Chapter 8: Regression Discontinuity

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Mathematical Econometrics I
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Fall 2023

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  - For example, whether you are admitted to a school or university may depend on whether your test score exceeds a certain cutoff score
- In such cases, RD compares outcomes for people with scores just above the threshold to people with scores just below the threshold

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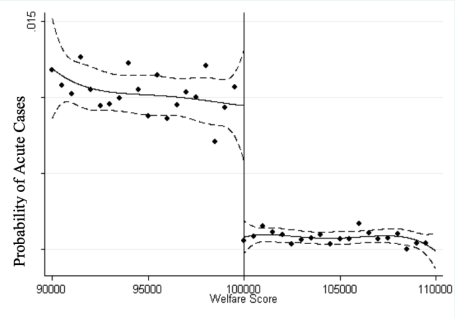
- Key idea: people with scores just above 100,000 may be very similar to people with scores just below 100,000
    - Except those below 100,000 had the health insurance treatment!

Figure 3: The Effect of MAP on Utilization of Acute Surgeries/In-Patient Services



Note: This figure plots probability of utilization of acute surgeries/inpatient services against welfare scores. Each dot is the average probability within 500 intervals of welfare scores. Solid lines are fitted values from 4^{th} order polynomial regressions on either side of the discontinuity. Dotted lines are 95% confidence intervals.

- This plot shows outcomes (acute surgeries) as a function of the score

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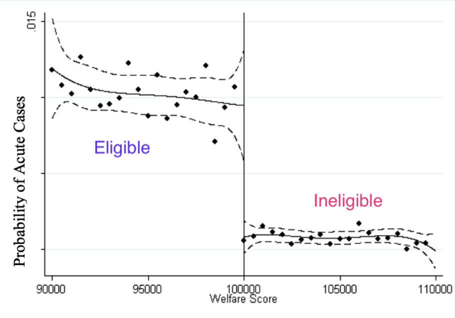

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- People to the left of the vertical line (at 100,000) receive health insurance from the government

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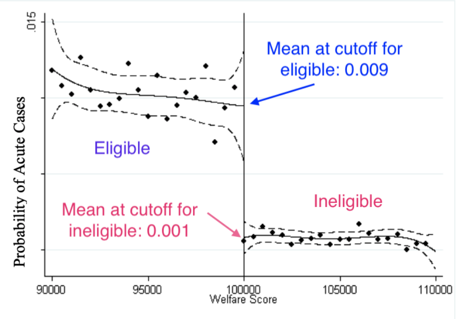

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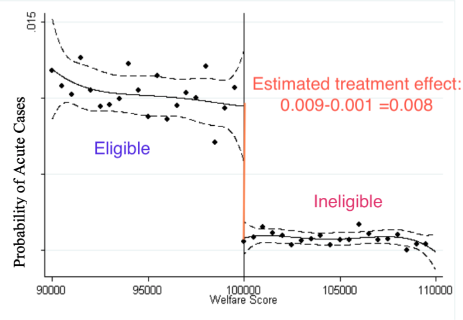

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- But people just below the threshold seem to get a lot more surgeries!
- If everything else is continuous at the threshold, the difference is the causal effect (for people at the threshold)!

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$$\tau_{RD} = \underbrace{\lim_{r \downarrow c} E[Y_i|R_i = r]}_{\text{Limit from above the cutoff}} - \underbrace{\lim_{r \uparrow c} E[Y_i|R_i = r]}_{\text{Limit from below the cutoff}}$$

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- When does τ_{RD} correspond to a causal effect?

Take it to the Limit

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the average treatment effect for people at the cutoff!

