin{bmatrix} 0 \\ 2 - j2 \end{bmatrix}$$

(e) Solve the systems of linear equations you derived in part (d) with any method you prefer and then find $i_c(t)$.

Solution:

The inverse of a 2×2 matrix is given by:

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc}\begin{bmatrix} d & -b \\ -c & a \end{bmatrix}.$$

$$\mathbf{A}^{-1} = \begin{bmatrix} -6 + j3 & 2 - j1 \\ -2 + j1 & 1.5 - j0.5 \end{bmatrix}$$

With that, we find

$$\begin{bmatrix}\widetilde{V}_1\\ \widetilde{V}_2\end{bmatrix} = \mathbf{A}^{-1}\vec{b} = \begin{bmatrix} 2 - j6 \\ 4 - j2 \end{bmatrix} = \begin{bmatrix} \sqrt{40}e^{-j1.249} \\ \sqrt{20}e^{-j0.464} \end{bmatrix}$$

$$I_C = \frac{\widetilde{V}_1}{-j5} = \frac{j}{5}\widetilde{V}_1 = \frac{\sqrt{40}}{5}e^{j0.322} = 1.265e^{j0.322}$$

Transforming I_C back to time domain, we get

$$i_C(t) = 1.265\cos(100t + 0.322)$$

4. Analyzing Mic Board Circuit

In this problem, we will work up to analyzing a simplified version of the mic board circuit. In lab, we will address the minor differences between the final circuit in this problem and the actual mic board circuit.

The microphone can be modeled as a frequency-dependent current source, $I_{MIC} = k\sin(\omega t) + I_{DC}$, where I_{MIC} is the current generated by the mic (which flows from VDD to VSS), I_{DC} is some constant current, k is the force [2] to current conversion ratio, and ω is the signal's frequency (in $\frac{\text{rad}}{\text{s}}$). VDD and VSS are 5 V and −5 V, respectively.

[2] The force is exerted by the soundwaves on the mic's diaphragm.

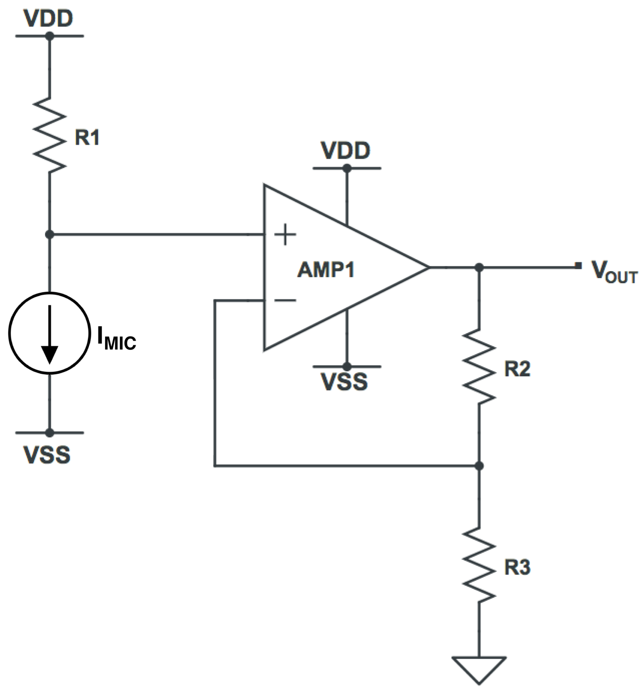


Figure 2: Step 1. The microphone is modeled as a DC current source.

(a) **DC Analysis** Assume for now that $k = 0$ (so that we can examine just the "DC" response of the circuit), find V_{OUT} in terms of I_{DC}, R_1, R_2, and R_3 (Hint: You do not need to worry about V_{ss} in your calculations).

