

Assignment 1

Ayush Kumar Singh - AI20BTECH11028

Download all python codes from

[https://github.com/ayush-2321/AI1103/blob/main/assignment1/problem\(a\).py](https://github.com/ayush-2321/AI1103/blob/main/assignment1/problem(a).py)

and latex-tikz codes from

<https://github.com/ayush-2321/AI1103/blob/main/assignment1/assignment1.txt>

PROBLEM(A)(PROB. MISC. 5.8)

On a multiple choice examination with 3 possible answers of each of the five questions, what is the probability that the candidate would get four or more correct answers just by guessing

1 SOLUTION

Let $X_i \in (0, 1)$ be a random variable where $X_i = 1$ represents a successful guess and $X_i = 0$ represents unsuccessful guess on the i^{th} question.

$$p = \frac{1}{3}$$

$$X = \sum_{i=1}^n X_i \quad (1.0.1)$$

where n is the total number of questions. So, X has a binomial distribution.

$$\Pr(X \geq r) = \sum_{k=r}^n \binom{n}{k} p^k (1-p)^{n-k} \quad (1.0.2)$$

Now, in this case n=5 and r=4. From (1.0.2)

$$\Pr(X = 4) = \frac{10}{243}$$

$$\Pr(X = 5) = \frac{1}{243}$$

Hence, required probability = $\frac{11}{243}$

PROBLEM(B)

Five cards are drawn successively from a well shuffled deck of 52 cards. What is the probability that

- All the five cards are spades.
- only 3 cards are spade.
- no cards are spade.

2 SOLUTION

Let $X_i \in (0, 1)$ be a random variable which denotes whether spade is drawn at the i^{th} draw.

$$X = \sum_{i=1}^5 X_i \quad (2.0.1)$$

where X denotes the number of spades obtained.

Since, $\Pr(x) = \frac{\text{number of favourable outcome}}{\text{total number of outcomes}}$

$$\Pr(x) = \frac{{}^{13}C_x \times {}^{39}C_{5-x}}{{}^{52}C_5} \quad (2.0.2)$$

X	0	1	2	3	4	5
P(X)	$\frac{{}^{13}C_0 \times {}^{39}C_5}{{}^{52}C_5}$	$\frac{{}^{13}C_1 \times {}^{39}C_4}{{}^{52}C_5}$	$\frac{{}^{13}C_2 \times {}^{39}C_3}{{}^{52}C_5}$	$\frac{{}^{13}C_3 \times {}^{39}C_2}{{}^{52}C_5}$	$\frac{{}^{13}C_4 \times {}^{39}C_1}{{}^{52}C_5}$	$\frac{{}^{13}C_5 \times {}^{39}C_0}{{}^{52}C_5}$

TABLE 3: Probabilities of each case

So, using table and (2.0.2)

i. $X = 5$

$$\Pr(X = 5) = \frac{{}^{13}C_5}{{}^{52}C_5}$$

$$\Rightarrow \Pr(X = 5) = 0.0049$$

ii. $X = 3$

$$\Pr(X = 3) = \frac{{}^{13}C_3 \times {}^{39}C_2}{{}^{52}C_5}$$

$$\Rightarrow \Pr(X = 3) = 0.081$$

iii. $X = 0$

$$\Pr(X = 3) = \frac{{}^{39}C_5}{{}^{52}C_5}$$
$$\implies \Pr X = 3 = 0.081$$