CSCI 5561 HW2

1.

Linearity:

let $y[x] = \alpha f[x] + \beta g[x]$,

$\Rightarrow T\{\alpha f[x] + \beta g[x]\} = T\{y[x]\}$, *by assumption of* $y[x]$

$= y[x-n]$, *by def of* $T\{\}$

$= \alpha f[x-n] + \beta g[x-n]$, *by def of* $y[x]$

$= \alpha T\{f[x]\} + \beta T\{g[x]\}$, *by def of* $T\{\}$

$T\{\alpha f[x] + \beta g[x]\} = \alpha T\{f[x]\} + \beta T\{g[x]\}$, so is Linear.

Shift-Invariance:

let $f_s[x] = Shift\{f[x]\} = f[x-x_0]$, $h[x] = T\{f[x]\} = f[x-n]$,

$\Rightarrow T\{f_s[x]\} = f[x-x_0-n]$, *by def of* $T\{\}$

$= Shift\{h[x]\}$, *by def of* $Shift\{\}$

$T\{f_s[x]\} = Shift\{h[x]\}$, so is Shift Invariant.

2.

Linearity:

let $y[x] = \alpha f[x] + \beta g[x]$,

$\Rightarrow T\{\alpha f[x] + \beta g[x]\} = T\{y[x]\}$, *by assumption of* $y[x]$

$= c_1 y[x] + c_2 y[x-1] + c_3 y[x-2]$, *by def of* $T\{\}$

$= c_1(\alpha f[x] + \beta g[x]) + c_2(\alpha f[x-1] + \beta g[x-1]) + c_3(\alpha f[x-2] + \beta g[x-2])$, *by def of* $y[x]$

$= \alpha(c_1 f[x] + c_2 f[x-1] + c_3 f[x-2]) + \beta(c_1 g[x] + c_2 g[x-1] + c_3 g[x-2])$, *by associativity*

$= \alpha T\{f[x]\} + \beta T\{g[x]\}$, *by def of* $T\{\}$

$T\{\alpha f[x] + \beta g[x]\} = \alpha T\{f[x]\} + \beta T\{g[x]\}$, so is Linear.

Shift-Invariance:

let $f_s[x] = Shift\{f[x]\} = f[x-x_0]$, $h[x] = T\{f[x]\} = c_1 f[x] + c_2 f[x-1] + c_3 f[x-2]$,

$\Rightarrow T\{f_s[x]\} = c_1 f[x-x_0] + c_2 f[x-x_0-1] + c_3 f[x-x_0-2])$, *by def of* $T\{\}$

$= c_1 f[x-x_0] + c_2 f[x-1-x_0] + c_3 f[x-2-x_0])$, *by commutativity*

$= Shift\{h[x]\}$, *by def of* $Shift\{\}$

$T\{f_s[x]\} = Shift\{h[x]\}$, so is Shift Invariant.

3.

Linearity:

let $y[x] = \alpha f[x] + \beta g[x]$,

$\Rightarrow T\{\alpha f[x] + \beta g[x]\} = T\{y[x]\}$, *by assumption of* $y[x]$

$= y[x^2]$, *by def of* $T\{\}$

$= \alpha f[x^2] + \beta g[x^2]$, *by def of* $y[x]$

$= \alpha T\{f[x]\} + \beta T\{g[x]\}$, *by def of* $T\{\}$

$T\{\alpha f[x] + \beta g[x]\} = \alpha T\{f[x]\} + \beta T\{g[x]\}$, so is Linear.

Shift-Invariance:

let $f_s[x] = Shift\{f[x]\} = f[x - x_0]$, $h[x] = T\{f[x]\} = f[x^2]$,

$\Rightarrow T\{f_s[x]\} = f[(x-x_0)^2] = f[x^2 - 2x_0 x + x_0^2]$, *by def of* $T\{\}$

$\neq Shift\{h[x]\} = f[x^2 - x_0]$, *by def of* $Shift\{\}$

$T\{f_s[x]\} \neq Shift\{h[x]\}$, so is **not** Shift Invariant.

4.

Linearity:

let $y[x] = \alpha f[x] + \beta g[x]$,

$\Rightarrow T\{\alpha f[x] + \beta g[x]\} = T\{y[x]\}$, *by assumption of* $y[x]$

$= y^2[x]$, *by def of* $T\{\}$

$= (\alpha f[x] + \beta g[x])^2$, *by def of* $y[x]$

$= \alpha^2 f^2[x] + 2\alpha\beta f[x]g[x] + \beta^2 g^2[x]$

$\alpha T\{f[x]\} + \beta T\{g[x]\} = \alpha f^2[x] + \beta g^2[x]$, *by def of* $T\{\}$

$T\{\alpha f[x] + \beta g[x]\} \neq \alpha T\{f[x]\} + \beta T\{g[x]\}$, so is **not** Linear.

