



# Multi class Logistic Regression

$$\hat{y} = \sigma(z) : \text{prob}$$
$$z = w^T x$$

prob > thresh  $\rightarrow$  class A  
 $\rightarrow$  class B

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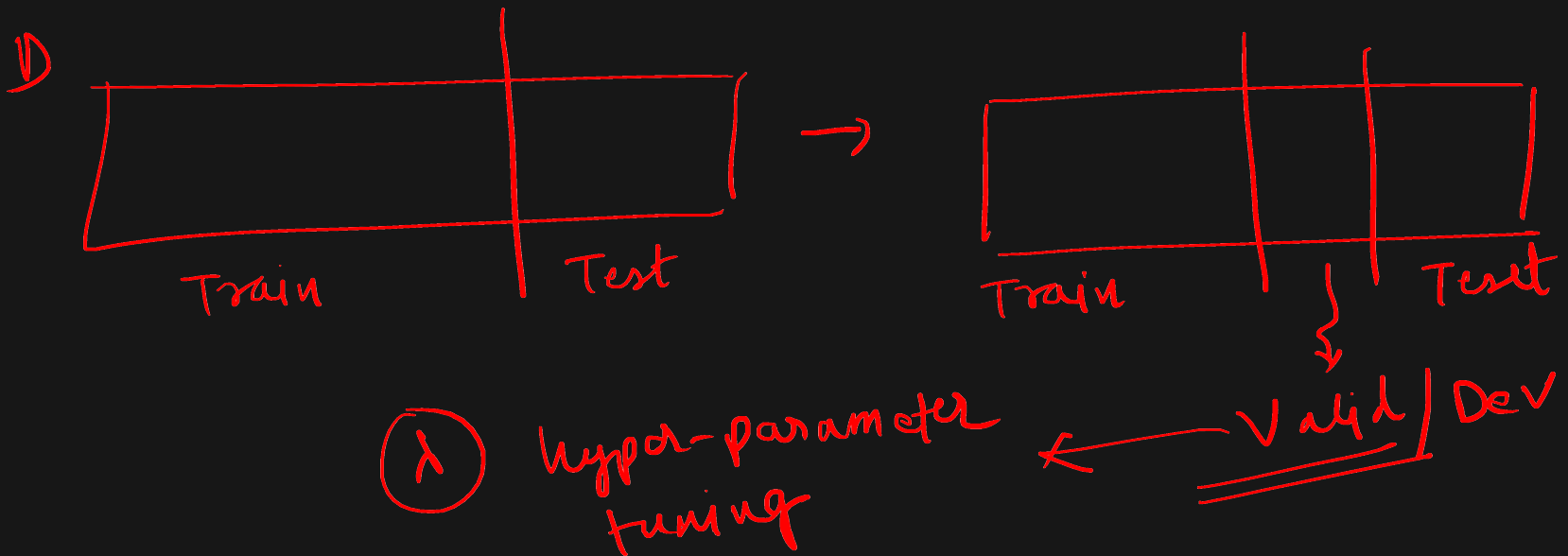
Q1. For logistic regression, the amount of L2 regularization  $\lambda$  should be chosen to maximize accuracy on the test set.

True

✓ False

should be  
completely  
unseen

$\lambda$   
hyper-parameter



MSQ

Q2. For which of the following models, it is necessary to keep the training data stored in memory

a. Linear Regression

✓ b. k-Nearest Neighbors

c. ~~Support Vector~~ Logistic Regression

d. None

$(w, b)$   $+ x$

Q3. What is the derivative of the sigmoid function  $\sigma(z) = \frac{1}{1+e^{-z}}$ ?

✓ A)  $\sigma(z)(1 - \sigma(z)) = \sigma'(z)$

B)  $\frac{1}{1+e^{-z}}$

C)  $\sigma(z)(\sigma(z) - 1)$

D)  $e^{-z}$

$\sigma'(2) = 0.25$

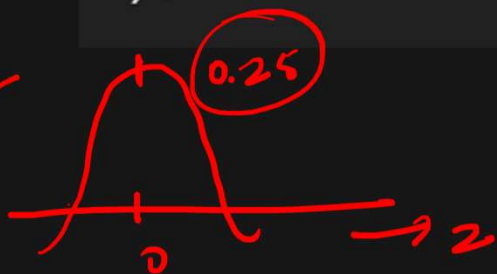
let  $\sigma(z) = t$

$t' = t(1-t)$

$t' = (t - t^2)$

$(1 - 2t) = 0$   
 $t = \frac{1}{2}$

$0 > -2$



$t = \frac{1}{2}$   
 $\sigma(z) = \frac{1}{2}$

$\frac{1}{1+e^2} = \frac{1}{2}$

Q4) Given  $x_t = [1, -2]$  and three different models.  
say, for each model of logistic regression  
 $w_1 = [1, 2]$  ,  $w_2 = [1, -2]$   $w_3 = [-1, 2]$