Solution: The current in the left branch is equal to I_{DC} since no current flows into the op-amp.

$$V_{in} = V_{DD} - V_{R1} = 5 - (I_{DC} \cdot R_1)$$

$$V_{out} = (1 + \frac{R_2}{R_3}) \cdot V_{in} = (1 + \frac{R_2}{R_3}) \cdot (5 - (I_{MIC} \cdot R_1))$$

(b) Now, let's include the sinusoidal part of I_{MIC} as well. We can model this situation as shown below, with I_{MIC} split into two current sources so that we can analyze the whole circuit using superposition. Let $I_{\text{AC}} = k\sin(\omega t)$. Find and plot the function $V_{\text{OUT}(t)}$.

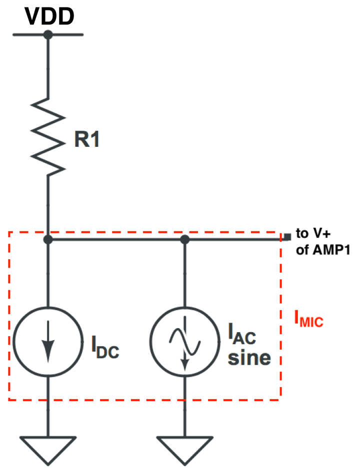


Figure 3: Step 2. The microphone is modeled as the superposition of a a DC and a sinusoidal ("AC") current source.

Solution: Doing superposition, we null each of the sources and add the results. Let's use superposition to find V_{in}. Note, here when we do superposition we have 3 sources that affect V_{in}: V_{DD}, I_{DC}, and I_{AC}. Nulling both current sources, we see that $V_{in_1} = V_{DD}$ because there is no current flowing in our circuit there is no change in voltage over the resistor. Nulling V_{DD} and I_{AC}, we get a similar expression to part (a) except there is no 5 volt source: $V_{in_2} = -R_1 \cdot I_{DC}$. And finally, nulling V_{DD} and I_{DC}, we get a similar expression to our last one: $V_{in_3} = -R_1 \cdot I_{AC}$
Putting these together and plugging in our expression for I_{AC} we get:

$$V_{in} = V_{in_1} + V_{in_2} + V_{in_3} = 5 - R_1 \cdot (ksin(\omega t) + I_{DC})$$

This then goes through a noninverting amplifier for our final answer:

$$V_{out} = (1 + \frac{R_2}{R_3}) \cdot (5 - R_1 \cdot (ksin(\omega t) + I_{DC}))$$

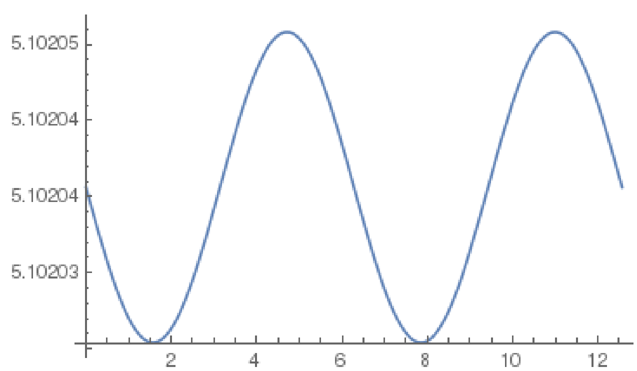


Figure 4: $V_{out}(t)$ when $R_1 = 10\,\text{k}\Omega, R_2 = 2040\,\Omega, R_3 = 100\,\text{k}\Omega, I_{DC} = 10\,\mu\text{A}, k = 10^{-9}$

(c) Given that $V_{DD} = 5\,\text{V}$, $V_{SS} = -5\,\text{V}$, $R_1 = 10\,\text{k}\Omega$, and $I_{\text{DC}} = 10\,\mu\text{A}$, find the maximum value of the gain G of the noninverting amplifier circuit for which the op-amp would not need to produce voltages greater than VDD or less than VSS (i.e, find the maximum gain G we can use without causing the op-amp to clip).

