Assignment 2

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Download all python codes from

```
https://github.com/ayush−2321/AI1103/tree/main/
    assignment%203
```

and latex-tikz codes from

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https://github.com/ayush−2321/AI1103/tree/main/
    assignment%203
```

PROBLEM 54 (GATE MA 2007)

Consider two identical boxes $B1$ and B_2 where the box $B(i = 1, 2)$ contains $i + 2$ red balls and $5 - i - 1$ white balls.A fair die is cast. Let the number shown on the top face of die be N. If N is even or 5, then two balls are drawn with replacement from the box b_1 otherwise two balls are drawn with replacement from box B_2.The probability that two balls drawn are of different colors is

(a) $\frac{7}{25}$

(b) $\frac{9}{25}$

(c) $\frac{12}{25}$

(d) $\frac{16}{25}$

1 SOLUTION

Let $X \in \{1, 2\}$ be a discrete random variable which denotes whether the ball has been drawn from box B_1 or B_2.

Let $Y \in \{0, 1\}$ be a discrete random variable which denotes whether the drawn ball from a box B_1 is of same color or not respectively.

Let $Z \in \{0, 1\}$ be a discrete random variable which denotes whether the drawn ball from a box B_2 is of same color or not respectively.

$$\text{Total number of red balls in box } B_{i=1} = 3$$
$$\text{Total number of white balls in box } B_{i=1} = 3$$
$$\text{Total number of red balls in box } B_{i=2} = 4$$
$$\text{Total number of white balls in box } B_{i=2} = 2$$
$$\Pr(X = 1) = \frac{2}{3} \tag{1.0.1}$$
$$\Pr(X = 2) = \frac{1}{3} \tag{1.0.2}$$
$$\Pr(Y = 1) = \frac{1}{2} \tag{1.0.3}$$
$$\Pr(Z = 1) = \frac{4}{9} \tag{1.0.4}$$

For draw of different colors following cases are possible.

1) X=1,Y=1
$$\Pr(X = 1, Y = 1) = \Pr(X = 1) \times \Pr(Y = 1)$$
$$\text{Since, both events are independent}$$
$$\Pr(X = 1, Y = 1) = \frac{1}{3}, \text{using (1.0.1) and (1.0.3)}$$

2) X=2, Z=1
$$\Pr(X = 2, Z = 1) = \Pr(X = 2) \times \Pr(Z = 1)$$
$$\text{Since, both events are independent}$$
$$\Pr(X = 2, Z = 1) = \frac{4}{27}, \text{using (1.0.2) and (1.0.4)}$$

$$\text{So, required probability } = \frac{1}{3} + \frac{4}{27} = \frac{13}{27}$$