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- #1: confounding factors change discontinuously at the cutoff
- #2: people can manipulate scores to get just above/below the cutoff

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- Example: Holmes (1998) is interested in how “right-to-work laws,” which weaken labor unions, affect businesses
- He uses an RD to compare the density of manufacturing employment on both sides of borders between states that have/ don’t have right to work laws

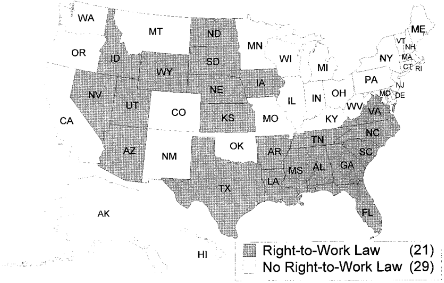

Fig. 1.—Geography of right-to-work laws

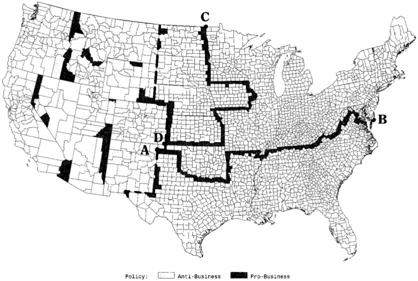

Fig. 3.—Counties within 25 miles of the policy change border

TABLE 1

Manufacturing Employment Shares and Growth Rates: Cross-County Averages by Distance from Border and Side of Border

Miles from Border	Coal Region Included: Share of 1992 Total (1)	Coal Region Included: Growth Rate, 1947–92 (2)	Coal Region Excluded: Share of 1992 Total (3)	Coal Region Excluded: Growth Rate, 1947–92 (4)
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75–100	25.9	67.5	25.0	68.2
50–75	23.1	62.7	25.0	80.9
25–50	23.2	82.0	24.7	88.8
0–25	21.0	62.4	22.1	77.2
	B. Probusiness Side of Border			
0–25	28.6	100.7	27.9	104.2
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- There does appear to be more manufacturing just to the RTW side of state borders
- But are RTWs the only thing that vary at state borders?! Could it be that RTW laws are correlated with other policies that affect manufacturing?

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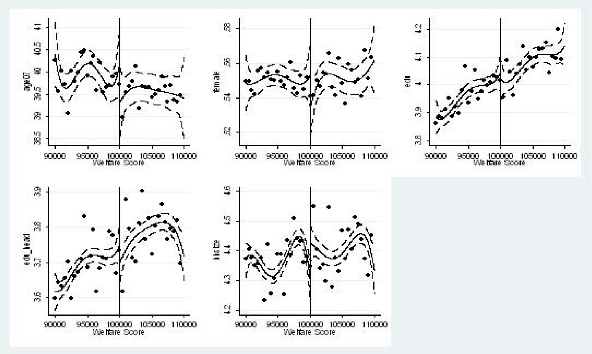

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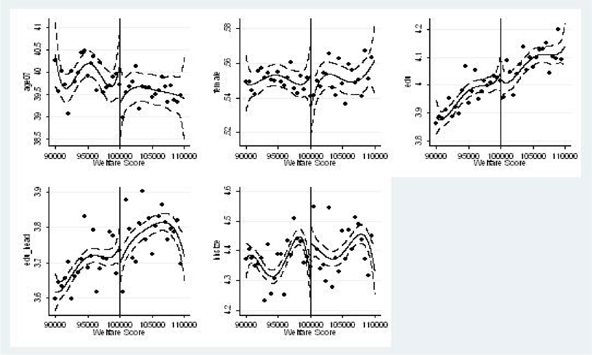

- But as usual, there may still be concern about other unobserved confounding factors varying at the cutoff

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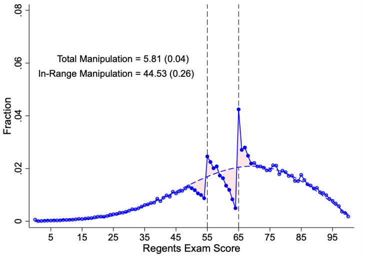

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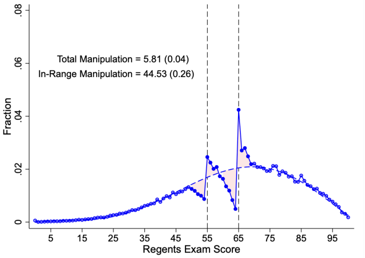

- Likely reason: teachers cheat to bump students over the threshold

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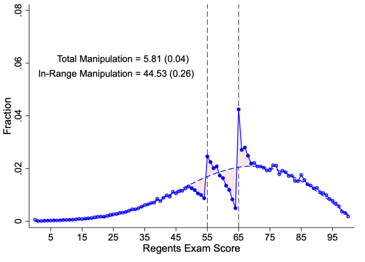