Solution: Since the signal is centered around $5 - R_1 I_{DC} = 4.9\text{V}$, we know that V_{DD} will limit the amplitude of the signal first.
Using our expression for V_{out} from part (b):
VDD side:

$$G \cdot (5 - R_1 I_{DC} + R_1 \max(k \sin(\omega t)) \leq V_{DD}$$
$$G \cdot (5 - 10^4 \cdot 10^{-5} + 10^4 k)) \leq 5$$
$$G \leq \frac{5}{4.9 + k \cdot 10^4}$$

(d) We have modified the circuit as shown below to include a high-pass filter so that the term related to I_{DC} is removed before we apply gain to the signal. Provide a symbolic expression for V_{OUT} given that that $\text{VDD}_0 = 5\,\text{V}$, $\text{VSS}_0 = -5\,\text{V}$, $\text{VDD}_1 = 3.3\,\text{V}$, $\text{VSS}_1 = 0\,\text{V}$. Show your work.

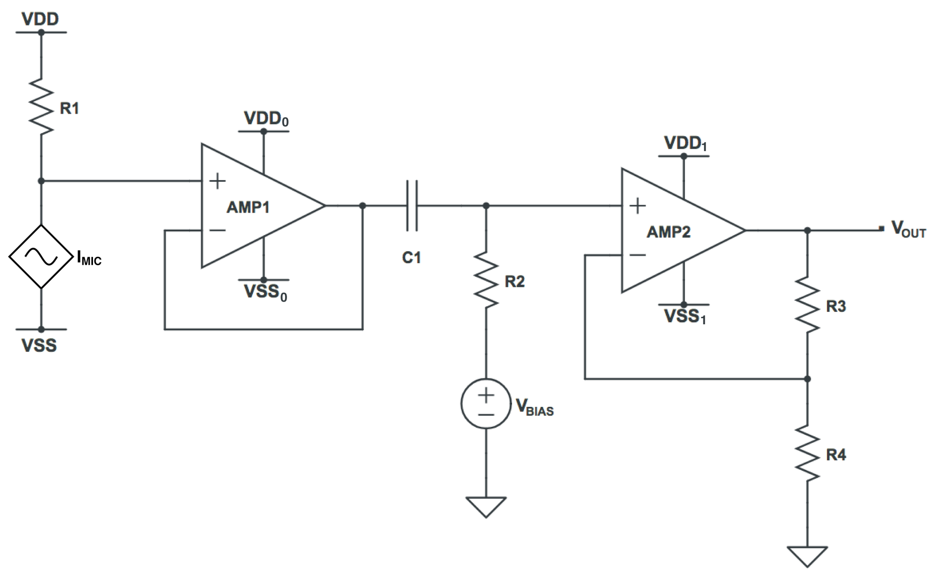


Figure 5: Step 3. Approaching the real mic board circuit. The microphone is still modeled as the superposition of a a DC and a sinusoidal ("AC") current source.

Solution: Since the high-pass filter removes the DC portion of the mic signal (the portion contributed by I_{DC}), the voltage going into the noninverting terminal of AMP2 is $(R_1 k \sin(\omega t) + V_{\text{BIAS}}$, a sinusoid centered around V_{BIAS}. From there, the gain of the noninverting amplifier circuit is $V_{OUT} = (1 + \frac{R_3}{R_4})$, which yields:

$$V_{OUT} = \left(1 + \frac{R_3}{R_4}\right)\left(-R_1 k \sin(\omega t) + V_{\text{BIAS}}\right)$$

(e) We would now like to choose V_{BIAS} so that we can get as much gain G out of the non-inverting amplifier circuit (AMP2) as possible without causing AMP2 to clip (i.e, the output of AMP2 must stay between 0V and 3.3V). What value of V_{BIAS} will achieve this goal? If $k = 10^{-5}$ and $R_1 = 10\,\text{k}\Omega$, what is the maximum value of G you can use without having AMP2 clip?