Shift-Invariance:

let $f_s[x] = Shift\{f[x]\} = f[x - x_0]$, $h[x] = T\{f[x]\} = f^2[x]$,

$\Rightarrow T\{f_s[x]\} = (f[x - x_0])^2$, *by def of* $T\{\}$

$= Shift\{h[x]\} = (f[x - x_0])^2$, *by def of* $Shift\{\}$

$T\{f_s[x]\} = Shift\{h[x]\}$, so is Shift Invariant.

5.

Linearity:

let $y[x] = \alpha f[x] + \beta g[x]$,

$\Rightarrow T\{\alpha f[x] + \beta g[x]\} = T\{y[x]\}$, *by assumption of* $y[x]$

$= cy[x] + d$, *by def of* $T\{\}$

$= c(\alpha f[x] + \beta g[x]) + d$, *by def of* $y[x]$

$= c\alpha f[x] + c\beta g[x] + d$

$\alpha T\{f[x]\} + \beta T\{g[x]\} = \alpha(cf[x] + d) + \beta(cf[x] + d)$, *by def of* $T\{\}$

$= \alpha cf[x] + \beta cg[x] + \alpha d + \beta d$

$T\{\alpha f[x] + \beta g[x]\} \neq \alpha T\{f[x]\} + \beta T\{g[x]\}$, so is **not** Linear.

Shift-Invariance:

let $f_s[x] = Shift\{f[x]\} = f[x - x_0]$, $h[x] = T\{f[x]\} = cf[x] + d$,

$\Rightarrow T\{f_s[x]\} = cf_s[x] + d$, *by def of* $T\{\}$

$= cf[x - x_0] + d$, *by def of* f_s

$= Shift\{h[x]\}$, *by def of Shift{}*

$T\{f_s[x]\} = Shift\{h[x]\}$, so is Shift Invariant.

6.

Linearity:

let $y[x] = \alpha f[x] + \beta g[x]$,

$\Rightarrow T\{\alpha f[x] + \beta g[x]\} = T\{y[x]\}$, *by assumption of* $y[x]$

$= e^{y[x]}$, *by def of* $T\{\}$

$= e^{\alpha f[x] + \beta g[x]}$, *by def of* $y[x]$

$\alpha T\{f[x]\} + \beta T\{g[x]\} = \alpha e^{f[x]} + \beta e^{g[x]}$, *by def of* $T\{\}$

$T\{\alpha f[x] + \beta g[x]\} \neq \alpha T\{f[x]\} + \beta T\{g[x]\}$, so is **not** Linear.

Shift-Invariance:

let $f_s[x] = Shift\{f[x]\} = f[x - x_0]$, $h[x] = T\{f[x]\} = e^{f[x]}$,

$\Rightarrow T\{f_s[x]\} = e^{f_s[x]}$, *by def of* $T\{\}$

$= e^{f[x - x_0]}$, *by def of* f_s

$= Shift\{h[x]\}$, *by def of Shift{}*

$T\{f_s[x]\} = Shift\{h[x]\}$, so is Shift Invariant.

7.

Linearity:

let $y[x] = \alpha f[x] + \beta g[x]$,

$\Rightarrow T\{\alpha f[x] + \beta g[x]\} = T\{y[x]\}$, *by assumption of* $y[x]$

$= x + cy[x]$, *by def of* $T\{\}$

$= x + c(\alpha f[x] + \beta g[x])$, *by def of* $y[x]$

$= x + c\alpha f[x] + c\beta g[x]$

$\alpha T\{f[x]\} + \beta T\{g[x]\} = \alpha(x + cf[x]) + \beta(x + cg[x])$, *by def of* $T\{\}$

$= \alpha x + \beta x + \alpha c f[x] + \beta c g[x]$

$T\{\alpha f[x] + \beta g[x]\} \neq \alpha T\{f[x]\} + \beta T\{g[x]\}$, so is **not** Linear.

Shift-Invariance:

let $f_s[x] = Shift\{f[x]\} = f[x - x_0]$, $h[x] = T\{f[x]\} = x + cf[x]$,

$\Rightarrow T\{f_s[x]\} = x + f_s[x]$, *by def of* $T\{\}$

$= x + cf[x - x_0]$, *by def of* f_s

$Shift\{h[x]\} = (x - x_0) + cf[x - x_0]$, *by def of Shift{}*

$T\{f_s[x]\} \neq Shift\{h[x]\}$, so is **not** Shift Invariant.