HW1 An An Yu

Implementation:

- \triangleright Rotate the 4 corners of I_1 (the original image). The height and width spanned by the resulting 4 points are the minimal dimensions of I_2 (the rotated image).
- $ightharpoonup p_f$ is the rotated p_0 expressed in the new frame. The new frame is defined as follows (assuming x-axis points down and y-axis points right): x-axis is the vertical line that passes through the leftmost point in the rotated image, and y-axis is the horizontal line that passes through the topmost point in the rotated image.

$$p_f = -min(p_{cr}), p_{cr} = R(\theta)(p_c - p_o)$$
 (1)

 \succ For every pixel in I_2 , calculate its corresponding coordinates in I_1 by reverse mapping:

$$p_{I1} = p_0 + R(-\theta)(p_{I2} - p_f) \tag{2}$$

Find the four neighbors of p_{I1} and do bilinear interpolation on them, set the resulting value to p_{I2}

Compare with 'imwarp':

Bilinear interpolation produces fuzzier edges in the image. Taking a weighted average of pixel values from 4 neighbors is similar to apply an size 2x2 averaging/blurring window on the original image, only difference is the weight is defined as the relative euclidean distance from each neighbor.



figure 1. rotate_image(60° , (128,128))



figure 2. imwarp(60°, 'linear')

 \bullet p_0 doesn't affect the result of our rotated image because p_f always cancels its effect, aligning with the x and y axes.

$$\begin{split} p_{I1} &= p_0 + R(-\theta)(p_{I2} - p_f) \text{ where } p_f = -\min(R(\theta)(p_c - p_o)) \\ &\implies p_{I1} = p_0 + R(-\theta)p_{I2} + R(-\theta)\min(R(\theta)(p_c)) - R(-\theta)R(\theta)p_0 \\ &\implies p_{I1} = R(-\theta)p_{I2} + R(-\theta)\min(R(\theta)(p_c)) \quad \text{(independent of } p_0 \text{)} \\ \text{The first arrowhead holds because } \min \text{ is taken over } p_c \text{, which is independent of } p_o \text{.} \end{split}$$

Results:







figure 3. rotate_image, po = (0,0)







figure 4. rotate_image, po = (40,100)





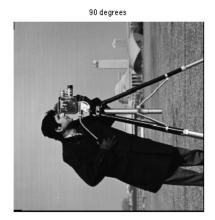


figure 5. rotate_image, po = (128,128)



figure(6). rotate_image(135, (40,100))



figure(7). rotate_image(225, (128,128))