Solution: Since the sinusoidal term has zero mean, we want to put it in the middle of AMP2's range. In other words, we want the output of AMP2 to have a mean of $\frac{3.3-0}{2} = 1.65\,\text{V}$. Since the non-inverting amplifier has a gain of $G = 1 + \frac{R_3}{R_4}$, in order to achieve this we need to set $V_{bias} = \frac{1.65\,\text{V}}{G}$
Therefore, we should choose $V_{\text{BIAS}} = \frac{3.3V-0V}{2} = 1.65V$ as the optimum V_{BIAS}.

$$V_{\text{OUT}} = G(-R_1 k \sin(\omega t) + V_{\text{BIAS}})$$

Letting $V_{\text{BIAS}} = \frac{1.65\,\text{V}}{G}$:

$$3.3V - 1.65V = -GR_1 k \sin(\omega t)$$
$$1.65V = -GR_1 k \sin(\omega t)$$

Letting $\sin(\omega t) = -1$, its maximum value:

$$1.65V = GR_1 k$$
$$1.65V = G(10^4\Omega)(10^{-5}\text{A})$$
$$G = \frac{1.65}{0.1} = 16.5$$

5. Color Organ Filter Design

In the fourth lab, we will design low-pass, band-pass, and high-pass filters for a color organ. There are red, green, and blue LEDs. Each color will correspond to a specified frequency range of the input audio signal. The intensity of the light emitted will correspond to the amplitude of the audio signal.

(a) First, you realize that you can build simple filters using a resistor and a capacitor. Design the first-order **passive** low and high pass filters with following frequency ranges for each filter using 1 µF capacitors. ("Passive" means that the filter does not require any power supply.)

- Low pass filter – 3-dB frequency at 2400 Hz = $2\pi \cdot 2400 \frac{rad}{sec}$
- High pass filter – 3-dB frequency at 100 Hz = $2\pi \cdot 100 \frac{rad}{sec}$

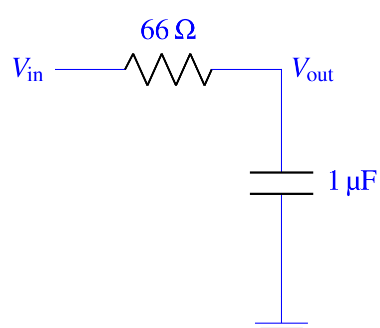
Draw the schematic-level representation of your designs and show your work finding the resistor values. Also, please mark V_{in}, V_{out}, and ground nodes in your schematic. Round your results to two significant figures.

Solution:

i. Low-pass filter

$$f_{3\,\text{dB}} = \frac{1}{2\pi RC} = 2400\,\text{Hz}$$

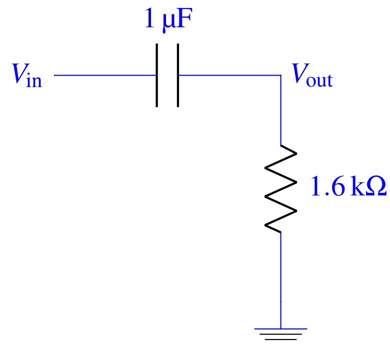
Therefore, we need a 66 Ω resistor.



ii. High-pass filter

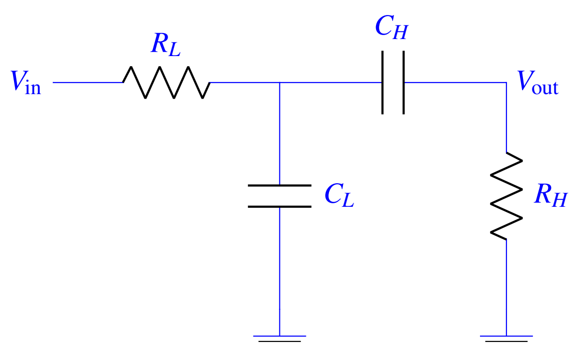
$$f_{3\,\text{dB}} = \frac{1}{2\pi RC} = 100\,\text{Hz}$$

Therefore, we need a $1.6\,\text{k}\Omega$ resistor.