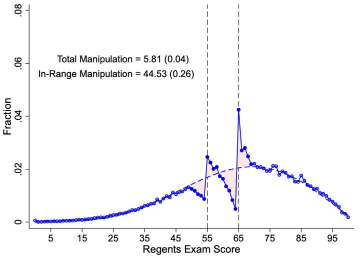
- Likely reason: teachers cheat to bump students over the threshold
- Do we think students who get bumped over the threshold versus not are similar?

Example - Manipulation

- In NYC, students must take the Regents exams and get a score of at least 55 to get a diploma (and 65 to get a more prestigious diploma)

- Might be tempted to use this to study diploma effects... But it turns out there are way more students with scores just above the thresholds

Figure 1
Test Score Distributions for Core Regents Exams, 2004-2010



- Likely reason: teachers cheat to bump students over the threshold

- Do we think students who get bumped over the threshold versus not are similar? Likely not, if teachers tend to bump better students...

Testing for Manipulation

- To test for such RD manipulation, it is common to check whether there are a similar number of units on both sides of the cutoff. This is often called a McCrary test.
- If there is bunching on one side of the cutoff, this is typically interpreted as evidence of manipulation
- The continuity assumption will usually be much more questionable if there is bunching.

Estimating RD

- Under continuity, we have identification:

$$\lim_{r \downarrow c} E[Y_i | R_i = r] - \lim_{r \uparrow c} E[Y_i | R_i = r] = E[Y_i(1) - Y_i(0) \mid R_i = c]$$

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- How do we estimate CEFs? OLS!

RD with Linear Regression

- Suppose the CEF is piecewise linear:

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E[Y_i|R_i = r] &\approx \alpha_0 + \alpha_1 r && \text{if } r < c \\
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- We can estimate these regression coefficients via OLS to estimate the effect of the treatment

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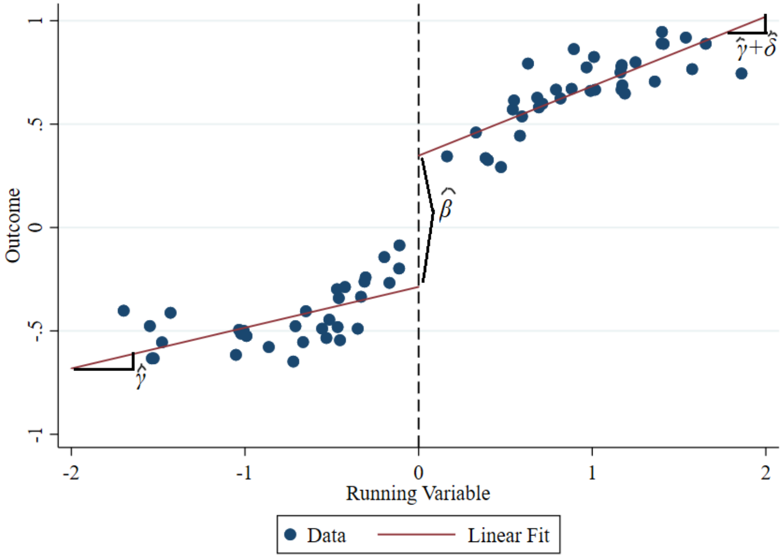
Outcome
1
.5
0
-.5
-1
-2
-1
0
1
2
Running Variable
$\hat{\gamma}$
$\hat{\beta}$
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Data
Linear Fit

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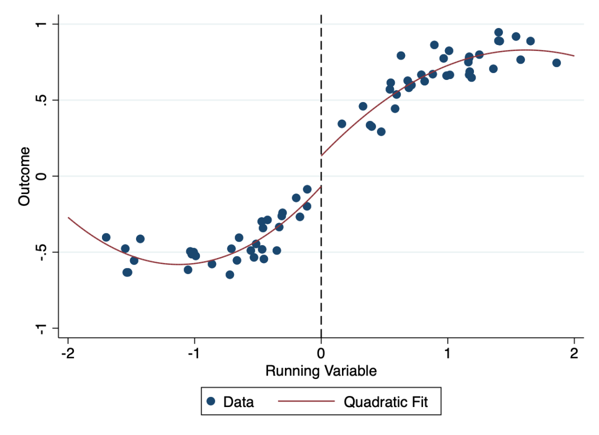
- As before, we can also add higher-order terms to the regression
- Here, for example, we fit a quadratic on each side:



