CS771:Machine learning: tools, techniques, applications
Assignment #4: SVM, Convex sets

Due on: 18-03-2015, 23.00 11-03-2015
MM: 170

1. Consider a slight variant of the soft-margin SVM discussed in class as follows:

$$\min \frac{\|\mathbf{w}\|^2}{2} + \frac{C}{2}\sum_{i=1}^{n}\xi_i^2$$
$$\text{subj. to: } y_i[\mathbf{w}^T\mathbf{x}_i + w_0] \geq 1 - \xi_i \quad i = 1..n$$

   Note that the constraints $\xi_i \geq 0$ have been dropped. This is called the ℓ_2 problem (the one discussed in class was the ℓ_1 problem). a) Argue that the optimal value does not change whether we retain or drop the constraints b) set up the Lagrangian for this problem c) Minimize it and use the KKT conditions to set up the dual problem d) say how you will find $\mathbf{w}$ and w_0.

   [40]

2. Consider the $\nu-$SVM formulation where the margin ρ is introduced explicitly into the objective function. Let $\mathbf{w}^T\mathbf{x} + w_0 = \pm\rho$ be the two margin planes then the optimization problem can be written as:

$$\min f(\mathbf{w}, w_0, \xi, \rho) = \frac{\|\mathbf{w}\|^2}{2} - \nu\rho + \frac{1}{n}\sum_{i=1}^{n}\xi_i$$

   where $0 \leq \nu \leq 1$. The constraints are:

$$y_i[\mathbf{w}^T\mathbf{x}_i + w_0] \geq \rho - \xi_i \quad i = 1..n$$
$$\xi_i \geq 0 \quad i = 1..n$$
$$\rho \geq 0$$

   a) Set up the Lagrangian and minimize it b) use the KKT conditions to set up the dual problem c) say how $\mathbf{w}$ and w_0 will be calculated. [30]

3. For the Spambase data set (`http://archive.ics.uci.edu/ml/datasets/Spambase`) in the UCI repository use a soft margin SVM to classify it and report five fold cross validated accuracies. For a list of SVM libraries in different languages see `http://www.support-vector-machines.org/SVM_soft.html`.

   [50]

4. This question is about convex sets. It asks you to work out justifications for some of the observations made in class:

   (a) Let $S \subseteq \mathbb{R}^n$ be a convex set. Argue that a convex combination of vectors from S will be in S. That is show that $\theta_1\mathbf{x}_1 + \ldots + \theta_k\mathbf{x}_k \in S$, where $\mathbf{x}_i \in S,\ i = 1..k$ and $\sum_{i=1}^{k}\theta_i = 1$. Note that the definition of convexity is the case $k = 2$.

   (b) Show that the set $\{\mathbf{x} = (x_1, x_2) \in \mathbb{R}^{+2} \mid x_1x_2 \geq 1\}$ is convex. Generalize this to the set $\{\mathbf{x} = (x_1, \ldots, x_n) \in \mathbb{R}^{+n} \mid \prod_{i=1}^{n} x_i \geq 1\}$.

(c) Show that S is convex iff its intersection with any line is convex. Also, argue that a set is affine iff its intersection with any line is affine.

(d) Show that the convex hull, $cnvsh[S]$, is the intersection of all convex sets that contain S. Briefly, argue that the same holds on for affine hulls and conic hulls.

(e) Is the set $S = \{\mathbf{x} \,|\, \|\mathbf{x} - a\|_2 \leq \theta\|\mathbf{x} - b\|_2\}$, where $a,\ b$ are fixed points and $0 \leq \theta \leq 1$, convex? Justify.

[10×5=50]