Discussion 11A

See also attached ipython notebook.

Notes

Interpolation with Basis Functions

Assume there exists a set of functions $\phi_i(x)$ such that

$$\phi_i(x_i) = 1 \quad \text{and} \quad \phi_i(x_j) = 0 \text{ when } j \neq i$$

We can interpolate between the data points (x_i, y_i) with the function

$$f(x) = \sum_{k=1}^{n} y_k \phi_k(x) \quad \text{because } f(x_i) = \sum_{k=1}^{n} y_k \phi_k(x_i) = y_i$$

We call this set of functions "basis functions".

Sampling theorem

Let f be a signal bandlimited by frequency ω_{max}, and we sample with a period of Δ then we can write the sinc-interpolated signal $\hat{f}$

$$\hat{f} = \sum_{n=-\infty}^{\infty} y[n]\Phi(x - n\Delta)$$

Where $\Phi(x) = \operatorname{sinc}\left(\frac{x}{\Delta}\right)$

Then we can recover the signal, i.e. $f = \hat{f}$, if $\omega_{max} < \frac{\pi}{\Delta}$

Questions

1. Interpolation

Samples from the sinusoid $f(x) = \sin(0.2\pi x)$ are shown in Figure 1. Draw the results of interpolation using each of the following three methods:

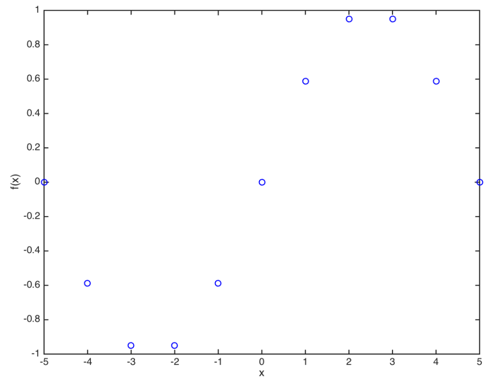

Figure 1: Samples of $f(x)$.

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(a) Zero order hold interpolation.

(b) Linear interpolation.

(c) Sinc interpolation assuming the Nyquist limit has been satisfied.

2. Sampling Theorem basics

Consider the following signal, $f(x)$ defined as,

$$f(x) = \cos(2\pi x)$$

(a) Find the maximum frequency, $\omega_{\max}$, in radians per second? In Hertz? (From now on, frequencies will refer to radians per second.)

(b) What is the smallest sampling Δ that would result in an imperfect reconstruction?

(c) If I sample every Δ_s seconds, what is the sampling frequency?

3. More Sampling

Let's sample the signal from the previous question f with sampling period $\Delta_m = \frac{1}{4}$s and $\Delta_n = 1$s and perform sinc interpolation on the resulting samples. Let the reconstructed functions be g_m and g_n.

(a) Have we satisfied the Nyquist limit (i.e. the sampling theorem) in any case?

(b) What is the highest frequency we can reconstruct with the sampling rate Δ_n?

(c) Based on this answer, can you think of any periodic function that has a frequencies less than or equal to π that samples the same as g_n?

4. Aliasing

Consider the signal $f(x) = sin(0.2\pi x)$.

(a) At what period T should we sample so that sinc interpolation recovers a function that is identically zero?

(b) At what period T should we sample so that sinc interpolation recovers the function $g(x) = -\sin\left(\frac{\pi}{15}x\right)$?