Quiz 1

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Download latex codes from

https://github.com/ayush−2321/EE3900/new/main

Problem

Q2.30(a):- For the following system determine whether the system is (1) stable, (2) casual (3) linear , (4) time invariant.

$$T(x|n) = (cos(\pi n))x[n] \tag{0.0.1}$$

Solution

Definition 1. ***Stable*** *A system is said to be stable if the response to a bounded input is always bounded.*

Let,

$$T(x|n) = (\cos(\pi n))x[n] = y[n] \tag{0.0.2}$$

Since, $\cos(\pi n)$ = either 1 or -1.

$$y[t] = (-1)^n x[t] \tag{0.0.3}$$

Now,

$$|y[n]| = |(-1)^n x[n]| \tag{0.0.4}$$
$$= |x[n]| \tag{0.0.5}$$

Since, x[n] is bounded, therefore $|x[n]|$ is bounded. Hence, y[n] is **stable**.

Definition 2. ***Casual*** *The output at any instant does not depend on the future inputs i.e, for at n_0 $y[n_0]$ does not depend on $x[n]$ for $n > n_0$.*

Since, y[n] = $|(-1)^n x[n]|$, depends only on the current value of x[n].
Therefore, it is **casual**.

Definition 3. ***Linear*** *The response to an arbitary linear combination of input signals is always the same linear combinations of the individual responses to these signals*

Let,

$$x_1[n] \implies (-1)^n x_1[n] \tag{0.0.6}$$
$$x_2[n] \implies (-1)^n x_2[n] \tag{0.0.7}$$

Now,

$$y_1[n] + y_2[n] \implies (-1)^n x_1[n] + (-1)^n x_2[n] \tag{0.0.8}$$
$$\implies (-1)^n (x_1[n] + x_2[n]) \tag{0.0.9}$$

Hence, it is **linear**.

Definition 4. ***Time Invariant*** *The response to an arbitrary translated set of inputs is always the response to the original set, but translated by the same amount.*
If

$$x[n] \implies y[n] \tag{0.0.10}$$

then

$$x[n - n_0] \implies y[n - n_0] \tag{0.0.11}$$

for all x and n_0.

Let,

$$x[n] \implies (-1)^n x[n] = y[n] \tag{0.0.12}$$
$$x[n - n_0] \implies (-1)^n x[n - n_0] = y[n - n_0] \tag{0.0.13}$$

Since, the output is translated by the same amount. Hence it is **time invariant**.