CS188: Exam Practice Session 6 Solutions

Q1. Probabilities

(a) Fill in the circles of **all** expressions that are equal to **1**,
given no independence assumptions:

● $\sum_a P(A = a \mid B)$
○ $\sum_b P(A \mid B = b)$
● $\sum_a \sum_b P(A = a, B = b)$
○ $\sum_a \sum_b P(A = a \mid B = b)$
● $\sum_a \sum_b P(A = a)\ P(B = b)$
○ $\sum_a P(A = a)\ P(B = b)$
○ None of the above.

Probability distributions, including conditional distributions, sum to one. We are testing this axiom when applied to multivariate distributions and conditionals.

(b) Fill in the circles of **all** expressions that are equal to $\mathbf{P(A, B, C)}$,
given no independence assumptions:

● $P(A \mid B, C)\ P(B \mid C)\ P(C)$
○ $P(C \mid A, B)\ P(A)\ P(B)$
● $P(A, B \mid C)\ P(C)$
● $P(C \mid A, B)\ P(A, B)$
○ $P(A \mid B)\ P(B \mid C)\ P(C)$
○ $P(A \mid B, C)\ P(B \mid A, C)\ P(C \mid A, B)$
○ None of the above.

We are testing the chain rule when applied to more than two variables.

(c) Fill in the circles of **all** expressions that are equal to $\mathbf{P(A \mid B, C)}$,
given no independence assumptions:

● $\frac{P(A,B,C)}{\sum_a P(A=a,B,C)}$
● $\frac{P(B,C|A)\ P(A)}{P(B,C)}$
● $\frac{P(B|A,C)\ P(A|C)}{P(B|C)}$
○ $\frac{P(B|A,C)\ P(A|C)}{P(B,C)}$
○ $\frac{P(B|A,C)\ P(C|A,B)}{P(B,C)}$
○ $\frac{P(A,B|C)}{P(B|A,C)}$
○ None of the above.

This is Bayes' rule applied to distributions over multiple variables. $P(A \mid B, C) = P(A, B, C)/P(B, C)$

(d) Fill in the circles of **all** expressions that are equal to $\mathbf{P(A \mid B)}$,
given that $\mathbf{A \perp\!\!\!\perp B \mid C}$:

○ $\frac{P(A|C)\ P(B|C)}{P(B)}$
○ $\frac{P(A|C)\ P(B|C)}{P(B|C)}$
● $\frac{\sum_c P(A|C=c)\ P(B|C=c)\ P(C=c)}{\sum_{c'} P(B|C=c')\ P(C=c')}$
○ $\frac{P(A|B,C)}{P(A|C)}$
● $\frac{\sum_c P(B|A,C=c)\ P(A,C=c)}{P(B)}$
○ $\frac{\sum_c P(A,C=c)\ P(B|C=c)}{\sum_{c'} P(A,B,C=c')}$
○ None of the above.

Apply Bayes' rule to get $P(A \mid B) = P(A, B)/P(B) = \sum_c P(A, B \mid C = c)P(C = c)/P(B)$ and conditional independence $P(A, B \mid C) = P(A \mid C)\ P(B \mid C)$

(e) Fill in the circles of **all** expressions that are equal to $\mathbf{P(A, B, C)}$, **given that** $\mathbf{A \perp\!\!\!\perp B \mid C}$ **and** $\mathbf{A \perp\!\!\!\perp C}$:

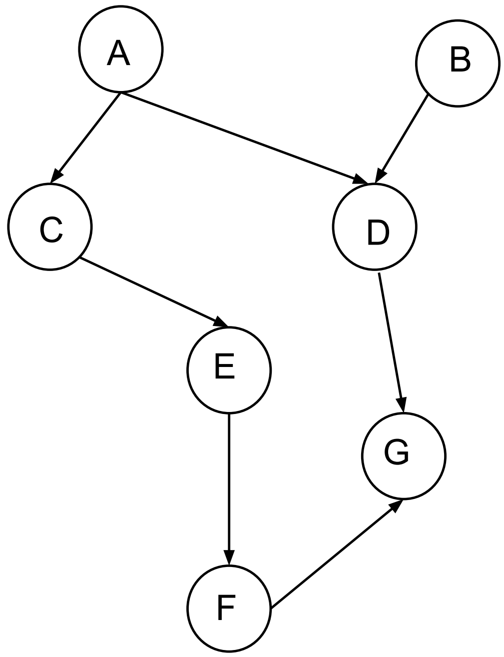
- ○ $P(A)\ P(B)\ P(C)$
- ● $P(A)\ P(B, C)$
- ● $P(A \mid B)\ P(B \mid C)\ P(C)$
- ○ $P(A \mid B, C)\ P(B \mid A, C)\ P(C \mid A, B)$
- ● $P(A \mid C)\ P(B \mid C)\ P(C)$
- ○ $P(A \mid C)\ P(B \mid C)$
- ○ None of the above.

If $A \perp\!\!\!\perp B \mid C$ and $A \perp\!\!\!\perp C$ it can be proven that $A \perp\!\!\!\perp B$ but not that $B \perp\!\!\!\perp C$. Here is the proof:

$$\begin{aligned}
P(A, B) &= \sum_c P(A, B \mid C = c) P(C = c) \\
&= \sum_c P(A \mid C = c) P(B \mid C = c) P(C = c) \\
&= P(A) \sum_c P(B \mid C = c) P(C = c) \\
&= P(A) \sum_c P(B, C = c) \\
&= P(A)\ P(B)
\end{aligned}$$

Q2. Bayes Nets: Independence

Consider a Bayes Net with the following graph:



Which of the following are guaranteed to be true without making any additional conditional independence assumptions, other than those implied by the graph? (Mark all true statements)

● $P(A \mid C, E) = P(A \mid C)$

○ $P(A, E \mid G) = P(A \mid G) * P(E \mid G)$

● $P(A \mid B = b) = P(A)$

○ $P(A \mid B, G) = P(A \mid G)$

○ $P(E, G \mid D) = P(E \mid D) * P(G \mid D)$

● $P(A, B \mid F) = P(A \mid F) * P(B \mid F)$

This question deals with (conditional) independence of a Bayes Net.
Option 1: $A \perp\!\!\!\perp E \mid C$, since with C observed no path between A and C is active.
Option 2: there's no conditional independence between A and E given G, since A-C-E is active.
Option 3: $A \perp\!\!\!\perp B$, since no path between A and B is active.
Option 4: there's no conditional independence between A and B given G, since the path A, B, G is an active.
Option 5: there's no conditional independence between E and G, since E-F-G is active.
Option 6: $A \perp\!\!\!\perp B \mid F$, since A, B, F is active.