Problems with Higher-Order Polynomials

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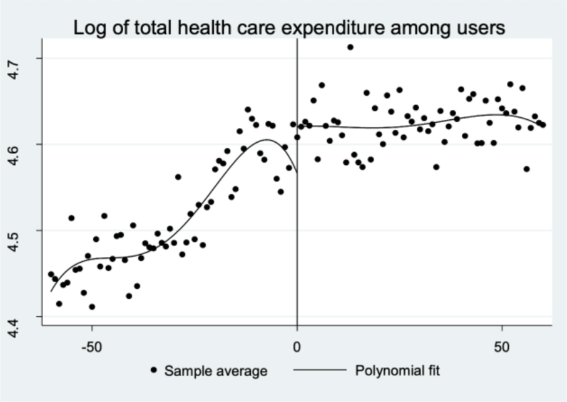
- Is there really a discontinuity in this picture?



Stick to Linear?

The overfitting problem has led a certain Nobel laureate (and Brown graduate) to take a strong stand!

Why High-Order Polynomials Should Not Be Used in Regression Discontinuity Designs

Andrew Gelman
Department of Statistics and Department of Political Science, Columbia University, New York, NY, 10027 (*gelman@stat.columbia.edu*)

Guido Imbens
Graduate School of Business, Stanford University, Stanford, CA 94305, and NBER, Stanford University, Stanford, CA 94305 (*imbens@stanford.edu*)

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- Clearly, there are tradeoffs between having a rich enough approximation to the CEF and over-fitting the data

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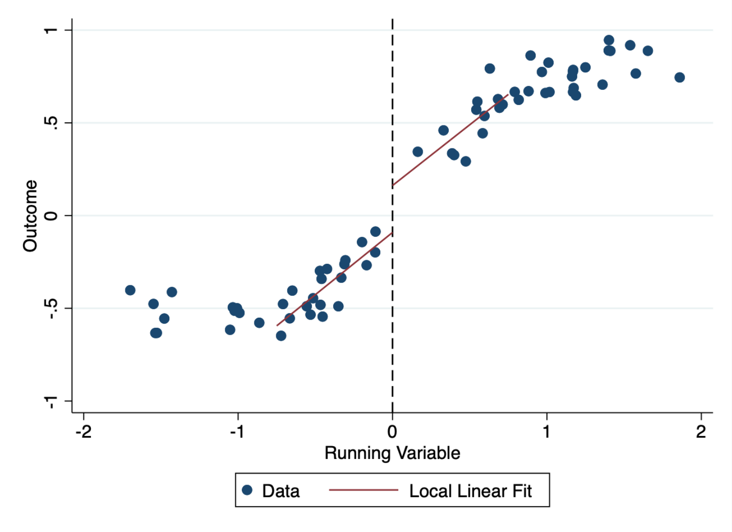
- This is easy to implement with the `rd` package in Stata

Local Linear Regression



RD Estimation is Tricky!

- Local linear regression is a relatively good medium between being rich enough and over-fitting
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- Local linear regression is a relatively good medium between being rich enough and over-fitting
- But it's not perfect! In general, difficult to distinguish between a very non-linear CEF and a discontinuity
- Good to trust your eyes – does it look like there's a discontinuity on the plot?
- The most convincing RDs are obvious from the plot and don't need any fancy econometrics

Fuzzy RD

- Sometimes crossing a threshold doesn't completely determine treatment status, but discontinuously increases treatment probability
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- The basic idea is similar to IV — we estimate the effect of being above the threshold on the outcome, then divide this by the effect on the treatment (i.e. the change treatment probability)
- Under certain conditions, this will identify the LATE at the cutoff

Example – Bleemer and Mehta (2022)

