Assignment 1

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Download all python codes from

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https://github.com/ayush−2321/AI1103/blob/main/
    assignment1/problem(a).py
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and latex-tikz codes from

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https://github.com/ayush−2321/AI1103/blob/main/
    assignment1/asignment1.txt
```

Problem(a)(prob. misc. 5.8)

On a multiple choice examination with 3 possible answers of each of the five questions, what is the probability that the candidate would get four or more correct answers just by guessing

1 Solution

Let $X_i \in (0, 1)$ be a random variable where $X_i = 1$ represents a successful guess and $X_i = 0$ represents unsuccessful guess on the i^{th} question.

$$p = \frac{1}{3}$$

$$X = \sum_{i=1}^{n} X_i \tag{1.0.1}$$

where n is the total number of questions. So, X has a binomial distribution.

$$\Pr(X \geq r) = \sum_{k=r}^{n} \binom{n}{r} p^k (1-p)^{n-k} \tag{1.0.2}$$

Now, in this case n=5 and r=4

$$\Pr(X = 4) = \frac{10}{243}$$
$$\Pr(X = 5) = \frac{1}{243}$$

Hence, required probability= $\frac{11}{243}$

problem(b)

Five cards are drawn successively from a well shuffled deck of 52 cards. What is the probability that

i. All the five cards are spades.
ii. only 3 cards are spade.
iii. no cards are spade.

2 solution

Let $X_i \in (0, 1)$ be a random variable which denotes whether spade is drawn at the i^{th} draw.

$$X = \sum_{i=1}^{i=5} X_i \tag{2.0.1}$$

where X denotes the number of spades obtained.

Since, $\Pr(x) = \frac{\text{number of favourable outcome}}{\text{total number of outcomes}}$

$$\Pr(x) = \frac{\binom{13}{x}\binom{39}{(5-x)}}{\binom{52}{5}}$$

TABLE

X	0	1	2	3	4	5
P(X)	$\frac{{}_{13}C_0 \times {}_{39}C_5}{{}_{52}C_5}$	$\frac{{}_{13}C_1 \times {}_{39}C_4}{{}_{52}C_5}$	$\frac{{}_{13}C_2 \times {}_{39}C_3}{{}_{52}C_5}$	$\frac{{}_{13}C_3 \times {}_{39}C_2}{{}_{52}C_5}$	$\frac{{}_{13}C_4 \times {}_{39}C_1}{{}_{52}C_5}$	$\frac{{}_{13}C_5 \times {}_{39}C_0}{{}_{52}C_5}$

i. $X = 5$

$$\Pr(X = 5) = \frac{{}_{13}C_5}{{}_{52}C_5}$$
$$\implies \Pr(X = 5) = 0.0049$$

ii. $X = 3$

$$\begin{aligned} \Pr(X = 3) &= \frac{{}_{13}C_3 \times {}_{39}C_2}{{}_{52}C_5} \\ \implies \Pr X = 3 &= 0.081 \end{aligned}$$

iii. $X = 0$

$$\begin{aligned} \Pr(X = 3) &= \frac{{}_{39}C_5}{{}_{52}C_5} \\ \implies \Pr X = 3 &= 0.081 \end{aligned}$$