

Computer Science 70: Celebrity Induction Problem

Sinho Chewi

Fall 2016

1 The Problem

Consider a party of n people. A celebrity is defined to be a person satisfying:

- Everyone else at the party knows the celebrity.
- The celebrity knows no one at the party.

You are allowed to ask questions of the form “does person A know person B ?” Prove that you can determine the celebrity’s identity, or none if no celebrity exists, by asking no more than $3n - 4$ questions.

2 The Solution

For $n = 2$, we are allowed 2 questions, which is sufficient to ask if A knows B and if B knows A .

Pick a random pair (A, B) of individuals at the party and ask if A knows B . If the answer is yes, then A cannot be the celebrity; if the answer is no, then B cannot be the celebrity. In either case, we have narrowed down the search to $n - 1$ candidates. Applying the recursive algorithm to the remaining $n - 1$ candidates will take $3(n - 1) - 4 = 3n - 7$ questions, and the algorithm will tell us one of the following:

- There is no celebrity among the remaining $n - 1$ candidates. But we already determined that the last person cannot be a celebrity, so we know there are no celebrities at the party.
- There is a celebrity among the remaining $n - 1$ candidates. In this case, we must verify that the candidate remains a celebrity when the n th person is included. But we can simply ask if the n th person knows the candidate, and if the candidate knows the n th person. This requires two extra questions, which when combined with the original question and the $3n - 7$ questions from the recursive algorithm, sum to $3n - 4$ total questions.

By induction, the proof is complete.