(b) You decide to build a bandpass filter by simply cascading the first-order low-pass and high-pass filters you designed in part (a). Connect the V_{out} node of your low-pass filter directly to the V_{in} node of your high pass filter. The V_{in} of your new band-pass filter is the V_{in} of your old low-pass filter, and the V_{out} of the new filter is the V_{out} of your old high-pass filter. What is H_{BPF}, the transfer function of your new band-pass filter? Use R_L, C_L, R_H, and C_H for low-pass filter and high-pass filter components, respectively. Show your work.

Solution:



$$\left(\frac{1}{j\omega C_H} + R_H\right) \parallel \frac{1}{j\omega C_L} = \frac{\left(\frac{1}{j\omega C_H} + R_H\right)\frac{1}{j\omega C_L}}{\frac{1}{j\omega C_L} + \frac{1}{j\omega C_H} + R_H} = \frac{1 + j\omega R_H C_H}{-\omega^2 R_H C_L C_H + j\omega(C_H + C_L)}$$

Therefore, the transfer function from V_{in} of **the low pass filter** to V_{out} of **the low pass filter** is

$$H_{LPF} = \frac{\left(\frac{1}{j\omega C_H} + R_H\right) \parallel \frac{1}{j\omega C_L}}{R_L + \left(\frac{1}{j\omega C_H} + R_H\right) \parallel \frac{1}{j\omega C_L}} = \frac{1 + j\omega R_H C_H}{1 - \omega^2 R_L R_H C_L C_H + j\omega(R_H C_H + R_L C_L + R_L C_H)}$$

And, the transfer function from V_{out} of **the low pass filter** to V_{out} of **the high pass filter** is

$$H_{HPF} = \frac{j\omega R_H C_H}{1 + j\omega R_H C_H}$$

The overall transfer function is

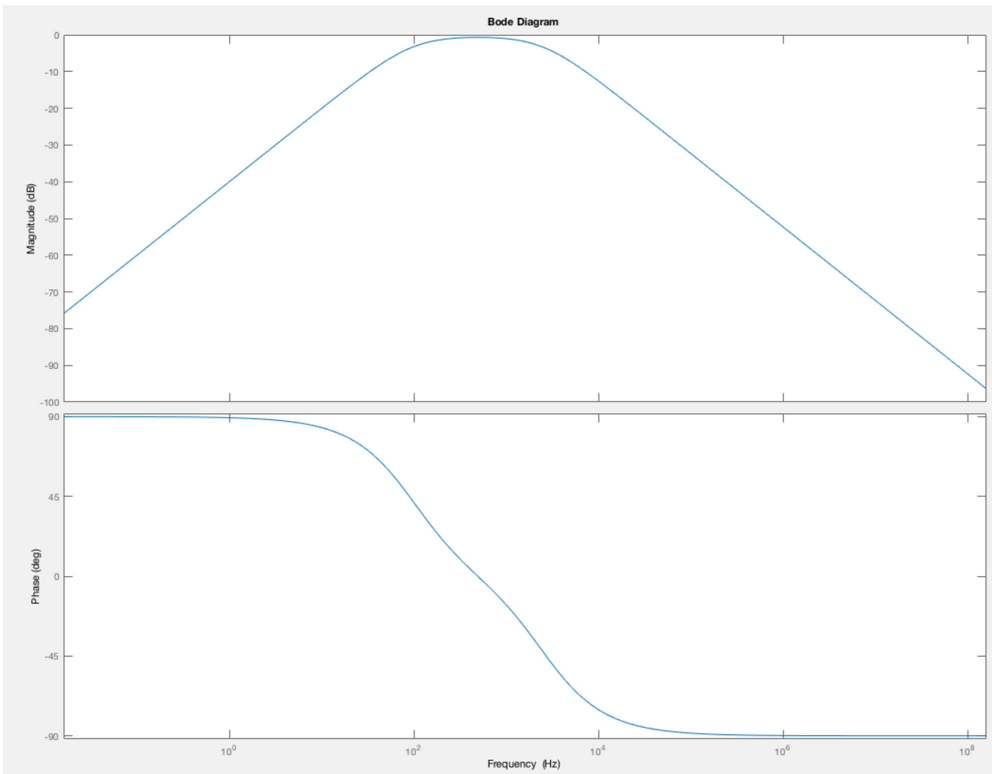
$$H_{BPF} = H_{LPF} \cdot H_{HPF} = \frac{j\omega R_H C_H}{1 - \omega^2 R_L R_H C_L C_H + j\omega(R_H C_H + R_L C_L + R_L C_H)}$$

