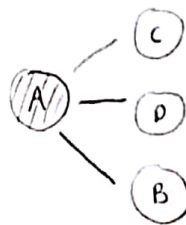
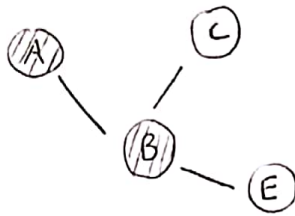


Q1. a) i.



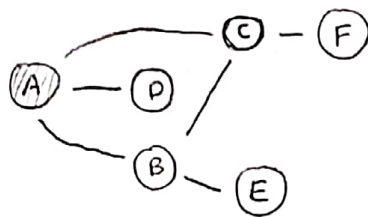
$C \rightarrow A$   
 $D \rightarrow A$   
 $B \rightarrow A$

ii.



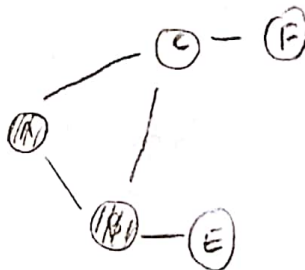
$C \rightarrow B$   
 $E \rightarrow B$

iii.



$B \rightarrow A$   
 $D \rightarrow A$   
 $C \rightarrow A$   
 $\vdots$   
 $F \rightarrow C$   
 $E \rightarrow B$

iv.



$C \rightarrow B$   
 $E \rightarrow B$   
 $\vdots$   
 $F \rightarrow C$

$D \rightarrow A$  doesn't appear  
 bc  $A \rightarrow C, A \rightarrow B$  never appear  
 (A is assigned so can't be tail)

- AC-3: enforce binary constraints
- 3rd algo in a series of optimizations
- either as preprocessing step or propagation during search
- assigned vars can only be the head of an arc
- runtime =  $O(ed^3)$ 
  - enforcing arc =  $O(d^2)$
  - enforcing arc is called at most  $O(ed)$
  - bc modifying domain of size  $d$  removes at least 1 val and adds at most  $e$  arcs to the queue

start with arcs that have assigned var as head

Tree CSP:  
 if no empty domains,  
 there is a set of valid assignments

b) i. A-B-C-D-E:0

- enforce AC
- assign A
- assign B (in same valid assignment as A)
- enforce AC
- assign C (in same valid assignment as A, B)
- assign D (in same valid assignment as A, B, C)
- ...

\* Same as Tree

Z-Pass Algo.

→ No backtracking

- Backtracking:
  - iterate through vars
  - only consider vals consistent with previous assignments

## Tree 2-Pass Algo

- (backward part)
- enforce  $D \rightarrow E, C \rightarrow D, B \rightarrow C, A \rightarrow B$  (subset of AC algo rules)  $O(ed^2)$
- (forward assignment)
- assign  $A, B, C, D, E$  in that order  $O(n)$
  - general: pick root, perform Topo Sort, enforce  $\text{Par}(X_i) \rightarrow X_i$  for  $i = n, \dots, 2$ ,  
assign  $X_i$  for  $i = 1, \dots, n$

$C - B - D - E - A: 0$  Same as Tree 2-Pass

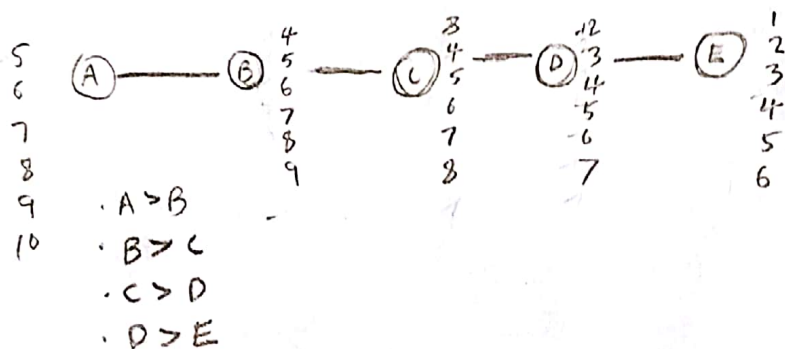


Topo sort: each node doesn't have  $\geq 2$  edges from left

$A - E - B - D - C: 2(d-1)$

- enforce AC (if no empty domain, there is a set of valid assignments)
- assign A
- assign E (might not be on same valid assignment as A)
  - worst case: backtrack through all values in domain of E except for correct assignment ( $d-1$  backtracks)
- ...
- assign D (same as E,  $d-1$  backtracks)
- observe: E is not connected to previous var A (may lead to conflict),  
D is not connected to previous var B,  
B/C are assigned immediately after AC

### Example



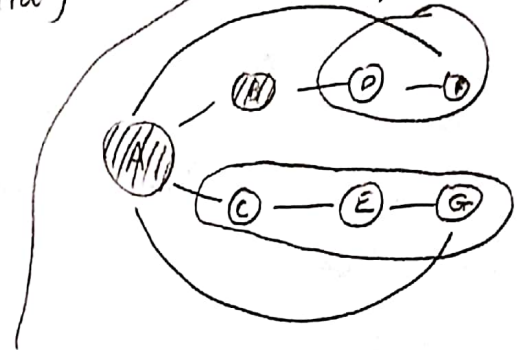
- Assign  $A = 5$
  - Assign  $E = 6$
  - Backtrack after AC
  - Assign  $E = 5$
  - Backtrack after AC
  - ...
- $d-1$  backtracks

ii.  $A-B-C-D-E-F-G: d^2 - 1$

- enforce AC3 (doesn't ensure 0 backtracks)
- assign A (might not be valid)
  - CSP becomes 2 trees
  - next AC3 ensures 0 backtracks
- assign B (might not be valid)

$\begin{matrix} X & A \rightarrow ? \\ & B \rightarrow ? \end{matrix}$

A, B will never be tails



- enforce AC (empty domain)
- backtrack A, B
- enforce AC (empty domain)
- backtrack A, B
- ...

- enforce AC (non-empty domain, Tree CSP  $\rightarrow$  0 backtracks) & topo ordering
- ...

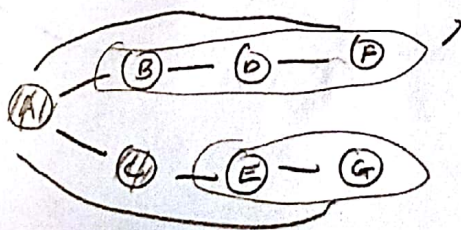
$d^2 - 1$   
backtracks  
•  $d^2$  total combos  
• at least 1 valid

$F-D-B-A-C-G-E: d^4 - 1 + d - 1$

- $F-D-B-A \rightarrow AC3$  in Tree CSP =  $d^4 - 1$  backtracks
- $C-G \rightarrow G$  might not be in same valid assignment as C =  $d - 1$

$C-A-F-E-B-G-D: d^2 - 1 + d - 1$

- enforce AC
- assign C
- assign A
- backtrack  $d^2 - 1$  times on C, A while enforcing AC
- finally perform AC with non-empty domains in Tree CSP



Assignment Order: F, B, D

↓ not topological!

↓ but AC3 occurs between F and B!  
↓ no backtracks



Q2. • Search ALL solutions

- LCV doesn't matter

- only changes order of exploring domain of an unassigned var

- MRV affects order of node exploration

- matters if there are edges

Final Notes

- CSP problems tend to be mechanical

- Understand runtime, Tree  $\mathcal{R}$ -Pass, AC, Forward-checking