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- Bleemer and Mehta ask a very important question (for you): does majoring in economics make you rich?
- They study this Q in the context of UC Santa Cruz (UCSC), where the econ department only allowed people with GPA below 2.8 in intro classes to major in econ "at the discretion of the department"

First Stage

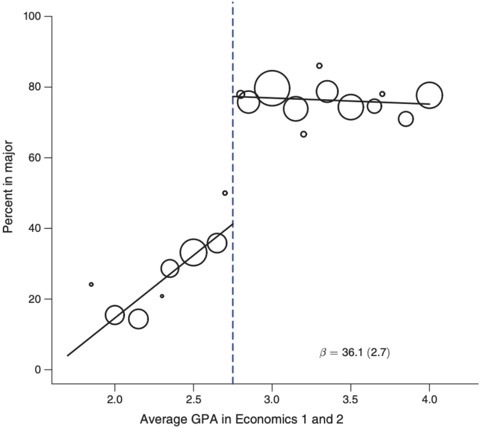


FIGURE 1. THE EFFECT OF THE UCSC ECONOMICS GPA THRESHOLD ON MAJORING IN ECONOMICS

Notes: Each circle represents the percent of economics majors (y-axis) among 2008–2012 UCSC students who earned a given *EGPA* in Economics 1 and 2 (x-axis). The size of each circle corresponds to the proportion of students who earned that *EGPA*. *EGPA*s below 1.8 are omitted, leaving 2,839 students in the sample. Fit lines and beta estimate (at the 2.8 GPA threshold) from linear RD specification; standard error (clustered by *EGPA*) in parentheses.

Students above the threshold about 36 pp more likely to major in econ

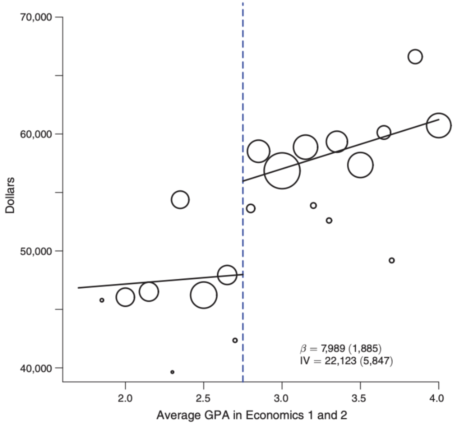
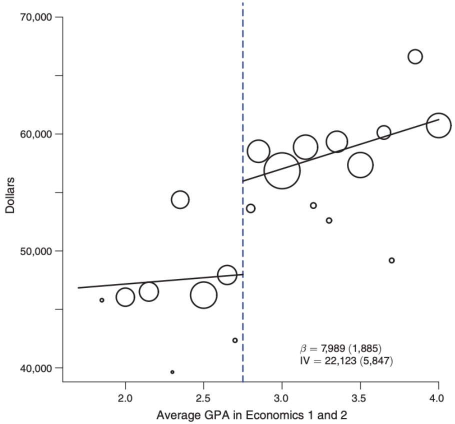


FIGURE 2. THE EFFECT OF THE UCSC ECONOMICS GPA THRESHOLD ON ANNUAL WAGES

- Students about the threshold earn about $8K more



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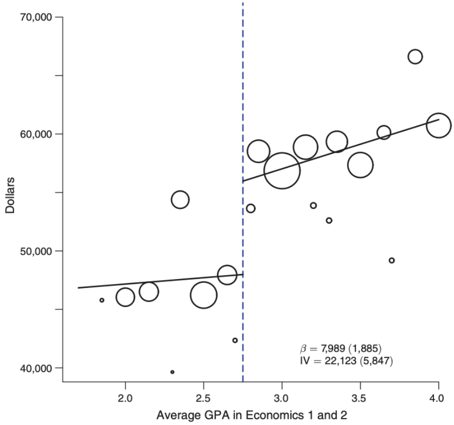

Figure 2. The Effect of the UCSC Economics GPA Threshold on Annual Wages

- Students about the threshold earn about \$8K more
- Aren't you glad you took this course?!
- The fuzzy RD estimate of the effect of majoring in econ is then $\$8K/0.36 \approx 22K$, or 40% of mean earnings

When does Fuzzy RD Give a LATE?