(c) Plug the component values you found in (a) into the transfer function H_{BPF}. Using MATLAB or IPython, draw a Bode plot from 0.1 Hz to 1 GHz. If you use iPython, you may find the function `scipy.signal.bode` useful. What are the frequencies of the poles and zeros? What is the maximum magnitude of H_{BPF} in dB? Is that something that you want? If not, explain why not and suggest a simple way (either adding passive or active components) to fix it.

Solution:

$$H_{BPF} = \frac{j\omega(1.6 \cdot 10^{-3})}{1 - \omega^2(1.1 \cdot 10^{-7}) + j\omega(1.7 \cdot 10^{-3})}$$

The Bode plot is as below.



There are two poles and one zero at 100 Hz, 2.4 kHz, and DC, respectively. The maximum magnitude (around 500 Hz = $3.14 \times 10^3 \ \frac{\text{rad}}{\text{s}}$) is

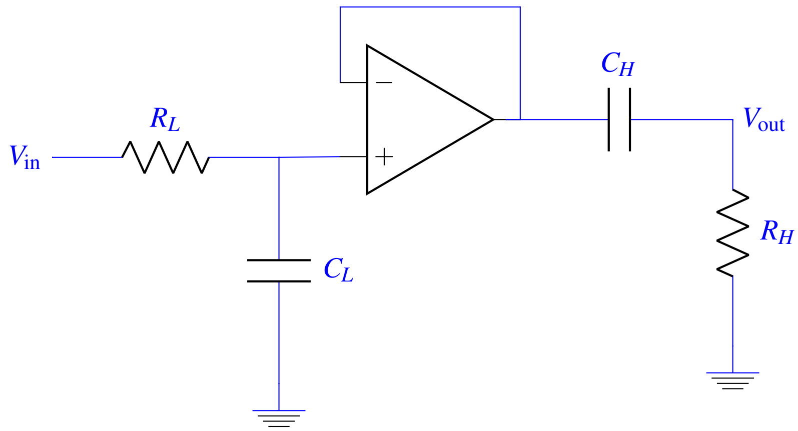
$$\left| \frac{j(3.14 \cdot 10^3)(1.6 \cdot 10^{-3})}{1 - (3.14 \cdot 10^3)^2(1.1 \cdot 10^{-7}) + j(3.14 \cdot 10^3)(1.7 \cdot 10^{-3})} \right| = 0.94 \, \frac{\text{V}}{\text{V}} = -0.52 \, \text{dB}$$

This is pretty similar to what we wanted. The gain, $|H_{BPF}|$, is close to 0 dB at its maximum. However, the transfer function of the bandpass filter that we likely intended to get by cascading the two filter circuits was:

$$\begin{aligned} H_{\text{ideal BPF}} &= \frac{j\omega R_H C_H}{(1 + j\omega R_H C_H)(1 + j\omega R_L C_L)} \\ &= \frac{j\omega R_H C_H}{1 - \omega^2 R_H C_H R_L C_L + j\omega(R_L C_L + R_H C_H)} \end{aligned}$$

Therefore, in our circuit, only the $j\omega R_L C_H$ term is added at the denominator. Because $R_L = 66\,\Omega$ is small, it did not cause any significant problem in our case. $j\omega R_L C_H$ is added because the low pass filter is experiencing impedance loading from the high pass filter, leading to a change in H_{LPF}. However, to be safe, a simple solution is to place a voltage buffer between the filters as below.