The order of prob assigned by respective model  
would be

- a.  $m1 > m2 > m3$
- ☒ b.  $m2 > m1 > m3$
- c.  $m3 > m1 > m2$

☒ d. The model output should not be compared as they are labels in case of logistic regression

→ do you understand  
what is o/p in case of  
log Reg model

→  $z = \text{logit}$   
→  $\sigma(wx) = \text{prob} > \text{thr}$

$$x[1, -2] \quad w_1 = [1, 2]$$



$$x @ w_1 = (1 \times 1) + (-2 \times 2)$$

$$= 1 + (-4)$$

$$x @ w_1 = -3$$

# Binary cross-entropy

$\begin{pmatrix} 1 \\ 0 \end{pmatrix} \rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$y_i \in \{0, 1\}$

$$- \left( \sum_{i=1}^N y_i \log(h_i) + (1 - y_i) \log(1 - h_i) \right)$$

$y_i$  (circled)  $\rightarrow$  true label  
 $h_i$  (circled)  $\rightarrow$   $\hat{y}_i$   
 $(1 - y_i)$  (circled)  $\rightarrow$   $e \in [0, 1]$

$N$ : # eg/ sample data-point

$h_i$  (circled) = predic<sup>n</sup> of model  
 $= \sigma(w^T x)$  :  $\hat{y}_i$

1	$\rightarrow$ class 1	(1)
0	$\rightarrow (1 - 1) = 0$	(0)

# Multi-class - cross entropy

CATEGORICAL

$y [0, 0, 1, 0]$

$$x = [ \dots ]$$

$$\downarrow$$

$$\frac{\text{sigmoid}(xw)}{[ \dots ]}$$

$$\sum_{i=1}^N \sum_{j=1}^{C=4} (y_j^i * -\log(h_j^i))$$

actual label

neg log prob

$[0, 0, 0]$

wtd -ve log prob

$N$ : #eg

$C$ : #classes

true-labels  $(y)$ :  $[1 \ 0]$

class A, B, C

$x^1$   
 $x^2$   
 $x^3$

A  
B  
C

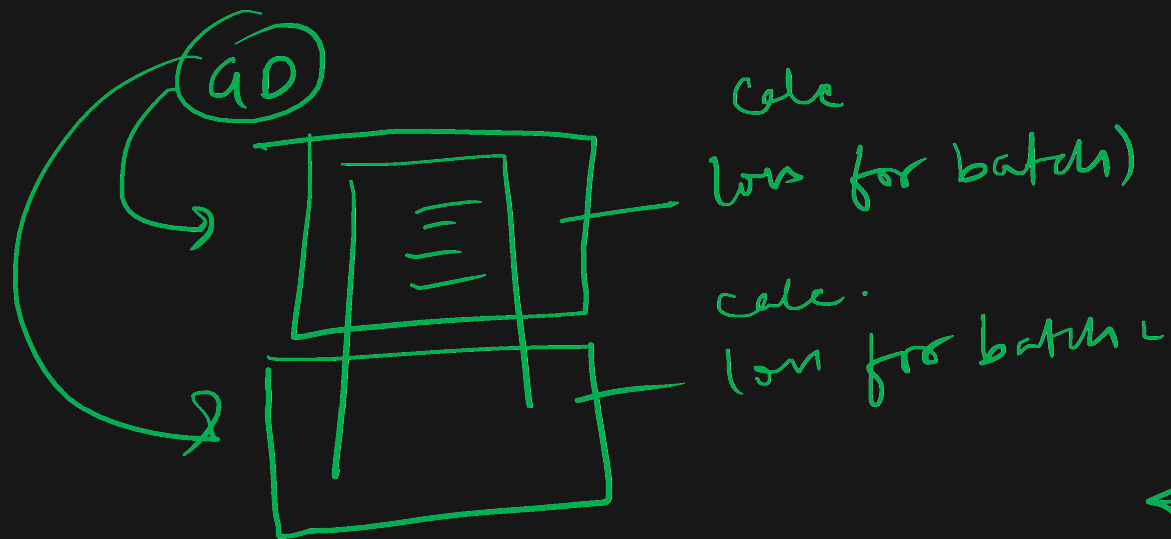
This assumes one-hot encoded labels

one-hot encoding

$y^1$	1	0	0
$y^2$	0	1	0
$y^3$	0	0	1

$C \geq 2$





model