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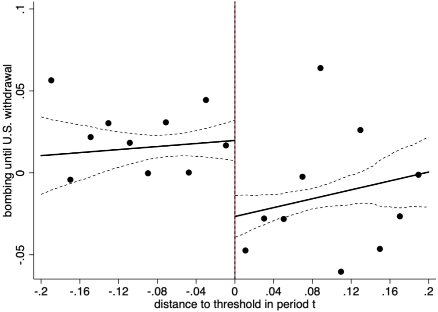
- Under similar assumptions to those for IV, fuzzy RD gives a LATE for compliers at the cutoff
- **Continuity**. Need that $E[Y_i(d)|R_i]$ is continuous at the cutoff for $d = 0, 1$
- **Exclusion**. Being just above/below the cutoff affects outcomes only through its effect on treatment
- **Relevance**. There is a discontinuity in treatment takeup at the cutoff.
- **“Local” Monotonicity**. No defiers who only take treatment if below the cutoff.

Dell and Querubin (2017)

- Dell and Querubin exploit the fact that during the Vietnam War, the US airforce selected bombing targets based on a risk score formed using 169 security/political/economic characteristics of villages

- The algorithm produced a continuous score which was rounded to the nearest integer before being given to generals → Dell and Querubin exploit the discontinuity from rounding

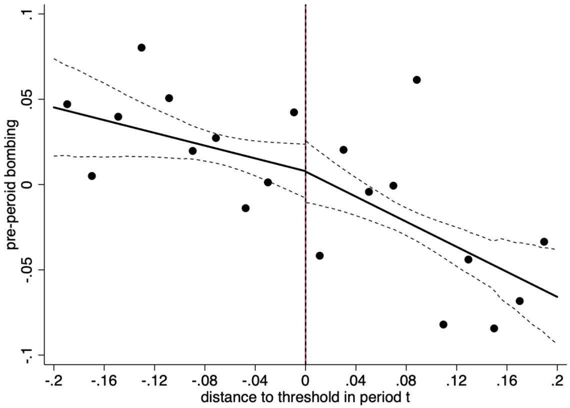
(b) Cumulative First Stage



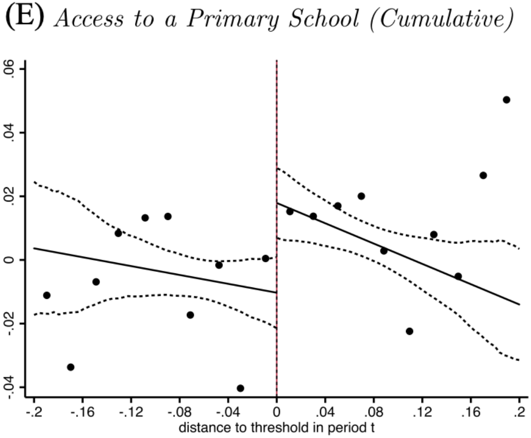
Checking for Balance

- Areas above/below the threshold have similar characteristics prior to bombing decision
- E.g., they have similar # of bombings prior to when score was used

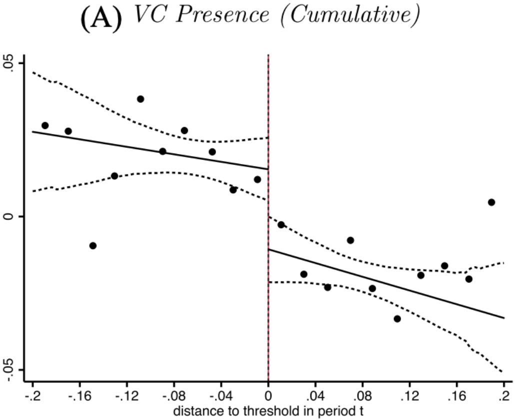
(c) All Prior Quarters Bombing



- Bombing appears to be bad for villages → fewer schools



- Bombing also appears to be bad for military objectives $\rightarrow$ more long-run Vietcong (VC) activity



So to Conclude...

- Bombing is bad ✓
- Econometrics is good ✓