Note that the ideal voltage buffer has infinite input impedance and zero output impedance. This blocks any load effects from the following stage, and the next stage will see the op-amp output as an ideal voltage source.



(d) Now that you know how to make filters and amplifiers, we can finally build a system for the color organ circuit below. Before going into the actual schematic design, you must first set specifications for each block. The goal of the circuit is to divide the input signal into three frequency bands and turn the LEDs on based on the input signal's frequency.

In this problem, assume that the mic board is a 3-pole 2-zero system. Poles are located at 10 Hz, 100 Hz, and 10 000 Hz. Zeros are at DC and 200 Hz. This means that the frequency response at the mic board output can be modeled as follows.

$$V_{MIC} = K_{MIC} \frac{j\omega\left(1+\frac{j\omega}{\omega_{z1}}\right)}{\left(1+\frac{j\omega}{\omega_{p1}}\right)\left(1+\frac{j\omega}{\omega_{p2}}\right)\left(1+\frac{j\omega}{\omega_{p3}}\right)}$$

where K_{MIC} is a constant gain, ω_{z1}, ω_{p1}, ω_{p2}, and ω_{p3} are the zero and poles. Note that $j\omega$ term in the numerator denotes the zero at DC. Also note that poles are always in $\frac{rad}{sec}$: for example, $\omega_{p1} = 2\pi \cdot 10\text{Hz}$. The magnitude of the voltage at the mic board output is 1 V peak-to-peak at 40 Hz. (*Hint:* You can use this information to calculate K_{MIC}.)

Suppose that the three filters have transfer functions as below.

- Low pass filter

$$H_{LPF} = \frac{2}{1+\frac{j\omega}{200\pi}}$$

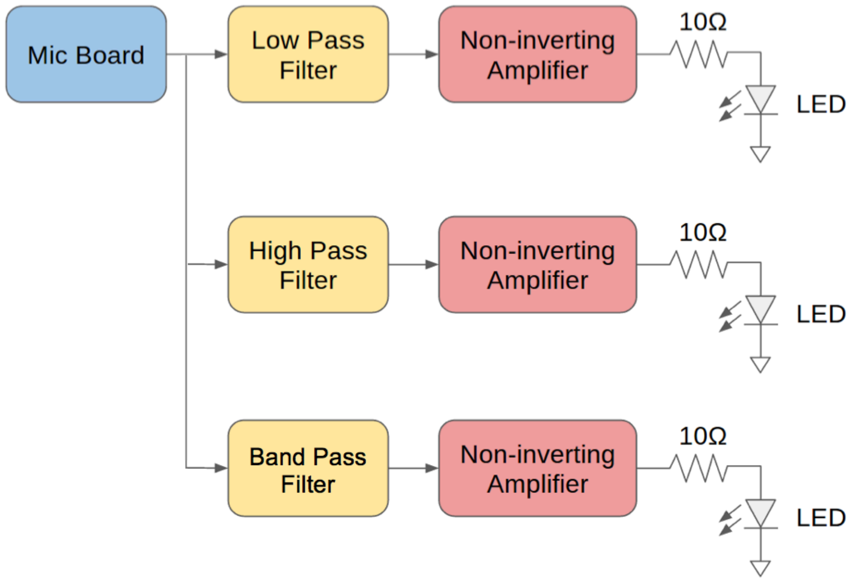
- Band pass filter

$$H_{BPF} = \frac{4.54 \cdot 10^{-4} j\omega}{\left(1+\frac{j\omega}{400\pi}\right)\left(1+\frac{j\omega}{4000\pi}\right)}$$

- High pass filter

$$H_{HPF} = \frac{\frac{j\omega}{8000\pi}}{1+\frac{j\omega}{8000\pi}}$$

What are the phasor voltages at the output of each filter as a function of ω? To clarify, $\frac{3(1+j\omega(1.5\cdot10^3))}{1+j\omega(2\cdot100)}$ would be a valid phasor voltage at the output of some filter. Assume that there are ideal voltage buffers before and after each filter.



