Assignment 1

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Download all python codes from

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https://github.com/ayush−2321/AI1103/blob/main/
    assignment1/problem(a).py
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and latex-tikz codes from

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https://github.com/ayush−2321/AI1103/blob/main/
    assignment1/asignment1.txt
```

Problem(a)(prob. misc. 5.8)

On a multiple choice examination with 3 possible answers of each of the five questions, what is the probability that the candidate would get four or more correct answers just by guessing

Solution

Let $X_i \in (0,1)$ be a random variable where $X_i = 1$ represents a successful guess and $X_i = 0$ represents unsuccessful guess on the i^{th} question.

$$p = \frac{1}{3}$$

$$X = \sum_{i=1}^{n} X_i \tag{0.0.1}$$

where n is the total number of questions. So, X has a binomial distribution.

$$\Pr(X \geq r) = \sum_{k=r}^{n} \binom{n}{r} p^k (1-p)^{n-k}$$

Now, in this case n=5 and r=4

$$\Pr(X = 4) = \frac{10}{243}$$

$$\Pr(X = 5) = \frac{1}{243}$$

Hence, required probability= $\frac{11}{243}$

problem(b)

Five cards are drawn successively from a well shuffled deck of 52 cards. What is the probability that

i. All the five cards are spades.
ii. only 3 cards are spade.
iii. no cards are spade.

solution

Let $X_i \in (0,1)$ be a random variable which denotes whether spade is drawn at the i^{th} draw.

$$X = \sum_{i=1}^{i=5} X_i$$

where X denotes the number of spades obtained. Since, $\Pr(x) = \frac{\text{number of favourable outcome}}{\text{total number of outcomes}}$

$$\Pr(x) = \frac{\binom{13}{x}\binom{39}{(5-x)}}{\binom{52}{5}}$$

X	0	1	2	3	4	5
P(X)	$\frac{\binom{13}{0}\binom{39}{5}}{\binom{52}{5}}$	$\frac{\binom{13}{1}\binom{39}{4}}{\binom{52}{5}}$	$\frac{\binom{13}{2}\binom{39}{3}}{\binom{52}{5}}$	$\frac{\binom{13}{3}\binom{39}{4}}{\binom{52}{5}}$	$\frac{\binom{13}{4}\binom{39}{1}}{\binom{52}{5}}$	$\frac{\binom{13}{5}\binom{39}{0}}{\binom{52}{5}}$

i. $X = 5$

$$\Pr(X = 5) = \frac{\binom{13}{5}}{\binom{52}{5}}$$

$$\implies \Pr(X = 5) = 0.0049$$

ii. $X = 3$

$$P = \Pr(X = 3) = \frac{\binom{13}{3}\binom{39}{2}}{\binom{52}{5}}$$

$$\implies \Pr X = 3 = 0.081$$

iii. $X = 0$

$$\Pr(X = 0) = \frac{\binom{39}{5}}{\binom{52}{5}}$$

$$\implies \Pr(X = 0) = 0.221$$