

Assignment 2

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Download all python codes from

<https://github.com/ayush-2321/AI1103/tree/main/assignment2>

and latex-tikz codes from

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PROBLEM 54 (GATE MA 2007)

Consider two identical boxes B_1 and B_2 where the box $B(i = 1, 2)$ contains $i + 2$ red balls and $5 - i - 1$ white balls. A fair die is cast. Let the number shown on the top face of die be N . If N is even or 5, then two balls are drawn with replacement from the box B_1 otherwise two balls are drawn with replacement from box B_2 . The probability that two balls drawn are of different colors is

- (a) $\frac{7}{25}$
- (b) $\frac{9}{25}$
- (c) $\frac{12}{25}$
- (d) $\frac{16}{25}$

1 SOLUTION

Let $X \in \{1, 2\}$ be a discrete random variable which denotes whether the ball has been drawn from box B_1 or B_2 .

Let $Y \in \{0, 1\}$ be a discrete random variable which denotes whether the drawn ball from a box B_1 is of same color or not respectively.

Let $Z \in \{0, 1\}$ be a discrete random variable which denotes whether the drawn ball from a box B_2 is of same color or not respectively.

Total number of red balls in box $B_{i=1} = 3$

Total number of white balls in box $B_{i=1} = 3$

Total number of red balls in box $B_{i=2} = 4$

Total number of white balls in box $B_{i=2} = 2$

$$\Pr(X = 1) = \frac{2}{3} \quad (1.0.1)$$

$$\Pr(X = 2) = \frac{1}{3} \quad (1.0.2)$$

$$\Pr(Y = 1) = \frac{1}{2} \quad (1.0.3)$$

$$\Pr(Z = 1) = \frac{4}{9} \quad (1.0.4)$$

For draw of different colors following cases are possible.

1) $X=1, Y=1$

$$\Pr(X = 1, Y = 1) = \Pr(X = 1) \times \Pr(Y = 1)$$

Since, both events are independent

$$\Pr(X = 1, Y = 1) = \frac{1}{3}, \text{ using (1.0.1) and (1.0.3)}$$

2) $X=2, Z=1$

$$\Pr(X = 2, Z = 1) = \Pr(X = 2) \times \Pr(Z = 1)$$

Since, both events are independent

$$\Pr(X = 2, Z = 1) = \frac{4}{27}, \text{ using (1.0.2) and (1.0.4)}$$

$$\text{So, required probability} = \frac{1}{3} + \frac{4}{27} = \frac{13}{27}$$