Solution:

Because we know that we have 1 V_{pp} at 40 Hz, we can plug $2\pi\cdot40$ into ω to get K_{MIC}.

$$1 = \left| K \cdot \frac{j(80\pi)\left(1+\frac{j(80\pi)}{\omega_{z1}}\right)}{\left(1+\frac{j(80\pi)}{\omega_{p1}}\right)\left(1+\frac{j(80\pi)}{\omega_{p2}}\right)\left(1+\frac{j(80\pi)}{\omega_{p3}}\right)} \right|$$

Therefore, $K = 0.017$. Finally, the phasor voltages at the output of each filter are as below.

$$V_{LPF} = 0.034 \cdot \frac{j\omega\left(1+\frac{j\omega}{400\pi}\right)}{\left(1+\frac{j\omega}{20\pi}\right)\left(1+\frac{j\omega}{200\pi}\right)^2\left(1+\frac{j\omega}{20000\pi}\right)}$$

$$V_{BPF} = 7.72\cdot10^{-6} \cdot \frac{(j\omega)^2}{\left(1+\frac{j\omega}{20\pi}\right)\left(1+\frac{j\omega}{200\pi}\right)\left(1+\frac{j\omega}{4000\pi}\right)\left(1+\frac{j\omega}{20000\pi}\right)}$$

$$V_{HPF} = 0.017 \cdot \frac{\frac{(j\omega)^2}{8000\pi}\left(1+\frac{j\omega}{400\pi}\right)}{\left(1+\frac{j\omega}{20\pi}\right)\left(1+\frac{j\omega}{200\pi}\right)\left(1+\frac{j\omega}{8000\pi}\right)\left(1+\frac{jw}{20000\pi}\right)}$$

(e) For 50 Hz, 1000 Hz, and 8000 Hz, what is the voltage gain required of each non-inverting amplifier such that the output peak to peak voltage measured right before the 10 Ω resistor is 5 V_{pp}?

Solution:

i. Low pass filter path

At $\omega = 100\pi$,

$$|V_{LPF}| = \left| 0.034 \cdot \frac{j100\pi\left(1+\frac{j100\pi}{400\pi}\right)}{\left(1+\frac{j100\pi}{20\pi}\right)\left(1+\frac{j100\pi}{200\pi}\right)^2\left(1+\frac{j100\pi}{20000\pi}\right)} \right| = 1.73$$

Therefore, the non-inverting amplifier gain should be 2.9 $\frac{\mathrm{V}}{\mathrm{V}}$ (or 9.24 dB).

ii. Band pass filter path
At $\omega = 2000\pi$,

$$|V_{BPF}| = \left| \frac{7.72 \cdot 10^{-6} \cdot (j2000\pi)^2}{\left(1+\frac{j2000\pi}{20\pi}\right)\left(1+\frac{j2000\pi}{200\pi}\right)\left(1+\frac{j2000\pi}{4000\pi}\right)\left(1+\frac{j2000\pi}{20000\pi}\right)} \right| = 0.27$$

Therefore, the non-inverting amplifier gain should be 18.5 $\frac{\mathrm{V}}{\mathrm{V}}$ (or 25.3 dB).

iii. High pass filter path
At $\omega = 16000\pi$,

$$|V_{HPF}| = \left| \frac{0.017 \cdot \frac{(j16000\pi)^2}{8000\pi}\left(1+\frac{j16000\pi}{400\pi}\right)}{\left(1+\frac{j16000\pi}{20\pi}\right)\left(1+\frac{j16000\pi}{200\pi}\right)\left(1+\frac{j16000\pi}{8000\pi}\right)\left(1+\frac{j16000\pi}{20000\pi}\right)} \right| = 0.37$$

Therefore, the non-inverting amplifier gain should be 13.5 $\frac{\mathrm{V}}{\mathrm{V}}$ (or 22.6 dB).