Quiz 2

Ayush Kumar Singh - AI20BTECH11028

Download latex codes from

https://github.com/ayush−2321/EE3900/new/main

Problem

Q3.23(b):- Consider an LTI system that is stable and for which H(z), the z-transform of the impulse response is given by:

$$H(z) = \frac{(1 - \frac{1}{2}z^{-2})}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})} \tag{0.0.1}$$

Solution

Theorem 1. ***z-transform linearity***

$$ax_1[n] + bx_2[n] \overset{\mathcal{Z}}{\leftrightarrow} X_1[n] + bX_2[n] \tag{0.0.2}$$

Now,

$$H(z) = \frac{(1 - \frac{1}{2}z^{-2})}{(1 - \frac{1}{2}z^{-1})(1 - \frac{1}{4}z^{-1})} \tag{0.0.3}$$

Using partial fraction method

$$H(z) = -4 + \frac{(5 + \frac{7}{2}z^{-1})}{1 - \frac{3}{4}z^{-1} + \frac{1}{8}z^{-2}} \tag{0.0.4}$$

$$H(z) = -4 - \frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{7}{1 - \frac{1}{4}z^{-1}} \tag{0.0.5}$$

Now,

$$h(n) = \mathcal{Z}^{-1}\left(-4 - \frac{2}{1 - \frac{1}{2}z^{-1}} + \frac{7}{1 - \frac{1}{4}z^{-1}}\right) \tag{0.0.6}$$

Using theorem 1,

$$h(n) = -4\delta[n] - 2(\frac{1}{2})^n u[n] + 7\frac{1}{4}^n \tag{0.0.7}$$

Since, we know that $y[n] = h[n] * x[n]$
Hence, the difference equation is:

$$y[n] - \frac{3}{4}y[n-1] + \frac{1}{8}y[n-2] = x[n] - \frac{1}{2}x[n-2] \tag{0.0.8}$$